

## Learning and the Volatility of Exchange Rates

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This paper investigates the implications for the volatility of exchange rates of specifying market uncertainty as parameter uncertainty. Private agents learn by means of Bayes rule about a parameter of the stochastic process generating the exogenous variables. Learning is shown to magnify the reaction of exchange rates to random shocks to the 'market fundamentals'. This magnification effect of learning can explain the rejections of the econometric tests on the variance bounds and on the absence of bubbles that have been reported in the literature.

Perhaps the most striking aspect of the experience with a regime of floating exchange rates since 1973 has been the very large volatility of both nominal and real exchange rates. This volatility has not been excessive relative to that of other asset prices—see for instance Frenkel (1981). However, both the volatility of exchange rates and more generally the volatility of asset prices cannot be easily explained by existing rational expectations models: the variance bounds implied by the most common of these models have often been rejected; and some recent specification tests designed to check the hypothesis of no bubbles in these same models have met with a similar fate.<sup>1</sup>

This failure to explain the variability of asset prices and exchange rates has recently been attributed, by a number of economists, to the simplistic way in which financial-market uncertainty is generally specified in existing models. In most of such models, for instance, policy variables are assumed to follow simple linear stochastic processes, and private agents are supposed to have full current information and to know with certainty the parameters of the underlying processes. Flood and Hodrick (1986), Obstfeld and Rogoff (1986), Singleton (1987), among others, have suggested that potential changes in government policies, modelled as stochastic process switches, can account for some of the unexplained variability in exchange rates and asset prices.

This paper considers another respecification of market uncertainty that can account for some of the unexplained volatility of exchange rates: namely, uncertainty about the parameters of the stochastic process generating the exogenous variables. This specification of uncertainty is very close in spirit to the idea of stochastic process switching. It differs from it in the way in which it

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accounts for the endogenous evolution of market expectations: whereas in the case of process switching this evolution is governed exclusively by the properties of the exogenous stochastic process, in the case of parameter uncertainty the evolution of market expectations reflects the underlying subjective learning process of private agents.

The paper analyzes a simple monetary model of the exchange rate. Private agents ignore the true value of a parameter in the stochastic process generating the policy variables. Their learning process is described by means of Bayes rule. Eventually the learning process converges to the true parameter value. However, while learning goes on, exchange rates exhibit more volatility. The two main results of the paper are that:

1. Learning always increases the unconditional variance of exchange rates.
2. In the presence of learning, it is no longer possible to interpret the rejection of the variance bounds and specification tests mentioned above as an indication of bubbles: learning, rather than bubbles, can account for the rejection of these tests.

The paper outline is as follows. Section I describes the model and derives the solution. The learning process is illustrated in Section II. Section III analyzes the effects of learning on the volatility of exchange rates. The implications of learning for the econometric tests on bubbles are discussed in Section IV. Section V contains the conclusions.

### I. The Model

The monetary approach to exchange rates, together with the assumptions of open interest parity and equality of money demand coefficients in the domestic and foreign countries, implies the following equation (see for instance Bilson, 1978):

$$\langle 1 \rangle \quad s_t = x_t + \alpha_2(s_{t+1,t} - s_t) + u_t, \quad \alpha_2 > 0$$

where  $s_t$  is the log of the nominal exchange rate;  $x_t = m_t - \alpha_1 y_t$ , and  $m_t, y_t$  are the logs of money supply and output respectively (in the form of deviations from foreign variables);  $s_{t+1,t}$  is the expectation of the nominal exchange rate next period, based on the information available at time  $t$ ;  $u_t$  is a random shock that can be interpreted as either a deviation from purchasing power parity or as a shock to money demand; and  $\alpha_2$  is the interest semi-elasticity of money demand. Throughout the paper it will be assumed that  $u_t$  is a random walk:  $u_t = u_{t-1} + e_t$ ,  $e_t \sim N(0, \sigma_e^2)$ .

