Monetary Policy and Bond Prices with Drifting Equilibrium Rates and Diagnostic Expectations *

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Abstract

We propose a framework that reconciles drifting Treasury bond prices with stationary and predictable bond returns. Bond prices are drifting because they reflect the drift in average expected monetary policy rates over the life of the bonds. In our framework, deviations of bond prices from their drift should be stationary when the drift is correctly modelled; they can originate from either term premia or temporary deviations from rational expectations in a behavioral framework. Empirically, modeling the drift in monetary policy rates using demographics and productivity trends, plus long-term inflation expectations, leads to stationary deviations of bond prices from their drift that predict future bond returns. Through our model, we detect a significant role of temporary deviations from the rational expectations in determining the cyclical properties of yields at all maturities.

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1 Introduction

Bond prices are drifting: they are non-stationary.[1] In turn, when the stochastic trend has been removed from yields, cyclical (i.e., stationary) components naturally emerge. What are the drivers of the stochastic trend in yields? And to what extent term premium can be identified with the movements in the cyclical components of yields? Indeed, the cycles in yields could be related to term premium within a no-arbitrage framework (Bauer and Rudebusch, 2020), and/or to expectation errors (about the short-term rate) in a behavioral model where the hypothesis of Full Information Rational Expectations (FIRE) does not hold (Piazzesi et al., 2015; Cieslak, 2018). In this paper, we propose a novel and simple modeling approach which is explicit about the drivers of the trend in yields, and that permits to test the importance of deviations from rational expectations (in the form of diagnostic expectations) for the cyclical components of yields.

The fact that bond prices are drifting has important implications for modeling monetary policy, the term structure of interest rates and holding period excess bond returns.[2] However, these implications have been so far overlooked since both standard factor models for the term structure and (empirical models built on) monetary policy rules are designed for stationary variables. Only recently, the non-stationarity of bond yields has been acknowledged (Kozicki

[1]Bond prices have been drifting in the last forty years because their secular drivers have been drifting. As we shall see later, we find that demographics, productivity, and long-term inflation expectations jointly capture the stochastic trend in yields.

[2]The relevance of investigating the drift in the term structure of yields is not restricted to Treasury bonds. For example, Farhi and Gourio (2018) propose a macro-finance neoclassical growth model to account for drifting real rates and stable return to private capital. van Binsbergen (2020) finds that accounting for secular trends in interest rates is fundamental for assessing long duration dividend risk. Campbell and Sigalov (2020) derive a model of reaching for yield and show that agents take more risk when the real interest rate declines while the risk premium remains constant. Also, see a general discussion on the importance of drifting prices for long-term investing at https://www.nber.org/lecture/long-term-investing-nonstationary-world.

and Tinsley, 2001) and modeled (Bauer and Rudebusch, 2020).[3] We show that a simple monetary policy rule with an equilibrium rate driven by productivity, demographics factor, and long-term inflation expectations goes a long way in capturing the stochastic trend in yields. Despite not imposing no-arbitrage restrictions, our cyclical components are highly correlated with the term premia estimates provided by Bauer and Rudebusch (2020). One interpretation of this finding is that no-arbitrage restrictions are empirically of second order importance. More aggressively, we find that deviations from rational expectations can be an important driver for the fluctuations in the cyclical component of yields. When we test for the role of Diagnostic Expectations (overreaction of agents to deviations of the monetary policy rate from its trend), we find that on average 17% of the fluctuations in yield cycles can indeed be attributed to this mechanism for bonds with maturity from 2 to 10 years. However, the importance of diagnostic expectations declines with the maturity of the bond, leaving a potential important role to term premia. At a minimum, however, we strongly reject any evidence of non-stationary term premia.

More specifically, we start by showing that the drift in monetary policy rates can be successfully modeled by fluctuations in productivity, demographics and long-term inflation expectations. Indeed, our monetary policy rule tracks well the evolution of the short-term rate both in- and out-of-sample. Importantly, by being explicit about the non-stationary drivers of rates, our model is purposely transparent and simple (i.e., not involving any filtering). Furthermore, through the lens of our modeling approach, monetary inertia could be heavily overestimated if the drivers of the drifting equilibrium policy rates are not included in the monetary reaction function.

[3] An important literature (most notably, Cieslak and Povala (2015) and Jørgensen (2018)) has documented the importance of slow-moving component in yields for bond return predictability; however, these papers work within a stationary environment, and do not address how to model non-stationary yields.

Then, we derive the implications of our monetary policy rule specification for the entire term structure of Treasury bond yields. Our approach decomposes bond yield at each maturity into a drifting component, the average expected sequence of monetary policy rates over the life of the bond, and a residual cyclical component (namely, the deviation of yields from their drift). We show that our framework with drifting bond prices implies a battery of mis-specification tests such as parametric restrictions on yields and their drift that are analogous to the restriction between prices and dividends in the Campbell and Shiller (1988) present-value model. E.g., when the (non-stationary) drivers of the monetary policy rates have been correctly specified, deviations of bond prices from their estimated drift should be stationary with a co-integrating vector of $(1, \ -1)$, and generate the cyclical components of yields. In the data, our proposed model passes all these (mis-specification) tests. Specifically, we show that deviations of bond prices from their drift are indeed cyclical.

Having analyzed the statistical properties of our model, and having confirmed it is is well-behaved, we turn to the economic interpretation of the cyclical components. In particular, the presence of Diagnostic Expectations on the monetary policy rate is a statistically significant driver of its fluctuations. Interestingly, the cycle components also comove strongly with state-of-the-art term premium estimates like the one proposed by Bauer and Rudebusch (2020). This is interesting since our framework does not impose no-arbitrage. Thus, our evidence suggests that a sizable fraction of what is deemed risk premium may instead reflect temporary deviations from rational expectations.

Finally, we show that a framework with drifting bond prices implies the presence of bond predictability. Specifically, we formally show that (stationary) deviations of bond prices from their drift should predict excess bond returns. Empirically, our model generates large R^2 of about 30% (10%) when it is used to predict the one-year (one-quarter) ahead excess returns on bond with maturities ranging from 2 to 10 years.

To sum up, our paper proposes a general framework that reconciles, in a parsimonious way, drifting bond prices with stationary and predictable holding period returns; through our model we detect a significant role of temporary deviations from the FIRE hypothesis in determining the cyclical properties of yields at all maturities.

Related Literature. Our evidence that bond prices are drifting is in line with several papers documenting a slow-moving component common to the entire term structure (see, for example, Balduzzi et al., 1998 and Fama, 2006).

Stationarity of returns and non-stationarity of prices is common to many asset classes. In the equity space, standard factor models focus on returns and leave prices undetermined. In a related paper focusing on stock prices, Favero et al. (2020) show that modeling the drift in stock prices leads to an equilibrium correction term in a model relating returns to factors; however, this term is invariably omitted in standard factor model of stock prices. Interestingly, in the fixed income space, standard factor models concentrate on bond prices rather than on holding period returns but ignore their drifts. The evidence in this paper shows that a stationary (factors) framework cannot be adopted for yields-to-maturity. In this regard, our analysis supports the literature that models Treasury yields using shifting endpoints (Kozicki and Tinsley, 2001), vector autoregressive models (VAR) with common trends (Negro et al., 2017), and slow-moving averages of inflation (Cieslak and Povala, 2015) and consumption (Jørgensen, 2018).

Standard ATSMs for bond yields assume stationarity, thus ruling out (by design) the drift in bond prices. Hence, our evidence is in line with Bauer and Rudebusch (2020) who propose a term structure model for interest rates with four state variables, one of which being an (unobserved) stochastic trend common across Treasury yields. Importantly, none of the above cited papers explores the implications of drifting equilibrium rates for monetary policy, Treasury yields, and bond returns predictability within a cohesive framework.[4]

Finally, our paper fits into the literature that studies the role played by (shifts in) the monetary conduct in determining the dynamics of bond yields. Berardi et al. (2020) show that the stance of monetary policy—as proxied by the difference between the natural rate of interest and the current level of short term rate—contains valuable information for bond predictability. Ang et al. (2011) show that the evolution of the Fed's response to inflation affect long-term yields. Similarly to Ang et al. (2011), we propose to model monetary policy and the term structure of interest rates jointly. However, our modeling of the policy rule with a drifting equilibrium rate is different from their model with time-varying policy coefficients. In turn, our approach has implications for interest rates comovement and bond returns predictability induced by deviations of bond prices from their drift. These testable implications are unique to our framework and not shared by Ang et al. (2011).

