Time-varying Cost of Distancing: Distancing Fatigue, Holidays and Lockdowns

Christoph Carnehl* Satoshi Fukuda† Nenad Kos‡

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Abstract

We study an SIR model with endogenous behavior and a time-varying cost of distancing. We show that a steep increase in distancing cost is necessary for a second wave of an epidemic to arise. As a special case of the model with changing cost, we study distancing fatigue—the distancing cost increases in past distancing—and show that it cannot generate a second wave. Moreover, we characterize the change in the distancing cost necessary for the slope of prevalence to change its sign. This characterization informs policymakers: (i) of the required strictness of mitigation policies to cease the increase of prevalence, (ii) when policies can be lifted without causing a second wave, and (iii) whether public holidays are likely to generate another wave of the epidemic. Finally, we illustrate the implementation of desirable time-varying transmission rates through time-varying distancing cost with endogenous equilibrium distancing.

Keywords: Social Distancing; Distancing Cost; Distancing Fatigue; Second Wave
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*Bocconi University, Department of Economics and IGIER. Email: christoph.wolf@unibocconi.it.
†Bocconi University, Department of Decision Sciences and IGIER. Email: satoshi.fukuda@unibocconi.it.
‡Bocconi University, Department of Economics, IGIER and CEPR. Email: nenad.kos@unibocconi.it.

1 Introduction

Social distancing, whether individuals' voluntary behavior or public policy measures, reduces the spread of an infectious disease. The cost of distancing plays the crucial role in the individuals' decisions on whether to engage in social interactions or isolate themselves to reduce the chance of exposure. We study how changes in the distancing cost affect the epidemic's dynamics through individual distancing behavior.

The reasons for such changes in distancing cost are numerous. Religious festivals such as Christmas and Holi or seasonal festivals such as Thanksgiving and the Chinese New Year make it more difficult for people to avoid social interactions and, thus, correspond to a sudden and short-term rise in distancing cost.[1] Distancing fatigue—the phenomenon that individuals find it harder to limit social interactions when they have been deprived of them previously—leads to a gradual increase in distancing cost as individuals continue to socially distance.[2] Franzen and Wöhner (2021) document distancing fatigue among young adults in Switzerland during the COVID pandemic. Petherick et al. (2021) document a decline in the adherence to protective behaviors against COVID-19 in 2020 from a sample of 14 countries. On a more basic level, there is a long line of research documenting how social groups increase the well-being of individuals by offering safety and increased odds of survival (Harlow and Zimmermann, 1959; Bowlby, 1969; Baumeister and Leary, 1995; Eisenberger, 2012).[3] Finally, some government policies enacted during an epidemic can be thought of as decreases in the distancing cost. For example, closures of restaurants and cinemas reduce the availability of activities with individuals interacting and thereby facilitate them to engage in social distancing.

We explore an SIR epidemiological model in which myopic individuals choose how much to distance at each point in time while the distancing cost may change over time. Distancing of myopic individuals is studied in Avery (2021), Dasaratha (2020) and Carnehl, Fukuda, and Kos (2021).[4] These papers show that with the constant cost of distancing, the prevalence is single-peaked. Here we show that for a second peak of an epidemic to arise, the cost of distancing would have to rise extremely rapidly at some point after the first peak.[5] We then proceed to analyze a framework with distancing

[1] See for instance, Hillstrom (2011) for the historical importance of Thanksgiving in the US society.

[2] The World Health Organization Regional Office for Europe defines "pandemic fatigue" as demotivation to follow recommended protective behaviors, emerging gradually over time and affected by several emotions, experiences, and perceptions as well as the cultural, social, structural, and legislative environment (WHO, 2020).

[3] Matthews et al. (2016) show that after 24 hours of isolation mice search for social interaction and dopamine neurons in mice brain show similar patterns of activation as in other cravings.

[4] To the best of our knowledge, there is no characterization of equilibrium distancing behavior in an SIR model with far-sighted individuals.

[5] We provide a precise definition in terms of the semi-elasticity of the distancing cost.

fatigue in which the cost of distancing increases in the amount of distancing in the past and depreciates over time; for an axiomatization, see Baucells and Zhao (2019). Roughly, the current distancing cost is a "discounted value" of past distancing—the further in the past the distancing, the more it is discounted. We show that, in the model with distancing fatigue, prevalence has a single peak. After the prevalence peaks first, the growth of fatigue slows down up to the point at which fatigue starts decreasing. As a consequence, a second wave of the infection cannot arise. This suggests that distancing cost also is single-peaked and that distancing cost peaks at least as late as prevalence. While our result shows that distancing fatigue alone cannot qualitatively change the shape of prevalence, it can still play an important role quantitatively; for quantitative effects, see Goldstein, Yeyati, and Sartorio (2021).[6]

While distancing fatigue cannot cause a second wave of an epidemic, a sharp, sudden rise in the cost of distancing can. Such sharp rises are generated by public holidays and festivities—when it becomes challenging for individuals to keep social interactions low—or by the end of a mitigation policy—when the distancing cost discretely rises. To better understand such sharp rises, we characterize a threshold distancing cost function: By how much at each point in time must the distancing cost increase or decrease instantaneously to change the sign of the slope of prevalence.

The threshold distancing cost is particularly useful for two purposes. First, when the prevalence is decreasing, it determines the largest amount by which the distancing cost could increase without causing a second wave. This threshold is important when a policy maker considers lifting a mitigation policy or when public holidays are coming. If the authorities believe that an increase in the distancing cost shall arise, they can try to counteract it. While it would be intuitive that lifting a mitigation policy may cause a second wave of an epidemic, our analysis reveals that if the policy maker introduces a mitigation measure that causes the prevalence to decline, the threshold distancing cost also declines endogenously. Consequently, even partially lifting of the measure may cause a second wave. The threshold distancing function provides the bound on distancing cost under which a second wave does not arise.

Second, when the prevalence is increasing, our characterization shows how much the cost of distancing would have to fall for the prevalence to start decreasing. This information is of utmost importance for policymakers weighing the harshness of non-pharmaceutical interventions to implement in attempting to reverse the course of an epidemic.

[6] Moreover, if politicians base their decisions to lift policies based on current prevalence and immunity alone while ignoring distancing fatigue, an unintended second wave may arise.

Finally, we illustrate that our equilibrium model with time-varying cost can be useful for other purposes. Several papers have analyzed optimal mitigation policies in a reduced form by assuming that a planner controls the transmission rate of a disease over time (see, for example, Acemoglu et al., 2021, Alvarez, Argente, and Lippi, 2021, Farboodi, Jarosch, and Shimer, 2021, and Kruse and Strack, 2022). However, they do not explicitly incorporate endogenous responses to such policies and how these transmission rates would be implemented. We show how different, time-varying transmission rates can be implemented in equilibrium with endogenous distancing by controlling the distancing cost, such as closing restaurants or restricting the occupation rate of public indoor places. However, this analysis reveals that papers studying optimal mitigation should consider alternative cost functions of transmission reduction taking prevalence into account. When the prevalence is high, endogenous distancing already leads to a substantial reduction in the transmission rate.

Related Literature. To the best of our knowledge, this is the first paper to comprehensively study the effects of a time-varying cost of distancing in an SIR model with behavior. However, our model builds on work developed over the last century. We do our best to give credit to these foundations and other related work.

The building blocks of the SIR model were set by the seminal work of Ross and Hudson (1917) and Kermack and McKendrick (1927). The incorporation of preventive behavior in such models, however, is a more recent endeavor. Reluga (2010), Fenichel et al. (2011), Chen (2012), and Fenichel (2013) introduced social distancing into SIR models and provided numerical analyses of equilibrium trajectories.

Models of distancing with myopic agents, are analyzed by Dasaratha (2020), Avery (2021), Engle et al. (2021), McAdams (2020), and Carnehl, Fukuda, and Kos (2021).[7] The last two papers establish the single-peakedness of equilibrium prevalence. Dasaratha (2020) analyzes a model where the individuals are uncertain whether they are infected. Avery (2021) studies the interplay between distancing behavior and the willingness to get vaccinated. In relation to distancing fatigue, Avery (2021) models fatigue as an increase in a cost of distancing after a certain amount of time, independently of the previous amount of distancing. His analysis revolves around the effects of fatigue on the adoption of vaccines. In McAdams (2020), each individual's distancing cost varies over time because it depends on other non-infected individuals' distancing.

