Monetary Policy and the Extensive Margin of Exports*

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Abstract: This paper develops a two country Dynamic Stochastic General Equilibrium model to analyze the extensive margin of exports and international transmission of monetary policy. Heterogeneity in productivity and fixed costs for exporting generate endogenous tradability of goods - the extensive margin of exports. Sunk costs for entry give rise to an endogenous number of firms. Monetary policy affects the extensive margin of exports through movements in the terms-of-trade, the real interest rate, and wages. When exports are priced in the producers' currency, monetary shocks generate negative co-movement in extensive margins. When exports are priced in local currency, monetary shocks generate positive co-movement in extensive margins.

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1. Introduction

In this paper, I provide a theory of monetary policy and the extensive margin of exports by developing a two-country sticky-price DSGE model which features fixed costs for exporting and firm heterogeneity in productivity.[1] The main result is that co-movement in the extensive margin of exports depends on the extent of exchange rate pass-through. When exports are priced in producer currency, and there is full exchange rate pass-through, monetary shocks generate negative co-movement in extensive margins. When exports are priced in local currency, and pass-through is limited, monetary shocks generate positive co-movement. The international transmission of monetary policy along the extensive margin occurs through three channels. Two of these channels - the terms-of-trade and real interest rate - are often stressed in traditional models of macroeconomic interdependence.[2] The the third channel of international transmission - wages - arises because exporters require resources to cover fixed costs.

Monetary shocks affect the extensive margin of exports in the following way. Positive monetary shocks raise the number of exporters (expand the extensive margin of exports) across both economies by stimulating global aggregate demand. Monetary shocks also change the terms-of-trade and this affects the competitiveness of exports. Greater competitiveness results in potentially higher export profits available

[1] The model builds on recent developments in international trade. For example, see Melitz (2003).

[2] The transmission of monetary policy in the Mundell-Fleming model and the more recent analysis of Obstfeld and Rogoff (1995) relies on movements in the terms-of-trade (expenditure switching) and real interest rate (expenditure shifting).

for a country's firms and increased profits provide an incentive for weaker firms to enter the export market. Entry by weaker firms raises costs for all exporters, and as costs rise there is a fall in the number of exporters. When this mechanism is linked to a theory of monetary policy based on nominal rigidities, the international transmission of monetary shocks depends on the currency in which exports are priced and the extent of exchange rate pass-through. Greater pass-through strengthens the competitiveness effect, and when this dominates, a positive monetary shock in one country generates a fall in the number of exporters from that country and negative co-movement internationally. When pass-through is limited, the global aggregate demand channel dominates, and monetary shocks generate positive co-movement in the extensive margin of exports.

There is an important difference between the mechanism that generates co-movement in the extensive margin of exports in this paper and the mechanism that generates co-movement in the intensive margin (volume) of exports when the number of exporters is given. Fixed costs - which generate a simple theory of exporting - interact with sunk costs - which give rise to an endogenous number of firms.[3] Firms only enter the domestic market if the present discounted value of their total expected profit is greater than the sunk cost. Monetary policy affects firm entry via the real interest rate and wages because household saving decisions are tied to the purchase of shares in a mutual fund of firms and because the sunk cost of entry is labor intensive.

[3] See Bilbiie *et al.* (2007), Bergin and Corsetti (2008) and Lewis (2009) for recent closed economy models with monetary shocks and firm entry. The latter two papers also provide empirical evidence on firm entry and monetary policy over the business cycle.

Monetary policy affects the export decision via the terms-of-trade and wages because the former determines the competitiveness of exports and the latter the fixed cost of exporting. All else equal, a drop in resource costs raises export participation and firm entry. However, increased firm entry raises costs and reduces export participation. In this case, the terms-of-trade, real interest rate, and wages jointly determine the response of the extensive margin of exports to monetary shocks.

A key advantage of my approach is that I am able to generate analytical results despite integrating firm heterogeneity and nominal rigidities. I do this by taking advantage of the idea that there is international trade in intermediate inputs where consumed (final) products can be thought of undergoing two distinct stages of production. Firms at stage one of production are heterogeneous in productivity and export subject to fixed costs. Firms at stage two use domestic and foreign stage one goods as inputs, take input prices as given, and set their output prices either in producer or local currency terms. When prices are preset one period in advance the analytical results for extensive margins can be interpreted in terms of exchange rate pass-through. I also generate quantitative results by assuming prices are set following the staggered structure of Calvo (1983). The quantitative results are consistent with those when prices are preset but also show that monetary shocks imply sizable movements in the extensive margin of exports.

There is empirical evidence that can help understand the response of the extensive margin to monetary shocks in this paper. Theoretically, the key point is that monetary shocks and sticky-prices interact to generate movements in the terms-of-trade and real exchange rate. Empirically, both Bernard and Jensen (2004) and Berman *et al.* (2011) show that an appreciation of the domestic currency reduces the probability of exporting for US and French firms, respectively.[4] This suggests favorable movements in the real exchange rate expand a country's extensive margin of exports which is consistent with limited exchange rate pass-through in my model. Alessandria and Choi (2008) provide descriptive statistics showing that the number of exported products is counter-cyclical whereas the number of imported products is pro-cyclical. In my model, this is consistent with greater exchange rate pass-through.[5] Finally, in other research, Campbell and Lapham (2004) present empirical evidence that changes in the real exchange rate induce entry at business cycle frequencies and Bergin and Lin (2010) emphasize the importance of the extensive margin for understanding the trade effects of monetary unions using product-level data.

There is small theoretical literature that incorporates elements of firm heterogeneity into open economy monetary models of the macroeconomy. Naknoi (2008) studies a monetary version of the Dornbusch *et al.* (1977) model of comparative advantage and decomposes real exchange rate movements into those associated with fluctuations in the relative price of traded goods and traded to non-traded goods, under different exchange rate regimes.[6] Lewis (2010) uses a static framework to focus on a firms

[4]These studies show that export participation decisions are correlated with unconditional movements in the real exchange rate (industry specific and aggregate, respectively) and are silent on the possible sources of exchange rate variation. For a recent discussion on the effects of monetary policy on the exchange rate see Scholl and Uhlig (2005).

[5]The model I develop assumes each firm produces a single product, or variety.

[6]In an earlier paper, Kimbrough (1992) introduces money into the Dornbusch *et al.* (1977)

decision to export or undertake multinational production. Lewis' (2010) results highlight the importance of price-setting - only when exports are producer currency priced and multinational production is local currency priced does increased nominal volatility reduce multinational sales relative to exports. My analysis also highlights the role of price-setting behavior for exporting, but I focus on co-movement in extensive margins over the business cycle. The model developed in this paper is also related to Ghironi and Melitz (2005) who assume firms face sunk entry and fixed exporting costs. However, they consider a flexible price environment where aggregate technology shocks generate real exchange rate dynamics. Alessandria and Choi (2007) analyze a flexible price model in which a given number of firms face sunk and fixed continuation costs of exporting to understand the cyclical behavior of net exports.[7]

The remainder of the paper is organized as follows. Section 2 develops a two-country monetary model with heterogeneous firms and sticky prices. Section 3 provides analytical results and computes impulse responses for a quantitative version of the model. Section 4 concludes.

2. The Model Economy

The world consists of a home and foreign economy each populated by a unit mass of identical, infinitely lived households. Each household supplies labor to domestic

framework through a cash-in-advance assumption to study the international transmission of monetary policy and the extensive margin of exports.

[7]Bergin and Glick (2009) develop a static model where goods are heterogeneous in terms of iceberg trade costs and argue that endogenous tradability is an explanation for the low degree of volatility in the relative price of non-traded goods.

firms, holds domestic money balances, a mutual fund of domestic firms, and a one period bond. Households consume an aggregate (final) good comprised of domestic and foreign goods. A differentiated good for consumption requires two stages of production. Stage one (S-1) production requires labor. Firms that produce S-1 goods are heterogenous in productivity and each potential S-1 firm faces a sunk cost of entry and a fixed cost of exporting. Stage two (S-2) production requires home and foreign S-1 goods as inputs. Below, goods produced in the home country are subscripted with an h, while those produced in the foreign country are subscripted with an f. Prices and quantities in the foreign country are denoted with an asterisk.

2.1. Home Economy S-2 Firms

S-2 firms use the following technology,

$$y_{2h,t}(z) + y^{\star}_{2h,t}(z) = \Gamma Y^{\gamma}_{1h,t} Y^{1-\gamma}_{1f,t} \equiv \mathcal{Y}_{2,t}(z)$$

where $\Gamma \equiv \gamma^{\gamma}(1-\gamma)^{1-\gamma}$. The variable $y_{2h,t}(z)$ is S-2 production for sale domestically, $y^{\star}_{2h,t}(z)$ is S-2 production for export, and $\mathcal{Y}_{2,t}(z)$ is an aggregate of the two composite S-1 goods: $Y_{1h,t} = \mathfrak{n}_{h,t}\left(\int_{n_{h,t}} y_{1h,t}(z,a)^{(\theta-1)/\theta}\, da\right)^{\theta/(\theta-1)}$ is purchased from domestic S-1 firms and $Y_{1f,t} = \mathfrak{n}_{f,t}\left(\int_{n_{f,t}} y_{1f,t}(z,a^{\star})^{(\theta-1)/\theta}\, da\right)^{\theta/(\theta-1)}$ is imported from foreign S-1 firms, where $\mathfrak{n}_{h,t} = n_{h,t}^{\upsilon-[\theta/(\theta-1)]}$ and $\mathfrak{n}_{f,t} = n_{f,t}^{\upsilon-[\theta/(\theta-1)]}$ control variety effects in the model.

