CONDITIONAL OFFERS IN TAKEOVERS WITH A LARGE SHAREHOLDER
Prelimenary and Incomplete

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We study tender offers for a firm owned by one large shareholder who holds less than half of the total shares, and many small shareholders each of whom holds a unit share. Each shareholder is privately informed, yet uncertain, about the raiders ability to improve the value of the firm, whereas the raider is uninformed. We show that, a raider would be indifferent between making an unconditional offer and a conditional offer for the shares of the company.

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1. Introduction

The market for corporate control may be inefficient due to a free-riding problem, if the ownership of the company is widely dispersed (see Grossman and Hart (1980), Bagnoli and Lipman (1988)). A small shareholder's impact on the success probability of the takeover is very small. Therefore, if the raider's price offer is smaller than the post-takeover value of a share, then the shareholder who anticipates that the takeover will succeed refuses to sell his share, which in turn causes the takeover to fail. Consequently, the small shareholders rip the profits brought about by the new management, while the raider ends up empty-handed.

In Ekmekci and Kos (2012), we showed that, if there is a large shareholder who holds a minority stake in the company, then the raider cannot make profit. However, if in addition to the large shareholder, there is uncertainty about the raider's ability to increase the value of the firm, and if the shareholders have private information about the raider's ability, then the raider may make strictly positive profit. In other words, the existence of a large shareholder together with asymmetric information facilitates profitable takeovers.

More specifically, Ekmekci and Kos (2012) study a model with incomplete information and a large shareholder. In their model, with some probability the state is low and the value-added from the takeover is zero. With the complementary probability, the state is high and the value-added is positive if the takeover is successful. The value of the company is unchanged if the takeover is unsuccessful. Each shareholder observes a private and imperfectly informative signal about the state. The raider, who has no information beyond the common prior about the state of the world, submits an unconditional offer for the equity shares of the firm by specifying a price per share. Then, each small shareholder decides either to accept the offer and tender his share or reject the offer and keep his share, while the large shareholder decides how many shares to tender. The takeover succeeds only if the raider acquires at least half of all the shares.

In this paper, we study a similar model in which the raider submits a conditional offer to the shareholders. In particular, the raider buys all shares tendered as long as the fraction of shares tendered is at least 50% of the firm, and buys no shares if the fraction of shares tendered is less than 50% of the firm. We show that, both conditional and unconditional offers yield the same equilibrium profits for the raider. Moreover, the profits after any price offer, including off-equilibrium prices, in the conditional offer model are identical to those in the unconditional offer model.

2. Model

We study the takeover game with a continuum of shares. There is a continuum of shares represented by the interval $[0, 1]$, where a mass of $(1 - x)$ is held by a continuum of small shareholders of size $(1-x)$, and a mass x of the shares is held by a large shareholder. The larger shareholder is not large enough to facilitate the takeover by selling all of his shares, i.e. $x < 1/2$.

The raider who wants to acquire a firm offers a conditional price offer p for the shares of the firm. If the total fraction of tendered shares is greater than 1/2, then the raider gets the tendered shares, pays the price p for every share, and takes over the control of the firm. If the total fraction of tendered shares is less than 1/2, then the raider does not buy any shares, and does not takeover the control of the firm. If the fraction of the tendered shares is exactly 1/2, then nature chooses from a lottery, whether the takeover is successful or not, and the raider pays price p for every share only if the takeover is successful.

2.1. Information There are two states of the world, $\omega \in \{h, l\}$, and the common prior belief that $\omega = h$ is $\lambda \in (0, 1)$. The value of the firm is 1 only if the takeover is successful and the state of the world is h, and is 0 otherwise. Each small shareholder observes a signal conditionally i.i.d signal $s \in [0, 1]$ drawn from a distribution $F(s|\omega)$ with the density function $f(s|\omega)$, where ω is the true state of the world. Large shareholder, instead, observes a signal drawn from distribution $H(s|\omega)$, with the density $h(s|\omega)$ and we assume that his signal is independent of the small shareholders' signals. We assume that the monotone likelihood ratio property (MLRP) holds for the shareholders' distributions, i.e. $\frac{f(s|h)}{f(s|l)}$ and $\frac{h(s|h)}{h(s|l)}$ are strictly increasing on the interval $[0, 1]$.[1,2]

