# Pricing and Consumer Surplus in Monopoly with Product Design*

Kyungmin Kim† Nenad Kos‡

January 16, 2024

**Abstract**

The model considers a monopolist who optimally chooses the design and price of a product on the Hotelling line. We characterize the set of prices and consumer surplus that can arise in the model across all distributions of tastes. In a stark departure from the monopoly model without product design, the seller never offers a price below a certain threshold. Moreover, the maximal consumer surplus is strictly smaller than in the absence of product design. It is attained by a distribution that renders the seller indifferent over a set of design/price combinations. Notably, the distribution does not exhibit unit elasticity given any fixed design.

JEL Classification Numbers: D42, D80, L12.

Keywords: Product Design; Hotelling; Pricing; Consumer Surplus.

# 1 Introduction

Consumer surplus plays a pivotal role in the theory of monopoly by shedding light on the economic implications of market power and pricing strategies employed by monopolistic firms. The notion that consumers might be willing to pay more than the offered price, resulting in consumer surplus, and that said consumer surplus corresponds to the area below the demand and above the price was

*We thank Christoph Carnehl, Nima Haghpanah, Vijay Krishna, Ali Shourideh, and seminar audiences at Bocconi, CMU/Pittsburgh, Emory, and Penn State for many helpful comments. Part of this project was completed while Nenad was visiting EconTribute center at Bonn University. He is very grateful for their hospitality.

†Department of Economics, Emory University, Email: kyungmin.kim@emory.edu

‡Bocconi University, Department of Economics, IGIER and CEPR, nenad.kos@unibocconi.it.

introduced by Dupuit (1844) and expanded upon by Marshall (1890).[1] Despite its importance for the analysis of monopoly, how the range of achievable consumer surplus depends on the demand function (equivalently, distribution of valuations) was characterized only recently by Condorelli and Szentes (2020). Their model takes the product as given and studies how information/demand shapes consumer surplus. Our paper asks the question of how the seller's ability to design the object to conform to consumers' tastes affects the size of the surplus accruing to consumers. Product design has been recognised as a vital part of product placement at least since the seminal work of Hotelling (1929). Hotelling used an example of a producer choosing the sweetness of cider with consumers' locations on the line representing their preferred sweetness.[2]

Formally, consumers have heterogeneous tastes represented by the distribution $F$ over the product space $[-1, 1]$ and the seller chooses price $p$ and design $\ell \in [-1, 1]$.[3] A consumer's willingness to pay for the product is $1 - c(|x - \ell|)$, where $x$ is the consumer's prefered design and $c(\cdot)$ an increasing convex (disutility) function. A consumer's value of the product is $1$ if it perfectly matches his taste and decreases as the distance between $\ell$ and $x$ grows. Moving $\ell$ corresponds to determining horizontal characteristics of the product, raising some consumers' willingness to pay but lowering others'. The standard monopoly model can be interpreted as a special case of our model in which the seller's position is exogenously fixed: The further away consumers are from the seller, the less they are willing to pay for the good. The distribution of consumers' tastes, thus, induces a distribution of willingness to pay and the corresponding demand function.

High consumer surplus is achieved at the coincidence of high social welfare and low produer surplus. The former requires that consumers' preferred designs are close to the seller's design (in order to have low disutility from the mismatch). The latter, that the seller charges a low price. Low prices are, however, optimal only when consumers are sufficiently spread out. In fact, we show that very low prices are never optimal for the seller: the seller never offers a price strictly below $1/3$, regardless of the disutility function or the distribution of tastes. The result stems from the observation that the design and the price of the product determine the interval of consumer tastes that are willing to buy the good. The seller can split this interval into two intervals of the same width and cover only one of them—the one with more consumers. This guarantees that the seller reaches at least half of the initial demand at a higher price. Due to the convexity of the disutility

[1] Although consumer surplus is nowadays a widely employed metric of consumers' benefits from trade, the discussion around its usefulness and ability to capture compensating variations was raging for a long time; see for example Willig (1976).

[2] For more on the importance of product design, see, e.g., Lancaster (1966), Johnson and Myatt (2006) and, more recently, Bar-Isaac et al. (2023).

[3] The model also allows for the interpretation that the position of a consumer represents his physical location on a linear city and the seller chooses a physical location.

the price needed to cover half of the interval is more than double the initial price if the latter is below $1/3$. This is in stark contrast with the standard monopoly model where any price in $[0, 1]$ can be optimal. At its core, the ability to design a product empowers the seller to maximize the demand at any given price. Without the element of design, the sole avenue for the seller to boost demand is by reducing the price.

The consumer-optimal distribution strikes the balance between the opposing forces of welfare maximization and profit minimization. It requires that the seller be indifferent over a set of design/price pairs; each design paired with a different price. The characterization of the distribution of tastes leads to several important implications. Firstly, unlike in the standard monopoly model, consumer demand given the seller's equilibrium location $\ell$ is *not* unit-elastic; see Figure 3 for the comparison of the two distributions. Indeed, any distribution of consumers' tastes that would create unit-elastic demand for some fixed design makes it suboptimal for the seller to choose that design. An immediate implication is that the seller's ability to design the product reduces the maximal consumer surplus.

Second, when our methodology is applied to the standard monopoly model, it provides a simple constructive proof of the distribution of valuations that maximizes consumer surplus. The idea is to maximize consumer surplus along all distributions that induce the seller to offer some price $p$. Since consumer surplus increases as valuations above the price rise, the maximum is achieved when the seller is indifferent between offering $p$ and any price above it. The indifference yields a Pareto distribution. Maximization over $p$ yields the distribution that maximizes consumer surplus.

Finally, among all increasing and convex disutilities $c(\cdot)$, consumer surplus is maximized when the disutility is linear. The maximal consumer surplus with product design is, however, strictly smaller than the maximal consumer surplus without design identified in Condorelli and Szentes (2020). The ability to design the product is advantageous for the seller but may have a detrimental effect on consumers.

**Related Literature.** The study of consumer surplus in monopoly has recently garnered significant attention. Our paper is most closely related to Condorelli and Szentes (2020), who (among other things) characterize the distribution of valuations that maximizes consumer surplus in the standard monopoly setting.[4] Bergemann et al. (2015) characterize the set of consumer (and producer) surplus that can arise when the seller can engage in third-degree price discrimination based on information about consumers.[5] Roesler and Szentes (2017) consider a monopoly problem when

[4]See also Condorelli and Szentes (2022), in which they extend their analysis to the Cournot oligopoly model.

[5]A recent study by Terstiege and Vigier (2023) discovers an interesting connection between Condorelli and Szentes (2020) and Bergemann et al. (2015); they show that a multi-product version of the latter's problem can be approximated

the buyer chooses to acquire information about his value and the seller sets a price after observing the buyer's information acquisition strategy (signal). Their problem can be interpreted as the case where the set of feasible demand (distribution) functions is restricted by the belief-consistency (mean-preserving contraction) condition relative to an underlying distribution of buyers' values. As opposed to our paper, this body of work takes the design of the product as exogenously given.

Optimal product design has been studied extensively, typically utilizing the Hotelling framework. In fact, Hotelling (1929) himself considers the question of optimal/equilibrium positioning (product design). Most existing studies examine an oligopoly setting with the uniform distribution of tastes. How design and pricing in duopoly are affected by more general distributions of tastes is investigated by, for example, Anderson et al. (1997).[6]

Monopoly in a Hotelling context is studied by Hidir and Vellodi (2021), Bar-Isaac et al. (2023), and Kim and Kos (2023). Hidir and Vellodi (2021) study consumer optimal information revelation in a model of monopolistic product design.[7] Bar-Isaac et al. (2023) analyze a model of product design where consumers are distributed on a circle while the monopolist chooses a design in a circle. In both of these papers the distribution of tastes is assumed to be uniform. Kim and Kos (2023) study the seller's optimal design and pricing strategy when the seller has no information about the distribution of tastes; a robustness problem.

The remainder of this paper is organized as follows. Section 2 formally introduces our model. Section 3 studies the benchmark case where the design of the product is exogenously fixed. Section 4 and Section 5 provide our main results: Section 4 determines the set of implementable prices, and Section 5 characterizes the maximal achievable consumer surplus and a distribution that produces it. Section 6 concludes.

# 2 The Model

A monopolist is facing a unit mass of consumers whose heterogeneous tastes are distributed over $[-1, 1]$ according to some distribution $F$. Given $F$, the seller chooses a product design (position) $\ell$ and a price $p \geq 0$. Each consumer's willingness-to-pay for the seller's product depends on the distance between his and the seller's positions. Given $\ell$, a consumer at $x \in [-1, 1]$ values the product at $1 - c(|x - \ell|)$, where $c : \mathcal{R}_+ \to \mathcal{R}_+$ is strictly increasing and weakly convex with

by the former's as the number of products grows.

[6] Rhodes and Zhou (2022) examine how personalized pricing and uniform pricing affect consumer surplus under various distributions of tastes in a Hotelling oligopoly, albeit they do not allow for product design.

[7] Ali et al. (2023) study information disclosure in (among other things) duopoly on a Hotelling line, but without product design.

$c(0) = 0$. A consumer's willingness-to-pay for the preferred design is normalized to 1, while $c(|x - \ell|)$ captures disutility from preference misalignment (or "transportation cost"). The seller's marginal cost of production is 0.

