A dynamic model of competition in retail banking

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Abstract. This paper presents a two-period dynamic model of branching behaviour and competition in retail banking. It is an extension of a static model, that has already been published and used to study competition in Italian local banking markets, from which it borrows most of its theoretical underpinnings. It however adds some information on banks branching strategies under the (realistic) assumption that it takes time and possibly strategic adjustments for a bank to reach its optimal branching network size.
The empirical counterpart of the model may be estimated and it becomes possible to compute measures of competition and, more interestingly, measures of benefits and costs by bank and by markets and their evolution in time . It results that between 2004 and 2006 competition among banks tended to decrease in intensity, while banks increased their efficiency through adjustments in their branching size and location. There is evidence of crosstime subsidisations: banks are willing to operate at suboptimal branching sizes, with marginal benefits lower than marginal costs at some point in time, in order to reach "optimality" in the future.

Keywords: Banking industry, Strategic Branching Behaviour, Competition, Dynamic Probit.

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Introduction

A static model of bank branching behaviour and competition in retail banking is presented in Cerasi et al. 2000, 2002, 2009 and Chizzolini 2007, together with its empirical counterpart and some estimation results.

Theory states that a profit maximising bank will enter a market if its profits are at least equal to entry costs and it will expand its branching network up to the point where marginal benefits equate marginal costs, given its “expected” profit function. The profit, entry and branching cost functions are specified, in the theoretical model, as simple functions of observable and unobservable variables, while the time dimension is not really dealt with: it is assumed that at each period in time each bank in a market takes its decision and (immediately) adjusts its branching network to its optimal size.

When turning to the data and the empirical testing of the model, however, time becomes very much an issue and the static model is transformed into a dynamic one, or at least partially so.

In the way the empirical test is designed, there exists a time 0 when a bank takes its entry and branching decisions, based on the existing branching network size and on “expected” profits and branching costs at time 1, and the econometrician observes ex post (at time 1) if the bank is in the market and if it has expanded or shrunk its branching size. From theory, if the number of branches has increased or stayed the same from t=0 to t=1 then it must be that the bank faced additional branching costs equal or lower than additional branching benefits, viceversa costs must have been larger than benefits if the bank closed down branches or decided to keep just one branch open in that market. Given the assumptions made on the functional form of both the profit and the cost functions and assumptions on the stochastic properties of the unobservable (latent) variables in those functions, it is possible to estimate all the relevant variables: profits, marginal benefits and marginal costs, as well as some measures of the degree of competition among banks in local markets.

There are two issues to be discussed within this empirical approach.

1) Is the distance between t=0 and t=1, in terms of actual years relevant? How much time does opening or closing a branch take?
2) Is it really plausible that decisions on structural factors such as the number of branches are taken by looking at just time 1 profits?

The two questions are related but the second one seems the more relevant and the dynamic model presented below is a first solution to the problem. It assumes that banks take their entry and branching decisions on the present value of the flow of future periods profits. At this stage the analysis is limited to the case of a two periods present value profits. The extension to an infinite planning horizon has not been tackled yet.

Section 1 deals with the dynamic extension of the model of bank branching competition, both in theory and in its empirical application, using data on Italian bank groups by province over the 1999-2004-2006 years. In Section 2, the dynamic model is modified to take into account mergers. Preliminary estimates of the relevant parameters as well as of the "cost" of a merger are presented. Conclusions close the paper.

1. The dynamic extension

1.1 A model of bank branching behaviour over time

Table 1.1 summarizes the functional forms, relevant variables of the representative bank's profit and cost functions and the profit maximizing conditions in the static model of bank behaviour (see Cerasi et al . 2002, 2009 for a more detailed description of the model and its elements). The same functions will enter the dynamic extension of the model, together with the same assumptions: banks behave as non cooperative monopolistic competitors in retail markets and compete in both prices (interest rates) and size and location of their branching network. Competition on interest rates yields, for each bank, its "expected" profit as a function of market size and number of competitors as well as of its branching network size. The latter is then determined according to the profit maximising conditions in Table 1.1.