2.2. Strategies A strategy for the raider is a price offer $p \in [0, \infty)$. Every price offer induces a tender subgame that we describe below. In a tender subgame with a fixed price offer $p \geq 0$, a small shareholder either sells his share or keeps it, while the large shareholder decides how many shares to sell. In particular, a mixed strategy for a small shareholder, σ, specifies the probability with which he tenders his share for every signal he observes:

$$\sigma : S \to [0, 1].$$

The term $\sigma(s)$ denotes the probability that a small shareholder tenders his share conditional on receiving signal s. A strategy for the large shareholder is σ_L, which is a right continuous and weakly increasing mapping $\sigma_L : S \times [0, 1] \to [0, 1]$, which denotes the cumulative distribution function of the fraction of the shares he tenders, and whose marginal on its first coordinate coincides with the distribution of signals. Modeling the strategy as a cumulative distribution function helps avoid measurability problems (see Milgrom and Weber (1985)). The function

[1]Notice that the strictly increasing condition implies that all the densities are larger than zero and finite for all $s \in (0, 1)$.

[2]We assume strict MLRP solely for expositional purposes. All of our results go through if we assume the weak version of MLRP, i.e., $\frac{f(s|h)}{f(s|l)}$ and $\frac{h(s|h)}{h(s|l)}$ are weakly increasing in s. Weak MLRP accommodates, among other things, a formulation with finitely many signals.

σ_L is weakly increasing in both of its arguments, and for every $s \in [0,1]$ satisfies

$$\sigma_L(s,1) = \lambda H(s|h) + (1-\lambda)H(s|l). \tag{1}$$

The above condition ensures that the marginal of σ_L on its first coordinate is equal to the signal distribution. The set of strategies of the large shareholder, Σ_L, is the set of all strategies (distributions) satisfying equality 1. In addition, we introduce the conditional distributional strategies, $\sigma_L(s,r|\omega)$ as:

$$\sigma_L(\bar{s},\bar{r}|\omega) := \int_{s=0}^{\bar{s}} \int_{r=0}^{\bar{r}} \frac{h(s|\omega)}{\lambda h(s|h) + (1-\lambda)h(s|l)} d\sigma(s,r).$$

Note that $\sigma_L(s,1|\omega) = H(s|\omega)$, for every $s \in [0,1]$.

A strategy for a small shareholder, σ, is a threshold strategy if there exists a signal $s^* \in S$ such that $\sigma(s) = 1$ for every $s < s^*$ and $\sigma(s) = 0$ for every $s > s^*$.

A strategy profile is a collection $\{\sigma_i\}_{i \in [0,1-x] \cup L}$, and a strategy profile is *symmetric* if $\sigma_i = \sigma_j$ for every $i, j \in [0, 1-x]$. A typical symmetric strategy profile is a tuple (σ, σ_L).

2.3. Payoffs A small shareholder's expected payoff from tendering his share is the price p. The expected payoff from keeping his share depends on his beliefs $q, q_t \in [0,1]$ that the tender is successful in state h, and that the tender is successful in state l, respectively.[3] Let

$$\beta(s) := \frac{\lambda f(s|h)}{\lambda f(s|h) + (1-\lambda) f(s|l)},$$

be the posterior belief of a small shareholder that the state is h given his signal s. In particular,

$$u(p, s, q, q_t, keep) = \beta(s) q,$$

and

$$u(p, s, q, q_t, sell) = p[\beta(s) q + (1-\beta(s)) q_t].$$

Similarly, let the posterior belief that the state is h for the large shareholder with signal s be $\beta_L(s) := \frac{\lambda h(s|h)}{\lambda h(s|h) + (1-\lambda) h(s|l)}$. For a collection of beliefs $q_L := \left(q(r)_{r\in[0,1]}, q_t(r)_{r\in[0,1]}\right)$, the expected payoff from tendering a fraction $r \in [0,1]$ of his shares is given by:

$$u_L(p, s, q_L, r) = x\left(rp[q_t(r)(1-\beta_L(s)) + q(r)\beta_L(s)] + (1-r)q(r)\beta_L(s)\right).$$

[3]The roles and the meanings of the concepts as the beliefs q and q_t introduced here will be clearer when we define the equilibrium concept below.

