Web Appendix to Democratic Capital: The nexus of Political and Economic Change

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I.

PROOF OF LEMMA 1. — Maximizing equation (3) in the text w r t k_t and setting $k_t = k_t^*$, the equilibrium capital stock in period t satisfies:

$$(1) \qquad -V_c(w - k_t^*) + (1 + (1 - p_t^*)\theta) = 0 \; .$$

Applying the implicit function theorem to (1) completes the proof. The boundary conditions on $V_c(\cdot)$ and concavity of $V(\cdot)$ imply a unique equilibrium with positive capital.

DERIVATION OF LEMMA 2. — What is the equilibrium behavior of agents at stage 2, when nature has instigated a coup or uprise at stage 1? The cost of participation is borne irrespective of whether the coup fails or not (uprise succeeds or not). Suppose that agent j observes a noisy signal of this cost:

$$m_t^j = \mu_t + v_t^j \; ,$$

where μ_t is the true individual cost and v_t^j is normally distributed noise. Each agent holds the (improper) prior that μ_t has a uniform distribution on the real line.

The benefit b_t is enjoyed only if the defense succeeds. Hence, the expected benefit from participation is $b_t s_t$, where s_t is the probability that democracy succeeds, and it coincides with the fraction of old who participate in defense of democracy. Each individual old agent treats the probability of success as independent of her own participation. When individual j does not participate, she bears no cost and gets no social benefit of the defense. Thus, we normalize the utility from non-participation to 0.

In this notation, the expected net gain from participation in defense of democracy for individual j is:

$$E(b_t - \mu_t) = b_t s_t - m_t^j \; .$$

Under these assumptions, old individuals play a global game with incomplete information, which fulfills the conditions **A1-A5** in Morris and Shin (2002, Section 2.2.1). By their results, all individuals follow an identical strategy $\sigma(m_t^j)$ of participating ($\sigma = 1$), or not ($\sigma = 0$), based

upon a unique cutoff value for their signal:

$$\sigma(m_t^j) = \begin{cases} 1 & \text{if } m_t^j < \mu_t^* = \frac{b_t}{2} \\ 0 & \text{if } m_t^j \geq \mu_t^* = \frac{b_t}{2} \, . \end{cases}$$

This strategy reflects a strategic complementarity, but the game nevertheless has a unique equilibrium. In this equilibrium, the fraction of old who defend democracy is:

$$(2) \qquad s_t^* = \text{Prob}(v < b_t/2 - \mu_t) \equiv \Phi(b_t/2 - \mu_t) \,,$$

where $\Phi(\cdot)$ is the c.d.f. of the normally distributed noise v.

Substituting equation (5) in the text into (2), the equilibrium probability of autocracy becomes

$$(3) \qquad p_t^* = \begin{cases} \chi(1 - E_{t-1}[s_t^*|k_t^*; \, \theta, \, d]) & \text{if } a_{t-1} = 0 \\ 1 - \chi E_{t-1}[s_t^*|k_t^*; \, \theta, \, d] & \text{if } a_{t-1} = 1 \,, \end{cases}$$

where the expectations operator is taken over the random variable μ_t, conditional on a_{t-1} and (perfectly foreseen) value of k_t^*. Finally, we can rewrite (3) to define:

$$(4) \qquad p_t^* = P(k_t^*, a_{t-1}; \, \theta, d) \,,$$

where the properties of P listed in Lemma 2 follow from definition (5) in the text and the monotonicity of (2).

PROOF OF LEMMA 3. — Applying the implicit function theorem to equation (4) in the text and (4), we have:

$$\begin{aligned} \tilde{P}_d &= \frac{P_d}{\Delta} & \tilde{K}_d &= \frac{P_d}{\Delta} \frac{\theta}{V_{cc}} \\ \tilde{P}_w &= \frac{P_k}{\Delta} & \tilde{K}_w &= \frac{1}{\Delta} \,, \end{aligned}$$

where

$$\Delta = 1 - P_k \, K_p \,.$$

If $|P_k| \, |K_p| < 1$, $\Delta > 0$. By Lemmas 1 and 2, parts (i) and (ii) of Lemma 3 follow. To prove part (iii), repeat the above procedure to get:

$$(5) \qquad \tilde{P}_\theta = -\frac{1}{\Delta} \left\{ \frac{(1 - p_t^*)P_k}{V_{cc}} + \frac{\chi k_t^* E_{t-1}\phi(b_t/2 - \mu_t)}{2} \right\} \,,$$

where the derivation of (4) implies:

$$(6) \qquad P_k = -\chi \theta E_{t-1} \phi(b_t/2 - \mu_t)/2 \,.$$

Inserting (5) in (6), we obtain:

$$\tilde{P}_\theta = \frac{\chi}{2\Delta} E_{t-1} \phi(b_t/2 - \mu_t)/2 \left\{ \frac{(1 - p_t^*)\theta}{V_{cc}} - k_t^* \right\} .$$

Thus, $\tilde{P}_\theta < 0$ if $\Delta > 0$ and $\theta > \hat{\theta}_t \equiv \frac{k_t^* V_{cc}}{1 - p_t^*}$, where $\hat{\theta}_t < 0$. Finally, part (iv) follows directly from equation (7) in the text. QED

DERIVATION OF LEMMA 4. — To study the dynamics of sorting, it is useful to adapt the notation introduced in the text. Define as $\overline{p}^a$ (respectively $\underline{p}^a$) the equilibrium probability that a country with $\theta = \overline{\theta}$ (resp. with $\theta = \underline{\theta}$) is autocratic in t, given that in $t-1$ it was in regime $a_{t-1} = 0, 1$. By Lemma 3 and the law of large numbers:

$$(7) \qquad \begin{aligned} \overline{p}^a &= \widetilde{P}(a_{t-1}; \overline{\theta}, d, w), \ a_{t-1} = 0, 1 \\ \underline{p}^a &= \widetilde{P}(a_{t-1}; \underline{\theta}, d, w), \ a_{t-1} = 0, 1 . \end{aligned}$$

These probabilities are constant over time. Lemma 3 says that $\overline{p}^a < \underline{p}^a$ unless $\underline{\theta} < 0$ is not too far below $\hat{\theta} < 0$, which we have assumed.[1] In words, starting from any political regime, the probability of autocracy is higher for countries economically better off in that regime. Finally, the persistence result in Lemma 3 says that $\overline{p}^1 = \overline{p}^0 + (1 - \chi)$, and similarly, $\underline{p}^1 = \underline{p}^0 + (1 - \chi)$.

How do countries with different values of θ sort themselves into political regimes over time? Recall that the fraction $\overline{n}_t^a$ (resp. $\underline{n}_t^a$) of countries with $\theta = \overline{\theta}$ (resp $\theta = \underline{\theta}$) have regime a in period t, and that the $\overline{\theta}$ countries sum to λ, while the $\underline{\theta}$ countries sum to $1 - \lambda$. Given that countries can only be in one regime, we have:

$$(8) \qquad \begin{aligned} \overline{n}_t^0 &= \lambda - \overline{n}_t^1 \\ \underline{n}_t^0 &= 1 - \lambda - \underline{n}_t^1 . \end{aligned}$$

Hence, it is sufficient to characterize the law of motion for each productivity type in one regime, say autocracy. The dynamics of the shares within autocracy is:

$$(9) \qquad \begin{aligned} \overline{n}_t^1 &= \overline{n}_{t-1}^1 \overline{p}^1 + (\lambda - \overline{n}_{t-1}^1)\overline{p}^0 \\ \underline{n}_t^1 &= \underline{n}_{t-1}^1 \underline{p}^1 + (1 - \lambda - \underline{n}_{t-1}^1)\underline{p}^0 . \end{aligned}$$

For each productivity type, the first term on the right hand side corresponds to former autocracies that remain under autocracy, and the second term corresponds to former democracies that switch

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[1]If $\hat{\theta} < \underline{\theta}$, we always have $\overline{p}^a < \underline{p}^a$ by Lemma 3. If $\underline{\theta} < \hat{\theta} < 0$, we have

$$\overline{p}^a = \underline{p}^a + \int_{\underline{\theta}}^{\hat{\theta}} P_\theta(\theta)d\theta + \int_{\hat{\theta}}^{\overline{\theta}} P_\theta(\theta)d\theta .$$

The first positive integral is dominated by the second negative intergral unless the distance between $\underline{\theta}$ and $\hat{\theta}$ is large.