If at time  $t$  private agents can observe  $s_t$  and  $x_t$ , then equation  $\langle 1 \rangle$  can be iterated forward to obtain the following no-bubbles solution

$$\langle 2 \rangle \quad s_t - x_t - u_t = \sum_{i=1}^{\infty} \gamma^i \Delta x_{t+i,t}, \quad \gamma = \frac{\alpha_2}{1 + \alpha_2},$$

where  $\Delta x_{t+i,t} = x_{t+i,t} - x_{t+i-1,t}$ . Equation  $\langle 2 \rangle$  expresses the exchange rate as a linear function of the expected discounted present value of the exogenous variables  $\Delta x_{t+i,t}$ . A relationship analogous to  $\langle 2 \rangle$  is at the core of all variance bounds and specification tests for bubbles, in exchange rates or in any other asset prices.<sup>2</sup>

Following Meese (1986), I assume that the 'market fundamentals,'  $x_t$ , are driven

by a simple stochastic process on first differences:

$$\langle 3 \rangle \quad \Delta x_t = \delta + \beta \Delta x_{t-1} + v_t, \quad \delta \begin{matrix} \geq \\ \leq \end{matrix} 0, \quad |\beta| < 1,$$

$$v_t \sim N\left(0, \frac{1}{\tau}\right), \quad \tau > 0.$$

For simplicity, it is assumed that  $v_t$  and  $e_t$  are mutually and serially uncorrelated. More general stochastic processes would complicate the notation, without affecting the results in any respect.

Finally, and in contrast to the existing literature, private agents are assumed to be uncertain about some parameters of the model. Specifically, and so as to simplify the analysis, private agents are assumed to be uncertain about the true value of the parameter  $\delta$  in  $\langle 3 \rangle$ .<sup>3</sup> This form of parameter uncertainty can naturally be interpreted as arising from private agents having incomplete information about some crucial element of the policy-making decision process in either country, for instance, about the policy-maker's preferences or political constraints.

Based on their knowledge of the true model, private agents gradually learn about  $\delta$  from their current observations. This learning process will be described in Section II below. For now, we will simply denote the expected value of  $\delta$ , based on the market posterior beliefs updated to and including period  $t$ , with  $d_t$ .

On the basis of these assumptions, the closed form solution to equation  $\langle 2 \rangle$  can easily be shown to be:<sup>4</sup>

$$\langle 4 \rangle \quad s_t - x_t - u_t = \frac{\gamma}{1 - \gamma\beta} \left[ \beta \Delta x_t + \frac{1}{1 - \gamma} d_t \right].$$

Equation  $\langle 4 \rangle$  illustrates one of the central points of the paper. Because of the uncertainty about one of the parameters in the exogenous stochastic process  $\langle 3 \rangle$ , the set of fundamentals to which the equilibrium exchange rate reacts includes the market beliefs, as measured by the variable  $d_t$ . Any change in the market beliefs is reflected, through equation  $\langle 4 \rangle$ , in a change in the equilibrium value of the exchange rate. As a result, learning can add volatility to exchange rates. The next two sections will show that this is indeed the case.

## II. The Learning Process

The observation of the exogenous variable,  $\Delta x_t$ , enables financial markets to learn gradually about the unknown parameter  $\delta$ .<sup>5</sup> This section describes the learning process by means of Bayes rule.

As a preliminary step, rewrite equation  $\langle 3 \rangle$  as:

$$\langle 5 \rangle \quad \delta + v_t = z_t,$$

where  $z_t = \Delta x_t - \beta \Delta x_{t-1}$  summarizes the information about  $\delta$  that becomes available to financial markets in the course of period  $t$ . Now suppose that the market beliefs about  $\delta$  at the beginning of period  $t$  (*i.e.*, before the observation of  $z_t$  but after the observation of  $z_{t-1}$ ) have a normal prior distribution, with mean  $d_{t-1}$  and precision  $h_{t-1}$ . Then it follows from Bayes theorem that their posterior distribution at the end of period  $t$  (*i.e.*, after the observation of  $z_t$ ) is normal, with

mean

$$\langle 6 \rangle \quad d_t = \frac{d_{t-1}b_{t-1} + \tau z_t}{b_{t-1} + \tau}$$

and precision  $b_t = b_{t-1} + \tau$  (see, for instance, Cyert and De Groot, 1974).

Via  $\langle 5 \rangle$  and  $\langle 6 \rangle$ , it can be shown that  $\text{plim}_{t \rightarrow \infty} d_t = \delta$ , so that eventually the market beliefs will converge in probability to the true parameter value. However, any change in the policy regime or in the political constraints of the government, or any other permanent shock to some unobservable component of the stochastic process generating  $\Delta x_t$ , will give rise to new parameter uncertainty. Financial markets will then have to start their learning process again from a new set of prior beliefs.

### III. Learning and the Volatility of Exchange Rates

The results of the previous two sections will now be combined to explore the implications of learning for the volatility of exchange rates. To simplify the notation, throughout the rest of the paper it is assumed that  $\delta = 0$ .

