

Dynamically Complete Experimental Asset Markets

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Abstract

We compare prices and portfolio choices in complete and incomplete experimental financial markets. The incomplete-markets treatment differs from the complete-markets one in that we shut down one market, and that we announce, halfway through trading, which of three states will not occur. The information structure in the incomplete markets is such that these markets satisfy the *necessary* condition to be *dynamically complete*. If they are indeed dynamically complete – a property that depends on the preferences of experiment participants, the individual holdings and asset prices in the incomplete-market treatment must be equivalent to those in the complete-market treatment. This is our finding. The distribution of asset holdings is undistinguishable among treatments, and state-price probability rankings coincide and are equilibrium rankings, except for one case, where incomplete markets achieve close-to-equilibrium ranking more often than complete markets.

1 Introduction

In this paper we investigate the experimental validity of the theory of dynamically complete asset markets. The main conclusion of this theory is that under mild assumptions, complete markets for the trade of short-lived securities, can be replicated

with markets for long-lived securities, given a flow of information in time. The relevance of this conclusion comes from two sources. The first is the well-known result that complete competitive markets for aggregate-wealth risk, produce trades to efficient final outcomes. The second is the fact that the space of possible aggregate wealth outcomes is potentially very large, thus requiring an absurd number of securities to achieve the aforementioned efficiency. Dynamic completeness implies that efficiency in an uncertain world can be achieved through trade and re-trade of a small number of long-lived assets.¹

In our experiments we compare complete and incomplete markets for the trade of the same form of aggregate risk. In the incomplete markets, securities are long-lived and information is refined in time, before the liquidation of assets. For most indicators we consider, the two setups are undistinguishable. In particular, the distribution of final individual holdings of wealth in each state of the world is undistinguishable. This is an important finding, since it is easy to manipulate participants' beliefs such that this will not be the case (see Bossaerts et al. [2007]), and "beliefs" are in fact an important part of the formation of prices in the dynamic, incomplete markets we implement experimentally.

Since the comparison of the two treatments is noisy at best, we go further, to pose the hypothesis of equilibration. If the incomplete markets are indeed dynamically complete, and equilibrium is achieved, then certain conditions must hold for prices and individual holdings, for a wide range of individual preferences. Specifically, if agents are risk averse, state-price probabilities (the prices of Arrow-Debreu securities that are implied by the prices of the tradable assets) must be ranked in inverse order of aggregate wealth. That is, the price of consumption in a *wealthy* state of the world, must be lower than the price of consumption in a *poor* state of the world. We find that state-price probabilities in the incomplete-market treatment, are ranked in accordance to the above criterion. In the complete-market treatment,

¹How small, and how much smaller than the number of short-lived securities, depends on the nature of the flow of information about aggregate risk in the market.

state-price probabilities are mostly correctly ranked, but a systematic divergence from this hypothesis arises, which we discuss in the paper.

In both treatments we consider, there are three states of the world. In the complete-market treatment there are three securities with linearly independent distribution of dividends. There is one period for the trade of these securities, after which the securities are liquidated. In the incomplete-market treatment, although participants are endowed with identical holdings of the three securities traded in the complete market, they are allowed to trade only two of them. This is an insufficient number for the market to be complete. However, in this treatment there are two trading periods. One initial period happens with the original information about the distribution of states of the world. The second period happens after an announcement is made that correlates with the true realized state of the world, and thus, refines the information of the participants. In fact, the announcement reduces the number of possible states of the world to two.

The markets *after announcement* are always complete. However, whether the market before announcement is such that the entire situation is dynamically complete, depends on the prices at which participants trade in the second period. This means that whether this treatment is dynamically complete or not, depends on the preferences of participants. A result by Kreps [1982] allows us to rule out the possibility that our incomplete-market treatment is *not* dynamically complete, as a knife-edge situation. The theoretical predictions we test here are also summarized in this work, Kreps [1982], for discrete-time markets and information, as is the case in our experimental situation. The theory is developed for continuous-time information and price processes, in Duffie and Huang [1985].

It is important to note that the precise knowledge of the information flow and the distribution of dividends, are key to the tests we perform in this paper. These are privileges of the experimental paradigm. Two important features were taken into account when designing the experiment we describe in what follows. The first

is that the computation of state-price probabilities requires more information than is available in any single experimental period. Aggregate uncertainty is manipulated to bypass this constraint and compute state-price probabilities with the information of one period only. The second is that the assumption of mean-variance preferences, which is used in setting up some of our hypotheses, may imply efficiency *without* completeness (or dynamic completeness). Again, the experiment is designed such that the assumption of mean-variance efficiency does not trivially imply the validity of the tests we use (the tests relate to efficiency) and thus, render the exercise meaningless.

The following section introduces the theoretical background and notation used in the remainder of the paper. It also contains the description of the experiment, both in a separate subsection, subsection 2.3, and in examples used to illustrate the notation. Section 3 presents and discusses experimental results that relate to prices and state-price probabilities. Section 4 presents and discusses results that relate to individual asset holdings. Section 5 concludes.

2 Notation and Experimental Setup

In this section we give the setup necessary to describe the experiment and hypotheses that are the focus of this paper. A general framework for the study of dynamic completeness in discrete-time multi-period markets can be found in Kreps [1982]. We give a more specific description, where we include probabilities for information sequences, since these are part of the experimental setup.

2.1 Notation

We consider asset markets for long-lived securities that liquidate in consumption period T , after T trading periods, 0 to $T - 1$. Periods are indexed $t \in \{0, \dots, T\}$. There is a single consumption good (e.g., money), which is also the numeraire for

asset dividends and prices. An asset market is defined by a matrix and probability distribution over asset liquidating dividends, a vector of total asset supply, a set of possible information sequences, and a statement of what assets can be traded. We will restrict attention to dynamic markets with a simple linear information structure that we specify below. An asset market is thus defined by

$$\{\tilde{R}, R, w, \mathcal{M}, \pi_0, \}, \quad (1)$$

where \tilde{R} is a $S \times \tilde{K}$ matrix of risky asset dividends. There are \tilde{K} risky assets paying dividends that depend on the realization of a random variable called *state of the world*. There are S possible realizations of the state of the world, denoted $s \in \{1, \dots, S\}$. The vector $w = (w_1, \dots, w_{\tilde{K}})$ identifies supply of each asset. Let $\tilde{w} = (\tilde{w}_1, \dots, \tilde{w}_S)$ be the *endowment* or total supply of the unique consumption good in each state of the world. Clearly, $\tilde{w} = \tilde{R}w$. The matrix R of dividends of tradable assets, has dimension $S \times K$, $K \leq \tilde{K}$. Without loss of generality, we assume that whenever $K < \tilde{K}$, securities $\{1, \dots, K\}$ are tradable, while securities $\{K + 1, \dots, \tilde{K}\}$ cannot be traded.

\mathcal{M} is a set of possible sequences that define how information evolves in time. Each sequence $\{M_t\}_{t=1}^T \in \mathcal{M}$, has an ex-ante probability of occurrence, $\pi_0(\{M_t\}_{t=1}^T)$. Sequences are mutually exclusive. The probability distribution over final realizations of the state of the world is implicit in the structure of \mathcal{M} . At every period $t > 0$ M_t determines the “surviving” return matrix, R_t , from the previous period return matrix, R_{t-1} , through the simple formula

$$R_t = M_t R_{t-1}.$$

Information at time $t = 0$ is always $R_0 = R$, and at time $t = T$, uncertainty is resolved in a single state of the world. That is, $R_T = r_s$, where r_s is the s th row of matrix R , for some $s \in \{1, \dots, S\}$. Every matrix M_t is composed of zeros and ones.

Along a sequence in \mathcal{M} , the number of rows and columns of M_t is non-increasing in t , and M_T always has only one row. Each column of a matrix M_t is either a column of zeros or a basis vector e_i , containing a 1 in the i th row, and zeros everywhere else. M_t contains zero-vectors in column-positions corresponding to the states “eliminated” at time 1 along the information sequence being considered. The remaining columns are e_1, \dots, e_{S_t} , entered in ascending index order (skipping the positions where there is a vector of zeros).

The distribution π_0 defines the ex-ante probabilities of occurrence of each sequence in \mathcal{M} . Probabilities can be updated in the obvious way, as time and information evolve. We will use $\{M_\tau\}_{\tau=1}^t$ to denote the statement, “up to time t information has followed the path given by $\{M_1, \dots, M_t\}$.”

Example 1 (Experimental markets). *In our experiment we consider two markets. In both cases there are three states of the world with names “X”, “Y”, and “Z”, and three securities called A, Bond, and B. The markets differ in the number of periods and the tradable securities.*

Using the notation given above, the so-called complete market treatment has $T = 1$ and is described as follows:

1. $\tilde{R} = R = \begin{bmatrix} 0.5 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0.5 \end{bmatrix}$.
2. $w = (8, 4, 2) \Rightarrow \tilde{w} = (6, 6, 12)$.²
3. $\mathcal{M} = \left\{ \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \right\}$.
4. $\pi_0 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right)$.³

²The market portfolio differed across experimental sessions. Exact numbers are given in Table 3.

³During an experimental session, states of the world were drawn without replacement, causing variation in the probability distribution. We explain this further in the next section. Probabilities in all periods remained close to uniform across states of the world.

The incomplete-market treatment has $T = 2$ and is described as follows:

$$1. \tilde{R} = \begin{bmatrix} 0.5 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0.5 \end{bmatrix}, R = \begin{bmatrix} 0.5 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

$$2. w = (8, 4, 2) \Rightarrow \tilde{w} = (6, 6, 12).$$

3. $\mathcal{M} = \{\{M_{11}, M_{21}\}, \{M_{11}, M_{22}\}, \{M_{12}, M_{23}\}, \{M_{12}, M_{24}\}\}$, where

$$M_{11} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, M_{12} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and

$$M_{21} = M_{23} = \begin{bmatrix} 1 & 0 \end{bmatrix}, M_{22} = M_{24} = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

In words, the first sequence in \mathcal{M} corresponds to “at $t = 1$ it is revealed that the state is not Z , and at time $t = 2$ it is revealed that the state is X ”. The other sequences have analogous meanings

$$4. \pi_0 = \left(\frac{1}{3}, \frac{1}{6}, \frac{1}{6}, \frac{1}{3}\right).$$

Let $\mathcal{M}^s \subset \mathcal{M}$ be the set of all sequences such that $R_T = M_T R_{T-1} = r_s$. The ex-ante ($t = 0$) probability of occurrence of state of the world s is inferred from the probability over information-transformation sequences as follows:

$$\pi^s = \sum_{\{M_t\}_{t=1}^T \in \mathcal{M}^s} \pi_0(\{M_t\}_{t=1}^T).^4 \quad (2)$$

For $t > 0$, a pair $(t, \{M_\tau\}_{\tau=1}^t)$ is called a *node*. At $t = 0$ there is a unique *initial node* denoted 0. The successors of node 0 are all the period-one nodes defined by

⁴It is evident from this statement that our notation is inconsistent with the general treatment of information sequences as sequences of finer partitions of the state space. In that treatment, each terminal node must correspond to a separate state. In our example, that would mean that there is an additional state, call it Y' , in which the assets pay the same dividends as in state Y .

sequences in \mathcal{M} . That is,

$$\Phi(0) = \{(1, M_1) \mid \{M_1, \dots, M_T\} \in \mathcal{M}\}.$$

For $t > 0$, the set of *successors* of a node, $(t, \{\bar{M}_\tau\}_{\tau=1}^t)$ is defined as follows:

$$\Phi(t, \{\bar{M}_\tau\}_{\tau=1}^t) = \{(t+1, \{M_\tau\}_{\tau=1}^{t+1}) \mid \{M_\tau\}_{\tau=1}^t = \{\bar{M}_\tau\}_{\tau=1}^t \text{ and } \{M_\tau\}_{\tau=1}^T \in \mathcal{M}\}.$$

Example 2. *In both treatments of our experiment, the probabilities of final states of the world are $(\pi^X, \pi^Y, \pi^Z) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.*

In the incomplete market treatment, we have the following sets of successors:

$$\begin{aligned} \Phi(0) &= \{(1, M_{11}), (1, M_{12})\} \\ \Phi(1, M_{11}) &= \{(2, \{M_{11}, M_{21}\}), (2, \{M_{11}, M_{22}\})\} \\ \Phi(1, M_{12}) &= \{(2, \{M_{12}, M_{23}\}), (2, \{M_{12}, M_{24}\})\}. \end{aligned}$$

The complete market treatment has only one set of successors, that of the matrix R , which is composed of the three vectors r_s .