Large shareholders payoff comes from two sources, from the shares he offers for sale to the raider and from the shares he withholds.

Finally, the raider's payoff when he offers the price p and believes the tender to be successful as specified by the vector of probabilities $q_L := \left(q(r)_{r\in[0,1]}, q_t(r)_{r\in[0,1]}\right)$ is given by:

$$\begin{aligned} U_R(p,\sigma,\sigma_L,q_L) &= \lambda \int_{r,s} q(r)\left(xr + (1-x)\int_{s\in[0,1]} \sigma(s) f(s|h) ds\right) d\sigma_L(s,r|h) \\ &- \lambda p \int_{r,s} q(r)\left(xr + (1-x)\int_{s\in[0,1]} \sigma(s) f(s|h) ds\right) d\sigma_L(s,r|h) \\ &- (1-\lambda) p \int_{r,s} q_t(r)\left(xr + (1-x)\int_{s\in[0,1]} \sigma(s) f(s|l) ds\right) d\sigma_L(s,r|l). \end{aligned}$$

2.4. Equilibrium A tuple $T = (\sigma, \sigma_L, q, q_t, q_L)$ is a *symmetric* equilibrium of a tender sub game with a price offer p if the following conditions hold:

$$u(p,s,q,q_t,\sigma(s)) \geq u(p,s,q,q_t,a),\ \forall a \in \{keep, sell\}\ ,\ \forall s \in [0,1], \tag{2}$$

$$\int_{s\in[0,1], r\in[0,1]} u_L(p,s,q_L,r) d\sigma_L(s,r) \geq \int u_L(p,s,q_L,r) d\bar{\sigma}_L(s,r),\ \forall \bar{\sigma}_L \in \Sigma_L, \tag{3}$$

$$q(r) = \begin{cases} 0, & \text{if } (1-x)\int_0^1 \sigma(s) dF(s|h) + xr < 1/2 \\ 1, & \text{if } (1-x)\int_0^1 \sigma(s) dF(s|h) + xr > 1/2 \\ \in [0,1], & \text{if } (1-x)\int_0^1 \sigma(s) dF(s|h) + xr = 1/2, \end{cases} \tag{4}$$

$$q_t(r) = \begin{cases} 0, & \text{if } (1-x)\int_0^1 \sigma(s) dF(s|l) + xr < 1/2 \\ 1, & \text{if } (1-x)\int_0^1 \sigma(s) dF(s|l) + xr > 1/2 \\ \in [0,1], & \text{if } (1-x)\int_0^1 \sigma(s|l) dF(s|l) + xr = 1/2, \end{cases} \tag{5}$$

$$q = \int_{s\in[0,1], r\in[0,1]} q(r) d\sigma_L(s,r|h). \tag{6}$$

$$q_t = \int_{s\in[0,1], r\in[0,1]} q_t(r) d\sigma_L(s,r|l). \tag{7}$$

The first two conditions simply require that the small shareholders and the large shareholder behave optimally given their beliefs. Conditions (4) and (5) describe the large shareholder's beliefs about the probability of the successful takeover conditional on the high state and the unconditional belief given the strategies of the small shareholders and given that he himself tenders fraction r of his shares. The expected number of shares to be tendered by the small shareholders in the high state, given the strategy σ, is certain and is equal to $(1-x)\int_0^1 \sigma(s)dF(s|h)$. Therefore, if the large shareholder tenders fraction r of his shares and $(1-x)\int_0^1 \sigma(s)dF(s|h) + xr > (<) 1/2$ the tender succeeds (fails) with certainty. The troublesome case is when $(1-x)\int_0^1 \sigma(s)dF(s|h) + xr = 1/2$. We leave the large shareholder's beliefs to be determined in equilibrium in such a knife-edge case. Finally, conditions (6) and (7) require that the small shareholders' beliefs, q and q_t, about the success of the takeover in the high and low states is the expected value of the large shareholder's beliefs.

