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MONEY, DEBT AND DEFICITS IN A DYNAMIC GAME*

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This paper analyzes a dynamic linear-quadratic game between the fiscal and monetary authorities. The equilibrium outcome of the game determines the time path of money creation, fiscal deficits and public debt. The game is solved for different equilibrium concepts. The equilibria are then compared and interpreted with reference to alternative institutional arrangements. The main results are: (i) Coordination increases the speed of adjustment towards the steady state and takes the steady state value of public debt closer to the desired target. (ii) Precommitments take the non-cooperative equilibrium closer to the coordinated outcome. (iii) Increasing the weight that each policymaker assigns to his own private objectives slows down the adjustment process, places more burden on the opponent, but has ambiguous effects on the steady state value of public debt.

1. Introduction

The government budget constraint provides a dynamic link between fiscal deficits, the creation of monetary base and the time path of public debt. In most industrial countries, the size of fiscal deficits and the growth of monetary base are selected by two relatively independent authorities. Since these two authorities are subject to different political incentives and constraints, it is plausible to conjecture that they may have different objectives. In this case, strategic considerations are bound to play a major role in shaping monetary and fiscal policies. This paper investigates how this strategic interaction between the monetary and fiscal authorities manifests itself in the determination of the time path of public debt.

The paper focuses on a situation particularly relevant for a number of industrial countries since the beginning of the 1980s: The stock of public debt as a proportion of national income is assumed to be above the time path optimal for the two authorities. In this scenario, both authorities face a dilemma: Whether to adjust their policy instruments so as to slow down the

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rate of growth of public debt towards its optimal path; but in so doing they would be foregoing other (monetary or fiscal) objectives. Or not to adjust, in the hope that the burden of the adjustment will be borne mainly by the other policymaking authority; but in this case the adjustment of public debt may be slower or it may never come about.

These issues are analyzed here in the setting of a dynamic linear-quadratic game between the fiscal and monetary authorities. The equilibrium outcome of the game determines the time path of public debt, fiscal deficits and the monetary base. The game is solved for different equilibrium concepts. The equilibria are then compared and interpreted with reference to alternative institutional arrangements.

The main findings are: (i) In all the equilibria, monetary and fiscal policies are contingent on the stock of public debt outstanding. As a result, the time path of public debt over nominal income can be stable even if the real rate of interest exceeds the rate of growth of real output. (ii) Coordination of monetary and fiscal policies brings about a smaller steady state stock of public debt, and a more rapid adjustment of all variables towards the steady state. (iii) Enabling both policymakers to undertake binding commitments has the positive effect of moving the uncoordinated outcome closer to the cooperative benchmark. (iv) Increasing the weight that either player assigns to its own private (monetary or fiscal) objectives relative to the common goal of stabilizing public debt – i.e., increasing the degree of decentralization – has the following effects: (a) it brings about a slower adjustment; (b) it places a bigger burden of adjustment on the opponent; (c) it has ambiguous effects on the steady state stock of public debt, which depend on the nature of the equilibrium. Thus, for instance, increasing the weight that the central bank assigns to its (private) monetary objectives causes: Smaller monetization, smaller fiscal deficits, a slower adjustment, and either a smaller or a larger steady state stock of public debt, depending on the nature of the equilibrium and on the parameter values.

The outline of the paper is as follows. Section 2 sets up the basic model. Section 3 characterizes the general nature of the solution. Section 4 derives a closed-form expression for the feedback-Nash equilibrium. The gains from coordination are illustrated in section 5. Section 6 derives the Cournot–Nash equilibrium, in which both players can undertake binding commitments. The consequences of changing the degree of decentralization between the two players are investigated in section 7. Section 8 contains the conclusions.

2. The model

In order to focus on the strategic interaction between the two policymakers in the choice of public financial policies, the private sector is left out of the picture. Thus, the model presented in this section differs substantially from some of the recent literature on games of macroeconomic policy, which instead

has emphasized the strategic interaction between one or both policymakers and the private sector – see the surveys by Cukierman (1985), Fischer (1986) and Rogoff (1986). Modifying the simple model presented below by adding a strategic role for the private sector may add new insights into other issues, but should not affect the nature of the results.

The lack of an explicit description of the private sector behavior is reflected in two special features of the model presented below, which greatly simplify the analysis: the linearity of the law of motion of public debt and the particular specification of the loss functions of the two policymakers.

