



Università Commerciale
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Paper: “Equity premium prediction: the role of economic and statistical constraints”

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July 2016

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Advanced Financial Econometrics III

February 2020

Introduction

This paper shows that the **equity premium** is **predictable OOS** for short predictable horizon when we use a predictive **regression** that conditions on a large set of **economic fundamentals**, subject to:

1. **economic constraints** on the sign of coefficients and return forecasts
2. **statistical constraints** imposed by shrinkage estimation

- The key to establishing equity premium predictability is implementing a predictive framework based on 3 aspects:
 - i. using a single predictive regression that condition on a large number of predictors → **kitchen-sink regression**
 - ii. imposing **economic constraints** on the sign of coefficients and return forecasts
 - iii. using a **shrinkage estimator** designated to improve performance by reducing the effects of less informative predictors in OOS forecasting

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Introduction: the 3 aspects more in detail

- i. Specify a **kitchen-sink regression** → provides a direct way of **pooling information** by construction a “super” model which nests each of the models that condition on a single predictor
- ii. Imposing **economic constraints** on the sign of coefficients and return forecasts → impose **economic theory** on the predictive regressions and usually improve performance
- iii. Implement a **shrinkage estimation** of the kitchen-sink regression → kitchen-sink regression transforms from being the worst model when estimated with OLS to being the best model when estimated with a shrinkage estimator. Shrinkage estimation produces biased parameter estimates by shrinking all estimate toward 0, which is the value implied by the bmk historical mean model.

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Predicting equity premium: kitchen-sink regression

- **Equity premium** → continuously compounded return on the S&P 500 index including dividend *minus* the T-bill rate
- **Monthly Economic fundamentals** used for predicting the monthly equity premium for the period Jan. 1927 to Dec. 2014
 - use **11 monthly predictors** primarily based on stock characteristics and interest rates: dividend yield, earnings-price ratio, book-to-market ratio, net equity expansion, stock variance, T-Bill rate, term spread, long-term rate of return, default yield spread, default rate spread, inflation
- The model used for predicting the equity premium is based on the **kitchen sink regression**:

$$r_{t+1}^e = \alpha + \sum_{j=1}^N \beta_j x_{j,t} + \varepsilon_{t+1}, \text{ where}$$

$r_{t+1}^e = r_{t+1} - r_f$ is equity premium at $t+1$, r_{t+1} is total return on the S&P 500 at $t+1$, r_f is the T-bill rate; $x_{j,t}$ → is the $j \leq N$ predictor at time t ; ε_{t+1} → normal error term; α and $\beta = \{\beta_j\}$ are constant parameters to be estimated

KS regression allows to capture all available info in a single regression → way of pooling info into a single forecast

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- KS regression allows to capture all available info in a single regression \rightarrow way of pooling info into a single forecast

Statistical constraints due to shrinkage estimation

Authors **estimate** the **KS regression** with a **shrinkage estimator** because the OOS performance of the KS regression estimated with OLS is very poor relative to the historical mean bmk

- Implement the shrinkage estimation → **shrink the regression coefficients** towards 0 (i.e. the value implied by the historical mean bmk) in a way that directly **minimizes the OOS mean squared error**
 - In contrast to the OLS estimator that is unbiased, a shrinkage estimator is biased but may have lower variance and lower MSE than OLS

- Shrink the regression coefficients by estimating the KS regression with the *elastic-net* estimator, which solves:

$$\min_{\beta} \frac{1}{2} \sum_{t=1}^{T-1} (r_{t+1}^e - \alpha - \sum_{j=1}^N \beta_j x_{j,t})^2 \text{ s.t. } \sum_{j=1}^N |\beta_j| < s_1, \sum_{j=1}^N \beta_j^2 < s_2$$

s_1, s_2 positive constants, estimated in a way that minimizes MSE of forecasts

- Special case of the elastic-net estimator:
 - *ridge regression* → $s_1 = \infty$ (first constraint is unbounded)
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Economic constraints