Table 1.1 – The functions in the static model

Profit function:	$\pi_{ij} \equiv \pi(k_{ij}; S_j, cci_j, N_j) = S_j \dfrac{k_{ij}^{c_j}}{\sqrt{N_j}}$	[1]
Cost function, linear:	$s_{ij} = a_{ij} + \varepsilon_{ij}\left(k_{ij} - 1\right)$	[2]
Marginal branching benefits:	$MB_{ij} = \dfrac{d\pi(k_{ij})}{dk_{ij}} = \dfrac{S_j k_{ij}^{c_j - 1}}{\sqrt{N_j}}\left(c_j - \dfrac{k_{ij}}{2N_j}\right)$	[3]
Marginal branching costs:	$MC_{ij} = \dfrac{ds_{ij}}{dk_{ij}} = \varepsilon_{ij}$	[4]
Entry condition:	$\pi_{ij} \geq s_{ij}$	[5]
Profit maximizing condition:	$a) k_{ij} = 1 \rightarrow MB_{ij} < MC_{ij}$ $b) k_{ij} > 1 \rightarrow MB_{ij} = MC_{ij}$	[6]
Where:		

k_{ij} =Number of branches of bank *i* operating in market *j*
S_j = Market size (Deposits in the empirical version)
$N_j = k_{ij} + \sum_{i \neq o} k_{oj}$ = Total number of bank branches operating in market *j*
c_j = competition parameter (it must be strictly less than 1.5 for the profit function to be concave).
An increase in c_j implies a decrease in toughness in competition among banks in market *j*.
s_{ij} = Entry and branching costs
a_{ij} = Fixed entry costs for a unit branch bank, non observable variable
ε_{ij} = Constant marginal branching cost for bank *i* in market *j*, non observable variable

Now introduce time and assume that banks decide at time 0 whether to enter a market and their branching size at time 1 and time 2 by comparing their expected profits over the 2 time periods to sunk entry and branching costs.

$$E(\pi_i) = \frac{S_1 k_{i1}^{c_1}}{\sqrt{N_1}} + \delta\left(\frac{S_2 k_{i2}^{c_2}}{\sqrt{N_2}}\right) = \frac{S_1 k_{i1}^{c_1}}{\sqrt{N_1}} + \delta\left(\frac{S_2 (k_{i1} + \Delta k_{i2})^{c_2}}{\sqrt{N_2}}\right) \text{ where } \delta \text{ is a discounting factor} \qquad [1.1]$$

$$C_i = a + \varepsilon_{i1}(k_{i1} - 1) + \varepsilon_{i2}(k_{i2} - k_{i1}) = a + \varepsilon_{i1}(k_{i1} - 1) + \varepsilon_{i2}(\Delta k_{i2}) \qquad [1.2]$$

where *a* is the sunk fixed entry cost with one branch and $\varepsilon_{\iota\tau}$ is the bank's marginal branching costs at time t = 1, 2, independent between the two periods. In the definition of second period costs, $\varepsilon_{\iota 2}$ might also be different between the cases of Δk positive or negative. Let's assume at first ε_{i2} to be equal in the two cases. (ε_2 is the already discounted second period marginal branching costs)
Disregarding entry decisions and focusing only on branching, for each incumbent bank there are two decision variables: branching network size in period 1 and in period 2. Using the definition where second period branching network consists of first period network plus the second period decision variable, Δk, the first order profit maximising conditions become:

$$\begin{cases} MB_{i1} = \dfrac{\partial \pi_i}{\partial k_{i1}} = \dfrac{S_1 k_{i1}^{c_1 - 1}}{\sqrt{N_1}}\left(c_1 - \dfrac{k_{i1}}{2N_1}\right) + \delta\left(\dfrac{S_2 (k_{i1} + \Delta k_{i2})^{c_2 - 1}}{\sqrt{N_2}}\left(c_2 - \dfrac{k_{i1} + \Delta k_{i2}}{2N_2}\right)\right) = \varepsilon_1 = MC_{i1} \\ MB_{i2} = \dfrac{\partial \pi_i}{\partial \Delta k_{i2}} = \delta\left(\dfrac{S_2 (k_{i1} + \Delta k_{i2})^{c_2 - 1}}{\sqrt{N_2}}\left(c_2 - \dfrac{k_{i1} + \Delta k_{i2}}{2N_2}\right)\right) = \varepsilon_2 = MC_{i2} \end{cases} \qquad [1.3]$$