S-2 firms take input (i.e., S-1) prices - $p_{1h,t}(a)$ and $p_{1f,t}(a)$ - as given. Cost minimization implies the unit cost function for S-2 firms is, $P_{1,t} = P^{\gamma}_{1h,t} P^{1-\gamma}_{1f,t}$, where $P_{1h,t} = \frac{1}{\mathfrak{n}_{h,t}}\left(\int_{n_{h,t}} p_{1h,t}(a)^{1-\theta}\, da\right)^{1/(1-\theta)}$ and $P_{1f,t} = \frac{1}{\mathfrak{n}^{\star}_{f,t}}\left(\int_{n^{\star}_{f,t}} p_{1f}(a^{\star})^{1-\theta}\, da^{\star}\right)^{1/(1-\theta)}$

are price indices associated with S-1 domestic and imported goods, respectively. The demands for home and foreign S-1 goods are given by,

$$y^{d}_{1h,t}(a) = \mathfrak{n}^{\theta-1}_{h,t}\left(\frac{P_{1h,t}}{P_{1,t}}\right)^{-1}\left(\frac{p_{1h,t}(a)}{P_{1h,t}}\right)^{-\theta}\gamma\int_{0}^{1}\mathcal{Y}_{2,t}(z)\,dz$$

and,

$$y^{d}_{1f,t}(a^{\star}) = \mathfrak{n}^{\theta-1}_{f,t}\left(\frac{P_{1f,t}}{P_{1,t}}\right)^{-1}\left(\frac{p_{1f,t}(a^{\star})}{P_{1f,t}}\right)^{-\theta}(1-\gamma)\int_{0}^{1}\mathcal{Y}_{2,t}(z)\,dz$$

Demand depends on movements in relative prices.[8] For example, in the first expression, $P_{1h,t}/P_{1,t}$ reflects the domestic S-1 aggregate industry price relative to the overall price index, which includes imported S-1 goods, and $p_{1h,t}(a)/P_{1h,t}$ reflects the price of an S-1 individual good relative the industry price level.

S-2 firms compete in a monopolistically competitive domestic and export market. I consider two possibilities for S-2 firms in terms of price setting. They can set prices in producer currency terms, in which case the law of one price holds. Or they can set prices in local currency terms, in which case firms are assumed to segment markets internationally and the law of one price may fail. Under producer currency pricing (PCP), an S-2 firm chooses the price $p_{2h,t}(z)$. The foreign currency price of the home S-2 good is determined by a law of one price condition, $p^{\star}_{2h,t}(z) = p_{2h,t}(z)/e_t$. Under local currency pricing (LCP) an S-2 firm chooses both $p_{2h,t}(z)$ and $p^{\star}_{2h,t}(z)$. In either

[8] This setting is similar to Huang and Liu (2007) who also study a sticky-price model with international trade in intermediate inputs. They argue that staggered price setting and trade in intermediate inputs is important for generating international co-movement in aggregate quantities when there are monetary shocks.

case, a firm will maximize the present discounted value of expected profits,

$$D_{2,t}(z) = \mathbb{E}_t \sum_{j=t}^{\infty} \alpha^{j-t} \mathcal{M}_{t,j} \left[(p_{2h,j}(z) - P_{1,j}) \, y^d_{2h,j}(z) + \left(e_j p^{\star}_{2h,j}(z) - P_{1,j} \right) y^{\star d}_{2h,j}(z) \right]$$

where α is the probability that the firm can reset it's price and $\mathcal{M}_{t,j}$ is a stochastic discount factor. S-2 firms take the unit cost of production, the nominal exchange rate, and demand in the domestic and export markets as given. The resulting first-order conditions for this problem can be expressed in the following way. For PCP,

$$\mathbb{E}_t \sum_{j=t}^{\infty} \alpha_h^{j-t} \mathcal{M}_{t,j} \left(\frac{p_{2h,t}(z)}{P_{1,j}} - \Phi \right) y^d_{2h,j}(z) = 0 \tag{1}$$

and the law of one price condition. For LCP, equation (1) and,

$$\mathbb{E}_t \sum_{j=t}^{\infty} \alpha_h^{\star j-t} \mathcal{M}_{t,j} \left(\frac{e_j p^{\star}_{2h,t}(z)}{P_{1,j}} - \Phi \right) y^{\star d}_{2h,j}(z) = 0 \tag{2}$$

where $\Phi = \theta / (\theta - 1)$ is a monopoly mark-up. Since foreign S-2 firms are identical, under local currency pricing, I assume the location of consumption determines the degree of nominal price stickiness; i.e., $\alpha_h = \alpha^{\star}_f$ and $\alpha^{\star}_h = \alpha_f$, so that within a country price rigidities are the same for imported and domestically produced S-2 goods. This case is relevant because it generates a pricing-to-market element in the firms price-setting decision. Finally, flexible prices correspond to $\alpha = 0$.

2.2. Home Economy S-1 Firms

There is a competitive fringe of potential S-1 firms. Prior to entry, S-1 firms face a sunk entry cost, $f_e > 0$, in units of labor. Upon entry, S-1 firms draw productivity level, a, from a Pareto distribution, $g(a) = \kappa a^{-(\kappa+1)}$, where κ measures the dispersion of productivity draws. S-1 firms that enter the domestic market compete under monopolistic competition and each firm produces a differentiated product. If they choose to export, S-1 firms pay a fixed cost, $f_h^\star > 0$ - also in terms of units of labor - and melting-iceberg trade costs, $\tau \geq 1$.[9]

S-1 firms use the following technology,

$$y_{1h,t}(a) + y_{1h,t}^\star(a) = al_{1h,t}(a) + al_{1h,t}^\star(a) \tag{3}$$

where $l_{1h,t}(a)$ and $l_{1h,t}^\star(a)$ represent the labor used in the production of goods for the domestic and export markets. The costs of production are, $W_t\left(al_{1h,t}(a) + al_{1h,t}^\star(a)\right)$, and cost minimization implies the unit cost of production for S-1 firms is, W_t/a. S-1 firms also choose prices in the domestic and export markets - $p_{h,t}(a)$ and $p_{h,t}^\star(a)$ - to maximize profits, $D_{1,t}(a) = D_{1h,t}(a) + D_{1h,t}^\star(a)$, which can be expressed as,

$$D_{1,t}(a) = \left(p_{1h,t}(a) - W_t\right) y_{1h,t}^d(a) + \left(e_t p_{1h,t}^\star(a) - W_t\right) y_{1h,t}^{\star d}(a) - W_t f_h^\star$$

S-1 firms take the unit cost of production, the nominal exchange rate, and demand in each market as given. The resulting first-order conditions for this problem are,

$$p_{1h,t}(a) = \Phi \frac{W_t}{a} \quad \text{and} \quad p_{1h,t}^\star(a) = e_t^{-1}\tau p_{1h,t}(a) \tag{4}$$

Given these pricing rules, an S-1 firms' nominal profits in each market are,

$$D_{1h,t}(a) = \left(\frac{\mathfrak{n}_{h,t}^{\theta-1}}{\theta}\right)\left(\frac{p_{1h,t}(a)}{P_{1h,t}}\right)^{1-\theta} P_{1,t}\gamma \int_0^1 \mathcal{Y}_{2,t}(z)\,dz$$

and,

$$D_{1h,t}^\star(a) = \left(\frac{\mathfrak{n}_{h,t}^{\star\theta-1}}{\theta}\right)\left(\frac{p_{1h,t}^\star(a)}{P_{1h,t}^\star}\right)^{1-\theta} Q_{1,t}P_{1,t}(1-\gamma) \int_0^1 \mathcal{Y}_{2,t}^\star(z)\,dz - W_t f_h^\star$$

[9]The fixed export cost assumes the use of domestic factors of production only, consistent with the idea that resources must be set aside to acquire information about entry into foreign markets.

where $Q_{1,t} \equiv e_t P^{\star}_{1,t}/P_{1,t}$ is the real exchange rate associated with trade between S-1 and S-2 firms. S-1 firm profits from domestic sales are always positive. However, due to the fixed cost of exporting, it is possible that export profits are zero. Profits depend on firm level productivity, a, and I define a zero export profit productivity cut-off, $a^{\star}_{h,t} = \inf \left\{ a : D^{\star}_{1h,t}(a) > 0 \right\}$.