For each $p \leq 1$, let $\Delta(p)$ denote the maximal distance from the product design at which a consumer is still willing to buy the product; that is, $\Delta(p) := c^{-1}(1-p)$. Given $(\ell, p)$, the seller's demand and profit are, respectively, given by

$$D(\ell, p; F) := F(\ell + \Delta(p)) - F_{-}(\ell - \Delta(p)) \text{ and } \pi(\ell, p; F) := pD(\ell, p; F),$$

where $F_{-}(x) := \lim_{x' \uparrow x} F(x)$. The seller chooses $(\ell, p)$ to maximize her profit:

$$\pi(F) := \max_{(\ell,p)} \pi(\ell, p; F),$$

with $B(F)$ denoting the set of the solutions: $B(F) := \{(\ell, p) : \pi(\ell, p; F) = \pi(F)\}$. We write $CS(F)$ for the maximal consumer surplus under the distribution $F$ given that the seller maximizes profit:

$$CS(F) := \max_{(\ell,p) \in B(F)} \int \max\{1 - c(|x - \ell|) - p, 0\} dF(x).$$

Our primary goal is to characterize a distribution that maximizes $CS(F)$.

**Relation to the standard monopoly model.** Our model differs from the standard monopoly model, in that consumers have horizontally differentiated *tastes*, rather than vertically different values, and the seller designs the good. The two models, however, are much closer than these differences seem to suggest. In fact, the standard monopoly model can be interpreted as a special case of our model in which the product's design is exogenously given. To see this formally, fix the product's design $\ell \in [-1, 1]$. Since a consumer's value for the product is $1 - c(|x - \ell|)$, the distribution of consumers' willingness-to-pay, denoted $G$, is given by

$$G(p) := 1 - Pr\{1 - c(|x - \ell|) > p\} = 1 - \int_{(\ell - \Delta(p), \ell + \Delta(p))} dF(x),$$

where $\Delta(p)$ is as defined above. The resulting demand function is

$$D(p) := 1 - G_{-}(p) = F(\ell + \Delta(p)) - F_{-}(\ell - \Delta(p)). \tag{1}$$

The seller's problem is to choose $p$ that maximizes $pD(p)$, just as in the standard monopoly problem.

Conversely, consider the standard monopoly model with a downward-sloping demand function $D(p)$. Fix the design to $\ell = 0$, and take any continuous and strictly increasing function $c$ such that $c(1) \geq 1$.[8] The given demand function can be produced by the distribution of tastes $F$ such that

$$F(y) = \frac{D(\Delta^{-1}(y)) + 1}{2}, \text{ for all } y \in [0, 1] \tag{2}$$

and $F$ is symmetric around 0 (i.e., $F(-y) = 1 - F_{-}(y)$ for all $y \in [0, 1]$). The following result summarizes the argument.

**Proposition 1** *Fix a disutility function $c$ that is continuous and strictly increasing with $c(1) \geq 1$.*[9] *The problem given the distribution of tastes $F$ and the design $\ell = 0$ has the same solution as the standard monopoly problem given a non-increasing demand function $D : [0, 1] \to [0, 1]$ when $F$ and $D$ satisfy* (1) *(or* (2)*).*

# 3 Maximal Consumer Surplus for a Fixed Design

We start by examining the case where the design of the product is exogenously fixed at $\ell = 0$. Let $G$ denote the distribution of consumers' willingness-to-pay induced by the distribution of tastes $F$ given the design $\ell = 0$. Then, the problem of maximizing $CS(F)$ reduces to the following problem of choosing $G$:

$$\max_{p, G \in \Delta([0,1])} \int_p^1 (v - p) dG(v) \text{ s.t. } p \in \arg\max_v v \cdot (1 - G_{-}(v)), \tag{3}$$

which is identical to the problem studied by Condorelli and Szentes (2020). We provide a constructive solution method that extends to the problem with product design. It consists of two steps: (i) For each price $p$, we identify $G$ that maximizes consumer surplus while maintaining the seller's incentive to offer $p$. (ii) We then find the price $p$ that maximizes consumer surplus.

[8] Given $\ell = 0$, the consumers that are farthest away from the seller (those at $-1$ or 1) are willing to pay $1 - c(1)$. Therefore, the assumption $c(1) \geq 1$ ensures that consumers can have arbitrarily small willingness-to-pay for the seller's product.

[9] For this proposition, this condition can be relaxed to $c(2) \geq 1$ by assuming that the design is given at $-1$ or 1. We fix $\ell$ to 0 so as to make this result directly applicable to the subsequent analysis.

For the first step, a key observation is[10]

$$\begin{aligned}\int_p^1 (v-p)dG(v) &= \int_p^1 (1-G(v))dv \\ &\leq \int_p^1 \frac{p(1-G(p))}{v}dv \\ &= -\log(p)p(1-G(p)).\end{aligned}$$

The first equality is through integration by parts, while the inequality uses the fact that $p$ is an optimal price given $G$ and thus for each $v$, $v(1-G(v)) \leq p(1-G(p))$. The upper bound $-\log(p)p(1-G(p))$ is maximized when $G(p)=0$ and attained by the distribution $G$ such that $v(1-G(v)) = p(1-G(p))$ for all $v \geq p$. It follows that the optimal distribution given $p$ is such that its support is $[p,1]$ and the seller is indifferent over all prices in the interval.

Intuitively, given price $p$, consumer surplus increases as the distribution $G$ rises in the sense of first-order stochastic dominance. However, it is subject to the constraint that $p$ should be the seller's optimal price. In particular, no price above $p$ can be a profitable deviation for the seller. This limits the extent to which one can stochastically raise $G$. For any value $v$ above $p$, probability mass above $v$ can be assigned only up to the point where the seller is indifferent between charging $p$ and $v$. This fully determines the optimal distribution given $p$.

The second step is to maximize $-\log(p)p$. Solving it leads to the following result, which corresponds to Theorem 1 of Condorelli and Szentes (2020).

**Theorem 1** *If the product's design is fixed at $\ell = 0$, the maximally attainable consumer surplus is equal to $1/e$ when $1-c(1) \leq 1/e$ and $-(1-c(1))\ln(1-c(1))$ otherwise.*[11] *It can be achieved with the following distribution:*

$$G_0(v) = \begin{cases} 1 - \frac{p_0}{v} & \text{if } v \in [p_0, 1) \\ 1 & \text{if } v \geq 1, \end{cases}$$

*where $p_0 = 1/e$ if $1-c(1) \leq 1/e$, and $p_0 = 1-c(1)$ otherwise.*

Figure 1 depicts a symmetric distribution of tastes $F$ that induces the distribution of willingness to pay $G_0$ in Theorem 1 (left) and its density conditional on $x \neq 0$ (right); probability mass at $\ell = 0$

[10] For ease of notation, we restrict attention to continuous $G$. But, the argument can apply even if $G$ is not continuous, because any distribution $G$ is monotone and so can be approximated through continuous distributions.

[11] The second case arises because $1-c(1)$ is the lowest willingness to pay by consumers. This limits the extent to which the firm can be incentivized to lower its price, rendering the maximal consumer surplus $1/e$ out of reach.

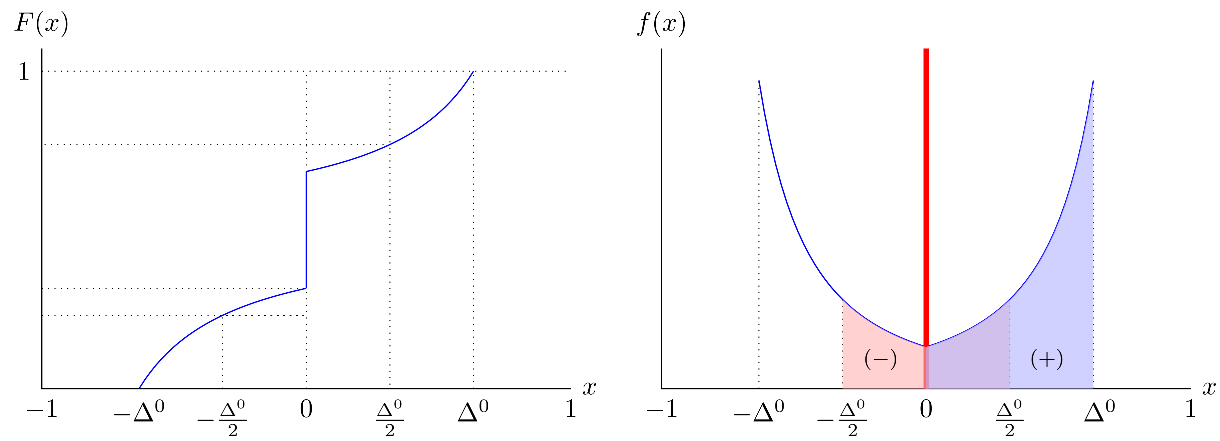


Figure 1: The symmetric distribution $F$ associated with $G_0$ in Theorem 1. The left panel depicts the cumulative distribution function, while the right panel draws the probability density function, excluding the mass point at 0 (represented as the red line on 0). In this figure, $c(|x-\ell|) = |x-\ell|$.

is represented by the solid red pillar. We first examine the symmetric $F$ as it lends itself to simple graphical arguments. An important feature of the distribution $F$ is that its density increases fast as $x$ moves away from $\ell = 0$ (i.e., as $|x|$ increases). The property is required for the seller to stay indifferent over a set of prices conditional on $\ell = 0$. The indifference over the prices and the shape of the density, however, render the distribution vulnerable to deviations in product design. In particular, despite the symmetric structure of $F$, $\ell = 0$ is *not* the seller's optimal product design. Therefore, if the seller can choose the design, consumer surplus is smaller than in Theorem 1.