Gualtieri and Hecht (2021) and MacDonald, Browne, and Gulbudak (2021) study

[7] Rachel (2020a) and Toxvaerd (2020) analyze the model with non-myopic agents and offer two sets of results that contradict each other; both arguments that the behavior under analysis is equilibrium behavior are incomplete.

distancing fatigue (or epidemic fatigue) in a non-behavioral SIR model. They divide the susceptible population into two compartments, where susceptible individuals in one, the "fatigue" compartment, compartment have a higher transmission rate than those in the other, the "quarantine", compartment. Due to distancing fatigue, a susceptible individual in the quarantine compartment moves to the fatigue compartment at a constant rate of time (MacDonald, Browne, and Gulbudak, 2021) or discretely after some predetermined time (Gualtieri and Hecht, 2021).

Other papers have studied possibilities and reasons behind second waves. Rachel (2020b) studies the likelihood of a second wave of an epidemic in a behavioral SIR model and argues that lifting of a mitigation policy can lead to a second wave. Numerical projections for the COVID-19 pandemic in Giannitsarou, Kissler, and Toxvaerd (2021) suggest that waning immunity can cause several waves.

2 Model

We study how individuals' social distancing behavior during an epidemic is affected by a time-varying cost of distancing and how this affects the dynamics of an epidemic. A continuum of agents, indexed by $i \in [0, 1]$, is infinitely lived with time labeled by $t \in [0, \infty)$. The population is divided into three compartments: susceptible (S), infected (I) and recovered (R).[8] Susceptible individuals can get infected by meeting an infected individual. Infected individuals recover at rate $\gamma > 0$; this implies that it takes on average $1/\gamma$ units of time to recover. After recovery, individuals acquire permanent immunity and cannot get infected again.

Individuals are responsive to the threat of infection and thus might try to avoid it. We capture this by letting a susceptible individual choose the level of exposure to the infection $\varepsilon(t) \in [0, 1]$ at each point in time. A susceptible individual who chooses exposure $\varepsilon(t)$ at time t gets infected at rate $\beta\varepsilon(t)I(t)$, where $\beta > \gamma$ is the transmission rate of the disease. Less exposure, i.e., lower $\varepsilon(t)$, thus, decreases the chance of infection. Conversely, we define distancing as $d(t) := 1 - \varepsilon(t)$. We assume that getting infected comes at a cost $\eta \geq 0$ while being susceptible generates a flow payoff of π_S. The cost of infection being constant over time is an assumption akin to assuming that the individuals are myopic.[9] A reduction in exposure comes at a cost $\frac{1}{2}c(t)(1 - \varepsilon(t))^2$, therefore, in the absence of the epidemic, individuals would go about their daily business with $\varepsilon(t) = 1$. The main

[8] Our model abstracts from deaths for the simplicity of presentation.

[9] For other models with myopic individuals, see Dasaratha (2020), Engle et al. (2021), and Carnehl, Fukuda, and Kos (2021).

novelty of our model is that we allow the distancing cost $c(t)$ to vary over time.

For each susceptible individual i, the distancing cost is a piece-wise continuously differentiable function $c_i : [0, \infty) \to [\underline{c}, \infty)$ with the following three properties: (i) there exists a lower bound $\underline{c} > 0$ such that $c_i(t) \geq \underline{c}$ for all t; (ii) there are at most a finite number of jump discontinuities of c_i, which are common for all individuals i, at $t_1 < \cdots < t_N$ such that, on each interval (t_n, t_{n+1}) with $n \in \{1, \ldots, N\}$,[10] $\dot{c}_i(t)$ is a continuous function satisfying

$$\dot{c}_i(t) = F(t, c_i(t), d_i(t)), \tag{1}$$

where $F(t, \cdot, \cdot)$ is a function of the current distancing cost $c_i(t)$ and the current distancing level $d_i(t)$ of individual i, and (iii) at each t_n with $n \in \{1, \ldots, N\}$, c_i is right-continuous. At $t = 0$ and at any point of jump discontinuity of c_i, we assume that $c_i(t)$ is given.[11] We assume that $c_i(0)$ is independent of i and denote it c_0. A clarification is in order: While $c_i(t)$ may depend on the identity of an individual i, the environment is symmetric due to the common law of motion F and the common initial cost c_0. Differences in the cost among agents might, however, arise due to variations in the choice of distancing.

A susceptible individual i determines her exposure level at each time by solving:

$$\max_{\varepsilon_i(t) \in [0,1]} \pi_S - \frac{c_i(t)}{2}(1 - \varepsilon(t))^2 - \beta \eta I(t) \varepsilon_i(t). \tag{2}$$

At each time t, the susceptible individual i takes the value of $c_i(t)$ as given, while the resulting exposure level affects the slope of the distancing cost, $\dot{c}_i(t)$.[12]

Let the average exposure be $\varepsilon(t) = \frac{1}{S(t)} \int \varepsilon_i(t) di$ where the integral is taken over the susceptible individuals. The disease dynamics are governed by the following system of differential equations:

$$\dot{S}(t) = -\beta \varepsilon(t) I(t) S(t), \tag{3}$$

$$\dot{I}(t) = I(t)(\beta \varepsilon(t) S(t) - \gamma), \tag{4}$$

$$\dot{R}(t) = \gamma I(t), \tag{5}$$

for all except possibly a finite number of t, with the initial condition $(S(0), I(0), R(0)) = (S_0, I_0, 0)$ with $I_0 \in (0, 1)$ and $S_0 = 1 - I_0$. The size of the population is constant over

[10] For ease of exposition, let $t_{N+1} = \infty$.

[11] We allow for such discrete jumps to accommodate holidays (in which the distancing costs discretely jump) or changes in social-distancing policies (that also discretely affect the distancing costs).

[12] Intuitively, consider a discrete-time model in which, at the start of each period, a susceptible individual takes her distancing cost at that time as given. Her resulting exposure level affects her distancing cost in the next period. Our model would correspond to the continuous-time limit of such a model.

time: $S(t) + I(t) + R(t) = 1$ for all $t \geq 0$.

Definition 1. An equilibrium is a tuple of functions $(S, I, R, (c_i, \varepsilon_i)_i)$ with the following three properties: (i) (S, I, R) are continuous functions that satisfy (3), (4) and (5) with the initial condition $(S(0), I(0), R(0)) = (S_0, I_0, 0)$, where ε is the average exposure;[13] (ii) each ε_i solves (2), that is, ε_i is a best response to (S, I, R) given c_i; and (iii) the distancing cost function c_i satisfies (1), where $d_i = 1 - \varepsilon_i$. An equilibrium is symmetric if $\varepsilon = \varepsilon_i$ for all i.

The first order condition of the individual's problem yields

$$\varepsilon_i(t) = \max\left(0, 1 - \frac{\beta\eta I(t)}{c_i(t)}\right). \tag{6}$$

An agent chooses a lower exposure (that is, she socially distances more) when the prevalence is higher. Distancing is increasing in the cost of infection η and the transmission rate β, and decreasing in the cost of distancing $c_i(t)$. In equilibrium, $\varepsilon_i = \varepsilon$ and $c := c_i$ for all i due to (6) differing across individuals only in the distancing cost $c_i(t)$ and the fact that $c_i(0) = c_0$ for all i. Therefore, any equilibrium is symmetric:

$$\varepsilon(t) = \max\left(0, 1 - \frac{\beta\eta I(t)}{c(t)}\right). \tag{7}$$

Plugging the expression for exposure (7) into the system of differential equations (3), (4), (5), and (1) leads to the system of differential equations characterizing the equilibrium.

Proposition 1. *An equilibrium exists, is unique and symmetric. In the unique equilibrium, the system (S, I, R) satisfies $I_\infty := \lim_{t\to\infty} I(t) = 0$, $S_\infty := \lim_{t\to\infty} S(t) \in \left(0, \frac{\gamma}{\beta}\right)$, and $\lim_{t\to\infty} \varepsilon(t) = 1$.*

Throughout most of the analysis, we focus on the case in which the prevalence is increasing at the outset, $\dot{I}(0) > 0$. This occurs whenever

$$\beta S_0 \left(1 - \frac{\beta\eta I_0}{c_0}\right) - \gamma > 0. \tag{8}$$

The above inequality is satisfied as long as $\beta > \gamma$ and the initial seed of infection I_0 is

[13] The assumption of continuity of (S, I, R) is innocuous in light of our application. Discontinuities could only arise at discontinuities of $c(t)$. However, $c(t)$ only affects behavior and the changes in I and S over an interval of time of length $\Delta > 0$ is bounded from above by the SIR dynamics without behavior and from below by a path in which no individual is infected in this interval. Both paths are continuous as Δ goes to zero and so are our modified dynamics.

small enough. Put differently, given the fixed values of other parameters, there exist $\underline{\beta}$ and $\overline{\beta}$, with $\gamma < \underline{\beta} < \overline{\beta}$, such that $\dot{I}(0) > 0$ if and only if $\beta \in (\underline{\beta}, \overline{\beta})$.

