Interest Rates Under Falling Stars

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Macro-finance theory implies that trend inflation and the equilibrium real interest rate are fundamental determinants of the yield curve. However, empirical models of the term structure of interest rates generally assume that these fundamentals are constant. We show that accounting for time variation in these underlying long-run trends is crucial for understanding the dynamics of Treasury yields and predicting excess bond returns. We introduce a new arbitrage-free model that captures the key role that long-run trends play in determining interest rates. The model also provides new, more plausible estimates of the term premium and accurate out-of-sample yield forecasts.
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Researchers have made numerous attempts to connect macroeconomic variables to the term structure of interest rates using a variety of approaches ranging from reduced-form no-arbitrage models to fully fledged dynamic macro models.[1] Despite both theoretical and empirical progress, there is no clear consensus about how macroeconomic information should be incorporated into yield curve analysis. One important link between the macroeconomy and the yield curve that has been largely overlooked is the connection between the time-varying long-run means of macroeconomic series and interest rates, that is, between their stochastic trends. Specifically, macroeconomic data and models can shed light on estimates of the trend in inflation (π_t^*) and the equilibrium real interest rate (r_t^*), while finance theory—from the Fisher equation to modern no-arbitrage theory—implies that such macroeconomic trends must be reflected in interest rates. Of course, the key outstanding issue is whether there is empirically significant variation in these long-run trends over time. Almost all term structure analyses in finance assume constant long-run means. Instead, in this paper, we document that accounting for changes in π_t^* and r_t^* is essential for modeling the term structure, estimating bond

    [1]See, among many others, Ang and Piazzesi (2003), Diebold, Rudebusch and Aruoba (2006), Bikbov and Chernov (2010), and Rudebusch and Swanson (2012). A detailed survey is Gürkaynak and Wright (2012).

risk premia, and forecasting yields. We quantify the importance of time-varying macroeconomic trends for interest rates in two steps. First, using a variety of empirical proxies for these shifting endpoints, we establish some stylized facts about macroeconomic and interest rate trends. One of these stylized facts is the importance of such trends for predicting excess bond returns. Second, to account for these stylized facts, we introduce a new dynamic term structure model that embeds a stochastic interest rate trend. This no-arbitrage model provides new estimates of the long-run stochastic trend in Treasury yields and the term premium and accurately predicts interest rates out of sample. This analysis bridges an important gap between macroeconomics and finance by incorporating shifting macro-finance trends.
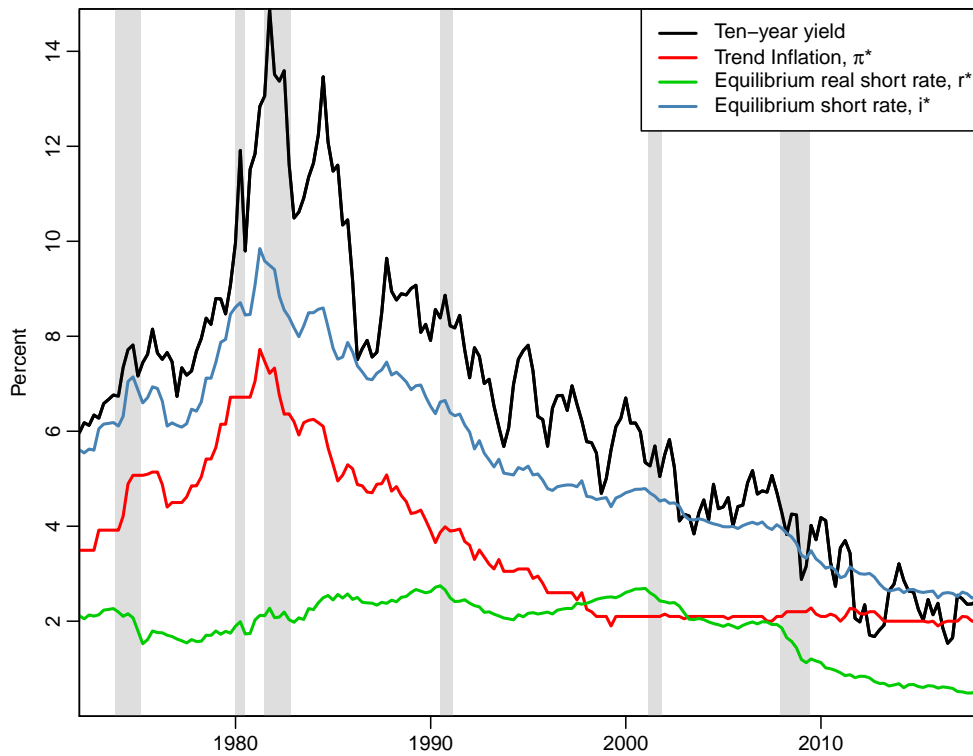
FIGURE 1. TEN-YEAR YIELD AND MACROECONOMIC TRENDS

Note: Ten-year Treasury yield and estimates of trend inflation, π_t^* (the mostly survey-based PTR measure from the Federal Reserve), the equilibrium real rate, r_t^* (an average of all filtered and real-time estimates, see Section I), and the equilibrium short rate, $i_t^* = \pi_t^* + r_t^*$. Shaded areas are NBER recessions. The data are quarterly from 1971:Q4 to 2018:Q1.

A simple illustration of the potential importance of macro trends for U.S. inter-

est rates is provided in Figure 1. The underlying drivers of the apparent secular decline in longer-term Treasury yields since the early 1980s remain contentious. Standard finance (no-arbitrage) models of the yield curve assume that interest rates are stationary, i.e., mean-reverting—see Kim and Wright (2005), Joslin, Singleton and Zhu (2011), and Adrian, Crump and Moench (2013), among many others. This mean reversion implies that expected short-term rates are invariably close to their historical average at horizons beyond a few years ahead. That is, the expectations component of long-term interest rates varies only modestly. Consequently, low-frequency variation in interest rates must be captured by the term premium, which is the residual difference between a long-term interest rate and its expectations component (e.g., Wright, 2011). Instead, our analysis stresses the important role of the time-varying trends underlying interest rates. To illustrate this role, Figure 1 displays observable proxies of these trends (described further in Section I. Trend inflation, π_t^*, measured using long-horizon survey forecasts, declined almost 6 percentage points from the Volcker disinflation of the early 1980s to the late 1990s and then remained stable around 2 percent. In contrast, the trend or equilibrium real interest rate, r_t^*—shown here as the average of different estimates—changed little early on but declined notably over the past two decades, likely reflecting structural changes in the global economy.[2] As a result, the equilibrium nominal short-term interest rate, $i_t^* = \pi_t^* + r_t^*$, which reflects movements in both underlying macro trends, accounts for much of the low-frequency movements in the ten-year Treasury yield through variation in the expectations component. This close connection suggests that the early downward trend in π_t^* and the more recent decline in r_t^*—that is, an environment of falling stars—played a major role in the secular decline in nominal interest rates.

This link between macro trends and interest rate trends is at the core of our analysis. We first establish two stylized facts about macro-finance trends and yields using various observable proxies for these trends. First, variation in both the inflation trend and the equilibrium real interest rate is responsible for the very high persistence of interest rates. It has long been recognized that nominal interest rates of different maturities contain a slow-moving common trend component and are best modelled as cointegrated (Campbell and Shiller, 1987; Hall, Anderson and Granger, 1992). We show that this common trend is i_t^* and that it is driven by quantitatively important fluctuations in both underlying macroeconomic trends, π_t^* and r_t^*. Accounting only for the inflation trend on its own, as in Kozicki and Tinsley (2001) and Cieslak and Povala (2015), leaves a highly persistent component of interest rates unexplained. We show that it is necessary to include movements in r_t^* to fully capture the trend component in interest rates.

Our second stylized fact is that macro trends are crucial for understanding bond risk premia. Relative to the canonical predictive regressions for excess bond

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[2] We use existing estimates of r_t^* from Laubach and Williams (2003, 2016), Kiley (2015), Holston, Laubach and Williams (2017), Del Negro et al. (2017), and Johannsen and Mertens (2016, 2018). For robustness, we also consider four r_t^* estimates that we create, which range from a simple moving-average of the real rate to a multivariate estimate.

returns that forecast future excess bond returns with current yields (Fama and Bliss, 1987; Campbell and Shiller, 1991; Cochrane and Piazzesi, 2005), we find that adding π_t^* and r_t^* as regressors provides notable gains in predictive power. Our results provide evidence against the *spanning hypothesis* implied by most asset pricing models, which posits that the current yield curve contains all the relevant information for predicting future interest rates (e.g., Duffee, 2013; Bauer and Rudebusch, 2017; Bauer and Hamilton, 2018). The predictive gains we find are economically large, so, for example, when interest rates adjust to a lower trend, long-term bond holders benefit substantially, just as they did during the adjustment to a lower r_t^* following the 2008 financial crisis. Indeed, including the equilibrium real rate is especially important later in our sample when the inflation trend shows less variation.[3]

To account for these two stylized facts, we introduce a new dynamic term structure model (DTSM) with a shifting endpoint for interest rates. This novel no-arbitrage specification parsimoniously models the Treasury yield curve using four state variables: three "yield factors" which are simply linear combinations of nominal bond yields, and one common stochastic trend, i_t^*, which captures movements in the underlying macroeconomic trends. The existence of a common trend in interest rates stands in stark contrast to the usual stationary DTSMs in which yield factors follow a stationary VAR(1). Furthermore, in our model, the trend is *unspanned* or hidden, so it cannot be backed out from the cross section of yields. This specification is consistent with our evidence showing that adding i_t^* improves predictions of bond returns. In addition, an unspanned i_t^* need not follow Treasury yields in lockstep and can play a similar role as the trend component in an unobserved components model. For robustness, we employ two different estimation methodologies, which provide similar results. First, we estimate the model using an observed proxy for i_t^* in order to carry out a model-based analysis using the same real-time interest rate trend we employed in establishing the stylized facts about macro-finance trends. Second, we use Bayesian estimation of our model to infer i_t^* using only Treasury yields, imposing a certain degree of smoothness on our trend estimate, similar to Del Negro et al. (2017).

We then show that simulated data from these empirical specifications are able to match the stylized facts that we established about the role of trends for the yield curve. The model-based estimates of the trend and cycle components of interest rates are consistent with the cointegration regression results from the actual data. Also, the model accounts for the additional predictive power for future bond returns provided by the trend underlying interest rates as was evident in our historical data sample. Specifically, predictive regressions using simulated data from our model demonstrate notable increases in R^2 when the interest rate trend is added as a predictor—just as in the actual data.

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[3]The improvement we document using trends to predict excess bond returns dovetails with recent research (e.g., Cieslak and Povala, 2015; Brooks and Moskowitz, 2017; Garg and Mazzoleni, 2017; Jorgensen, 2017) showing predictive gains from including slow-moving averages of past inflation, real output growth, or consumption growth as predictors—variables that should be closely related to π_t^* and r_t^*.

We also use our shifting-endpoint model to investigate two important applications of DTSMs: (1) decomposing long-term interest rates into an expectations component and a term premium and (2) forecasting interest rates. For the former, our model provides markedly different decompositions of long-term rates than the usual stationary DTSM. A stationary DTSM implies a quite stable expectations component and attributes most of the secular decline in interest rates to the residual term premium, as discussed in critiques by Kim and Orphanides (2012) and Bauer, Rudebusch and Wu (2014). Instead, our shifting-endpoint model decomposition attributes the majority of that secular decline to a fall in long-term expectations driven by i_t^*. Consequently, our term premium estimates exhibit only a modestly decreasing trend and more pronounced cyclical swings, similar to other risk premia in asset prices (Fama and French, 1989). As for out-of-sample interest rate forecasting, previous researchers have struggled to find models that consistently outperformed the random walk forecast, which simply predicts that a future yield will equal the current yield. But we find that forecasts implied by our shifting-endpoint DTSM are generally more accurate than this martingale benchmark and are also better than long-range projections from the Blue Chip survey of professional forecasters. By contrast, the standard stationary DTSM (corresponding to the special case of our model with a constant i^*) forecasts substantially worse than the random walk. The improvements in forecast accuracy from allowing a slow-moving shifting endpoint are both economically and statistically significant, and they are consistent with yields returning over time to their underlying macro trend.[4]

Our analysis is related to several different strands of the macro and finance literatures. Some studies have described a link between the inflation trend and nominal yields (Kozicki and Tinsley, 2001; van Dijk et al., 2014; Cieslak and Povala, 2015), but this leaves unexplained the continuing decline in yields over the past 20 years. Time variation in r_t^* has so far received little attention in finance, despite growing evidence of a sizable decline in the equilibrium real interest rate.[5] Our paper comprehensively documents the empirical importance of both relevant macro trends, π_t^* and r_t^*, for the dynamics of the yield curve.

No-arbitrage models of the yield curve typically rule out time-varying trends by specifying that the dynamics of yields are stationary. However, such models are ill-suited to capture the extreme persistence of interest rates, and a variety

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[4]In related work, van Dijk et al. (2014) documented forecast improvements relative to a random walk by including shifting endpoints based on their proxy of π_t^*. We demonstrate how to achieve even greater forecast gains in an arbitrage-free DTSM framework that accounts for time-varying π_t^* and r_t^*.

[5]The economic forces reducing the equilibrium real interest rate likely include lower productivity growth, changing demographics, a decline in the price of capital goods, and strong precautionary saving flows from emerging market economies, which have tended to increase global savings, reduce desired investment, and push down the steady-state real interest rate. Discussions include Summers (2014), Kiley (2015), Rachel and Smith (2015), Hamilton et al. (2016), Laubach and Williams (2016), Johannsen and Mertens (2016, 2018), Christensen and Rudebusch (2019), Del Negro et al. (2017), Holston, Laubach and Williams (2017) and Lunsford and West (2019). In macroeconomics, r_t^* is often labeled the neutral or natural rate of interest although, as noted below, there can be subtle dfferences among various definitions.

of remedies have been suggested.[6] Our results suggest that allowing for shifting long-run trends is a better solution. The idea of a shifting-endpoint term structure model goes back to Kozicki and Tinsley (2001), who specified a restrictive two-factor DTSM that imposed the expectations hypothesis and identified i_t^* with a distant-horizon forward rate (with a unit root under both the risk-neutral and the real-world measure). Some macro-finance models of the yield curve allow for changes in the inflation trend but still assume a constant equilibrium real rate (Hördahl, Tristani and Vestin, 2006; Rudebusch and Wu, 2008). The model of Campbell, Sunderam and Viceira (2017), which is designed to capture the changing comovement of stock and bond returns, is an asset pricing model with a time-varying π_t^* and constant r^*. A few no-arbitrage models allow for changes in r_t^*, but they also make strong assumptions about this trend by tightly linking it to π_t^* (Dewachter and Lyrio, 2006) or trend output growth (Dewachter and Iania, 2011). Our model allows both stochastic macroeconomic trends to affect term structure dynamics. We also depart from the existing literature by treating i_t^* as unspanned, which allows this trend to exhibit some variation independent of contemporaneous yields.

Three recent papers propose statistical models of interest rates with time-varying long-run means and are thus closely related to ours. Del Negro et al. (2017) estimate a Bayesian common-trend VAR for interest rates and inflation that allows for time variation in π_t^* and r_t^*. We use their estimates of r_t^*, along with others, in our analysis establishing stylized facts. Johannsen and Mertens (2018) also propose a time series model for yields and a Bayesian estimation framework that allows for stochastic volatility, and we also include their real-time r_t^* among our trend proxies. Finally, Crump, Eusepi and Moench (2018) allow for shifting macroeconomic trends in a common-trend VAR for interest rates and inflation with i_t^*, r_t^*, and π_t^* pinned down from surveys of professional forecasters. These three papers provide complementary time series analyses to our approach. However, our arbitrage-free specification, which jointly captures the time series and cross section of Treasury yields, allows us to investigate the role of trends for the yield curve, the term premium and expected bond returns.