With perfect foresight, present value marginal benefits of the first period branching network, k_{i1}, include discounted second period marginal benefits and the equilibrium conditions state that end of time 1 observed marginal benefits might be negative if the additional net marginal benefits incurred in the second period are expected to be larger than first period ones.[2]

Assume now that, conditionally on a set of exogenous bank and market specific variables, ε_1 and ε_2 are two identically distributed <u>independent</u> random variables with known distribution F_ε. Given the initial values of its branching network, $k_{i0} \geq 0$, bank *i*'s decision path and present value two-periods profit may be summarized in the following two way table:

		2			
		Entry ($\Delta k_{i2} \geq 0$, $k_{i2} > 1$)		exit or unit bank ($\Delta k_{i2} < 0$, $k_{i2} > 1$; $k_{i2} = 1$)	
1	Entry ($\Delta k_{i1} \geq 0$, $k_{i1} > 1$)	$\pi_{e,e}$	$p_1 * p_2$	$\pi_{e,ne}$	$p_1 * (1-p_2)$
	exit or unit bank ($\Delta k_{i1} < 0$, $k_{i1} > 1$; $k_{i1} = 1$)	$\pi_{ne,e}$	$(1-p_1) * p_2$	$\pi_{ne,ne}$	$(1-p_1) * (1-p_2)$

Where p_1 is $F_\varepsilon(MB_{i1})$ and p_2 is $F_\varepsilon(MB_{i2})$.

The econometrician observes at time 2 the entry and branching decisions taken by the bank in each period and sets up the following problem:

$$\begin{aligned} p_1 &= \Pr(k_1 > 1, \Delta k_1 \geq 0) = \Pr(\varepsilon_1 \leq MB_1) \\ 1 - p_1 &= \Pr(k_1 > 1, \Delta k_1 < 0) + \Pr(k_1 = 1) = \Pr(\varepsilon_1 > MB_1) \end{aligned} \qquad [1.4.1]$$

and

$$\begin{aligned} p_2 &= \Pr((k_1 + \Delta k_2) > 1, \Delta k_2 \geq 0) = \Pr(\varepsilon_2 \leq MB_2) \\ 1 - p_2 &= \Pr((k_1 + \Delta k_2) > 1, \Delta k_2 < 0) + \Pr((k_1 + \Delta k_2) = 1) = \Pr(\varepsilon_2 > MB_2) \end{aligned} \qquad [1.4.2]$$

Under lognormality of ε_1 and ε_2 , and the assumption that for t = 1, 2, $\varepsilon_t = MC_t \upsilon_t \therefore \ln(\varepsilon_t) = \ln(MC_t) + \ln(\upsilon_t) = mc_t + v_t$, and $v_i = \ln(\varepsilon_t) - mc_t \approx N(0,1)$, the loglikelihood function for this problem is:

$$\begin{aligned} \ln L = & \sum_{ij \in \pi_{ee}} \ln\left[\Phi(\ln(MB_{ij1}) - mc_{ijt}) * \Phi(\ln(MB_{ij2}) - mc_{ij2})\right] \\ & + \sum_{ij \in \pi_{e,ne}} \ln\left[\Phi(\ln(MB_{ij1}) - mc_{ij1}) * (1 - \Phi(\ln(MB_{ij2}) - mc_{ij2}))\right] \\ & + \sum_{ij \in \pi_{ne,e}} \ln\left[(1 - \Phi(\ln(MB_{ij1}) - mc_{ij1})) * \Phi(\ln(MB_{ij2}) - mc_{ij2})\right] \\ & + \sum_{ij \in \pi_{ne,ne}} \ln\left[(1 - \Phi(\ln(MB_{ij1}) - mc_{ij1})) * (1 - \Phi(\ln(MB_{ij2}) - mc_{ij2}))\right] \end{aligned} \qquad [1.5]$$

[2] If the planning horizon were longer, each period's decision about changes in the size of the branching network would be made upon the flow of discounted marginal benefits from that period onwards. Some terminal condition should be included.