2 Modeling Monetary Policy

Monetary policy rates are drifting. This fact is overlooked in standard specification of the monetary policy rules.

Monetary policy rules specify the dynamics of the short-term rate, $y_t^{(1)}$. The following specification is general and encompasses most of the rules that have been proposed in the

[4]Also, in our framework stationarity of bond returns naturally co-exists with non-stationary bond prices. Bond returns are predicted by the stationary deviations of bond prices from their drift. Interestingly Bauer and Rudebusch (2020) note that, even when no-arbitrage is imposed, the loading of returns on the unobserved common stochastic trend is an order of magnitude smaller than the loading of prices. They also report that predictive regressions of yields on de-trended yields and trend proxies lead to coefficients on the trend that are not significantly different from zero.

literature:

$$y_t^{(1)} = y_t^* + \beta' X_t + u_t^{(1)} \quad (1)$$
$$u_t^{(1)} = \rho u_{t-1}^{(1)} + \varepsilon_t^{(1)},$$

where y_t^* is the equilibrium monetary policy rate,[5] X_t is a vector of stationary monetary policy drivers, and ρ is a parameter usually interpreted as monetary policy persistence. Arguably, the most famous special case of this specification is the Taylor (1993) rule. In this case, the vector X_t is composed of the output gap and the percentage deviation of inflation from its target. Furthermore, the Taylor (1993) rule assumes a constant equilibrium policy rate and, thus, provides a natural benchmark for our analysis.

With a constant equilibrium rate, the typical estimate of ρ is often close to one. This is to be expected since, if monetary policy rates are drifting, any attempt to model them only by means of stationary factors such as the output and inflation gaps naturally leads to a (close to) unit root process for u_t. What it is commonly interpreted as a monetary policy smoothing parameter can very well measure the mis-specification generated by modeling a drifting variable as mean reverting around a constant. Furthermore, if (1) ρ is high but smaller than one, and (2) only stationary factors are employed in the policy rule, then long-term forecast of monetary policy rates with a Taylor rule will inevitably (slowly) converge to the sample mean over the estimation period.[6]

Interest rates are sometimes modeled in first-difference which removes the stochastic trend

[5]The "natural" level of real interest rates is often referred to as the "natural", "equilibrium" or "neutral" real rate of interest. Interestingly, the possibility of a non-stationary equilibrium rate is rarely entertained in the traditional literature. See Giammarioli and Valla (2004) and Kiley (2015) for a review of the various concepts and estimation methods adopted in the literature.

[6]Rudebusch (2002) highlights the contradiction between apparent high-persistence and low-predictability of policy rates.

in policy rate at the cost of leaving the equilibrium level of the policy rate undetermined (e.g., Orphanides, 2003). The model in first-difference is a special case of our general specification when $\rho = 1$. Specifying the monetary policy rule in first-difference comes with benefits and costs.[7] The benefit of making the rule independent from the challenging estimation of the level of the equilibrium rate has to be traded-off against the cost of accepting that any monetary policy shock (i.e., any deviation from the rule) has a permanent effect on policy rates. Indeterminacy is a major concern for long-term forecasting, because as the unconditional distribution of policy rates is not defined, the long-run policy rate is also left undetermined.

We propose a "cointegrating" approach to drifting policy rates, where the stationarity of residuals of the monetary policy reaction function is taken as an indication of a valid specification for y_t^*. Equivalently, a valid specification for the equilibrium rate requires that y_t^* is the stochastic trend that drives drifting policy rates.[8]

In particular, we propose to model drifting policy rates as follows:[9]

$$\begin{aligned} y_t^{(1)} &= y_t^* + \beta_1 E_t(\pi_{t+1} - \pi_{t+1}^*) + \beta_2 E_t(x_{t+1}) + u_t^{(1)} \\ y_t^* &= \gamma_1 MY_t + \gamma_2 \Delta x_t^{pot} + \gamma_3 \pi_t^* \\ u_t^{(1)} &= \rho_1 u_{t-1}^{(1)} + \varepsilon_t^{(1)} \end{aligned} \tag{2}$$

where $y_t^{(1)}$ is the one-period (three-month) yield, y_t^* is the equilibrium nominal rate, π_t is the percentage annual log change in Personal Consumption Expenditures (PCE), π_t^* is the Fed

[7] Cochrane (2007) provides a thorough discussion on the effects of specifying a model in level vs. first-difference to compute long-term yield-curve decomposition.

[8] Our approach is in line with, e.g., Woodford (2001) who observed that the optimal policy response to real disturbances requires including a time-varying real rate in monetary policy rules.

[9] We consider a forward-looking version of the policy rule as, for example, in Clarida et al. (2000).

perceived target rate (PTR), and x_t is the output gap (log percentage difference between real GDP and potential GDP). The drivers of the equilibrium real rate are the age structure of population and potential output growth.[10] We obtain the nominal equilibrium rate by adding the central bank inflation target π_t^*. Appendix A provides details on the data source.

Following Geanakoplos et al. (2004) and Favero et al. (2016), the age structure of the population is described by the ratio of middle-aged (40-49) to young (20-29) population in the U.S. (labelled as MY). Potential output growth is the percentage annual log change in potential output.

MY_t, Δx_t^{pot}, and π_t^* are non-stationary (i.e., their mean changes over time) and they represent the drivers of the drifting equilibrium rate in our cointegrated specification.)[11,12]

Finally, in all our tests, we always compare the results from our baseline (drifting) model to the results of a restricted model that, inspired by the large body of literature on the classical Taylor (1993) rule, does not model the drift in monetary policy:[13]

$$\begin{aligned} y_t^{(1)} &= y^* + \beta_1 E_t(\pi_{t+1} - \pi_{t+1}^*) + \beta_2 E_t(x_{t+1}) + u_t^{(1)} \\ u_t^{(1)} &= \rho_1 u_{t-1}^{(1)} + \varepsilon_t^{(1)}. \end{aligned} \tag{3}$$

[10]See Lunsford and West (2019) for a comprehensive work on the drivers of the U.S. real equilibrium rate.

[11]Our specification is compatible with yields being non-stationary or yields appearing non-stationary from the perspective of a model that does not include regime-shifts. What matters for the validity of our specification is that the deviations of actual rates from equilibrium rates are stationary.

[12]We test MY_t, Δx_t^{pot} and π_t^* for the presence of unit roots. In our sample, the p-values from the Phillips and Perron (1988) unit root test for the three variables are respectively 0.95, 0.13, and 0.69; thus, we cannot reject the null of the series being integrated of order 1.

[13]We deviate in two respects from a standard empirical Taylor rule. First, the model in (3) is forward-looking. Second, we specify the inflation gap as deviations of inflation from a time-varying inflation target (π_t^*) rather than from a constant inflation target (e.g., 2%). These two modifications ease the comparison with the model featuring drifting equilibrium rates. Considering a standard empirical Taylor rule would not affect our conclusions.

2.1 Empirical Results

Panel A of Figure 1 displays the realized nominal short-term rate, the fitted rates from our cointegrated monetary rule (c.f. Equation (2)), and the fitted monetary policy rates from a version of our model which restricts the equilibrium rate to be constant (c.f. Equation (3)). Panel B plots the monetary policy residuals implied by our proposed monetary policy rule and its restricted version. Table 1 reports the estimation results for these two rules.[14]

Figure 1–Panel A shows that our monetary rule with a drifting equilibrium rate tracks well the short-term rate movements throughout the sample. Indeed, the R^2 for the cointegrated specification is about 95% whereas that of a model with constant equilibrium rate is just 11% (c.f. Table 1).[15,16] Figure 1–Panel B shows that the residuals implied by our drifting monetary policy rule are mean reverting. On the other hand, the residuals from a rule with constant equilibrium rates display a close-to-unit root behavior. This is confirmed in Table 1: the residuals from the rule with drifting (constant) equilibrium rates have an autoregressive coefficient equal to 0.67 (0.95).