Authors impose also 2 **constraints** motivated by **economic theory**

- **1st constraint** → **equity premium > 0 in every period**

How? Replace negative forecasts with zero

Campbell and Thompson (2008) argue that a reasonable investor would not have used a model to forecast a negative equity premium

- **2nd constraint** → constraint sign of the slope coefficients to be consistent with economic theory

How? Set value of 0 for a coefficient that does not have the theoretically motivated sign

- In this study: slope coefficient is positive for all predictors except net equity expansions, T-bill rate and inflation

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Other predictive regressions

- The **Bmk** → historical sample mean for the equity premium: case of $\beta_j = 0$
- **Individual predictors** → predictive regressions that condition on a single predictor (in this case, 11 regressions each conditioning on 1 predictor)
- **Combinations of predictors**
 - “**Model selection**” (MS) approach: estimate regressions with all possible combinations of predictors and at each point in time select the one forecast that has performed the best (lowest cumulative MSE up to that point) → choose one among 11^2 models
 - **Principal Component Analysis** (PCA): estimate a set of PCs that parsimoniously incorporate info from the 11 predictors
- Estimate **kitchen-sink** regression with **OLS**
- **Forecasts combination** → combine the forecasts of several predictive regressions that condition on one predictor. Implement 2 approaches:
 - **Mean combination**: compute equally-weighted average of all forecasts at each point in time
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Other predictive regressions

- **Grand combinations of pooling information and pooling forecasts**
 - Authors predictive framework provides a way of **pooling information** since it is based on a **kitchen-sink regression** that directly conditions on a large number of predictors
 - in contrast, **combined forecasts** are designed to **pool forecasts** rather than pool information

Pooling forecasts involves two stages:

1. estimate several predictive regressions each conditioning on one predictor
2. combine the individual forecasts into one forecast combination.

In theory, this two-stage process introduces an **efficiency loss** and ignores the correlations between the predictors.

Therefore, it is often argued that **pooling information** is **optimal relative to pooling forecasts**

Other predictive regressions

- **Grand combinations of pooling information and pooling forecasts**
 - in addition to just pooling information or just pooling forecasts, authors also form a “**grand**” **combination** of the two approaches.
 - This provides a framework for assessing whether pooling forecasts adds to the predictive ability of pooling information and vice versa.
 - The grand combination is an interesting addition to the model set because, the combined forecasts are the closest competitor to our predictive framework.
 - **Authors form equally-weighted grand combinations of forecasts** by combining:
 - (i) the e-net KS forecasts with the mean combined forecasts;
 - (ii) the e-net KS forecasts with the MSE combined forecasts;
 - (iii) the lasso KS forecasts with the mean combined forecasts;
 - (iv) the lasso KS forecasts with the MSE combined forecasts.

Out-of-sample analysis

All empirical models are **evaluated OOS** relative to the historical mean bmk. Authors generate OOS forecast with **rolling predictive regressions** using a 20y estimation window (1st forecast is for Jan 1947, last for Dec. 2014)

- The main statistical criterion for evaluating the OOS predictive ability of the model is **OOS R^2 statistic**

$$R_{OOS}^2 = 1 - \frac{MSE(\hat{r}_{t+1|t}^e)}{MSE(\bar{r}_{t+1|t}^e)} = 1 - \frac{\sum_{t=1}^{T-1} (r_{t+1}^e - \hat{r}_{t+1|t}^e)^2}{\sum_{t=1}^{T-1} (r_{t+1}^e - \bar{r}_{t+1|t}^e)^2}$$

- It compares:
 - the unconditional 1-month ahead forecast $\bar{r}_{t+1|t}^e$ of the historical mean bmk
 - to the conditional forecast $\hat{r}_{t+1|t}^e$ of the alternative model
- **$R_{OOS}^2 > 0$** \rightarrow alternative model outperforms the bmk by means of lower MSE

Empirical Results

Access the OOS performance of the empirical models by reporting the R_{OOS}^2

- Table displays the R_{OOS}^2 for predictive models of the monthly equity premium against the null of the historical mean. The R_{OOS}^2 is for models that impose a sign constraint on the slope coefficients and a positivity constraint on the forecasts.
- In addition to the full sample, are reported also results for the 2 subsamples of **expansions** and **recessions**