Let $\{q(t, \{M_\tau\}_{\tau=1}^t)\}_{t=0}^{T-1}$ be a sequence of prices of assets defined for every node in market $\{\tilde{R}, R, w, \mathcal{M}, \pi_0, \}$. $q(t, \{M_\tau\}_{\tau=1}^t)$ is a $K \times 1$ vector of prices for each tradable asset. We can now define market *completeness* and *dynamic completeness*.

We say that the asset market $\{\tilde{R}, R, w, \mathcal{M}, \pi_0\}$ is **complete** if $\text{rank } R = S$.

We say that the asset market $\{\tilde{R}, R, w, \mathcal{M}, \pi_0\}$ with asset prices $\{q(t, \{M_\tau\}_{\tau=1}^t)\}_{t=0}^{T-1}$ is **dynamically complete** if, for every sequence $\{M_t\}_{t=1}^T$, the following statements are true:

1. $\text{rank } R_{T-1} = S_{T-1}$, where $R_{T-1} = (\Pi_{t=1}^{T-1} M_t) R$.
2. For $t \in \{0, \dots, T-2\}$, the price vectors for every successor of $(t, \{M_\tau\}_{\tau=1}^t)$ are

linearly independent. Let

$$q(\Phi(t, \{M_\tau\}_{\tau=1}^t)) = \begin{bmatrix} q(t+1, \{M_1, \dots, M_t, M_{t+1}\}) \\ \dots \\ q(t+1, \{M_1, \dots, M_t, M'_{t+1}\}) \end{bmatrix},$$

be the matrix of price vectors for all successors of $(t, \{M_\tau\}_{\tau=1}^t)$. The requirement that these prices be linearly independent is equivalently stated as

$$\text{rank } q(\Phi(t, \{M_\tau\}_{\tau=1}^t)) = |\Phi(t, \{M_\tau\}_{\tau=1}^t)|.$$

2.2 State-Price Probabilities

An asset market, $\{\tilde{R}, R, w, \mathcal{M}, \pi_0, \}$, has an associated *Arrow-Debreu market*,

$$\{I^S, \tilde{w}, \mathcal{M}, \pi_0\},$$

where I^S denotes the $S \times S$ identity matrix, indicating that the tradable assets are units of contingent consumption for each state of the world.⁵

Consumers embedded in a market structure $\{I^S, \tilde{w}, \mathcal{M}, \pi_0\}$ will be able to achieve efficient consumption in equilibrium, in spite of the uncertainty about future supply of the consumption good. However, it may be the case that S is very large, thus implying a very large number of tradable securities to achieve efficient outcomes. The main result of Duffie and Huang [1985] is that, given the information structure implied by \mathcal{M} , efficient consumption in equilibrium can be achieved in a *dynamically complete* asset market with K significantly smaller than S .

Let there be N consumers indexed by $n \in \{1, \dots, N\}$. Consumer n has preferences v_n over consumption at time T . Each consumer is endowed with $\tilde{w}_{ns} \geq 0$ units of the consumption good in state of the world s . Consumers are embedded in an

⁵We omit the matrix of tradable assets in the description, because in the Arrow-Debreu markets all assets given in I^S can be traded.

Arrow-Debreu market structure with $\tilde{w} = \sum_{n=1}^N \tilde{w}_n$ (where $\tilde{w}_n = (\tilde{w}_{n1}, \dots, \tilde{w}_{nS})$).

Definition 3. An allocation $(x_1^*, \dots, x_N^*) \in \mathfrak{R}^{N \times S}$ and a vector of prices for contingent consumption, (p_1^*, \dots, p_S^*) is an Arrow-Debreu equilibrium in market $\{I^S, \tilde{w}, \mathcal{M}, \pi_0\}$ if:

- i) For every n , x_n^* is maximal for v_n in agent n 's budget set.
- ii) Contingent markets clear, that is

$$\sum_{n=1}^N x_n^* = \tilde{w}.$$

Definition 4. Asset holdings $(z_1^*(t, \{M_\tau\}_{\tau=1}^t), \dots, z_N^*(t, \{M_\tau\}_{\tau=1}^t))$ and asset prices $q^*(t, \{M_\tau\}_{\tau=1}^t)$, constitute a Radner equilibrium in market $\{\tilde{R}, R, w, \mathcal{M}, \pi_0, \}$ if:

- i) For every n , $\tilde{R}z_n^*(T-1, \{M_\tau\}_{\tau=1}^{T-1})$ is maximal for v_n in agent n 's budget set, in every state of the world, and under the condition that $z_{nk}^*(t, \{M_\tau\}_{\tau=1}^t) = w_{nk}$ for $k > K$ (non-tradable assets), and all n, t .
- ii) At every node, asset markets clear,

$$\sum_{n=1}^N z_n^*(t, \{M_\tau\}_{\tau=1}^t) = w.$$

If the asset market $\{\tilde{R}, R, w, \mathcal{M}, \pi_0\}$ with prices $q^*(t, \{M_\tau\}_{\tau=1}^t)$, is dynamically complete, then the prices, p^* , and contingent consumption, x_n^* , can be uniquely determined from $q^*(\cdot)$ and $z_n^*(\cdot)$, respectively.

Moreover, if market $\{\tilde{R}, R, w, \mathcal{M}, \pi_0\}$ with prices $q(t, \{M_\tau\}_{\tau=1}^t)$ (not necessarily equilibrium prices), is dynamically complete, then a unique vector of prices of Arrow-Debreu securities can be determined from the prices $q(t, \{M_\tau\}_{\tau=1}^t)$. This vector satisfies

$$q_0(R, \pi_0) = R'p, \tag{3}$$

and is called the vector of *state-price probabilities*.

If, in addition, market $\{\tilde{R}, R, w, \mathcal{M}, \pi_0\}$ is complete, p can be uniquely determined as the solution to equation (3), since R is invertible.

Prices of contingent consumption can be determined from asset prices at every node. We will call these *conditional state prices*, and denote them $p(t, \{M_\tau\}_{\tau=1}^t)$. An equation analogous to equation (3) holds for conditional state prices.

We have not yet been specific about the nature of consumers' preferences, v_n . The hypotheses that direct our experimental findings are based on specific assumptions about v_n . First we assume v_n admits an expected utility representation and that all consumers are risk averse (decreasing marginal utility over final consumption). Under these assumptions, the ranking of prices in an Arrow-Debreu equilibrium is fully determined by the ranking of endowments in different states of the world.

Theoretical Prediction 1: Given an Arrow-Debreu market $\{I^S, \tilde{w}, \mathcal{M}, \pi_0\}$ and risk averse expected utility-maximizing consumers, the Arrow-Debreu equilibrium prices of consumption in every state of the world, p^* satisfy

$$\tilde{w}_s > \tilde{w}_{s'} \Rightarrow \frac{p_{s'}^*}{\pi^{s'}} > \frac{p_s^*}{\pi^s}, \quad (4)$$

for all $s, s' \in \{1, \dots, S\}$.

The above also holds for Arrow-Debreu equilibrium prices determined for nodes different from 0, or *conditional state prices*. As mentioned before, if the asset market $\{\tilde{R}, R, w, \mathcal{M}, \pi_0\}$ with equilibrium prices $q^*(t, \{M_\tau\}_{\tau=1}^t)$ is dynamically complete, state-price probabilities and conditional state-price probabilities can be uniquely determined, and will satisfy the same ranking condition.

Theoretical Prediction 2: If $\{\tilde{R}, R, w, \mathcal{M}, \pi_0\}$ with equilibrium prices $q^*(t, \{M_\tau\}_{\tau=1}^t)$ is dynamically complete and consumers are risk-averse, expected utility maximizers, the associated state-price probabilities and conditional state-price probabilities,

$p^*(t, \{M_\tau\}_{\tau=1}^t), t \in \{0, \dots, T-1\}$ satisfy

$$\tilde{w}_s > \tilde{w}_{s'} \Rightarrow \frac{p_{s'}^*(t, \{M_\tau\}_{\tau=1}^t)}{\pi^{s'}(t, \{M_\tau\}_{\tau=1}^t)} > \frac{p_s^*(T, \{M_\tau\}_{\tau=1}^t)}{\pi^s(t, \{M_\tau\}_{\tau=1}^t)}, \quad (5)$$

for every $s, s' \in \{1, \dots, S\}$ such that $\pi^s(\cdot) > 0, \pi^{s'}(\cdot) > 0$, and where $\pi^s(t, \{M_\tau\}_{\tau=1}^t)$ denotes the updated probability of state s at node $(t, \{M_\tau\}_{\tau=1}^t)$.

Analogous results hold for asset holdings. We present them as theoretical predictions 3 and 4 below.

Theoretical Prediction 3: If $\{\tilde{R}, R, w, \mathcal{M}, \pi_0\}$ with prices $q^*(t, \{M_\tau\}_{\tau=1}^t)$ is dynamically complete, then the Radner equilibrium holdings, $z_n^*(t, \{M_\tau\}_{\tau=1}^t)$ and the Arrow-Debreu equilibrium holdings of the associated Arrow-Debreu market, satisfy:

$$\tilde{R}z_n^*(T-1, \{M_\tau\}_{\tau=1}^{T-1}) = x_n^*, \forall n. \quad (6)$$

The next prediction requires a much tighter restriction of consumers' preferences.⁶ Under the assumption that consumers' preference-maximizing choice of asset holdings equals the choice of an agent maximizing mean-variance utility over asset holdings, in equilibrium all consumers will have relative holdings of assets equal to the market portfolio (if all assets can be traded). The *market portfolio* in the Arrow-Debreu market, is given by \tilde{w} , while it is w in the asset market.

Theoretical Prediction 4: If consumers have preferences that can be represented with mean-variance utility over asset holdings, then the Arrow-Debreu equilibrium holdings satisfy

$$\frac{x_{ns}^*}{x_{ns'}^*} = \frac{\tilde{w}_s}{\tilde{w}_{s'}}, \forall s, s' \in \{1, \dots, S\}.$$

If $\{\tilde{R}, R, w, \mathcal{M}, \pi_0\}$ with prices $\{q^*(t, \{M_\tau\}_{\tau=1}^t)\}$ is dynamically complete, then the

⁶It is not strictly correct to call this a restriction on preferences, since this restriction on representation of consumers' choices of asset holdings depends both on preferences and the probability distribution of states of the world (see Berk [1997] for a thorough discussion of the subject).

Radner equilibrium holdings, $z_n^*(t, \{M_\tau\}_{\tau=1}^t)$, satisfy:

$$\frac{\sum_{k=1}^{\bar{K}} r_{sk} z_{nk}^* (T-1, \{M_\tau\}_{\tau=1}^{T-1})}{\sum_{k=1}^{\bar{K}} r_{s'k} z_{nk}^* (T-1, \{M_\tau\}_{\tau=1}^{T-1})} = \frac{\tilde{w}_s}{\tilde{w}_{s'}}, \forall s, s' \in \{1, \dots, S\}, \quad (7)$$

where r_{sk} is the dividend on asset k in state of the world s .