Given the distribution of productivity draws, I re-write all S-1 firm level variables in averages, denoted by an upper-bar. In any period there are $n_{h,t}$ S-1 firms. For all S-1 firms, I define the average level of productivity as $\overline{a} \equiv \left(\int_1^{\infty} a^{\theta-1} f(a)\, da \right)^{1/(\theta-1)}$. In this case, the average price for all S-1 firms is $\overline{p}_{1h,t} = P_{1h,t} n_{h,t}^{1/(\theta-1)}$, where $\overline{p}_{1h,t} \equiv p_{1h,t}(\overline{a})$. Using the same definition, it follows that domestic S-1 output is $Y_{1h,t} = \overline{y}_{1h,t} n_{h,t}^{\theta/(1-\theta)}$. Finally, domestic S-1 profits are $\overline{D}_{1h,t} = D_{1h,t} n_{h,t} > 0$. Because it is possible to earn zero profit from exporting, only a fraction, $n^{\star}_{h,t}$, of the S-1 firms export. I define the average productivity of S-1 exporting firms (i.e., those that earn non-zero profits) as, $\overline{a}^{\star}_{h,t} \equiv \left(\frac{1}{1-G\left(a^{\star}_{h,t}\right)} \int_{a^{\star}_{h,t}}^{\infty} a^{\theta-1} g(a)\, da \right)^{1/(\theta-1)}$, where $1 - G\left(a^{\star}_{h,t}\right)$ is the ex-post probability of successful exporting and $n^{\star}_{h,t}/n_{h,t} = 1 - G\left(a^{\star}_{h,t}\right)$ is the ratio of S-1 exporters to all S-1 firms. As with prices and output in the domestic market, in the export market, $P^{\star}_{1h,t} = \overline{p}^{\star}_{1h,t} n_{h,t}^{\star 1/(1-\theta)}$ and $Y^{\star}_{1h,t} = \overline{y}^{\star}_{1h,t} n_{h,t}^{\star \theta/(1-\theta)}$. The assumption of a Pareto distribution over productivity draws implies the relationships between the cut-off level of productivity, $a^{\star}_{h,t}$, and the average productivity of S-1 exporting firms, and the ratio of exporting S-1 firms to all S-1 firms are,

$$\overline{a}^{\star}_{h,t} = \kappa \left[\kappa - (\theta - 1)\right]^{1/(1-\theta)} a^{\star}_{h,t} \quad \text{and} \quad n^{\star}_{h,t}/n_{h,t} = \left\{ \kappa \left[\kappa - (\theta - 1)\right]^{1/(1-\theta)} / a^{\star}_{h,t} \right\}^{\kappa} \tag{5}$$

Finally, total average profits, $\overline{D}_{1,t}$, and the cut-off level of export profits, $\overline{D}^{\star}_{1h,t}$, are,

$$\overline{D}_{1,t} = \overline{D}_{1h,t} + \left(n^{\star}_{h,t}/n_{h,t}\right)\overline{D}^{\star}_{1h,t} \quad \text{and} \quad \overline{D}^{\star}_{1h,t} = \left[\frac{\theta-1}{\kappa-(\theta-1)}\right] f^{\star}_{h} W_t \tag{6}$$

where $\kappa > (\theta - 1)$.

New S-1 firms at time t start producing at time $t+1$. Prospective S-1 firms are forward looking, and correctly anticipate their future expected profits as well as the probability of incurring an exit shock, δ, at the end of each period after they produce. S-1 firms expected post-entry value, $\overline{V}_t$, is given by the present discounted value of expected profits, $\overline{V}_t = \mathbb{E}_t \mathcal{M}_{t,t+1}\overline{D}_{1,t+1} + \mathbb{E}_t \sum_{j=t+1}^{\infty} (1-\delta)^{j-t} \mathcal{M}_{t,j+1}\overline{D}_{1,j+1}$, where $1-\delta$ represents the fraction of S-1 firms that produce and survive each period. New S-1 firms enter as long as they can cover sunk costs and the following condition holds,

$$\overline{V}_t \geq f_e W_t \tag{7}$$

The timing of entry and production also implies the number of S-1 firms during period t is,

$$n_{h,t} = (1-\delta)n_{h,t-1} + n_{e,t-1} \tag{8}$$

where $n_{e,t-1}$ is the mass of new S-1 firms.

2.3. Home Economy Households

The representative household consumes C_t units of a final good, supplies labor, L_t, holds nominal cash balances, M_t, shares in a mutual fund of S-1 firms, F_t, and has access to a one period bond, B_t. The households intertemporal utility function is,

$$\mathbb{U}_t = \mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} \left[\ln C_j + \frac{\chi_m}{1-\sigma_m}\left(\frac{M_j}{P_{2,j}}\right)^{1-\sigma_m} - \frac{\chi_L}{1+\sigma_L} L_j^{1+\sigma_L} \right]$$

where $\beta \in (0,1)$ is a subjective discount factor, $1/\sigma_l > 0$ is the Frisch elasticity of labor supply to wages, and σ_m is a measure of the interest elasticity of money demand. Utility is maximized subject to the sequence of constraints,

$$P_{2,j}C_j + \Delta M_j + B_j I_j + \left(\overline{D}_{1,j} + \overline{V}_j\right) n_{h,j} F_j + D_{2,j} = W_j L_j + B_{j+1} + T_j + \overline{V}_{j+1} n_j F_{j+1}$$

where $n_j \equiv n_{h,j} + n_{e,j}$ is the total number of S-1 firms prior to the exit shock and T_j is a lump-sum government transfer. The following conditions are associated with the household's optimization problem,

$$1/I_j = \beta \mathbb{E}_j \frac{C_j P_{2,j}}{C_{j+1} P_{2,j+1}} \tag{9}$$

$$\overline{V}_j = \beta \mathbb{E}_j \left(\frac{C_j P_{2,j}}{C_{j+1} P_{2,j+1}} \right) \left[(1-\delta) \overline{V}_{j+1} + \overline{D}_{1,j+1} \right] \tag{10}$$

$$\left(\frac{M_j}{P_{2,j}} \right)^{\sigma_m} = \chi_m C_j \left(\frac{I_j}{I_j - 1} \right) \tag{11}$$

$$\frac{W_j}{P_{2,j}} = \chi_L C_j L_j^{\sigma_l} \tag{12}$$

Equation (9) is a consumption Euler equation, (10) is an Euler equation for shares, and (11) and (12) characterize money demand and the labor-leisure trade-off.

The final (consumption) good, $Y_{2,t}$, is a Cobb-Douglas aggregate of home and foreign S-2 goods, $Y_{2,t} = 2Y_{2h,t}^{1/2} Y_{2f,t}^{1/2}$, where $Y_{2i,t} = \left(\int_0^1 y_{2i,t}(z)^{(\theta-1)/\theta} dz \right)^{\theta/(\theta-1)}$ for $i = \{h, f\}$ are composites of home S-2 and imported S-2 goods. The parametrization of this function implies that nominal expenditure on domestic and foreign S-2 goods is equal and the demand functions for good z are, $y_{2i,t}^d(z) = \frac{1}{2} \left(\frac{p_{2i,t}(z)}{P_{2i,t}} \right)^{-\theta} \left(\frac{P_{2f,t}}{P_{2h,t}} \right)^{1/2} Y_{2,t}$. Demand for good z depends on two relative prices, where $P_{2i,t} = \left(\int_0^1 p_{2i,t}(z)^{1-\theta} dz \right)^{1/(1-\theta)}$

is the price index of S-2 goods, either produced and sold domestically or imported, and the overall price level is an average of the two price indices, $P_{2,t} = P_{2h,t}^{1/2} P_{2f,t}^{1/2}$.

2.4. Foreign Economy and Equilibrium

In the foreign economy, there are $n_{f,t}^{\star}$ S-1 firms and $n_{f,t}$ S-1 goods are exported to home S-2 firms.[10] The average productivity of foreign S-1 firms that export is $\overline{a}_{f,t}$ and the cut-off level of export profits is $\overline{D}_{1f,t} = \{(\theta - 1) / [\kappa - (\theta - 1)]\} f_f W_t^{\star}$. Foreign manufacturers enter their domestic market only if they can cover a sunk cost, and produce with a one period lag, subject to being hit by an exit shock. This implies $\overline{V}_t^{\star} = \mathbb{E}_t \mathcal{M}_{t,t+1}^{\star} \overline{D}_{1,t+1} + \mathbb{E}_t \sum_{j=t+1}^{\infty} (1-\delta)^{j-t} \mathcal{M}_{t,j+1}^{\star} \overline{D}_{1,j+1}^{\star} \geq f_e^{\star} W_t^{\star}$ and $n_{f,t}^{\star} = (1-\delta) n_{f,t-1}^{\star} + n_{e,t-1}^{\star}$.