	Predictor	R_{OOS}^2 (%)		
		Full Sample	Expansion	Recession
3 KS regression using \neq shrinkage estimation	<i>E-net KS</i>	1.64***	0.89***	3.44**
	<i>Lasso KS</i>	1.77***	0.40**	5.05***
	<i>Ridge KS</i>	1.17***	0.90***	1.83**
Forecast combination	<i>Mean</i>	0.82***	0.86***	0.72
	<i>MSE</i>	0.87***	0.86***	0.91
4 grand combinations	<i>E-net KS + Mean</i>	1.69***	1.28***	2.65**
	<i>E-net KS + MSE</i>	1.66***	1.28***	2.56**
	<i>Lasso KS + Mean</i>	1.70***	1.01***	3.34***
	<i>Lasso KS + MSE</i>	1.67***	1.02***	3.24***
	<i>MS</i>	-1.73	-3.27	1.94**
	<i>PCA</i>	-0.06	0.92***	-2.39
	<i>OLS KS</i>	-55.84	-72.17	-16.90

	Predictor	R_{OOS}^2 (%)		
		Full Sample	Expansion	Recession
OLS predictive regressions that condition on 1 predictor at time	<i>dy</i>	0.72***	1.07***	-0.11
	<i>epr</i>	0.03**	1.13***	-2.58
	<i>bm</i>	0.05	0.10**	-0.08
	<i>ntis</i>	-0.26	-0.32	-0.11
	<i>svar</i>	-2.38	-2.62	-1.83
	<i>tbl</i>	0.35**	-0.43	2.20**
	<i>ltr</i>	0.39**	-0.57	2.67**
	<i>tms</i>	0.08***	-1.12	2.94***
	<i>dfy</i>	-1.57	-1.04	-2.84
	<i>dfr</i>	-0.76	-0.17	-2.16
	<i>infl</i>	-0.28	-0.61	0.51

** , and *** denote statistical significance at the 5%, and 1% level

Empirical Results

The **3 shrinkage estimators** of the **KS regression** deliver an R_{OOS}^2 that is **positive, significant** and **higher** than all other models. Indeed, the shrinkage models are the only models that have a positive and significant R_{OOS}^2 in both expansions and recessions

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- The **closest competitors** to the shrinkage models are the **combined forecasts**. However, the shrinkage models exhibit a **much higher R^2_{oos}** for the full sample, which becomes even higher in **recessions**

→ **Evidence** strongly **favors** authors' approach of **pooling info** to the standard approach of pooling forecasts
- The **OLS KS model** is by far the **worst performing model**, which indicates that the way authors estimate the KS regression is of critical importance

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- Turning to the **grand combinations** of pooling info and pooling forecast, authors find that they deliver similar performance to the plain shrinkage model
 → **grand combinations do not add to the predictive ability** of authors' approach

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- Among the **single predictors**, the best performing model is dividend yield (*dy*). However, the *dy* has an R_{oos}^2 that is less than half of the value of the e-net or lasso and it is only positive in expansions, while being negative in recession
- None of the 11 predictors have a significantly positive R_{oos}^2 in both expansions and recessions.

Empirical Results

To summarize, evidence based on R_{OOS}^2 indicates that the **best model** for **predicting the equity premium OOS** is a **kitchen-sink regression** that conditions on a large set of predictors and imposes *statistical constraints* through shrinkage estimation together with *economic constraints* on slope coefficients and forecasts.