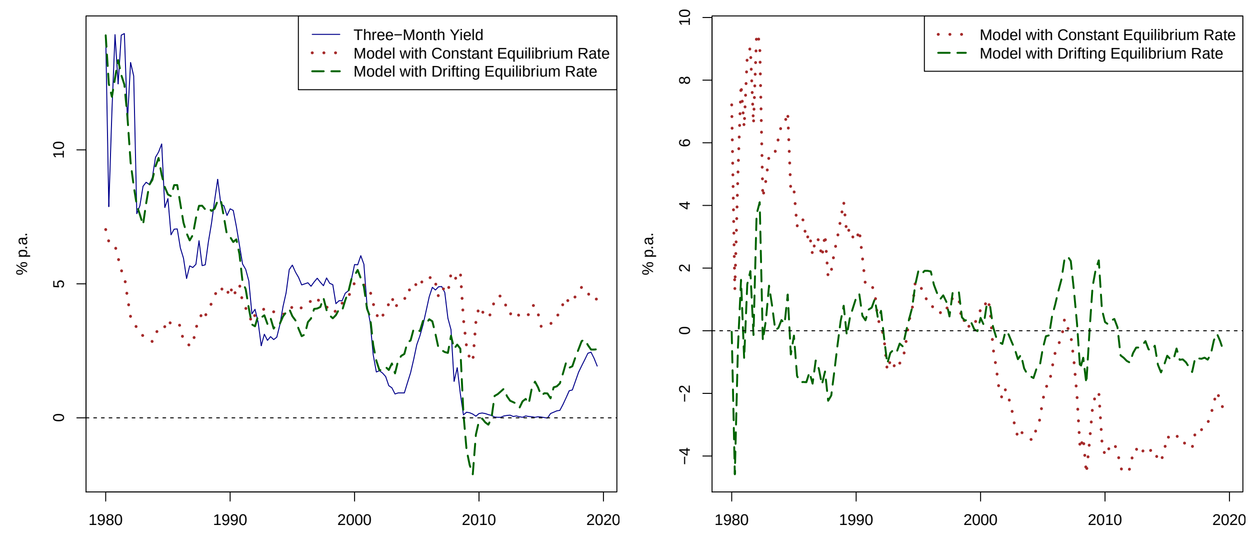
[14]Our estimate of the loading on π_t^* is in line with parameter values reported in Bauer and Rudebusch (2020, Table 1) despite the difference in the maturity of the bond analyzed (their Table 1 analyzes the 10-year bond, whereas we focus on the 3-month Treasury bill).

[15]Furthermore, a regression of the three-month yield on the fitted values implied by the two monetary rules (dotted and dashed lines in Figure 1–Panel A) delivers an estimate of zero on the rule with constant equilibrium rates (3), and a statistically significant estimate not different from one on the drifting rule (2).

[16]Positing the following cointegration framework where the equilibrium real rate r_t^* is estimated first, i.e.,

$$\begin{aligned} y_t^{(1)} &= \alpha_1 r_t^* + \alpha_2 \pi_t^* + \beta_1 E_t(\pi_{t+1} - \pi_{t+1}^*) + \beta_2 E_t(x_{t+1}) + u_t^{(1)} \\ r_t^* &= \gamma_1 MY_t + \gamma_2 \Delta x_t^{pot} \\ u_t^{(1)} &= \rho u_{t-1}^{(1)} + \epsilon_t^{(1)} \end{aligned}$$

leaves our conclusions unaltered. See Appendix Figure B.1.



(a) Rates and fitted values from policy rules.

(b) Residual $u_t^{(1)}$ from policy rules.

Figure 1: Actual vs Fitted Short-Term Rate. Panel (a) shows actual three-month yield and fitted values for our (cointegrated) model with drifting equilibrium rates (c.f. equation (2); see green dashed line) as well as for a model that restricts the equilibrium rate to be constant (c.f. equation (3); see brown dotted line). Panel (b) shows the differences between actual three-months yield and the fitted values. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

Figure 2 displays the forecasts implied by the two monetary rules. The rule with constant equilibrium rates generates forecasts that converge fast to the unconditional mean. On the other hand, the drifting monetary policy rule tracks well the future evolution of the short rate for each of the three out-of-sample periods considered in the figure. Appendix Figure B.2 confirms that allowing for inertia in the restricted rule would not alter our conclusion.

Finally, we observe that an accurate modeling of the trend alleviates concerns related to the zero lower bound: the fitted short rate in Figure 1(a) falls below zero only for a very short period of time, and the forecasts in Figure 2 never hit the bound.

Table 1: Short-term rate models with and without drifting equilibrium rate

This table reports the estimates for our (cointegrated) model with drifting equilibrium rates (c.f. equation (2); see column (2)) as well as estimates for a model that restricts the equilibrium rate to be constant (c.f. equation (3); see column (1)). We estimate the two rules by instrumental variables, where the instruments are lags of inflation gap and output gap. The last row reports OLS estimates for the monetary policy residuals' persistence. Values in parenthesis are GMM standard errors that correct for autocorrelation in the residuals. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

	Three-Month Yield	
	(1)	(2)
MY		-2.652^{***}
		(0.726)
Δx_t^{pot}		0.932^{***}
		(0.317)
π_t^*		1.656^{***}
		(0.177)
$E_t(\pi_{t+1} - \pi_{t+1}^*)$	0.721	0.709^{***}
	(0.519)	(0.244)
$E_t(x_{t+1})$	0.086	0.389^{***}
	(0.481)	(0.137)
Constant	4.656^{***}	
	(1.036)	
Observations	160	160
Adjusted R^2	0.036	0.950
ρ	0.949^{***}	0.673^{***}
	(0.022)	(0.110)