I aggregate across households and impose the following equilibrium conditions, $F_t = F_t^{\star} = 1$, $B_t = B_t^{\star} = 0$ and $C_t = Y_{2,t}$ and $C_t^{\star} = Y_{2,t}^{\star}$. In each economy the labor market clears (there are four sources of labor demand - domestic production, export production, exporting costs, domestic entry costs. In the home economy, for example, $L_t = n_{h,t} \bar{l}_{h,t} + n_{h,t}^{\star} \bar{l}_{h,t}^{\star} + n_{h,t}^{\star} f_h^{\star} + n_{e,t} f_h$) and the free entry condition holds with equality. A monetary authority uses lump-sum transfers to inject money into the economy, so $T_t = M_t + M_{t-1}$ and $T_t^{\star} = M_t^{\star} + M_{t-1}^{\star}$. Trade is balanced and two sector specific net export equations capture the world trade in goods. These are,

$$nx_{1,t} = n_{h,t}^{\star} e_t \overline{p}_{1h,t}^{\star} \overline{y}_{1h,t}^{\star} - n_{f,t} \overline{p}_{1f,t} \overline{y}_{1f,t} \quad \text{and} \quad nx_{2,t} = e_t p_{2h,t}^{\star} y_{2h,t}^{\star} - y_{2f,t} p_{2f,t} \tag{13}$$

[10] S-2 firms choose output prices - $p_{2f,t}^{\star}(z)$ and $p_{2f,t}(z)$ - to maximize profits, taking input prices - $p_{1h,t}^{\star}(a)$ and $p_{1f,t}^{\star}(a)$ - and the exchange rate as given. S-1 firms choose prices - $p_{1f,t}^{\star}(a)$ and $p_{1f,t}(a)$ - to maximize profits also with the input price and exchange rate taken as given.

Finally, equilibrium requires that a resource constraint is satisfied,

$$W_t L_t + n_{h,t}\overline{D}_{1,t} + D_{2,t} = P_{2,t}Y_{2,t} + \overline{V}_t n_{e,t} \tag{14}$$

with an analogous version for the foreign economy. The right-hand side of (14) represents expenditure - on consumption and investment - and the left-hand side income - from labor and firm profits.

3. Analytical Results for the Extensive Margin of Exports

In this section I study a simplified version of the model for which analytical results are available. I solve the model by linearizing around a steady state with flexible prices. Linearized variable are in sans-serif fonts. To isolate the role of monetary policy shocks on the extensive margin of exports I assume there is an unanticipated and permanent change in the level of the home money supply, such that, at date $t = 0$, M_t jumps from zero to M. This is equivalent to a one period change in the rate of home money growth. All S-2 firms set the price of their product one period in advance and are able to reset their price each period. This implies prices in periods $t \geq 1$ adjust completely to the shock.[11] For example, for the home economy I replace the price setting equations given in (1) and (2) with,

$$\mathsf{P}_{2h,t} = \mathbb{E}_{t-1}\left\{\mathsf{P}_{1,t}\right\} \quad \text{and} \quad \mathsf{P}^{\star}_{2h,t} = \mathbb{E}_{t-1}\left\{\mathsf{P}_{1,t} - \mathsf{e}_t\right\}$$

Despite this simplification, the law of one price need not hold at date $t = 0$ because firms can discriminate across international markets. The law of one price holds after the initial period because prices set in periods $t \geq 1$ are consistent with flexible

[11]In periods $t \geq 1$ all variables have reached their long-run levels and money shocks are neutral.

prices. Firms that enter in period $t = 0$ only produce in period $t = 1$, the variety effect is eliminated by assuming $\upsilon = 1$, as in Bergin and Corsetti (2008) and Jaimovich and Floetotto (2008), and domestic and foreign intermediate inputs are used equally. Finally, the preferences of the household are such that, (i) $\sigma_m = 1$ and there is no liquidity effect from monetary policy, and (ii), $\sigma_L \to \infty$ and total labor supply is constant.[12]

3.1. Understanding the Extensive Margin of Exports

I start by deriving a reduced form expression for the home economy's extensive margin of exports. The decision to export for an individual firm is driven by profitability, which depends on firm level productivity. Firm level productivity is drawn upon entry and remains fixed thereafter. Average productivity (i.e., across exporting S-1 firms) changes with the proportion of firms that decide to export and the allocation of resources across the industry (i.e., across all S-1 firms). Recall that the total number of firms in period $t = 0$ is fixed because there is a time-to-build lag in production. Thus, the extensive margin of exports and average industry level productivity are linked by $\mathsf{n}^{\star}_{h,0} = -\kappa \bar{\mathsf{a}}^{\star}_{h,0}$, where κ is the dispersion of productivity draws. In turn, average productivity is determined by a zero-profit cut-off condition. Accounting for export demand, a reduced form for the extensive margin of exports can be written as,

$$\mathsf{n}^{\star}_{h,0} = \left[\left(\mathsf{Q}_{2,0} - \mathsf{P}^{\star}_{2,0} \right) - \frac{1}{2} \left(\mathsf{P}_{2f,0} - \mathsf{P}_{2h,0} \right) \right] + \frac{1}{2} \left(\mathsf{C}_0 + \mathsf{C}^{\star}_0 \right) - \left(\mathsf{w}_0 - \mathsf{P}^{\star}_{1,0} \right) \tag{15}$$

[12]When I calibrate a more general version of the model (see section 4) these simplifications do not affect the qualitative results derived here.

where $\mathsf{Q}_{2,0} = \mathsf{e}_0 + \mathsf{P}^\star_{2,0} - \mathsf{P}_{2,0}$ is the real exchange rate associated with trade between S-2 firms and consumers. Equation (15) shows that movements in the extensive margin of exports arise through movements in relative consumer prices. When consumer prices are preset in the currency of the producer, then $\left[\left(\mathsf{Q}_{2,0} - \mathsf{P}^\star_{2,0}\right) - \frac{1}{2}\left(\mathsf{P}_{2f,0} - \mathsf{P}_{2h,0}\right)\right] = 0$. When prices are preset in local currency terms, then $-\mathsf{P}^\star_{2,0} = \frac{1}{2}\left(\mathsf{P}_{2f,0} - \mathsf{P}_{2h,0}\right)$, which implies $[...] = \mathsf{Q}_{2,0} > 0$. This suggests a rise in the home money supply has a positive impact on the extensive margin of exports. The extensive margin of exports is also affected through changes in global aggregate demand, $\frac{1}{2}\left(\mathsf{C}_0 + \mathsf{C}^\star_0\right)$, which is the weighted sum of consumption across the two economies. All else equal, higher world aggregate demand leads to an expansion in the extensive margin of exports.

The relationship between costs - captured by $\left(\mathsf{w}_0 - \mathsf{P}^\star_{1,0}\right)$ - and the extensive margin of exports depends on the pattern of trade in intermediate inputs between S-1 and S-2 firms. The marginal cost of stage one production in the home economy is the real wage, $\mathsf{w}_0 \equiv \mathsf{W}_0 - \mathsf{P}_{2,0}$. A rise in the real wage reduces the number of firms that export, which is also consistent with higher fixed costs of exporting. Changes in S-1 costs also manifest themselves through the cost of production for foreign S-2 firms, $\mathsf{P}^\star_{1,0}$. Using the pricing equations for S-1 firms, given by (4), the unit cost of foreign S-2 production can be written as, $\mathsf{P}^\star_{1,0} = \frac{1}{2}\left(\mathsf{w}_0 + \mathsf{w}^\star_0\right) - \frac{1}{2}\left(\mathsf{Q}_{2,0} - 2\mathsf{P}^\star_{2,0}\right) + \left(\frac{1}{2\kappa}\right)\mathsf{n}^\star_{h,0}$. This shows, (i), there is a feedback effect onto the extensive margin of exports from movements in stage two unit costs as $\mathsf{n}^\star_{h,0}$ enters the right-hand side of this condition, and (ii), changes in relative consumer prices affect unit costs, via $\mathsf{Q}_{2,0}$. If we temporarily ignore these two mechanisms we can write $-\left(\mathsf{w}_0 - \mathsf{P}^\star_{1,0}\right) = -\frac{1}{2}\mathsf{w}^R_0$, where $\mathsf{w}^R_0 \equiv \left(\mathsf{w}_0 - \mathsf{w}^\star_0\right)$ is the relative real wage. This is important because it implies higher home marginal costs reduce the home extensive margin as they raise home fixed costs, but higher foreign stage one marginal costs expand the home extensive margin because there is trade in intermediates between S-1 and S-2 firms. All else equal, changes in wages across the two economies, generate countervailing pressures on both home and foreign extensive margins.