We denote the raider's continuation payoff from offering p when the tuple $T = (\sigma_s, \sigma_L, q, q_t, q_L)$ is played in the tender sub game as:

$$\Pi(p, T) := U_R(p, \sigma, \sigma_L, q_L).$$

Note that, a price $p > 1$ can never result in a positive payoff for the raider, therefore we restrict his price offer to the interval $[0, 1]$. We say that the collection $(p, T(p')_{p' \in [0,1]})$ is an equilibrium of the takeover game, if each $T(p')$ is an equilibrium of the tender sub game with price offer p', and if $p \in argmax_{p' \in [0,1]} \Pi(p', T(p'))$

When there is a unique equilibrium of a tender sub game for a given price p, we simply use $\Pi(p)$ to refer to the payoff of the raider when all other players play the unique equilibrium of the tender sub game.

3. Equilibrium Characterization

Note that, in any sub game with a price offer $p > 0$, there is a symmetric equilibrium where no one tenders any share. This is because, not tendering any share is a best reply when the probability of a successful takeover is zero. And when no one tenders any share, then the probability of a successful takeover is indeed zero, making this an equilibrium outcome. Such equilibria result in zero payoff for the raider. We will from hereon consider non-trivial equilibria where either $q_t > 0$ or $q > 0$.

We start by observing that the small shareholders use threshold strategies in all the tender sub games.

Lemma 1 *In any equilibrium of the tender subgame where $p > 0$, the small shareholders use a threshold strategy.*

PROOF: MLRP implies that a small shareholder's belief $\beta(s)$ is a strictly increasing function. Fix an equilibrium of the tender sub game for some $p > 0$, $T = (\sigma, \sigma_L, q, q_t, q_L)$. The payoff of a small shareholder from tendering a share is $p(\beta(s)q + (1-\beta(s))q_t)$, while the payoff from keeping a share is $q\beta(s)$. Therefore, if $\sigma(s) > 0$ for some s, then for every $s' < s$, $q\beta(s') < p(\beta(s')q + (1-\beta(s'))q_t)$ and hence $\sigma(s') = 1$. Similarly, if $\sigma(s) < 1$ for some s, then $q\beta(s') > p(\beta(s')q + (1-\beta(s'))q_t)$ for all $s' > s$, and therefore $\sigma(s') = 0$ for every $s' > s$. □

In the following development, we refer to equilibrium strategies of the small shareholders with the threshold signal of their strategy. I.e., from hereon $\sigma \in [0,1]$ denotes the threshold signal of the small shareholders' equilibrium strategy.

The pivotal type, $s^* \in [0,1]$, is the type such that if the small shareholders use the threshold s^*, and if the large shareholder tendered all his shares, then the fraction of shares tendered equals 1/2. Since $x < 1/2$, pivotal type $s^* \in (0,1)$ is uniquely defined by

$$F(s^*|h)(1-x) + x = 1/2.$$[4]

The price $\bar{p}$ is the price that would keep a pivotal type indifferent between tendering her share and keeping it, if she believed that the takeover is successful with probability one in both states. In particular, $\bar{p} := \beta(s^*)$.

THEOREM 1 *(Characterization) For any $p > 0$, there is a unique non-trivial equilibrium of the tender sub game, $T = (\sigma, \sigma_L, q, q_t, q_L)$.*

(i) $q_t = 1$.

(ii) *If $p \leq \bar{p}$, then $\sigma = s^*$, $q = \frac{1-\beta(s^*)}{\beta(s^*)}\frac{p}{1-p}$, $q_t = 1$, $\sigma_L(s,r) = 0$ for every $s \in [0,1]$ and $r < 1$. Moreover, the raider's profit is:*

$$\Pi(p) = \lambda\frac{q}{2} - p\left(\lambda\frac{q}{2} + (1-\lambda)[(1-x)F(s^*|l) + x]\right) \tag{8}$$

$$= p\left(\frac{(1-\beta(s^*))\lambda}{2\beta(s^*)} - (1-\lambda)[(1-x)F(s^*|l) + x]\right). \tag{9}$$