The paper is structured as follows. Section I describes key concepts and our initial proxies for the macroeconomic trends. Section II presents stylized facts about the macroeconomic trends and Treasury yields. Section III introduces our no-arbitrage model, shows how it captures the stylized facts, and presents novel estimates of the term premium as well as out-of-sample forecast results. Section IV concludes. The Appendix contains further details, and a separate Online Appendix[7] contains supplemental material.

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[6]Proposed remedies include adding survey information (Kim and Wright, 2005; Kim and Orphanides, 2012), restricting risk prices (Joslin, Priebsch and Singleton, 2014; Bauer, 2018), correcting for small-sample bias (Bauer, Rudebusch and Wu, 2012, 2014), and using near-integration or long memory processes (Jardet, Monfort and Pegoraro, 2013; Goliński and Zaffaroni, 2016).

[7]The Online Appendix is available at https://www.michaeldbauer.com/files/br_trends_online_appendix.pdf.

I. Interest rate trends: theory and estimates

The role of trends for interest rates can be illustrated with the usual decomposition of any n-period yield, $y_t^{(n)}$, into expectations of short-term rates and a term premium, $TP_t^{(n)}$:

$$(1) \qquad y_t^{(n)} = \frac{1}{n}\sum_{j=0}^{n-1} E_t i_{t+j} + TP_t^{(n)} = i_t^* + \frac{1}{n}\sum_{j=0}^{n-1} E_t i_{t+j}^c + TP_t^{(n)},$$

where i_t is the nominal short rate, $y_t^{(1)}$.[8] The term premium compensates bond investors for the duration risk in longer-term bonds, though in practice, it also captures liquidity and financial market frictions. The expectations theory of interest rates assumes that the term premium is zero (strong form) or constant (weak form), whereas modern finance theories of interest rates explicitly model the time variation in the term premium, as described in Section III. The second equality in equation (1) introduces the trend or equilibrium short rate, i_t^*, and the short rate cycle, $i_t^c = i_t - i_t^*$. We define i_t^* as the Beveridge-Nelson trend in the short-term nominal interest rate,

$$(2) \qquad\qquad i_t^* \equiv \lim_{j\to\infty} E_t i_{t+j}.$$

We employ this stochastic trend concept throughout this paper and assume no deterministic time trends. All nominal yields are affected by changes in i_t^*, and if this trend is time-varying, then yields are nonstationary with a common trend. In this case, Kozicki and Tinsley (2001) fittingly call i_t^* a "shifting endpoint." The Fisher equation suggests that two macroeconomic trends determine this endpoint:

$$i_t = r_t + E_t \pi_{t+1} \quad \Rightarrow \quad i_t^* = r_t^* + \pi_t^*,$$

where r_t is the real short rate, π_t is inflation, and the trends in r_t and π_t are defined analogously to i_t^*. If inflation or the real interest rate contain long-run trend components, then nominal interest rates will also be driven by these same macroeconomic trends.

The remainder of this section describes the empirical proxies of these trends that we employ to establish stylized facts about macro-finance trends in interest rates.[9] For the long-run inflation trend, the prevailing consensus in empirical macroeconomics models inflation as an $I(1)$ process—in order to produce accurate forecasts and correctly capture the evolution of inflation expectations (Stock and

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[8]This intuitive definition of the term premium—the expected excess log return on the long-term bond held to maturity over a strategy of rolling over short-term bonds—is common in macroeconomics. It differs from the definition used in DTSMs by a Jensen inequality term, but in homoskedastic models such as the one proposed in this paper, this variance correction term is a constant.

[9]The Appendix provides an overview of all trend estimates.

Watson, 2007; Faust and Wright, 2013). From a macroeconomic perspective, the resulting time-varying π_t^* can be viewed as the perceived inflation target of the central bank. Empirical proxies for trend inflation have been constructed from surveys, statistical models, or a combination of the two—see, for example, Stock and Watson (2016) and the references therein. We employ a well-known, widely used survey-based measure of long-run inflation expectations, the Federal Reserve's series on the perceived target rate (PTR) for inflation.[10] Figure 1 shows that, from the beginning of our sample to the late 1990s, this estimate generally mirrored the rise and fall of the ten-year yield.[11] Since then, however, it has remained close to 2 percent, which has been the explicit longer-run inflation goal of the Federal Reserve since 2012. Given our focus on forecasting, real-time availability is an important issue, and for most of our sample (since 1979) the PTR estimate was available in real time from the Survey of Professional Forecasters.

As a proxy for r_t^*, we use a variety of estimates to avoid reliance on one specific model or estimation approach. All of these estimates are consistent with our definition of r_t^* as the stochastic trend in the real short rate, analogous to equation (2). The first two r_t^* estimates are the long-run real rate trends from time series models identified with Bayesian methods: Del Negro et al. (2017)(DGGT) estimate a linear state space model with common trends, and Johannsen and Mertens (2016, 2018)(JM) employ a nonlinear state space model with a shadow rate. Three additional estimates are based on simple New Keynesian macro models and use the Kalman filter to infer the *neutral* real interest rate, that is, the level of the real rate consistent with real output at potential and inflation at target: Laubach and Williams (2003, 2016) (LW) implemented this concept empirically using the Rudebusch and Svensson (1999) model; Holston, Laubach and Williams (2017) (HLW) use a slight simplification of the LW model; and Kiley (2015) augments the LW model to account for changes in financial conditions. The implementation of these macro models assumes that the neutral rate follows a martingale; thus, their r_t^* estimates are consistent with the long-run concept we employ. We also construct three model-based estimates of our own. The first is based on a simple univariate unobserved components model, and the second uses the same model augmented with moving averages of real GDP and labor force growth that appear to be correlated with the real rate trend (Lunsford and West, 2019). The third estimate uses the observed nominal short rate and inflation in a state-space model with a latent real rate and r_t^*. We estimate these models using Bayesian methods, and like Del Negro et al. (2017), we employ a prior with a low innovation variance for r_t^*. Finally, as a simple, model-free estimate of r_t^*, we include an exponentially-weighted moving average of past real rates.[12]

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[10]PTR is used in the Fed's FRB/US model, see Federal Reserve Board of Governors (2019*b*).

[11]Other estimates of the inflation trend, such as the long-range forecasts in the Blue Chip survey, the moving-average estimate of Cieslak and Povala (2015), or various model-based estimates (Clark and Doh, 2014; Stock and Watson, 2016) exhibit a similar pattern.

[12]Our smoothing parameter of 0.98 follows other research estimating macro trends (Malmendier and Nagel, 2011; Orphanides and Williams, 2005).
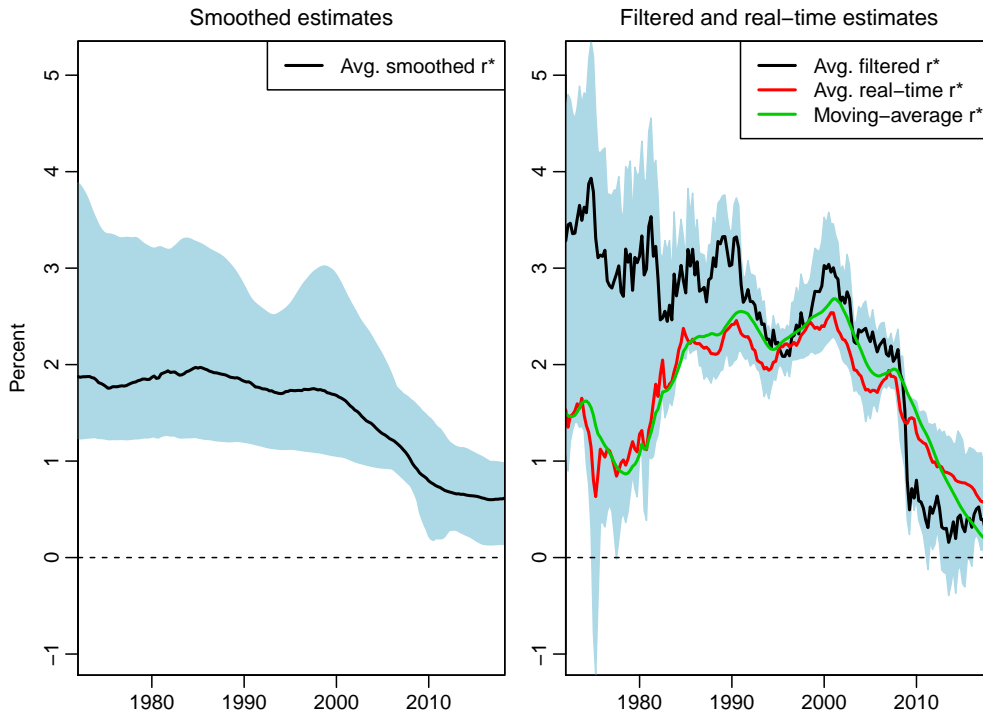
FIGURE 2. MEASURES OF THE EQUILIBRIUM REAL INTEREST RATE

Note: Smoothed (two-sided), filtered (one-sided), and real-time estimates of the equilibrium real interest rate, r_t^*. Left panel: range and average of seven smoothed estimates, including the four estimates by Del Negro et al. (2010) (DGGT), Johannsen and Mertens (2018) (JM), Laubach and Williams (2016) (LW), and Kiley (2015), as well our own three model-based estimates. Right panel: range and average of filtered and real-time estimates. Filtered estimates are the estimates of LW, Holston, Laubach and Williams (2017), and Kiley. Real-time estimates are those of DGGT, JM, our own three real-time model-based estimates, and the model-free moving-average estimate, which is also shown separately. For details, see Section I and the Appendix. The data are quarterly from 1971:Q4 to 2018:Q1.

Figure 2 summarizes these r_t^* estimates. The left panel shows the average and the range of the smoothed (two-sided) estimates, which use the full data sample for inference about the parameters and state variables. These show a relatively stable r_t^* until the late 1990s, but then exhibit a pronounced decline. To better capture what bond market investors could have known about macro trends contemporaneously, our analysis will focus on filtered (one-sided) and real-time estimates of r_t^*, which are shown in the right panel of Figure 2. Filtered estimates—from the LW, HLW, and Kiley models—infer latent state variables at time t using data available up to time t. However, they also use model parameter estimates from the full sample, which would not have been available to investors in real time. Therefore, to avoid the pitfall of look-ahead bias in our analysis of predictive power, we consider (pseudo) real-time estimates that only use data through period t to construct r_t^*. Such real-time estimates are available for the JM and DGGT models, our own three models, and the moving-average of past real rates.[13] As shown in Figure 2, the average of these six real-time estimates increased from about one to two percent early in the sample, moved sideways for about 15 years, and then around 2000 started a substantial, steady decline until the end of the sample. This pattern closely follows the movements of the moving-average estimate of r_t^*, which has the added benefit of not relying on a model or estimation technique unavailable to investors historically. Finally, the average of all nine filtered and real-time r_t^* estimates, which is displayed in Figure 1, also shows a similar pattern.

Substantial estimation and model uncertainty underlie estimates of any long-run trends, as suggested by the range of estimates in Figure 2. To reduce the uncertainty inherent in individual estimates, we take averages of r^* estimates across models. While this approach is not a panacea for estimation uncertainty and measurement error, model-averaging is well-known to improve forecast accuracy (Timmermann, 2006).[14] Our baseline estimate of i_t^* for the analysis in Section II will be the sum of the PTR estimate of π_t^* and the average of all six real-time estimates of r_t^*.[15]

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[13]These estimated trends all rely on revised inflation data, and the multivariate model also uses revised output data. Although the use of real-time data vintages can be important for near-term forecasting, it is arguably less relevant for estimation of long-run trends. Also, Laubach and Williams (2016) argue that, for the past decade, their truly real-time r_t^* estimates have turned out to be similar to filtered estimates.

[14]Because we find a close link between the yield curve and our noisy estimates of the macro trends, our results are likely somewhat conservative regarding the tightness of the connection between the yield curve and the *true* underlying trends (as discussed in the Online Appendix).

[15]An alternative proxy for i_t^* could be obtained from surveys, such as the long-range forecast for the three-month T-bill rate from the Blue Chip Financial Forecasts. This series, which is only available semiannually starting in 1986, fluctuates around our baseline estimate of i_t^*. We consider survey forecasts in Section III.F.

II.   Macro trends and yields: stylized facts

A.   *The common trend in nominal yields*

We first investigate whether macroeconomic trends account for the high persistence of interest rates. We then assess whether accounting for just the inflation trend is sufficient or if a real rate trend also materially contributes to movements in nominal bond yields. Our proxies for these macro trends are the PTR survey-based estimate of π_t^* and the three alternative estimates of r_t^*: the average of three filtered estimates, the average of the six real-time estimates, and a simple (exponentially weighted) moving average of past real rates. We proxy for the equilibrium nominal short rate i_t^* with the sum of π_t^* and the real-time average estimate of r_t^* (but obtain similar results with the filtered or moving-average measures).

We find that variation in the macro trends accounts for the persistence of interest rates, and the trend and cycle components of interest rates can be identified using our proxy i_t^*. This is illustrated in Figure 3, which shows the (demeaned) ten-year yield and two detrended versions of that yield.[16] The first detrended yield is the simple difference between the yield and i_t^*, which should be stationary if the yield is indeed driven by this trend and the loading on i_t^* is unity. In comparison to the ten-year yield, this series exhibits much less of a trend, for example, declining by less than half as much from the early 1980s to the end of our sample. Still, a notable trend remains in this simple difference, which indicates that the yield trend component moves more than one-for-one with i_t^*. By definition, the expectations component of yields moves one-for-one with i_t^*, so the ten-year term premium must contain a trend component positively related to i_t^*. To account for this relationship, the second detrended series—discussed in detail below—is the residual from a (Dynamic OLS) cointegration regression of the yield on i_t^*. This cointegration residual shows no discernible trend, indicating that changes in i_t^*, properly scaled, can fully capture the trend in the ten-year Treasury yield. The fitted value of the cointegration regression is the trend component of the ten-year yield, for which we also obtain a model-based estimate in Section III.C, and the residual—shown in Figure 3—corresponds to the interest rate cycle.

It has long been known that nominal interest rates are highly persistent.[17] Summary statistics and unit root tests reported in the Online Appendix confirm that Treasury yields are best modeled as having a stochastic trend in our sample. We examine whether this persistence is driven by a common macro trend. Figure 3 suggests a likely cointegration relationship with a loading on the trend larger than unity. To formally investigate this relationship, including the individual

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[16]We use end-of-quarter zero-coupon Treasury yields from Gürkaynak, Sack and Wright (2007) with maturities from one to 15 years and three-month and six-month Treasury bill rates from the Federal Reserve's H.15 release (Federal Reserve Board of Governors, 2019a), over the period from 1971:Q4 to 2018:Q2.