Example 5 (State-price probabilities ranking in experiment). *The markets considered in the experiment have total supply of consumption good, $\tilde{w} = (6, 6, 12)$. Given that all three states have equal probability of occurrence, the Arrow-Debreu equilibrium prices should be ranked as follows:*

$$p_X^* = p_Y^* > p_Z^*,$$

where we have replaced the numerical indices with letters denoting the states. Moreover,

$$\frac{3}{2} p_X^* (1, \{M_{11}\}) = 3 p_Y^* (1, \{M_{11}\}), \text{ and } 3 p_Y^* (1, \{M_{12}\}) > \frac{3}{2} p_Z^* (1, \{M_{12}\}).$$

The incomplete market treatment clearly does not implement a complete market, but prices may be such that the market is dynamically complete. If it is, then the above rankings of state-price probabilities must hold also in the incomplete market treatment.

2.3 Experimental Implementation of Asset Markets

In our experiments there are three securities, Asset A, Asset B, and Bonds, as well as cash. In the *complete market* treatment, all three securities can be traded by participants, while in the *incomplete market* treatment, only Asset A and Bonds can be traded. Securities can be sold short, while cash can be held only in non-negative amounts. A bankruptcy rule explained below is used to prevent agents from committing to trades that require negative cash holdings.

		State of the world		
		X	Y	Z
ASSET	A	0.5	0	1
	B	0	1	0.5
	Bond	1	1	1

Table 1: Dollar dividends of assets, in each state of the world.

An experimental session lasts approximately $2\frac{1}{2}$ hours, of which approximately 30 minutes are dedicated to a review of instructions, another 30 minutes are composed of three practice periods to familiarize participants with the trading software, and the remaining time is split over 8 payoff-relevant periods. The *complete market* treatment, which serves as control, is run in three of the payoff-relevant periods, lasting 6 minutes each. The remaining periods last 8 minutes each, and are *incomplete market-treatment* periods.

At the beginning of a period, participants are endowed with holdings of the risky assets A and B, the risk-free Bonds, and cash. During a *complete market* period, participants can trade these holdings in parallel markets for all three assets, to achieve new positions. During an *incomplete market* period, participants can only trade asset A and Bonds, in two parallel markets. In both treatments trading is done through an electronic open book continuous market, or double auction, which is implemented with jMarkets software. At the end of the period, dividends of the securities are realized, and participants' earnings are added to their cumulative experimental earnings.

Dividends of assets A and B are governed by the realization of the state of the world, which can be X, Y, or Z. During an experimental session, states of the world for each period are drawn without replacement from an urn that starts the session containing 6 balls marked X, 6 marked Y, and 6 marked Z. All accounting is done in dollar units, including the values of dividends. Dividends in different states of the world were given in Example 1, and are repeated in Table 1 below.

At the beginning of each period, a participant knows his own holdings, the prob-

<i>If the state is:</i>	X	Y	Z
<i>Then the announcement is</i>	“The state is not Z” with probability 1	“The state is not X” or “The state is not Z” with equal probabilities	“The state is not X” with probability 1

Table 2: Distribution of the announcement conditional on the state of the world.

abilities of the states of the world, and the payoff table (Table 1). At the beginning of an *incomplete market* period participants also know that half-way through the period (after exactly 4 minutes) they will be given a piece of information - which we will call the “announcement” - that correlates with the state of the world. The distribution of the announcement conditional on the state of the world is also known (see Table 2), and the distribution of the state of the world conditional on each possible announcement can be readily computed with the information available to participants. Midway an *incomplete market* period, participants learn the realization of the announcement. At the end of every period, participants learn the realization of the state of the world, their period earnings, and their total earnings for the experimental session up to that point.

Figure 1 illustrates the incomplete market treatment, and gives conditional probabilities for states at every node. Short-hand notation is used to denote the nodes for the experiment. Nodes $(1, M_{11})$ and $(1, M_{12})$ are called \bar{Z} and \bar{X} respectively, referring to the announcement of what state *did not occur*.

The restriction that participants have non-negative cash holdings is implemented through a bankruptcy rule, automated and incorporated in the experimental software. Whenever a participant attempts to submit an order to buy or sell an asset, her cash holdings after dividends are computed for all states of the world, given her current asset holdings, and her outstanding orders (orders awaiting trade) that are *likely* to trade, including the order she is trying to submit. If these hypothetical cash holdings turn out negative for some state of the world, she is not allowed to submit the order.

More detailed information about the experimental setup and the software used

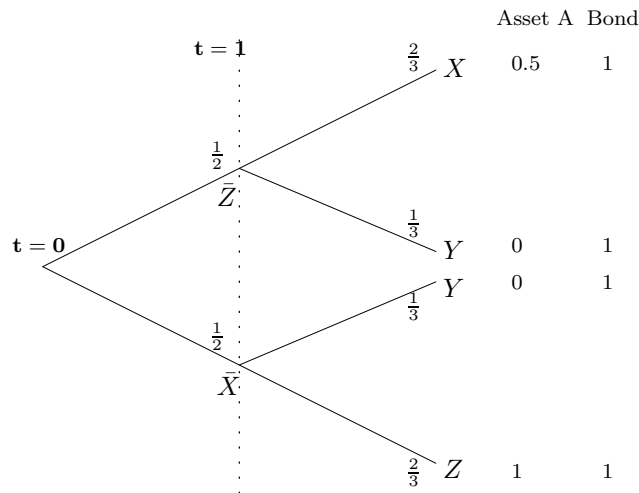


Figure 1: Time-event tree for the incomplete market periods.

to implement the markets, can be found in the experiment instructions on the web, at <http://clef.caltech.edu/exp/dc/index.html>

Two features of the experimental design deserve further comment. The first, is that we create incomplete markets by shutting down one market, not by having only two securities to start with. The second is that the expected payoff of the two risky assets in our experimental markets are equal.

Shutting down a market instead of starting with only two assets has two relevant implications. On one hand, it permits us to have identical market portfolios (not only equal state-holdings) in complete and incomplete market periods. On the other hand, by having participants hold a risky asset that they cannot trade, we can produce a situation where they cannot *trade to the market* before the announcement. They must trade after the announcement is made in order to reproduce the payoffs to the market portfolio in every state. We discuss this further in section ??.

The second feature, the fact that assets A and B have the same expected payoff, does not mean that they should be equally priced. Asset A is twice as abundant as

asset B, and has a lower price in the presence of risk aversion. This gives a clean test for the presence of risk aversion.

We report results taken from four experimental sessions, run in the Fall of 2005. Experimental sessions are identified by their date (yymmdd). Experiment 051005 had 12 participants, experiment 051022 had 22 participants, and experiments 051122 and 051201 had 31 participants each. Earnings per session per participant averaged approximately \$45, with a standard error of \$9. We give general statistics on trading behavior and prices in the Appendix.

3 Result: State-price probability rankings

In this section we look at theoretical predictions 1 and 2 (see Subsection 2.2). We first give the methodology used to compute state-price probabilities for the incomplete-market periods, given the limited information available. The methodology for computing conditional state-price probabilities in the incomplete-market periods and state-price probabilities in the complete-market periods is also briefly reviewed.

3.1 A Full Rank System to Compute State-Price Probabilities

If the experimental markets are dynamically complete, we know that the following holds:

$$q_0 = R'p. \tag{8}$$

Dynamic completeness and state-price probabilities are theoretical notions. Whether dynamic completeness is true and what state price probabilities are implied depend on all interim prices – but interim prices are not observed in an given period. One possibility to verify dynamic completeness and construct p from the sequence of asset prices is to combine asset prices from different experimental periods, where different information sequences are realized. The alternative, which we present here, is to

treat the quest of computing state-price probabilities from the experimental prices as a simple algebra problem.

If the market is dynamically complete, we know that equation (8) holds, but state-price probabilities cannot be computed from this equation because it is an underdetermined system. In other words, since there are three possible states of the world and two independent tradeable assets, R is not invertible, and p is not uniquely defined by the above equation. However, it will be uniquely defined in the presence of one more, independent conditions to define p . Our parameter specification is such that we have an additional condition on p . Specifically, states X and Y have equal aggregate wealth. Therefore, in equilibrium, states X and Y must have equal *adjusted* state-price probabilities (the adjusted state-price probability of state s is the state-price probability of state s divided by the exogenous probability of occurrence of state s).

In other words, take a third equation:

$$\frac{p_X}{\pi^X} - \frac{p_Y}{\pi^Y} = 0 \Leftrightarrow \begin{bmatrix} \frac{1}{\pi^X} & -\frac{1}{\pi^Y} & 0 \end{bmatrix} \begin{bmatrix} p_X \\ p_Y \\ p_Z \end{bmatrix} = 0,$$

We use the above equation to construct a new system of equations,

$$\begin{bmatrix} q_1(0) \\ q_2(0) \\ 0 \end{bmatrix} = \mathbf{R}p,$$

where

$$\mathbf{R} = \begin{bmatrix} R'_{(2 \times 3)} \\ \frac{1}{\pi^X} & -\frac{1}{\pi^Y} & 0 \end{bmatrix},$$

$q_1(0)$ is the price of asset A at time 0, and $q_2(0)$ is the price of the Bond at time 0.

The matrix \mathbf{R} of coefficients is invertible, and p can be computed, *given that* $\frac{p_X}{\pi^X} = \frac{p_Y}{\pi^Y}$. The results we present below are computed using this method.

3.2 Other methodological remarks

The computation of state-price probabilities from asset prices in experimental periods with a complete market is obvious. We use equation (8), where R is now that corresponding to complete market periods, and is thus invertible. Solving for p is straightforward.

We proceed similarly with the computation of conditional state-price probabilities in the two conditioning nodes (\bar{X}, \bar{Z}) . In this case, the following equation is true:

$$q(1\{M_{11}\}) = R'_{11}p$$

at node \bar{Z} , and an analogous equation is true at \bar{X} . R_{11} and R_{12} are both invertible matrices, implying that $p(1, \cdot)$ can be uniquely solved for.

In the experimental markets, trade is not instantaneous, but happens over a period of time during which the market is active. The time during which it is active before announcement is denoted $t = 0$, while $t = 1$ refers to the time when the market is open after the announcement. For each of these theoretical times there are, in the experiment, many trading prices. In describing the theoretical framework and the experiment (including the methodology), we have referred to asset prices as a single magnitude at every time. However, there are many prices at every time. In computing the state-price probabilities that we report, we will take an asset's price at a given time to be either the final price in the experimental period or the average over all trading prices in the period. We are now ready to state the first results.

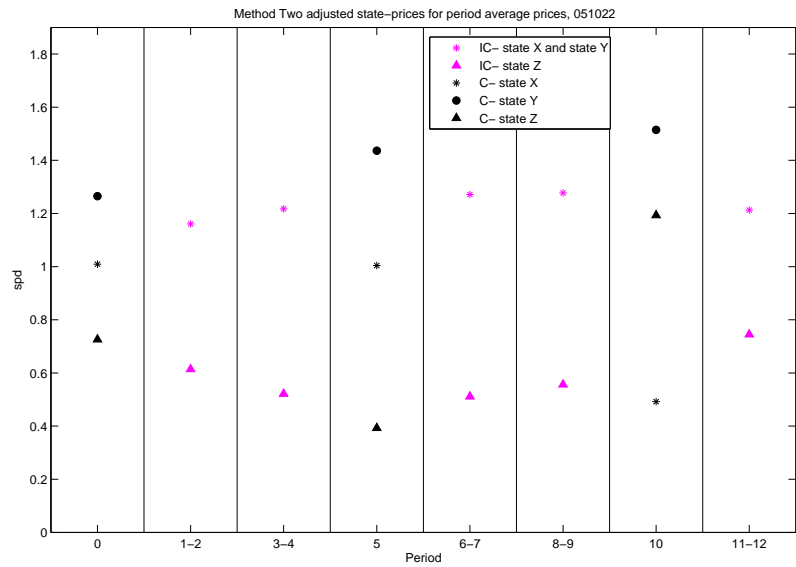
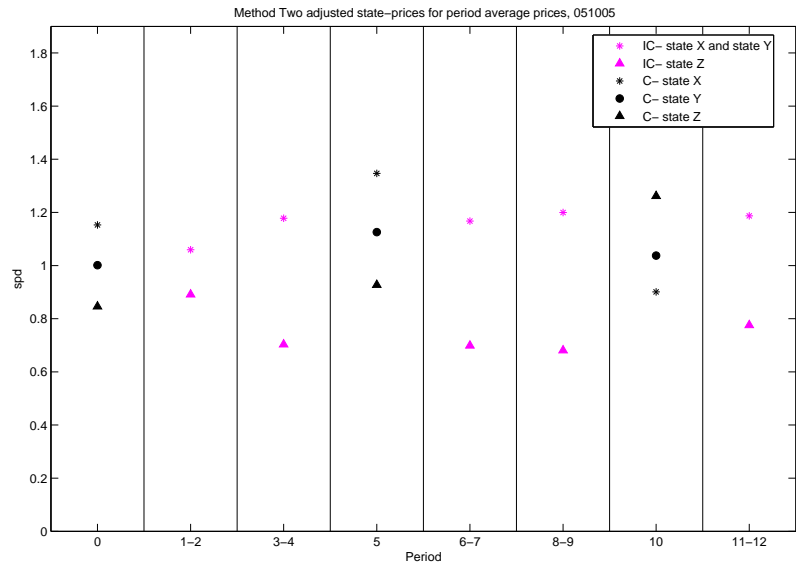


Figure 2: Adjusted state-price probabilities computed using average prices in each experimental period.