To see why $\ell = 0$ is not the seller's optimal design under $F$, let $\Delta_0 := c^{-1}(1-p_0)$ and fix $p = 1 - c\left(\frac{\Delta_0}{2}\right)$. Conditional on $\ell = 0$, the seller is indifferent between $p = 1 - c\left(\frac{\Delta_0}{2}\right)$ and $p_0 = 1 - c(\Delta_0)$. Consider the seller's deviation to $(\ell, p) = \left(\frac{\Delta_0}{2}, 1 - c\left(\frac{\Delta_0}{2}\right)\right)$; that is, suppose the seller chooses the same price $p = 1 - c\left(\frac{\Delta_0}{2}\right)$ but moves her position to $\frac{\Delta_0}{2}$.[12] As visualized in the right panel of Figure 1, the deviation makes the seller lose consumers in $\left[-\frac{\Delta_0}{2}, 0\right)$ but gain those in $\left(\frac{\Delta_0}{2}, \Delta_0\right]$. Due to the shape of the distribution, the latter gain is larger than the former loss, thus raising the quantity sold.[13] Since the price stays the same, the deviation is strictly profitable for the

[12] We use this deviation to show that $\ell = 0$ is not optimal. However, it can be shown that $(\ell, p) = \left(\frac{\Delta_0}{2}, 1 - c\left(\frac{\Delta_0}{2}\right)\right)$ is, in fact, the seller's optimal strategy given the specific $F$.

[13] For this argument, it suffices that the density function $g_0$ of $G_0$ strictly increases in $y$. The property holds whenever $c$ is strictly increasing and weakly convex, because

$$g_0(y) = (1 - c(\Delta_0)) \frac{c'(y)}{(1-c(y))^2} \text{ for any } y \in (0, \Delta_0).$$

seller.

While the above argument only shows that $l = 0$ is not optimal under the symmetric distribution of tastes $F$ that generates $G_0$, the result holds more generally.

**Proposition 2** *The product design $\ell = 0$ is not optimal for any distribution of tastes $F$ that induces the willingness to pay $G_0$.*

**Proof.** See the appendix. ■

# 4 Pricing

We return to the main model where the seller chooses both the design of the product and the price. Before characterizing consumer surplus we explore the seller's pricing.

For any price $p < 1$, the seller can design a product that appeals to a strictly positive mass of consumers. Price $0$ is therefore never optimal. We will show that similar logic extends to all prices below some strictly positive threshold. The following terminology will be of use.

**Definition 1** *A price $p$ is* implementable *if there exists a distribution $F$ of tastes and a design $\ell$ such that $(\ell, p)$ is optimal for the seller given the distribution $F$.*

Suppose that given the distribution $F$, it is optimal for the seller to choose a design $\ell$ and price $p$; let $\Delta$ be the corresponding reach. A *necessary* condition for $p$ to be an optimal price is that the following deviation is not profitable. The seller could choose to serve only $[\ell-\Delta, \ell]$ or $[\ell, \ell+\Delta]$—a strategy that entails a higher price while allowing the seller to capture at least a half of consumers on $[\ell - \Delta, \ell + \Delta]$ by choosing the more populous side. For the deviation not to be profitable, it must be that

$$(1 - c(\Delta)) \left(F(\ell + \Delta) - F_{-}(\ell - \Delta)\right) \geq \left(1 - c\left(\frac{\Delta}{2}\right)\right) \frac{F(\ell + \Delta) - F_{-}(\ell - \Delta)}{2},$$

which simplifies to $1 - c(\Delta) \geq 1/2(1 - c(\Delta/2))$. In words, the price cannot more than double. The following result ensues.

**Proposition 3** *Let $\overline{\Delta}$ be the maximal value of $\Delta \in [0, 1]$ such that $1 - c(\Delta) \geq 1/2(1 - c(\Delta/2))$. No price $p < 1 - c(\overline{\Delta})$ is implementable.*

**Proof.** In the appendix, we show that $\overline{\Delta}$ is well defined, and $1 - c(\Delta) \geq 1/2(1 - c(\Delta/2))$ holds if and only if $\Delta \leq \overline{\Delta}$. ■

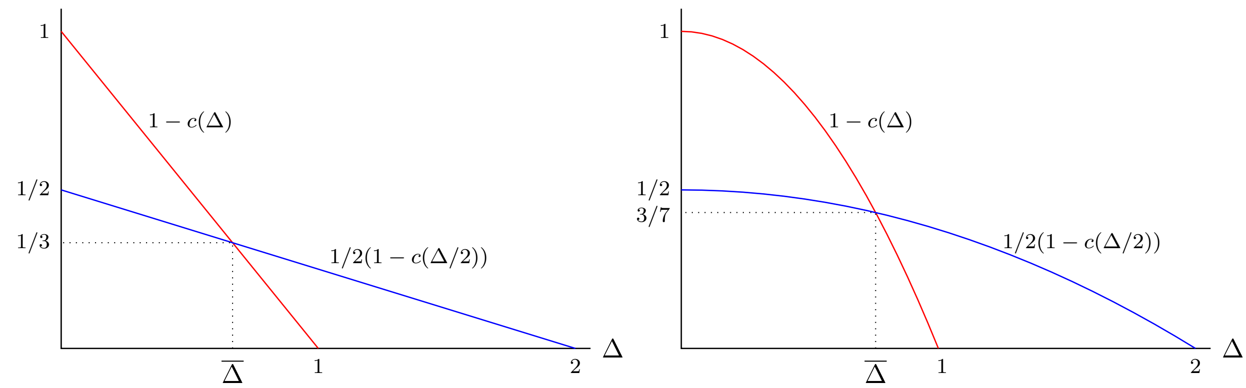


Figure 2: This figure illustrates Proposition 3. The left panel depicts the case where $c(y) = y$ (so $1 - c(y) = 1 - y$), while the right panel is for the case where $c(y) = y^2$.

The result holds because $1 - c(\Delta)$ decreases faster than $1/2(1 - c(\Delta/2))$ at any $\Delta$, so they can cross at most once; see Figure 2. If crossing occurs before $\Delta = 1$ then $\overline{\Delta}$ corresponds to the crossing point. Otherwise (i.e., $1 - c(\Delta)$ is uniformly above $1/2(1 - c(\Delta)/2)$ over $[0, 1]$), $\overline{\Delta} = 1$.

In Section 5, we establish the converse to Proposition 3: any $p \in [1-c(\overline{\Delta}), 1]$ is implementable. In consequence, price $p$ is implementable if and only if $p \in [\underline{p}, 1]$ where $\underline{p} := 1 - c(\overline{\Delta})$. We refer to $\underline{p}$ as the *minimal implementable price*. The following result establishes distribution-free bounds on $\underline{p}$.

**Corollary 1** *The following holds:*

$$1/3 \leq \underline{p} \leq \max\{1/2, 1 - c(1)\}.$$

**Proof.** By its definition, $\overline{\Delta}$ satisfies

$$1 - c(\overline{\Delta}) \geq \frac{1}{2}\left(1 - c\left(\frac{\overline{\Delta}}{2}\right)\right).$$

Since $c$ is convex and $c(0) = 0$, $c\left(\overline{\Delta}/2\right) \leq c\left(\overline{\Delta}\right)/2$. Therefore, the above inequality implies that

$$1 - c(\overline{\Delta}) \geq \frac{1}{2}\left(1 - \frac{c(\overline{\Delta})}{2}\right)$$

and thus

$$1 - c(\overline{\Delta}) \geq \frac{1}{3}.$$

The second result holds because either $\overline{\Delta} = 1$ or

$$1 - c(\overline{\Delta}) = \frac{1}{2}(1 - c(\overline{\Delta}/2)) \leq \frac{1}{2}.$$

■

The lower bound in the above result is particularly striking: a price below $1/3$ can never be optimal for the seller. Note that the result holds regardless of the distribution $F$ and the disutility function $c$. The result lends itself to a geometric interpretation. Suppose $c$ is linear and consider a price $p < 1/3$ combined with some design $\ell$. The seller sells to consumers on $[\ell - \Delta, \ell + \Delta]$ where $\Delta = c^{-1}(1 - p)$. The seller could instead sell only to consumers on $[\ell - \Delta, \ell]$ or $[\ell, \ell + \Delta]$ and guarantee to reach at least a half of the demand. The corresponding price is $(p + 1)/2$ due to linear disutility. If $p < 1/3$, the price more than doubles, while the demand at most halves. Examining power disutility functions further elucidates the result.

*Power Disutility.* Suppose $c(y) = ty^{\alpha}$ for some $t > 0$ and $\alpha \geq 1$. Then

$$\overline{\Delta}_{\alpha} = \min\left\{ \left( \frac{1}{t\,(2 - 1/2^{\alpha})} \right)^{1/\alpha}, 1 \right\}.$$

The resulting minimal implementable price is

$$\underline{p}_{\alpha} = \max\left\{ \frac{2^{\alpha} - 1}{2^{\alpha+1} - 1}, 1 - t \right\}.$$

Observe that this minimal price is equal to $1/3$ when $\alpha = 1$; the lower bound of $\underline{p}$ in Corollary 1 is achievable when $c$ is linear. It increases and converges to $\max\{1/2, 1 - t\}$ as $\alpha$ tends to $\infty$, thus establishing that the upper bound in Corollary 1 is also binding.