The law of motion of public debt is given by the government budget constraint:

$$\dot{d}(t) = rd(t) + f(t) - m(t), \tag{1}$$

where all variables have been scaled to nominal income, and where $d \equiv$ stock of nominal outstanding public debt, $f \equiv$ fiscal deficit net of interest payments, $m \equiv$ creation of monetary base against liabilities of the Treasury,[1] and where r can be shown to be the difference between the real rate of interest net of taxes and the rate of growth of real income. The hypothesis that r is a parameter implies that there is a flat demand for public debt. This assumption is needed in order to have a linear dynamics, and can be interpreted as saying that the economy is small and faces perfect international markets.

The loss functions of both policymakers are taken to be quadratic in the state variable, d, and in the policy instruments that enter the government budget constraint, m and f. This specification is consistent with the main goal of the paper, namely to analyze the conflict between the two policymakers over the determination of the time path of public debt.[2] However, it presupposes that each policymaker considers the relationship between its instruments and the ultimate macroeconomic targets as invariant to changes in the instruments of the other policymaker.[3]

Specifically, it is assumed that the monetary authorities (M) choose the time path of $m(t)$ to minimize

$$V^{M}(d(t)) = \tfrac{1}{2}\int_{t}^{\infty}\left[(m(s)-\bar{m})^2 + \tau d^2(s)\right]e^{-\beta(s-t)}\,ds, \tag{2}$$

$$\tau > 0.$$

[1] That is, $m(t) = (dM(t)/dt)/Y(t)$, where $M(t)$ is the stock of monetary base corresponding to liabilities of the Treasury and where $Y(t)$ is nominal income.

[2] See Pindyck (1976) for a game theoretic analysis of the coordination of monetary and fiscal policies in the pursuit of standard macroeconomic objectives.

[3] This specification of the policymakers' loss function is also subject to the Lucas critique: in the following sections, alternative equilibria are compared. The different equilibria reflect different institutional settings. Yet, the analysis neglects the fact that changes in the institutions may also change the policymakers' goals, $\bar{m}$ and $\bar{f}$, indirectly, through changes in the private sector behavior.

subject to the government budget constraint, eq. (1), to the process generating the time path of $f(t)$ – yet to be specified – and to an initial condition on $d(t)$. The fiscal authorities (F) choose the time path of $f(t)$ to minimize

$$V^{\mathrm{F}}(d(t)) = \tfrac{1}{2}\int_t^{\infty}\left[(f(s)-\bar{f})^2 + \lambda d^2(s)\right]\mathrm{e}^{-\beta(s-t)}\mathrm{d}s, \qquad \lambda > 0, \tag{3}$$

also subject to the government budget constraint and to some hypothesis about the time path of $m(t)$.

Eq. (2) states that M attempts to minimize deviations of changes in the domestic component of base money from a given constant target, $\bar{m}$, and deviations of the stock of outstanding public debt from a desired value, for notational convenience taken to be zero. The parameter τ indicates the relative weight assigned by M to the two objectives. Similarly, eq. (3) states that F attempts to minimize deviations of fiscal deficits net of interest payments from a given target, $\bar{f}$, and deviations of the stock of outstanding public debt from zero. The parameter λ indicates the relative weight assigned to the two objectives. Recall that all variables are expressed in proportion to nominal income.

The final objectives of monetary and fiscal policies can be thought of as being implicit in the desired targets for m, f and d. The target for m is presumably determined with reference to balance of payments or exchange rate objectives, or, in the case of a closed economy, to the desired rate of growth of nominal income or prices. The preferences about f, the value of fiscal deficits net of interest payments, could reflect both standard macroeconomic objectives and political considerations. The preferences about d, for both fiscal and monetary authorities, can be justified by appealing to the fact that, in the absence of lump-sum taxes, a larger stock of public debt implies larger tax distortions in order to pay interest on the debt. Moreover, whenever the interest rate fluctuates, the fluctuations of taxes needed to pay for the debt are larger the larger is the stock of public debt outstanding. In addition, higher levels of d can be associated with lower rates of capital accumulation or with higher levels of external debt – see, for instance, Blanchard (1985).

The model could be modified by adding some preferences for money growth in the loss function of F or some preferences about budget deficits in the loss function of M. This modification would not change the nature of the results, as long as the model retained its basic feature: namely, that both players would like to see a reduction in the stock of public debt outstanding; but M would like this to occur mainly through a reduction in fiscal deficits, whereas F would like it to come about mainly through an increase in debt monetization.