3 Continuous Distancing Cost with an Application to Distancing Fatigue

This section studies the case in which the distancing cost is a continuous function of time. First, Section 3.1 provides a sufficient condition for the epidemic being single-peaked under a general specification of distancing cost. Section 3.2 introduces the effect of past distancing behavior on the current distancing cost, which we interpret as distancing fatigue, and shows that prevalence peaks at most once.

3.1 Sufficient Condition for Single-Peaked Epidemics

In this part, we examine conditions on the distancing cost under which the prevalence peaks at most once. We define a peak prevalence to be a strict local maximum.

Proposition 2. *Let c be a (continuously) differentiable function such that $\dot{c}$ is given by (1) for all $t > 0$. If*

$$\frac{\dot{c}(t)}{c^2(t)} < \frac{\varepsilon^2(t)}{\eta} \text{ for all } t > 0, \tag{9}$$

then, in equilibrium, prevalence I has a single peak. A sufficient condition for the above inequality to be satisfied is

$$\frac{\dot{c}(t)}{c^2(t)} < \frac{\gamma^2}{\eta\beta^2 S_0^2} \text{ for all } t > 0. \tag{10}$$

If the cost of distancing is growing slowly, more precisely, if $\dot{c}/c^2$ is small, then I has only one local maximum. The prevalence either immediately starts decreasing and never picks up or increases from the outset until it reaches the peak and decreases thereafter. In this case, it takes on a familiar shape as in the SIR model without distancing. The proof argues that for the second wave to arise, I would first need to attain a local minimum at some time $t > 0$ for which to occur $\dot{I}(t) = 0$ and $\ddot{I}(t) \geq 0$ are necessary. By taking the derivative of $\dot{I}$, the latter requirement can be shown to be equivalent to

$$\frac{\dot{c}(t)}{c^2(t)} \geq \frac{\varepsilon^2(t)}{\eta}.$$

Therefore, if $\dot{c}(t)/c^2(t) < \varepsilon^2(t)/\eta$ for all $t > 0$, there can be no local minimum and consequently no second wave. This result nests $c(t) = c$ as the special case; an environment which was previously analyzed in Carnehl, Fukuda, and Kos (2021).

In contrast, notice that if $\dot{c}(t)/c^2(t) > \varepsilon^2(t)/\eta$ for all $t > 0$, then any stationary point of I is a local minimum. This contradicts the existence of a differentiable cost function such that $\dot{I}(0) > 0$, as $I_\infty = 0$. Indeed, a similar result can be established directly. Let us examine more closely the inequality $\dot{c}(t)/c^2(t) > 1/\eta$, which is sufficient for the above inequality to be satisfed. Moreover, let

$$\sigma(t) := \frac{\dot{c}(t)}{c(t)}$$

be the semi-elasticity of c. The above inequality can be rewritten as $\sigma(t) > \frac{1}{\eta}c(t)$; that is, it can be interpreted as the requirement that the semi-elasticity of a function dominates the function itself (multiplied by some positive constant).[14] The next result establishes that no differentiable function that is everywhere positive satisfies this property.

Lemma 1. *There does not exist a continuously differentiable function $c : [0, \infty) \to [\underline{c}, \infty)$ with $c(t) \geq \underline{c}$ for all t, such that $\sigma(t) > \frac{1}{\eta}c(t)$ for all $t > 0$.*

The inequality $\sigma(t) > c(t)/\eta$, which is necessary for a second wave to arise, requires that the distancing cost c grows extremely fast. To build further intuition, fix a $c_0 > 0$. A solution to the differential equation $\sigma(t) = c(t)/\eta$ on $t \in (0, \eta/c_0)$ is

$$c(t) = \frac{1}{\frac{1}{c_0} - \frac{t}{\eta}},$$

which grows towards infinity as t goes towards η/c_0. This analysis establishes that if there is to be a continuous distancing cost function that generates more than one wave, such a cost function needs to satisfy $\dot{c}(t)/c^2(t) < \varepsilon^2(t)/\eta$ for low values of t. More precisely it must do so at the first stationary point, followed by a period of time where $\dot{c}(t)/c^2(t) > \varepsilon^2(t)/\eta$. The second period must, however, be limited in duration as otherwise there does not exist a distancing cost function that satisfies these conditions.

In the following, we apply Proposition 2 to show that the prevalence of a disease is single-peaked for several natural cost-function specifications. An example illustrates the usefulness of Proposition 2 as it does not require an analysis of the potentially complicated disease dynamics but only an inspection of the cost function itself. In the following subsection, we move on to a more realistic but more complicated evolution of the distancing

[14]Note that the right-hand side of the inequality, the ratio between the cost of the disease and the distancing cost, is inversely proportional to equilibrium distancing.

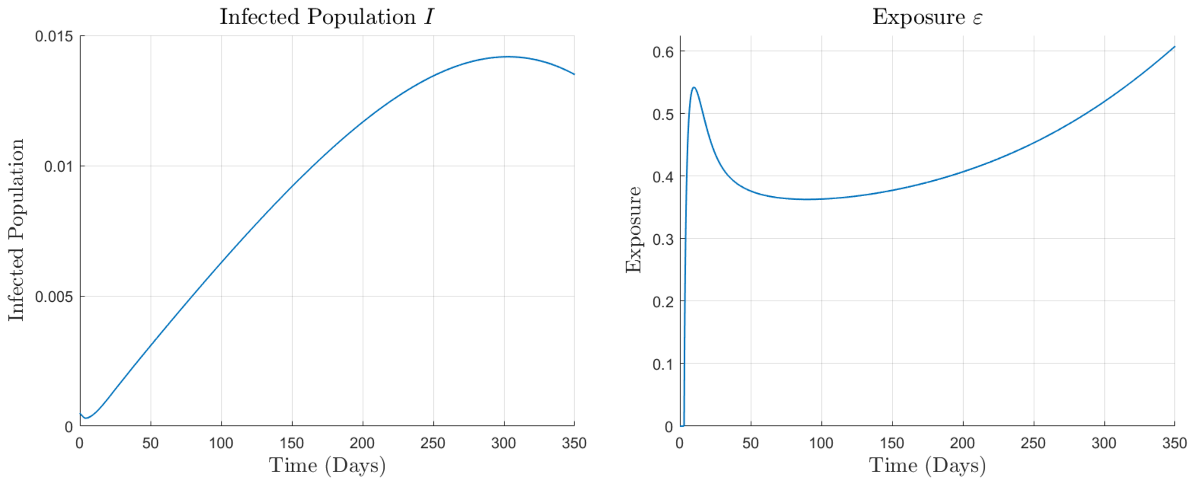

Figure 1: Example 1. The left panel depicts the infected population I over time. While I initially decreases, it starts to increase at around $t = 5$. The right panel depicts the exposure level ε over time.

cost based on distancing fatigue.

Example 1. Suppose that the cost function is linear in time

$$c(t) = c_0 + k \cdot t,$$

where $c_0 > 0$ and $k > 0$. In this case, the term $\dot{c}(t)/c^2(t) = k/c^2(t)$ is non-increasing over time, therefore by the above arguments there do not exist two points in time t_1 and t_2 such that $\dot{I}(t_1) = \dot{I}(t_2) = 0$ and $\ddot{I}(t_1) < 0$ and $\ddot{I}(t_2) < 0$. Consequently, if $\dot{I}(0) > 0$, prevalence is single-peaked.

However, when the prevalence is initially decreasing, $\dot{I}(0) < 0$, the behavior of the model with linear distancing cost can differ substantially from the standard SIR model without distancing or the SIR model with constant distancing cost. In those models, if the prevalence is initially decreasing, then it is decreasing throughout. Here, in contrast, it may occur that, even though the prevalence is decreasing at the outset, there exists a time $\hat{t} > 0$ when the prevalence is increasing, i.e., with $\dot{I}(\hat{t}) > 0$. For this to occur, the conditions for (i) $\dot{I}(0) < 0$—i.e., the reverse inequality in (8)—, and for (ii) $\dot{c}(0)/c^2(0) < \varepsilon^2(0)/\eta$—i.e., the necessary condition for the existence of some $\underline{t} > 0$ with $\dot{I}(\underline{t}) = 0$ and $\ddot{I}(\underline{t}) > 0$—must be satisfied. Figure 1 illustrates an environment with linearly increasing distancing cost where the prevalence increases eventually, $\dot{I}(t) > 0$ for some $t > 0$, despite it being decreasing at the outset, $\dot{I}(0) < 0$.[15] □

[15]The parameters are $(\beta, \gamma, I_0, \eta, c_0, k) = (0.3 + \frac{1}{7}, \frac{1}{7}, 10^{-4}, 2761.63, 0.05, 0.005)$. The parameters $(\beta, \gamma, I_0, \eta)$ are calibrated for the onset of COVID-19. See Carnehl, Fukuda, and Kos (2021).