The above power utility example suggests that the minimal implementable price depends on convexity of the disutility function $c$. The following result shows that the result indeed holds generally. We adopt the standard definition that a function $\tilde{c}$ is more convex than another function $c$ if there exists an increasing and convex function $\psi$ such that $\tilde{c} = \psi(c)$.

**Proposition 4** *Let $\underline{p}$ be the minimal implementable price under $c$ and $\underline{\tilde{p}}$ the minimal implementable price under $\tilde{c}$. If $\underline{p} > 1 - c(1)$, $\underline{\tilde{p}} > 1 - \tilde{c}(1)$,*[14] *and $\tilde{c}$ is more convex than $c$, then $\underline{\tilde{p}} \geq \underline{p}$.*

**Proof.** See the appendix. ■

*Relation to the standard monopoly model.* In the monopoly model without product design, the monopolist distorts the outcome by offering a price above the marginal cost. Depending on the demand (and the cost function), the price can be arbitrarily close to or even $0$. Corollary 1 establishes a stark departure from this in the model with design. The seller designs a product in such a way that the demand for the given design makes it optimal for the seller to charge a price of at least $1/3$. The combination of pricing and design, therefore, guarantees that the distortions from pricing are substantial, as the marginal cost of production in our model is nil.

# 5 Consumer Surplus

In this section, we characterize the range of consumer surplus that the model with product design can generate. The minimal consumer surplus, zero, is attained when the distribution of tastes is a Dirac distribution. The seller designs the product all the consumers prefer and extracts full surplus by charging price $1$. Most of the subsequent analysis revolves around characterizing the maximal attainable consumer surplus. Any level between the minimal and the maximal can be realized too.

To derive the maximal consumer surplus, we introduce a class of distributions of tastes, characterize their properties, and show that for every implementable price, a distribution in this class maximizes consumer surplus. Then we characterize the maximal consumer surplus and the Pareto payoff frontier.

## 5.1 A Class of Design-Robust Distributions

In Section 3 we showed that facing a distribution of tastes that produces the maximal consumer surplus given $\ell = 0$ (which makes the seller indifferent over an interval of prices) the seller does not find the design $\ell = 0$ optimal. With that in mind, we define a class of distributions that make the seller indifferent over a set of design-price pairs.

**Definition 2** *For each $\Delta \in [0, \overline{\Delta}]$, let $F_\Delta$ denote a distribution such that*

$$F_\Delta(x) = 1 - F_\Delta(-x) = \frac{1 - c(\Delta)}{1 - c\left(\frac{x+\Delta}{2}\right)} \text{ for } x \in [0, \Delta].$$

[14] If the minimal implementable price is $1 - c(1)$ (i.e., $\overline{\Delta} = 1$) then it simply depends on $c(1)$.

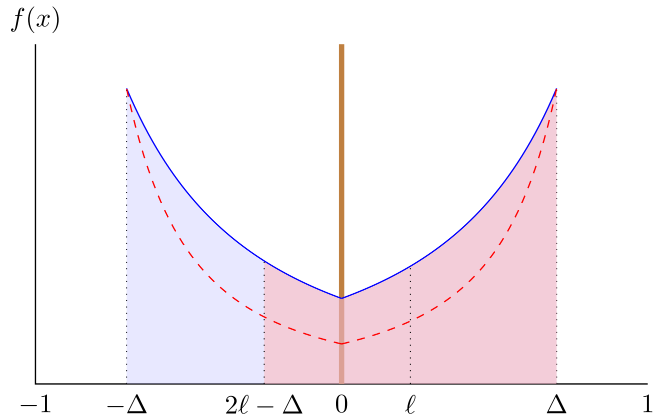


Figure 3: This figure depicts the density function of $F_\Delta$ in Definition 2 (blue solid) and that of the optimal distribution in Theorem 1 (red dashed), each conditional on $x \neq 0$. The brown solid pillar represents probability mass at 0. In this figure, $c(|x-\ell|) = |x-\ell|$ and $\Delta = \frac{e-1}{e} \approx 0.6321$.

Note that distribution $F_\Delta$ assigns probability mass of size

$$\lim_{x\to 0} F_\Delta(x) - F_\Delta(-x) = 2\left(\frac{1-c(\Delta)}{1-c(\Delta/2)} - 1\right) = \frac{1-2c(\Delta)+c(\Delta/2)}{1-c(\Delta/2)}$$

to 0. This probability is non-negative and thus well defined if and only if $\Delta \in [0, \overline{\Delta}]$ (see Proposition 3).

Figure 3 shows how $F_\Delta$ (blue solid) compares to the symmetric consumer-optimal distribution under a fixed design (at $\ell = 0$) from Theorem 1, denoted $F^0$ (red dashed). Just like $F^0$, $F_\Delta$ is symmetric around 0 with probability mass (only) at 0, and its density is increasing as $x$ moves further away from 0. However, $F_\Delta$ is less concentrated around 0: for each $x \in (0, \Delta)$, $F_\Delta(x) - F_\Delta(-x) < F^0(x) - F^0(-x)$. This is necessary for the following property, which is a counterpart to unit elasticity of $F^0$.

**Lemma 1** *Under $F_\Delta$, for any $\ell \in [0, \Delta/2]$, $\pi(0, 1-c(\Delta)) = \pi(\ell, 1-c(\Delta-\ell))$.*

**Proof.** If $\ell \in [0, \Delta/2]$ and $p = 1 - c(\Delta - \ell)$ then, as visualized in Figure 3, the seller covers

$[2\ell - \Delta, \Delta]$. Therefore,

$$\begin{aligned}\pi(\ell, 1-c(\Delta-\ell)) &= (1-c(\Delta-\ell))(1-F_\Delta(\Delta-2|\Delta-\ell|)) \\ &= (1-c(\Delta-\ell))F_\Delta(\Delta-2\ell) \\ &= (1-c(\Delta-\ell))\frac{1-c(\Delta)}{1-c\left(\frac{\Delta-2\ell+\Delta}{2}\right)} \\ &= 1-c(\Delta) \\ &= \pi(0, 1-c(\Delta)),\end{aligned}$$

where the second equality is because $F$ is symmetric around $0$ (i.e., $F(-x) = 1 - F(x)$) and the third equality due to the definition of $F_\Delta$. ■

Under $F_\Delta$, the seller is indifferent over a set of design-price combinations: $(\ell, 1-c(\Delta-\ell))$ for any $\ell \in [0, \Delta/2]$.[15] Equivalently, the seller is indifferent between covering $[-\Delta, \Delta]$ and covering $[2\ell - \Delta, \Delta]$ for every $\ell \in [0, \Delta/2]$. This is in stark contrast with—and the main difference from—$F^0$ under which the seller is indifferent over covering $[-\Delta, \Delta]$ for different values of $\Delta$.

The following result shows that given $F_\Delta$ no other deviation is profitable, and thus $(\ell, p) = (0, 1-c(\Delta))$ is the firm's optimal strategy.

**Proposition 5** *Under $F_\Delta$, $\pi(0, 1-c(\Delta)) \geq \pi(\ell, p)$ for any $(\ell, p)$.*

**Proof.** See the appendix. ■

For the intuition behind Proposition 5, fix $p = 1 - c(\widehat{\Delta})$ for some $\widehat{\Delta} \in \left[\frac{\Delta}{2}, \Delta\right]$,[16] and consider the deviation to $(\ell, p)$ for some $\ell$. By Lemma 1, the seller is indifferent between $(0, 1-c(\Delta))$ and $(\widehat{\ell}, p)$ where $\widehat{\ell} = \Delta - \widehat{\Delta}$. Therefore, it suffices to show that $\widehat{\ell}$ is the seller's optimal design given that she chooses $p = 1 - c(\widehat{\Delta})$, which is equivalent to $D(\widehat{\ell}, p; F_\Delta) \geq D(\ell, p; F_\Delta)$ for any $\ell$. This property follows from the shape of $F_\Delta$: as shown in Figure 3, $f_\Delta$ is increasing in $|x|$. Therefore, choosing $\ell < \widehat{\ell}$ can never increase the seller's demand—the area below $f_\Delta$ over $[\ell - \Delta, \ell + \Delta]$ plus probability mass at $0$.

The fact that Proposition 5 holds for any $\Delta \in [0, \overline{\Delta}]$ implies the following result.

**Corollary 2** *For any $\Delta \in [0, \overline{\Delta}]$, there exists a distribution under which the seller chooses $p = 1 - c(\Delta)$. Consequently, the distribution $F_{\overline{\Delta}}$ is a price-minimizing distribution among all possible distributions over $[-1, 1]$.*

[15] Because of symmetry the seller is also indifferent between $(0, 1-c(\Delta))$ and $(-\ell, 1-c(\Delta-\ell))$ for any $\ell \in [0, \Delta/2]$.

[16] In the appendix, we show that choosing $\widehat{\Delta} < \Delta/2$ is strictly unprofitable for the seller, regardless of her design.