[17]See, for example, Nelson and Plosser (1982), Rose (1988), and King et al. (1991).
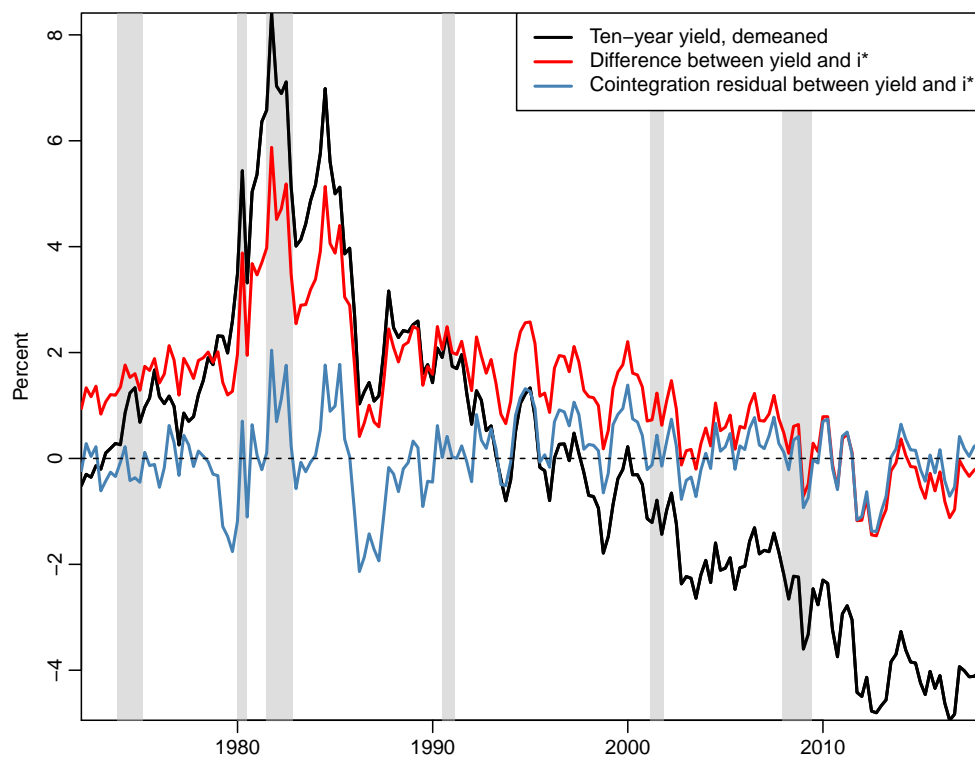
FIGURE 3. DETRENDING THE TEN-YEAR YIELD

Note: Ten-year Treasury yield and two detrended yield series using as the proxy for the equilibrium interest rate i_t^* the sum of the PTR estimate of π_t^* and the real-time estimate of r_t^* shown in Figure 2. The first detrended yield series is the difference between the yield and i_t^*. The second detrended series is the residual from the (Dynamic OLS) cointegration regression of the yield on i_t^*, estimates of which are reported in the last column of Table 1. Shaded areas are NBER recessions. The data are quarterly from 1971:Q4 to 2018:Q1.

contributions of π_t^* and r_t^*, we use cointegration regressions of the form $y_t^{(40)} = \beta_0 + \beta_1' X_t^* + u_t$, where $y_t^{(40)}$ is the ten-year yield, the vector X_t^* contains one or two macro trend proxies, and u_t is a stationary residual under cointegration. We estimate the coefficients β_0 and β_1 using the Dynamic OLS estimator of Stock and Watson (1993) by including four leads and lags of $\Delta y_t^{(40)}$ and ΔX_t^*.[18]

Table 1 shows the results of our cointegration analysis, including coefficient estimates, Newey-West standard errors (using six lags) and the regression R^2. The first regression specification includes only π_t^*, and the results echo those of Cieslak and Povala (2015), who used simple OLS regressions but a similar inflation trend proxy. The inflation trend coefficient is somewhat higher than one and strongly significant, and the R^2 is very high. However, trend inflation does not fully account for the trend behavior of yields, as evident from the persistence of the cointegration residuals $\hat{u}_t = y_t^{(40)} - \hat{\beta}_0 + \hat{\beta}_1 \pi_t^*$. While $\hat{u}_t$ is much less persistent than the ten-year yield, it is still quite persistent. As shown in the second column, for this series, unit root tests—the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests—do not reject the null, and the low-frequency stationary test of Müller and Watson (2013) rejects stationarity at the 5 percent level.[19] The inadequacy of detrending with only π_t^* is also supported by a Johansen (1991) trace test, which does not reject the null of absence of cointegration among the yield and the inflation trend.[20] Finally, an error-correction model, where $\Delta y_t^{(40)}$ is regressed on $\hat{u}_{t-1}$ and four lags each of the first-differenced yield and the inflation trend, indicates only modest equilibrating forces pushing yields toward the inflation trend.

The missing driver of the level of the yield curve is the real rate trend. The remaining columns of Table 1 show that there is strong evidence for cointegration once r_t^* is added. For all three r_t^* proxies, the estimated coefficients are at or above one and strongly significant. Adding the real rate trend increases the R^2 by 8 to 11 percentage points, and the cointegration residual now appears to be stationary. For the real-time and moving-average estimates of r_t^*, the Johansen test indicates a cointegration rank of one, and the error-correction model shows that yield dynamics quickly eliminate deviations from trend. The coefficients on the two macro trends are roughly equal, and the last column of Table 1 shows that i_t^* can fully capture the trend component in the ten-year yield, since the evidence for cointegration is as strong as when both macro trends are included individually. The cointegration coefficient is 1.67, suggesting a positive loading of the term premium on i_t^*, a result replicated by our model in Section III.C. The error-correction coefficient is estimated as -0.45 and is strongly significant.

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[18]In the Online Appendix, we report results for the level of the yield curve, which are similar to those for the ten-year yield. Also, standard OLS as well as the reduced-rank maximum likelihood VAR estimation of Johansen (1991) lead to the same conclusions as Dynamic OLS (not reported).

[19]For the unit root tests of cointegration residuals, we omit intercepts in the ADF and PP regressions and use the critical values provided by Phillips and Ouliaris (1990), which vary with the number of regressors (as detailed in the Online Appendix.)

[20]The test is based on a VAR representation with four lags as selected using information criteria.

TABLE 1—COINTEGRATION REGRESSIONS AND TESTS

	Yield	(1)	(2)	(3)	(4)	(5)
constant	6.48	0.65	-0.22	-1.82	-0.64	-2.17
	(0.55)	(0.54)	(0.33)	(0.43)	(0.39)	(0.30)
π_t^*		1.65	1.26	1.53	1.47	
		(0.11)	(0.08)	(0.07)	(0.07)	
r_t^*			0.99	1.76	1.18	
			(0.15)	(0.16)	(0.13)	
i_t^*						1.67
						(0.06)
R^2		0.85	0.93	0.96	0.96	0.95
Memo: r^*			filtered	real-time	mov. avg.	real-time
SD	2.94	1.31	1.09	0.70	0.87	0.70
$\hat{\rho}$	0.97	0.88	0.85	0.65	0.75	0.64
Half-life	26.4	5.6	4.3	1.6	2.5	1.5
ADF	-1.13	-2.60	-3.94**	-5.32***	-4.33**	-5.37***
PP	-3.11	-18.32*	-26.73**	-68.47***	-46.30***	-70.30***
LFST	0.00	0.03	0.16	0.72	0.23	0.71
Johansen $r=0$		13.34	33.08*	46.83***	45.49***	30.69***
Johansen $r=1$		1.29	5.92	11.57	9.14	0.73
ECM $\hat{\alpha}$		-0.11	-0.18	-0.44	-0.49	-0.45
		(0.03)	(0.05)	(0.08)	(0.09)	(0.08)

Note: Dynamic OLS regressions of the ten-year yield, $y_t^{(40)}$, on macroeconomic trends, including four leads and lags of $\Delta y_t^{(40)}$ and differenced trend variables. Newey-West standard errors using six lags are in parentheses. The r^* estimates are described in Section I, and the long-run nominal short rate i_t^* is the sum of π_t^* and the real-time estimate of r_t^*. For the cointegration residuals (and, in the first column, for the ten-year yield itself), the second panel reports standard deviations (SD), first-order autocorrelation coefficients ($\hat{\rho}$), half-lives ($\ln(0.5)/\ln(\hat{\rho})$), Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root test statistics, and p-values for Mueller-Watson low-frequency stationary test (LFST). The table also reports the Johansen trace statistic which tests whether the cointegration rank (r) among the yield and the macro trends is zero/one against the alternative that it exceeds zero/one, using four lags in the VAR. For ADF, PP and Johansen tests *,** and *** indicate significance at 10%, 5%, and 1% level. Estimates of the coefficient α (with White standard errors) on the cointegration residual in the error-correction model (ECM) for $\Delta y_t^{(40)}$ that also includes an intercept, four lags of $\Delta y_t^{(40)}$, and four lags of differenced macro trends. The data are quarterly from 1971:Q4 to 2018:Q1.

Accordingly, when the ten-year yield is high relative to i_t^*, it tends to revert quickly toward this trend, with almost half of the equilibrium error on average reversed within one quarter. Therefore, knowledge of the macro trend appears quite useful when predicting future yields. In sum, these results indicate that the low-frequency behavior of Treasury yields is explained by movements in a common underlying macroeconomic trend, i_t^*, in which both π_t^* and r_t^* play important roles.

B. *Predicting excess bond returns with macro trends*

We now consider whether macroeconomic trends can help predict future excess bond returns. The one-quarter return on a bond with maturity n in excess of the risk-free rate is

$$rx_{t+1}^{(n)} \equiv p_{t+1}^{(n-1)} - p_t^{(n)} - y_t^{(1)} = -(n-1)y_{t+1}^{(n-1)} + ny_t^{(n)} - y_t^{(1)},$$

where $p_t^{(n)}$ denotes the log-price of a zero-coupon bond. Since Fama and Bliss (1987) and Campbell and Shiller (1991), it is well-known that the yield curve contains useful information for predicting excess bond returns. The key question is whether the current yield curve contains *all* of the information relevant for predicting future returns, that is, whether the spanning hypothesis holds. Cieslak and Povala (2015) showed that adding a proxy for the inflation trend significantly improved predictive power—an apparent violation of the spanning hypothesis. Bauer and Hamilton (2018) found that even after correcting for the small-sample econometric distortions associated with highly persistent regressors, the trend inflation proxy remains a relatively robust predictor and thus is unspanned by the yield curve.[21]

To investigate this result more broadly, we examine the gains from including π_t^*, r_t^*, and i_t^* in predictive regressions for excess bond returns. We predict the one-quarter excess return averaged across maturities from two to 15 years.[22] Table 2 reports results for six different predictive regressions. The first column is the usual baseline specification that includes only a constant and the first three principal components (PCs) of yields.[23] The specification in the second column adds π_t^*, and columns three through five include π_t^* as well as one of our three proxies of r_t^* in order to capture the effects of both trends. The final column adds only i_t^* instead of two separate macro trends. We report White standard errors as well as small-sample p-values for the macro trends using the parametric bootstrap of Bauer and Hamilton (2018) to obtain accurate small-sample tests even though

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[21]Other empirical results contradict spanning in different ways. For example, Cochrane and Piazzesi (2005) find that lagged forward rates add predictive power to current forward rates.

[22]We use one-quarter holding periods to avoid the econometric problems resulting from overlapping observations. The one-quarter returns are calculated with the usual approximation $y_{t+1}^{(n-1)} \approx y_t^{(n)}$. We have obtained qualitatively similar results with annual returns (not reported). In addition, the Online Appendix provides estimates for individual bond maturities.

[23]We scale the PCs such that they correspond to common measures of level, slope, and curvature, as in Joslin, Priebsch and Singleton (2014). For example, the loadings of yields on PC1 sum to one.

TABLE 2—PREDICTIVE REGRESSIONS: YIELDS AND MACRO TRENDS

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Full sample, 1971:Q4–2018:Q1						
PC1	0.08	0.98	1.39	2.38	2.04	2.47
	(0.17)	(0.26)	(0.39)	(0.67)	(0.56)	(0.61)
PC2	0.43	0.47	0.43	0.67	0.68	0.70
	(0.17)	(0.17)	(0.17)	(0.15)	(0.15)	(0.15)
PC3	-2.37	-1.79	-1.92	-0.92	-0.90	-0.86
	(1.34)	(1.27)	(1.22)	(1.39)	(1.43)	(1.35)
π_t^*		-1.95	-2.21	-4.40	-3.89	
		(0.44)	(0.47)	(1.10)	(0.92)	
		[0.00]	[0.00]	[0.00]	[0.00]	
r_t^*			-1.19	-3.89	-2.71	
			(0.59)	(1.47)	(1.04)	
			[0.14]	[0.07]	[0.04]	
i_t^*						-4.50
						(1.05)
						[0.00]
R^2	0.09	0.16	0.18	0.21	0.20	0.21
Memo: r^*			filtered	real-time	mov. avg.	real-time
Panel B: Subsample, 1985:Q1–2018:Q1						
PC1	0.25	0.59	1.67	2.65	2.38	1.93
	(0.16)	(0.22)	(0.47)	(0.57)	(0.51)	(0.47)
PC2	0.41	0.50	0.49	0.53	0.65	0.58
	(0.15)	(0.16)	(0.16)	(0.15)	(0.15)	(0.15)
PC3	-1.09	-0.97	0.14	1.74	2.11	0.56
	(1.14)	(1.12)	(1.30)	(1.48)	(1.55)	(1.19)
π_t^*		-1.05	-1.95	-3.44	-3.34	
		(0.73)	(0.75)	(0.87)	(0.83)	
		[0.38]	[0.10]	[0.01]	[0.01]	
r_t^*			-2.03	-5.80	-4.11	
			(0.82)	(1.54)	(1.08)	
			[0.07]	[0.01]	[0.01]	
i_t^*						-3.08
						(0.91)
						[0.02]
R^2	0.08	0.10	0.14	0.19	0.18	0.16
Memo: r^*			filtered	real-time	mov. avg.	real-time

Note: Predictive regressions for quarterly excess bond returns, averaged across two- to 15-year maturities. The predictors are the first three principal components of yields (PC1, PC2, PC3) and estimates of the inflation trend π_t^*, the real-rate trend r_t^*, and the long-run nominal short rate i_t^*. The long-run nominal short rate i_t^* is the sum of π_t^* and the real-time estimate of r_t^*. The macro trend estimates are described in Section I. Numbers in parentheses are White standard errors and in squared brackets are small-sample p-values obtained with the bootstrap method of Bauer and Hamilton (2018). The data are quarterly from 1971:Q4 to 2018:Q1.

the regressors are persistent and correlated with lagged forecast errors.[24] Panel A uses the full sample of data, while Panel B considers a subsample from 1985 to 2018 for robustness. There is some evidence of a shift in the conduct of U.S. monetary policy in the mid-1980s, which would likely lead to a change in interest rate dynamics, so such a subsample is often considered more appropriate for yield curve analysis (e.g., Rudebusch and Wu, 2007; Joslin, Priebsch and Singleton, 2014).

In the full sample, the inclusion of the inflation trend increases the predictive power substantially compared with using only yield curve information. Both the inflation trend and the level of yields (PC1) are highly significant (in addition to the slope, PC2), as in Cieslak and Povala (2015). However, adding r_t^* to the regression leads to further impressive gains in predictive power. Notably, for the moving-average estimate of r_t^*, the R^2 jumps, the coefficients and significance for π_t^* and PC1 rise, and the coefficient on r_t^* is large and significant.[25] Not surprisingly given Figure 1, the relative predictive power of the two macro trends is different in the more recent period as the variation in r_t^* became more pronounced. In the subsample starting in 1985, the inflation trend by itself is not statistically significant. Only with the addition of the equilibrium real rate do both trends matter for bond risk premia: the coefficients on π_t^* and PC1 more than double, the R^2 increases substantially, and the coefficients on π_t^* and r_t^* are statistically significant. These results imply that the trend in the real interest rate is as important as—and recently is more important than—the trend in inflation.