To further understand the role of relative wages in determining the response of the extensive margin of exports to monetary shocks, recall that firms enter the domestic market until entry costs rise above the presented discounted value of total expected profits. Firm entry is financed by share issue and households hold shares in a mutual portfolio of domestic firms. The return on shares in each country is tied to the return on bonds by a no-arbitrage condition between domestic bonds and shares. In the case where firms only produce for one period, the value of the firm reduces to the future (i.e. $t = 1$) level of profits, discounted by the current real interest rate. For the home economy, the value of S-1 firms is, $\bar{\mathsf{v}}_0 = \mathsf{C}_0 - \mathsf{C}_1 + \bar{\mathsf{d}}_{1,1}$, and because future S-1 profits are consistent with flexible prices, total profits can be written as, $\bar{\mathsf{d}}_{1,1} = \frac{1}{2}\left(\mathsf{C}_1 + \mathsf{C}_1^\star\right) - \mathsf{n}_{e,0}$. Here $\mathsf{n}_{e,0} = \mathsf{n}_{h,1}$ measures the entry of new S-1 firms. Imposing free entry, the relative number of new entrants is,

$$\mathsf{n}_{e,0}^R = -\left(\mathsf{w}_0^R + \mathsf{r}_0^R\right) \tag{16}$$

where I have used the consumption Euler equation to introduce the relative real interest rate, r_0^R. An expansionary monetary shock tends to raise wages and lower the real interest rate. However, from the perspective of understanding the extensive margin of exports, the relative pattern of firm entry and origin country of the shock are what matter. Eliminating $\mathsf{P}^{\star}_{1,0}$ in (15), all else equal, higher relative wages, w^{R}_{0}, lead to a contraction in the home extensive margin of exports, $\mathsf{n}^{\star}_{h,0}$. This is appealing because a higher relative wage reflects higher home S-1 costs. Using (16), the same negative relationship holds for domestic entry, and so there is a positive relationship between relative firm entry and the home extensive margin. Moreover, since (16) holds for both economies, relative firm entry and the foreign extensive margin, $\mathsf{n}_{f,0}$, are negatively related. In this case, relatively more entrants at home (abroad) generates an expansion in the home (foreign) extensive margin of exports. The connection between the extensive margin of exports and firm entry is profits. Greater profits expand both entry and exporting but each activity requires resources. Whilst relative domestic firm entry depends on future relative S-1 total profits, $\overline{\mathsf{d}}^{R}_{1,1} = -\mathsf{n}^{R}_{e,0}$, export decisions depend on current relative S-1 export profits, $\overline{\mathsf{d}}^{\star}_{1h,0} - \overline{\mathsf{d}}_{1f,0} = \mathsf{w}^{R}_{0}$. The wedge between these two sets of profits is then reflected in the relative real interest rate.

3.2. Linking The Extensive Margin to Monetary Policy Shocks

I now account for S-2 firm price setting assumptions and generate explicit solutions for the extensive margin of exports. Different price setting assumptions allow me to understand the role of short-run exchange rate pass-through in determining the impact of monetary shocks on the extensive margin of exports. In period $t = 0$, when S-2 firms engage in producer currency pricing (PCP), domestic output prices do not react to shocks. Because there is full exchange rate pass-through, the foreign currency price of the home good is, $\mathsf{P}_{2f,0} = \mathsf{e}_0$, and the domestic currency price of the foreign good is, $\mathsf{P}^{\star}_{2h,0} = -\mathsf{e}_0$. When firms engage in local currency pricing (LCP), consumer price indices, $\mathsf{P}_0 = \frac{1}{2}\left(\mathsf{P}_{2h,0} + \mathsf{P}_{2f,0}\right)$ and $\mathsf{P}^{\star}_0 = \frac{1}{2}\left(\mathsf{P}^{\star}_{2h,0} + \mathsf{P}^{\star}_{2f,0}\right)$, do not react to shocks. Using short-run money demand, I pin-down consumption in each country, for each price setting regime,

$$\begin{aligned} \text{PCP} &: \quad \mathsf{C}_0 = \mathsf{M} - \frac{1}{2}\mathsf{e}_0 \quad \text{and} \quad \mathsf{C}^{\star}_0 = \frac{1}{2}\mathsf{e}_0 \\ \text{LCP} &: \quad \mathsf{C}_0 = \mathsf{M} \quad \text{and} \quad \mathsf{C}^{\star}_0 = 0 \end{aligned} \tag{17}$$

where $\mathsf{M}^{\star} = 0$. A direct result of LCP is that consumption is not correlated across countries.[13] This is essentially the point made by Betts and Devereux (2000). With LCP, because national consumer price levels are pre-determined, adjustment to monetary shocks works through consumption. As exchange rate pass-through rises, the effect of a nominal depreciation of the domestic currency is to lower $\mathsf{P}^{\star}_{2,0}$, which generates a rise in $\mathsf{C}^{\star}_0$. Since higher M boosts aggregate demand I think of money shocks as aggregate demand shifters. Combing equations (15) and (16) with money demand, I derive the following relationships between extensive margins and monetary policy.

Figure 1: Extensive Margin of Exports and Relative Firm Entry

In figure 1, $\theta > 1$ and $\partial\theta/\partial\kappa < 0$ so that extensive margins are more sensitive to changes in relative firm entry and monetary shocks as the dispersion of productivity rises (recall, κ is the shape parameter of the Pareto distribution, and as κ rises the

[13]Implicit in these conditions is the result that there are no dynamics in the nominal interest rate from monetary shocks ($\mathsf{I}_0 = 0$). This is due to preferences and the type of monetary experiment I consider.

dispersion of firms productivity is concentrated nearer to 1). Most clearly, home monetary policy shocks have a direct and positive effect on the foreign extensive margin of exports. Fixing entry and monetary policy, a higher real exchange rate expands (contracts) the home (foreign) extensive margin of exports, but only under LCP, in which case $\mathsf{Q}_{2,0} = \mathsf{e}_0$. Empirical evidence - such as Berman *et al.* (2011) - points to a depreciation of the domestic currency leading to an expansion in the extensive margin. This is exactly what figure 1 shows. However, this point fails to account for the entry of new firms into domestic markets, for which there is separate empirical evidence that monetary policy shocks play an important role (Bergin and Corsetti, 2008).

Relative firm entry ($\mathsf{n}^R_{e,0}$) and the nominal exchange rate (e_0) are jointly determined by economy-wide resources (summing profits across sectors) and the trade account. As I show in the Appendix, when firms engage in PCP, there is trade between sectors and the nominal exchange rate rises proportionately less than the change in the money supply. This leads to a drop in relative entry across the two economies. That is, $\mathsf{n}^R_{e,0} < 0$ and $\mathsf{e}_0 < \mathsf{M}$. Using these results in figure 1, I find the following negative relationship between the extensive margins: $\mathsf{n}^\star_{h,0} = \frac{\theta}{2}\mathsf{n}^R_{e,0} < 0$ and $\mathsf{n}_{f,0} = \theta\mathsf{M} - \mathsf{n}^\star_{h,0} > 0$, where the change in the home extensive margin is smaller in magnitude than the change in the foreign margin. The basic intuition is that monetary shocks and changes in aggregate demand are consistent with the increased competitiveness of home firms only when firms engage in PCP. With increased competitiveness more (less) home (foreign) firms want to export. In the home economy, this causes an increase in the demand for labor, first, because of increased production, and second, because exporters need to pay a fixed cost. In response to the increase in labor demand, wages rise, increasing fixed costs. Only the most productive manufacturers survive the increase in costs, and resources are reallocated to fewer, more productive home firms. The opposite happens in the foreign economy, and the foreign extensive margin of exports expands.

Now consider the transmission of shocks when firms engage in LCP. In this case, the reaction of firm entry is identical across the two countries, and $\mathsf{n}_{e,0}^{R} = 0$. The nominal exchange rate changes proportionally with the shock, and $\mathsf{e}_0 = \mathsf{M}$. The result is that the extensive margin of exports co-move positively, and $\mathsf{n}_{h,0}^{\star} = \mathsf{n}_{f,0} = \theta\mathsf{M} > 0$, which is also consistent with the stimulus to global aggregate demand. Part of the explanation for positive co-movement is that there is no net trade between sectors, i.e., $\mathsf{nx}_{1,t} = \mathsf{nx}_{2,t} = 0$, but as stressed above, a natural interpretation of the differences in the reaction of extensive margins under PCP and LCP are the movements in relative prices, both between S-1 and S-2 firms, and S-2 firms and consumers. Under PCP, where an expenditure switching effect operates for S-2 goods, $\mathsf{Q}_{2,0} = 0$, and the real exchange rate between firms is, $\mathsf{Q}_{1,0} = \frac{1}{2\kappa}\left(\mathsf{n}_{h,0}^{\star} - \mathsf{n}_{f,0}\right)$. Under LCP, this result is reversed, and the real exchange rate between firms is independent of the shock, i.e., $\mathsf{Q}_{1,0} = 0$, but the consumer level real exchange rate is $\mathsf{Q}_{2,0} = \mathsf{M} > 0$. LCP therefore magnifies the response of the consumer real exchange rate to monetary shocks, as noted in many studies, but eliminates the traditional terms of trade effect. In this model, it also eliminates movements in the real exchange rate relevant for trade between firms. This points to the importance of a competition effect for generating differential movements in the extensive margin of exports.