3.2 Distancing Fatigue

It is in human nature to socialize and to maintain social distancing over a long time horizon becomes increasingly difficult. This phenomenon, pandemic fatigue, and its importance have been well-documented empirically during the COVID-19 pandemic; see, for example, Petherick et al. (2021). A reduction in distancing behavior while lockdowns remain in place is documented in Joshi and Musalem (2021) and attributed to increasing distancing fatigue. Moreover, they show that larger reductions in mobility are associated with higher levels of fatigue.

We model distancing fatigue by having individuals' cost of distancing depend cumulatively on all the previous distancing decisions

$$c(t) = c_0 + k \int_0^t e^{-r(t-\tau)}(1 - \varepsilon(\tau))d\tau, \tag{11}$$

where $k > 0$ is some positive constant and $r \geq 0$ is the fatigue recovery rate. The above function captures two important properties of fatigue. Past distancing increases an individual's distancing cost. At the same time, the effect of past distancing choices on the future cost of distancing decays over time. Baucells and Zhao (2019) provide an axiomatization of the fatigue utility model of this form. When $r = 0$, there is no decay, and thus the agent's cost of distancing is increasing over time.

In terms of the marginal change in the distancing cost, equation (11) is written as

$$\dot{c}(t) = k(1 - \varepsilon(t)) - r(c(t) - c_0), \tag{12}$$

with the initial condition $c(0) = c_0$. This differential equation is a special case of (1) and therefore the existence and uniqueness of equilibrium follow from Proposition 1.

In the following, we analyze the disease dynamics for the model with distancing fatigue. We start by developing two preliminary results. First, once individuals expose themselves to the disease to some extent, they will never fully distance afterward (i.e., their exposure is strictly positive afterward). Second, we show that, at the moment at which the distancing cost attains a local maximum, the prevalence is non-increasing.

Lemma 2. *In equilibrium, if $\varepsilon(t') > 0$ for some t', then $\varepsilon(t) > 0$ for all $t \geq t'$.*

Intuitively, the result follows because exposure is inversely related to $I(t)/c(t)$. Therefore, should exposure fall to a very low level, $I(t)$ would become decreasing and the distancing cost increasing—the fatigue effect. These two effects would lead the exposure to grow, thereby preventing it from falling to 0. In other words, $\varepsilon(t)$ can be equal to 0 only at the outset of an epidemic. In that case, $\dot{I}(0) < 0$. A simple assumption that guarantees $\varepsilon(t)$ to be always strictly positive is $\varepsilon(0) = 1 - \beta\eta I_0/c_0 > 0$, which in turn is satisfied if I_0 is small enough.

Next, we show that any critical point of the distancing cost is directly related to the prevalence dynamics at the critical point.

Lemma 3. *Suppose c is given by (11) and $\dot{I}(0) > 0$. In equilibrium, if c attains a local maximum (minimum) at $t > 0$, then $\dot{I}(t) \leq 0$ (≥ 0).*

If the distancing cost function has a local maximum, it must be at a time when the prevalence is declining. To see this, suppose that the maximum of the distancing cost function was attained when I was increasing. At the same point, the exposure would decrease due to c being locally flat and the prevalence increasing. However, then it cannot be that fatigue, and therefore the distancing cost function, is already maximized. This leads us to the main result of the section.

Proposition 3. *Suppose c is given by (11). If $\dot{I}(0) > 0$, I is single-peaked.*

Distancing fatigue itself cannot cause a second wave. For a second peak to arise, the prevalence would have to attain a local minimum first and then start increasing again. However, when the prevalence is falling, the growth of fatigue slows down and the fatigue may even start to decrease. As a consequence, distancing is not decreasing fast enough to jump-start another wave.

The single-peakedness of prevalence has important implications for other objects in the model. We start by taking a closer look at fatigue, which we define as the endogenous, distancing-induced increase in the cost of distancing. First, as long as the fatigue recovery rate is strictly positive, $r > 0$, fatigue is bounded above:

$$c(t) - c_0 = k \int_0^t e^{-r(t-\tau)}(1 - \varepsilon(\tau))d\tau \leq \frac{k}{r}.$$

Moreover, when $r > 0$, fatigue cannot be strictly increasing in equilibrium over the entire time horizon. Since the cost of distancing is bounded from below by c_0, distancing will dissipate with the eventually vanishing prevalence. The fatigue recovery rate implies that, as time passes, fatigue will vanish too. The following result establishes that fatigue is single-peaked as well.

Proposition 4. *Suppose c is given by (11). If $\dot{I}(0) > 0$ and $r > 0$, then c is single-peaked. Moreover, c cannot peak before I.*

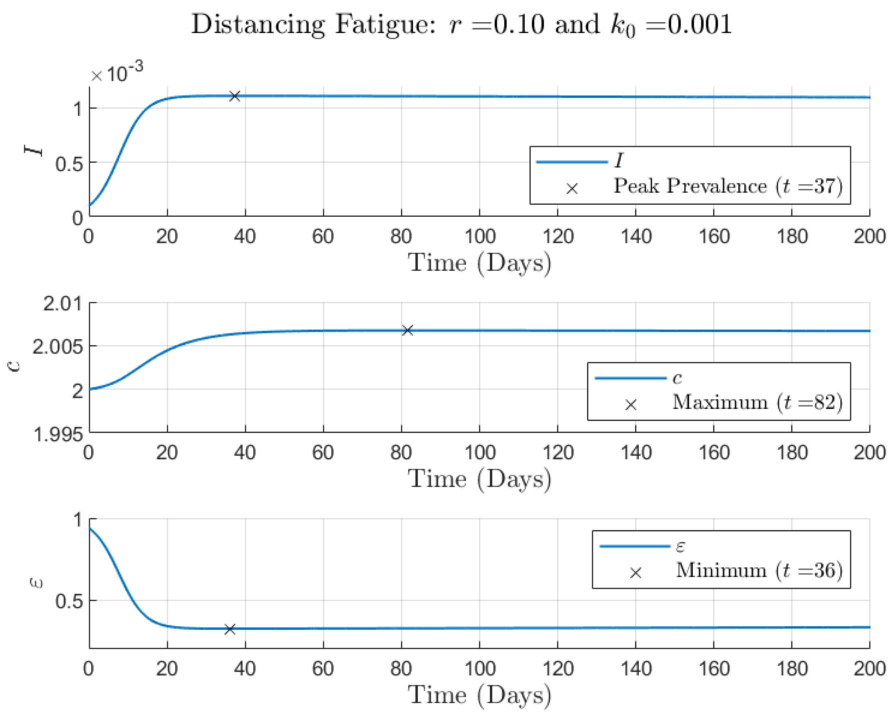

Figure 2: Distancing Fatigue. The figure depicts the infected population I, the distancing cost c, and the exposure level ε over time.

Figure 2 illustrates Propositions 3 and 4 when $r = 0.1$ and $k_0 = 0.01$. Prevalence initially increases relatively quickly which causes individuals to engage in more social distancing. Consequently, the distancing cost increases due to fatigue accumulating. As the cost increases and prevalence approaches its peak, individuals start increasing their exposure level again. After some time, this causes fatigue, and thus, the distancing cost, to slowly decrease again.[16]

Note that, as time goes to infinity, the distancing cost converges to $c_0 = \lim_{t\to\infty} c(t)$. In the limit, the infection dies out and the exposure level goes to full exposure. Fatigue $c(t) - c_0$ also goes back to zero because

$$\lim_{t\to\infty} \int_0^t e^{-r(t-\tau)}(1 - \varepsilon(\tau))d\tau = 0,$$

and consequently, the distancing cost returns to its initial value.

To conclude, our findings suggest that distancing fatigue does not affect qualitative

[16]Note that prevalence remains at an almost constant level in our model after the peak. This pattern has been empirically documented for the COVID-19 pandemic (see, for example, Atkeson, Kopecky, and Zha, 2020; Gans, 2022). In fact, Gans (2022) proposes a behavioral SIR model in which prevalence is *assumed* to be constant to simplify the analysis.

features of the prevalence trajectory. This, however, is not to say that distancing fatigue cannot play an important role in an epidemiological model. It may very well have critical quantitative implications. Goldstein, Yeyati, and Sartorio (2021), for example, show that after four months of lockdown, NPIs had a significantly lower effect on reducing fatalities.

4 Discontinuous Distancing Cost: Policies and Holidays

The cost of social distancing depends not only on previous exposure decisions but also on other factors, such as holidays or public health policies. The opportunity cost of social distancing sharply increases during holiday seasons when social gatherings have high value or during vacation times. Mehta et al. (2021) report an increase in travel and social activity during Thanksgiving 2020. Schlosser et al. (2020) documents an increase in travel during school and public holidays. In contrast, business closures due to a governmental lockdown discretely lower the opportunity cost of social distancing. When public health policies are lifted or holidays pass, the cost of distancing returns to its original level.[17] In this section, we examine the effects of discontinuous changes in the cost of distancing for a fixed amount of time. While the analysis to follow is cleanest with discontinuous changes, the results do not rely as much on the discontinuity as they do on sudden rapid changes in the cost of distancing.