3. The general solution

The next three sections will characterize the solution of the dynamic game under different hypotheses about the strategy spaces and the rules of the game,

but always maintaining the assumptions of infinite horizon and complete information. Since the game is linear-quadratic, it is natural to consider only solutions of the following form:

$$m(t) = \theta_0 + \theta_1 d(t), \tag{4a}$$

$$f(t) = \pi_0 - \pi_1 d(t), \tag{4b}$$

$$\dot{d}(t) = (r - \pi_1 - \theta_1)d(t) + (\pi_0 - \theta_0), \tag{4c}$$

where θ_i and π_i are coefficients to be determined in equilibrium and which depend on the original parameters of the model. In all the cases considered, it will be shown that $\theta_1, \pi_1 > 0$. Thus, a sudden increase in the stock of public debt outstanding, due for instance to an unexpected cyclical deficit, will lead immediately to some debt monetization and to a reduction of fiscal deficits net of interest payments. Moreover, with $\theta_1, \pi_1 > 0$, the time path of public debt can be stable even if $r > 0$ (i.e., even if the real interest rate exceeds the rate of growth of real output). The presumption that $r < 0$ was necessary for the stability of the time path of public debt has played a major role in some recent discussions concerning the consequences of monetary and fiscal policy – see, for instance, Sargent and Wallace (1981). Eq. (4) highlights the more general result that the stability of public debt hinges on the size of r relative to that of θ_1 and π_1, rather than just on the sign of r. As shown in the following sections, if r is large relative to λ and τ, then it is possible that $r - \theta_1 - \tau_1 > 0$, so that the equilibrium dynamics are unstable. In this case, the transversality conditions that are derived below will be violated, and the equilibrium may fail to exist.

Indicating the stable root of the dynamic system by $-\gamma = r - \pi_1 - \theta_1$, whenever it exists, and integrating (4c), we have

$$d(t) = (d_0 - d^s)\mathrm{e}^{-\gamma t} + d^s,$$

where d_0 is the initial condition (i.e., the stock of public debt outstanding when the game is started) and where d^s is the steady state value of public debt. Throughout the paper only the case $d_0 > d^s$ will be considered, so that when the game is started, d is above its steady state value. Using (4), it follows that

$$m(t) = m^s + (m_0 - m^s)\mathrm{e}^{-\gamma t},$$

$$f(t) = f^s + (f_0 - f^s)\mathrm{e}^{-\gamma t},$$

where m^s and f^s are the steady state values of m and f, and where $m_0 = \theta_0 + \theta_1 d_0$ and $f_0 = \pi_0 - \pi_1 d_0$. From the hypothesis that $d_0 > d^s$ it then follows that $m_0 > m^s$ and $f_0 < f^s$. Thus, if the system is stable, over time monetary policy becomes more restrictive and fiscal policy more expansionary while the stock of public debt converges towards its steady state value.

4. Feedback-Nash equilibrium

This equilibrium is defined as follows: Each player's decision rule, as defined in (4), is the optimal response to the opponent's decision rule. It is the natural non-cooperative equilibrium to look at if both players move simultaneously and cannot commit to a specific course of action. Each player is taking into account that its own current actions will influence the future actions of the opponent through their effect on the time path of public debt – i.e., the opponent's decision rule, and not its future actions, is taken as given. Moreover, since the equilibrium strategies can be computed by means of dynamic programming methods, and since they are contingent on the current value of the state variable, they are based on optimal future behavior both on and off the equilibrium path. Hence this feedback-Nash equilibrium is subgame perfect and, *a fortiori*, it is also dynamically consistent. Since the game is linear-quadratic, this is also the only subgame perfect Nash equilibrium defined on the memoryless strategies given in eq. (4).

Here the solution is derived by means of optimal control methods, rather than by means of dynamic programming, in order to simplify the algebra. The proof that this solution is identical to the one obtained from dynamic programming, and hence that the equilibrium is subgame perfect, is available upon request.

Consider the optimization problem of the monetary authorities first. They know that future deficits will be set according to (4b); thus, substituting (4b) into (1), we obtain the following current value Hamiltonian:

$$H_{M} = \tfrac{1}{2}(m-\bar{m})^2 + \tfrac{1}{2}\tau d^2 + \mu_1(\pi_0 + (r-\pi_1)d - m), \tag{5}$$

where μ_1 is the costate variable associated with (1); the first-order conditions are

$$m = \bar{m} + \mu_1,$$
$$\dot{\mu}_1 = (\beta + \pi_1 - r)\mu_1 - \tau d. \tag{6}$$