The inflation and real-rate trends have roughly equal coefficients (similar to the cointegration regression). This suggests they play similar roles for yield dynamics and that their sum, i_t^*, can capture their relevant information. Indeed, as shown in the final column of Table 2, including just i_t^* as a predictor provides the same strong predictive gains. The coefficient on i_t^* is of course negative: If the trend component increases, then interest rates also rise in response, producing losses for bond holders.[26] The coefficient is also larger in magnitude than the coefficient on the yield curve level (PC1), consistent with the evidence in Sections II.A and III that the trend component of yields moves more than one-for-one with changes in i_t^*.

An earlier literature showed that a high yield curve *slope* predicts positive excess bond returns. Cochrane and Piazzesi (2005) also found additional predictive power in other aspects of the shape of the yield curve—captured in mid-curve forward rates or higher-order PCs. Our findings, by contrast, document the

[24]We simulate 5000 bootstrap samples for yields and predictors under the spanning hypothesis, using separate (bias-corrected) VAR(1) models for yield factors and predictors. The small-sample p-values are the fractions of simulated samples in which the t-statistics of the macro trends are at least as large (in absolute value) as in the actual data.

[25]Changes in R^2 can be difficult to interpret in the presence of highly persistent predictors, and the Online Appendix confirms these results using our bootstrap.

[26]Numerically, an increase in i_t^* by 0.1 percentage point predicts a decrease in the next quarter's excess bond return by 0.45 percentage point. The Online Appendix shows that this coefficient rises with bond maturity.

importance of the *level* of the yield curve as a powerful predictor. While in yields-only regressions, such as specification (1) in Table 2, the level appears irrelevant, it becomes a strong predictor once i_t^* is accounted for. This result implies that the level of interest rates matters for expected future bond returns, but that it needs to be viewed relative to the underlying trend in yields. Furthermore, accounting for i_t^* makes the slope of the yield curve an even more powerful predictor.

TABLE 3—PREDICTIVE REGRESSIONS: DETRENDED YIELDS

	Full sample: 1971–2018				Subsample: 1985–2018			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
PC1	0.08	0.98	1.25	1.36	0.33	0.41	1.14	0.81
	(0.17)	(0.25)	(0.51)	(0.50)	(0.16)	(0.22)	(0.39)	(0.38)
PC2	0.43	0.48	0.76	0.78	0.34	0.44	0.62	0.62
	(0.17)	(0.17)	(0.16)	(0.16)	(0.14)	(0.14)	(0.14)	(0.14)
PC3	-2.37	-1.77	-0.79	-0.73	-1.04	-0.96	1.53	0.45
	(1.34)	(1.26)	(1.38)	(1.33)	(1.01)	(1.01)	(1.32)	(1.05)
R^2	0.09	0.15	0.18	0.20	0.08	0.08	0.17	0.14

Note: Predictive regressions for quarterly excess bond returns, averaged across two- to 15-year maturities. In specification (1) the predictors are the first three principal components of observed yields (PC1, PC2, PC3). In specifications (2), (3), and (4) the predictors are the first three principal components of detrended yields, that is, of residuals in regressions for yields on π_t^*, π_t^* and r_t^*, or i_t^*, respectively. The long-run nominal short rate i_t^* is the sum of π_t^* and the real-time estimate of r_t^*. The macro trend estimates are described in Section I. Numbers in parentheses are White standard errors with six lags. The data are quarterly from 1971:Q4 to 2018:Q1.

In Table 3, we further investigate the importance of viewing interest rates relative to their stochastic trend by using stochastically detrended yields as predictors. The use of such predictors with low persistence has the added benefit that conventional, large-sample inference is more reliable (Bauer and Hamilton, 2018). We use the first three PCs of the 17 detrended yields as predictors, where each detrended yield is obtained as the cointegration residual from a regression of the yield on macro trends. As before, we consider three types of detrending: using only π_t^*, using both π_t^* and (real-time) r_t^*, as well as using i_t^*. In the full sample, all three detrending methods lead to substantial predictive gains compared to the baseline regression that has just the first three PCs of the 17 original yields. Notably, the detrended level of the yield curve (PC1) becomes strongly significant. Detrending with i_t^* works particularly well, giving the largest gain of 11 percentage points. In the post-1985 sample, the R^2 does not increase when detrending with only π_t^*, but it increases by 9 and 6 percentage points, respectively, when detrending with either both π_t^* and r_t^* or with only i_t^*. These estimates provide additional support for the importance of macro trends in predicting bond returns.

In sum, the key to predicting bond returns and estimating bond risk premia is to account for the long-run trend in interest rates. Both underlying macro

trends have strong incremental predictive power for bond returns relative to the observed yield curve. That is, the long-run trend cannot be inferred from the cross-section of yields alone, so our results contradict the spanning hypothesis. Finally, for detrending interest rates, it is crucial to use i_t^* instead of π_t^*.

III. A no-arbitrage model with a stochastic trend

To account for the stylized facts establishing an important low-frequency macro-finance interplay, we introduce and estimate a new DTSM. While existing DTSMs generally assume stationarity, our new formulation incorporates a shifting end-point for interest rates, i_t^*, as a common stochastic trend for yields. This novel specification allows us to explain the stylized facts, provide new estimates of the term premium, and more accurately forecast interest rates.

A. Model specification

The state variables of our model of yield dynamics include N linear combinations of yields in the vector $\mathbf{P}_t$, and we use $N = 3$ such yield factors.[27] The key feature of our model is the presence of a single stochastic trend τ_t common to the three factors. The common trends representation of the dynamic system is:

$$(3) \qquad \mathbf{P}_t = \bar{\mathbf{P}} + \boldsymbol{\gamma}\tau_t + \tilde{\mathbf{P}}_t, \quad \tau_t = \tau_{t-1} + \eta_t, \quad \tilde{\mathbf{P}}_t = \boldsymbol{\Phi}\tilde{\mathbf{P}}_{t-1} + \tilde{\mathbf{u}}_t,$$

where $\boldsymbol{\gamma}$ is an N-vector with the loadings on the common trend, η_t is *iid* normal with variance σ_η^2, $\tilde{\mathbf{P}}_t$ are the N cycle components that have *iid* normal innovations $\tilde{\mathbf{u}}_t$ with covariance matrix $\widetilde{\boldsymbol{\Omega}}$, and $\boldsymbol{\Phi}$ is a mean-reversion matrix that has eigenvalues with modulus less than one, so $\tilde{\mathbf{P}}_t$ is stationary. Accordingly, the state variables $\mathbf{Z}_t = (\tau, \mathbf{P}_t')'$ are trivially cointegrated.[28] We assume that innovations to trend and cycle are orthogonal, $E(\eta_t \tilde{\mathbf{u}}_t) = \mathbf{0}$, as is commonly done with unobserved components models (Watson, 1986; Del Negro et al., 2017). Our assumptions imply that the long-run trend components of $\mathbf{P}_t$ are

$$(4) \qquad \mathbf{P}_t^* \equiv \lim_{j\to\infty} E_t\mathbf{P}_{t+j} = \bar{\mathbf{P}} + \boldsymbol{\gamma}\tau_t.$$

The short-term interest rate is taken to be an affine function of $\mathbf{P}_t$:

$$(5) \qquad i_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{P}_t.$$

Finally, we assume absence of arbitrage opportunities so that there exists a "risk-neutral" probability measure, denoted by $\mathbb{Q}$, which prices all financial assets (Har-

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[27]Let the vector $\mathbf{Y}_t$ contain J model-implied yields. Then $\mathbf{P}_t = \mathbf{W}\mathbf{Y}_t$, where $\mathbf{W}$ is an $N \times J$ coefficient matrix.

[28]The Appendix gives the VAR representation of the dynamic system. Innovations to $\mathbf{P}_t$ are $\mathbf{P}_t - E_{t-1}\mathbf{P}_t \equiv \mathbf{u}_t = \boldsymbol{\gamma}\eta_t + \tilde{\mathbf{u}}_t$ with covariance matrix $\boldsymbol{\Omega} = \boldsymbol{\gamma}\boldsymbol{\gamma}'\sigma_\eta^2 + \widetilde{\boldsymbol{\Omega}}$.

rison and Kreps, 1979). The risk-neutral dynamics are specified as

$$(6) \qquad \mathbf{P}_t = \boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}} \mathbf{P}_{t-1} + \mathbf{u}_t^{\mathbb{Q}},$$

where the innovations $\mathbf{u}_t^{\mathbb{Q}}$ are *iid* normal (under $\mathbb{Q}$) with covariance matrix $\boldsymbol{\Omega}$. We assume that $\mathbf{P}_t$ is stationary under $\mathbb{Q}$ by imposing that $\boldsymbol{\Phi}^{\mathbb{Q}}$ has eigenvalues less than one in absolute value, thus forward rates, yields, and bond convexity remain finite in the limit as maturity increases. Equations (5) and (6) imply that yields are affine functions of the yield factors, $\mathbf{Y}_t = \mathbf{A} + \mathbf{B}\mathbf{P}_t$. The loadings in $\mathbf{A}$ and $\mathbf{B}$ are determined by the risk-neutral parameters.[29]

Two types of normalizations are required for the state variables and parameters to be identified. The first set of normalizations, required in any affine term structure model, uniquely pins down the yield factors $\mathbf{P}_t$. In their absence, an affine transformation of $\mathbf{P}_t$ would leave implied interest rates $\mathbf{Y}_t$ unchanged (Dai and Singleton, 2000). We use the normalization of Joslin, Singleton and Zhu (2011), which imposes restrictions only on the $\mathbb{Q}$-dynamics and is ideally suited for the use of linear combinations of yields as yield factors. In this normalization, only four parameters determine the $\mathbb{Q}$-dynamics: a scalar $k^{\mathbb{Q}}$ and an N-vector $\boldsymbol{\lambda}^{\mathbb{Q}}$ containing the eigenvalues of $\boldsymbol{\Phi}^{\mathbb{Q}}$ which we assume to be real, distinct, and less than one. These four parameters and $\boldsymbol{\Omega}$ fully determine the yield loadings $\mathbf{A}$ and $\mathbf{B}$. Details are in the Online Appendix.

The second normalization is needed to identify the unobserved trend τ_t. Without it, an affine transformation of τ_t would leave the yield factors $\mathbf{P}_t$ unchanged. For identification, we require one linear constraint on each of the vectors $\bar{\mathbf{P}}$ (to fix the level) and $\boldsymbol{\gamma}$ (to fix the scale).[30] We identify τ_t as the long-run mean of the short rate, $\tau_t = i_t^* = \delta_0 + \boldsymbol{\delta}_1' \mathbf{P}_t^*$, using the normalizations $\delta_0 + \boldsymbol{\delta}_1' \bar{\mathbf{P}} = 0$ and $\boldsymbol{\delta}_1' \boldsymbol{\gamma} = 1$. In total, the parameters of the model are $k^{\mathbb{Q}}, \boldsymbol{\lambda}^{\mathbb{Q}}, \boldsymbol{\gamma}, \bar{\mathbf{P}}, \boldsymbol{\Phi}, \boldsymbol{\Omega}, \sigma_\eta^2$, and σ_e^2, where $\boldsymbol{\gamma}$ and $\bar{\mathbf{P}}$ each have two degrees of freedom.

The key innovation of this model relative to existing affine DTSMs is that the equilibrium nominal interest rate i_t^* is allowed to be time-varying. This implies that bond yields have a common stochastic trend and are $I(1)$. Imposing a unit root in this fashion is a convenient way to account for a very persistent process in a dynamic model, and it crystalizes the arguments regarding slow-moving interest rate trends. Taken literally, a unit root specification for interest rates is unlikely because nominal interest rates appear to have remained bounded over the centuries. However, in finite samples, a persistent stationary process can always be approximated arbitrarily well by a unit root process, and it is well-known that doing so can often be beneficial for forecasting (e.g., Campbell and Perron, 1991).[31] To illustrate the importance of the time-varying trend in

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[29]The Appendix provides expressions for excess returns and the term premium and more details on the role of the long-run trend for yields and risk premia.

[30]For further discussion of identification requirements in multivariate unobserved components (common trends) models, see, for example, Harvey (1990).

[31]For example, a unit root in inflation may not be literally true, but assuming so often improves

interest rates, we will compare our model, which we refer to as the "shifting endpoint" (SE) model, to a conventional DTSM with stationary dynamics, a "fixed endpoint" (FE) model. The latter specification corresponds to a restricted special case of our model where $i_t^* = i^*$, i.e., $\sigma_\eta^2 = 0$, which is equivalent to the popular three-factor affine DTSM of Joslin, Singleton and Zhu (2011).

Another important aspect of our model is that the long-run trend is *unspanned* by the yield curve, that is, the cross section of interest rates at time t is not deterministically related to i_t^*. Asset pricing models generally imply spanning, since in the absence of knife-edge restrictions the state variables can be backed out from the yield curve.[32] In our model, such inference is not possible because the trend does not *directly* affect yields: Conditional on $\mathbf{P}_t$, yields do not load on i_t^*. Therefore, the model can match the evidence against the spanning hypothesis documented in Section II.B, as we demonstrate below.[33] Another important rationale for an unspanned trend is to allow it to be unobserved. To incorporate an unobserved components specification within a dynamic term structure model, it is necessary for the trend component to be unspanned; otherwise, i_t^* would be completely determined by the time-t yield curve and thus observed. A trend hardwired to current interest rates would also result in excessively volatile trend estimates.[34]

Our parsimonious specification can accurately fit the cross section of yields, is consistent with absence of arbitrage and time-varying risk premia, and includes a stochastic trend in line with much evidence showing that nominal interest rates are effectively $I(1)$. Importantly, changes in i_t^* may affect expectations of future short rates as well as the term premium, and our estimates below will assess the empirical importance of a trend in the term premium. While our DTSM framework is quite flexible, it is kept deliberately simple by focusing on the nominal yield curve instead of separately modeling inflation and real yields. Such a separation, in the spirit of Christensen, Lopez and Rudebusch (2010), would be a useful future extension that could differentiate r_t^* and π_t^*, but it is unnecessary for our purposes of estimating nominal risk premia and forecasting nominal Treasury yields.

forecast performance (Faust and Wright, 2013).

[32]See Duffee (2013). Starting with Duffee (2011), a number of papers have built no-arbitrage models of the yield curve that impose such knife-edge restrictions and therefore allow for unspanned factors.

[33]A regression of i_t^* on *contemporaneous* yields provides an alternative perspective on spanning. For our preferred i^* proxy, such a regression gives an R^2 of 95%, which might seem to suggest that the trend is effectively spanned, but this high R^2 is misleading because of the presence of stochastic trends. Furthermore, this regression does not reveal whether any unspanned information is relevant for risk premia, as noted by Joslin, Priebsch and Singleton (2014).

[34]While sufficiently flexible *spanned* DTSM can generally match the data equally well as the corresponding *unspanned* DTSM, such spanned models require many more parameters (Bauer and Rudebusch, 2017). Furthermore, this result applies only to models in which the unspanned factors are observed, as in Joslin, Priebsch and Singleton (2014), whereas we allow for the possibility that the trend is unobserved. We have estimated term structure models with spanned trend factors and obtained implausibly volatile trend estimates, as movements in long-term yields are tightly linked to movements in the trend factor.