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Empirical Results

■ Effect of *economic constraints* on performance

- The table demonstrates the effects of constraints on the R_{OOS}^2 of models
- *First* → isolate the effect of statistical constraints in the absence of economic constraints

Predictor	R_{OOS}^2 (%)			
	No Constraint	Slope Constraint	Forecast Constraint	Both Constraints
<i>E-net KS</i>	-0.46**	0.97***	1.22***	1.64***
<i>Lasso KS</i>	-0.63**	0.65***	1.15***	1.77***
<i>Ridge KS</i>	-0.15**	0.42***	1.29***	1.17***
<i>MS</i>	-3.60	-3.33	-1.18	-1.73
<i>PCA</i>	-0.32	-0.32	-0.06	-0.06
<i>OLS KS</i>	-11.86	-479.91	-5.28	-55.84

The unconstrained OLS KS delivers an R_{OOS}^2 of -11.86% (insignificant) but when imposing the lasso constraint the R_{OOS}^2 rises to -0.63% (significant)

- **Statistical constraint** due to shrinkage estimation deliver a massive improvement on predictability relative to OLS, but **alone** they **fail short** of producing a **positive R_{OOS}^2**

Empirical Results

- **Effect of *economic constraints* on performance**
 - It is the **combination** of **economic** and **statistical constraints** in the context of kitchen-sink regression that delivers the most powerful results

<i>Predictor</i>	R_{oos}^2 (%)			
	<i>No Constraint</i>	<i>Slope Constraint</i>	<i>Forecast Constraint</i>	<i>Both Constraints</i>
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Imposing the economic constraints raises the lasso R_{OOS}^2 from -0.63% to 1,77% which is now positive, significant at 1% and the highest of all models

- The **economic constraints** seem to be **very effective** when applied to many predictors in a single regression estimated with shrinkage.

Empirical Results

- Effect of *economic constraints* on performance
- It is the **combination** of **economic** and **statistical constraints** in the context of the kitchen-sink regression that delivers the **most powerful results**.

The table displays the out-of-sample R_{oos}^2 in percent for predictive models of the monthly equity premium against the null of the historical mean. The R_{oos}^2 is for four types of models: models that impose no constraints; models that impose only the sign constraint on the slope coefficients; models that impose only the positivity constraint on the forecasts; and models that impose both constraints. The out-of-sample monthly forecasts are obtained using a 20-year rolling window for the sample period of January 1927 to December 2014. **, and *** denote statistical significance at the 5%, and 1% level, respectively, using the Clark and West (2006, 2007) one-sided t -statistic.

Predictor	R_{oos}^2 (%)			
	No Constraint	Slope Constraint	Forecast Constraint	Both Constraints
<i>E-net KS</i>	△ -0.46**	0.97***	1.22***	△ 1.64***
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<i>Ridge KS</i>	-0.15**	0.42***	1.29***	1.17***
<i>Mean</i>	0.67**	0.79***	0.64***	0.82***
<i>MSE</i>	0.60**	0.75***	0.64***	0.87***
<i>E-net KS + Mean</i>	0.83**	1.57***	1.36***	1.69***
<i>E-net KS + MSE</i>	0.88**	1.59***	1.36***	1.66***
<i>Lasso KS + Mean</i>	0.62**	1.35***	1.21***	1.70***
<i>Lasso KS + MSE</i>	0.66**	1.37***	1.21***	1.67***
<i>MS</i>	-3.60	-3.33	-1.18	-1.73
<i>PCA</i>	-0.32	-0.32	-0.06	-0.06
<i>OLS KS</i>	△ -11.86	-479.91	-5.28	-55.84
<i>dy</i>	0.44**	0.91***	0.51**	0.72***
<i>epr</i>	-1.65	-2.76	-0.24	0.03**
<i>bm</i>	-1.42	-0.21	-0.49	0.05
<i>ntis</i>	-0.72	-0.74	-0.14	-0.26
<i>svar</i>	-3.16	-2.39	-2.67	-2.38
<i>tbl</i>	-1.75	-0.98	-0.47	0.35**
<i>ltr</i>	-0.20**	0.14**	0.05**	0.39**
<i>tms</i>	-0.62**	-0.46**	-0.09**	0.08***
<i>dfy</i>	-1.84	-2.07	-0.64	-1.57
<i>dfr</i>	-1.48	-1.10	-1.06	-0.76
<i>infl</i>	-0.30	-0.70	-0.10	-0.28