Similarly, the current value Hamiltonian for the fiscal authorities is

$$H_F = \tfrac{1}{2}(f-\bar{f})^2 + \tfrac{1}{2}\lambda d^2 + \mu_2(-\theta_0 + (r-\theta_1)d + f), \tag{7}$$

where μ_2 is the costate variable associated with (1); the first-order conditions yield

$$f = \bar{f} - \mu_2,$$
$$\dot{\mu}_2 = (\beta + \theta_1 - r)\mu_2 - \lambda d. \tag{8}$$

In addition, the transversality conditions give the following sufficient condition [see Arrow and Kurz (1970), Cohen and Michel (1984)]:

$$\lim_{t\to\infty} \mu_i(t)d(t) = 0, \qquad i = 1, 2. \tag{9}$$

The equilibrium solution can be computed by means of the method of undetermined coefficients. Section A.1 of the appendix shows that θ_1 and π_1 must satisfy

$$\theta_1 = \frac{\tau}{\beta + \gamma^c + \pi_1 - r},$$
$$\pi_1 = \frac{\lambda}{\beta + \gamma^c + \theta_1 - r}, \tag{10}$$

where $\gamma^c = \theta_1 + \pi_1 - r$ is the rate of adjustment of the dynamic system to its steady state in this feedback-Nash equilibrium in which the players are choosing closed-loop strategies.

As in the existing related literature, it is not possible to show that an equilibrium exists for all parameter values. For r positive and large relative to λ and τ, the dynamic system characterized by (1), (6) and (8) may be unstable. In this case, the transversality condition (9) is violated, and an equilibrium may fail to exist. Papavassilopoulos et al. (1979) have shown that a sufficient condition for a feedback-Nash equilibrium to exist in a linear-quadratic set up is that the uncontrolled system be stable (here, that $r < 0$).

5. The benefits of cooperation

How does the non-cooperative equilibrium computed in the previous section differ from a set up in which monetary and fiscal policy decisions are coordinated by a single decision unit? The issue is very important, for such a cooperative equilibrium has a very natural interpretation in terms of institutional settings: Congress could be in charge of both fiscal and monetary policy decisions, for instance by setting operative guidelines for the time path of fiscal deficits, revenue from money creation and public debt for a prolonged interval of time.[4]

The answer is quite simple: from an analytical point of view, the dynamic game is transformed into an optimal control problem, by merging the two objective functions into a single one:

$$V(d(t)) = \min_{m(t),f(t)} \tfrac{1}{2}\int_t^\infty \Big[(f(s)-\hat{f})^2 + \omega(m(s)-\bar{m})^2 + (\lambda+\tau\omega)d^2(s)\Big]e^{-(s-t)\beta}\,ds, \tag{11}$$

where ω is the weight given to the original central bank objectives relative to those of the fiscal authority. The optimization problem is now solved by a single agent who controls both $m(t)$ and $f(t)$ and is subject to the government budget constraint, eq. (1). The first-order conditions are

$$f-\hat{f} = \omega(\bar{m}-m) = -\mu, \tag{12a}$$

$$\dot{\mu} = (\beta-r)\mu - (\lambda+\omega\tau)d, \tag{12b}$$

where μ is the costate variable associated with eq. (1). In addition, the transversality condition (9) and the initial condition on d still hold as before.

Eq. (12a) implies that the deviations of fiscal and monetary policy from their respective targets are proportional to each other, the constant of proportionality being ω, the weight assigned to the original central bank objectives. This makes intuitive sense: in a cooperative equilibrium both players internalize the costs borne by the opponent; thus the cost of deviations of f and m from their desired targets, weighted by ω, are equated at the margin.

The closed-form solution can be easily calculated either with the method of undetermined coefficients, or by solving the characteristic equation of the dynamic system consisting of (1) and (12). Section A.2 of the appendix contains the computations and shows that

$$\theta_1 = \frac{\tau+\lambda/\omega}{\gamma^p+\beta-r}, \qquad \pi_1 = \frac{\lambda+\omega\tau}{\gamma^p+\beta-r}, \tag{13}$$

where γ^p is the rate of adjustment to the steady state in this cooperative equilibrium.

[4] Institutional reforms of this kind have been advocated in the past and in recent times by a large number of economists. See, for instance, Friedman (1962) and Leijonhufvud (1984).

A comparison of (10) and (13) yields the following:

Proposition 1. The cooperative equilibrium has always a higher speed of adjustment, γ, and a lower steady state value of public debt, than the feedback-Nash equilibrium.

Proof. See section A.3 of the appendix.