B. Model estimation

The shifting trend model can be cast in state-space form for estimation. The measurement equation is $\mathbf{Y}_t^o = \mathbf{A} + \mathbf{B}\mathbf{P}_t + \mathbf{e}_t$, with $\mathbf{e}_t \overset{iid}{\sim} N(\mathbf{0}, \sigma_e^2 \mathbf{I}_J)$. For our data set, $\mathbf{Y}_t^o$ includes $J = 17$ yields. The state equation follows from (3).

The common trend in our DTSM is both unspanned and latent, unlike the observed unspanned factors in models like Joslin, Priebsch and Singleton (2014). Our specification is therefore effectively an unobserved components model. While this is desirable for matching the stylized facts of trends in interest rates, we face the usual estimation problem that empirically uncovering long-run trends using limited sample sizes is intrinsically difficult (Watson, 1986). For a very persistent time series with relatively infrequent business cycle fluctuations, even several decades of data typically contain limited information that can pin down the underlying trend. Simply estimating an unobserved components model with maximum likelihood methods and no external additional information is problematic and unreliable.[35] Therefore, in the literature, two broad approaches have been used to facilitate estimation of unobserved trends from time series data: the first includes additional data or proxies for the trend, the second incorporates prior information about properties of the trend such as its smoothness. For robustness, we consider each of these two methods for estimating our stochastic trend DTSM.

The first method adds data that can directly help pin down the trend estimate. For example, long-run survey expectations of inflation are often used to inform estimates of π_t^* (Kozicki and Tinsley, 2012; Del Negro et al., 2017). Similarly, Crump, Eusepi and Moench (2018) use surveys of professional forecasters to estimate i_t^* in a factor model for the yield curve. In this spirit, we use the empirical proxy for i_t^* equal to the sum of the PTR estimate of π_t^* and our real-time estimate of r_t^*. Section II showed that this trend proxy captures low-frequency movements of yields and predicts excess bond returns. Because we can now treat both i_t^* and $\mathbf{P}_t$ as observed, estimation is greatly simplified.[36] We denote the resulting estimates as the "observed shifting endpoint" (*OSE*) model. To guard against the problems that the zero lower bound (ZLB) on nominal interest rates poses for the estimation of affine yield curve models (Bauer and Rudebusch, 2016), we estimate the parameters of the *OSE* model using a sample that ends in 2007:Q4, before the height of the financial crisis and the ZLB period. Once the parameter estimates are obtained, we proceed to use the model over the entire sample period.[37] We find that the *OSE* model fits observed yields well, with a root mean-

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[35]The likelihood is essentially flat in some dimensions of the parameter space, and numerical optimization leads to local optima with very different trend estimates, depending on starting values.

[36]Details are in the Online Appendix. Treating i_t^* as observed is justified because maximum likelihood estimation of a state-space model with an additional measurement equation for i_t^* results in a noise variance essentially equal to zero. Treating the yield factors $\mathbf{P}_t$ as observed was proposed and justified by Joslin, Singleton and Zhu (2011). If estimation of the state-space model is desired, excellent starting values for the model's parameters can be obtained from a first-step estimation with observed state variables.

[37]An extension of our model to explicitly account for the zero lower bound, for example with a shadow-

squared error of 8 basis points over the full sample. The $\mathbb{Q}$-dynamics are highly persistent, with the largest eigenvalue of $\mathbf{\Phi}^{\mathbb{Q}}$ equal to 0.997. For the real-world dynamics, the mean reversion to the common trend is relatively quick, with the largest eigenvalue of $\mathbf{\Phi}$ equal to 0.933.

Our second estimation method of the long-run trend DTSM uses only Treasury yield data while making use of prior information in a Bayesian framework. In this regard, an effective approach to estimating unobserved trends imposes a tight prior distribution for the variance of the trend innovations. Such priors are commonly used in a variety of macroeconomic contexts to produce smooth trend estimates, consistent with the underlying macroeconomics that such trends are determined by gradual variation in structural factors such as demographic change. For interest rates, Del Negro et al. (2017) used such a smoothness prior for estimation of r_t^*. Similarly, we specify a tight inverse-gamma prior for σ_η^2 with a mean of $0.06^2/400$, implying that the standard deviation of the change in i_t^* over a century is six percent. In light of the observed variation in interest rates and our trend proxies, we view this as a conservative choice in the sense that it limits the amount of yield variation that can be attributed to the long-run trend. Also, we follow Del Negro et al. (2017) by combining this prior with the stationary version of a Minnessota VAR-prior for the cycle components $\tilde{\mathbf{P}}_t$ with the distribution for the diagonal elements of $\mathbf{\Phi}$ centered around zero. We call this representation the "estimated shifting endpoint" (*ESE*) model. We use largely uninformative priors for the remaining parameters, and design a Markov chain Monte Carlo (MCMC) algorithm to simulate draws from the joint posterior distribution of the latent state variables and parameters (as described in detail in the Online Appendix). To account for the ZLB, we treat short-term yields as missing observations when they were constrained.[38]

Figure 4 plots the resulting *ESE* model estimate of i_t^*—specifically, the posterior mean and 95 percent credibility intervals. The *ESE* model i_t^* rose until the early 1980s to over eight percent and then gradually declined over the following decades to just below three percent at the end of our sample. For comparison, the ten-year Treasury yield and the real-time proxy estimate of i_t^* are also plotted. The *ESE* model estimate matches the proxy fairly closely and can account for much of the low-frequency variation in the ten-year yield. As usual for unobserved trend models, the uncertainty around the estimated trend is quite large. From 2008 to 2015, the estimation intervals widen a bit as uncertainty increases because the yield curve is partially constrained by the ZLB and is less informative about the underlying trend.

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rate specification as in Kim and Singleton (2012) and Bauer and Rudebusch (2016), is conceptually straightforward but beyond the scope of this paper.

[38]Specifically, three- and six-month yields are assumed to be missing from 2008:Q4 to 2015:Q3, the one-year yield is omitted from 2011:Q4 to 2014:Q3, and the two-year yield is omitted from 2012:Q4 to 2013:Q1. The start dates of these periods are based on results in Swanson and Williams (2014) while the end dates are when the yields lifted off from the ZLB.
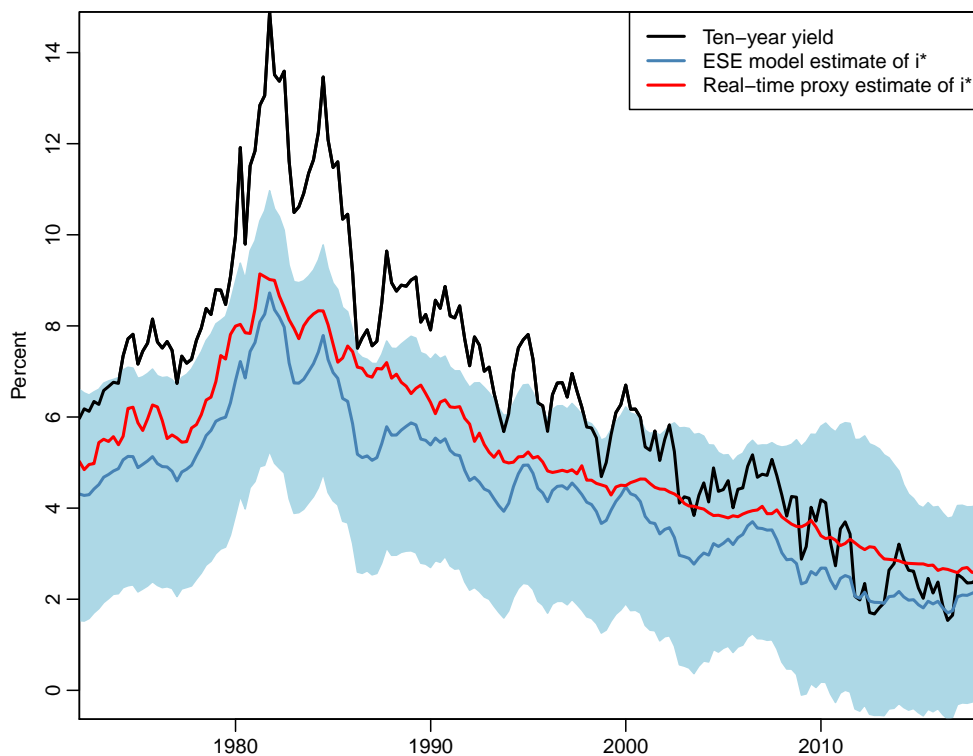
FIGURE 4. MODEL-BASED ESTIMATE OF EQUILIBRIUM INTEREST RATE

Note: Bayesian (MCMC) estimate—posterior mean and 95%-posterior credibility intervals—of the equilibrium nominal short rate, i^*, from the estimated shifting endpoint (ESE) dynamic term structure model. Also shown is a proxy estimate of i^*, the sum of the PTR estimate of π^* and the real-time macro estimate of r^*. The data are quarterly from 1971:Q4 to 2018:Q1.

C. *Implications for trend components of yields*

Section II.A showed that proxies for i_t^* were important drivers of yields, particularly in terms of cointegration. Now we interpret this evidence in the context of our model.
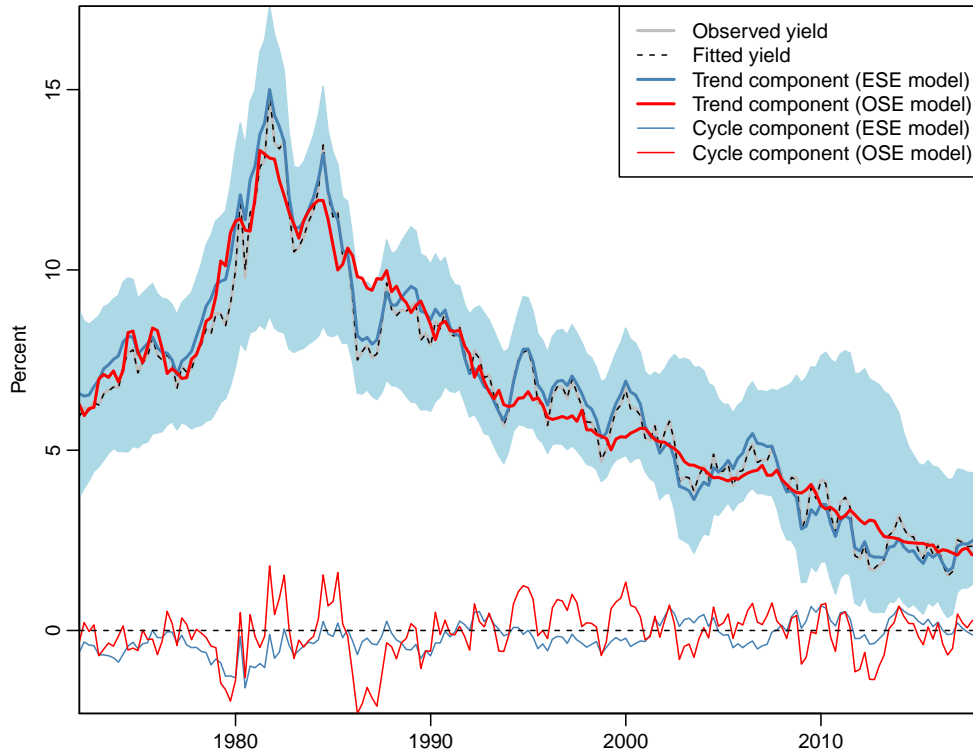

FIGURE 5. TREND AND CYCLE COMPONENTS OF TEN-YEAR YIELD

Note: Model-based decomposition of ten-year yield into trend and cycle components. Blue lines show trend and cycle (posterior means) for the model with estimated shifting endpoint (*ESE*), shaded area shows 95%-posterior credibility intervals for trend component; red lines show trend and cycle for model with observed shifting endpoint (*OSE*). The data are quarterly from 1971:Q4 to 2018:Q1.

First, note that our shifting-endpoint DTSM can decompose yields into trend and cycle components:

$$(7) \qquad \mathbf{Y}_t = \mathbf{Y}_t^* + \tilde{\mathbf{Y}}_t = \mathbf{A} + \mathbf{B}\bar{\mathbf{P}} + \mathbf{B}\boldsymbol{\gamma}i_t^* + \mathbf{B}\tilde{\mathbf{P}}_t, \quad \mathbf{Y}_t^* \equiv \lim_{j \to \infty} E_t \mathbf{Y}_{t+j}.$$

Figure 5 shows the (actual and model-fitted) ten-year yield and its trend and cycle components. The trend component accounts for most of the low-frequency

movements in the yield, while the cycle component—the model-based analog of the regression-based interest rate cycles discussed in Section II.A—exhibits clear mean reversion around zero and little to no apparent trend. This confirms that our model accounts for the long-run trend in interest rates.
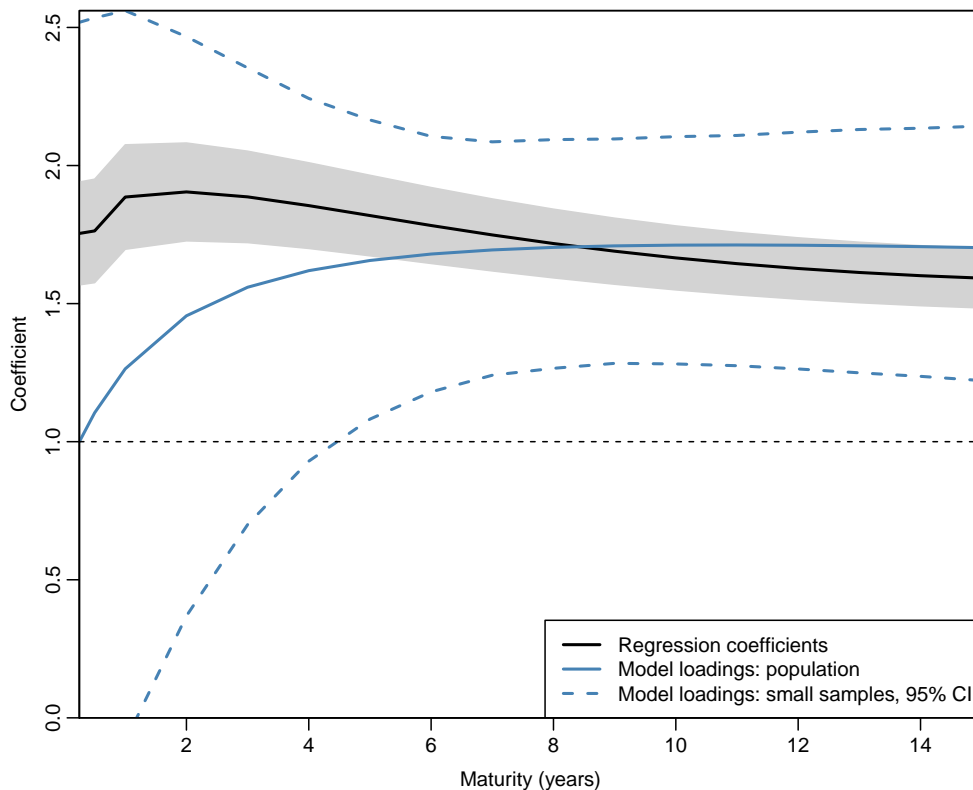

FIGURE 6. LOADINGS OF YIELDS ON THE EQUILIBRIUM INTEREST RATE

Note: Comparison of loadings of Treasury yields on the equilibrium nominal short rate i_t^* in the data and in the shifting-endpoint dynamic term structure model. Data: regression coefficients (with 95%-confidence intervals) for cointegration regressions of yields on the proxy-estimate of i_t^* (the PTR estimate of π^* plus the real-time macro estimate of r^*). Model: loadings of yields on i_t^* in population and 95%-Monte Carlo intervals for regression coefficients in small samples simulated from the model with an observed shifting endpoint (OSE).