We characterize the threshold on the cost of distancing such that if $c(t)$ is above the threshold, the slope of prevalence is positive, and if $c(t)$ is below the threshold, the slope of prevalence is negative. The difference between the threshold and the actual $c(t)$ is the largest instantaneous change in c that will not change the sign of the slope of $I(t)$.

To formulate the next result, we denote by $(S_1, I_1, R_1, c_1, \varepsilon_1)$ the equilibrium corresponding to an exogenously given distancing cost function c_1. That is, (S_1, I_1, R_1) are continuous functions that satisfy (3), (4) and (5) with the initial condition $(S_1(0), I_1(0), R_1(0)) = (S_0, I_0, 0)$, where ε_1 is the average exposure that satisfies (7) with $c = c_1$. Note that the unique equilibrium is symmetric.

Proposition 5. *Let c_1 be a time-dependent piece-wise continuously-differentiable distancing cost and let $(S_1, I_1, R_1, c_1, \varepsilon_1)$ be the corresponding equilibrium. Define the threshold*

[17] Hatchett et al. (2007) and Caley et al. (2008) report that relaxations in non-pharmaceutical interventions caused a new surge of cases due to increased social activity during the 1918 influenza pandemic. Similarly, Nguyen et al. (2020) finds an increase in mobility of 6-8% soon after US-states reopened during the COVID-19 pandemic.

distancing cost $\bar{c}$ *as follows: for each* $\tilde{t} \geq 0$,

$$\bar{c}(\tilde{t}) := \begin{cases} \frac{\beta^2 I_1(\tilde{t}) S_1(\tilde{t}) \eta}{\beta S_1(\tilde{t}) - \gamma}, & \text{if } S_1(\tilde{t}) > \frac{\gamma}{\beta} \\ \infty, & \text{if } S_1(\tilde{t}) \leq \frac{\gamma}{\beta} \end{cases}.$$

For any $\tilde{t} > 0$ *and a time-dependent piece-wise continuously-differentiable distancing cost* c_2 *with corresponding equilibrium* $(S_2, I_2, R_2, c_2, \varepsilon_2)$ *and the property that* $c_2(t) = c_1(t)$ *for all* $t < \tilde{t}$, *the following holds:*

$$\dot{I}_2(\tilde{t}_+) := \lim_{s \downarrow \tilde{t}} \dot{I}_2(s) < 0 \text{ if and only if } c_2(\tilde{t}) < \bar{c}(\tilde{t}).$$

In words, fix a distancing cost c_1 and its implied equilibrium and consider an alternative distancing cost c_2 that coincides with c_1 up to time $\tilde{t}$. There exists a function $\bar{c}(t)$, which prescribes at each point in time t, the largest value of the distancing cost under which the right-limit of the derivative of $I(t)$ is negative.[18]

Whenever $c_1(t)$ is such that $I(t)$ is single-peaked in equilibrium, $\bar{c}(t)$ intersects $c_1(t)$ once and from below. In particular, as long as $\dot{I}_1(t) > 0$, $\bar{c}(t) < c_1(t)$ and conversely so if $\dot{I}(t) > 0$. In addition, when $S_1(t)$ approaches γ/β from above, $\bar{c}(t)$ grows towards infinity.

The difference $\bar{c} - c_1$ plays an important role. When the prevalence is decreasing, the cost difference informs by how much the cost can instantaneously increase without the prevalence starting to increase. Conversely, when the prevalence is already increasing, the difference $c_1 - \bar{c}$ establishes by how much the cost of distancing must decrease for the prevalence to start falling. This is of particular interest to policymakers who are trying to establish the strictness of public health policies required to reduce the prevalence immediately. Conversely, it can be used to establish whether lifting a policy will lead to a second wave. During the 1918 influenza pandemic, relaxations in non-pharmaceutical interventions caused a new surge of cases (see, for example, Caley et al., 2008 and Hatchett et al., 2007).

We present two examples and depict how the threshold $\bar{c}$ evolves for different underlying scenarios. First, we consider an example with a constant underlying distancing cost c and an increase in distancing cost due to a holiday season. Second, we consider a scenario with a lockdown that reduces the constant distancing cost c for a period of time. These examples highlight the importance of taking endogenous distancing and time-varying cost seriously when a second wave is desired to be avoided.

[18]Note that $\bar{c}(t)$ depends on the equilibrium path $(S_1, I_1, R_1, c_1, \varepsilon_1)$.

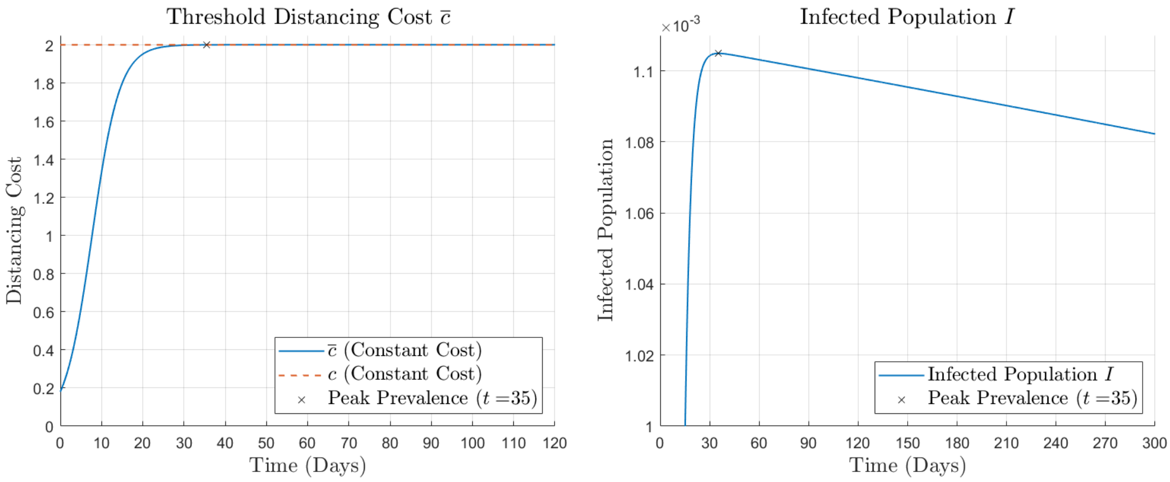

Figure 3: Constant distancing cost. The left panel depicts the threshold distancing cost function $\bar{c}$ over time. The right panel depicts the infected population I over time.

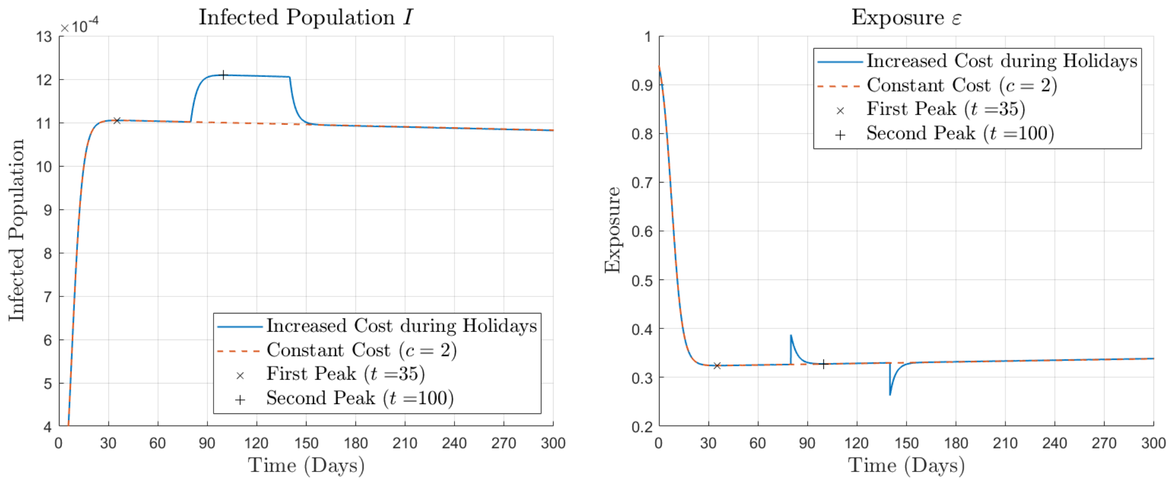

Figure 4: Increased distancing cost during holidays. The left panel depicts the infected population I and especially the second wave. The right panel depicts the exposure level ε.