The intuition behind this result can best be grasped by means of the following analogy. The model can be thought of as a game between two agents producing the same public good. The public good is the reduction of the time path of public debt towards its optimal value; the cost of producing it consists in the deviations of the policy instruments from their desired targets, $\hat{f}$ and $\bar{m}$. The parameter γ is then the rate at which the public good is produced. If the two agents cooperate, they internalize the benefit of the public good to the opponent, hence the quantity produced is larger than in the non-cooperative equilibrium (i.e., γ^p is larger than γ^c). As a result, in the steady state, the stock of public debt is closer to the desired target. Similar results are obtained, through numerical simulations and in different contexts, by Cooper (1969) and Hughes Hallett (1986).

6. The benefits of symmetric commitments

Here we compare the equilibria computed in the previous two sections with the following non-cooperative equilibrium: Monetary and fiscal policies are decentralized between two different authorities, but now these two authorities can simultaneously precommit to a future course of action. We can think of this set up as one in which two decentralized policymakers simultaneously submit a plan of their future course of action to a legislative authority, who then enforces such plans in the form of binding commitments, without attempting to bring about a coordinated outcome. Investigating the properties of this set up may be of interest, in the light of some recent results suggesting that precommitments may be desirable in a situation where the policymaker is playing a game against the private sector – see, for instance, Kydland and Prescott (1977). In the terminology of game theory, such a set up corresponds to that of a Cournot–Nash equilibrium (or an open-loop Nash equilibrium), in which each player takes as given the current and future actions of its opponent. More precisely, this Nash equilibrium is defined as follows: The time path of actions to which each player commits himself is an optimal response to the time path to which his opponent has committed.

The current value Hamiltonian for the monetary authorities now is

$$H_M = \tfrac{1}{2}(m-\bar{m})^2 + \tfrac{1}{2}\tau d^2 + \mu_1(rd+f-m). \tag{14}$$

The first-order conditions, in addition to (1), are

$$m = \bar{m} + \mu_1,$$
$$\dot{\mu}_1 = (\beta - r)\mu_1 - \tau d. \tag{15}$$

The same procedure yields the first-order conditions for the fiscal authorities:

$$f = \bar{f} - \mu_2,$$
$$\dot{\mu}_2 = (\beta - r)\mu_2 - \lambda d. \tag{16}$$

In addition, the transversality condition is still given by (9) in section 4.

The closed-form solution can be computed by means of the method of undetermined coefficients, repeating the same steps illustrated in section A.1 of the appendix, to obtain

$$\theta_1 = \frac{\tau}{\beta + \gamma^o - r},$$
$$\pi_1 = \frac{\lambda}{\beta + \gamma^o - r}, \tag{17}$$

where γ^o is the speed of adjustment in the open-loop Nash equilibrium.[5]

A comparison of (17) with (10) and (13) yields:

Proposition 2. The Cournot–Nash equilibrium has a higher speed of adjustment and a lower steady state public debt than the feedback-Nash equilibrium, but a lower speed of adjustment and a higher steady state public debt than the cooperative equilibrium.

Proof. See section A.3 of the appendix.

The analogy between reducing the stock of debt outstanding and producing a collective good can again provide some intuition. In the feedback-Nash, both players take into account that, whenever one of them reduces its production of the collective good, its opponent will find it optimal to step up its own production to some extent. In the Cournot–Nash, they are both prevented from taking advantage of this fact. As a result the equilibrium production is

[5] Here, unlike in the feedback-Nash equilibrium, γ^o is always a real number. It can be shown that a sufficient condition for the controlled dynamic system to be stable (and hence for the equilibrium to exist) is that $\lambda + \tau > r(\beta - r)$.

closer to the cooperative outcome (i.e., it is larger) in the Cournot than in the feedback-Nash equilibrium.[6]

7. Changing the degree of decentralization

This section analyses the consequences of changes in the parameters λ and τ in the loss functions of the two policymakers. As λ and τ are reduced towards zero, both players tend to care more about their private fiscal and monetary objectives, relative to the common goal of stabilizing public debt. Therefore, we can think of reductions in λ and τ as increasing the divergence in the policymakers' objectives.

Section A.4 of the appendix proves the following:

Proposition 3. Increasing the weight that either player assigns to its own private objectives relative to the common goal of stabilizing public debt (i.e., reducing λ or τ) has the effect of: (i) reducing the speed of adjustment, γ, both in the Cournot–Nash and in the feedback-Nash equilibria; (ii) placing a larger burden of the adjustment of the opponent (i.e., $\partial\theta_1/\partial\lambda, \partial\pi_1/\partial\tau < 0$), both in the Cournot–Nash and in the feedback-Nash equilibria; (iii) increasing the steady state value of public debt in the Cournot–Nash equilibrium.