Relative firm entry alone also plays an important role in understanding how monetary policy impacts the extensive margin of exports. In both the PCP and LCP settings, firm entry co-moves positively. To understand this result, note that current export profits always rise by more in the country in which the monetary shock originates, or $\overline{\mathsf{d}}^{\star}_{1h,0} - \overline{\mathsf{d}}_{1f,0} = \mathsf{w}^R_0 > 0$. This implies the sunk cost of entry rises at home by more than abroad in both PCP and LCP cases, and with free entry, $\mathsf{w}_0 = \overline{\mathsf{v}}_0 = \overline{\mathsf{d}}_{1,1} - \mathsf{r}_0 > 0$ and $\mathsf{w}^{\star}_0 = \overline{\mathsf{v}}^{\star}_0 = \overline{\mathsf{d}}^{\star}_{1,1} - \mathsf{r}^{\star}_0 > 0$. We also already know that relative total future profits are, $\overline{\mathsf{d}}^R_{1,1} = -\mathsf{n}^R_{e,0} = \mathsf{w}^R_0 + \mathsf{r}^R_0$, so when firms engage in PCP, $\overline{\mathsf{d}}^R_{1,1} > 0$, whereas with LCP, $\overline{\mathsf{d}}^R_{1,1} = 0$. We only then need the response of the real interest rate to understand the domestic entry decision. Under PCP, the fall in the home real interest rate is greater than the foreign real interest rate (i.e., $\mathsf{r}_0 < 0$ and $\mathsf{r}^{\star}_0 < 0$), and $\mathsf{r}^R_0 < 0$, so that relative wages rise by more than future expected total S-1 profits, i.e., $\overline{\mathsf{d}}^R_{1,1} > \overline{\mathsf{v}}^R_0$. Under LCP, monetary shocks imply $\overline{\mathsf{d}}^R_{1,1} = 0$ and $\mathsf{w}^R_0 = -\mathsf{r}_0 > 0$, where $\mathsf{r}^{\star}_0 = \mathsf{0}$ (recall also foreign consumption is independent of the shock), and so, $\mathsf{w}^{\star}_0 = \overline{\mathsf{v}}^{\star}_0 = \overline{\mathsf{d}}^{\star}_{1,1} > 0$. Thus, in either case, total future profits are pro-cyclical, which explains positive co-movement in firm entry, and firm entry is stronger in the country that receives the shock only when movements in aggregate demand are consistent with greater competitiveness of home firms in export markets.

4. A Quantitative Model with Extensive Margins

I now analyze the transmission of monetary policy shocks in a more general setting.[14] Although the model I present features firm entry and an endogenous extensive margin for exports, it is consistent with a large class of open economy models that feature a fixed number of varieties - such as Kollmann (2001) and Chari *et al.* (2002). In these papers movements in the real exchange rate can arise from three sources. Deviations from the law of one price for traded goods, fluctuations in the relative prices of non-traded to traded goods, and fluctuations in the relative prices of multiple traded goods when consumption baskets differ across countries. In my model, if firms engage in LCP, the law of one price fails in the S-2 sector. The non-tradability of S-1 goods is endogenous and each economy is (symmetrically) biased towards domestic S-1 goods. The real exchange rates relevant for consumers and firms are therefore a consequence of the interaction of S-2 firm pricing decisions and the use of S-1 goods. The following expression, which holds when firms engage in LCP, captures this linkage,

$$\mathsf{Q}_{1,t} = (1-2\gamma)\left(\mathsf{w}_t^R - \mathsf{Q}_{2,t}\right) - \varsigma \mathsf{n}_t^R + (1-\gamma)\,\xi \mathsf{n}_t^{\star R} \tag{18}$$

where $\mathsf{n}_t^R \equiv \mathsf{n}_{h,t} - \mathsf{n}_{f,t}^{\star}$ is the relative supply of products (number of firms) and $\mathsf{n}_t^{\star R} \equiv \mathsf{n}_{h,t}^{\star} - \mathsf{n}_{f,t}$ is the relative number of exports (extensive margin) and the parameters $\varsigma \equiv \left[\gamma(1-\upsilon) + \left(\frac{1-\gamma}{\kappa}\right)\right]$ and $\xi \equiv \left[\frac{\kappa(1-\upsilon)+1}{\kappa}\right]$ are positive, where $\gamma \in (0,1)$ measures the use of intermediate inputs and $\upsilon > 0$ the variety effect. Equation (18) is similar to that derived in Ghironi and Melitz (2005), where the term $\left(\mathsf{w}_t^R - \mathsf{Q}_{2,t}\right)$ represents

[14] In this case, I express the equations of the model as $\mathcal{Z}_t = \underline{A}_1 \mathcal{Z}_{t+1} + \underline{A}_2 \mathcal{Z}_{t-1} + \underline{A}_3 u_t$, where u_t is the monetary policy shock, $\underline{A}_i$, $i = 1, 2, 3$, are matrices containing structural parameters, and $\mathcal{Z}_t$ is a vector of the composite world and relative variables. I apply the solution method for linear rational expectations models outlined in Binder and Pesaran (1995).

the inverse 'terms of labor' - i.e., the relative cost of effective units of labor across countries. In my analysis, movements in the terms of labor affect the S-1 real exchange rate depending on the mix of intermediate inputs. In the special case analyzed in section 3, when the use of intermediates is equal (i.e., $\gamma = 1/2$) only the relative supply of products and number of exported goods (positively) affect the S-1 real exchange rate. Equation (18) also embeds the differences in pricing decisions when there are nominal rigidities, because when firms engage in PCP, $\mathsf{Q}_{2,t} = 0$, and the terms of labor reduce to $\mathsf{w}_t^R = \mathsf{W}_t^R - \mathsf{e}_t$, for $\gamma \neq 1/2$.

Dynamics are generated through two main mechanisms. The proportion of firms that continue to produce each period and price setting. When firms engage in PCP, two dynamic price equations determine the extent of global price dynamics at the consumer level,

$$\Delta\mathsf{P}_{2h,t} = \beta\mathbb{E}_t\Delta\mathsf{P}_{2h,t-1} + \digamma\left(\mathsf{P}_{1,t} - \mathsf{P}_{2h,t}\right) \quad \text{and} \quad \Delta\mathsf{P}^\star_{2f,t} = \beta\mathbb{E}_t\Delta\mathsf{P}^\star_{2f,t+1} + \digamma\left(\mathsf{P}^\star_{1,t} - \mathsf{P}^\star_{2f,t}\right)$$

where $\digamma \equiv (1-\alpha)(1-\alpha\beta)/\alpha\beta$. The consumer price indices in the home and foreign economy are given by, $\mathsf{P}_{2,t} = \frac{1}{2}\left(\mathsf{P}_{2h,t} + \mathsf{P}^\star_{2f,t} + \mathsf{e}_t\right)$ and $\mathsf{P}^\star_{2,t} = \frac{1}{2}\left(\mathsf{P}_{2h,t} - \mathsf{e}_t + \mathsf{P}^\star_{2f,t}\right)$, where the nominal exchange rate is pinned down by resources, trade, and money demand. When firms engage in LCP, the law of one price fails to hold, and I map price dynamics into four composite variables, $\Delta\mathsf{P}^R_{2,t}$ and $\Delta\mathsf{P}^W_{2,t}$ and $\Delta\mathsf{T}^R_{2,t}$ and $\Delta\mathsf{T}^W_{2,t}$, where $\mathsf{T}_{2,t} \equiv \mathsf{P}_{2f,t} - \mathsf{P}_{2h,t}$ and $\mathsf{T}^\star_{2,t} \equiv \mathsf{P}^\star_{2f,t} - \mathsf{P}^\star_{2h,t}$, as emphasized in Benigno (2004). These prices are driven by marginal costs and the two real exchange rates, given in (18). In the LCP case, two parameters, $\digamma \neq \digamma^\star$, determine the persistence of relative prices, given change in marginal costs.

To get an idea of the quantitative implications of my model, I use calibrated parameter values to compute impulse response functions to a one-off shock to the home money growth,

$$\Delta \mathsf{M_t} = \rho \Delta \mathsf{M_{t-1}} + \varepsilon_t \tag{19}$$

where $\rho \in (0,1)$ and ε_t captures the home monetary policy shock. I assume that one year after the shock the home money stock rises by 1%. Figure 2 presents the parameter values for the calibrating the model.

Figure 2: Calibrated Parameters for Quantitative Analysis

I interpret a model time period as a quarter of a year. I set $\beta = 0.99$, which implies a steady-state interest rate of 4%. For households, I set the (inverse) Frisch elasticity of labor supply to wages at $\sigma_L = 0.47$, and I assume $\sigma_m = 12$, which implies the semi-elasticity of money demand with respect to the interest rate is $\frac{1}{\sigma_m}\left(\frac{\beta}{1-\beta}\right) = 8.25$. For S-1 firms, I follow Ghironi and Melitz (2005) and set $\delta = 0.025$ - to match the U. S. empirical level of 10% job destruction per year - and $\theta = 3.8$, which, given the standard deviation of log US plant sales, implies $\kappa = 3.4$. For S-2 firms I assume $\upsilon = \theta/(\theta - 1)$ and (symmetric) bias for domestic S-1 goods, with $\gamma = 0.8$. For price setting, in the PCP case, I set $\alpha = 0.75$. In the LCP case I allow for a pricing-to-market element to S-2 firms price setting decisions and assume $\alpha_h = 0.7$ and $\alpha_h^\star = 0.8$. Finally, given a calibrated value of $\rho = 0.68$ for the home money growth process, a 1% rise in the home money stock after four quarters implies setting $\varepsilon_0 = 0.42$, with $\varepsilon_t = 0$ for $t \geq 1$.