Example 2. To illustrate the threshold distancing cost $\bar{c}$, we consider a simple example in which the distancing cost c is constant over time. The left panel of Figure 3 depicts the threshold function $\bar{c}$ (solid curve) for the corresponding constant cost function $c = 2$ (dashed line). The right panel depicts the infected population over time. Notably, the peak prevalence is attained around day 35.[19]

Suppose that the distancing cost jumps to $c(t) = 2.2$ during a holiday season: for

[19] While the prevalence theoretically declines towards 0 in this constant-distancing case, the right panel suggests that the decline is slow. The shape of the threshold distancing function $\bar{c}$ reflects this feature. Namely, as the left panel shows, $\bar{c}$, while growing, stays close to 2 for an extended period of time after the peak. That is, although it is difficult to see from the figure, c and $\bar{c}$ intersect only once (at around day 35).

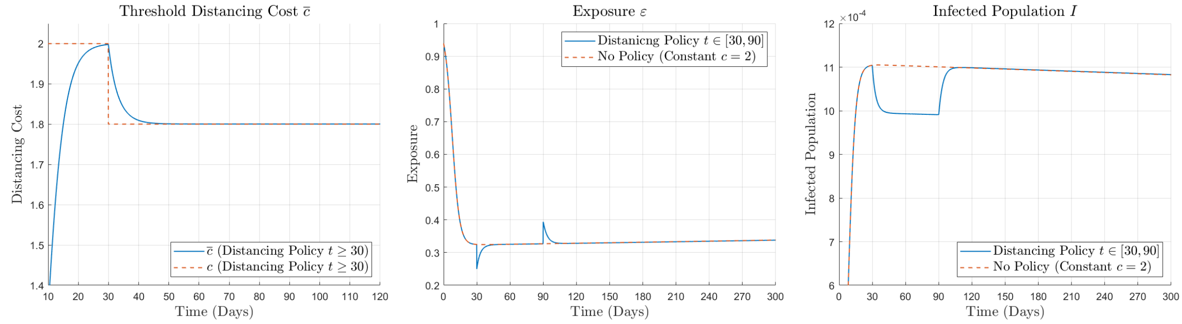

Figure 5: Social-Distancing Policy. The left panel depicts the threshold distancing cost function $\bar{c}$ over time. The central panel depicts the exposure level ε over time. The right panel depicts the infected population I over time.

the purpose of illustration, the holiday season lasts for two months between days 80 and 139. One can infer from the left panel of Figure 3 that the new distancing cost function c jumps above the original threshold $\bar{c}$ and thus induces prevalence to increase. The left panel of Figure 4 illustrates the ensuing second wave of infection. When the holiday season starts, individuals instantaneously best respond to the higher distancing cost by increasing their exposure levels. Increased exposure, in turn, leads to higher prevalence, to which then the individuals respond by decreasing their exposure again. The right panel shows that individuals' average exposure level eventually returns to the level close to (but slightly above) the case in which the distancing cost is fixed throughout.[20] After roughly 20 days, at around day 100, the infection peaks for the second time. □

Example 3. Here, we consider the introduction of a social-distancing policy. Letting the baseline distancing cost be $c(\cdot) = 2$, recall that the left panel of Figure 3 shows how much the distancing cost c must be decreased to decrease the infected population before it would reach its peak otherwise. The threshold cost $\bar{c}$ is 1.8 at around day 15 and the peak prevalence is attained on day 35. Suppose that the social-distancing measure, which decreases distancing cost to $c = 1.8$ is introduced on day 30.

The left panel in Figure 5 gives a new threshold distancing cost function $\bar{c}$ when the distancing cost function satisfies $c(t) = 1.8$ for $t \geq 30$ (the solid curve). After the introduction of the social-distancing measure, the infected population decreases, and the new threshold $\bar{c}$ endogenously decreases as well. The figure shows that on day 50, 20 days after the social-distancing measure is introduced, the threshold cost $\bar{c}$ is close to (but above) the distancing cost $c = 1.8$.

To understand the new threshold distancing cost function, consider how individuals respond to the social-distancing measure. As the central panel shows, individuals best

[20]This means that the (10%) increase in the distancing cost is numerically close to the order in which the infected population at the second wave is higher than that at the first peak.

respond to a lowered distancing cost by decreasing their exposure levels. The increase in distancing lowers the prevalence, which leads to a feedback effect of increasing exposure. The prevalence nevertheless decreases but individuals' responses slow down its decrease. The right panel, which depicts the infected population over time, illustrates this point.

After day 50, virtually any easing of the social-distancing measure causes the second wave.[21] For instance, assume that the distancing measure is lifted in its entirety after two months (i.e., on day 90). The infection resurges, and around day 111, the infected population almost coincides with the case in which no distancing measure is introduced; see the right panel. □

5 Implementing Optimal Transmission Rates with Distancing Cost

Several papers have analyzed optimal mitigation policies in a reduced form by assuming that a planner controls the path of the disease (see, for example, Acemoglu et al., 2021, Alvarez, Argente, and Lippi, 2021, Farboodi, Jarosch, and Shimer, 2021, and Kruse and Strack, 2022). That is, they assume that a planner directly controls the level of the infectiousness parameter, $\beta(t)$, or social interactions, $\varepsilon(t)$. Both cases can be viewed as a planner controlling an effective transmission rate $\tilde{\beta}(t) = \beta(t)\varepsilon(t)$, in which either $\beta(t)$ is controlled and $\varepsilon(t)$ is constant or in which β is constant and $\varepsilon(t)$ is controlled.

While mask mandates can directly affect the transmission rate, the attainable levels of the transmission rate are limited by such a policy alone. Many countries have introduced additional policies beyond mask mandates to reduce transmission during the COVID pandemic. Such policies aim at reducing the spread of the infection via reduced social interactions, such as bar and restaurant closures. However, to understand the effect of such policies—which affect the individuals' incentives to socially distance—, behavior should be modeled explicitly because they are only effective through individuals' endogenous choices, which, in turn, depend on the current state of the epidemic.[22]

[21] Anderson et al. (2020), in discussing mitigation measures for suppressing the course of the COVID-19 pandemic, point out that interventions that reduce transmission greatly, which make the epidemic curve longer and flatter, cause the risk of a resurgence when interventions are lifted. Our analysis sheds light on a behavioral mechanism: how such measures also lower the threshold distancing cost to suppress the infection so as not to rise again.

[22] Carnehl, Fukuda, and Kos (2021) show that policies that affect distancing incentives via reductions in the transmission rate and changes in the cost of distancing have qualitatively different effects on the path of an epidemic.

Nevertheless, the results obtained in these papers are important to understand the desirable epidemic paths of a planner who optimizes constrained by macroeconomic or other cost considerations. Therefore, we show how our equilibrium distancing model with a time-varying cost can be used to back out the path of policies affecting the cost directly to induce a desirable $\tilde{\beta}(t)$. That is, given an optimal path $\tilde{\beta}(t)$, which was obtained without explicitly modeling behavior, we derive the cost function $c(t)$ that can implement the time-varying transmission path when endogenous behavioral responses are taken into account.

Suppose that a desirable time-varying transmission rate $\tilde{\beta}(t)$ is given when the primitive of the non-behavioral SIR model is given (i.e., β, γ, I_0, and $S_0 = 1 - I_0$). The dynamics of the disease under the desirable transmission rate function $\tilde{\beta}$ is given by the system of equations (3), (4), and (5) where $\beta\varepsilon(t)$ is replaced by $\tilde{\beta}(t)$. Then, we can use our model to solve for the time-varying distancing cost function $\tilde{c}$ implementing such the transmission rate function $\tilde{\beta}(t) = \beta\varepsilon(t)$ via

$$\tilde{c}(t) := \frac{\beta^2 \eta I(t)}{\beta - \tilde{\beta}(t)}.$$

It should be noted that there is an endogenous upper bound on the implementable $\tilde{\beta}(t) = \beta$, which derives from individuals' endogenous distancing without policy interventions. Unless meetings can be subsidized during an epidemic—that is, more exposure encouraged than individuals would voluntarily engage in—, $\tilde{\beta}(t) > \beta$ cannot be attained.

An important observation is that the strictness of the policies in place depends not only on the transmission rate to be implemented but also on current prevalence. If the prevalence or the cost of becoming infected is high, policies do not have to be as strict to induce a certain transmission rate as if they were low. This suggests that in models studying the optimal control of a transmission rate during an epidemic, the cost function of reducing the transmission rate should depend on the current prevalence to take endogenous distancing decisions into account.