The likelihood function in 1.5 needs to be maximized with respect to the parameters that enter both *MB* functions and both *mc* functions: c_1 and c_2, in MB_1 and MB_2 respectively, the measures of the degree of competition by market by year, and mc_t , the logarithm of the average marginal branching costs by bank, market and year. MB_t and MC_t themselves can be computed given the estimated parameters as well as *MB/MC* ratios and pseudo-lerner indexes, (*(MB_MC)/MB*), by bank, province and year.

1.2 Estimation results: dynamic two-periods model

The available sample consists of data on branching networks by bank group, *i*, by province, *j*, in years *t*=1999, 2004 and 2006 (the number of branches by bank by province by year are labeled k_{ijt})[3]. Table 1.2 lists the bank groups in the sample and their frequency by year. The frequency actually gives information on the number of provinces in which the groups opened branches. Only the first 6 groups in the table are national banks, that operate in (almost) all the 103 Italian provinces. The remaining groups are large banks that operate in subsets of provinces, usually located in well defined areas within Italy.

Table 1.2 – Frequency of banks by year

Name	Code	Frequency
BNL	1005	103
SANPAOLO	1025	102
MPS	1030	102
INTESA	3069	103
UNICREDITO	3135	100
CAPITALIA	3207	101
POP.UNITE	5026	62
ANTONVENETA	5040	88
BIPIELLE	5164	64
POP.NO-VR	5188	67
POP. E.R.	5387	59
BPM	5584	39
OTHERS	9999	103

Together with data on branching networks, the data set contains data on total deposits by province for years 2004 and 2006 (S_{jt}), total loans and population by province in years 2004 and 2006, and total number of branches by province by year, N_{jt}, actually computed as the sum of k_{ijt} over *i* for each *j,t*. For t_1=2004, Δk has been computed on 1999, for t_2=2006, Δk has been computed on 2004

[3] The source for these data is Banca d'Italia and ISTAT. See Chizzolini 2007 for a more detailed description of the data

Table 1.3 shows the estimation results for the two-period dynamic model. It must be made clear that in the specification of the empirical model c_1 and c_2 are assumed to be market specific parameters that depend on some measure of market economic activity (*lpc*, loans per capita) and on other market characteristics (dbigpro, dummy for most densely populated provinces). Marginal branching costs, instead are assumed to be bank specific thus they only depend on bank dummies, on constants by year and on having entered at least one of the markets (provinces) only in 2006 (dnew2).

Table 1.3 - Estimates by maximum likelihood (BHHH method) – 1093 observations (Convergence achieved in 64 iterations)

	Coefficient	Std. Error	z-Statistic	Prob.
BNL	-0.644	0.166	-3.884	0.000
SAN PAOLO	-0.988	0.183	-5.385	0.000
MPS	-0.689	0.171	-4.022	0.000
INTESA	0.389	0.137	2.845	0.004
UNICREDITO	0.024	0.160	0.152	0.879
CAPITALIA	-0.081	0.145	-0.557	0.578
BANCHE POP UNITE	-0.371	0.167	-2.225	0.026
ANTONVENETA	-0.016	0.155	-0.103	0.918
BIPIELLE	-0.526	0.182	-2.899	0.004
POP NO + VR	-0.318	0.174	-1.826	0.068
POP EMILIA ROMAGNA	-0.251	0.179	-1.403	0.161
BPM	0.536	0.174	3.077	0.002
_cons2004	6.243	0.210	29.698	0.000
_cons2006	5.362	0.180	29.870	0.000
dnew2	1.178	0.299	3.947	0.000
_cons2004	1.204	0.141	8.520	0.000
dbigpro2004	-0.264	0.115	-2.290	0.022
lpc2004	-0.002	0.003	-0.614	0.539
_cons2006	1.319	0.040	32.593	0.000
dbigpro2006	-0.317	0.041	-7.696	0.000
lpc2006	-0.004	0.001	-4.768	0.000
Log likelihood	-1022.920		Akaike info criterion	1.910
Av. loglikelihood	-0.936		Schwarz criterion	2.006

The most relevant features in table 1.3 are the estimated constants for 2004 and 2006 both for the the marginal costs and for the competition parameters components of the model. Marginal branching costs decrease on average from 2004 to 2006 (the estimated parameters that refer to the average of the logarithm of *MC* fall, significantly, from 6.24 in 2004 to 5.36 in 2006), while the competition parameter increases in time, on average (from 1.2 to 1.3). These estimates suggest that banks have, on average increased their benefit-cost margins from 2004 to 2006, both because a reduction in competition and an increase in branching "efficiency". This result is confirmed in table 1.4: the average over the whole sample for c_1 is 1.16 while for c_2 it is 1.23 , for MC_1 it is 433.9 while for MC_2 it is 187.9 (1=2004 and 2=2006).