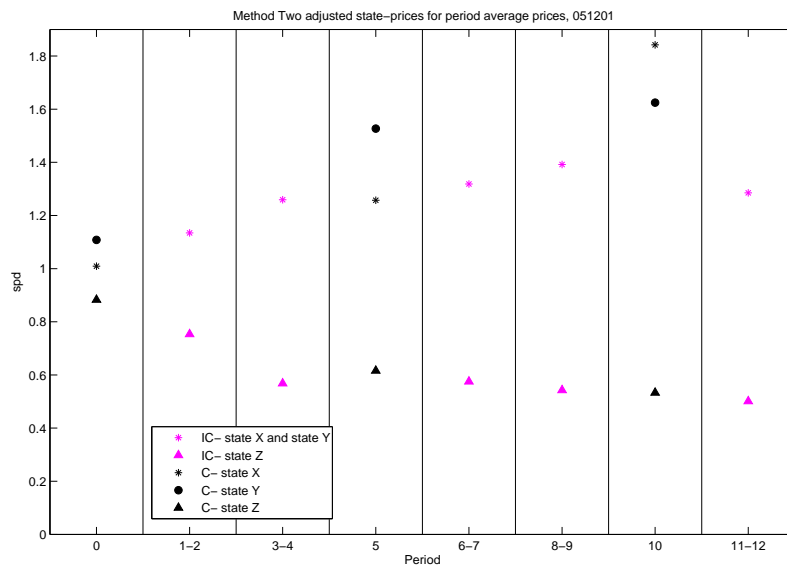
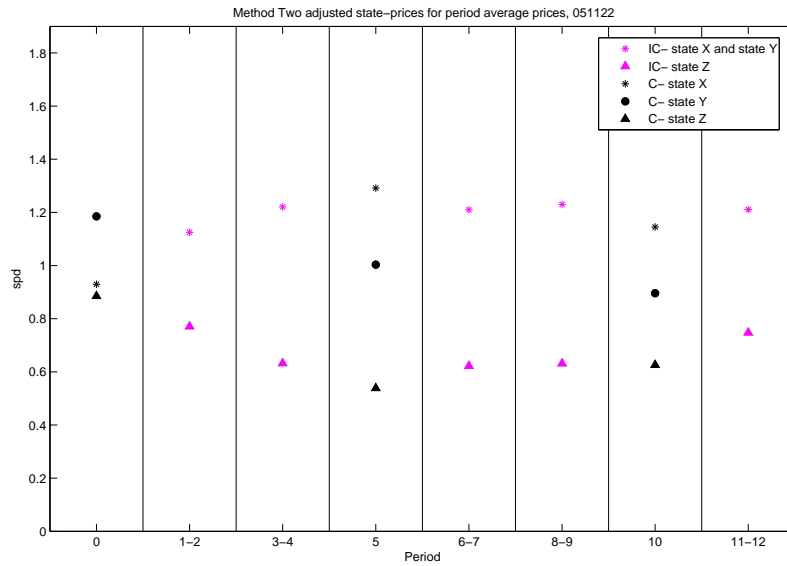


Figure 3: Adjusted state-price probabilities computed using average prices in each experimental period.

3.3 State-Price Probabilities for All States

Figures 2 and 3 show the adjusted state-price probabilities for each period, computed using average prices - before and after announcement, for the incomplete market periods. An analogous set of figures, where adjusted state-price probabilities are derived from end-of-period prices, is presented in figures 4 and 5.

State-price probabilities are *adjusted*, meaning that they are divided by each state's probability of occurrence. This is done because the states of the world are not always equally likely, since during one experimental session, states are drawn without replacement.

Given the total supply of assets A and B, and Bonds, and given the payoff matrix of the experiment, the theoretical prediction is that states X and Y have the same adjusted state-price probability, and that this value be higher than the adjusted state-price probability of state Z .⁷

In the figures, state Z is ranked lowest in all but two experimental periods. Thus, $p_Z < p_X$ and $p_Z < p_Y$, as predicted by theory. The relative ranking of states X and Y is assumed away in periods with incomplete markets. In complete market periods the prediction that $p_X = p_Y$ does not hold. Typically, $p_Y > p_X$.

3.4 Ranking of Conditional State-Price Probabilities

It is perfectly possible that correct ranking of state-price probabilities is obtained in spite of incorrect ranking of conditional state-price probabilities. This can happen by coincidence - prices at $t = 0$ and at $t = 1$ violate equilibrium theory predictions, but the algebra works in such a way as to produce correct state-price probability ranking - or because participants correctly anticipate the incorrectly ranked conditional state-price probabilities, but equilibrium prices at $t = 0$ are still such that the state-price probabilities are correctly ranked.

⁷This is only approximately the case in experimental sessions 051122 and 051201, where the wealth in state X is 6, it is 5.8 in state Y , and 12.05 in state Z .

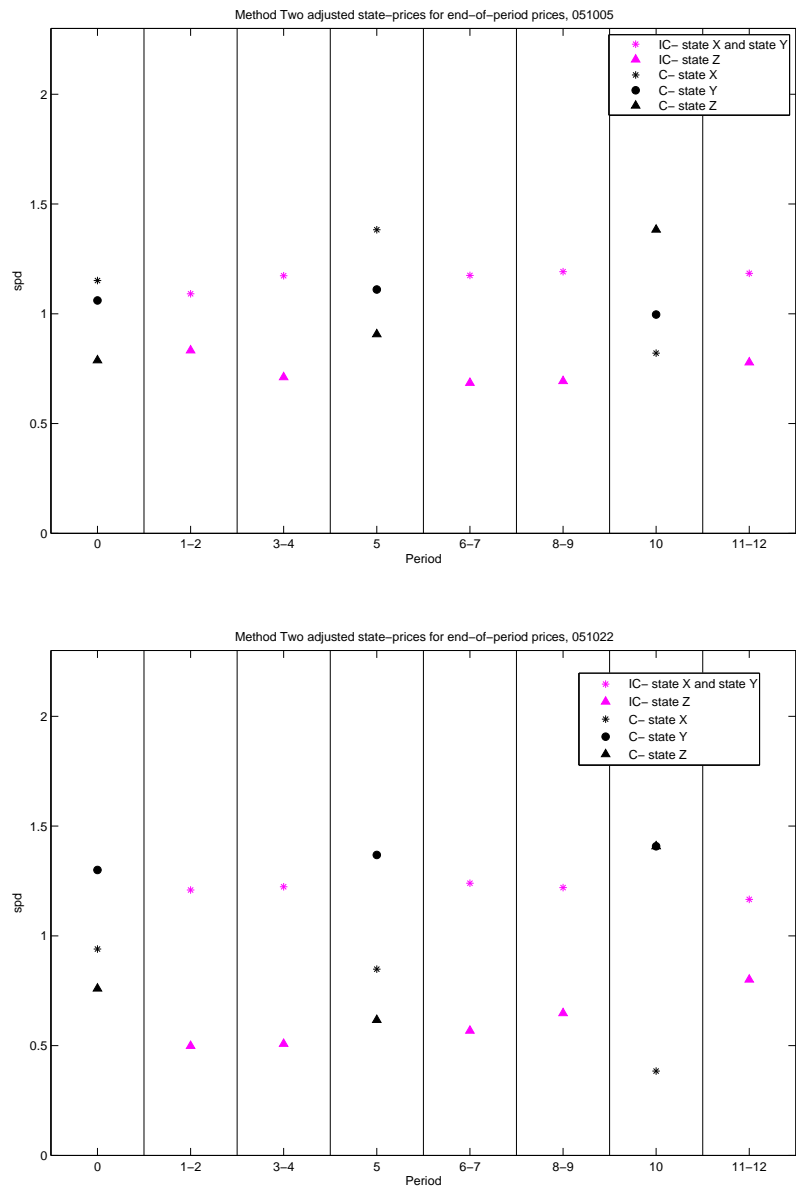


Figure 4: Adjusted state-price probabilities computed using final prices for each experimental period.

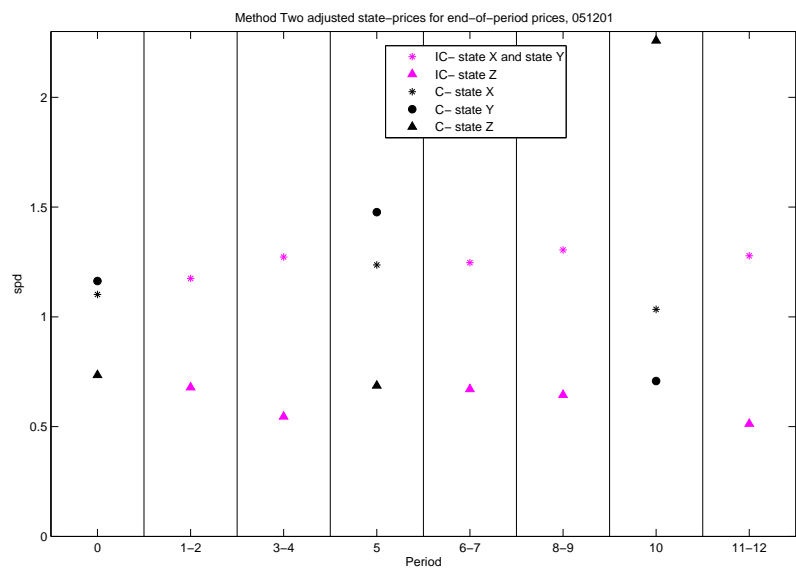
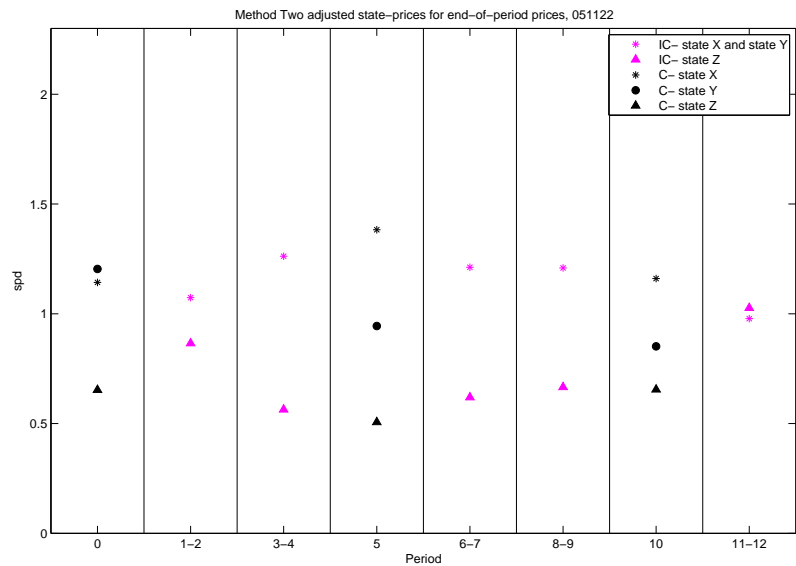


Figure 5: Adjusted state-price probabilities computed using final prices for each experimental period.