to autocracy. As already noted, $\overline{p}^1 = \overline{p}^0 + (1 - \chi)$. Solving (9) for a steady-state with constant shares, we have:

$$(10) \qquad\qquad \overline{n}^1 = \frac{\lambda \overline{p}^0}{\chi}, \qquad \underline{n}^1 = \frac{(1 - \lambda)\underline{p}^0}{\chi} \ .$$

Note that $\overline{n}^1$ is always smaller than λ, because $\overline{p}^1 = \overline{p}^0 + (1 - \chi)$ implies $\chi > \overline{p}^0$. Moreover, (9) implies that the steady state is dynamically stable with monotonic convergence.

As the probability of autocracy is higher for the countries more productive in that regime ($\overline{p}^a < \underline{p}^a$), countries sort themselves accordingly over time. Thus, relative to the world average, low-θ countries will be over-represented among the autocracies and under-represented among the democracies, and vice versa for high-θ countries. In our notation, the world ratio of high-θ to low-θ countries is $\lambda/(1 - \lambda)$. Monotonic convergence to the steady state directly imply Lemma 4, because a random initial allocation corresponds to the odds ratio $\lambda/(1 - \lambda)$ in both political regimes.

DERIVATION OF PREDICTION 1. — Prediction (1a) follows from Lemma 3, as a higher value of d reduces p_t^* irrespective of the value of θ. Prediction (1b) is about the relative effects of w in democracy and autocracy. By Lemma 3, $\widetilde{P}_w \lesseqqgtr 0$ as $\theta \gtreqqless 0$. A higher value of w raises k_t^*, which raises the value of defending democracy if $\theta = \overline{\theta} > 0$, but has the opposite effect if $\theta = \underline{\theta} < 0$. By sorting, however, more $\overline{\theta}$ countries find themselves under democracy than under autocracy, so (by Lemma 4) the weight on the first term in equation (9) in the text is bigger under democracy. Since the weights in each regime add up to one and $\widetilde{P}_w(0; \theta, d, w) = \widetilde{P}_w(1; \theta, d, w)$ (by Lemma 3), the algebraic effect is smaller in the sample of democracies than in autocracies. Finally, prediction (1c) is about the *sign* of lagged income in equation (9) in the text. Again, the derivative with respect to w has a negative first term and a positive second term. The weight on the negative term ($\frac{\overline{n}_{t-1}^a}{\overline{n}_{t-1}^a + \underline{n}_{t-1}^a}$) is increasing in λ, while the weight on the positive term ($\frac{\underline{n}_{t-1}^a}{\overline{n}_{t-1}^a + \underline{n}_{t-1}^a}$) is decreasing in λ. By continuity, the exists a value of λ, say λ^a, such that the two effects exactly cancel out. For $\lambda > \lambda^a$, $\frac{\partial p_t^a}{\partial w} < 0$. By part (b), $\lambda^1 > \lambda^0$.

VARIABLE DEFINITIONS AND DATA SOURCES. — The following is a list of the variables we use and their sources:

Africa: Regional dummy variable, equal to 1 if a country is in Africa, 0 otherwise.

Asia_middle east: Regional dummy variable, equal to 1 if a country is in the Middle East, 0 otherwise.

Current domestic democratic capital: Domestic democratic capital accumulated during the current democratic spell. It equals 0 over autocratic spells. Over democratic spells, the variable is equal to the difference between the value of *Domestic democratic capital* in the current year and its value at the end of the previous spell. Source: authors' calculations on Polity IV Project.

Democracy: Binary variable that captures the state of democracy of country i in year t. This measure is defined in two ways. In columns 1 to 4 of Table 2 and in Tables 3 and 6,

Democracy is equal to 1 if the variable $polity2$ in the Polity IV data set is strictly positive, and zero if $polity2$ is 0 or negative. In columns 5 and 6 of Table 2, the *Democracy* index is defined as in Boix and Rosato's (2001) extension of the index constructed by Przeworski et al (2000). This definition emphasizes the turnover of political power in fair and free elections, and is available from 1800 until 1994. Sources: Boix and Rosato (2001); Polity IV Project.