(iii) *If $\beta(1) \geq p > \bar{p}$, then σ satisfies*

$$\beta(\sigma) = p,$$

and $q = 1$. Moreover, there is a threshold signal $s_L \in [0,1]$ for the large shareholder such that, if his signal s is larger than s_L, then the large shareholder tenders a < 1 fraction of his shares, whereas if $s < s_L$, he tenders all of his shares. The raider's profit *is:*

$$\Pi(p) = \lambda\left[(1-x)F(\sigma|h) + x(a(1-H(s_L|h)) + H(s_L|h))\right] \\ -p\lambda\left[(1-x)F(\sigma|h) + x(a(1-H(s_L|h)) + H(s_L|h))\right] \\ -p(1-\lambda)[(1-x)F(\sigma|l) + x(a(1-H(s_L|l)) + H(s_L|l))]. \tag{10}$$

PROOF: See the Appendix. □

REMARK 1 *Note that the raider's profit function* $\Pi(p)$ *is identical to the profit function in Ekmekci and Kos (2012). Therefore the raider is indifferent between making a conditional offer versus an unconditional offer in the continuum shares model.*[5]

Next we characterize the raider's equilibrium price offers. As an intermediate step we show that the raider makes zero profit if he makes a price offer of zero.

LEMMA 2 $\Pi(0, T) = 0$ *for any equilibrium of the tender sub game where* $p = 0$.

PROOF: When $p = 0$, $q = 0$. If to the contrary $q > 0$, it would be optimal for the small shareholders not to tender their shares and obtain a positive payoff, contradicting $q > 0$. Therefore, no matter how many shares are tendered, the raider pays zero. Since the success probability is zero in the high state, his payoff from a successful takeover is zero. □

The next theorem characterizes the raider's optimal behavior. The raider, depending on the parameters of the environment, either offers a price zero and makes profit zero, or the price $\bar{p}$, in which case he makes a positive profit.

THEOREM 2 *The raider's profit is maximized at either* $p = 0$ *or* $p = \bar{p}$. *If*

$$\Pi(\bar{p}) := \lambda\frac{1}{2} - \bar{p}\left(\lambda\frac{1}{2} + (1-\lambda)[(1-x)F(s^*|l) + x]\right) > 0,$$

then the raider offers the price $\bar{p}$, *and the takeover is successful with probability one in both states. If instead* $\Pi(\bar{p}) < 0$, *then the raider offers price zero, and the takeover is unsuccessful in high state.*

PROOF: See Ekmekci and Kos (2012). □

[5] Some caution is required in comparison. For a given p, the q from this paper and the one from Ekmekci and Kos (2012) differ.

A. Proofs for the Incomplete information Model

Proof of Theorem 1: Suppose that there exists an equilibrium of the tender sub game, $T = (\sigma, \sigma_L, q, q_t, q_L)$. In the following development, we will fix this candidate equilibrium, and characterize its properties. Then we will verify that such an equilibrium exists.

Before we characterize the equilibrium of the tender subgame, we remind the reader of two definitions from main text. The pivotal type, $s^* \in [0,1]$ is the type such that if the small shareholders use the threshold strategy s^*, and if the large shareholder tendered all his shares, then the fraction of tendered shares is $1/2$. Since $x < 1/2$, there is indeed an interior threshold signal $s^* \in (0,1)$, which satisfies $F(s^*|h)(1-x) + x = 1/2$. The price $\bar{p}$ is the price that makes the shareholder observing the threshold signal $s^* \in (0,1)$ indifferent between tendering her share or keeping it, if she believes that the takeover is successful with probability one in both states. In particular, let $\bar{p} := \beta(s^*)$.

The first claim shows that the probability of the success of the takeover in the high state is larger than zero in any equilibrium.

Claim 1 *$q > 0$.*

Proof: Suppose, on the way to a contradiction that $q = 0$. Since T is a non-trivial equilibrium, $q_t > 0$. Then, not tendering yields an expected payoff of zero, while tendering yields $p(1 - \beta(s))q_t > 0$, therefore $\sigma = 1$. Consequently, at least $1 - x$ of shares are sold regardless of what the large shareholder does, resulting in $q(r) = 1$ for every $r \in [0,1]$. In turn $q = 1$, which contradicts our hypothesis that $q = 0$. □

Next we establish a lower bound on the threshold used in an equilibrium by small shareholders.