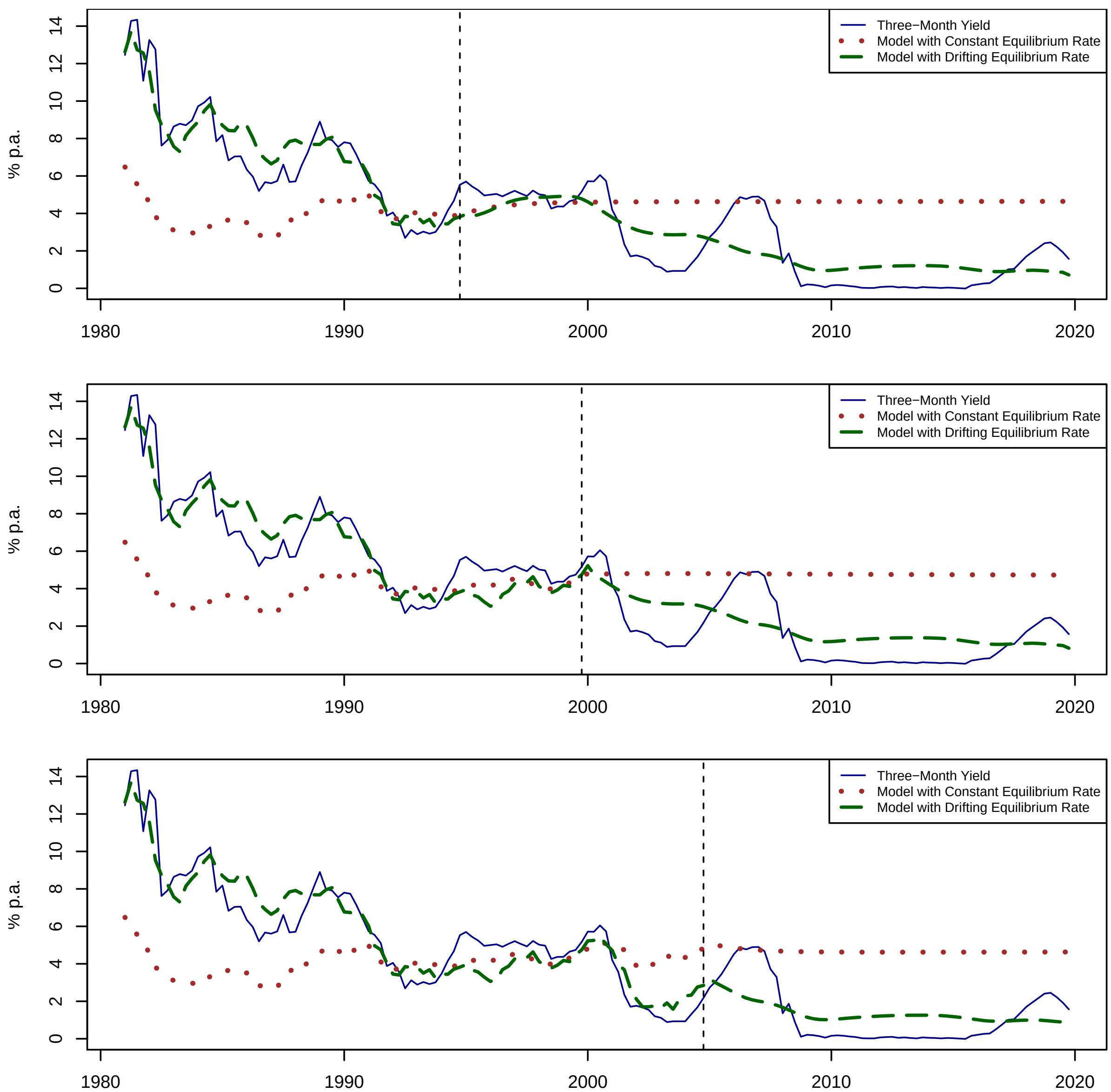


Figure 2: Short-Term Rate Forecasts. This figure shows actual three-month yield and predicted rates implied by our (cointegrated) model with drifting equilibrium rates (c.f. equation (2); green dashed line) and by a model that restricts the equilibrium rate to be constant (c.f. Equation (3); brown dotted line). The forecast of the drifting rule exploits the exogeneity of the demographic variable (MY) and of potential output (Δx^{pot}). In particular, the rule is estimated until 1995, 2000, and 2005 in the top, mid, and bottom panels, respectively. We then use the coefficients estimates, the projections of MY and Δx^{pot} (see also Appendix A), and the forecast of inflation and output gap from a VAR(1) as in equations (9) and (10). π^* is modeled as a random walk. Dotted vertical lines represent the end of in-sample estimation period. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

3 Modeling a Drifting Term Structure

The entire term structure is drifting.[17] Models that parsimoniously describe the term structure by projecting rates on a set of factors and by modeling the dynamics of the factors with a VAR will be inevitably confronted with the problem generated by the presence of unit roots in the VAR. Highly persistent VAR generate imprecise forecasts at long-horizons (e.g., Giannone et al., 2019). This feature can explain mixed results from the forecasting performance of affine term structure models (see, for example, Duffee, 2002; Sarno et al., 2016). Remarkably, this problem has not been fully acknowledged until very recently (see Bauer and Rudebusch (2020), Cieslak and Povala (2015), Favero et al. (2016)). We use the drift in monetary policy rates to model the drift in the entire term structure:

$$\begin{aligned} y_t^{(n)} &= y_t^{(n),*} + \delta_0 + u_t^{(n)} \\ u_t^{(n)} &= \rho_n u_{t-1}^{(n)} + \epsilon_t^{(n)} \\ y_t^{(n),*} &= \left(\frac{1}{n}\right) \sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)}] \end{aligned} \tag{4}$$

Yields at all maturities are decomposed into a trend, $y_t^{(n),*}$, and a cyclical component, $\delta_0 + u_t^{(n)}$. The trend is the average of expected monetary policy rates over the duration of the bond, while the cyclical component is the stationary residuals from the $(1, -1)$ cointegrating relationship between yields and their drift. We consider as valid any model of the term structure that delivers cointegration between $y_t^{(n)}$ and $y_t^{(n),*}$ with a $(1, -1)$ cointegrating vector and, therefore, a stationary $u_t^{(n)}$, i.e., $\mid \rho_n \mid < 1$.

[17]This fact is consistent with and supported by a large literature documenting a slow-moving component common to the entire term structure (e.g., Balduzzi et al., 1998; Fama, 2006; Cieslak and Povala, 2015; Bauer and Rudebusch, 2020).

3.1 No-Arbitrage versus Diagnostic Expectations

We do not impose any restrictions on the stationary cyclical component. Next, we justify this choice.

Under the Rational Expectations-No Arbitrage (RE-NA) approach the cyclical component would be identified with the term premium of the n-period bond. Consistently with our approach, Dai and Singleton (2002) argues that it is not plausible to consider the risk premium as a non-mean reverting component. However, a stationary $u_t^{(n)}$ does not necessarily provide support for the RE-NA framework. In fact, a stationary $u_t^{(n)}$ is also consistent with, e.g., temporary deviations from Rational Expectations generated within a Diagnostic Expectations framework (see Gennaioli and Shleifer, 2018) where long rates over-react relative to change in expectations about short rates. Following Bordalo et al. (2018) and d'Arienzo (2020), diagnostic expectations about policy rates can be represented as follows:

$$E^D\left[y_{t+i}^{(1)} \mid I_t\right] = E\left[y_{t+i}^{(1)} \mid I_t\right] + \theta\left(E\left[y_{t+i}^{(1)} \mid I_t\right] - E\left[y_{t+i}^{*} \mid I_t\right]\right) \ . \tag{5}$$

Diagnostic expectations, $E^D\left[y_{t+i}^{(1)} \mid I_t\right]$, differ from rational expectations, $E\left[y_{t+i}^{(1)} \mid I_t\right]$, by a shift in the direction of the information received at time t on deviations of monetary policy from its (stochastic) trend. Under the diagnostic expectations hypothesis agents over-react to the stationary deviations of monetary policy from its trend.

Interestingly, if agents use Diagnostic Expectations, the correct specification for the drift in yields would be given by:

$$y_t^{(n),*} = \left(\frac{1}{n}\right)\sum_{i=0}^{n-1} E_t^D[y_{t+i}^{(1)}]$$

In this case, Equation (4) can then be re-written as:

$$y_t^{(n)} = \left(\frac{1}{n}\right)\sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)}] + \delta_0 + \underbrace{\left(\frac{1}{n}\right)\sum_{i=0}^{n-1}\left(E_t^D[y_{t+i}^{(1)}] - E_t[y_{t+i}^{(1)}]\right)}_{u_t^{(n)}} \tag{6}$$

Thus, the (stationary) component $u_t^{(n)}$ can in principle be explained by the over-reaction induced by diagnostic expectation: i.e., $u_t^{(n)}$ can be justified also if term premia are constant or even absent.

Consistently with these different interpretations of the stationary component of yields, we do not impose NA restrictions when estimating our model. Thus, our estimation strategy runs the cost of losing efficiency if NA holds to gain consistency in the case NA is violated. At the same time, the flexibility of our approach will permit to quantify the relative importance of diagnostic expectations relative to explanations based on rational term premiums.

Our full term structure model is specified as follows:

$$\begin{aligned}
y_t^{(1)} &= y_t^* + \beta_1 E_t(\pi_{t+1} - \pi_{t+1}^*) + \beta_2 E_t(x_{t+1}) + u_t^{(1)} && (7)\\
y_t^* &= \gamma_1 MY_t + \gamma_2 \Delta x_t^{pot} + \gamma_3 \pi_t^* \\
u_t^{(1)} &= \rho_1 u_{t-1}^{(1)} + \epsilon_t^{(1)} \\
y_t^{(n)} &= y_t^{(n),*} + \delta_0 + u_t^{(n)} && (8)\\
u_t^{(n)} &= \rho_n u_{t-1}^{(n)} + \epsilon_t^{(n)} \\
y_t^{(n),*} &= \left(\frac{1}{n}\right)\sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)}] \\
(\pi_t - \pi_t^*) &= \theta_{1,1}\left(\pi_{t-1} - \pi_{t-1}^*\right) + \theta_{1,2} x_{t-1} + \theta_{1,3}\left(y_{t-1}^{(1)} - y_{t-1}^*\right) + v_{1,t} && (9)\\
x_t &= \theta_{2,1}\left(\pi_{t-1} - \pi_{t-1}^*\right) + \theta_{2,2} x_{t-1} + \theta_{2,3}\left(y_{t-1}^{(1)} - y_{t-1}^*\right) + v_{2,t} && (10)
\end{aligned}$$

where we assume $Cov(v_{1,t}, u_t^{(1)}) = Cov(v_{2,t}, u_t^{(1)}) = 0$.