Table 1.4 - Overall descriptive statistics

	c_1	c_2	MC_1	MC_2	MB_1	MB_2	LERNER1	LERNER2	PV_PROFIT	PV_COST
Mean	1.16	1.23	433.90	187.91	453.18	574.21	-0.23	0.55	42622.82	13635.22
Median	1.18	1.25	399.94	196.72	391.10	483.60	-0.02	0.66	6897.90	3406.80

Statistics by bank of the same variables (competition parameters, and more interesting, *MC* and *MB*) are in table 1.5. A couple of interesting cases pop up: Banca INTESA with very high marginal cost in the first time period, due to the recent merger between INTESA (CARIPLO already merged with AMBROVENETA) and Banca Commerciale Italiana that ended up with the new INTESA having to close many branches both because of internal strategical reasons and because compelled by Antitrust rulings, that was able to recoup at least partially in the second time period. At the other end of the spectrum SANPAOLO and MPS that are very profitable in both time periods: the interpretation of this result based on the underlying model is that they are both expanding their branching networks in "profitable" provinces, where branching benefits still largely exceed branching costs. It may be worthwhile to remember that, because of ante 1990 banking regulation, many local markets (provinces) are/were still "underbranched" given their size. (Guiso et al. 2006)

Table 1.5 - Statistics by bank

	MC1	MC2	C1	C2	MB/MC1	MB/MC2	MB1	MB2	LER.1	LER.2	PV_PROF	PV_COST
BNL	269.96	111.96	1.17	1.24	1.45	4.07	392.46	456.18	0.11	0.67	8433.72	1937.33
SAN PAOLO	191.48	81.16	1.17	1.24	2.48	7.91	474.04	628.79	0.47	0.82	46087.87	6378.82
MPS	258.24	116.54	1.17	1.24	1.66	5.01	429.89	542.85	0.20	0.69	22338.73	4739.48
INTESA	758.45	321.42	1.17	1.24	0.61	1.89	464.30	594.79	-1.17	0.19	35348.24	21300.43
UNICREDITO	526.82	218.48	1.17	1.23	0.90	2.76	475.42	602.67	-0.41	0.53	42952.04	16259.19
CAPITALIA	474.33	196.72	1.17	1.23	0.92	2.79	435.01	548.86	-0.39	0.52	27037.64	9084.37
BANCHE POP UNITE	354.83	157.83	1.16	1.22	1.31	3.88	464.67	576.88	0.01	0.62	29363.07	7085.25
ANTONVENETA	506.04	231.32	1.16	1.23	0.84	2.39	425.45	510.09	-0.56	0.37	15390.34	5738.27
BIPIELLE	303.76	134.83	1.16	1.22	1.40	3.95	425.91	507.42	0.09	0.66	11429.97	2559.46
POP NO + VR	373.95	155.09	1.16	1.22	1.27	3.67	476.25	569.49	0.02	0.65	25727.54	6657.87
POP EMILIA ROMAGNA	399.94	229.09	1.16	1.23	1.02	2.98	407.41	532.05	-0.35	0.35	26143.07	7714.43
BPM	879.13	364.60	1.14	1.19	0.56	1.54	494.06	562.19	-1.17	0.17	22825.44	15336.27
OTHERS	514.16	213.24	1.17	1.24	1.03	3.62	530.87	772.33	-0.21	0.64	194261.40	60438.48

The evolution in time of the competition and cost parameters of some of the more representative provinces is shown in table 1.6. A few comments: the statistics in **bold** refer to some of the larger towns/provinces in Italy. In ***bold italic***, Milano and Rome: look at the situation in Milano the only case where the lerner index decreases in the second period, definitely because of an increase in competition (*c* decreases). In *italic* some of the poorest provinces. For the other provinces in this summary table: look at how much more profitable they are in the second period (2006) against very low lerner indexes in 2004. This may mean that second period benefits subsidize first period losses. (Most lerner indexes are negative in the first period. Not shown in this table.)