Predictability and asset allocation

Authors assess the economic value of equity premium predictability using a **dynamic asset allocation strategy**

- The strategy involves **monthly rebalancing** of a **ptf** invested in the S&P 500 index (the **risky asset**) and the T-bill (**riskless asset**)
- Authors consider a mean-variance investor with a 1-month ahead horizon, who determines optimal weights by implementing a maximum expected utility rule:

$$\max_{w_t} E_t[U(r_{p,t+1})] = r_{p,t+1|t} - \frac{\gamma}{2} \sigma_{p,t+1|t}^2$$

$$\text{s.t. } r_{p,t+1|t} = w_t r_{t+1|t} + (1 - w_t) r_f \text{ and } \sigma_{p,t+1|t}^2 = w_t^2 \sigma_{t+1|t}^2$$

$r_{p,t+1|t}$ is the $t+1$ forecast of ptf return conditional on time t information, γ is the investor's degree of relative risk aversion, $\sigma_{p,t+1|t}^2$ is the $t+1$ forecast of ptf variance made at time t , $r_{t+1|t}$ is the $t+1$ forecast of the S&P 500 index return made at time t , r_f is the risk-free rate of return and $\sigma_{t+1|t}^2$ is the $t+1$ forecast of the variance of the S&P 500 index return made at time t

- The solution gives the risky asset weight: $w_t = \frac{1}{\gamma} \frac{r_{t+1|t} - r_f}{\sigma_{t+1|t}^2}$, $w_t \in [0, 1.5]$ (ie. short selling not allowed and leveraging is limited to no more than 50%)

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Predictability and asset allocation

Authors **evaluate** the **performance** of the ptf generated by a given set of equity premium forecasts using the **Sharpe ratio** (SR) and the **certainty equivalent return** (CER)

- **SR** \rightarrow average excess return of a ptf divided by the standard deviation of ptf returns
- **CER** $\rightarrow CER = (\bar{r}_p - \frac{\gamma}{2} \bar{\sigma}_p^2)$ where \bar{r}_p = mean ptf return and $\bar{\sigma}_p^2$ = ptf variance over the forecast evaluation period

The CER can be interpreted as the performance fee the risk-averse investor is willing to pay for switching from the riskless asset to the risky ptf.

Focus on:

$$\begin{aligned} \Delta CER &= CER^{Ptf \text{ generated by forecasts of alternative model}} \\ &\quad - CER^{Ptf \text{ generated by the historical mean bmk}} \end{aligned}$$

ΔCER measures the performance fee the risk-averse investor is willing to pay for switching from the risky ptf generated by the bmk model to the risky ptf generated by the alternative model

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Portfolio performance

Assess the performance of dynamically rebalanced ptf generated by the monthly forecasts of the predictive models

- Table shows the **OOS ptf performance** for a **mean-variance investor**, who each month rebalances the ptf by investing in one risky asset (S&P 500) and the riskless asset (T-bill). The investor has a degree of **risk aversion** equal to 5 and follows a maximum utility strategy. The OOS monthly forecasts are obtained using a 20y rolling window for the sample period Jan. 1927 to Dec. 2014. For the historical mean bmk, the level of CER is reported
- $\Delta CER \rightarrow$ is the gain in percent annualized CER for switching from the forecasts of the bmk to the forecasts generated by the alternative model

	ΔCER (%)		
	Full Sample	Expansion	Recession
Historical mean	5.82	8.33	-6.81
<i>E-net KS</i>	2.52	0.90	10.56
<i>Lasso KS</i>	2.71	0.71	12.71
<i>Ridge KS</i>	1.71	0.80	6.16
Mean	0.41	0.50	-0.07
MSE	0.50	0.54	0.24
<i>E-net KS + Mean</i>	1.97	1.05	6.44
<i>E-net KS + MSE</i>	1.91	1.02	6.24
<i>Lasso KS + Mean</i>	1.94	0.86	7.28
<i>Lasso KS + MSE</i>	1.88	0.82	7.08
MS	1.46	-0.84	13.00
PCA	1.36	1.05	2.78
OLS KS	0.64	-0.12	4.37