Figures 6 to 9 show conditional state-price probabilities conditional on the announcement. They are computed using average prices after announcement. If anything, conditional state-price probability ranking is closer to the theoretical prediction than state-price probability ranking in complete market periods. In periods where the announcement is \bar{Z} , $\frac{p_X(\bar{Z})}{\pi^X(\bar{Z})}$ is very close to $\frac{p_Y(\bar{Z})}{\pi^Y(\bar{Z})}$, as theoretically predicted. In periods where the announcement is \bar{X} , $\frac{p_Y(\bar{X})}{\pi^Y(\bar{X})}$ is bigger than $\frac{p_Z(\bar{X})}{\pi^Z(\bar{X})}$, as predicted by theory.

4 Individual Asset Holdings

This section deals with theoretical predictions 3 and 4. In our experiments individuals do not hold the market portfolio, regardless of whether we look at a complete or an incomplete market period. However, this is not different from previous (complete market) asset pricing experiments. Moreover, Bossaerts, Plott, and Zame [2005] shows that approximate CAPM equilibrium pricing (and even more so just correct state-price probability ranking) can arise in a model where individuals do not hold the market portfolio, but the *average* individual does.⁸

Figures 10 to 13 show the relative state-holdings of participants in our experiment. We call state holding the value of a participant's holdings in a given state (this is \tilde{w}_n in our original notation). When \bar{Z} is announced, we compute the relative state holdings for state X , $\frac{\tilde{w}_{nX}}{\tilde{w}_{nX} + \tilde{w}_{nY}}$. When \bar{X} is announced, we compute the relative state holdings of state Z , $\frac{\tilde{w}_{nZ}}{\tilde{w}_{nY} + \tilde{w}_{nZ}}$. *Time-zero state holdings* can be computed with a denominator that sums over all three states. We do not report these, since they make no sense, as shown in example 6, but we will use the term in examples and tables that follow.

Before describing our findings, it is important that we illustrate what it means

⁸Their result says that a sample market taken from a population where the deviation of individual demands from CAPM demands has mean zero, will converge to CAPM pricing as the size of the market increases.

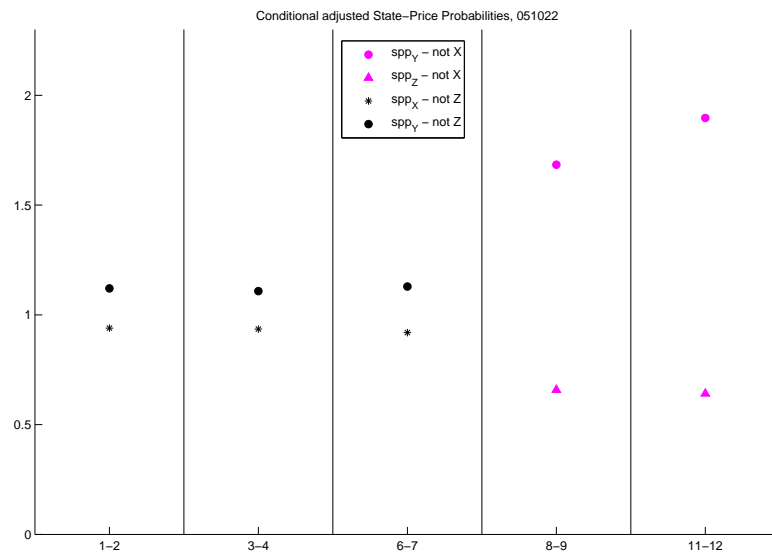
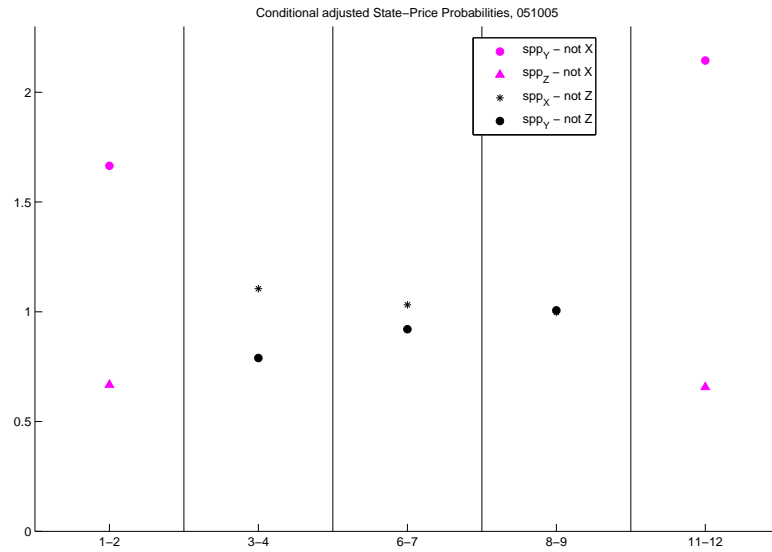


Figure 6: State-price probabilities conditional on announcement, computed using average prices after announcement.

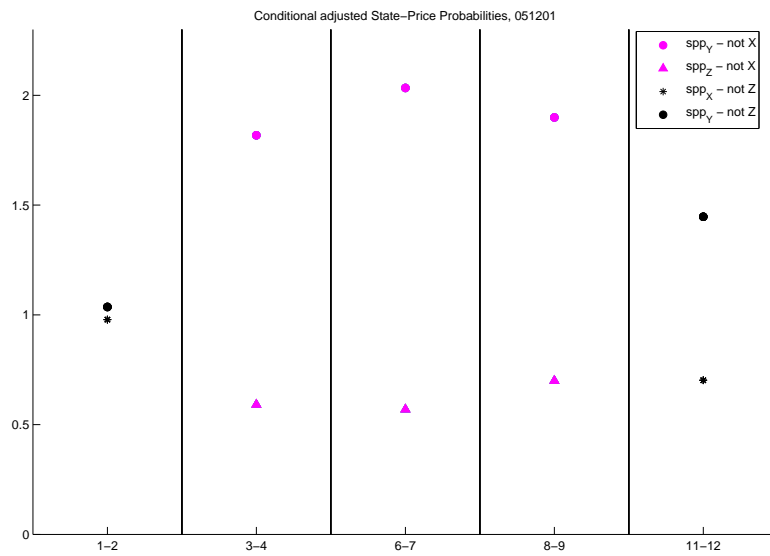
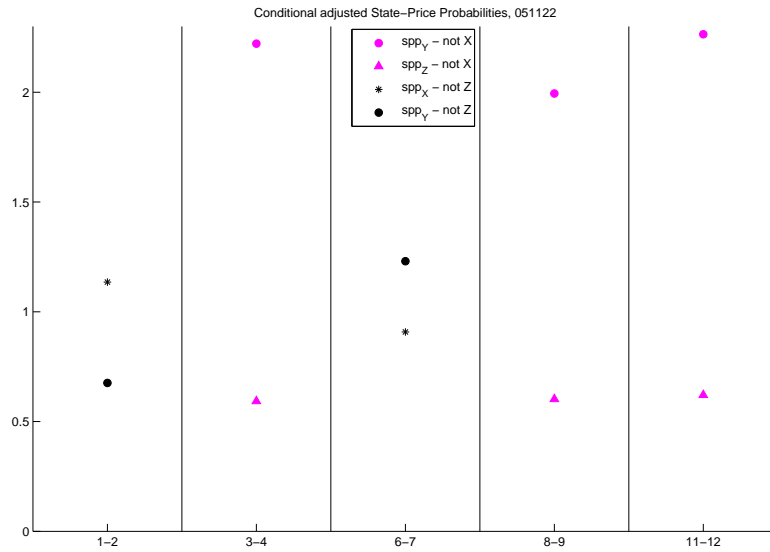


Figure 7: State-price probabilities conditional on announcement, computed using average prices after announcement.

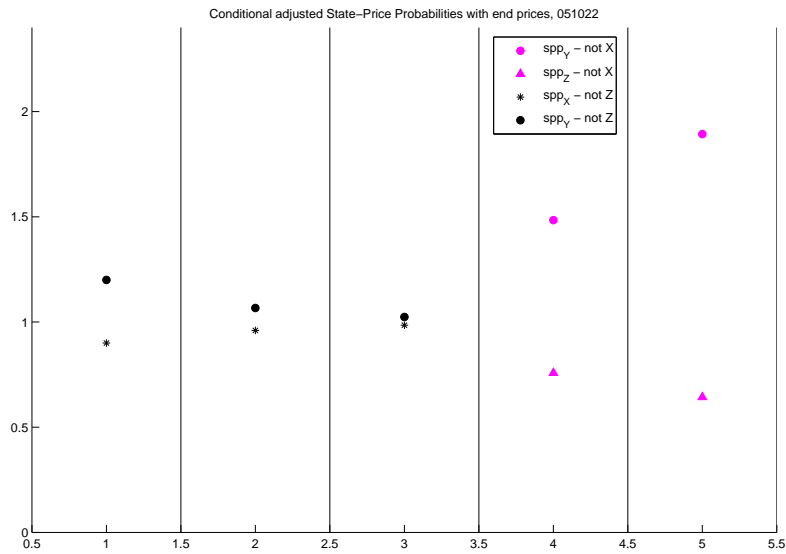
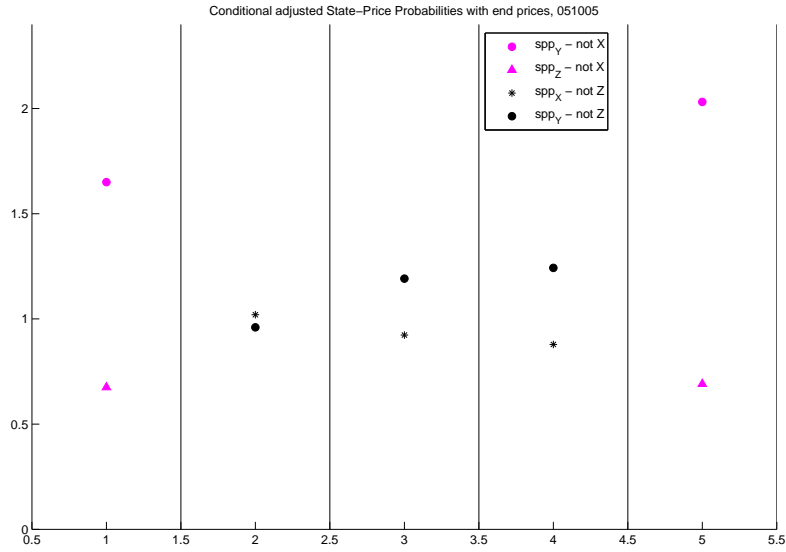


Figure 8: State-price probabilities conditional on announcement, computed using final prices of the period after announcement.