Second, we can compare the model-implied coefficients of yields on i_t^* to the empirical results we obtained from the cointegration regressions of yields on proxies of i_t^* in Section II.A. In the model, the trend i_t^* is unspanned and only indirectly affects yields through its relationship with $\mathbf{P}_t$. Equation (7) shows that the implied loadings of yields on i_t^* are $\mathbf{B}\boldsymbol{\gamma}$. Figure 6 plots these coefficients, which gradually rise from unity at the short end—since i^* is by definition the trend component of the short rate—to around 1.7. In the data, the coefficients (shown

with their 95 percent confidence intervals) are relatively flat around 1.6-1.9, with a slight hump around two years and a gradual decline thereafter. Note that the estimated coefficient for the ten-year yield corresponds to the value reported in the last column of Table 1. To gauge whether the model is consistent with the estimated cointegration coefficients we need to account for sampling uncertainty, hence we simulate 5,000 artificial samples of the trend and yields with the same size as the actual data and estimate the cointegration regressions in each simulated sample. The 95 percent Monte Carlo intervals of the coefficient estimates, shown as dashed lines in Figure 6, comfortably contain the values obtained with the actual data.[39]

Our results here and in Section II.A show that the trend component of long-term Treasury yields moves more than one-for-one with changes in i_t^*—the coefficients on i_t^* are larger than one. Because expectations of future short rates by definition load with a unit coefficient on i_t^*, coefficients above one indicate that the term premium positively responds to changes in i_t^*. (For a yield of maturity n, the term premium loading on i_t^* is $\mathbf{B}_n'\boldsymbol{\gamma} - 1$.) That is, our cointegration regressions and the model estimates suggest that the term premium responds positively to i_t^*. In Section III.E, we will show the resulting term premium estimates and quantify the relative importance of the trend in short-rate expectations and in the term premium. Our model has similar implications for the trend behavior of the *slope of the yield curve*. The slope, if measured as the simple difference between a long-term yield and the short rate, in fact has the exact same trend component and loading on i_t^* as the term premium in that long-term yield. While some macroeconomic time series models, such as DGGT, assume that separate trends drive short rates and the slope, in our no-arbitrage model the slope contains a trend due to the risk in i_t^*.

D. *Model-based explanation of excess return predictability*

A key stylized fact from Section II is that various measures of the long-run trends in interest rates have substantial predictive power for excess bond returns, and this additional information is not spanned by the first three principal components of yields. To explain this result with our shifting endpoint DTSMs, we first simulate 5,000 artificial samples from each of our three models: the *FE* model with stationary yields, the *OSE* model with an observed trend proxy, and the *ESE* model with an estimated trend.[40] We then run excess return regressions using the simulated data sets from the three models, and report the means and the 2.5th and 97.5th percentiles of the resulting Monte Carlo distributions.

Table 4 presents the R^2 results from the actual and simulated data. The top row

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[39]Because our model is stationary under $\mathbb{Q}$, the infinite-maturity yield is constant, so loadings of yields on all risk factors eventually decline to zero as maturity increases. For maturities of practical interest—say, less than 100 years—the loadings on i_t^* are notably larger than one (as discussed in the Online Appendix).

[40]For the *FE* model, we separately simulate i_t^* from a random walk process using the *OSE* parameters.

TABLE 4—MODEL-IMPLIED PREDICTABILITY OF EXCESS BOND RETURNS

	R^2 PCs only	R^2 with i_t^*	ΔR^2
Data	0.09	0.21	0.12
FE model	0.09	0.10	0.01
	[0.04, 0.17]	[0.04, 0.17]	[0.00, 0.04]
OSE model	0.10	0.19	0.09
	[0.04, 0.17]	[0.13, 0.26]	[0.02, 0.18]
ESE model	0.07	0.14	0.08
	[0.02, 0.13]	[0.05, 0.27]	[0.00, 0.20]

Note: The R^2 of predictive regressions for quarterly excess bond returns, averaged across maturities of 2 to 15 years. The R^2 in the data correspond to the full-sample estimates in Table 2 (first and last columns). The model-implied R^2 are based on 5,000 simulations of artificial data sets of the same size as the full sample. The table reports means and 95%-Monte Carlo intervals (in square brackets) of the R^2 of predictive regressions estimated in these simulated data.

reports the R^2 for the predictive regressions of excess bond returns using actual data with and without i_t^* as an additional predictor, as described in Section II.B (corresponding to the estimates in the first and last columns of Table 2). The reported ΔR^2 shows the substantial predictive gains from including the macro trend proxy.

The remaining rows of Table 4 provide results using data simulated from the *FE*, *OSE*, and *ESE* models. For the *FE* model, there are essentially no predictive gains on average from adding i_t^* to the yield principal components, with a mean ΔR^2 of 1 percentage point. This is not surprising because this model imposes the null hypothesis that i_t^* truly has no predictive power. The upper bound of the Monte Carlo interval (4 percentage points) is much lower than the actual improvement in R^2 from adding trends in the data (12 percentage points), consistent with our rejections of the spanning hypothesis in Section II.B using the bootstrap, which also simulated data under the null hypothesis.

For the *OSE* model, the predictive gains from adding i_t^* are large: the mean gain of 9 percentage points is close to the ΔR^2 of 12 percentage points in the actual data. Importantly, the Monte Carlo interval comfortably contains the value found in the data. For the *ESE* model, the results are similar, with large model-implied predictive gains from including the trend as an additional predictor. Overall, the shifting-endpoint models accurately replicate the actual predictive gains from adding i_t^* in regressions for excess bond returns.

In our shifting-endpoint model, shifts in i_t^* are unspanned by the yield curve by construction. Why are changes in the trend unspanned in the data? The intuitive reason is the following: Shifts in the level of the yield curve can occur because of either changes in the underlying trend (see the loadings in Figure 6) or because of movements in interest rates relative to the trend. These two drivers have opposite implications for expectations of future bond returns, and investors

cannot distinguish between them based on observed yields alone.[41] Therefore, yields and macro trends contain important separate pieces of information for predicting future interest rates.

To better understand the predictability of excess bond returns through the lens of our model, we examine model-implied expected excess bond returns analytically. Leaving aside constant terms that are irrelevant for predictability, bond risk premia are

$$
(8) \qquad E_t r x_{t+1}^{(n)} = \boldsymbol{\mathcal{B}}_{n-1}' \left(E_t \mathbf{P}_{t+1} - E_t^{\mathbb{Q}} \mathbf{P}_{t+1} \right)
$$

$$
(9) \qquad\qquad\qquad = \boldsymbol{\mathcal{B}}_{n-1}' \left[(\mathbf{I}_N - \boldsymbol{\Phi})\boldsymbol{\gamma}\tau_t + (\boldsymbol{\Phi} - \boldsymbol{\Phi}^{\mathbb{Q}})\mathbf{P}_t \right],
$$

where $\boldsymbol{\mathcal{B}}_n$ contains the affine loadings of the log bond price $p_t^{(n)}$ on the risk factors $\mathbf{P}_t$. Equation (8) shows that time variation in bond risk premia is due to differences in expectations of future yields under the physical and risk-neutral probability measures. In the presence of an unspanned trend in yields, as in our model, physical-measure expectations depend not only on current yields ($\mathbf{P}_t$) but also on the shifting endpoint i_t^*. Thus, bond risk premia depend on both as well, as evident in (9). Ignoring the trend misses an important part of the variation in $E_t \mathbf{P}_{t+1}$ and understates the predictability of bond returns.

A final point concerns the role of detrended yields, which we have shown in Section II.B to capture most of the variation in bond risk premia. Our model implies that the term premium and excess returns also have a small trend component, meaning that detrended yields cannot *fully* explain movements in bond risk premia. Empirically, however, when predicting excess bond returns it is of first-order importance to account for the trend in yields, and only of second-order importance to separately account for the trend when conditioning on detrended yields (as described in the Appendix).

E. *The term premium in long-term yields*

Another perspective on the pricing of risk in Treasury bonds is given by the term premium in long-term interest rates. As evident from the definition in equation (1), estimating the term premium requires long-horizon projections of future short-term rates, which will depend crucially on the nature of the long-run trend in interest rates.[42] Furthermore, because the term premium is the average of expected future excess bond returns, $TP_t^{(n)} = n^{-1}\sum_{j=0}^{n-2} E_t r x_{t+j+1}^{(n-j)}$, our results about the importance of a trend in interest rates for predictions of excess bond

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[41]Even in a term structure model with a spanned trend, the cross-sectional loadings of yields on i_t^* would differ only slightly from the loadings of the level factor in detrended yields, and small measurement errors would make them effectively indistinguishable (Jorgensen, 2017).

[42]In a DTSM, the term premium is defined as the difference between model-implied yields and risk-neutral yields, which differ from average expected short rates only by a convexity term. See the Appendix for details.

returns suggest that this trend should matter for the estimation of term premia as well.
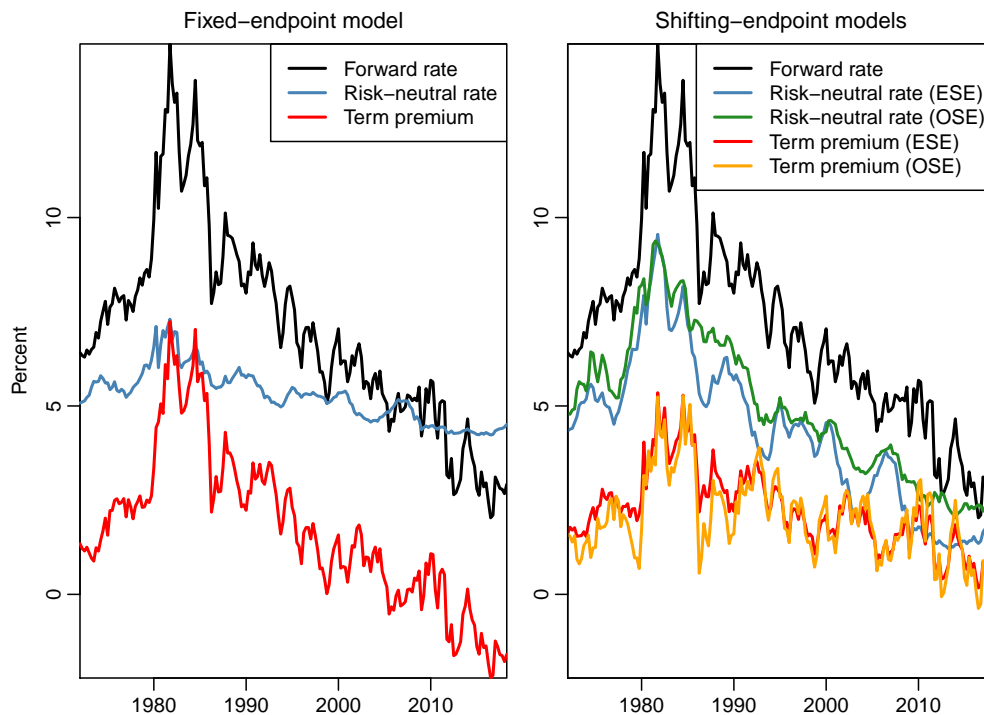

FIGURE 7. EXPECTATIONS AND TERM PREMIUM COMPONENTS IN LONG-TERM INTEREST RATES

Note: Five-to-ten-year forward rate with estimated expectations component (risk-neutral rate) and term premium. Left panel: conventional dynamic term structure model (DTSM) with a fixed endpoint i^* (Joslin, Singleton and Zhu, 2011). Right panel: DTSM with a common stochastic trend, where the shifting endpoint is either estimated (*ESE*, estimated shifting endpoint) or taken as observed using a proxy of i_t^* (*OSE*, observed shifting endpoint). The data are quarterly from 1971:Q4 to 2018:Q1.

Our baseline for comparison is the *FE* model in which the state variables follow a stationary VAR. This special case of our no-arbitrage model with a constant endpoint $i_t^* = i^*$ is representative of the conventional DTSM in the literature. The left panel of Figure 7 shows the five-to-ten-year forward rate as well as its expectations component, i.e., the "risk-neutral rate," and the term premium as estimated from the *FE* model.[43] The expectations component is estimated to be very stable, and it remains near the unconditional mean of the short rate, the fixed endpoint i^*. Therefore, the term premium, the residual, has to account for the trend in the long-term interest rate since the 1980s. As argued by Kim and Orphanides (2012) and Bauer, Rudebusch and Wu (2012), among others, such

[43]Results for the ten-year yield are qualitatively similar.

behavior by the expectations component and the term premium appears at odds with observed trends in survey-based expectations (Kozicki and Tinsley, 2001) and with the cyclical behavior of risk premia in asset prices (Fama and French, 1989). By allowing for a long-run trend in interest rates, we avoid such counterfactual decompositions of long-term rates into expectations and term premium. The right panel of Figure 7 shows the estimates for the risk-neutral rate and the term premium from our no-arbitrage *OSE* and *ESE* models.[44] For these shifting-endpoint models, short-rate expectations are anchored at i_t^*, and the cycle components $\tilde{\mathbf{P}}_t$ mean-revert relatively quickly; hence, the expectations component of the five-to-ten-year forward rate closely mirrors this trend. Movements in the expectations component account for the majority of the low-frequency variation in the observed interest rate. In particular, the estimates exhibit a pronounced downward trend since the 1980s due to the decline in i_t^*.

Our shifting-endpoint models attribute a majority of the secular decline in interest rates not to the term premium but instead to the decline in long-run expectations of future short-term rates. The drop in the forward rate from its average during 1980-1982 to its average during 2015-2017 was close to 10 percentage points. According to the stationary *FE* model, the estimated term premium accounts for 75 percent of this decline. In contrast, the shifting-endpoint models attribute only 35 percent (*OSE*) and 37 percent (*ESE*) to the term premium, while attributing the majority to a falling expectations component, in line with the substantial downward shift in the underlying macro trends.[45] Furthermore, the estimated term premium with a shifting endpoint exhibits more pronounced cyclical variation, in line with the notion that risk premia are countercyclical (Fama and French, 1993; Cochrane and Piazzesi, 2005). This stark difference demonstrates how accounting for the slow-moving trend component in interest rates fundamentally alters our understanding of the driving forces of long-term interest rates, and bridges the gap between the secular trends in macroeconomics and no-arbitrage models for the yield curve.

Although the shifting-endpoint models attribute most of the trend in the forward rate to the expectations component, the estimated forward term premium does exhibits some moderate low-frequency swings, declining from around 4-5 percent in the early 1980s to about 0-1 percent at the end of the sample. The reason is that long-term interest rates exhibited more pronounced secular movements than our estimates of i_t^*, and expectations move one-for-one with i_t^*, thus the estimated term premium also contains a trend component.[46] In other words, our estimates show that changes in the trend carry a positive risk premium. From

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[44]For the *OSE* model, the risk-neutral rate and term premium is calculated using point estimates of the parameters. For the *ESE* model, we calculate the means of the MCMC posterior distribution for these objects.

[45]Some researchers estimate stationary DTSMs with a sample beginning in, say, the mid-1980s to avoid the earlier shifts in inflation, but our comparison results were little changed using a post-1984 sample.