In the absence of learning,  $d_t = 0$  and equation  $\langle 4 \rangle$  can be written in moving average form as:

$$\langle 7 \rangle \quad s_t - x_t - u_t = \frac{\gamma}{1 - \gamma\beta} \left[ \sum_{i=0}^{t-1} \beta^{i+1} v_{t-i} + \beta^{t+1} \Delta x_0 \right],$$

where  $\Delta x_0$  is the initial condition for the stochastic process described in  $\langle 3 \rangle$ .

In the presence of learning, substituting  $\langle 6 \rangle$  into  $\langle 4 \rangle$  and expressing  $d_t$  and  $\Delta x_t$  as moving averages, we obtain:

$$\langle 8 \rangle \quad s_t - x_t - u_t = \frac{\gamma}{1 - \gamma\beta} \left[ \sum_{i=0}^{t-1} \left( \beta^{i+1} + \frac{\tau}{(1-\gamma)(b_0 + \tau t)} \right) v_{t-i} + \beta^{t+1} \Delta x_0 + \frac{b_0 d_0}{(1-\gamma)(b_0 + \tau t)} \right],$$

where  $d_0$  and  $b_0$  are the initial conditions for the prior mean and prior precision respectively. Since  $\tau/(1-\gamma)(b_0 + \tau t) > 0$ , a comparison of  $\langle 7 \rangle$  and  $\langle 8 \rangle$  yields immediately:<sup>6</sup>

*Proposition 1: Learning increases the unconditional variance of exchange rates.*

An example can provide some intuition. Recall that the higher is the true value of the unknown parameter  $\delta$ , the larger is the actual rate of growth of excess money supply in the domestic country relative to the foreign country. Hence, the higher is  $\delta$ , the more the exchange rate will depreciate in the future. Suppose that  $v_t > 0$  occurs so that financial markets observe a particularly large increase in the domestic (relative to foreign) money supply,  $\Delta x_t$ . They will interpret this high value of  $\Delta x_t$  as an indication that the true value of  $\delta$  is larger than they had previously thought. As they revise upwards their estimate of  $\delta$ , and hence of  $\sum_{i=1}^{\infty} \gamma^i \Delta x_{t+i|t}$ , the exchange rate depreciates. In other words, uncertainty about the parameter  $\delta$  induces financial markets to systematically overestimate the permanent component of random shocks. As a consequence, this form of uncertainty increases the reaction of exchange rates to contemporaneous and past shocks to the 'market fundamentals'.

This magnifying effect vanishes over time, since the term  $\tau/(1-\gamma)(b_0 + \tau t)$  becomes smaller as  $t$  grows. Intuitively: as market participants cumulate observations, their prior precision,  $b_t = b_0 + \tau t$ , becomes larger and larger. Accordingly, each new observation will carry a smaller weight, and the exchange rate will be driven closer and closer to its full information path.

#### IV. Learning and the Econometric Tests on Bubbles

Can the magnifying effect of learning described in the previous section explain the rejection of the econometric tests on the hypothesis of no bubbles? In this section it is shown that the answer to this question is indeed positive.

The presence of bubbles in exchange rates has generally been tested in two ways: the first and most common one is a variance bound test (see Huang, 1981; West, 1986). This test can be briefly illustrated as follows. Define  $\mu_{t+i} = \Delta x_{t+i} - \Delta x_{t+i|t}$  as the market forecast error in predicting  $\Delta x_{t+i}$ , based on the information set available at time  $t$ . Equation <2> can then be rewritten as:

$$\langle 9 \rangle \quad (s_t - x_t - u_t) + \epsilon_t = \sum_{i=1}^{\infty} \gamma^i \Delta x_{t+i}$$

where  $\epsilon_t = \sum_{i=1}^{\infty} \gamma^i \mu_{t+i}$  is the present discounted value of all forecast errors. In the absence of learning,  $\epsilon_t$  is uncorrelated with  $(s_t - x_t - u_t)$ . The following inequality can then be derived from <9>:

$$\langle 10 \rangle \quad \text{Var} \left[ \sum_{i=1}^{\infty} \gamma^i \Delta x_{t+i} \right] = \text{Var}(\epsilon_t) + \text{Var}(s_t - x_t - u_t) \geq \text{Var}(s_t - x_t - u_t).$$

The variance bound stated in <10>, or some variant of it, has been tested in the empirical literature quoted above and in note 1. Often such a test has been strongly rejected by the data. This rejection has then been interpreted as indicating the presence of bubbles.