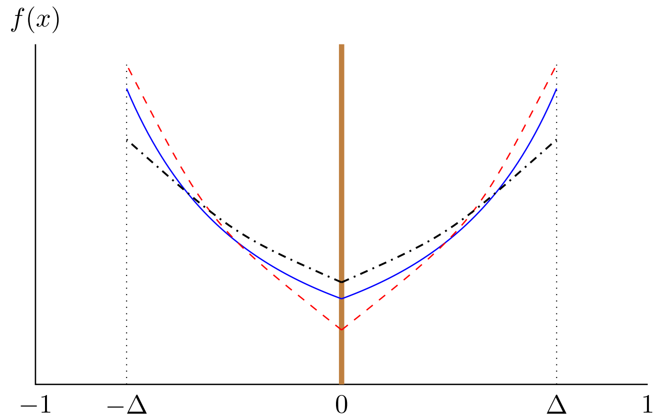


Figure 4: This figure depicts the density function of $F_\Delta$ in Definition 2 (blue solid), a density function that belongs to $\mathcal{F}(p)$ (red dashed), and one that does not belong to $\mathcal{F}(p)$ despite having the same support as $F_\Delta$ (dash-dotted), each conditional on $x \neq 0$. The brown solid pillar represents probability mass at 0. In this figure, $c(|x-\ell|) = |x-\ell|$ and $\Delta = \frac{e-1}{e} \approx 0.6321$.

## 5.2 Maximal Consumer Surplus

Let $\mathcal{F}(p)$ denote the set of all distributions under which it is optimal for the seller to choose price $p$ together with a certain location. The results so far imply that $\mathcal{F}(p)$ is non-empty if and only if $p = 1 - c(\Delta)$ for some $\Delta \in [0, \overline{\Delta}]$, and $F_\Delta \in \mathcal{F}(1 - c(\Delta))$. The following proposition argues that $F_\Delta$ is a consumer-optimal distribution among all distributions of tastes in $\mathcal{F}(1 - c(\Delta))$.

**Proposition 6** *For any $\Delta \in \left[0, \overline{\Delta}\right]$ and $p = 1 - c(\Delta)$, $F_\Delta$ maximizes consumer surplus in $\mathcal{F}(p)$.*

**Proof.** See the appendix. ■

To understand this result, fix $\Delta \in \left[0, \overline{\Delta}\right]$ and $p = 1 - c(\Delta)$, and consider any $F \in \mathcal{F}(p)$. Consumer surplus is larger, the more concentrated consumers are around the seller. If, however, too many consumers are in the seller's immediate vicinity, she would charge a higher price. For example, in Figure 4, given that the seller chooses $(\ell, p) = (0, 1 - c(\Delta))$, the black dash-dotted distribution yields more consumer surplus than the other two. However, due to a relatively high concentration of consumers around 0, the seller would not choose $(0, 1 - c(\Delta))$; it is more profitable for her to move to $\ell \in [0, \Delta/2)$ and charge a higher price.

The distribution $F_\Delta$ strikes a balance between the two effects: we show that for $(0, p)$ to be optimal, $F$ must assign at least as much mass as $F_\Delta$ above each $\ell \in \left[0, \frac{\Delta}{2}\right]$; see the red dashed curve in Figure 4 and compare it to $f_\Delta$ (blue solid).[17] This means that in the positive region $F$

[17]Recall that under $F_\Delta$, the seller is indifferent between $(0, p)$ and $(-(\Delta - \ell)/2, 1 - c((\Delta + \ell)/2))$ for any $\ell \in$

dominates $F_\Delta$ in the sense of first-order stochastic dominance. Applying the same logic in the negative region, $F$ is dominated by $F_\Delta$. This stochastic dominance comparison, together with the fact that both $F$ and $F_\Delta$ induce the same price $p$, implies that no $F \in \mathcal{F}(p)$ can produce larger consumer surplus than $F_\Delta$.

Proposition 6 suggests that for the purpose of maximizing consumer surplus we can restrict attention to the class of design-robust distributions in Definition 2. This reduces the infinite dimensional problem of finding a distribution that maximizes consumer surplus to a single-dimensional problem, yielding the following characterization.

**Theorem 2** *Let $\Delta^*$ be the value of $\Delta \in (0, \overline{\Delta}]$ that solves*

$$\max_{\Delta \in [0,\overline{\Delta}]} CS(F_\Delta) = -c(\Delta) + 2\int_0^\Delta \frac{1-c(\Delta)}{1-c\left(\frac{x+\Delta}{2}\right)} dc(x). \tag{4}$$

*Then, $F_{\Delta^*}$ maximizes consumer surplus in $\mathcal{F}$.*

To decipher this result, we consider the two canonical disutility functions, one in which $c$ is linear and the other in which $c$ is quadratic, and provide a more detailed characterization of consumer-optimal distributions and maximal consumer surplus in these two cases.

*Linear Disutility.* Suppose $c(y) = ty$ for all $y \geq 0$ and some $t > 0$. Then, $c'(x) = c'\left(\frac{x+\Delta}{2}\right) = t$, so $CS(F_\Delta)$ can be explicitly solved:

$$CS(F_\Delta) = -t\Delta + 4(1-t\Delta)\ln\left(\frac{1-t\frac{\Delta}{2}}{1-t\Delta}\right).$$

Observe that this expression depends only on $\eta := t\Delta$. The optimal value of $\eta$, denoted $\eta^*$, satisfies the following first order condition:

$$3 - 4\ln\left(\frac{1-\frac{\eta^*}{2}}{1-\eta^*}\right) - \frac{2(1-\eta^*)}{1-\frac{\eta^*}{2}} = 0.$$

$[0, \Delta/2]$ (Lemma 1). If $F \in \mathcal{F}(p)$ then the seller should weakly prefer $(0, p)$ to $(-(\Delta-\ell)/2, 1-c((\Delta+\ell)/2))$, which holds only when $F$ assigns larger probability above $\ell$ than $F_\Delta$.

The solution is $\eta^* \approx 0.5123$, and the resulting maximal consumer surplus is

$$\begin{aligned}\overline{CS} &:= -\eta^* + 4(1-\eta^*)\ln\left(\frac{1-\frac{\eta^*}{2}}{1-\eta^*}\right), \\ &\approx 0.3113.\end{aligned}$$

Since $\Delta \leq 1$, the optimal reach is given by $\Delta^* = \min\{\eta^*/t, 1\}$. Consequently, $CS(F_{\Delta^*}) = \overline{CS}$ if $\Delta^* < 1$, while $CS(F_{\Delta^*}) < \overline{CS}$ if $\Delta^* = 1$.

The following two facts about this result are of particular interest. First, for $t > \eta^*$ the price induced by the seller-optimal distribution is strictly larger than $1-c(\overline{\Delta})$—the lowest possible price. This means that consumer-optimal and price-minimizing distributions are distinct from each other, although they belong to the same design-robust class in Definition 2. Second, for $t \geq \eta^*$, the maximal consumer surplus does not depend on the cost parameter $t$; it is equal to $\overline{CS}$ regardless of $t(\geq \eta^*)$. If $t$ rises then the distribution $F_{\Delta^*}$ becomes proportionally contracted, so the resulting profit and consumer surplus stay constant.

*Quadratic Disutility.* Now suppose $c(y) = ty^2$ for all $y \geq 0$ and some $t > 0$. Unlike in the previous linear case, the integral in (4) cannot be solved in closed form. Numerically, it can be shown that $\Delta^* = \min\{\sqrt{0.4919/t}, 1\} (\leq \overline{\Delta} = \min\{\sqrt{4/(7t)}, 1\})$, $CS(F_{\Delta^*}) \approx 0.2908$ if $\Delta^* < 1$, and $CS(F_{\Delta^*}) < 0.2908$ if $\Delta^* = 1$.

*Consumer Surplus and Disutility.* Note that the maximal consumer surplus is strictly higher with linear disutility than with quadratic disutility. In fact, a stronger result holds: $\overline{CS} \approx 0.3113$ is a tight upper bound for consumer surplus, as formally stated in the following result.

**Proposition 7** *For any increasing and convex function $c$, $CS(F_\Delta) \leq \overline{CS} \approx 0.3113$.*

**Proof.** Since $c$ is convex, we have $c\left(\frac{x+\Delta}{2}\right) \leq \frac{c(x)+c(\Delta)}{2}$ for any $x \in [0, \Delta]$. This implies that

$$\begin{aligned} CS(F_\Delta) &= -c(\Delta) + 2(1-c(\Delta))\int_0^\Delta \frac{1}{1-c\left(\frac{x+\Delta}{2}\right)}dc(x) \\ &\leq -c(\Delta) + 2(1-c(\Delta))\int_0^\Delta \frac{1}{1-\frac{c(x)+c(\Delta)}{2}}dc(x) \\ &= -c(\Delta) + 4(1-c(\Delta))\ln\left(\frac{1-\frac{c(\Delta)}{2}}{1-c(\Delta)}\right), \end{aligned}$$

where the last equality is through direct calculus. As shown in the case of linear disutility, the last expression is maximized when $c(\Delta) = \eta^*$, and the maximized value is equal to $\overline{CS}$. ∎

Recall (from Theorem 1) that the maximal consumer surplus in Condorelli and Szentes (2020) is $1/e \approx 0.3679$, which is strictly larger than $\overline{CS} \approx 0.3113$. This means that consumer surplus in our model can never be as large as the maximal level in Condorelli and Szentes (2020),[18] demonstrating that the seller's ability to design her product, which is clearly beneficial to her, can be harmful to consumers.