Consider for instance a reduction in τ, the weight in the central bank loss function. This induces monetary policy to become less responsive to the stock of debt outstanding (i.e., θ_1 falls), and at the same time it forces the fiscal authority to become more responsive (i.e., π_1 increases). On balance, the first effect prevails over the second one, and the speed of adjustment of d towards the steady state is reduced. The same net effect occurs (for opposite reasons) when λ is reduced.

This result has an interesting economic interpretation, since presumably the parameters λ and τ reflect the feature of the institutional framework in which monetary and fiscal policies are chosen, such as the degree of decentralization between the two policymakers. In particular, we can think of τ as being a measure of the degree of independence of the central bank from the fiscal authority. If the fiscal authority could choose a specific value for τ, it would clearly set $\tau \to \infty$, for then $f = \bar{f}$ and $d = 0$. The fiscal authority would always be at its bliss point, and monetary policy would have the only role of financing the desired target for fiscal deficits, $\bar{f}$.[7] Two examples of such an institutional

[6] The result that commitments bring the Nash equilibrium closer to the cooperative outcome is not altogether new. A similar result was derived, in a different set up, by Reinganum and Stokey (1985).

[7] Note that letting $\tau \to \infty$ in the non-cooperative equilibria of sections 4 and 6 is formally equivalent to letting $\omega \to 0$ in the cooperative equilibrium of section 5. In other words, assigning zero weight to the monetary objectives in the cooperative equilibrium is equivalent to having a central bank which has completely lost its independence from the Treasury and whose only role is to provide sufficient revenue from money creation.

set up are provided by the United States before the 'Fed–Treasury accord' of 1951 and by Italy before the 1981 'divorce' between the Treasury and the Bank of Italy: In both situations, the monetary authorities had the explicit (in the US) or implicit (in Italy) role of sustaining the market price of Treasury Bills. Domestic monetary policy in these two cases was essentially determined by the choices of the fiscal authority concerning the budget deficit. The US accord and the Italian divorce shielded the central bank from political pressures and gave it some freedom to pursue its own monetary objectives. The consequences of this kind of institutional reform can be captured, within our model, by looking at how a reduction in τ affects the non-cooperative equilibrium of the game. Proposition 3 then suggests that, if the two policymakers move simultaneously (i.e., in a Nash equilibrium), an increase in the degree of independence of the central bank from the Treasury tends to reduce both the extent of debt monetization (i.e., θ_1 is reduced) and the size of fiscal deficits net of interest payments (i.e., π_1 is increased). The net effect is to reduce the speed of adjustment towards the steady state and, if both players choose open-loop strategies, to increase the steady state stock of public debt.

Note however that Proposition 3 cannot sign the effect that changes in λ or τ have on the steady state stock of public debt in the feedback-Nash equilibrium. This ambiguity also extends to equilibrium notions other than those considered so far. Specifically, a previous version of this paper contained a numerical simulation of a feedback-Stackelberg equilibrium, with the fiscal authorities as the leader. It was shown that a reduction in τ (i.e., an increase in the degree of independence of the central bank) would cause a reduction in the long-run values of both fiscal deficits and debt monetization, with ambiguous net effects on the long-run value of public debt, which depended on numerical values of the parameters.

8. Conclusions

In most industrial countries, monetary and fiscal policies are decentralized between two different authorities. Yet, both authorities are subject to the same dynamic budget constraint governing the time path of public debt. To the extent that decentralization gives rise to conflicting objectives, we can view the time path of debt, deficits and money creation as being the equilibrium outcome of a dynamic game played by the two policymakers.

This paper has derived the closed-form solution of such a game under alternative equilibrium concepts, interpreting each equilibrium with reference to a specific institutional arrangement. The main results are: (i) Coordination of monetary and fiscal policies brings about a more rapid adjustment and takes the steady state value of public debt closer to its desired target. (ii) Precommitments can be a substitute for coordination (albeit an imperfect one) in the sense that they take the non-cooperative equilibrium closer to the

coordinated outcome. (iii) Increasing the degree of decentralization between the two policymaking authorities slows down the adjustment towards the steady state, but has ambiguous effects on the steady state value of public debt. In particular, increasing the degree to which the central bank cares about its own private (monetary) objectives relative to the common goal of stabilizing public debt has the effects of: reducing debt monetization, reducing fiscal deficits, slowing down the adjustment process, with ambiguous effects on the steady state value of public debt.