However, in the presence of learning, the term  $(s_t - x_t - u_t)$  in <10> is always *negatively* correlated with the discounted present value of the forecast errors,  $\epsilon_t$  (a proof is available from the author). Therefore, in the presence of learning, the inequality stated in <10> can no longer be derived: learning always increases the likelihood of rejecting the inequality stated in <10>, independently of whether or not there are bubbles. Indeed, for some parameter values and if the learning process has not been going on for a long time, it is possible to show that the inequality stated in <10> is bound to be reversed by the presence of learning.<sup>7</sup>

A second test recently designed to investigate the presence of bubbles in exchange rates or in other asset prices is an application of the Hausman (1978) specification test (see Meese, 1986; West, 1985). This test compares the estimates of the parameter  $\gamma$  obtained from two alternative specifications of the same model. The first specification is given by my equation <4>, expressed in first differences, the analogue of which is Meese's (1986) equation <12>:

$$\langle 11 \rangle \quad \Delta s_t = \Delta x_t + \frac{\gamma\beta}{1-\gamma\beta} (\Delta x_t - \Delta x_{t-1}) + e_t + \frac{\gamma}{(1-\gamma\beta)(1-\gamma)} (d_t - d_{t-1}).$$

The second specification is obtained from equation <1>, also expressed in first differences, the analogue of which is Meese's equation <15>:

$$\langle 12 \rangle \quad \Delta s_t - \Delta x_t = \gamma(\Delta s_{t+1} - \Delta x_t) + [(1 - \gamma)e_t - \gamma(\eta_{t+1} - \eta_t)],$$

where  $\eta_{t+1} = s_{t+1} - s_{t+1|t}$  is the market forecast error of the exchange rate at time  $t+1$ , based on the information available at time  $t$ . In the absence of learning,  $\eta_{t+1}$  and  $\eta_t$  are uncorrelated with any element of the information set available to private agents at time  $t-1$  or earlier. Hence, any element of such a set which is also correlated with  $(\Delta s_{t+1} - \Delta x_t)$  is a valid instrument (see McCallum, 1976). Meese uses  $\Delta x_{t-1}$  as an instrument of  $(\Delta s_{t+1} - \Delta x_t)$  to estimate  $\gamma$ . This estimate of  $\gamma$  is then compared with the corresponding estimate obtained from a simple OLS regression of <11>, again under the hypothesis of no learning. The two estimates are found to be significantly different. Since the estimate of  $\gamma$  obtained from <11> is consistent only in the absence of bubbles, whereas the estimate obtained from <12> is always consistent, Meese concludes that the rejection of this test can indicate the presence of bubbles.

It is clear, however, that the rejection of this test can be interpreted as due to the presence of learning. With learning, both estimates of  $\gamma$  are inconsistent, whether or not there are bubbles. The estimate obtained from <12> is inconsistent because the change in the forecast error,  $(\eta_{t+1} - \eta_t)$ , is positively correlated with the instrument,  $\Delta x_{t-1}$ . And the estimate obtained from <11> is also inconsistent, since in the presence of learning the error term of <11> includes the variable  $(d_t - d_{t-1})$ , and this variable is positively correlated with the explanatory variable  $(\Delta x_t - \Delta x_{t-1})$ . Hence, in the presence of learning the Hausman specification test is not valid.

In Meese the rejection of the test is due to the fact that the estimate of  $\gamma$  derived from its counterpart to <11> is significantly larger than the estimate derived from its counterpart to <12>. Going through some tedious algebra, we can show that the asymptotic bias in the estimate of  $\gamma$ , which is due to learning, is positive for both estimation procedures. The relative size of these biases can only be assessed by means of numerical simulations. Therefore here, unlike in the case of the variance bounds tests, it is not possible to establish that the bias introduced by learning always makes the rejection of the test in Meese more likely. Whether or not this happens, depends on the numerical values of the parameters.

We can summarize the foregoing discussion as follows. Bubbles tests are joint tests of the model, of the stationarity of the driving process for  $\Delta x_t$ , and of the absence of bubbles. Learning introduces a nonstationarity in the driving process of  $\Delta x_t$ , as perceived by market participants. Hence, a rejection of the variance bounds tests or of the Hausman (1978) specification test can be interpreted as indicating the presence of learning, rather than bubbles. The authors of the bubble tests are well aware of the joint nature of their tests. Indeed, testing for the stationarity of the driving process for  $\Delta x_t$  is an important aspect of their research strategy. With learning, however, the *actual* driving process for  $\Delta x_t$  can be stationary, and yet the process as perceived by market participants would fail to be so. Thus, learning would introduce a particularly insidious kind of nonstationarity, which would generally not be detected by the battery of stationarity tests performed in the existing literature.