Finally, an analogous approach is feasible to implement desired levels of the effective reproduction number $\mathcal{R}^e(t)$ which measures how many secondary infections are caused by each infected individual.[23] Whenever $\mathcal{R}^e(t) > (<)1$, the prevalence is increasing (decreasing). For example, Budish (2020) considers $\mathcal{R}^e(t) \leq 1$ as a constraint for a planner without an explicit dynamic equilibrium model.[24] In our setting, this constraint corre-

[23]In the non-behavioral SIR model the effective reproduction number is given by $\frac{\beta}{\gamma}S(t)$.

[24]Note that while the effective reproduction number empirically has the tendency to be close to one for some time endogenously (see, for example, Atkeson, Kopecky, and Zha (2020) for data on the COVID-19 pandemic, and the discussion in Gans (2022)) policymakers have to take into account how changes in

sponds to the current policy satisfying the constraint that $c(t) \leq \bar{c}(t)$ as in Proposition 5.

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A Appendix

Proof of Proposition 1. At each time t, an individual's problem (2) is concave. Thus, the first-order condition (6) is sufficient. This pins down the individual's optimal distancing in the SIR dynamics.

Using the exposure obtained from (6) in the SIR dynamics together with the cost-function evolution yields

$$\dot{S}(t) = -\beta S(t)I(t)\max\left(1 - \frac{\eta\beta I(t)}{c(t)}, 0\right), \tag{13}$$

$$\dot{I}(t) = \beta S(t)I(t)\max\left(1 - \frac{\eta\beta I(t)}{c(t)}, 0\right) - \gamma I(t), \tag{14}$$

$$\dot{R}(t) = \gamma I(t) \tag{15}$$

$$\dot{c}(t) = F\left(t, c(t), \max\left(1 - \frac{\eta\beta I(t)}{c(t)}, 0\right)\right), \tag{16}$$

for all but possibly a finite number of t, at which at least one of the variables (S, I, R, c) is not differentiable. Let $t_1 < \cdots < t_N$ be the set of these points (this set may possibly be empty). Let $t_{N+1} = \infty$.

Thus, in any equilibrium, (S, I, R, c) is characterized by the system of differential equations $\frac{d}{dt}(S, I, R, c) = G(t, S, I, R, c)$, where G is defined by (13), (14), (15), and (16). The initial condition is $(S(0), I(0), R(0), c(0)) = (S_0, I_0, 0, c_0)$. Then, the initial value problem admits a unique solution (S, I, R, c) on $[0, t_1)$, as the system satisfies the conditions of the Picard-Lindelöf Theorem. Namely, the function G is continuous on the domain $D = [0, t_1) \times [0, 1]^3 \times [\underline{c}, \infty)$, and G is uniformly Lipschitz continuous in (S, I, R, c): there exists a Lipschitz constant L satisfying $\|G(t, S, I, R, c) - G(t, \tilde{S}, \tilde{I}, \tilde{R}, \tilde{c})\| \leq L\|(S, I, R, c) - (\tilde{S}, \tilde{I}, \tilde{R}, \tilde{c})\|$ for each $t \in [0, t_1)$. See, for example, Walter (1998). Since the equilibrium definition requires S, I and R to be continuous, we apply the same logic to the interval $[t_1, t_2)$ with the initial value $(S(t_1), I(t_1), R(t_1)) = \lim_{t \uparrow t_1} (S(t), I(t), R(t))$ and all the subsequent intervals. Now, $\varepsilon = \varepsilon_i$ is uniquely determined, and hence the model admits a unique and symmetric equilibrium.

Next, we show $\lim_{t \to \infty} I(t) = 0$. Since $R(\cdot) \in [0, 1]$ is weakly increasing, $\lim_{t \to \infty} R(\infty)$ exists in $[0, 1]$. By equation (15), we must have $0 = \lim_{t \to \infty} \dot{R}(t) = \gamma \lim_{t \to \infty} I(t)$, establishing $I_\infty = 0$.

Next, $\lim_{t \to \infty} \varepsilon(t) = 1$ follows from taking the limit of (7) because the distancing cost is bounded from below, $c(t) \geq \underline{c}$, and $\lim_{t \to \infty} I(t) = 0$.

Finally, having established $\lim_{t \to \infty} I(t) = 0$ and $\lim_{t \to \infty} \varepsilon(t) = 1$, $S_\infty < \frac{\gamma}{\beta}$ follows from the proof of Lemma 3 in Carnehl, Fukuda, and Kos (2021). □

Proof of Proposition 2. To have at least two peaks, there must be two local strict maxima of I at $t_2 > t_1 \geq 0$. Because I is continuous it has a minimum on $[t_1, t_2]$ by the extreme value theorem. Moreover, since t_1 and t_2 are local strict maxima, the minimum has to be attained at some $\hat{t} \in (t_1, t_2)$. The fact that in equilibrium S, I and R are continuous implies that I is differentiable and that its derivative is given by (4).

As I has a local minimum at $\hat{t}$, $\dot{I}(\hat{t}) = 0$ and thus $\beta\varepsilon(\hat{t})S(\hat{t}) = \gamma$. It follows that $\varepsilon(t) = 1 - \frac{\beta\eta I(t)}{c(t)} > 0$ and $S(t) > 0$ in the neighborhood of $\hat{t}$. Therefore, the function $\ddot{I}$ exists and is obtained by differentiating $\dot{I}$ at $t = \hat{t}$:

$$\ddot{I}(t) = \dot{I}(t)(\beta\varepsilon(t)S(t) - \gamma) + \beta I(t)(\dot{\varepsilon}(t)S(t) + \varepsilon(t)\dot{S}(t)).$$

Evaluating $\ddot{I}(t)$ with $\dot{I}(t) = 0$ yields

$$\ddot{I}(t)\Big|_{\dot{I}(t)=0} = \beta I(t)(\dot{\varepsilon}(t)S(t) + \varepsilon(t)\dot{S}(t)). \tag{17}$$

Substituting

$$\dot{\varepsilon}(t) = \frac{-\beta\eta I(t)}{c(t)}\left(\frac{\dot{I}(t)}{I(t)} - \frac{\dot{c}(t)}{c(t)}\right) \text{ and } \dot{S}(t) = -\beta\varepsilon(t)I(t)S(t)$$

into (17) results in

$$\ddot{I}(t)\Big|_{\dot{I}(t)=0} = \beta^2\eta I^2(t)S(t)\left(\frac{\dot{c}(t)}{c^2(t)} - \frac{\varepsilon^2(t)}{\eta}\right). \tag{18}$$

For no interior minimum to exist it is sufficient to show that $\ddot{I}(t)|_{\dot{I}(t)=0} < 0$ for all $t > 0$, which occurs precisely when (9) holds.

Since $\varepsilon(t) = \frac{\gamma}{\beta S(t)}$ when $\dot{I}(t) = 0$, (9) can be rewritten as

$$\frac{\dot{c}(t)}{c^2(t)} < \frac{\gamma^2}{\eta\beta^2 S^2(t)} \text{ for all } t > 0.$$

Due to S being non-increasing over time, a sufficient condition for the above condition is (10), as desired. □

Proof of Lemma 1. Suppose $\sigma(t) > \frac{1}{\eta}c(t)$ for all $t > 0$; that is,

$$\frac{1}{\eta} < \frac{\sigma(t)}{c(t)} = -\frac{d}{dt}\left(\frac{1}{c(t)}\right) \text{ for all } t > 0.$$

Since c is continuously differentiable, the right-hand side of the above expression is continuous. Integrating both sides from some $t_0 > 0$ to $t > t_0$, it follows from the fundamental theorem of calculus that

$$\frac{t - t_0}{\eta} + \frac{1}{c(t)} < \frac{1}{c(t_0)}.$$

Since $1/c(t_0)$ is finite, the inequality can obtain only if $c(t) < 0$ from some $\hat{t}$ on. □

Proof of Lemma 2. Suppose, to the contrary, that individuals choose $\varepsilon(t) = 0$ for some $t > t'$, and let $\underline{t} := \inf\{t \geq t' \mid \varepsilon(t) = 0\}$. Since I and c are continuous in equilibrium, so is ε. Thus, $\varepsilon(\underline{t}) = 0$ and consequently $\underline{t} > t'$. Towards the contradiction we will argue that in any small enough left neighborhood of $\underline{t}$, $\dot{\varepsilon}(t) > 0$.