These qualitative results have been obtained without imposing arbitrary restrictions on the parameters of the model, other than the linear-quadratic form for the dynamics and the loss functions respectively. Therefore, it is plausible to conjecture that they would readily extend, though with different economic interpretations, to other circumstances in which rational players interact strategically in a dynamic framework.

Appendix

A.1. Solution of the feedback-Nash equilibrium

From (4),

$$\dot{m} = \theta_1 \dot{d}, \qquad -\dot{f} = \pi_1 \dot{d}. \tag{A.1}$$

From (6) and (8), using (4),

$$\dot{m} = \dot{\mu}_1 = (\beta + \pi_1 - r)(\theta_0 + \theta_1 d - \bar{m}) - \tau d, \qquad -\dot{f} = \dot{\mu}_2 = (\beta + \theta_1 - r)(\bar{f} - \pi_0 + \pi_1 d) - \lambda d. \tag{A.2}$$

Putting together (A.1) and (A.2),

$$\dot{d} = \frac{1}{\theta_1}(\beta + \pi_1 - r)(\theta_0 + \theta_1 d - \bar{m}) - \tau d, \qquad \dot{d} = \frac{1}{\pi_1}(\beta + \theta_1 - r)(\bar{f} - \pi_0 + \pi_1 d) - \lambda d. \tag{A.3}$$

Equating coefficients of d in (A.3) with those of d in (1), using (4), we get

$$(\beta + \pi_1 - r) - \frac{\tau}{\theta_1} = r - \theta_1 - \pi_1 = (\beta + \theta_1 - r) - \frac{\lambda}{\pi_1}, \tag{A.4}$$

which immediately yields (10) in the text.

A.2. Solution of the cooperative equilibrium

The characteristic equation of the dynamic system made up of (1) and (12) is

$$Z^2-\beta Z+\left(r(\beta-r)-(\lambda+\omega\tau)\left(\frac{1+\omega}{\omega}\right)\right)=0,$$

whose negative (stable) solution is

$$-\gamma^p=\frac{\beta-\sqrt{(\beta-2r)^2+4(\lambda+\omega\tau)\left(\frac{1+\omega}{\omega}\right)}}{2}. \tag{A.5}$$

The initial condition at $t=0$, together with (12), yields the expressions for θ_1 and π_1 stated in (13).

A.3. Proof of Propositions 1 and 2

Substitute (17) in $\gamma^o=\theta_1+\pi_1-r$, to obtain

$$(\gamma^o)^2+\beta\gamma^o+(r(\beta-r)-(\tau+\lambda))=0.$$

Solving for γ^o and choosing the stable root [as implied by the transversality condition (9)] gives

$$\gamma^o=\frac{-\beta+\sqrt{(\beta-2r)^2+4(\tau+\lambda)}}{2}. \tag{A.6}$$

Comparing (A.6) and (A.5) yields that $\gamma^p>\gamma^o$.

In order to compare γ^c with γ^o, rewrite (A.4) as follows:

$$\theta_1^2+2\theta_1\pi_1+(\beta-2r)\theta_1-\tau=0,$$
$$\pi_1^2+2\theta_1\pi_1+(\beta-2r)\pi_1-\lambda=0. \tag{A.7}$$

Summing both terms and letting $x=\theta_1+\pi_1$, we obtain

$$x^2+(\beta-2r)x-(\tau+\lambda)=2\theta_1\pi_1<0. \tag{A.8}$$

In order to compare the solution of (A.8) with the open-loop Nash-equilibrium, notice that (17) implies

$$y^2+(\beta-2r)y-(\tau+\lambda)=0,$$

where $y=(\theta_1^o+\pi_1^o)$, and θ_1^o,π_1^o are the parameters of the open-loop Nash equilibrium given in (17) of the text. Since the right-hand side of (A.8) is negative, it follows that $x<y$. Recalling that $\gamma^o=y-r$ and that $\gamma^c=x-r$, we can conclude that $\gamma^o>\gamma^c$. Hence, a fortiori, $\gamma^p>\gamma^c$. This completes the proof of the statements in Propositions 1 and 2 concerning the speeds of adjustment.

The steady state equilibrium values of d can be computed from (1) and from the first-order conditions reported in the text, by setting $\dot d=\dot\mu_1=\dot\mu_2=0$. The comparison of the steady state values of d in the different equilibria then simply involves some tedious algebra.