[46]As discussed above in Section III.C, yields load on i_t^* with coefficients larger than unity, and the term premium consequently has a positive loading on this trend.

a macro-finance perspective, some downward trend in the term premium since the 1980s is plausible. Notably, inflation risk declined substantially over this period as average inflation declined, and the term premium includes compensation for this risk (Wright, 2011).[47] In sum, the decline in the term premium is much more moderate and more plausible with a shifting rather than a fixed endpoint.

Using a shifting-endpoint model with a slow-moving trend component solves the knife-edge problem of Cochrane (2007), who pointed out that assuming either stationarity or a random walk for the level of interest rates leads to drastically different implications for expectations and term premia, in both cases at odds with widely held views about interest rates. Imposing a random walk for the level is known to forecast well but leads to the implausible implication that the expectations component accounts for essentially all variation in yields. Imposing stationarity, as shown in the left panel of Figure 7, leads to the implausible implication that the term premium component accounts for all of the low-frequency variation in yields. Our approach of assuming that interest rates contain a slow-moving common trend avoids both extremes. It also delivers term premium estimates that are in line with the common macro-finance priors and produces interest rate forecasts that outperform the random walk, as we will show below.

In recent work, Crump, Eusepi and Moench (2018) estimate the term premium from a statistical model with shifting endpoints and find a somewhat more variable term premium than we do. Based on their estimates, the term premium accounts for just over half of the variance of long-term interest rates, whereas it accounts for about one-third according to our estimates.[48] While we use an arbitrage-free term structure specification, the different results largely reflect how the shifting endpoint is identified. Crump et al. use survey data, and their i_t^* closely mirrors long-run survey expectations of the three-month T-bill rate. This estimate of i_t^* is fairly stable and implies a somewhat more limited role for trends and long-run expectations and greater variability in the residual term premium.

Finally, we note that when interpreting model-based estimates of the term premium, it is important to keep in mind that they assume rational expectations, which may not represent the expectations of investors at each point in time. As shown by Piazzesi, Salomao and Schneider (2015) and Cieslak (2018), statistical estimates of risk premia are affected by persistent forecast errors by investors, i.e., deviations from rational expectations.[49] While this caveat also applies to our

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[47]Furthermore, in theory, a decline in the term premium is consistent with the fall in the covariance between nominal bond returns and stock returns since the 1980s, as emphasized by Campbell, Sunderam and Viceira (2017). A decline in real risk premia would also be implied by a change in business cycle dynamics that increased persistent risk relative to transitory risk, as described by Campbell (1986) and Beeler and Campbell (2012).

[48]Specifically, we calculated the same decomposition of the sample variance of forward rates as in Table 2 of Crump, Eusepi and Moench (2018). At maturities of five years and beyond, they find that 53 to 56 percent of the variance of level is explained by the forward term premium. By contrast, using their sample period, we find that only 35 percent of the variance is explained by the term premium.

[49]While some might recommend that estimates of term premia should therefore rely exclusively on survey expectations, there is no guarantee that expectations from surveys of professional forecasters are any closer to the true subjective expectations of bond traders than statistical models.

estimates, models like ours that allow for shifting endpoints are better suited to capture changes in long-run investor expectations that can result from learning about macroeconomic trends and policies. Estimates of such models are also much less sensitive to the choice of sample period than stationary DTSMs, which require inference about the unconditional mean of the short-term interest rate, assumed to be constant (Orphanides and Wei, 2012). For these reasons, our shifting-endpoint model is likely to be better able to recover true historical investor expectations and bond risk premia.

F. *Out-of-sample forecasts of interest rates*

We now turn to out-of-sample (OOS) forecasts of long-term interest rates using our new shifting-endpoint DTSM. Despite many advances in yield curve modeling, the random walk model has proven very hard to beat when forecasting bond yields, largely reflecting the extreme persistence of interest rates (Duffee, 2013). But our results so far suggest that one might be able to obtain more accurate forecasts by accounting for the interest rate trend.

We construct out-of-sample forecasts using the *OSE* model, which proxies for i_t^* with the sum of the PTR estimate for π_t^* and the real-time estimate of r_t^* described in Section I. We will compare these forecasts to those from the *FE* model and from a driftless random walk.[50] The models are recursively estimated—that is, using an expanding estimation window using all data available up to each forecast date—starting in 1976:Q1 when five years of data are available.[51] We focus on forecasts of the ten-year yield; we have found results for other maturities to be qualitatively similar. We forecast at horizons of 4, 10, 20, 30, and 40 quarters, and because our data end in 2018:Q2 the last forecast date is 2008:Q2, for a total of 127 (overlapping) forecasts.[52] The top panel of Table 5 reports the root mean-squared errors in percentage points. We also calculate *p*-values for tests of equal finite-sample forecast accuracy using the approach of Diebold and Mariano (1995) (DM).[53] We calculate the DM *p*-values using standard normal critical values for one-sided tests of the null hypothesis that the *OSE* model does not improve upon the *RW* or *FE* forecasts. We find that the forecasts from our

[50]We do not include estimates from the *ESE* model because the estimation of this model using our MCMC sampler takes a significant amount of time, which renders recursive estimation with each additional available observation period as required for out-of-sample forecasting computationally too burdensome.

[51]A rolling scheme, which uses only a fixed number of observations for parameter estimation, allows for an easier asymptotic justification of tests for predictive ability (Giacomini and White, 2006) but requires a specific choice of the window length. We have also obtained forecasts with such a scheme, using a variety of different window lengths, and found qualitatively similar results as when using a recursive scheme.

[52]The last date for which we re-estimate the models is 2007:Q4, consistent with Section III.B.

[53]Following common use, we construct the DM test with a rectangular window for the long-run variance and the small-sample adjustment of Harvey, Leybourne and Newbold (1997). Monte Carlo evidence in Clark and McCracken (2013) indicates that this test has good size in finite samples. However, for very long forecast horizons, the long-run variance is estimated with considerable uncertainty because there are only a few such non-overlapping observations in our sample.

TABLE 5—ACCURACY OF OUT-OF-SAMPLE FORECASTS FOR THE TEN-YEAR YIELD

Panel A: Quarterly sample, 1976:Q3-2008:Q1	Horizon in quarters				
(127 quarters)	4	10	20	30	40
Random walk (RW)	1.33	1.85	2.52	2.60	2.88
Fixed endpoint (FE)	1.42	2.25	3.28	3.72	4.19
Observed shifting endpoint (OSE)	1.17	1.76	2.37	2.39	2.60
p-value: $OSE \geq RW$	0.05	0.00	0.00	0.03	0.04
p-value: $OSE \geq FE$	0.00	0.00	0.01	0.03	0.05
Panel B: Blue Chip sample, 1988:Q1-2011:Q4	Horizon in years				
(48 Blue Chip surveys)	1	2	3	4	5
Blue Chip (BC)	1.06	1.39	1.59	1.79	1.99
Random walk (RW)	0.85	1.08	1.21	1.37	1.56
Fixed endpoint (FE)	1.53	2.08	2.52	2.96	3.34
Observed shifting endpoint (OSE)	0.87	0.95	1.04	1.18	1.37
p-value: $OSE \geq BC$	0.10	0.08	0.15	0.18	0.20
p-value: $OSE \geq RW$	0.58	0.05	0.01	0.04	0.08
p-value: $OSE \geq FE$	0.00	0.00	0.00	0.00	0.00

Note: Root mean-squared errors for forecasts for the ten-year Treasury yield. Method RW is a driftless random walk, FE is a common stationary dynamic term structure model (DTSM), which has a fixed endpoint, and OSE is a model with an observed shifting endpoint that uses the proxy $i_t^* = \pi_t^* + r_t^{*,RT}$. The data are quarterly from 1971:Q4 to 2018:Q1. Panel A: forecasts for horizons from 4 to 40 quarters made each quarter from 1976:Q3 (once five years of data are available) to 2008:Q1 for a total of 127 (overlapping) observations. Panel B: forecasts for horizons from 1 to 5 years (yearly averages of quarterly forecasts) made each quarter preceding a Blue Chip survey (long-range forecasts in Blue Chip Financial Forecasts) from 1988:Q1 to 2011:Q4 (48 surveys). Models are re-estimated each quarter up to 2007:Q4 (expanding window). Last rows in each panel report one-sided p-values for testing the null hypothesis of equal forecast accuracy against the alternative that method OSE is more accurate, using the method of Diebold and Mariano (1995) with a small-sample correction.

common-trend model *OSE* are substantially and statistically significantly more accurate than the *FE* model. For example, when forecasting 10 years ahead, model *OSE* lowers the RMSE by about 40 percent relative to the *FE* model. The improvements relative to the *RW* are smaller but still statistically significant.[54] These results document that, relative to conventional stationary models, more accurate Treasury yields forecasts can be obtained by allowing for an underlying common trend in interest rates. Gains are even possible relative to the random walk benchmark. In contrast to the random walk forecast, which simply assumes all changes are permanent, accounting for i_t^*, the underlying source of the highly persistent changes in interest rates, improves forecast accuracy.[55]

Finally, we compare the accuracy of our statistical models to that of professional forecasters. Since 1988, the Blue Chip Financial Forecasts (*BC*) survey has asked its respondents for long-range forecasts of interest rates twice a year (Wolters Kluwer, 2019). The respondents provide their average expectations of the target variable for each of the upcoming five calendar years and for the subsequent five-year period—we will focus on the five annual forecast horizons. We match the available information sets by using only data up to the quarter preceding the survey date for our model-based forecasts, and we exactly match the forecast horizons with the *BC* forecasts by taking averages of model-based forecasts over the relevant calendar years. The sample includes 48 forecast dates from March 1988 to December 2011. The bottom panel of Table 5 shows the RMSEs of the survey forecasts and the three model-based forecasts. Shifting-endpoint forecasts based on i_t^* improve over the *RW* forecasts in this sample as well, which are better than the *BC* forecasts. Because of the smaller sample size and substantial overlap, the gains of the *OSE* model are not as strongly significant, and for some horizons they are insignificant. The gains relative to the *FE* model are again strongly significant. The reason for the poor performance of the survey forecasts is that they consistently overpredict future yields, in particular at long horizons (results not shown). Other studies have documented the poor performance of survey forecasts of interest rates (e.g., van Dijk et al., 2014), which contrasts with the good performance of survey-based inflation forecasts (Ang, Bekaert and Wei, 2007; Faust and Wright, 2013). Our results suggest that the underlying reason for this poor performance is that professional forecasters have not sufficiently accounted for the shifting long-run trend component in interest rates.

IV. Conclusion

This paper bridges the gap between macroeconomic and finance views of long-run trends. We provide new evidence that interest rates and bond risk premia

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[54]Additional, unreported analysis—using plots of differences in cumulative sums of forecast errors over time—shows that the forecast gains of *OSE* are not driven by certain unusual subperiods but instead are a consistent pattern over most of our sample period.

[55]Earlier research by van Dijk et al. (2014) found that, when forecasting interest rates, it is beneficial to link long-run projections of interest rates to long-run expectations of inflation, but their forecasts do not account for time variation in r_t^* and are not based on an arbitrage-free model.

are determined over time by variation in the perceived trend in inflation and the equilibrium real rate of interest. Indeed, allowing for persistent shifts in π_t^* and r_t^* is crucial to account for yield dynamics. To understand and forecast long-term interest rates and bond returns requires a time-varying trend component in yields. In the recent period, variation in the equilibrium real rate is even more consequential than movements in the inflation trend. Our analysis started by establishing the links between macroeconomic trends and yields using trend estimates from surveys and macroeconomic and statistical models. However, to understand these results, we formulated and estimated a new no-arbitrage model that simultaneously models the long-run trend and term premium in yields—a key contribution of our analysis. In particular, when estimated with just yield curve data, this model, which embeds a non-stationary i_t^*, provides a new finance-based measure of the trend in nominal interest rates that complements estimates in the macroeconomics literature. It also provides for the first time an internally consistent formulation of that equilibrium trend and bond risk premia.

Instead of the ubiquitous stationary dynamic yield curve models with constant means, our results strongly support specifications that allow for slow-moving changes in the long-run means of nominal and real interest rates. Building upon our new common-trend dynamic term structure model, much work remains in jointly estimating macro trends and yield curve dynamics. Three future research avenues seem particularly promising. First, a joint model with shifting endpoints of real yields, nominal yields, and inflation expectations could provide separate estimates of π_t^* and r_t^*. Second, combining our shifting-endpoint DTSM specification with the shadow-rate paradigm (Bauer and Rudebusch, 2016) would allow for a sophisticated treatment of the effective lower bound on interest rates, which may provide additional insights into macro and yield-curve trends over post-crisis period. Third, one could consider time variation in the variability of trends by adding stochastic volatility (as in Stock and Watson, 2007) in order to assess how our (mostly unconditional) results about the relative importance of trends are affected by taking a conditional perspective.

Appendix

Trend estimates

Table A.1 provides an overview of all of the trend estimates used in our analysis. The first five estimates of r_t^* are obtained from models in published studies, and the following four estimates of r_t^* are new. In Section II, we use a "filtered" estimate and a "real-time" estimate of r_t^*, which are averages of the three filtered and six real-time estimates, respectively. The real-time estimate of i_t^* is the sum of the PTR estimate of π_t^* and the real-time estimate of r_t^*. The *ESE* model estimate of i_t^* is described in Section III.B.

TABLE A.1—OVERVIEW OF TREND ESTIMATES

Trend	Source	Real-time	Filtered	Smoothed
r^*	Del Negro et al. (2017) (DGGT)	✓		✓
	Johannsen and Mertens (2016) (JM)	✓		✓
	Laubach and Williams (2016) (LW)		✓	✓
	Holston, Laubach and Williams (2017) (HLW)		✓	
	Kiley (2015)		✓	✓
r^*	UC	✓		✓
	Proxies	✓		✓
	SSM	✓		✓
	MA	✓		
π_t^*	PTR	✓		
i^*	Real-time	✓		
	ESE			✓

Note: Overview of trend estimates used in the paper: five external sources of r^*, our own four estimates of r^*, the PTR estimate of π^*, and two i^* estimates. "Real-time" indicates (pseudo) real-time estimation of the trend proxy, "filtered" indicates estimates from a (one-sided) Kalman filter using full-sample parameter estimates, and "smoothed" indicates (two-sided) Kalman smoother or full-sample Bayesian estimate of the trend. All trend estimates are quarterly from 1971:Q4 to 2018:Q1.

Our proxy estimate of the inflation trend π_t^* is the Fed's PTR measure, the perceived target rate (PTR) of inflation, see Federal Reserve Board of Governors (2019b). PTR, which is part of the Fed's FRB/US macroeconomic model, measures expectations for inflation in the price index of personal consumption expenditures (PCE). Consistent with this definition, our estimates of r_t^* are based on real interest rates relative to PCE inflation. Since 1979, PTR corresponds to long-run inflation expectations from the Survey of Professional Forecasters (SPF). Before 1979, it is based on estimates from a learning model for expected inflation. For details see Brayton and Tinsley (1996). Data are available at https://www.federalreserve.gov/econresdata/frbus/us-models-package.htm.

The estimates of r_t^* are plotted and described in the Online Appendix. All of these estimates are based on the assumption that the real rate contains a stochastic trend, i.e., a stochastic endpoint.