Projections of the equilibrium policy rates depend on productivity and demographics, which we take as exogenous. The U.S. Census Bureau and the U.S. Congressional Budget Office provide ready-to-use projections respectively for MY and potential output. Equations (9) and (10) are used to compute the projections of inflation and output gaps. The dynamics of these two stationary variables depend on their own lags and on a third stationary variable: the deviation of the short-term rate from its trend. This cycle in monetary policy enters the dynamics of output and inflation gaps with a one-quarter lag; this is consistent with the delay with which monetary policy affects these variable in our specification of the forward looking policy rule (7).

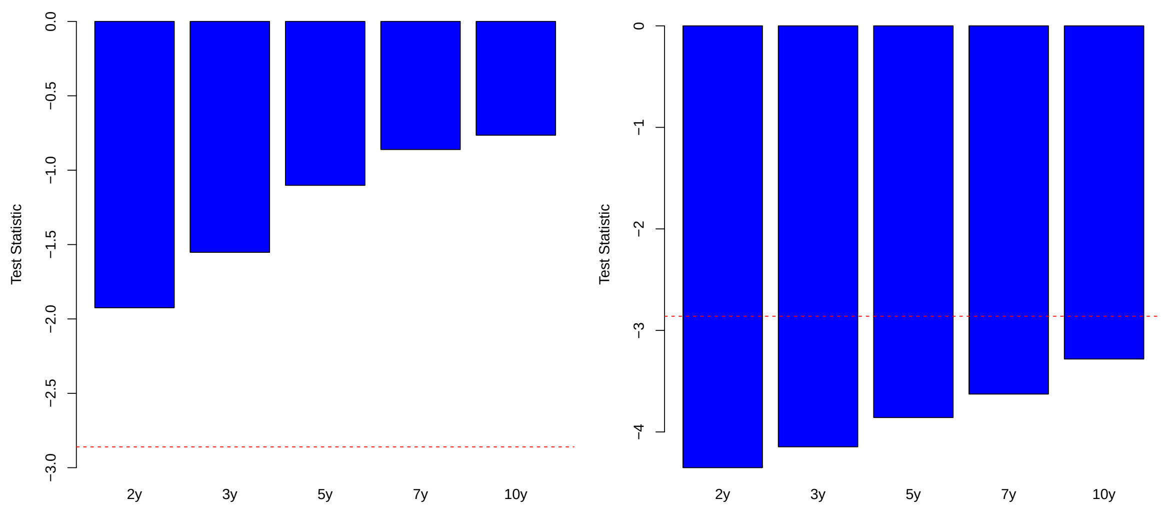
3.2 Empirical Results

3.2.1 Misspecification test for term structure models

The validity of a model with drifting monetary policy rates and bond prices can be assessed by checking the existence of cointegrating relationships with parameters $(1, -1)$ between $y_t^{(n)}$ and $y_t^{(n),*}$ (see Equation (8)). Thus, in this section we investigate the strength of the cointegrating relationship, the $(1, -1)$ parametric restriction, and the behavior of the residuals for our baseline model (see Equations (7)–(10)) as well as for its restricted version where the drift in monetary policy is assumed away (i.e., $y_t^* = y^*$).

Figure 3 reports the results for the (strength of the) cointegration relationship for five maturities ranging from 2- ($n = 8$ quarters) to 10-years ($n = 40$ quarters). The left panel is for the restricted model whereas the right panel is for our model with drifting equilibrium policy rates.

Our model provides overwhelming evidence to reject the null hypothesis of absence of



(a) Model with constant equilibrium rate.

(b) Model with drifting equilibrium rate.

Figure 3: **Engle and Granger (1987) Cointegration Test.**: This figure shows results for the Engle and Granger (1987) cointegration test for the residuals from regressing $y_t^{(n)}$ on $y_t^{(n),*}$ for different maturities. Panel (a) reports test statistics for a model that restricts the equilibrium rate to be constant (c.f. equation (3)). Panel (b) reports test statistics for our (cointegrated) model with drifting equilibrium rates (c.f. equations (7)–(10)). The null hypothesis is absence of cointegration. The dashed red line is the critical value at 5% level of significance as suggested by MacKinnon (2010). Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

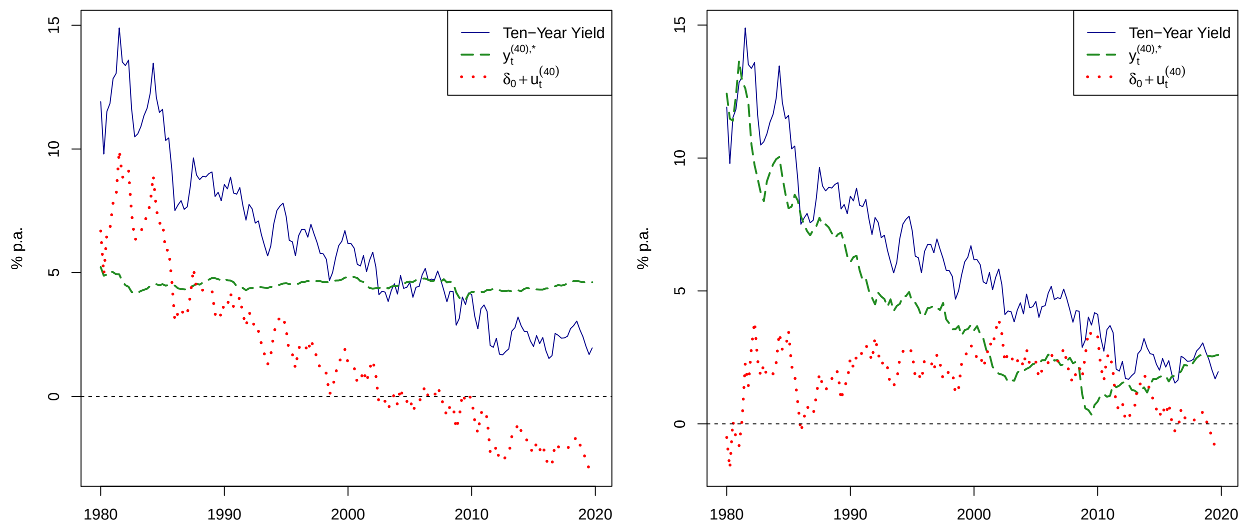
cointegrating relation between $y_t^{(n)}$ and $y_t^{(n),*}$ for all the considered maturities.

Furthermore, Appendix Table B.1 confirms that, within our framework with drifting policy rates, the parametric restriction $(1, -1)$ on the cointegrating relationship between yields and their drift is supported in the data for every maturities ranging from 2- to 10-years.

In all, our choice of the drivers for the equilibrium rate y_t^* provides also an accurate description of the stochastic trend underlying interest rates.

3.2.2 The dynamics of cyclical yields components

Next we study the behavior of the residual $u_t^{(n)}$. Specifically, Figure 4 shows the decomposition of the 10-year yield $y_t^{(40)}$ into $y_t^{(40),*}$ and $\delta_0 + u_t^{(40)}$, as per equation (8). As before, the left panel refers to the restricted model whereas the right panel refers to our benchmark model with drifting equilibrium policy rates. It is obvious that the two models have opposite implications: the residuals (dotted line) follow a random walk under the classical model with constant equilibrium rates, but are stationary in our model with drifting rates.[18,19]



(a) Model with constant equilibrium rates. (b) Model with drifting equilibrium rates.