A.4. Proof of Proposition 3

The statement that $\partial\gamma^o/\partial\tau,\ \partial\gamma^o/\partial\lambda>0$ follows immediately from the expression in (A.6). That same expression, together with (17), also implies that $\partial\theta_1/\gamma\tau,\ \partial\pi_1/\partial\lambda>0$, and that $\partial\theta_1/\partial\lambda,\ \partial\pi_1/\partial\tau<0$.

In order to prove that $\partial\gamma^c/\partial\tau,\ \partial\gamma^c/\partial\lambda>0$, use (A.4) and the fact that $\gamma^c=\theta_1+\pi_1-r$, to solve for θ_1 and π_1 in the feedback-Nash equilibrium

$$\theta_1=\frac{\beta+2\gamma^c-\sqrt{(\beta+2\gamma^c)^2-4\tau}}{2}>0,$$
$$\pi_1=\frac{\beta+2\gamma^c-\sqrt{(\beta+2\gamma^c)^2-4\lambda}}{2}>0, \tag{A.9}$$

where the negative sign in front of the square roots has been chosen in conformity with McCallum (1983) criterion of 'minimal set of state variables': when $\lambda=0$ we want $\pi_1=0$, and when $\tau=0$ we want $\theta_1=0$. If we substitute (A.9) back into $\gamma^c=\theta_1+\pi_1-r$, we obtain

$$g(\gamma^c,\tau)=2(\gamma^c+\beta-r)-\sqrt{(\beta+2\gamma^c)^2-4\tau}-\sqrt{(\beta+2\gamma^c)^2-4\lambda}$$
$$=0. \tag{A.10}$$

Applying the implicit function theorem to the function $g(\cdot)$ defined in (A.10) we then obtain that $\partial\gamma^c/\partial\tau,\ \partial\gamma^c/\partial\lambda>0$. Applying the same procedure to (A.8) yields that $\partial\theta_1/\partial\tau,\ \partial\pi_1/\partial\lambda>0$ and that $\partial\pi_1/\partial\tau,\ \partial\theta_1/\partial\lambda<0$. The statements of Proposition 3 concerning the steady state value of d can be proved with some algebra, by following the steps indicated at the end of the previous section.

References

Arrow, K. and M. Kurz, 1970, Public investment, the rate of return, and optimal fiscal policy (John Hopkins Press, Baltimore, MD).

Blanchard, O., 1985, Debt, deficits and finite horizons, Journal of Political Economy 93, 223–247.

Cohen, D. and P. Michel, 1984, Toward a theory of optimal precommitment, CEPREMAP working paper no. 8412.

Cooper, R., 1969, Macroeconomic policy adjustment in interdependent economies, Quarterly Journal of Economics, Feb., 1–24.

Cukierman, A., 1985, Central bank behavior and credibility: Some recent developments, Mimeo. (University of Tel Aviv, Tel Aviv).

Fischer, S., 1986, Time consistent monetary and fiscal policies: A survey, Mimeo. (M.I.T., Cambridge, MA).

Friedman, M., 1962, Should there be an independent monetary authority?, in: L. Yeager, ed., In search of a monetary constitution (Harvard University Press, Cambridge, MA).

Hughes Hallett, A.J., 1986, Autonomy and the choice of policy in asymmetrically dependent economies, Oxford Economic Papers, forthcoming.

Kydland, F. and E. Prescott, 1977, Rules rather than discretion: The inconsistency of optimal plans, Journal of Political Economy 85, 473–491.

Leijonhufvud, A., 1984, Constitutional constraints on the monetary powers of governments, Economia delle Scelte Pubbliche 1, 56–70.

McCallum, B., 1983, On non-uniqueness in rational expectations models: An attempt at perspective, Journal of Monetary Economics 11, 139–168.

Papavassilopoulos, G.P., J.V. Medanic and J.B. Cruz, 1979, On the existence of Nash strategies and solutions to coupled Riccati equations in linear-quadratic games, Journal of Optimization Theory and Applications 28, 49–76.

Pindyck, R., 1976, The cost of conflicting objectives in policy formulation, Annals of Economic and Social Measurement 5, 239–248.

Reinganum, N. and N. Stokey, 1985, Oligopoly extraction of a non-renewable common property resource: The importance of the period of commitment in dynamic games, International Economic Review 26, 161–174.

Rogoff, K., 1986, Reputational constraints on monetary policy: Credible rules or incredible theories?, Carnegie–Rochester Conference Series on Public Policy, forthcoming.

Sargent, T. and N. Wallace, 1981, Some unpleasant monetarist arithmetics, Federal Research Bank of Minneapolis Quarterly Review 5, 7–17.