Table 1.6 - Statistics by province

	MC1	C1	MB/MC1	MB1	LERNER1	MC2	C2	MB/MC	MB2	LERNER2	PV_PROF	PV_COST
Torino	**450.93**	**0.91**	**1.32**	**500.66**	**0.08**	**187.01**	**0.92**	**3.80**	**593.64**	**0.68**	**105542.20**	**42285.85**
Milano	***447.01***	***0.81***	***1.99***	***730.94***	***0.32***	***185.39***	***0.66***	***2.14***	***321.93***	***0.28***	***217829.60***	***102900.10***
Genova	**447.01**	**1.18**	**2.48**	**927.48**	**0.50**	**185.39**	**1.24**	**8.22**	**1269.71**	**0.84**	**97542.96**	**17837.08**
Venezia	454.69	1.17	1.91	716.98	0.31	188.57	1.24	6.02	930.03	0.76	76177.45	17625.47
Bologna	**447.01**	**1.15**	**2.63**	**985.34**	**0.54**	**185.39**	**1.18**	**8.04**	**1247.73**	**0.85**	**146598.00**	**26916.20**
Ferrara	447.01	1.18	1.01	389.66	-0.16	185.39	1.26	3.09	498.35	0.61	19408.67	7988.41
Firenze	**447.01**	**1.14**	**2.28**	**851.16**	**0.46**	**185.39**	**1.14**	**5.51**	**855.05**	**0.78**	**107223.50**	**22512.33**
Grosseto	460.24	1.18	0.65	260.78	-0.85	190.88	1.26	2.02	331.37	0.37	13329.08	6812.89
Roma	***447.01***	***0.87***	***2.13***	***793.88***	***0.42***	***185.39***	***0.82***	***4.24***	***655.70***	***0.70***	***232227.40***	***71255.91***
Isernia	*405.20*	*1.20*	*0.29*	*99.33*	*-3.53*	*238.79*	*1.30*	*0.80*	*115.64*	*-1.39*	*790.15*	*1035.23*
Napoli	447.01	0.93	1.38	532.29	0.16	185.39	0.97	4.11	652.14	0.72	76454.23	24608.07
Potenza	414.36	1.19	1.17	423.45	0.00	171.85	1.30	3.90	589.87	0.68	21306.12	7845.09
Enna	*409.55*	*1.20*	*0.40*	*138.43*	*-2.02*	*169.85*	*1.30*	*1.28*	*183.07*	*0.01*	*2910.80*	*3367.87*
Messina	422.58	1.19	1.26	455.25	0.07	175.26	1.29	4.40	661.82	0.73	27876.99	11246.31
Palermo	**415.81**	**1.19**	**2.44**	**878.44**	**0.52**	**172.45**	**1.28**	**9.01**	**1361.64**	**0.86**	**105769.00**	**18557.86**
Oristano	*424.17*	*1.20*	*0.41*	*151.65*	*-1.78*	*175.92*	*1.30*	*1.21*	*187.50*	*0.03*	*5080.65*	*4129.90*

2. Dynamic model and Mergers

2.1 The modified dynamic model: a sketch and very preliminary estimates.

Keeping all basic intertemporal profit and cost functions as in the previous case, assume further that a firm decides to merge or acquire another bank if the expected joint profit is larger than if the merger does not go through:

- Bank i - acquirer

$$E(\pi_i)=\frac{S_1 k_{i1}^{c_1}}{\sqrt{N_1}}+\delta\left(\frac{S_2(k_{i2}+k_{j2})^{c_2}}{\sqrt{N_2}}\right)=\frac{S_1 k_{i1}^{c_1}}{\sqrt{N_1}}+\delta\left(\frac{S_2(k_{i1}+k_{j1}+\Delta k_{i+j2})^{c_2}}{\sqrt{N_2}}\right)$$
$$\geq\frac{S_1 k_{i1}^{c_1}}{\sqrt{N_1}}+\delta\left(\frac{S_2(k_{i1}+\Delta k_{i2})^{c_2}}{\sqrt{N_2}}\right) \qquad [2.1]$$

where δ is a discounting factor

$$C_i=a+\varepsilon_{i1}(k_{i1}-1)+\varepsilon_{i2}(k_{i+j2}-k_{i1}-k_{j1})+\gamma k_{j1}=a+\varepsilon_{i1}(k_{i1}-1)+\varepsilon_{i2}(\Delta k_{i+j2})+\gamma k_{j1} \qquad [2.2]$$

The cost of acquiring bank j is assumed to be a constant proportion, γ, of bank j's branching network size.