### 5.3 Consumer Surplus and the Pareto Frontier

The above results can be used to characterize the full range of consumer surplus that can arise in our model. We characterized the maximal consumer surplus and a distribution that attains it. At the other extreme, the smallest consumer surplus is attained by Dirac distributions. One such Dirac distribution is $F_\Delta$ with $\Delta = 0$. Since the highest consumer surplus is attained by $F_{\Delta^*}$ for some $\Delta^* > 0$ and $CS(F_\Delta)$ is continuous in $\Delta$, the intermediate value theorem implies that any consumer surplus in the range $[0, CS(F_{\Delta^*})]$ can be achieved by some design-robust distribution.

**Proposition 8** *The set of attainable consumer surplus is* $[0, CS(F_{\Delta^*})]$.

*Pareto Frontier.* Recall that for each $\Delta \in [0, \overline{\Delta}]$, $F_\Delta$ maximizes consumer surplus among all distributions in $\mathcal{F}(p)$ where $p = 1 - c(\Delta)$ (Proposition 6). In addition, under $F_\Delta$, the seller serves all consumers, so her profit is $p = 1 - c(\Delta)$. Conditional on charging $p$, the seller's profit clearly cannot exceed $p$. This implies that $F_\Delta$ Pareto dominates all distributions in $\mathcal{F}(p)$. This observation allows us to obtain the following characterization of the Pareto payoff frontier—the upper envelope of payoff vectors attainable in our model.[19]

**Proposition 9** *The Pareto payoff frontier across all distributions is given by*

$$\left\{(p, \overline{CS}(p)) : p \in [1 - c(\Delta^*), 1]\right\},$$

*where* $\overline{CS}(p)$ *is defined as*

$$\overline{CS}(p) := \max\{CS(F_\Delta) : 1 - c(\Delta) \geq p\}.$$

[18] This comparison itself is a corollary of Proposition 2: Facing the distribution of tastes that, conditional on $\ell = 0$, produces the consumer-optimal distribution in Condorelli and Szentes (2020), the seller has no incentive to choose $\ell = 0$. Therefore, consumer surplus can never be $1/e$. Proposition 7, however, provides a tight upper bound.

[19] Our characterization of the frontier is similar in spirit to Bergemann et al. (2015) and Roesler and Szentes (2017), though the set of distributions here is not restricted by Bayes plausibility.

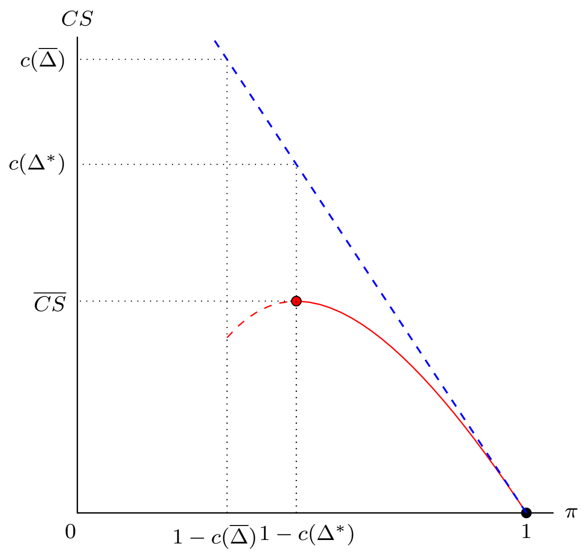


Figure 5: The maximal consumer surplus (red dot) and the Pareto frontier (red solid) when $c(y) = 0.8y$. The blue dashed line represents the case where full surplus of 1 is realized (i.e., $CS+\pi = 1$).

**Proof.** See the appendix. ■

Figure 5 illustrates Proposition 9. For the linear disutility function, consumer surplus from design-robust distributions, $CS(F_\Delta)$, is quasi-concave in $\Delta$, increasing until $\Delta^*$ and then decreasing. In this case, the Pareto payoff frontier is spanned by $\{F_\Delta : \Delta \in [0, \Delta^*]\}$. If $CS(F_\Delta)$ is not quasi-concave, then the Pareto frontier is spanned by a subset of $\{F_\Delta : \Delta \in [0, \Delta^*]\}$. In that case, for each $p \in [1 - c(\Delta^*), 1]$ it suffices to identify the highest attainable consumer surplus with a weakly higher price; note that $\overline{CS}(p)$ is necessarily quasi-concave.

The total available surplus is $1$, which is achieved when all consumers have the same taste and the seller chooses their favorite design. In that situation, however, the seller would extract full surplus by charging $p = 1$, minimizing consumer surplus. This implies that positive consumer surplus necessarily involves market inefficiency, that is, social surplus is strictly less than $1$ whenever consumer surplus is positive. In Figure 5, this is reflected in the fact that the Pareto payoff frontier lies strictly lower than the efficiency frontier (blue dashed).

# 6 Conclusion

We study how optimal pricing and consumer surplus depend on the monopolistic seller's ability to design the product. Product design gives the seller a strategic edge, rendering low prices never optimal. The seller never chooses a design/price combination with a price below $1/3$ regardless of the distribution of tastes. This is a striking departure from the standard monopoly model (without product design) where the seller can be induced to offer prices arbitrarily close to zero.

The seller's ability to design the product has significant implications for consumer surplus. Consumer surplus is maximized by a distribution of tastes that makes the seller indifferent over a certain set of design/price combinations. Unlike in the model without product design, the distribution is not unit elastic given a design. In fact, any distribution of tastes that induces unit-elastic demand for some fixed design makes that design suboptimal for the seller. Due to this difference, the maximal attainable consumer surplus is strictly lower in our model than in the model without product design.

While the Hotelling model has been used to study a wide variety of questions in industrial organization, little attention has been dedicated to how the distribution of tastes affects product design and, through that, market participants. Our model can be extended in various ways. For example, while degenerate distributions minimize consumer surplus in monopoly with product design, they induce Bertrand competition and thus produce maximal consumer surplus in the oligopoly setting. Less clear in that environment is what kind of distributions lead to low consumer surplus. A further avenue of research would be the value of information in the presence of product design, both for the seller(s) as well as consumers.

# References

**Ali, S Nageeb, Greg Lewis, and Shoshana Vasserman**, "Voluntary Disclosure and Personalized Pricing," *The Review of Economic Studies*, 2023, *90* (2), 538–571.

**Anderson, Simon P, Jacob K Goeree, and Roald Ramer**, "Location, location, location," *journal of economic theory*, 1997, *77* (1), 102–127.

**Bar-Isaac, Heski, Guillermo Caruana, and Vicente Cuñat**, "Targeted product design," *American Economic Journal: Microeconomics*, 2023, *15* (2), 157–186.

**Bergemann, Dirk, Benjamin Brooks, and Stephen Morris**, "The limits of price discrimination," *American Economic Review*, 2015, *105* (3), 921–957.

**Condorelli, Daniele and Balázs Szentes**, "Information design in the holdup problem," *Journal of Political Economy*, 2020, *128* (2), 681–709.

**_ and _** , "Surplus sharing in Cournot oligopoly," *Theoretical Economics*, 2022, *17* (3), 955–975.

**Dupuit, Jules**, “De la mesure de l’utilité des travaux publics,” 1844.

**Hidir, Sinem and Nikhil Vellodi**, “Privacy, personalization, and price discrimination,” *Journal of the European Economic Association*, 2021, *19* (2), 1342–1363.

**Hotelling, Harold**, “Stability in competition,” *Economic Journal*, 1929, *39* (153), 41–57.

**Johnson, Justin P and David P Myatt**, “On the simple economics of advertising, marketing, and product design,” *American Economic Review*, 2006, *96* (3), 756–784.

**Kim, Kyungmin and Nenad Kos**, “Robust Product Design and Pricing,” *mimeo*, 2023.

**Lancaster, Kelvin J**, “A new approach to consumer theory,” *Journal of political economy*, 1966, *74* (2), 132–157.

**Marshall, Alfred**, *Principles of Economics* 1890.

**Mas-Colell, Andreu, Michael Dennis Whinston, Jerry R Green et al.**, *Microeconomic theory*, Vol. 1, Oxford university press New York, 1995.

**Rhodes, Andrew and Jidong Zhou**, “Personalized pricing and competition,” *Available at SSRN 4103763*, 2022.

**Roesler, Anne-Katrin and Balázs Szentes**, “Buyer-optimal learning and monopoly pricing,” *American Economic Review*, 2017, *107* (7), 2072–80.