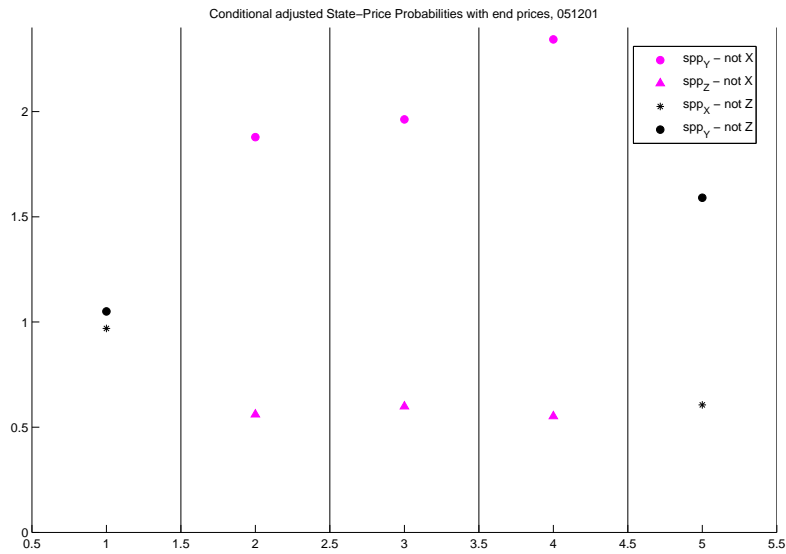
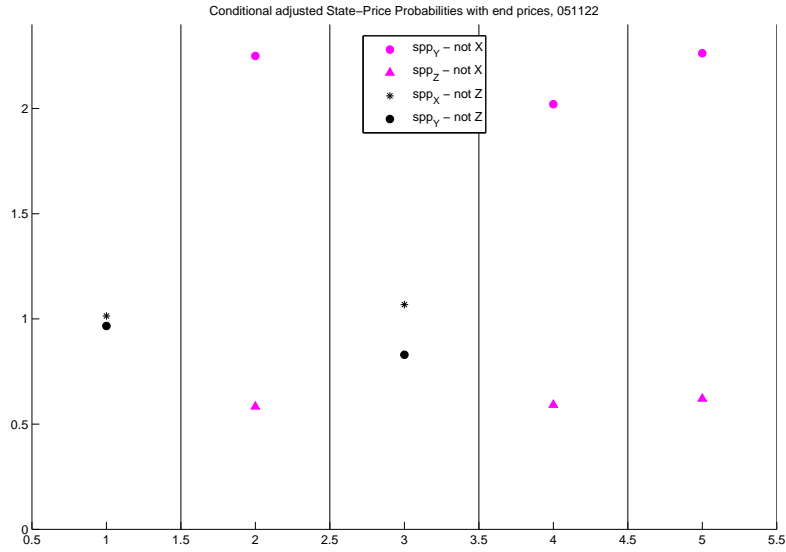


Figure 9: State-price probabilities conditional on announcement, computed using final prices of the period after announcement.

Initial Endowments			Market Portfolio in each session			
	Agent type			# agents type		
	<i>I</i>	<i>II</i>	<i>session</i>	<i>I</i>	<i>II</i>	<i>Mkt. Portfolio</i>
Asset A	14	2	051005	6	6	8A 4B 0Bonds \$2
Asset B	0	8	051022	11	11	8A 4B 0Bonds \$2
Bond	0	0	051122	16	15	8.2A 3.9B 0Bonds \$1.9
Cash	\$1	\$3	051201	16	15	8.2A 3.9B 0Bonds \$1.9

Table 3: Market portfolio in each experimental session. This is given by the initial endowments of each agent type, and the number of subject of each type.

to trade to the market’s state holdings in our setup. Table 3 reports the two types of initial endowments participants can hold in our experiments, and the market portfolio that follows, given the number of participants of each type we have in each experimental session. Table 4 shows the returns to endowments in each state (the state-holdings) for both types of initial endowments and the market, when there are equal numbers of participants of each type.

The following examples use the information in tables 3 and 4 to illustrate the exercise of “trading to the market” in our experiment.

Example 6 (Trading to the market before announcement). *This example shows that a participant of type I cannot trade to the market before announcement. Suppose a participant of type I tried to achieve the same state holdings as the market before the announcement, and given that she cannot trade asset B (which is the case in the incomplete market periods). The market has equal state-holdings in state X and state Y. Type I can only achieve this if she trades to 0 units of asset A. To see this, notice that in state Y, asset A pays 0, and type I has 0 units of asset B, which she cannot change. Hence, in state Y all her earnings come from risk-free sources (cash and bonds). Thus, in order to have the same holdings in state X, her earnings in state X must also come from risk-free sources only. This can only be achieved if she holds nothing of asset A. This means that type I must hold only bonds and cash (since she starts with nothing of asset B), which implies equal earnings in all three states. This yields the wrong relative state holdings for state Z.*

		State of the world		
		X	Y	Z
Type I	state-holdings	8	1	15
	rel. state-holdings	0.333	0.042	0.625
Type II	state-holdings	4	11	9
	rel. state-holdings	0.167	0.458	0.375
Market	state-holdings	6	6	12
	rel. state-holdings	0.25	0.25	0.5

Table 4: State holdings implied by the initial endowments of type I and type II participants, as well as state holdings implied by the market portfolio that obtains when there are equal numbers of participants of each type. Relative state holdings are the fraction of state holdings in one state over the sum of holdings in all states.

Example 7 (Trade to the market when “not Z” is announced). *The market gives equal state-holdings for states X and Y. From example 6 we know that this means that type I must hold 0 units of asset A, no matter how much she holds of bonds and cash. She starts with 14 units of asset A, which she must sell to achieve the same relative state-holdings as the market.*

To have the same relative state-holdings as the market when the announcement is “not Z”, agent type II must hold 16 units of asset A (double the amount he holds of asset B, and 8 times his initial endowment of A!). His holdings of bonds and cash are irrelevant.

When the announcement is “not Z”, in order to equal the market’s relative state holdings, both types must fine-tune their holdings of asset A with respect to their holdings of asset B, which are fixed. The following example shows that when the announcement is “not X”, the important relation is that of their asset A holdings with respect to their risk-free asset holdings (cash plus bonds).

Example 8 (Trade to the market when “not X” is announced). *The market holds twice as much in state Z as in state Y. Type I holds 0 of asset B, so all that matters are the returns to asset A in states Y and Z. Asset A yields 0 in state Y, and 1 – the same as the risk-free assets – in state Z. Knowing this, it is straightforward*

that she must hold the same number of units of asset A as of risk-free assets. Thus, depending on the price of asset A, type I must sell enough units of asset A to equal her holdings of cash and of asset A.

Type II holds 8 units of asset B, that give \$8 in state Y, and \$4 in state Z, which he cannot affect. Knowing this, it is easy to verify that his number of units of asset A must equal his number of units of risk-free assets plus 12. Depending on the price of asset A, he must buy asset A until this relation between bond and cash holdings and holdings of asset A is achieved.

The above examples convey how remarkable it is that one participant of either type trade to the market. We do not see many participants that do this, but we do see the average and median participant having the same state-holdings as the market. Moreover, participant state-holdings in incomplete market periods are indistinguishable from their holdings in complete market periods. These findings are in agreement with those of prior asset pricing experiments, and are compatible with correct state-price ranking within the framework of the CAPM+ ϵ model.

We display relative state-holdings for each announcement for all relevant periods, side by side in Figures 10 to 13. The figures support the findings mentioned above.

5 Conclusion

We have shown that state-price probability ranking according to theory obtains in experimental dynamically complete markets. Moreover, the data support the claim that this ranking obtains because participants are able to form the right expectations about future prices and the median participant trades towards the market portfolio.

It is important to stress that we design our experiments such that the above results obtain in an environment where participants can hold the market portfolio only if they re-trade after the announcement that completes the markets. Careful design of the experiment allowed us to recover state-price probabilities from this

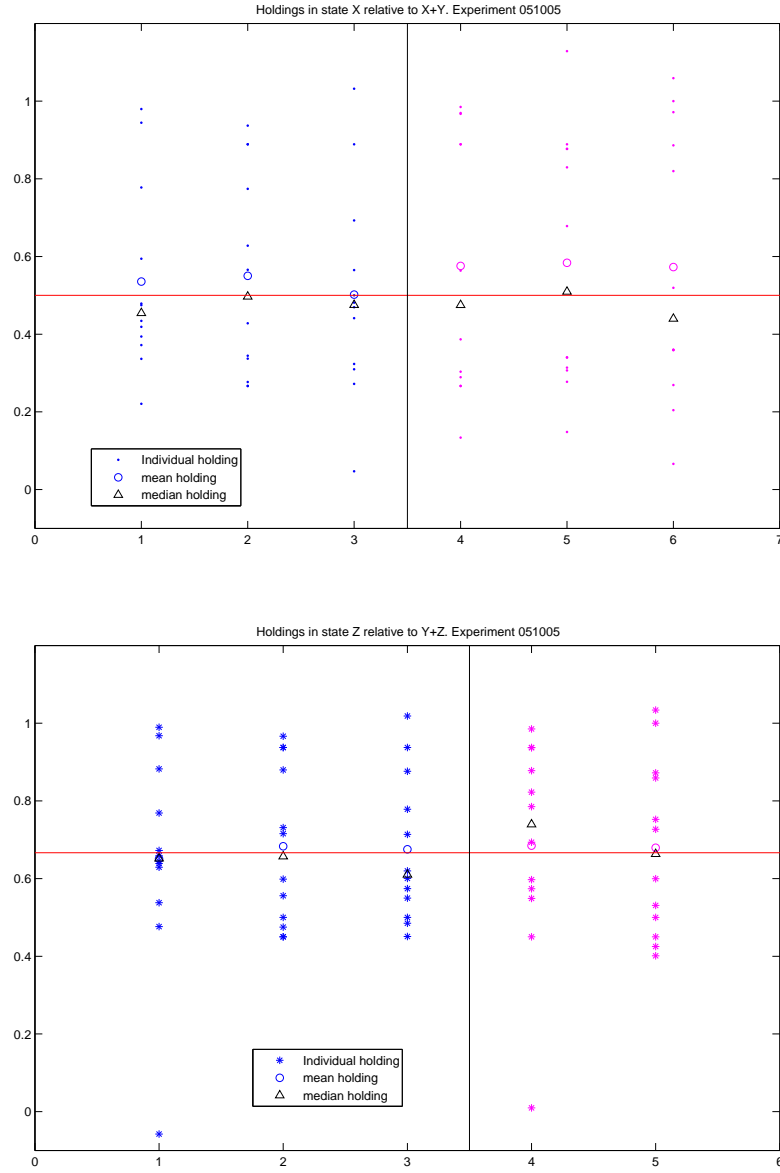


Figure 10: Relative state holdings for periods with announcement “not Z ” and announcement “not X ”. The same relative state holdings are computed for complete market periods and reported in the same figure for comparison. Periods left of the vertical line are complete market periods. The horizontal line represents the relative state holding given by the market portfolio.