The cost for bank i to acquire bank j's branches, γk_{j1}, becomes the beginning of second period revenues for bank j, and bank j will be willing to merge if this revenue exceeds the present value of second period profits

- Bank *j* - acquired

$$E\left(\pi_j^{ad}\right)=\frac{S_1 k_{j1}^{c_1}}{\sqrt{N_1}}+\gamma k_{j1}\geq\frac{S_1 k_{j1}^{c_1}}{\sqrt{N_1}}+\delta\left(\frac{S_2\left(k_{j1}+\Delta k_{j2}\right)^{c_2}}{\sqrt{N_2}}\right) \qquad [2.3]$$

$$C_j=a+\varepsilon_{j1}\left(k_{j1}-1\right) \qquad [2.4]$$

For both type of banks, the conditions above also state that profits must be larger or equal to sunk entry costs.

In this setup, the marginal conditions for profit maximisation relative to branching become:

- Bank *i* - acquirer

$$\begin{cases} MB^{ag}{}_{i1}=\dfrac{\partial\pi_i}{\partial k_{i1}}=\dfrac{S_1 k_{i1}^{c_1-1}}{\sqrt{N_1}}\left(c_1-\dfrac{k_{i1}}{2N_1}\right)+\delta\left(\dfrac{S_2\left(k_{i1}+k_{j1}+\Delta k_{i2}\right)^{c_2-1}}{\sqrt{N_2}}\left(c_2-\dfrac{k_{i1}+k_{j1}+\Delta k_{i2}}{2N_2}\right)\right)=\varepsilon_{i1}=MC_{i1} \\ MB^{ag}{}_{i2}=\dfrac{\partial\pi_i}{\partial\Delta k_{i+j2}}=\delta\left(\dfrac{S_2\left(k_{i1}+k_{j1}+\Delta k_{i+j2}\right)^{c_2-1}}{\sqrt{N_2}}\left(c_2-\dfrac{k_{i1}+k_{j1}+\Delta k_{i+j2}}{2N_2}\right)\right)=\varepsilon_{i2}=MC_{i2} \end{cases} \qquad [2.5]$$

- Bank *j* - acquired

$$MB^{ad}{}_{j1}=\frac{\partial\pi_j}{\partial k_{j1}}=\frac{S_1 k_{j1}^{c_1-1}}{\sqrt{N_1}}\left(c_1-\frac{k_{j1}}{2N_1}\right)+\gamma=\varepsilon_{j1} \qquad [2.6]$$

If the merging decision is not modelled and the status of a bank, acquiring or acquired, is observed ex-post at the end of period 2, the branching decisions of the two types of banks may be specified and estimated separately. The added assumption of independent ε_1 across banks is also needed.

For the acquiring bank, this empirical branching model does not change much relative to the simple dynamic model. Costs are different in this case, but for a fixed amount related to the size of the acquired branching network (that, it must be stressed again, is not a decision variable for the acquiring bank):

			2		
			Entry ($\Delta k_{i+j,2}\geq 0$, $k_{i+j,2}>1$)		exit or unit bank ($\Delta k_{i+j,2}<0$, $k_{i+j,2}>1$; $k_{i+j,2}=1$)
1	Entry ($\Delta k_{i1}\geq 0$, $k_{i1}>1$)	$\pi_{e,e}$	$p_1^{ag}*p_2^{ag}=\Pr\left(\varepsilon_1\leq MB_1^{ag}\right)*\Pr\left(\varepsilon_2\leq MB_2^{ag}\right)$	$\pi_{e,ne}$	$p_1^{ag}*(1-p_2^{ag})=\Pr\left(\varepsilon_1\leq MB_1^{ag}\right)*\Pr\left(\varepsilon_2>MB_2^{ag}\right)$
	exit or unit bank ($\Delta k_{i1}<0$, $k_{i1}>1$; $k_{i1}=1$)	$\pi_{ne,e}$	$\left(1-p_1^{ag}\right)*p_2^{ag}=\Pr\left(\varepsilon_1>MB_1^{ag}\right)*\Pr\left(\varepsilon_2\leq MB_2^{ag}\right)$	$\pi_{ne,ne}$	$\left(1-p_1^{ag}\right)*\left(1-p_2^{ag}\right)=\Pr\left(\varepsilon_1>MB_1^{ag}\right)*\Pr\left(\varepsilon_2>MB_2^{ag}\right)$