**Terstiege, Stefan and Adrien Vigier**, “Market Segmentation and Product Steering,” *mimeo*, 2023.

**Willig, Robert D**, “Consumer’s surplus without apology,” *The American Economic Review*, 1976, *66* (4), 589–597.

# Appendix: Omitted Proofs

**Proof of Proposition 2.** Consider any (asymmetric) distribution $F$ that induces $G_0$. Given $\ell = 0$, the seller is indifferent between charging $p_0 = 1 - c(\Delta_0)$ and $1 - c(\Delta_0/2)$. Therefore, we have

$$\begin{aligned}\pi(0, p_0) &= (1 - c(\Delta_0))(F(\Delta_0) - F_-(-\Delta_0)) \\ &= 1 - c(\Delta_0) \\ &= \pi(0, 1 - c(\Delta_0/2)) \\ &= (1 - c(\Delta_0/2))\left(F\left(\Delta_0/2\right) - F_-(-\Delta_0/2)\right).\end{aligned}$$

Similarly, the seller is indifferent between charging $p_0$ and $1$, so we also have

$$\pi(0, p_0) = 1 - c(\Delta_0) = \pi\left(0, 1\right) = F(0) - F_-(0).$$

***Claim 1*** *The following holds:*

$$\max\left\{F(\Delta_0) - F_-(0), F(0) - F_-(-\Delta_0)\right\} \geq 1 - \frac{c(\Delta_0)}{2}.$$

**Proof.** If the inequality fails then the following contradiction emerges:

$$\begin{aligned}1 &= F(\Delta_0) - F(-\Delta_0) \\ &= (F(\Delta_0) - F_-(0)) + (F(0) - F_-(-\Delta_0)) - (F(0) - F_-(0)) \\ &< 2 - c(\Delta_0) - (1 - c(\Delta_0)) = 1.\end{aligned}$$

■

***Claim 2*** *If $c$ is weakly convex then for any $y > 0$ we have*

$$(1 - c(y/2))\left(1 - \frac{c(y)}{2}\right) > 1 - c(y).$$

**Proof.** For any convex $c$, we have $c(y/2) \leq c(y)/2$. Therefore,

$$\begin{aligned}(1 - c(y/2))\left(1 - \frac{c(y)}{2}\right) &\geq \left(1 - \frac{c(y)}{2}\right)\left(1 - \frac{c(y)}{2}\right) \\ &= 1 - c(y) + \frac{c(y)^2}{4} \\ &> 1 - c(y).\end{aligned}$$

■

We proceed to show that it is a profitable deviation for the seller to move her location to either $-\Delta_0/2$ or $\Delta_2/2$ and charge $1 - c(\Delta_0/2)$. In Claim 1, without loss of generality, suppose $F(\Delta_0) - F_{-}(0) \geq 1 - c(\Delta_0/2)$. Then, we have

$$\begin{aligned}\pi(\Delta_0/2, 1 - c(\Delta_0/2)) &= (1 - c(\Delta_0/2))(F(\Delta_0) - F_{-}(0)) \\ &\geq (1 - c(\Delta_0/2))\left(1 - \frac{c(\Delta_0)}{2}\right) \\ &> 1 - c(\Delta_0) = \pi(0, 1 - c(\Delta_0)),\end{aligned}$$

where the weak inequality is due to Claim 1, while the strict inequality is due to Claim 2.

■

**Proof of Proposition 3.** Define $f(\Delta) := 2c(\Delta) - c(\Delta/2)$. Then, $\overline{\Delta}$ can be interpreted as the maximal value of $\Delta \in [0, 1]$ such that $f(\Delta) \leq 1$. We show that $f$ is continuous and strictly increasing; this implies not only that $\overline{\Delta}$ is well defined, but also that $1 - c(\Delta) \geq 1/2(1 - c(\Delta/2))$ holds if and only if $\Delta \leq \overline{\Delta}$.

Continuity of $f$ follows from that of $c$. For monotonicity, consider any $0 \leq \Delta < \Delta'$. Then,

$$\begin{aligned}f(\Delta') - f(\Delta) &= 2(c(\Delta') - c(\Delta)) - (c(\Delta'/2) - c(\Delta/2)) \\ &= c(\Delta') - c(\Delta) + [(c(\Delta') - c(\Delta)) - (c(\Delta'/2) - c(\Delta/2))] \\ &> c(\Delta') - c(\Delta) > 0,\end{aligned}$$

where the first inequality holds because $c$ is convex. ■

**Proof of Proposition 4.** Let $\phi(y, \varepsilon, c)$ denote the value ("probability premium") such that

$$c(y) = \left(\frac{1}{2} + \phi(y, \varepsilon, c)\right) c(y - \varepsilon) + \left(\frac{1}{2} - \phi(y, \varepsilon, c)\right) c(y + \varepsilon).$$

Since $c$ is strictly increasing and convex, $\phi(y, \varepsilon, c)$ is well defined and non-negative for any $\varepsilon \in (0, y]$. If $\tilde{c}$ is more convex than $c$, then $\phi(y, \varepsilon, \tilde{c}) \geq \phi(y, \varepsilon, c)$; see Proposition 6.C.2 in Mas-Colell et al. (1995).

Consider the case where $y = \varepsilon = \overline{\Delta}/2$, and let $\widehat{\phi}(c) := \phi(\overline{\Delta}/2, \overline{\Delta}/2, c)$. Then, we have

$$1 - c\left(\overline{\Delta}/2\right) = 1 - \left(\left(\frac{1}{2} + \widehat{\phi}(c)\right) c(0) + \left(\frac{1}{2} - \widehat{\phi}(c)\right) c(\overline{\Delta})\right)$$
$$= 1 - \left(\frac{1}{2} - \widehat{\phi}(c)\right) c(\overline{\Delta})$$

where the second equality holds because $c(0) = 0$. The assumption that $\overline{\Delta} < 1$ (equivalently, $\underline{p} > 1 - c(1)$) implies that $1 - c(\overline{\Delta}) = 1/2(1 - c(\overline{\Delta}/2))$. Combining this with the above equation leads to

$$2\left(1 - c(\overline{\Delta})\right) = 1 - \left(\frac{1}{2} - \widehat{\phi}(c)\right) c(\overline{\Delta}),$$

which can be rewritten as

$$\underline{p} = 1 - c(\overline{\Delta}) = 1 - \frac{1}{3/2 + \widehat{\phi}(c)}.$$

Since $\tilde{c}$ is more convex than $c$, $\widehat{\phi}(\tilde{c}) \geq \widehat{\phi}(c)$ and therefore $\underline{\tilde{p}} \geq \underline{p}$. ∎

**Proof of Proposition 5.** Consider $p = 1 - c(\widehat{\Delta})$ where $\widehat{\Delta} \in \left[\frac{\Delta}{2}, \Delta\right]$. By Lemma 1, $\pi(0, 1 - c(\Delta)) = \pi\left(\widehat{\ell}, p\right)$ where $\widehat{\ell} = \Delta - \widehat{\Delta}$. We show that $\pi(\widehat{\ell}, p) \geq \pi(\ell, p)$ for any $\ell \in [-1, 1]$. Since $supp(F_\Delta) = [-\Delta, \Delta]$, the inequality clearly holds if $\ell > \widehat{\ell}$ or $\ell < -\widehat{\ell}$. The result for $\ell \in [0, \widehat{\ell})$ follows from the fact that

$$D(\ell, p) - D(\widehat{\ell}, p) = \left(F(2\widehat{\ell} - \widehat{\Delta}) - F(2\widehat{\ell} - \widehat{\Delta} - (\widehat{\ell} - \ell))\right) - \left(1 - F(\Delta - (\widehat{\ell} - \ell))\right)$$
$$< 0,$$

where the inequality holds because the density function is symmetric around 0 and strictly increasing in $|x|$. A symmetric argument applies when $\ell \in (-\widehat{\ell}, 0)$.

Now, consider $p = 1 - c(\widehat{\Delta})$ for $\widehat{\Delta} \in \left[0, \frac{\Delta}{2}\right)$. Given the shape of $F_\Delta$, there are two cases to consider, one in which $\ell = \widehat{\Delta}$ and the other in which $\ell = \Delta - \widehat{\Delta}$. In the former case, the inequality

$$\begin{aligned}\pi(\ell, p) &= p(F_\Delta(2\widehat{\Delta}) - F_{\Delta-}(0)) \\ &\leq \pi(0, 1 - c(\Delta)) \\ &= 1 - c(\Delta)\end{aligned}$$

is equivalent to

$$\frac{1}{1 - c\left(\widehat{\Delta} + \frac{\Delta}{2}\right)} + \frac{1}{1 - c\left(\frac{\Delta}{2}\right)} \leq \frac{1}{1 - c(\Delta)} + \frac{1}{1 - c(\widehat{\Delta})},$$

which holds because convexity of $c(\cdot)$ implies that $\frac{1}{1-c(\cdot)}$ is strictly convex, $(\widehat{\Delta} + \frac{\Delta}{2}) + \frac{\Delta}{2} = \Delta + \widehat{\Delta}$, and $\max\{\frac{\Delta}{2}, \widehat{\Delta} + \frac{\Delta}{2}\} \leq \Delta$. In the latter case, inequality

$$\begin{aligned}\pi(\ell, p) &= p\left(F_\Delta(\Delta) - F_\Delta(\Delta - 2\widehat{\Delta})\right) \\ &\leq \pi(0, 1 - c(\Delta)) \\ &= 1 - c(\Delta)\end{aligned}$$

is equivalent to

$$\frac{1}{1 - c(\Delta)} \leq \frac{1}{1 - c(\widehat{\Delta})} + \frac{1}{1 - c(\Delta - \widehat{\Delta})}.$$

Since $\frac{1}{1-c(\cdot)}$ is strictly convex, the right-hand side is minimized when $\widehat{\Delta} = \frac{\Delta}{2}$. Therefore, the inequality holds for any $\widehat{\Delta} \in \left[0, \frac{\Delta}{2}\right)$ if and only if

$$\frac{1}{1 - c(\Delta)} \leq \frac{2}{1 - c\left(\frac{\Delta}{2}\right)},$$

which holds due to $\Delta \leq \overline{\Delta}$ (see Proposition 3). ■

**Proof of Proposition 6.** Let $\widetilde{F}$ denote a consumer-optimal distribution in $\mathcal{F}(p)$ and $\widetilde{\ell}$ denote the seller's optimal location under $\widetilde{F}$ at which $p$ is the seller's optimal price.