	ΔCER (%)		
	Full Sample	Expansion	Recession
<i>dy</i>	0.53	1.13	-2.70
<i>epr</i>	1.20	1.01	2.15
<i>bm</i>	-1.05	-0.44	-4.22
<i>ntis</i>	-0.02	-0.29	1.32
<i>svar</i>	-0.94	-0.35	-3.90
<i>tbl</i>	1.32	-0.62	11.06
<i>ltr</i>	0.81	0.03	4.55
<i>tms</i>	1.09	0.08	6.06
<i>dfy</i>	-0.79	-0.49	-2.47
<i>dfr</i>	-0.35	0.10	-2.54
<i>infl</i>	0.11	-0.35	2.39

Portfolio performance

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- The **historical mean** bmk delivers a CER of **5,82%** per year relative to the riskless investing. The CER rises to 8.33% in expansions but falls to -6.81% in recessions.
 - Historical mean is a **poor predictor** of equity premium in **recessions**
- Any of the 3 **shrinkage KS regressions performs better** than any of the other models
 - Ex: lasso regression delivers a ΔCER relative to the historical mean bmk that of **2.71%** per year, which becomes **12.71%** in recessions

Portfolio performance

	SR	ΔCER (%)
	Full	Full
Historical mean	Sample	Sample
	0.45	5.82
<i>E-net KS</i> ⁴	0.64**	2.52
<i>Lasso KS</i>	0.66**	2.71
<i>Ridge KS</i>	0.58	1.71
<i>Mean</i>	0.48	0.41
<i>MSE</i>	0.49	0.50
<i>E-net KS + Mean</i>	0.59**	1.97
<i>E-net KS + MSE</i>	0.59**	1.91
<i>Lasso KS + Mean</i>	0.59**	1.94
<i>Lasso KS + MSE</i>	0.59**	1.88
<i>MS</i>	0.47	1.46
<i>PCA</i>	0.55	1.36
<i>OLS KS</i>	0.47	0.64
<i>dy</i>	0.48	0.53
<i>epr</i>	0.53	1.20
<i>bm</i>	0.38	-1.05
<i>ntis</i>	0.45	-0.02
<i>svar</i>	0.40	-0.94
<i>tbl</i>	0.54	1.32
<i>ltr</i>	0.52	0.81
<i>tms</i>	0.54	1.09
<i>dfy</i>	0.43	-0.79
<i>dfr</i>	0.43	-0.35
<i>infl</i>	0.45	0.11

- Also considering the annualized **Sharpe ratio**, **shrinkage strategies** substantially outperforms the other models
- Overall, this is evidence that the **equity premium** is **predictable OOS** and, in the context of a dynamic mean-variance strategy, there is **high economic value** in using a **KS regression** with both **statistical** and **economic constraints**
 - For example, the economic gains of the **lasso** approach can be summarized into a performance fee of 2.71% per year before transaction costs, together with an increase in the Sharpe ratio from 0.45 to 0.66.

Conclusion

- Authors investigate the predictability of the equity premium using a **kitchen-sink regression** that conditions on a **large set of economic fundamentals**. The regression is estimated with a **shrinkage methodology** designated to maximize predictive performance
- Authors implement this framework using a long sample of monthly data and arrive at the empirical findings that:
 - equity premium is predictable OOS: authors' predictive framework consistently outperforms both the historical mean bmk and competing models
- Equity premium forecasts based on authors' predictive framework consistently outperform the historical mean bmk, especially during recessions
- The superior performance of the shrinkage estimators in recessions is very important because the predictive information of economic fundamentals is more valuable to an investor during recessions. This is true because during recessions the equity premium is on average negative with high volatility, which makes the historical mean a poor forecast.