Claim 2 *$(1-x)F(\sigma|h) + x \geq 1/2$.*

Proof: If the claim was not true, less than half the shares in total would be sold in the high state, even if the large shareholder were to tender all of his shares. Implying $q(r) = 0$ for every $r \in [0,1]$. Therefore, $q = 0$ which would contradict the above claim that $q > 0$. □

The above claim establishes that in any equilibrium the strategy of the small shareholders is such that the large shareholder could guarantee that at least half of the shares are sold in the high state.

In what follows we define the notation for the share of the large shareholder that needs to be tendered so that exactly half of the shares are tendered in the high state, given the equilibrium strategy of the small shareholders. Notice that such a share of the large shareholder exists in an equilibrium due to the previous claim.

DEFINITION 1 *Let*

$$a := \max\{0, \frac{1/2 - (1-x)F(\sigma|h)}{x}\}.$$

In the next claim we show, roughly speaking, that the large shareholder never sells less than fraction a of his shares. And moreover, if he is selling precisely a and $q(a) < 1$ then it better be the case that $a = 1$. Indeed, if the large shareholder were to sell $a < 1$ with positive probability and $q(a) < 1$, then he would be better off by selling just slightly more than a which would ensure the success of the takeover in the high state and yield a significantly larger payoff on the shares the large shareholder keeps.

CLAIM 3 *If $\sigma_L(s, r) > 0$ for some $r < 1$ and $s \in [0, 1]$, then $q(r) = 1$.*

PROOF: The proof is in two steps. First, if $\sigma_L(s, r) > 0$ for some $r < 1$, then $r \geq a$. To see this, note that $q(r) = 0$ for $r < a$, by the definition of a. Therefore when $r < a$ the large shareholder gets $pq_t(r)(1 - \beta_L(s))$ for the shares he tendered and zero for the ones he keeps. However, $q_t(r)$ is weakly increasing in r, and $q_t(a) = 1$. Consequently, tendering a fraction a of shares does strictly better than $r < a$. Hence, $\sigma_L(s, r) = 0$ for every $r < a$.

For the second step, we show that the claim is true under the two remaining cases: i) $r > a$ and ii) $r = a$.

i) When $r > a$, $q(r) = 1$ by the definition of a and by the equilibrium requirement on $q(r)$.

ii) Now suppose that $r = a$, $\sigma_L(s, a) > 0$, $r < 1$, and suppose contrary to the assertion of the claim, that, $q(a) < 1$. Then the large shareholder has a profitable deviation by tendering a fraction arbitrarily close to but higher than a, by which he pushes the probability of the success in the high state to 1. This contradicts the equilibrium condition that σ_L maximizes the large shareholder's payoff. □

If the small shareholders expect the success of the takeover in the high state to be less than certain, then it must be the case that the large shareholder is selling all of his shares. Otherwise he could increase the probability to one by selling just slightly more shares.

CLAIM 4 *If $q < 1$, then $\sigma_L(s, r) = 0$ for every $r < 1$ and $s \in [0, 1]$.*

PROOF: Let $q < 1$. If $a = 1$, then the claim is true because $\sigma_L(s, r) = 0$ for every $r < a = 1$ as shown in Claim 3.

Let $a < 1$, and on the way to a contradiction assume that $\sigma_L(s, r) > 0$ for some $r < 1$. Then $q(r) = 1$ for all such r, by Claim 3. Moreover, if $a < 1$, then $q(1) = 1$. Therefore $q = 1$, contradicting the supposition that $q < 1$. □

Claim 5 $q_t = 1$.

Proof: We have already shown that $q > 0$. If $q < 1$, then the large shareholder tenders all his shares at every signal, so in state l, he tenders all his shares. Moreover, the small shareholders use a cutoff strategy, therefore, the fraction of shares tendered by them in state l strictly exceeds the fraction tendered in state h. Therefore $q_t = 1$ if $q < 1$. If $q = 1$, then $q(r) = 1$ for every fraction r that is tendered with positive probability. But then, since in state l, the fraction of shares tendered by the small shareholders is strictly larger in state l than in state h, $q_t(r) = 1$ for such fractions that are tendered with positive probability. Therefore, $q_t = 1$. □

So far we have established several properties of the equilibrium. In what follows, we will complete the characterization under two cases: i) when $p < \bar{p}$ and ii) when $p \geq \bar{p}$.