Domestic democratic capital: Defined by expression (12) in the text, which ranges from 0 to 1. We calculated it for different values of the depreciation rate $(1 - \delta)$. For each country, the initial value (at the year of independence or at the year 1800, whichever comes last) of domestic democratic capital is assumed to be zero. Domestic democratic capital then accumulates in years of democracy and depreciates geometrically, at a rate $(1 - \delta)$, in years of autocracy. Source: authors' calculations on Polity IV Project.

Duration of current autocratic spell: defined as the difference between the current year and the starting year of the current spell. Source: authors' calculations on Polity IV Project.

Duration of current democratic spell: defined as the difference between the current year and the starting year of the current spell. Source: authors' calculations on Polity IV Project.

Esp_colony: Dummy variable equal to 1 if a country is a former Spanish colony, 0 otherwise. Source: Wacziarg (1996).

Foreign Democratic Capital: Defined by expression (13) in the text, divided by 10, such that its value ranges from 0 to 1. It is the weighted average of the continuous variable Polity2 in neighboring countries, taken from the Polity IV data set (see *Democracy*). The weights correspond to the distance between capitals. The index depends on the value of ρ, which identifies the boundaries of what is considered neighborhood. In the regressions showed in the text $\rho = 1$, i.e., all countries in the world are included in the neighborhood. Sources: authors' calculations on Polity IV Project.

Foreign income: Defined by expression (16) in the text. It is a weighted average of the log of real per capita output in the neighboring countries, with weights equal to the distance between capitals. Source: Maddison (2001)

Government Anti Diversion Policies: Index of government's anti-diversion policies, measured over the period 1986-95. It is an equal-weighted average of these five categories: i) law and order, ii) bureaucratic quality, iii) corruption, iv) risk of expropriation and v) government repudiation of contracts (each of these items has higher values for governments with more effective policies towards supporting production) and ranges from 0 to 1. Source: Hall and Jones (1999).

Human capital: Years of schooling of the population above 25 years of age. Annual measure constructed in Persson (2005) by interpolating the five-year observations from Barro and Lee. Sources: Persson, 2005; Barro and Lee, 2000

Initial constraints on the executive: Constraints in the executive in the year of independence (source: Polity IV)

Initial democracy score: $Polity2$ score in the year of independence, when democracy is defined as $polity2 > 0$ (source: Polity IV). Dummy variable equal to 1 if a democracy in the year of independence, when democracy is defined as in Boix and Rosato (2001).

More than five regime switches: Dummy variable equal to 1 for countries that had more than five regime switches between autocracy and democracy, or vice versa since independence.

Past domestic democratic capital: Democratic capital accumulated over previous spells.

For autocratic spells, the index is equal to the corresponding value of *Domestic democratic capital.*. For democratic spells, the index is equal to the value of *Domestic democratic capital* at the end of the previous spell, depreciating at a rate $(1 - \delta)$ over the current spell. Source: authors' calculations on Polity IV Project.

Per capita income: log of per real capita output adjusted for purchasing power parity. Source: Maddison (2001).

Period: linear time trend

Period Squared: quadratic time trend

Socialist legal origin: Dummy variable equal to 1 if a country's legal system has socialist origin, 0 otherwise. Source: La Porta et al. (1999)

Socialist transition: Dummy variable equal to 1 after 1989 for former socialist countries in Central and Eastern Europe and the Asian provinces of the former Soviet Union

Thinks democracy is best: Index of individuals' opinions on democracy, defined as the country average of the opinions on the statement "Democracy may have problems but it's better than any other form of government", as expressed in the World Values Survey (WWS) data set on a 4 point scale, from 1=strongly agree to 4=strongly disagree (question v163 in wave 3 and 4 of the survey). Missing and don't know answers were dropped and the average normalized, so that its value ranges from 0 to 1. Most observations are from the fourth wave of the WWS, in 1999-2000. For a few countries, data refer to the third wave, in 1995. Source: World Values Survey dataset (http://www.worldvaluessurvey.org/services/index.html)

UK_colony: Dummy variable equal to 1 if a country is a former British colony, 0 otherwise. Source: Wacziarg (1996).

War Dummy variable equal to 1 if a country is at war over a certain year, 0 otherwise. A war is defined as any kind of war (internal or external). Source: Correlates of War: http://www.correlatesofwar.org/

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