Figure 4: Decomposing long-term rates. Panel (a) shows the decomposition of the ten-year yield implied by a model which assumes away drifting monetary policy rates (i.e., $y_t^* = y^*$). Panel (b) shows the decomposition of the ten-year yield implied by our model with drifting equilibrium rates (see equations (7)–(10)). Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

[18]Replacing, in the restricted model, the perceived target rate π_t^* with a fixed target rate at 2%, leaves our conclusion unchanged: the 10-year residual is close to a random walk with an AR(1) coefficient of 0.98.

[19]Wright (2011) argue for term premiums to decline internationally over the sample 1990–2007. Bauer et al. (2014) and Wright (2014) discuss the extent to which small-sample bias in maximum likelihood estimates of affine term structure models alters the conclusions about term premia and its (a)cyclical properties. Our evidence is complementary: we do not focus on statistical biases but we stress the importance of modeling the economic determinants of equilibrium rates. Furthermore, our framework is flexible and allows, without imposing, to interpret the (stationary) deviations of bond prices from their drifts as term premia.

Importantly, Figure 5 shows that our estimated deviations of bond prices from their drifts comove strongly with state-of-the-art term premium estimates like the one proposed by Bauer and Rudebusch (2020).

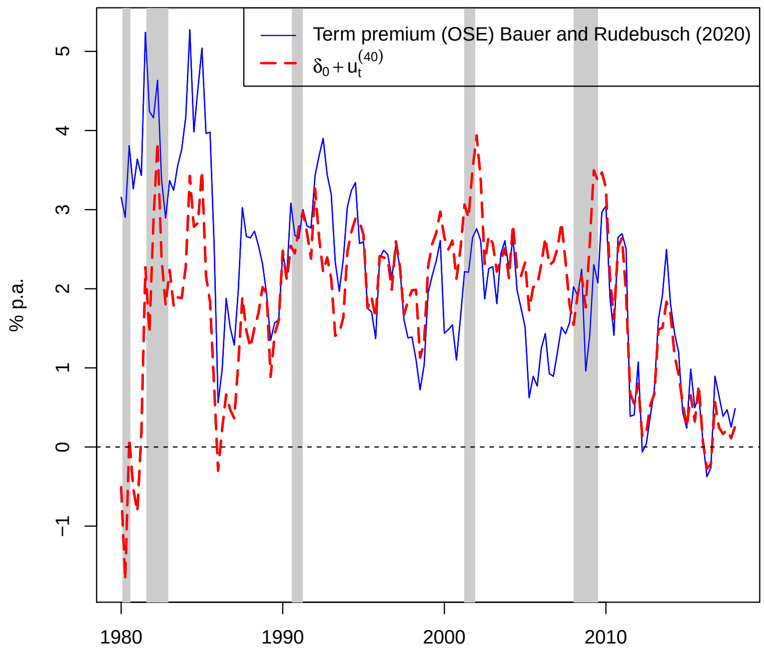


Figure 5: Cyclical component from model with drifting equilbrium rates vs. term premium estimate: This figure shows the term premium component for a 10-year Treasury bond estimated following the methodology (OSE, observed shifting endpoint) proposed by Bauer and Rudebusch (2020) together with deviations of the 10-year bond yields from their drift, $\delta_0 + u_t^{(40)}$, implied by our (cointegrated) model with drifting equilibrium rates (c.f., equations (7)–(10)). Quarterly observations. The sample period is 1980:Q1 to 2018:Q1.

Two conclusions can be drawn from this analysis. First, this result is reminiscent of Joslin et al. (2013) who find that the estimated joint distribution within a macro-finance term structure model with NA is nearly identical to the estimate from an economic-model-free factor vector-autoregression. The evidence in Figure 5 suggests that this conclusion is likely to hold true also in models that accommodate a drifting term structure.

Second, and more aggressively, the similarity between our cyclical component and term premiums estimates, together with the fact that our framework does not impose no-arbitrage, hints to the possibility that a non-trivial fraction of what is deemed to be risk premium is a mere reflection of temporary deviations from rational expectations. Motivated by this evidence, next we develop a formal test to quantify the relative contribution of rational term premium and of deviations from rational expectations to the cyclical components of yields.

3.3 Testing Diagnostic Expectations

We use our flexible framework to assess the role played by diagnostic expectations in explaining the stationary component of yields.

Suppose agents use diagnostic expectations for the short rate and the risk premium is constant. We have:

$$y_t^{(n)} = \left(\frac{1}{n}\right)\sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)}] + \delta_0 + \underbrace{\left(\frac{1}{n}\right)\sum_{i=0}^{n-1}\left(E_t^D[y_{t+i}^{(1)}] - E_t[y_{t+i}^{(1)}]\right)}_{u_t^{(n)}} \tag{11}$$

$$= \left(\frac{1}{n}\right)\sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)}] + \delta_0 + \left(\frac{1}{n}\right)\theta\sum_{i=0}^{n-1}\left(E\left[y_{t+i}^{(1)} \mid I_t\right] - E\left[y_{t+i}^{*} \mid I_t\right]\right) \tag{12}$$

where in the second row we exploit the expression for Diagnostic Expectations in equation (5): $E^D\left[y_{t+i}^{(1)} \mid I_t\right] - E\left[y_{t+i}^{(1)} \mid I_t\right] = \theta\left(E\left[y_{t+i}^{(1)} \mid I_t\right] - E\left[y_{t+i}^{*} \mid I_t\right]\right)$.

Given that the cyclical component of monetary policy rates is stationary with zero mean, we can write

$$\left(y_{t+1}^{(1)} - y_{t+1}^{*}\right) = \phi\left(y_t^{(1)} - y_t^{*}\right) + v_{t+1}, \tag{13}$$

where $\mid \phi \mid < 1$. Using the dynamics for $\left(y_t^{(1)} - y_t^{*}\right)$ to compute the expectations in equation

(12), we derive the following expression for $u_t^{(n)}$:

$$u_t^{(n)} = \left(\frac{1}{n}\right)\sum_{i=0}^{n-1}\left(E_t^D[y_{t+i}^{(1)}] - E_t[y_{t+i}^{(1)}]\right)$$
$$= \left(\frac{1}{n}\right)\theta\frac{1-\phi^n}{1-\phi}\left(y_t^{(1)} - y_t^*\right) \tag{14}$$

We observe that deviations from rational expectations depend on the parameter θ and on the persistence of the deviations of monetary policy rates from the trend. The stationarity of $\left(y_t^{(1)} - y_t^*\right)$ implies that, for large n (i.e., at long horizons), diagnostic expectations for the monetary policy rates will converge towards rational expectations.[20] More importantly, the significance of Diagnostic Expectations can be tested by projecting $u_t^{(n)}$ on $\left(y_t^{(1)} - y_t^*\right)$.

Table 2 displays the results for such test.

Table 2: Testing Diagnostic Expectations

This table reports OLS estimates for the regression $u_t^{(n)} = \beta\left(y_t^{(1)} - y_t^*\right) + \epsilon_t$, where $u_t^{(n)}$ is the deviation of a bond with maturity n-period from its drift (see Equation (8)) and $\left(y_t^{(1)} - y_t^*\right)$ is the difference between the observed short-term rate and the estimated drifting equilibrium rate at time t. θ is calculated as implied by Equation (14). Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors with automatic bandwidth selection procedure as described in Newey and West (1994). Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

	$u_t^{(8)}$ (1)	$u_t^{(12)}$ (2)	$u_t^{(20)}$ (3)	$u_t^{(28)}$ (4)	$u_t^{(40)}$ (5)
$\left(y_t^{(1)} - y_t^*\right)$	0.379***	0.337***	0.248***	0.189***	0.141**
	(0.058)	(0.059)	(0.064)	(0.065)	(0.068)
Observations	160	160	160	160	160
R^2	0.331	0.257	0.140	0.080	0.043
Implied θ	0.42	0.41	0.37	0.34	0.32

[20] Maxted (2019) considers a case in which convergence of DE to RE is not realized as the underlying process is non-stationary.