Figure 3 plots the response of home and foreign variables to a monetary shock under producer currency pricing.

Figure 3: Impulse Responses with Producer Currency Pricing

The solid line represents home economy variables and the starred (i.e., $\star$) impulse responses are for the foreign economy. In all cases, the dashed line represents the shock. The impulse response functions confirm the results derived analytically. A home money shock drives weaker home firms out of the export market (i.e., the home extensive margin of exports contracts) whilst the number of home imports expands. This is consistent with Alessandria and Choi's (2008) analysis of product-level data for the US and Canada. In particular, there is a 0.3% fall (1.1% rise) in the home (foreign) extensive margin on impact, given a 1% rise in the home money supply after 4 quarters (the impact change in the money supply is 0.42%). New domestic entrants in each economy embody investment by households and the total stock of firms represents the accumulated capital. Thus, we can think of investment in this model as also being at the extensive margin. As with exports, it is more common to consider international co-movement at the intensive margin, but the results on the extensive margin are surprisingly similar. Investment, which I measure as the real value of household investment in new firms, rises in the home economy, and there are strong positive spillovers onto the foreign economy, with positive international co-movement. Investment is also pro-cyclical. Consumption and the real interest rate also respond as they would do in a standard sticky-price model of the business cycle - consumption rises in both the home and foreign economy and the real interest rate falls.[15]

The international transmission of monetary policy along the extensive margin of exports rests on three channels, two of which are usually stressed when adjustment of exports takes place along the intensive margin: the terms-of-trade and real interest rate. These two channels imply that consumers switch expenditure towards the country in which the shock originated (due to changes in the terms of trade) and that aggregate demand rises in both countries (due to changes in real interest rates). Here, consumers switch between home and foreign S-2 production and firms change the mix of S-1 inputs. Changes in wages are the third channel that affect the extensive margin of exports, and are also present in standard models that feature a fixed number of varieties. However, in this case, wages only play a role in determining labor supply. With an endogenous number of exporters wages play an important role because they change the costs faced by firms when deciding on whether or not to export. Moreover, in the model I present, changes in wages and in the real interest rate also determine investment in new firms/firm entry, altering the channel through which movements in wages affect the extensive margin of exports.

In fixed-variety models, both the terms-of-trade and real interest rate channels of macroeconomic interdependence are reduced one LCP is assumed. This leads to a muted response of foreign macro-variables to home monetary shocks. Above,

[15] I have not plotted the response of inflation, but the reaction of home and foreign consumer price inflation is standard. Consumer prices rise by less in the foreign economy than in the home economy.

however, I show that the PCP/LCP distinction is perhaps even more important for the co-movement of the extensive margin of exports, and the same is true for a more general version of the model. Figure 4 plots the response of home and foreign variables to a monetary shock under local currency pricing.

Figure 4: Impulse Responses with Local Currency Pricing

Given the reaction of profits, costs (wages) and the real interest rates internationally, the intuition for co-movement in the extensive margin of exports is similar to that developed in the analytical section of the paper. That is, changes in global demand dominate the export decision under LCP and this results in both the home and foreign extensive margin of exports expanding. This expansion is always less than the expansion in the home extensive margin of exports under producer currency pricing, lending support to the idea that the competitiveness channel which operates under PCP has a strong effect on the extensive margin. One striking feature of the impulse responses is that whilst the reaction of most foreign variables to the shock is muted - in keeping with the result that LCP mitigates the international spillovers from monetary policy when the number of firms is given - there remain strong international spillovers from monetary shocks onto profits. Again, this is the same mechanism that operates in the analytical version of the model, making it clear that when there is trade in intermediate inputs between firms, a monetary stimulus will always raise profits in both countries, independent of PCP or LCP price setting. The overriding difference between the PCP and LCP cases, is that in the former, a smaller number of more productive firms share higher average profits, whereas under LCP, although profits rise, so does the number of firms. The reason for this is that the change in relative prices that monetary shocks induce under PCP falls on the side of the firms (recall, $\mathsf{Q}_{2,t} = 0$ under PCP), whereas under LCP the adjustment falls on the side of the consumer.

4. Conclusion

This paper develops a two-country, sticky price, DSGE model to understand how monetary policy shocks affect the extensive margin of exports. The effect of monetary policy shocks on the extensive margin is linked to exchange rate pass-through. When there is full exchange rate pass-through monetary shocks generate negative co-movement in extensive margins. With limited exchange rate pass-through monetary shocks generate positive co-movement. To understand the international transmission of monetary policy, three channels of interdependence are stressed - those normally associated with adjustment along the intensive margin - the real interest rate and terms of trade - and wages. The third channel links costs of domestic entry and fixed costs of exporting. Simple quantitative results show that the change in the extensive margin of exports is sizeable.

References

Alessandria, G., Choi, H., 2007. Do Sunk Costs of Exporting Matter for Net Export Dynamics?, Quarterly Journal of Economics 122, 289-336.

Alessandria, G., Choi, H., 2008. The Role of Exporting and Trade for Entry over the Business Cycle, mimeo.

Benigno, G., 2004. Real Exchange Rate Persistence and Monetary Policy Rules, Journal of Monetary Economics 51, 473-502.

Bergin, P., Corsetti, G., 2008. The Extensive Margin and Monetary Policy, Journal of Monetary Economics 55, 1222-1237.

Bergin, P., Glick, R., 2009. Endogenous Tradability and Macroeconomic Implications, Journal of Monetary Economics 56, 1086-1095.

Bergin, P., Lin, C., 2010. The Dynamic Effects of Currency Union on Trade, mimeo.

Berman, N., Martin, P., Mayer, T., 2011. How do Different Exporters React to Exchange Rate Changes? Theory, Empirics and Aggregate Implications, CEPR Discussion Paper 7493.

Bernard, A., Jensen, B., 2004. Why Some Firms Export, Review of Economics and Statistics 86, 561-569.

Bilbiie, F., Ghironi, F., Melitz, M., 2007. Monetary Policy and Business Cycles with Endogenous Entry and Product Variety, NBER Macroeconomics Annual, MIT Press.

Binder, M., Pesaran, H., 1995. Multivariate Rational Expectations Models and Macroeconomic Modelling: A Review and Some New Results. In: H. Pesaran and M.

Wickens, Editors, Handbook of Applied Econometrics: Macroeconomics, Blackwell (1995).

Calvo, G., 1983. Staggered Prices in a Utility-Maximizing Framework, Journal of Monetary Economics 12, 383-398.

Campbell, J. and B. Lapham, 2004. Real Exchange Rate Fluctuations and the Dynamics of Retail Trade Industries in the U.S.-Canada Border, American Economic Review 94, 1194-1206.

Chari, V., Kehoe, P., McGratten, E., 2002. Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?, Review of Economic Studies 69, 533-564.

Dornbusch, R., Fischer, S., Samuelson, P., 1977. Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods, American Economic Review 67, 823-39.

Ghironi, F., Melitz, M., 2005. International Trade and Macroeconomic Dynamics with Heterogeneous Firms, Quarterly Journal of Economics 120, 865-915.

Huang, K., Liu, Z., 2007. Business Cycles with Staggered Prices and International Trade in Intermediate Inputs, Journal of Monetary Economics 54, 1271-1289.

Jaimovich, N., Floetotto, M., 2008. Firm Dynamics, Markup Variations, and the Business Cycle, Journal of Monetary Economics 55, 1238-1252.

Kimbrough, K., 1992. Specialization, the Terms of Trade, and the International Transmission of Monetary Policies, Canadian Journal of Economics 25, 884-900.

Kollmann, R., 2001. Explaining International Comovements of Output and Asset Returns: The Role of Money and Nominal Rigidities, Journal of Economic Dynamics and Control 25, 1547-1583.

Lewis, V., 2009. Business Cycle Evidence on Firm Entry, Macroeconomic Dynamics 13, 605-624.

Lewis, L., 2010, Exports versus Multinational Production under Nominal Uncertainty, mimeo, University of Michigan.

Melitz, M., 2003. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity, Econometrica 71, 1695-1725.

Naknoi, K., 2008. Real Exchange Rate Fluctuations, Endogenous Tradability and Exchange Rate Regimes, Journal of Monetary Economics 55, 645-663.

Obstfeld, M., Rogoff, M., 1995. Exchange Rate Dynamics Redux, Journal of Political Economy 103, 624-60.

Scholl, A., Uhlig, H., 2005. New Evidence on the Puzzles: Results from Agnostic Identification on Monetary Policy and Exchange Rates, Journal of International Economics 76, 1-13.