For the acquired bank the branching decision must only be made for the first period, when however the present value of the branching network to be sold is taken into account (in MB_1^{ad}):

1	Entry ($\Delta k_{i1} \geq 0$, $k_{i1} > 1$)	π_e	$p_1^{ad} = \Pr\left(\varepsilon_1 \leq MB_1^{ad}\right)$
	exit or unit bank ($\Delta k_{i1} < 0$, $k_{i1} > 1$; $k_{i1} = 1$)	π_{ne}	$\left(1 - p_1^{ad}\right) = \Pr\left(\varepsilon_1 > MB_1^{ad}\right)$

Very preliminary estimation results of the competition parameters and of the marginal branching costs faced by acquirers in the first and second time period, as well as of the competition parameter, marginal costs of the acquired and of γ, the unit cost of a branch to be sold are shown in tables 2.1.ag and 2.1.ad respectively. All parameters are specified as constants for each subsample of banks.

Table 2.1.ag – Average c and log marginal costs for acquiring banks, time 1 and 2
Included observations: 721

	Coefficient	Std. Error	z-Statistic	Prob.
$mc_1^{ag} = \log\left(MC_1^{ag}\right)$	5.778100	0.125496	46.04228	0.0000
$mc_2^{ag} = \log\left(MC_2^{ag}\right)$	4.744066	0.122613	38.69139	0.0000
c_1	1.072173	0.087626	12.23585	0.0000
c_2	1.104606	0.028197	39.17407	0.0000
Log likelihood	-770.7827	Akaike info criterion		2.149189
Avg. log likelihood	-1.069047	Schwarz criterion		2.174602

Table 2.1.ad – Average c, log marginal costs for acquired banks, time 1, andγ
Included observations: 348

	Coefficient	Std. Error	z-Statistic	Prob.
$mc_1^{ad} = \log\left(MC_1^{ad}\right)$	6.270861	1.061024	5.910199	0.0000
c_1	1.111766	0.158564	7.011461	0.0000
γ	804.2747	1047.661	0.767686	0.4427
Log likelihood	-177.7592	Akaike info criterion		1.038846
Avg. log likelihood	-0.510802	Schwarz criterion		1.072055

From these estimates, banks that ended up being acquired in the second period, had, on average, higher marginal branching costs and faced less competition (c_1 higher: 1.11 vs. 1.07) in the first period than their counterparts, the acquiring banks. It is actually a result in line with the literature that states that target firms in acquisition deals are usually less efficient that the acquirers.

As for γ, the estimated number, 804, is the average unit price of acquired branches, in million euros. Given the definition of bank profits in this model as shares of total deposits by market, this number may also be interpreted as the average value of deposits by sold branch.

3. Conclusions

This paper presents a two-period dynamic model of branching behaviour and competition in retail banking. It is an extension of a static model, that has already been published and used to study competition in Italian local banking markets, from which it borrows most of its theoretical underpinnings. It however adds some information on banks branching strategies under the (realistic) assumption that it takes time and possibly strategic adjustments for a bank to reach its optimal branching network size.

The empirical counterpart of the model may be estimated and it becomes possible to compute measures of competition and, more interestingly, measures of benefits and costs by bank and by markets and their evolution in time . It results that between 2004 and 2006 competition among banks tended to decrease in intensity, while banks increased their efficiency through adjustments in their branching size and location. There is some indication of crosstime subsidisations: banks are willing to operate at suboptimal branching sizes, with marginal benefits lower than marginal costs at some point in time, in order to reach “optimality” in the future.

The paper also presents preliminary work on a dynamic model with mergers that allows the econometrician to estimate the average value per branch of the acquired bank in case of an M&A operation.

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