We find that diagnostic expectations explain between 4% and 33% of the variability in the cyclical components of yields. In line with our discussion of equation (14), the importance of diagnostic expectations decreases with the maturity of the bond. Remarkably, our estimates of the parameter θ are in line with previous values reported in the literature (e.g., Bordalo et al., 2020; d'Arienzo, 2020).

The empirical relevance of overreaction has been recently documented by Cieslak (2018) for the short end of the curve. Similarly, Piazzesi et al. (2015) provide evidence that realized survey (interest rates) forecast errors as well as forecast differences relative to VAR-based measure may be responsible for the time-variation in bond premia from statistical models. We have shown that these explanations may be important even in a model that accommodates a drifting term structure. However, the contribution of overeaction decreases at long maturities; this is consistent with deviations of monetary policy rates from the equilibrium rate being fast mean-reverting.

3.4 Trend-cycle yield decomposition: implications for hidden factors

Our evidence also contributes to the debate on the presence of hidden factors.

We start by reviewing the concept of hidden factors (Duffee, 2011). For yields of any maturity we can write

$$y_t^{(n)} = \underbrace{\left(\frac{1}{n}\right)\sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)}]}_{y_t^{(n),*}} + \delta_0 + u_t^{(n)} \tag{15}$$

A factor f_t is hidden if its effect on $y_t^{(n),*}$ and $\delta_0 + u_t^{(n)}$ exactly compensate so that the overall effect of f_t on $y_t^{(n)}$ is zero. Note that factors are taken as stationary variables. However, we have shown that—because of the drift in interest rates—both non-stationary variables (drivers) and stationary variables (factors) are needed to model properly the term structure of Treasury yields. Therefore, it is natural to ask what are the implications for hidden factors of a yield model with drifting prices.

To answer this question, we exploit our model with drifting equilibrium rate (see Equations (7)–(10)), and we re-write equation (15) as follows:

$$y_t^{(n)} = \left(\frac{1}{n}\right)\sum_{i=0}^{n-1} E_t[y_{t+i}^*] \tag{16}$$

$$+ \left(\frac{1}{n}\right)\sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)} - y_{t+i}^*] + \delta_0 + u_t^{(n)} \tag{17}$$

In words, long-term interest rates depend on three components: a trend component that reflects exclusively the drift in short-term rates (see (16)), and two cyclical components (see (17)). The first cyclical component, $\left(\frac{1}{n}\right)\sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)} - y_{t+i}^*]$, is related to fluctuations of short-term rates around their time-varying mean; the second cyclical component, $\delta_0 + u_t^{(n)}$, is related to term premia and possibly deviations from rational expectations (c.f. Section 3.3). We extend the definition of Duffee (2011), and say that a factor is hidden when its impact on these two cyclical components is equal but with opposite sign. Thus, we can test if a factor is hidden by running the following regression:

$$\left(y_t^{(n)} - \left(\frac{1}{n}\right)\sum_{i=0}^{n-1} E_t[y_{t+i}^*]\right) = \alpha + \beta f_t + \epsilon_t^{(n)} \tag{18}$$

and check for $\beta = 0$ (i.e., f_t is hidden).

Few comments are in order. First, according to our definition, the determinants of the trend component in monetary policy are - by construction – hidden by the yield curve. This is because productivity, demographics, and inflation trends do not affect yields directly but only trough the drift in monetary policy rates, giving rise to a coefficient $\beta = 0$ in (18). This is consistent with Bauer et al. (2014) who argued that "the trend component [in interest rates] is unspanned by yields."[21] Second, and more important, a test of hidden factor can be constructed *only once* the drift and cyclical components of yields at all maturities are identified so that the impact of factors on these two components can also be identified and estimated.

Next we investigate the hidden nature of the cyclical components of deviation of inflation from its long-run target $(\pi_t - \pi_t^*)$, the output gap x_t, and the monetary policy shocks $u_t^{(1)}$. We report the results in Table 3.[22] Panel A shows that deviation of inflation from its long-run target are by and large hidden by the yield curve. On the other hand, we observe in Panel B that the effect of x_t is strongly significant for maturities ranging from 2 to 5 years. Finally, in Panel C, we find strong evidence against monetary policy shocks being hidden.[23]

Since the seminal contribution of Joslin et al. (2014), a vast literature has thought of output (gap) and inflation (gap) as unspanned factors. A contribution of our analysis is to point to the importance of decomposing yields into trend and cycle before evaluating the (hidden) nature of (non-stationary) drivers and (stationary) factors. In fact, the question on the nature of spanned or unspanned of factors can be properly answered only after modeling

[21] The term $\left(\frac{1}{n}\right)\sum_{i=0}^{n-1} E_t[y_{t+i}^*]$ depends on the maturity n. However, the correlation across maturities ranging from 1- to 10-years is very high at 99.5%. Therefore, this term effectively captures the stochastic trend common across yields and cannot be inferred from the cross-section of interest rates.

[22] We run simple regressions since the output gap and the deviation of inflation from its long-run target display a mild positive correlation of 20%. On the other hand monetary policy shocks are orthogonal by construction to $(\pi_t - \pi_t^*)$ and x_t so the coefficient on u_t is identical in simple and multiple regressions.

[23] Interestingly, the cyclical component of yields, $u_t^{(n)}$, has a long-run unit coefficient on the deviations of monetary policy rates from equilibrium rates, i.e. $u_t^{(1)}$ (see Appendix Figure B.3). So no cancellation can occur, confirming further that monetary policy shocks cannot be hidden.

of the drift in the term structure with non-stationary drivers.

Table B.2 in the Appendix shows the empirical importance of de-trending yields before testing for hidden factors. When yields are not properly detrended, cyclical factors are (wrongly) labelled as hidden on the basis the projections of drifting yields on them.

Table 3: Hidden Factor Test for Detrended Yields

This table reports OLS estimates for the regression $\tilde{y}_t^{(n)} = \alpha + \beta f_t + \epsilon_t^{(n)}$, where $\tilde{y}_t^{(n)} \equiv \left(y_t^{(n)} - \left(\frac{1}{n}\right) \sum_{i=0}^{n-1} E_t[y^*_{t+i}] \right)$ is the cyclical component of an yield with maturity n as defined in Equation (17) and f_t are different factors to be tested. Panel A reports results for the deviation of inflation from its long-run target $(\pi_t - \pi_t^*)$. Panel B reports results for the output gap x_t. Panel C reports results for the monetary policy shocks $u_t^{(1)}$. Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors with automatic bandwidth selection procedure as described in Newey and West (1994). Constants are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

Panel A: The inflation gap.

	$\tilde{y}_t^{(8)}$	$\tilde{y}_t^{(12)}$	$\tilde{y}_t^{(20)}$	$\tilde{y}_t^{(28)}$	$\tilde{y}_t^{(40)}$
	(1)	(2)	(3)	(4)	(5)
$(\pi_t - \pi_t^*)$	0.626**	0.498	0.287	0.155	0.050
	(0.273)	(0.314)	(0.398)	(0.422)	(0.455)
Observations	159	159	159	159	159
R^2	0.117	0.077	0.029	0.009	0.001

Panel B: The Output Gap.

	$\tilde{y}_t^{(8)}$	$\tilde{y}_t^{(12)}$	$\tilde{y}_t^{(20)}$	$\tilde{y}_t^{(28)}$	$\tilde{y}_t^{(40)}$
	(1)	(2)	(3)	(4)	(5)
x_t	0.875**	0.853**	0.768**	0.693*	0.600
	(0.374)	(0.380)	(0.357)	(0.358)	(0.404)
Observations	159	159	159	159	159
R^2	0.248	0.246	0.222	0.191	0.148

Panel C: The Monetary Policy Shocks.