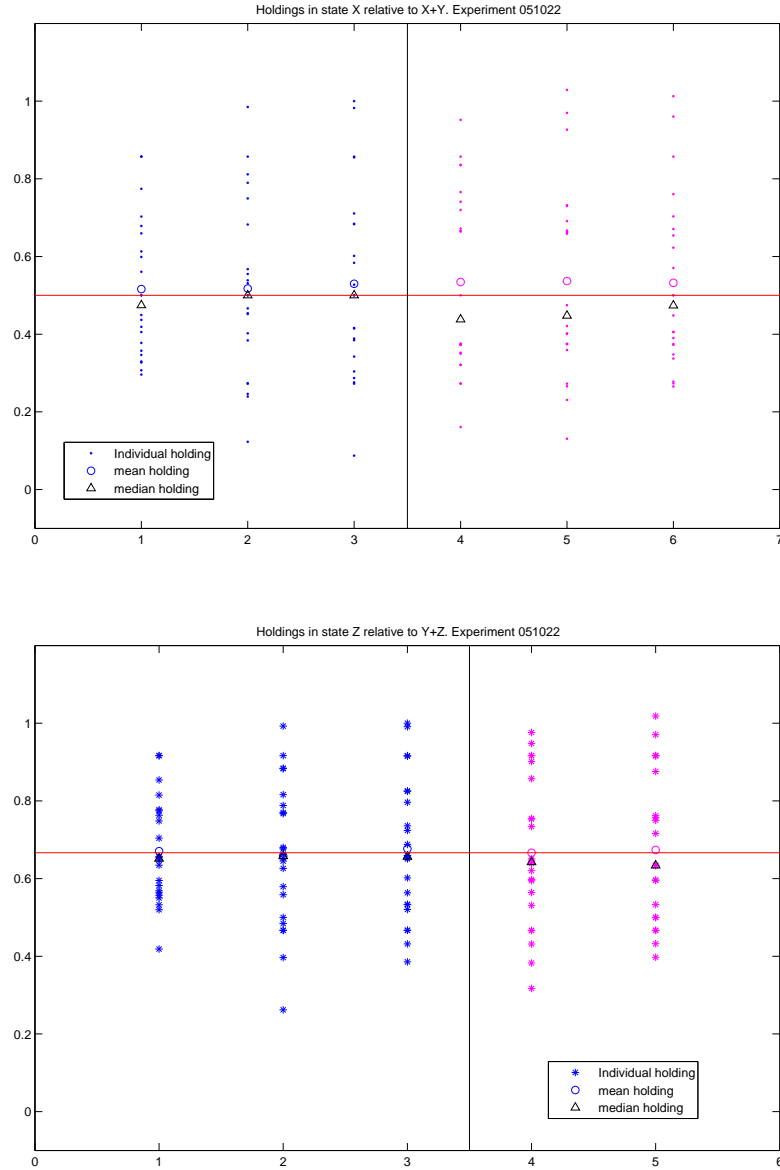


Figure 11: Relative state holdings for periods with announcement “not Z ” and announcement “not X ”. The same relative state holdings are computed for complete market periods and reported in the same figure for comparison. Periods left of the vertical line are complete market periods. The horizontal line represents the relative state holding given by the market portfolio.

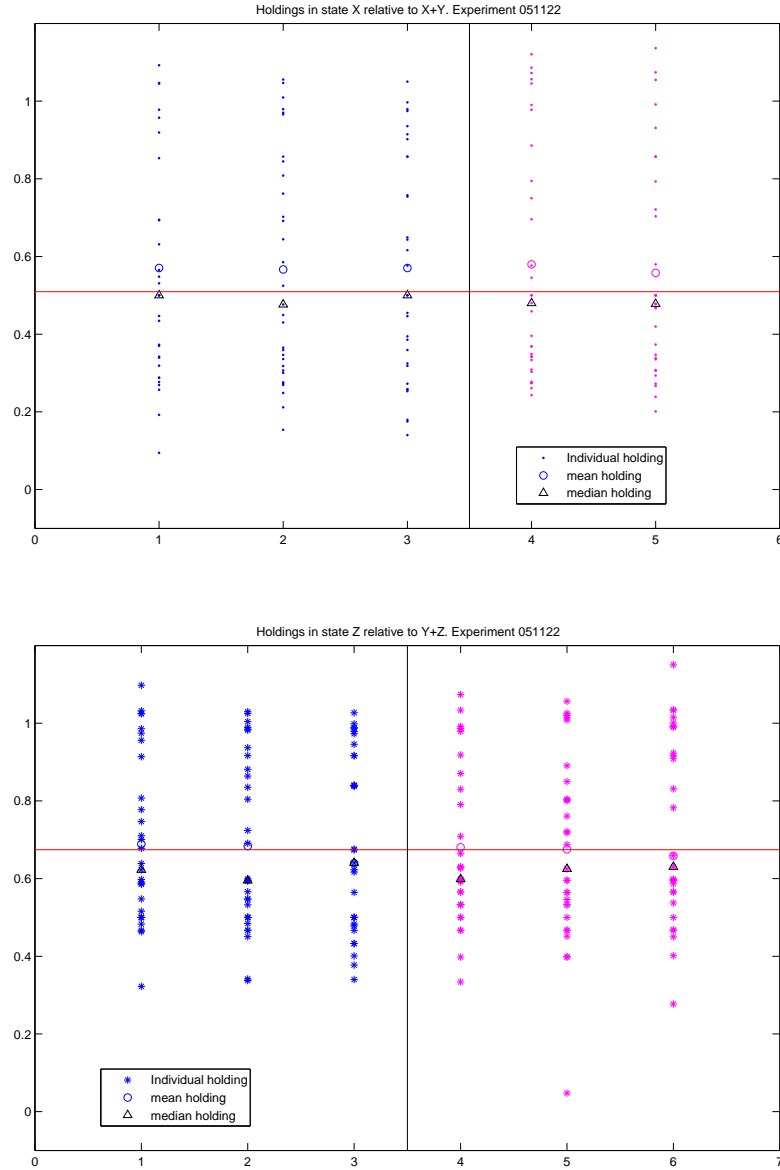


Figure 12: Relative state holdings for periods with announcement “not Z ” and announcement “not X ”. The same relative state holdings are computed for complete market periods and reported in the same figure for comparison. Periods left of the vertical line are complete market periods. The horizontal line represents the relative state holding given by the market portfolio.

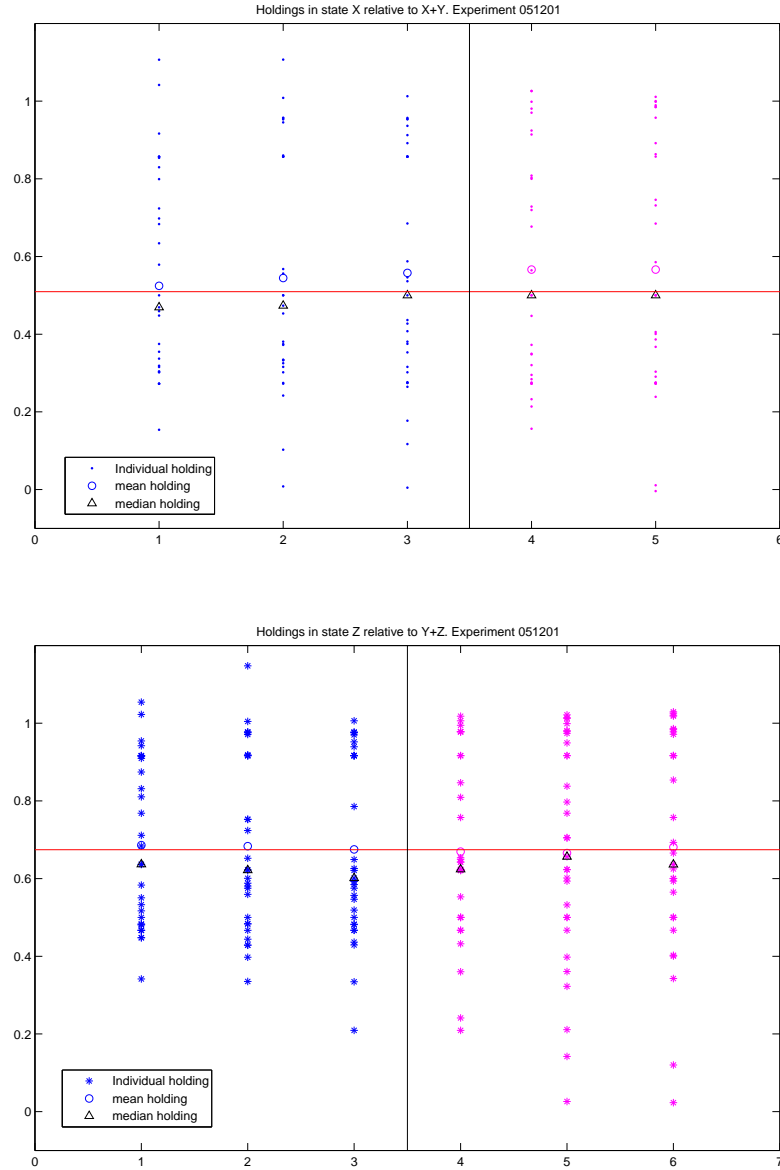


Figure 13: Relative state holdings for periods with announcement “not Z ” and announcement “not X ”. The same relative state holdings are computed for complete market periods and reported in the same figure for comparison. Periods left of the vertical line are complete market periods. The horizontal line represents the relative state holding given by the market portfolio.

environment where the theory is put to a very hard test.

Correct state-price probability ranking in the presence of median and mean trade towards the market portfolio is a result of the CAPM+ ϵ model proposed in Bossaerts, Plott, and Zame [2005] for a complete market environment. In view of our experimental results it is important to return to the theoretical framework and develop the dynamical structure necessary to evaluate the model with our data for dynamically complete markets.

Appendix

A Experimental Sessions

In this appendix we describe the details of the experimental sessions that we ran. We give statistics on trade and price behavior that are informative, but not central to the point we make in the paper.

Tables 5 and 6 contain statistics on trading volume. Table 5 contains per minute average trading volume for each experimental session. Because different experiments have different numbers of participants, the per minute values are divided by the number of participants and reported. The trading volume averages are taken over complete market periods, incomplete market periods *before* announcement, and incomplete market periods *after* announcement.

Table 6 reports the average number of trades that each participant is involved in, as well as the standard deviation across participants of this number (notice that each trade involves two participants, hence the discrepancy between numbers in the two tables, since Table 6 doubles the number of trades from Table 5). Standard deviation is computed for the experimental totals over complete market periods, and incomplete market periods, both before and after announcement. Naturally, standard deviations in single periods are significantly larger than standard deviations of the totals, which are reported here.

<i>Average TpM taken over:</i>	Experimental Session							
	051005		051022		051122		051201	
	<i>TpM</i>	$\frac{TpM}{\#subjects}$	<i>TpM</i>	$\frac{TpM}{\#subjects}$	<i>TpM</i>	$\frac{TpM}{\#subjects}$	<i>TpM</i>	$\frac{TpM}{\#subjects}$
Complete mkts.	9.6	0.8	14.2	0.65	28.4	0.9	21.6	0.7
Incomplete mkts. before announce	8	0.67	12.6	0.57	26.5	0.9	23	0.74
Incomplete mkts. after announce	5.8	0.48	8.8	0.4	19.3	0.6	16	0.52

Table 5: Average number of trades per minute (TpM) and number of trades per minute, per participant.

<i>Trades per subject in periods:</i>	Experimental Session							
	051005		051022		051122		051201	
	<i>Ave.</i>	<i>Std. Dev.</i>	<i>Ave.</i>	<i>Std. Dev.</i>	<i>Ave.</i>	<i>Std. Dev.</i>	<i>Ave.</i>	<i>Std. Dev.</i>
Complete mkts.	28.7	14.2	23.2	13.6	33	26.2	25	19.8
Incomplete mkts. before announce	26.7	15.4	22.9	16.7	34.1	28.9	29.7	27.3
Incomplete mkts. after announce	19.3	12.7	15.9	11.6	24.8	19.4	20.7	18

Table 6: Average and Standard deviation of the number of trades that a participant is involved in during an experimental session.

The evolution of trading prices of tradeable assets over time is summarized in Figures 14 to 17.