Appendix on Analytical Results under PCP and LCP

To understand firm entry, consider resources in the home economy. Accounting for the PCP pricing assumptions, we find,

$$\begin{aligned}
\mathsf{n}_{e,0} &= \left(\frac{wL}{\overline{v}n_e}-1\right)\mathsf{w}_0+\left(\frac{n_h\overline{d}_1}{\overline{v}n_e}\right)\overline{\mathsf{d}}_{1,0}+\left(\frac{d_2}{\overline{v}n_e}\right)\mathsf{d}_{2,0}-\left(\frac{Y_2}{\overline{v}n_e}\right)\mathsf{C}_0\\
\overline{\mathsf{d}}_{1,0} &= \mathsf{C}_0^W+\alpha\mathsf{P}_{1,0}+(1-\alpha)\,\mathsf{P}_{1,0}^{\star} \qquad \text{and} \qquad \mathsf{d}_{2,0}=\mathsf{C}_0^W-(\sigma-1)\,\mathsf{P}_{1,0}\\
\mathsf{w}_0 &= \mathsf{C}_0-\mathsf{n}_{e,0}
\end{aligned}$$

where steady state variables (i.e., those without a sub-script) are a function of the underlying parameters of the model. An equivalent set of conditions hold for $\mathsf{n}_{e,0}^{\star}$. Eliminating profits and wages,

$$\frac{wL}{\overline{v}n_e}\mathsf{n}_{e,0}^R=\left(\frac{n_h\overline{d}_1}{\overline{v}n_e}+\frac{d_2}{\overline{v}n_e}\right)\left(\mathsf{e}_0-\mathsf{M}^R\right)+\left[(1-\sigma)\left(\frac{d_2}{\overline{v}n_e}\right)-(1-2\alpha)\left(\frac{n_h\overline{d}_1}{\overline{v}n_e}\right)\right]\mathsf{P}_{1,0}^R$$

where $\mathsf{n}_{e,0}^R\equiv\mathsf{n}_{e,0}-\mathsf{n}_{e,0}^{\star}$. Thus, we are left needing to pin down prices and the exchange rate. The equation for the exchange rate comes from the trade balance and relative money demand, which can be expressed as,

$$0=\left(\frac{1}{\Phi}\right)\mathsf{P}_{1,0}^R-\left(\mathsf{M}^R-\mathsf{e}_0\right)$$

where $\Phi\equiv\frac{\sigma}{\sigma-1}$. Using the equations for the extensive margin of exports and relative prices at S-1,

$$\mathsf{P}_{1,0}^R=\left(\frac{\kappa+1/2}{\kappa-1/2}\right)\mathsf{w}_0^R+\left(\frac{\kappa}{\kappa-1/2}\right)\mathsf{e}_0 \quad \text{and} \quad \mathsf{P}_{1,0}^W=\mathsf{w}_0^W+\left(\frac{1/2}{\kappa-1/2}\right)\mathsf{M}^W$$

where I have used, $\mathsf{C}_0^W=\mathsf{M}^W$ and wages again depend on the free entry condition. Using all of these equations, the solution for $\widehat{n}_{e,0}^R$ is:

$$\mathsf{n}_{e,0}^R=2\kappa\left(\frac{\Omega}{\Omega-1}\right)\mathsf{M} \quad ; \quad \Omega\equiv\left[\frac{2\alpha\sigma+\left(\frac{1+\Phi}{\Phi}\right)}{2\alpha\sigma+(\beta-1)}\right]\left[\left(\frac{\Phi}{1+\Phi}\right)\left(\frac{1}{2\kappa-1}\right)\right]$$

First note that $\Phi/(1+\Phi)$ and $1/(2\kappa-1)$ are both less than 1. The term $2\alpha\sigma$, where $\alpha = \kappa/[\kappa+(\sigma-1)]$, is greater than 2 and thus the numerator and denominator and both positive in $\left[2\alpha\sigma + \left(\frac{1+\Phi}{\Phi}\right)\right]/[2\alpha\sigma+(\beta-1)]$. Although it is possible for this term to be large than 1, the parameter Ω is always between zero and one. In this case, $\mathsf{n}_{e,0}^{R} < 0$. Once we know this, it is possible to show $\mathsf{M} > \mathsf{e}_0 > 0$. Using these condition in the expressions for the extensive margin of exports, we can conclude, $\mathsf{n}_{h,0}^{\star} < 0$ and $\mathsf{n}_{f,0} > 0$.

For LCP, I repeat the analysis. In this case, there is a unit coefficient for M^{R} and $\mathsf{Q}_{2,0}$ in the relative resource constraint. I can use this to understand relative price fluctuations, $\mathsf{P}_{1,0}^{R}$. If we temporarily suppose $\mathsf{n}_{e,0}^{R} = 0$, it is immediate that $\mathsf{P}_{1,0}^{R}$ and M^{R} are also related by a unit coefficient. Since $\mathsf{n}_{e,0}^{R}$ is a function of $\mathsf{Q}_{2,0}$ and $\mathsf{P}_{1,0}^{R}$, it follows $\mathsf{n}_{e,0}^{R} = 0$.

Figure 1: Extensive Margin of Exports and Relative Firm Entry

	Home Export Margin ($\mathsf{n}^{\star}_{h,0}$)	Foreign Export Margin ($\mathsf{n}_{f,0}$)
PCP	$\left(\frac{\theta}{2}\right)\mathsf{n}^{R}_{e,0}$	$\theta\left(\mathsf{M}-\frac{1}{2}\mathsf{n}^{R}_{e,0}\right)$
LCP	$\frac{\theta}{2}\left(\mathsf{n}^{R}_{e,0}+\mathsf{e}_{0}\right)$	$\theta\left[\mathsf{M}-\frac{1}{2}\left(\mathsf{n}^{R}_{e,0}+\mathsf{e}_{0}\right)\right]$

Figure 2: Calibrated Parameters for Quantitative Analysis

Calibrated Parameters		
Parameter	Description	Value
β	Subjective discount factor	0.99
σ_L	Inverse Frisch elasticity of labor supply	0.47
σ_m	Coefficient of relative risk aversion in real balances	12
α	Calvo probability that prices are adjusted, PCP	0.75
α_h and $\alpha_h^{\star}$	Calvo probability that prices are adjusted, LCP	0.7 and 0.8
θ	Elasticity of substitution between goods	3.8
κ	Firm dispersion	3.4
γ	Use of intermediate inputs	0.8
δ	Exit shock	0.025
ρ	Auto-regressive parameter for money growth	0.68

Figure 3: Impulse Responses with Producer Currency Pricing

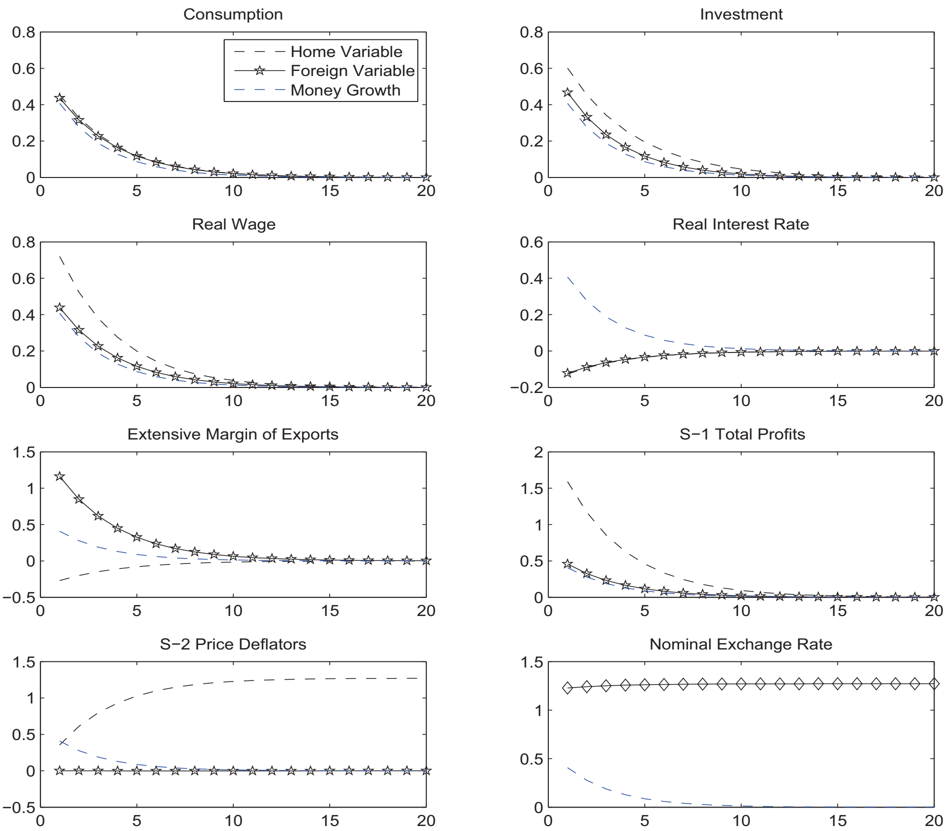

Figure 4: Impulse Responses with Local Currency Pricing