At any t where $\varepsilon(t) > 0$, ε is differentiable with derivative

$$\dot{\varepsilon}(t) = (1 - \varepsilon(t)) \left(\frac{\dot{c}(t)}{c(t)} - \frac{\dot{I}(t)}{I(t)} \right). \tag{19}$$

In addition, $\varepsilon(t) > 0$ on $(\underline{t}, t')$ implies

$$\begin{aligned} c(t) - c_0 &= k \int_0^t e^{-r(t-\tau)}(1 - \varepsilon(\tau))d\tau \\ &< k \int_0^t e^{-r(t-\tau)}d\tau \\ &< \frac{k}{r}. \end{aligned}$$

Since I and c are continuous in equilibrium, for any $\delta_1 > 0$, there exists $\delta_2 > 0$ such that $\varepsilon(t) < \delta_1$ if $t \in (\underline{t} - \delta_2, \underline{t}]$. But then, given that $c(t) - c_0 < \frac{k}{r}$, δ_1 can be chosen small enough so that $r(c(t) - c_0) < k(1 - \varepsilon(t))$. In other words, for δ_2 small enough, $\dot{c}(t) > 0$ for $t \in (\underline{t} - \delta_2, \underline{t})$. Moreover, equation (4) implies that $\dot{I} < 0$ whenever $\varepsilon < \frac{\gamma}{\beta}$. Therefore, δ_1 can be chosen so that $\dot{c}(t) > 0$ and $\dot{I}(t) < 0$. Consequently, due to (19), $\dot{\varepsilon}(t) > 0$ on $(\underline{t} - \delta_1, \underline{t})$. But this means that, whenever $\varepsilon(t)$ becomes very small, it starts increasing and thus cannot reach 0. □

Proof of Lemma 3. As $\dot{I}(0) > 0$, it must be the case that $\varepsilon(0) > 0$. By Lemma 2, $\varepsilon(t) > 0$ for all $t \geq 0$. By implication ε and therefore $\dot{c}$ are differentiable for all $t > 0$.

Suppose c attains a critical point at some t; thus, $\dot{c}(t) = 0$. Differentiating (12) and evaluating it at $\dot{c}(t) = 0$ yields

$$\ddot{c}(t)|_{\dot{c}(t)=0} = -k\dot{\varepsilon}(t) = k\frac{\beta\eta\dot{I}(t)}{c(t)}.$$

Thus, if c attains a local maximum (minimum) at t, it is necessary that $\dot{I}(t) \leq 0$ (≥ 0). □

Proof of Proposition 3. $\dot{I}(0) > 0$ and $I_\infty = 0$ imply that I peaks at least once for some $t > 0$. In addition, $\dot{I}(0) > 0$ implies $\varepsilon(0) > 0$ and thus by Lemma 2, $\varepsilon(t) > 0$ for all $t > 0$. As a consequence, $\dot{I}(\cdot)$ is differentiable.

Let t_1 be some t at which a local maximum of I is attained. Thus, $\dot{I}(t_1) = 0$ and $\ddot{I}(t_1) \leq 0$. Then by (18) it must be the case that

$$\frac{\dot{c}(t_1)}{c^2(t_1)} \leq \frac{\varepsilon^2(t_1)}{\eta}.$$

Let t_2 be the smallest $t > t_1$ such that $\dot{I}(t) = 0$ and $\ddot{I}(t) \geq 0$. If it exists, t_2 is the first local minimum after t_1. If there is no local minimum after the first local maximum, our result is proven.

We consider two cases. First, suppose $c(t_2) \geq c(t_1)$. Then:

$$\begin{aligned}\dot{c}(t_2) &= k(1 - \varepsilon(t_2)) - r(c(t_2) - c(0)) \\ &< k(1 - \varepsilon(t_1)) - r(c(t_1) - c(0)) \\ &= \dot{c}(t_1),\end{aligned}$$

where the inequality follows from the fact that at any t such that $\dot{I}(t) = 0$, $\varepsilon(t) = \gamma/(\beta S(t))$ and that $S(t)$ is decreasing. As a consequence,

$$\frac{\dot{c}(t_2)}{c^2(t_2)} < \frac{\dot{c}(t_1)}{c^2(t_1)} \leq \frac{\varepsilon^2(t_1)}{\eta} < \frac{\varepsilon^2(t_2)}{\eta},$$

which due to equality (18) contradicts the assumption that $\dot{I}(t_2) = 0$ and $\ddot{I}(t_2) \geq 0$.

Second, suppose $c(t_2) < c(t_1)$. By the definition of t_2, $I(t_1) > I(t_2)$ and I is decreasing on $[t_1, t_2]$. Since c is continuous on $[t_1, t_2]$, it attains a maximum and minimum on the interval by the extreme value theorem. Lemma 3 implies that if c attains an interior extremum, then it has to be a local maximum. Alternatively, c is decreasing on the whole interval. In either case $\dot{c}(t_2) \leq 0$. But then the inequality $\dot{c}(t_2)/c^2(t_2) < \varepsilon^2(t_2)/\eta$ is automatically satisfied and thus $\ddot{I}(t_2) < 0$, which contradicts the supposition.

Thus, there does not exist a time $t_2 \in (t_1, \infty)$ such that $\dot{I}(t_2) = 0$ and $\ddot{I}(t_2) \geq 0$. □

Proof of Proposition 4. The second assertion follows from Lemma 3. The proof of the first assertion contains two steps. The first step shows that c has a local maximum. The second step shows that c is single-peaked.

First, since $\dot{c}(0) = k(1 - \varepsilon(0)) > 0$, there exists t_1 such that $c(t_1) > c_0$. Otherwise, $\dot{c}(0) \leq 0$, a contradiction. Since $r > 0$ and $\lim_{t\to\infty} \varepsilon(t) = 1$, it follows that $\lim_{t\to\infty} c(t) = c_0$. Thus, there exists $t_2 \geq t_1$ such that $c(t) \leq c(t_1)$ for all $t \geq t_2$. Now, c admits a local maximum on $[0, t_2]$ by the extreme value theorem. By construction, the local maximum of c on $[0, t_2]$ is a local maximum on $[0, \infty)$.

Second, suppose to the contrary that there would exist t_1 and t_2 at which c attains local maxima and a $t' \in (t_1, t_2)$ such that $c(t') < \min(c(t_1), c(t_2))$. Since c is continuous, it has a minimum on the interval $[t_1, t_2]$ by the extreme value theorem. Let $\tilde{t} \in (t_1, t_2)$

be some t at which c is minimized over $[t_1, t_2]$. Then $\dot{c}(t_1) = \dot{c}(\tilde{t}) = \dot{c}(t_2) = 0$. Using equation (12), we obtain

$$1 - \varepsilon(t) = \frac{r}{k}(c(t) - c_0) \text{ for } t \in \{t_1, \tilde{t}, t_2\}.$$

As $r > 0$, the inequality $c(\tilde{t}) < \min(c(t_1), c(t_2))$ implies that $1 - \varepsilon(\tilde{t}) < \min(1 - \varepsilon(t_1), 1 - \varepsilon(t_2))$. In turn, the last two inequalities together with (7) imply that $I(\tilde{t}) < \min(I(t_1), I(t_2))$, which would contradict that I is single-peaked as established in Proposition 3. □

Proof of Proposition 5. Let $c_1(\cdot)$ be a cost function and $\tilde{t}$ and $\bar{c}(\tilde{t})$ be as in the statement of the proposition, moreover, let cost function $c_2(\cdot)$ coincide with c_1 for $t < \tilde{t}$ and suppose that $c_2(\tilde{t}) = \bar{c}(\tilde{t})$. Let $(S_2, I_2, R_2, c_2, \varepsilon_2)$ be the equilibrium under c_2. By the definition of the equilibrium, S_2 and I_2 are continuous. Notice that S_2 and I_2 coincide with S_1 and I_1 for $t \in [0, \tilde{t}]$.

If c_2 has any discontinuities for $t > \tilde{t}$, let t' be smallest $t > \tilde{t}$ where c_2 is discontinuous; otherwise set $t' = \infty$. Since I_2 and S_2 are continuous, $\dot{I}_2$ and $\dot{S}_2$ exist and are continuous on $(\tilde{t}, t')$. Notice that:

$$\beta\varepsilon_2(\tilde{t})S_2(\tilde{t}) - \gamma = 0.$$

By continuity of ε_2 and S_2 for every $\delta_1 > 0$ there exists a $\delta_2 > 0$ such that $|\varepsilon_2(\tilde{t})S_2(\tilde{t}) - \varepsilon_2(t)S_2(t)| < \delta_1$ for all t such that $t - \tilde{t} < \delta_2$. Consequently

$$\begin{aligned}\dot{I}_2(t) &= I_2(t)(\beta\varepsilon_2(t)S_2(t) - \gamma)\\ &< I_2(t)(\beta\varepsilon_2(\tilde{t})S_2(\tilde{t}) + \delta_1 - \gamma)\\ &= \delta_1 I_2(t),\end{aligned}$$

for all $t \in (\tilde{t}, \tilde{t} + \delta_2)$. Therefore the right limit of $\dot{I}_2(t)$ at $\tilde{t}$ is 0. It is then easy to see that if $c_2(\tilde{t}) < \bar{c}(\tilde{t})$ the derivative of I_2 would be smaller than 0. □