**(i)** Under $\widetilde{F}$, the seller should serve all consumers, that is, $\widetilde{F}\left(\widetilde{\ell} + \Delta\right) - \widetilde{F}_-\left(\widetilde{\ell} - \Delta\right) = 1$.

Suppose $\widetilde{F}\left(\widetilde{\ell}+\Delta\right)-\widetilde{F}_-\left(\widetilde{\ell}-\Delta\right)<1$. Then, consider the following alternative distribution:

$$\widehat{F}(x)=\begin{cases}0 & \text{if } x<\widetilde{\ell}-\Delta\\ \frac{\widetilde{F}(x)-\widetilde{F}_-\left(\widetilde{\ell}-\Delta\right)}{\widetilde{F}(\widetilde{\ell}+\Delta)-\widetilde{F}_-\left(\widetilde{\ell}-\Delta\right)} & \text{if } x\in[\widetilde{\ell}-\Delta,\widetilde{\ell}+\Delta]\\ 1 & \text{if } x\geq\widetilde{\ell}+\Delta.\end{cases}$$

In other words, $\widehat{F}$ takes all probability of $\widetilde{F}$ outside of $[\widetilde{\ell}-\Delta,\widetilde{\ell}+\Delta]$ and spreads it over $[\widetilde{\ell}-\Delta,\widetilde{\ell}+\Delta]$ proportionally to $\widetilde{F}$. By construction, $\widehat{F}$ preserves the firm's preferences over its strategies inside $[\widetilde{\ell}-\Delta,\widetilde{\ell}+\Delta]$ and makes them more profitable than the other strategies (overlapping with the interval below $\widetilde{\ell}-\Delta$ or the interval above $\widetilde{\ell}+\Delta$) relative to $\widetilde{F}$. Therefore, $(\widetilde{\ell},p)$ remains the seller's optimal strategy. It is straightforward that $\widehat{F}$ produces larger consumer surplus than $\widetilde{F}$, which is a contradiction.

(ii) Without loss of generality, assume $\widetilde{\ell}=0$. Then, $\widetilde{F}(x)\leq F_\Delta(x)$ for all $x\in(0,\Delta]$ and $\widetilde{F}(x)\geq F_\Delta(x)$ for all $x\in[-\Delta,0)$.

Fix $x\in(0,\Delta]$. Let $\Delta'=\frac{x+\Delta}{2}$ and notice that $\Delta'\in\left(\frac{\Delta}{2},\Delta\right]$. Then, by Lemma 1

$$\begin{aligned}\pi(x-\Delta',1-c(\Delta');F_\Delta)&=(1-c(\Delta'))F_\Delta(x)\\&=\pi(0,1-c(\Delta);F_\Delta)\\&=1-c(\Delta).\end{aligned}$$

If $\widetilde{F}(x)>F_\Delta(x)$ then

$$\begin{aligned}\pi(x-\Delta',1-c(\Delta');\widetilde{F})&=(1-c(\Delta'))\widetilde{F}(x)\\&>(1-c(\Delta'))F_\Delta(x)\\&=\pi(0,1-c(\Delta);F_\Delta)\\&=\pi(0,1-c(\Delta);\widetilde{F}).\end{aligned}$$

This implies that $p=1-c(\Delta)$ is not the seller's optimal price under $\widetilde{F}$, contradicting $\widetilde{F}\in\mathcal{F}(p)$. A symmetric argument applies in the straightforward fashion to the case when $\widetilde{F}(x)<F_\Delta(x)$ for some $x\in(-\Delta,0)$.

(iii) $\widetilde{F}=F_\Delta$ almost surely.

We show that $CS(\tilde{F}) \leq CS(F_\Delta)$. By (i) and the construction of $F_\Delta$, we have

$$CS(\widetilde{F}) = 1 - c(\Delta) - \int_{-\Delta}^{\Delta} c(|x|) d\widetilde{F}(x),$$

and

$$CS(F_\Delta) = 1 - c(\Delta) - \int_{-\Delta}^{\Delta} c(|x|) dF_\Delta(x),$$

yielding

$$CS(\widetilde{F}) - CS(F_\Delta) = \int_{-\Delta}^{0} c(-x) d(F_\Delta(x) - \widetilde{F}(x)) + \int_{0}^{\Delta} c(x) d(F_\Delta(x) - \widetilde{F}(x)).$$

(ii) implies that $\widetilde{F}$ (first-order) stochastically dominates $F_\Delta$ above $0$, so $\int_0^\Delta c(x)d(F_\Delta(x)-\widetilde{F}(x)) \leq 0$. To the contrary, below $0$, $F_\Delta$ stochastically dominates $\widetilde{F}$, so $\int_{-\Delta}^{0} c(-x)d(F_\Delta(x) - \widetilde{F}(x)) \leq 0$. The desired result follows from the fact that if $\widetilde{F}(x) \neq F_\Delta(x)$ over a positive measure of values then $CS(\widetilde{F}) < CS(F_\Delta)$, which contradicts the optimality of $\widetilde{F}$. ■

**Proof of Theorem 2.** By Proposition 3 , it suffices to consider prices in $[1 - c(\overline{\Delta}), 1]$. Then, by Proposition 6, a consumer-optimal distribution necessarily belongs to the class of design-robust distributions. This means that the problem reduces to

$$\max_{\Delta \in [0,\overline{\Delta}]} CS(F_\Delta) = \int_{-\Delta}^{\Delta} (c(\Delta) - c(|x|)) dF_\Delta(x).$$

In turn,

$$\begin{aligned}
&\int_{-\Delta}^{\Delta} (c(\Delta) - c(|x|)) dF_\Delta(x) \\
=\ & (c(\Delta) - c(0)) \lim_{x \to 0} (F_\Delta(x) - F_\Delta(-x)) + \int_{(-\Delta,0)\cup(0,\Delta)} (c(\Delta) - c(|x|)) dF_\Delta(x) \\
=\ & 2\left(F_\Delta(0) - \frac{1}{2}\right) c(\Delta) + 2\int_{(0,\Delta]} (c(\Delta) - c(x)) dF_\Delta(x) \\
=\ & 2\left(F_\Delta(0) - \frac{1}{2}\right) c(\Delta) + 2\left[(c(\Delta) - c(x))F_\Delta(x)|_0^\Delta + \int_{(0,\Delta]} F_\Delta(x) dc(x)\right] \\
=\ & -c(\Delta) + 2(1 - c(\Delta)) \int_0^\Delta \frac{dc(x)}{1 - c\left(\frac{x+\Delta}{2}\right)},
\end{aligned}$$

where the second equality is due to $c(0) = 0$ and symmetry of $F$, the third is through integration by parts, and the last uses the fact that $F_\Delta(0) = (1 - c(\Delta))/(1 - c(\Delta/2))$. ■

**Proof of Proposition 9.** If $\Delta = 0$ then $(\pi(F_\Delta), CS(F_\Delta)) = (1, 0)$, which is the seller-optimal point on the Pareto frontier. The consumer-optimal point on the Pareto frontier is given by $(\pi(F_{\Delta^*}), CS(F_{\Delta^*}))$: by Proposition 6 and Theorem 2, $F_{\Delta^*}$ is a consumer-optimal distribution, so $(\pi(F_{\Delta^*}), CS(F_{\Delta^*}))$ must be Pareto efficient.

For each $p \in [\pi(F_{\Delta^*}), 1]$, define $\overline{CS}(p) := \max\{CS(F_\Delta) : 1 - c(\Delta) \geq p\}$. By construction, $\overline{CS}(p)$ is weakly decreasing. Note that if $CS(F_\Delta)$ is strictly increasing in $\Delta \in [0, \Delta^*]$ then $\overline{CS}(1 - c(\Delta)) = CS(F_\Delta)$.

Pick any $\pi \in [\pi(F_{\Delta^*}), 1]$ such that $\overline{CS}$ is strictly decreasing over $[\pi, \pi + \varepsilon)$ for some small $\varepsilon > 0$. By construction, $\overline{CS}(\pi) = CS(F_{\Delta^\dagger})$ where $\Delta^\dagger = c^{-1}(1 - \pi)$. Since total demand cannot exceed $1$, the firm can obtain this profit only when its price is not smaller than $\pi$. By Proposition 6, for each $p = 1 - c(\Delta) \geq \pi$, $F_\Delta$ maximizes consumer surplus in $\mathcal{F}(p)$. In addition, $\pi(F_\Delta) = 1 - c(\Delta)$ (i.e., all consumers purchase), and $\overline{CS}(1 - c(\Delta))$—the maximal consumer surplus—is increasing in $\Delta$ (decreasing in $p$). This implies that given $\pi \in (\pi(F_{\Delta^*}), 1)$, consumer surplus cannot exceed $CS(F_{\Delta^\dagger})$.

■