## V. Concluding Remarks

Rational expectations models of exchange rates and asset prices generally embody a particular specification of market uncertainty: private agents ignore the future realizations of exogenous random variables; but they are supposed to know with certainty the parameters of the linear stochastic process generating these random variables.

According to a substantial body of empirical evidence, this class of models often fails to explain the large volatility of exchange rates and other asset prices. A common inference drawn from this failure is that financial markets are driven by bubbles, fads, or by other symptoms of more blatant irrationality on the part of market participants.

This paper has investigated the implications of an alternative specification of market uncertainty, namely, parameter uncertainty. This specification is obviously consistent with optimizing behavior on the part of private agents: the market reacts to this form of uncertainty by trying to learn about the unknown parameters from its current observations. The main result of the paper is that the market learning process magnifies the reaction of exchange rates to random shocks to the 'market fundamentals.' This magnification effect of learning can explain the rejections of the econometric tests on the variance bounds and on the absence of bubbles that have been reported in the literature.

Naturally, the proof of the pudding is, as always, in the eating. Thus, the most important implication to be drawn from the results of this paper is the potential worth of exploring the empirical relevance of alternative respecifications of market uncertainty along the lines illustrated in the previous pages. Indeed, one of the main advantages of the approach taken in this paper is that the formal description of the market learning process can rely on the modern tools of the theory of optimal statistical decisions. As a consequence, this specification of market uncertainty lends itself quite naturally to empirical investigations, such as those of Lewis (1986) and, in a different context, Baxter (1985).

### Notes

1. Variance bounds tests on asset prices were originally conceived by LeRoy and Porter (1981) and by Shiller (1979). See Merton (1985) for an up-to-date survey. Huang (1981) and West (1986) apply these tests to exchange rates. Meese (1986) and West (1985) carry out a specification test designed to detect the presence of bubbles in exchange rates and stock prices respectively.
2. The results presented in the text can easily be extended to a more general version of the same model, in which money demand includes a lagged term standing for partial adjustment behavior, as in Woo (1985).
3. Uncertainty about the remaining parameters,  $\beta$  in <3> and  $\alpha_1, \alpha_2$  in <1>, would introduce additional complications since  $\alpha_1, \alpha_2$  and  $\beta$  enter multiplicatively with other random variables rather than additively. Even though the intuition behind the results presented below would still be applicable (see p. 246 of the text), closed form expressions for both the exchange rate and its unconditional variance would be much more difficult to derive analytically.
4. Equation <4> has been derived by noting that

$$\Delta x_{t+i} = \beta^i \Delta x_t + d_t \sum_{j=0}^{i-1} \beta^j, \quad i \geq 1.$$

5. The results presented in this and the next section can easily be extended to the case where  $x_t$  is observed by market participants only with a time lag; in such a case, the market would also learn about  $\delta$  from its observation of the exchange rate.

6. Strictly speaking, Proposition 1 refers to the variance of exchange rates conditional on the initial conditions  $\Delta x_0, b_0, d_0$  but unconditional on any information that becomes available at  $t > 0$  through the observable variables  $x_t$  and  $s_t$ . That is, Proposition 1 describes the effects of learning from given prior beliefs,  $b_0$  and  $d_0$ , under the assumption that the only source of information about the unknown parameter  $\delta$  comes from the observations of the stochastic forcing variable  $x_t$ . If other random sources of information about  $\delta$  were available, then equation (8) would contain some additional ('extraneous') random variables. Such additional variables could add further variability to exchange rates. However, the comparison of (7) and (8) would be complicated by the presence of the covariance of these extraneous random variables with the forcing variable  $x_t$ .
7. Specifically, after going through some tedious algebra, we can show that the inequality stated in (10) will always be violated if the following condition holds:

$$\beta(1-\beta')(1-\gamma) > (1-\beta)(b_0 + \tau t).$$

which is more likely to be satisfied the smaller are the prior and sample precisions,  $b_0$  and  $\tau$ , and the smaller is  $t$ , the length of the time period for which learning has been going on.

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