A SHIFTING-ENDPOINT DYNAMIC TERM STRUCTURE MODEL

Equation (3) specifies the evolution of the state variables $\mathbf{Z}_t = (\tau, \mathbf{P}_t')'$ under the real-world/physical probability measure $\mathbb{P}$. It has the form of a common trends representation and shows that Z_t is trivially cointegrated with one common trend. The same holds for yields, see equation (7). The VAR representation is

(B.1) $$\mathbf{Z}_t = \boldsymbol{\mu}_Z + \boldsymbol{\Phi}_Z \mathbf{Z}_{t-1} + \mathbf{v}_t, \quad \mathbf{v}_t = (\eta_t, \mathbf{u}_t')',$$

with

$$\boldsymbol{\mu}_Z = \begin{pmatrix} 0 \\ (\mathbf{I}_n - \boldsymbol{\Phi})\bar{\mathbf{P}} \end{pmatrix}, \quad \boldsymbol{\Phi}_Z = \begin{pmatrix} 1 & \mathbf{0}_{1\times N} \\ (\mathbf{I}_N - \boldsymbol{\Phi})\boldsymbol{\gamma} & \boldsymbol{\Phi} \end{pmatrix},$$

$$\boldsymbol{\Omega}_v = E(\mathbf{v}_t\mathbf{v}_t') = \begin{pmatrix} \sigma_\eta^2 & \boldsymbol{\gamma}'\sigma_\eta^2 \\ \boldsymbol{\gamma}\sigma_\eta^2 & \boldsymbol{\Omega} \end{pmatrix}.$$

The innovations to $\mathbf{P}_t$ are $\mathbf{u}_t = \boldsymbol{\gamma}\eta_t + \tilde{\mathbf{u}}_t$ and have covariance matrix $\boldsymbol{\Omega} = E(\mathbf{u}_t\mathbf{u}_t') = \boldsymbol{\gamma}\boldsymbol{\gamma}'\sigma_\eta^2 + \widetilde{\boldsymbol{\Omega}}$.

Equation (6) specifies the evolution under the risk-neutral measure $\mathbb{Q}$. We assume that under $\mathbb{Q}$ the yield factors follow a *stationary* VAR. This specification avoids the counterfactual implication of a unit root under $\mathbb{Q}$ that yields and forward rates diverge to minus infinity with maturity.

From the short rate equation (5) and the $\mathbb{Q}$-dynamics (6), it follows that the trend τ_t is *unspanned by yields*, meaning that there is no deterministic mapping from $\mathbf{P}_t$ (or yields) to τ_t. Formally, the requirement for τ_t to be an unspanned risk factor is

$$E^{\mathbb{Q}}(i_{t+h}|\mathbf{P}_t, \tau_t) = E^{\mathbb{Q}}(i_{t+h}|\mathbf{P}_t), \quad \text{for all } h,$$

see Joslin, Priebsch and Singleton (2014), which holds true because neither the short rate i_t nor expectations of future yield factors $\mathbf{P}_{t+h}$ depend on τ_t once we condition on $\mathbf{P}_t$. The key features of the risk-neutral distribution for $\mathbf{P}_t$ are its stationarity and its independence of τ_t.

The dynamics under $\mathbb{P}$ and $\mathbb{Q}$ together imply a stochastic discount factor (SDF) and prices of risk. The SDF is the projection of the economy-wide SDF on the risk factors driving bond prices, $\mathbf{P}_t$, as in Joslin, Priebsch and Singleton (2014). As usual in affine DTSMs, the SDF is exponentially affine:

$$(\text{B.2}) \qquad M_{t+1} = \exp(-i_t - \frac{1}{2}\boldsymbol{\lambda}'_t\boldsymbol{\lambda}_t - \boldsymbol{\lambda}'_t\boldsymbol{\varepsilon}_{t+1})$$

with $\boldsymbol{\varepsilon}_t = \mathbf{L}^{-1}\mathbf{u}_t$, where $\mathbf{L}\mathbf{L}' = \boldsymbol{\Omega}$. The risk prices $\boldsymbol{\lambda}_t$ are affine in $\mathbf{Z}_t$:

$$(\text{B.3}) \qquad \boldsymbol{\lambda}_t = \boldsymbol{\Omega}^{-1/2}(\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1\mathbf{Z}_t),$$

where $\boldsymbol{\lambda}_0$ is a N-vector and $\boldsymbol{\lambda}_1$ is a $N \times (N+1)$ matrix of risk sensitivities. The Online Appendix shows that

$$(\text{B.4}) \qquad \boldsymbol{\lambda}_0 = (\mathbf{I}_n - \boldsymbol{\Phi})\bar{\mathbf{P}} - \boldsymbol{\mu}^{\mathbb{Q}}$$

and

$$(\text{B.5}) \qquad \boldsymbol{\lambda}_1 = \left[(\mathbf{I}_N - \boldsymbol{\Phi})\boldsymbol{\gamma}, \boldsymbol{\Phi} - \boldsymbol{\Phi}^{\mathbb{Q}}\right].$$

A "knife-edge" restriction is imposed so that τ_t does not to affect $E^{\mathbb{Q}}(\mathbf{P}_{t+1}|\mathbf{Z}_t)$. Namely, the risk prices λ_t respond exactly in the same way to changes in τ_t as do the expectations $E(\mathbf{P}_{t+1}|\mathbf{Z}_t)$, and the sensitivities in each case are $(\mathbf{I}_N - \boldsymbol{\Phi})\boldsymbol{\gamma}$. This restriction implies that movements in τ_t lead to changes in term premia that exactly offset the changes in expectations, leaving yields unchanged.

Prices of zero-coupon bonds are exponentially affine, so that for the log-price of an n-period bond we have $p_t^{(n)} = \mathcal{A}_n + \boldsymbol{\mathcal{B}}'_n\mathbf{P}_t$, and the coefficients follow the usual recursions (e.g., Ang and Piazzesi, 2003):

$$(\text{B.6}) \qquad \mathcal{A}_{n+1} = \mathcal{A}_n + \boldsymbol{\mathcal{B}}'_n\boldsymbol{\mu}^{\mathbb{Q}} + \frac{1}{2}\boldsymbol{\mathcal{B}}'_n\boldsymbol{\Omega}\boldsymbol{\mathcal{B}}_n - \delta_0$$

$$(\text{B.7}) \qquad \boldsymbol{\mathcal{B}}_{n+1} = (\boldsymbol{\Phi}^{\mathbb{Q}})'\boldsymbol{\mathcal{B}}_n - \boldsymbol{\delta}_1$$

with initial conditions $\mathcal{A}_0 = 0$ and $\boldsymbol{\mathcal{B}}_0 = 0$. Yields are affine functions of the factors:

$$y_t^{(n)} = A_n + \mathbf{B}'_n\mathbf{P}_t \quad A_n = -\frac{1}{n}\mathcal{A}_n, \quad \mathbf{B}_n = -\frac{1}{n}\boldsymbol{\mathcal{B}}_n.$$

If we denote the vector of J yields that are included in our model as $\mathbf{Y}_t$, we can write $\mathbf{Y}_t = \mathbf{A} + \mathbf{B}\mathbf{P}_t$ for J-vector $\mathbf{A} = (A_{n_1}, \dots, A_{n_J})'$ and $J \times N$ matrix $\mathbf{B} = (\mathbf{B}_{n_1}, \dots, \mathbf{B}_{n_J})'$. For forward rates $f_t^{(n)} = p_t^{(n)} - p_t^{(n+1)}$ the Online Appendix shows that

$$(\text{B.8}) \quad f_t^{(n)} = -\frac{1}{2}\boldsymbol{\mathcal{B}}'_n\boldsymbol{\Omega}\boldsymbol{\mathcal{B}}_n + \delta_0 - \boldsymbol{\mathcal{B}}'_n\boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\delta}'_1(\boldsymbol{\Phi}^{\mathbb{Q}})^n\bar{\mathbf{P}} + \boldsymbol{\delta}'_1(\boldsymbol{\Phi}^{\mathbb{Q}})^n\boldsymbol{\gamma}\tau_t + \boldsymbol{\delta}'_1(\boldsymbol{\Phi}^{\mathbb{Q}})^n\tilde{\mathbf{P}}_t.$$

The convexity term in the expression for forward rates is $-\frac{1}{2}\boldsymbol{\mathcal{B}}'_n\boldsymbol{\Omega}\boldsymbol{\mathcal{B}}_n$. Since the $\mathbb{Q}$-dynamics are stationary, $\boldsymbol{\mathcal{B}}_n$ converges to a finite limit as n tends to infinity, along with the convexity in yields and forward rates. If, by contrast, $\boldsymbol{\Phi}^{\mathbb{Q}}$ had an eigenvalue on the unit circle, convexity and therefore yields and forward rates would diverge to minus infinity. This fact, discussed in Campbell, Lo and MacKinlay (1997, p. 433), is an important reason why term structure models are usually specified with stationary risk-neutral dynamics, and we follow this tradition here.

Loadings of yields and forward rates on the trend τ_t are hump-shaped with respect to maturity. Short rate loadings equal one since $\tau_t = i_t^*$. For longer maturities in our sample, the term premium responds positively to τ_t, so yields move more than one-for-one with the trend. Eventually, loadings decline very gradually toward their limit of zero, and the infinite-maturity forward rate/bond yield is a constant. Further details are provided in the Online Appendix.

Model-implied excess bond returns are

$$(B.9) \qquad rx_{t+1}^{(n)} = -\frac{1}{2}\boldsymbol{\mathcal{B}}'_{n-1}\boldsymbol{\Omega}\boldsymbol{\mathcal{B}}_{n-1} + \boldsymbol{\mathcal{B}}'_{n-1}\left(E_t\mathbf{P}_{t+1} - E_t^{\mathbb{Q}}\mathbf{P}_{t+1}\right) + \boldsymbol{\mathcal{B}}'_{n-1}\mathbf{u}_{t+1}.$$

thus expected excess returns, adjusted for convexity, are

(B.10)
$$E_t rx_{t+1}^{(n)} + \frac{1}{2}Var_t rx_{t+1}^{(n)} = \boldsymbol{\mathcal{B}}'_{n-1}\left[\lambda_0 + (\mathbf{I}_N - \boldsymbol{\Phi})\boldsymbol{\gamma}\tau_t + (\boldsymbol{\Phi} - \boldsymbol{\Phi}^{\mathbb{Q}})\mathbf{P}_t\right]$$
$$(B.11) \qquad\qquad = \boldsymbol{\mathcal{B}}'_{n-1}\left[\lambda_0 + (\boldsymbol{\Phi} - \boldsymbol{\Phi}^{\mathbb{Q}})\bar{\mathbf{P}} + (\mathbf{I}_N - \boldsymbol{\Phi}^{\mathbb{Q}})\boldsymbol{\gamma}\tau_t + (\boldsymbol{\Phi} - \boldsymbol{\Phi}^{\mathbb{Q}})\tilde{\mathbf{P}}_t\right].$$

Equation (B.10) shows that when holding the yield factors $\mathbf{P}_t$ constant, the loadings on τ_t are $\boldsymbol{\mathcal{B}}'_{n-1}(\mathbf{I}_N - \boldsymbol{\Phi})\boldsymbol{\gamma}$, and our model estimates imply substantially negative values for these loadings, across all n, in line with the regression estimates in Section II.B. Conditional on current yields, an increase in the trend implies lower expected returns. Equation (B.11) shows the separate roles of trends and cycles for bond risk premia. If we hold the yield cycles $\tilde{\mathbf{P}}_t$ constant, the loadings on τ_t are $\boldsymbol{\mathcal{B}}'_{n-1}(\mathbf{I}_N - \boldsymbol{\Phi}^{\mathbb{Q}})\boldsymbol{\gamma}$. These loadings tend to be an order of magnitude smaller (in absolute value) than the loadings on τ_t in equation (B.10), consistent with the findings in Section II.B that predictive regressions with only detrended yields capture most of the predictability of excess bond returns. The loadings on τ_t in (B.11) are positive because risk premia are positively related to movements in the trend. Unreported results show that predictive regressions with both detrended yields and trend proxies lead to coefficients on the trend that are insignificantly different from both zero and the model-based loadings—consistent with the implications of our model.

A term premium is the difference between a long-term interest rate and an expectations component of the same maturity. For simplicity, the expectations component is often specified as the average of expected future short rates, as

in equation (1). In a no-arbitrage term structure model, it is advantageous to instead use "risk-neutral rates" which, like yields, account for the convexity of bond prices. Specifically, the risk-neutral yield on an n-period bond is

$$\tilde{y}_t^{(n)} = -\log E_t \exp\left(-\sum_{j=0}^{n-1} i_{t+j}\right)/n,$$

which differs from the average expected future short rate $\sum_{j=0}^{n-1} E_t i_{t+j}/n$ by a Jensen's inequality term that is constant in a homoskedastic model like ours. The risk-neutral yields are affine in $\mathbf{Z}_t$, and the loadings are calculated using recursions similar to those in equations (B.6)- (B.7), with $\boldsymbol{\mu}_Z$ and $\boldsymbol{\Phi}_Z$ replacing $\boldsymbol{\mu}^{\mathbb{Q}}$ and $\boldsymbol{\Phi}^{\mathbb{Q}}$, $\boldsymbol{\Omega}_v$ replacing $\boldsymbol{\Omega}$, and the $N+1$ vector $(0, \boldsymbol{\delta}_1')'$ replacing $\boldsymbol{\delta}_1$. Crucially, risk-neutral yields depend on both τ_t and $\mathbf{P}_t$, since τ_t affects real-world expectations of future short rates. The term premium on an n-period bond is then defined as

$$TP_t^{(n)} = y_t^{(n)} - \tilde{y}_t^{(n)}.$$

Because the real-world dynamics feature a unit root, risk-neutral rates diverge to minus infinity (and the term premium to infinity) as maturity increases. This issue is neither of theoretical nor practical concern: First, no tradeable securities have payoffs tied to these quantities. Second, at the maturities we focus on, due to the high persistence of the risk-neutral dynamics, the convexity in actual and risk-neutral yields is quite similar, and estimates of the term premium are not noticeably affected by convexity.

Risk-neutral forward rates are

(B.12) $$\tilde{f}_t^{(n)} = -\frac{1}{2}\tilde{\boldsymbol{\mathcal{B}}}_n'\boldsymbol{\Omega}_v\tilde{\boldsymbol{\mathcal{B}}}_n + \tau_t + \boldsymbol{\delta}_1'\boldsymbol{\Phi}^n\tilde{\mathbf{P}}_t,$$

where $\tilde{\boldsymbol{\mathcal{B}}}_n$ is the loading of the log risk-neutral bond price with maturity n on $\mathbf{Z}_t$. The intuition for equation (B.12) is that $\tilde{f}_t^{(n)} = E_t i_{t+n}+$convexity, and short-rate expectations move one-for-one with the trend. Equations (B.8) and (B.12) imply that the forward term premium $ftp_t^{(n)} = f_t^{(n)} - \tilde{f}_t^{(n)}$ is

(B.13) $$ftp_t^{(n)} = \text{constant} + \boldsymbol{\delta}_1'\left[(\boldsymbol{\Phi}^{\mathbb{Q}})^n - \mathbf{I}_N\right]\boldsymbol{\gamma}\tau_t + \boldsymbol{\delta}_1'\left[(\boldsymbol{\Phi}^{\mathbb{Q}})^n - \boldsymbol{\Phi}^n\right]\tilde{\mathbf{P}}_t.$$

The loadings on τ_t have a limit of -1 as n tends to infinity. The limiting-maturity forward rate is constant, as the limiting-maturity forward term premium exactly offsets movements in the limiting-maturity risk-neutral rate. However, for maturities relevant in the real world, forward rates and yields have loadings on τ_t greater than one and positive term premium loadings—consistent with the cointegration regressions reported in Section II.A.

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