CASE 1: Suppose that $p < \bar{p}$.

When the price p is below the threshold $\bar{p}$ the takeover cannot succeed with certainty in the high state.

Claim 6 *If $p < \bar{p}$, then $q < 1$.*

Proof: Suppose on the way to a contradiction that $q = 1$. Then, $p = q\beta(\sigma)$, $p < \bar{p} = \beta(s^*)$, and $\beta(\cdot)$ is strictly increasing imply $\sigma < s^*$. Therefore,

$$(1 - x)F(\sigma|h) + x < (1 - x)F(s^*|h) + x = 1/2,$$

hence $q(r) = 0$ for every $r \in [0, 1]$. Consequently $q = 0$, which contradicts the assumption we started with. □

Claim 1 and 6 put together tell us that if there is to exist an equilibrium it has to be the case that $q \in (0, 1)$. From the previous analysis we also know that in such an equilibrium the large shareholder would try to ensure the certain success. The only way he can be prevented is if he is already selling all of his shares.

Claim 7 *$\sigma_L(s, r) = 0$ for every $s \in [0, 1]$ and $r < 1$.*

Proof: From Claim 6, we know that $q < 1$. The result is then implied by Claim 4. □

In words, the large shareholder tenders all his shares regardless of his information.

Claim 8 *$a = 1$, $\sigma = s^*$ and $q = \frac{p(1-\beta(s^*))}{\beta(s^*)(1-p)}$.*

PROOF: If $a < 1$, then $q(1) = 1$, and since the large shareholder would be tendering all shares, $q = 1$. This would contradict the the statement of Claim that $q < 1$. Therefore $a = 1$, and using the definition of a, $\sigma = s^*$. Finally, since σ is the threshold type, $q\beta(s^*) = p(\beta(s^*)q + (1 - \beta(s^*))q_t)$. Using $q_t = 1$, we arrive at $q = \frac{p(1-\beta(s^*))}{(1-p)\beta(s^*)}$, completing the proof. □

Below, we summarize the characterization of the equilibrium when $p < \bar{p}$.

SUMMARY 1 *If $p < \bar{p}$, then*

$$\sigma = s^*, \qquad q = \frac{p(1 - \beta(s^*))}{\beta(s^*)(1 - p)}, \qquad \sigma_L(s, r) = 0,$$

for every $s \in [0, 1]$ and $r < 1$, and

$$q(1) = q, \qquad q(r) = 0,$$

for every $r < 1$. It is straightforward to verify that this is an equilibrium. We conclude that there is a unique equilibrium for $p < \bar{p}$.

CASE 2: Suppose that $p \geq \bar{p}$.

First we establish that for high prices the takeover succeeds with probability one in the high state in all equilibria.

CLAIM 9 *If $p \geq \bar{p}$ then $q = 1$.*

PROOF: On the way to a contradiction, assume that $q < 1$. Then by claim four, $\sigma_L(1, r) = 0$ for every $r < 1$, i.e., the large shareholder tenders all his shares. Moreover, $p \geq \bar{p}$ and $q < 1$ imply $\beta(\sigma) > \bar{p} = \beta(s^*)$ and therefore $\sigma > s^*$. But then $q = 1$, because the large shareholder tenders all his shares and $(1 - x)F(\sigma|h) + x > 1/2$. This contradicts the initial hypothesis that $q < 1$. □

REMARK 2 *Since $q = 1$, σ is found by the identity $\beta(\sigma) = p$. Since $p \geq \bar{p}$, $\sigma \geq s^*$.*

At low prices, $p < \bar{p}$, the takeover succeeds in the high state with a probability q smaller than one. The only way to prevent the large shareholder from increasing this probability to one is if he is already tendering all of his shares in the equilibrium. For the high price, $p \geq \bar{p}$, however, the probability of the takeover succeeding in the high state must be one, as shown in Claim 8. This leaves scope for the large shareholder to keep some of the shares if he deems them more valuable then the price the raider is offering.