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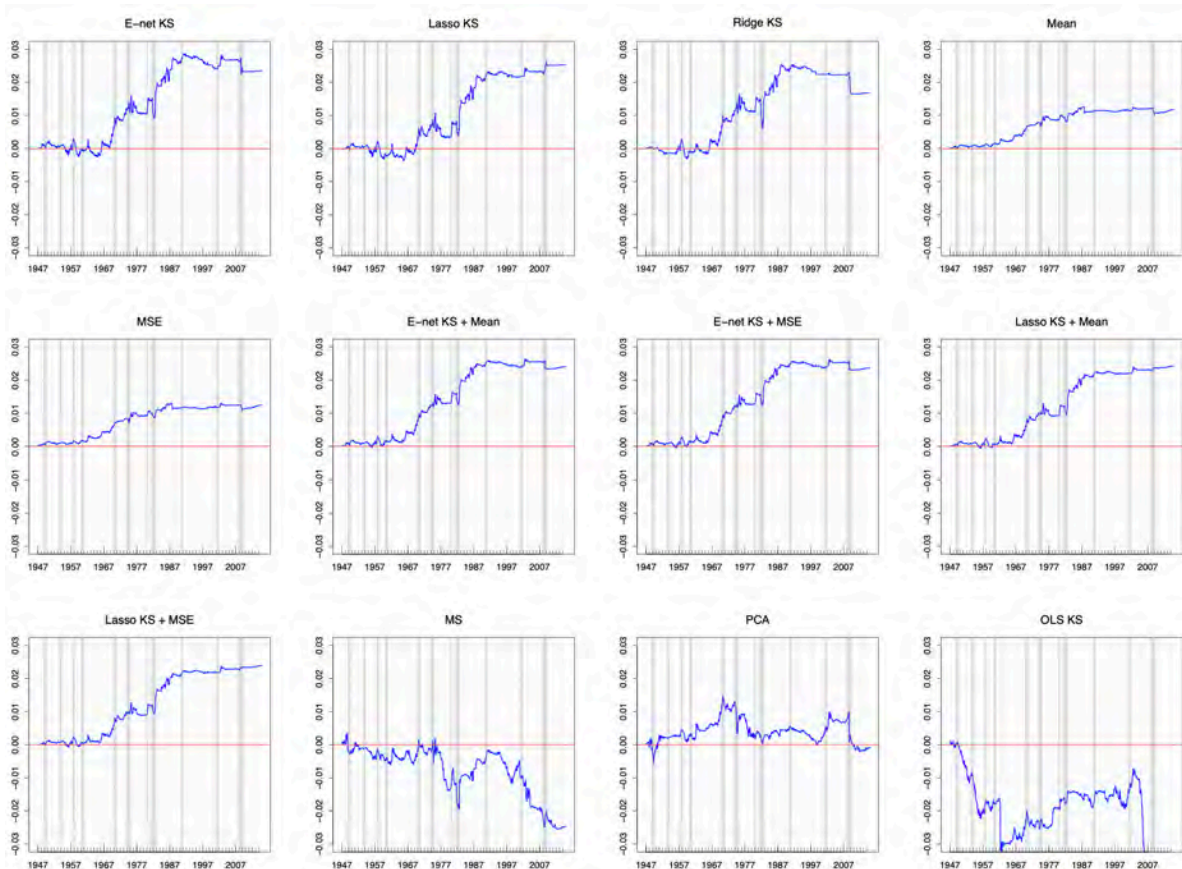
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- Equity premium forecasts based on **authors' predictive framework** consistently **outperform** the historical mean **bmk**, especially during recessions
- The **superior performance** of the shrinkage estimators in **recessions** is very **important** because the predictive information of economic fundamentals is more valuable to an investor during recessions. This is true because during recessions the equity premium is on average negative with high volatility, which makes the historical mean a poor forecast.

Assessing performance over

Figures plot the OOS performance of each model over time. They show that **shrinkage estimators** display the **most pronounced upward trend** in their OOS performance over time

- Figures plot the difference of the cumulative squared error of the null (historical mean) minus the cumulative squared error of alternative.



Assessing performance over

The good performance of shrinkage estimators is not due to a particular subsample but is systematic over a long sample spanning the full postwar period