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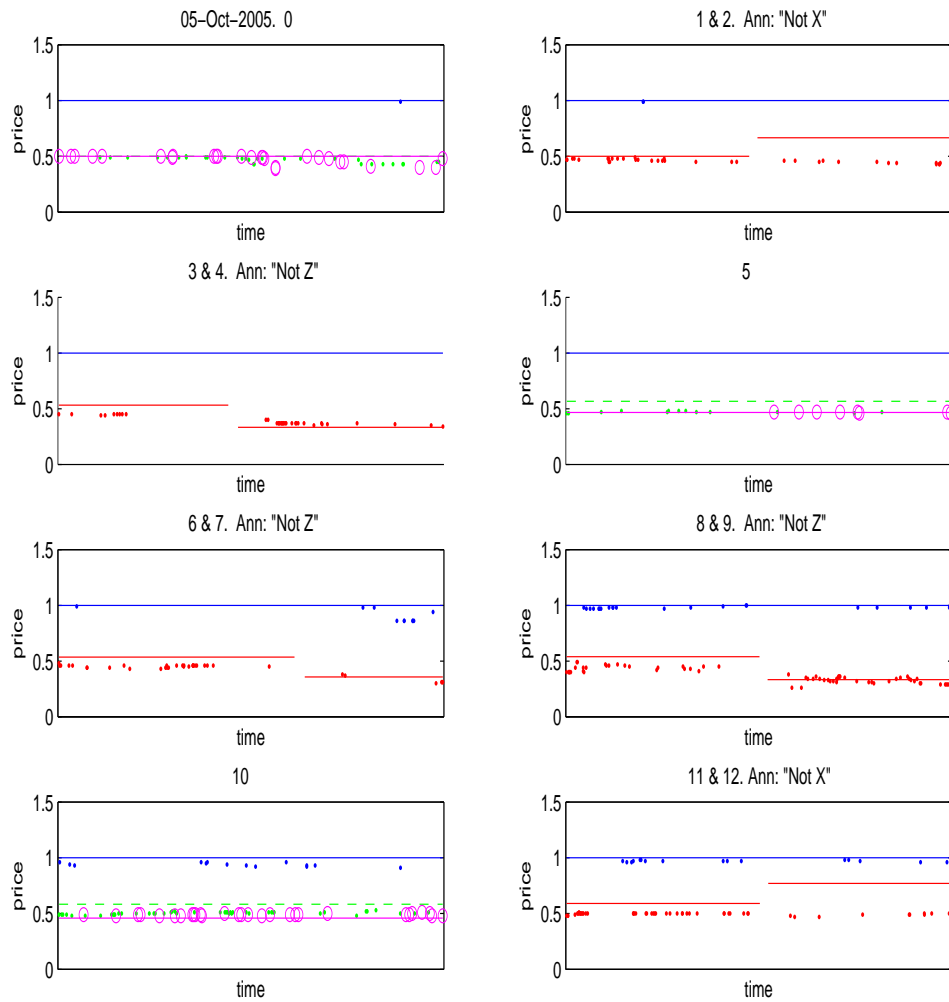


Figure 14: Price evolution in time for every period. Straight lines indicate expected values of security dividends, given the contents of the urn and the announcement in each period. In complete market periods, circles and a solid line correspond to asset A, while dots and a dashed line correspond to asset B.

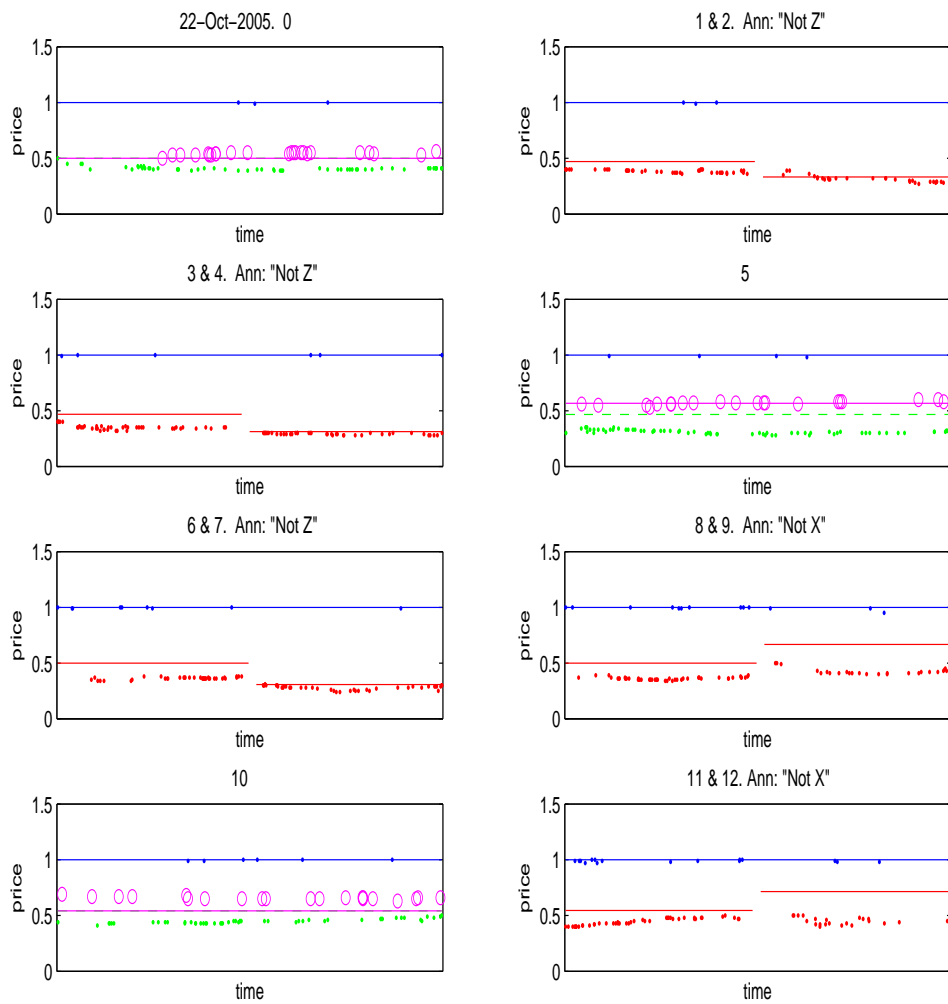


Figure 15: Price evolution in time for every period. Straight lines indicate expected values of security dividends, given the contents of the urn and the announcement in each period. In complete market periods, circles and a solid line correspond to asset A, while dots and a dashed line correspond to asset B.

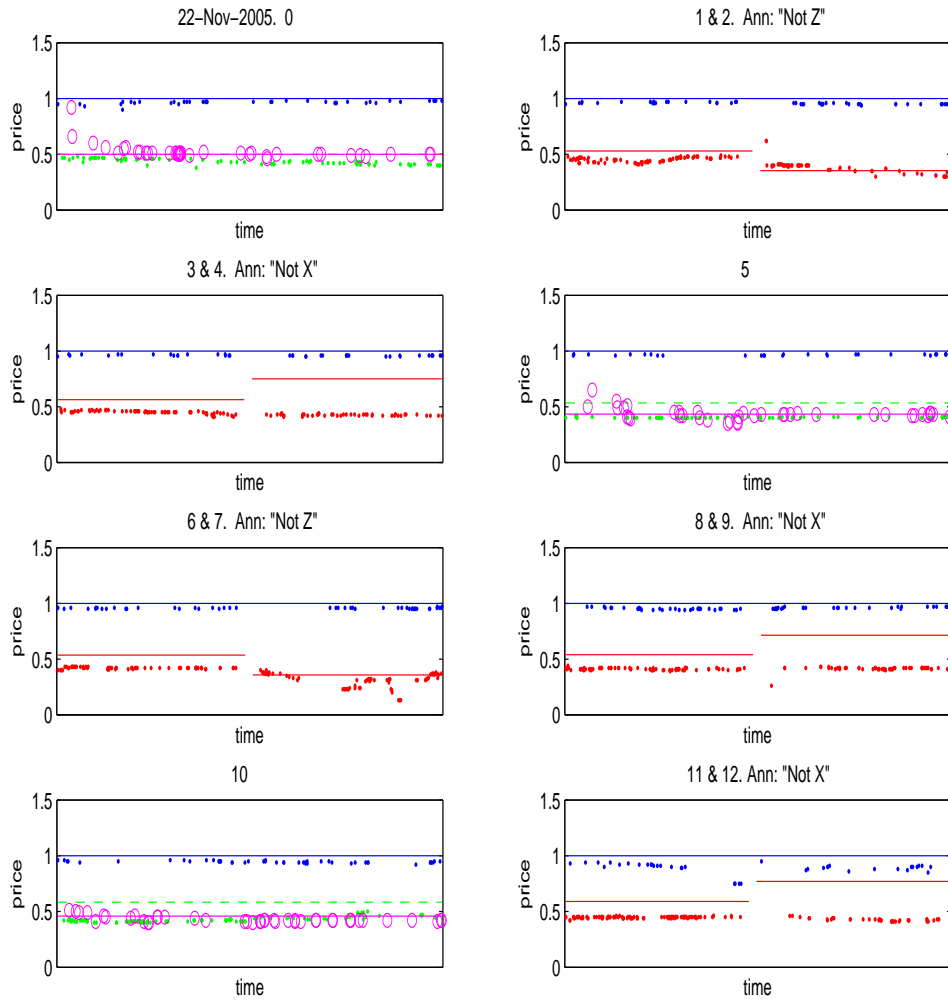


Figure 16: Price evolution in time for every period. Straight lines indicate expected values of security dividends, given the contents of the urn and the announcement in each period. In complete market periods, circles and a solid line correspond to asset A, while dots and a dashed line correspond to asset B.

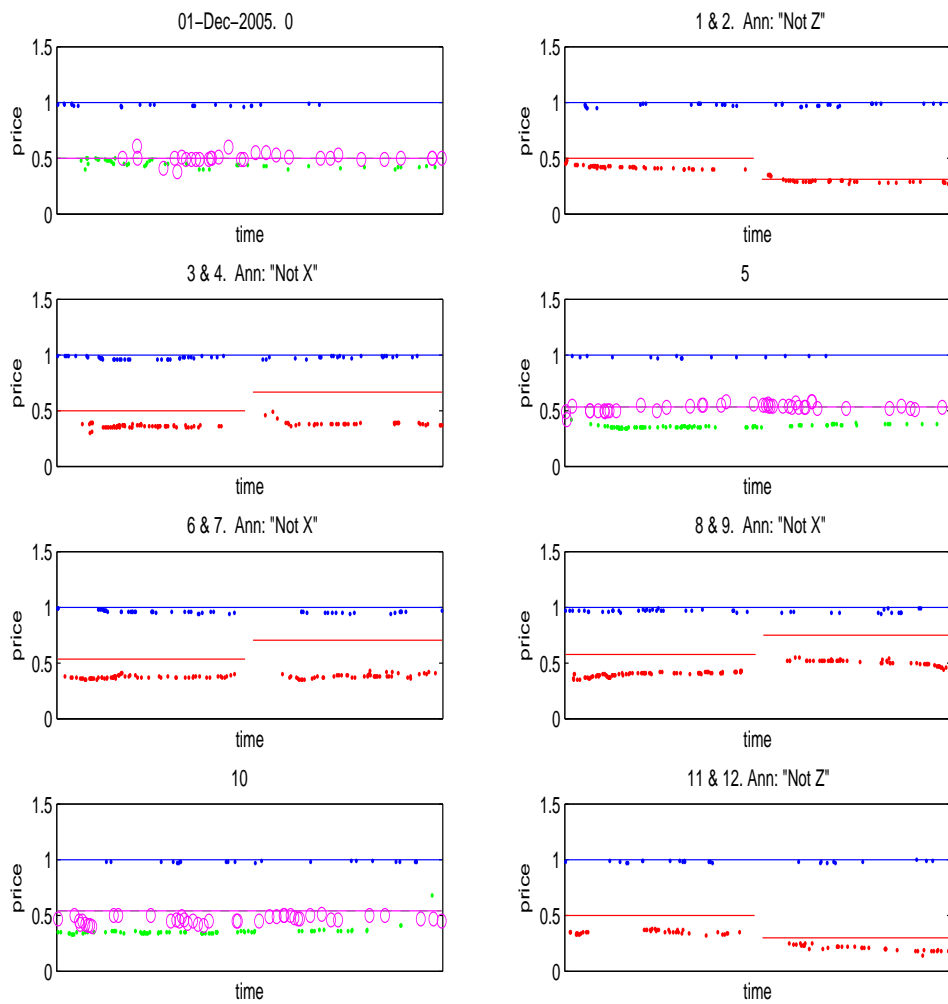


Figure 17: Price evolution in time for every period. Straight lines indicate expected values of security dividends, given the contents of the urn and the announcement in each period. In complete market periods, circles and a solid line correspond to asset A, while dots and a dashed line correspond to asset B.