Definition 2 *Let s_L be the signal that satisfies $\beta_L(s_L) = p$, if such a signal exists. Let $s_L := 0$ if $\beta_L(0) > p$ and let $s_L := 1$ if $\beta_L(1) < p$.*

The signal s_L is the one at which the large shareholder's expected value for each of his shares is equal to price, when he is expecting the takeover to succeed with probability one.

The following claim shows that the large shareholder tenders fraction a of his shares when his signal is high and all of his shares when the signal is low.

Claim 10 *i) $\sigma_L(s, r) = 0$ for $r < a$ and any $s \in [0, 1]$. In words, the large shareholder does not tender a fraction less than a.*

ii) For $s < s_L$ and $r < 1$: $\sigma_L(s, r) = 0$. For $s > s_L$ and $r \in [a, 1)$:

$$\sigma_L(s, r) = \lambda[F(s|h) - F(s_L|h)] + (1 - \lambda)[F(s|l) - F(s_L|l)].$$

Notice that $\sigma_L(s, 1) = \lambda F(s|h) + (1 - \lambda)F(s|l)$ for all $s \in [0, 1]$, by the definition of distributional strategies.

In words, the large shareholder tenders exactly fraction a if his signal is above the threshold signal s_L. He tenders all his shares if his signal is below s_L.

iii) If $s_L < 1$, then $q(a) = 1$.

Proof: Part i) follows directly from Claim 3.

We will argue part ii) by considering two cases: $\sigma = s^*$ and $\sigma > s^*$. If $\sigma = s^*$, then $a = 1$ by the definition of a and s^*. Since the large shareholder never tenders a fraction smaller than a, as in part i), $\sigma_L(s, r) = 0$ for every $r < 1$ and every $s \in [0, 1]$. $q = 1$ then implies that $q(a) = 1$.

If $\sigma > s^*$, then $a < 1$. When $s < s_L$, the price p is greater than $\beta_L(s)$, by the definition of s_L. Therefore, it is optimal for the large shareholder to tender all his shares. Hence, if $s < s_L$, then $\sigma_L(s, r) = 0$ for every $r < 1$.

If $s_L = 1$, then the proof is complete. So, let $s_L < 1$. A direct calculation shows that tendering any fraction $r > a$ is dominated by tendering fraction $\frac{a+r}{2}$ of shares. Moreover, if $q(a) < 1$, then tendering a fraction arbitrarily close to a from above is a profitable deviation. Therefore, for the optimal strategy of the large shareholder to exist it must be the case that $q(a) = 1$. Since $\sigma_L(s, r) = 0$ for $r < a$, in any equilibrium from part i), $\sigma_L(s, a) = \lambda[F(s|h) - F(s_L|h)] + (1 - \lambda)[F(s|l) - F(s_L|l)]$ for $s > s_L$, which concludes the proof. □

We have proven that all equilibria of the tender subgame have the following structure:

i) If $p < \bar{p}$, then $\sigma = s^*$, $q = \frac{p(1-\beta(s^*))}{\beta(s^*)(1-p)}$, $\sigma_L(s, r) = 0$ for every $s \in [0, 1]$ and $r < 1$. Moreover, the raider's profit is:

$$\Pi(p) = \lambda(q/2 - pq/2) + (1-\lambda)(-p[(1-x)F(s^*|l) + x]).$$

ii) If $p = \bar{p}$, then $\sigma = s^*$, $q = 1$, $\sigma_L(s,r) = 0$ for every $s \in [0,1]$ and $r < 1$; the raider's profit is:

$$\Pi(p) = \lambda(1/2 - p/2) + (1-\lambda)(-p[(1-x)F(s^*|l) + x]).$$

iii) If $p > \bar{p}$, then σ satisfies $\beta(\sigma) = p$, $q = 1$, $\sigma_L(s,r)$ satisfies the findings above; the raider's profit is:

$$\begin{aligned}\Pi(p) = \lambda\left[(1-x)F(\sigma|h) + x(a(1-H(s_L|h)) + H(s_L|h))\right] \\ -p\lambda\left[(1-x)F(\sigma|h) + x(a(1-H(s_L|h)) + H(s_L|h))\right] \\ -p(1-\lambda)[(1-x)F(\sigma|l) + x(a(1-H(s_L|l)) + H(s_L|l))].\end{aligned}$$

□

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