	$\tilde{y}_t^{(8)}$	$\tilde{y}_t^{(12)}$	$\tilde{y}_t^{(20)}$	$\tilde{y}_t^{(28)}$	$\tilde{y}_t^{(40)}$
	(1)	(2)	(3)	(4)	(5)
$u_t^{(1)}$	0.723***	0.673***	0.563***	0.489***	0.428***
	(0.111)	(0.111)	(0.111)	(0.116)	(0.118)
Observations	159	159	159	159	159
R^2	0.453	0.408	0.318	0.254	0.201

4 Predicting Holding Period Excess Returns

Predictability of interest rates on the basis of the cointegration between $y_t^{(n)}$ and $y_t^{(n),*}$, also implies predictability of holding period excess returns on the basis of the stationary deviations of bond yields from their drift.

To see this, write the expected excess returns obtained by holding for one period the n-period bond as:

$$\begin{aligned} E_t(rx_{t+1}^{(n)}) &= y_t^{(n)} n - (n-1) E_t(y_{t+1}^{(n-1)}) - y_t^{(1)} \\ &= y_t^{(n)} - (n-1)\left(E_t(y_{t+1}^{(n-1)}) - y_t^{(n)}\right) - y_t^{(1)} \\ &= y_t^{(n)} - y_t^{(1)} - (n-1)\left(E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1)}\right) - (n-1)\left(y_t^{(n-1)} - y_t^{(n)}\right), \end{aligned} \tag{19}$$

where $y_t^{(n)} - y_t^{(1)}$ is the slope of the term structure, $\left(y_t^{(n-1)} - y_t^{(n)}\right)$ is known as the roll-down, and $\left(E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1)}\right)$ is the expected change in prices of the $(n-1)$-maturity bond. Since the seminal contributions by Fama and Bliss (1987) and Campbell and Shiller (1991), the slope of the term structure has played a central role for forecasting bond returns. Indeed, it is common to assume away any predictability arising from $\left(E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1)}\right)$, since the level of the term structure is deemed to be close to unforecastable (see, e.g., Duffee, 2013).

Our proposed "cointegrated" specification of the monetary policy rule and the term structure suggests otherwise. Using Equation (8) and the autoregressive dynamics of the residual, one can express the expected price changes as

$$E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1)} = E_t\left(y_{t+1}^{(n-1),*} - y_t^{(n-1),*}\right) + \left(\rho_{(n-1)} - 1\right)\underbrace{\left(y_t^{(n-1)} - y_t^{(n-1),*} - \delta_0\right)}_{u_t^{(n-1)}}. \tag{20}$$

Therefore, in our model, persistent but stationary deviations of bond prices from their drift, $u_t^{(n-1)}$, show up as a natural predictor of excess bond returns.[24] This term has gone unrecognized since standard models start off with stationary factor (within our framework, this is equivalent to assume a constant equilibrium rate). In turn, this leads to a close-to-unit-root residual (c.f., Figure 4(a)), or $\rho_{(n-1)} - 1 \approx 0$ (and the level being a random walk $E_t(y_{t+1}^{(n-1)}) = y_t^{(n-1)}$).[25]

We start the evaluation of the predictive performance of our model with a drifting equilibrium rate by running the following regression:

$$rx_{t+4}^{(n)} = \alpha + \beta E_t(rx_{t+4}^{(n)}) + \epsilon_t \ , \tag{21}$$

where $rx_{t+4}^{(n)}$ is the realized one-year holding period excess return of a bond with maturity n-quarters. We denote with $E_t(rx_{t+4}^{(n)})$ the expected excess return implied by our specification that allows for stationary deviations of bond prices from their drifts.[26] We compare our specification to the classical model with a constant equilibrium rate.[27] Table 4 displays the results for the model with constant equilibrium rate in Panel A, and the results for our model with drifting bond prices in Panel B. We consider maturities ranging from 2 ($n = 8$

[24] More precisely, $u_t^{(n-1)}$ should forecast the price change component in bond returns. However, empirically the correlation between $rx_{t+4}^{(n)}$ and the price change term, $-\left(y_{t+4}^{(n-4)} - y_t^{(n-4)}\right)$, is high at 93%, 95%, 97%, 98%, and 99% for $n = 8, 12, 20, 28, 40$ quarters, respectively.

[25] Cieslak and Povala (2015) and Jørgensen (2018) predict bond returns using a de-trended (term structure) level factor. Using their proposed persistence-based Wold decomposition, Ortu et al. (2020) extract a cyclical component from the level of the yield curve and show that it contains information about future excess bond returns. To our knowledge, we are the first to show that a cyclical component of the level of the term structure emerges as a natural predictor within a cointegrated framework of bond prices.

[26] We exploit equations (7)–(10) together with the exogeneity of demographics and potential output to construct the expected change in constant-maturity yield $\left(E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1}\right)$ in equation (19).

[27] To make our results comparable to a large literature (e.g., Cochrane and Piazzesi, 2005; Cieslak and Povala, 2015) we focus on one-year excess returns. However, our conclusions are identical when we use one-quarter holding period returns.

quarters) to 10 years ($n = 40$ quarters). The regression of realized excess returns on the expected returns implied by our (cointegrated) model with drifting equilibrium rates delivers statistically significant estimates and coefficients of determination that are greater than 30% at all maturities.[28] On the other hand, a classical model with constant equilibrium rates leads to a coefficient not significantly different from zero and to small explanatory power.

We also highlight that the model with constant equilibrium rates performs worse than a (reduced-form) model based just on the slope. This is easily explained. The realized returns $rx_{t+4}^{(n)}$ on the left hand side of (21) are stationary whereas the expected returns $E_t(rx_{t+4}^{(n)})$ from the model with constant equilibrium rate is non-stationary since it inherits the drift from the residual component $u_t^{(n)}$ (c.f. Figure 4).

4.1 Dissecting Predictive Regressions

To further dissect the unique contribution coming from our cointegrated approach, Table 5 shows that the expected change in the $(n-1)$-maturity bond prices drives away the predictability of the slope (column (1)), and that deviations of bond prices from their drift, $u_t^{(n-1)}$, are the most important driver of such predictability (c.f. columns (3) and (4)). Also, the loading on the cyclical component $u_t^{(n-1)}$ is negative as predicted by our framework: if $0 < \rho_{(n-1)} < 1$, then next period returns are negative in times when bond prices are higher than those implied by their drift.

In the Appendix, we show that the relevance of such cyclical component for forecasting excess returns is not restricted to any specific maturity or holding period. Table B.3 reports results for the predictive regressions when we use bonds with maturities ranging from 2- to 7-years. Also, Table B.4 confirms that stationary deviations of bond prices from their drift

[28]The constant is not statistically significant for bond with maturities $n = 8, 12, 20$ quarters.

Table 4: Predictive Regressions across Different Maturities

This table reports OLS estimates for the regression $rx^{(n)}_{t+4} = \alpha + \beta E_t(rx^{(n)}_{t+4}) + \epsilon_t$ where $rx^{(n)}_{t+4}$ is the realized one-year holding period excess return of a bond with maturity n-period and $E_t(rx^{(n)}_{t+4})$ is the expected excess return implied by our specifications. Panel A reports results for the classical model with a constant equilibrium rate. Panel B reports results for our model with drifting equilibrium rates. Values in parenthesis are conservative standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

Panel A: Model with constant equilibrium rate.

	$rx^{(8)}_{t+4}$	$rx^{(12)}_{t+4}$	$rx^{(20)}_{t+4}$	$rx^{(28)}_{t+4}$	$rx^{(40)}_{t+4}$
	(1)	(2)	(3)	(4)	(5)
$E_t(rx^{(8)}_{t+4})$	0.146**				
	(0.069)				
$E_t(rx^{(12)}_{t+4})$		0.107			
		(0.078)			
$E_t(rx^{(20)}_{t+4})$			0.077		
			(0.077)		
$E_t(rx^{(28)}_{t+4})$				0.067	
				(0.075)	
$E_t(rx^{(40)}_{t+4})$					0.065
					(0.075)
Observations	156	156	156	156	156
R^2	0.107	0.062	0.035	0.029	0.028

Panel B: Model with drifting equilibrium rate.

	$rx^{(8)}_{t+4}$	$rx^{(12)}_{t+4}$	$rx^{(20)}_{t+4}$	$rx^{(28)}_{t+4}$	$rx^{(40)}_{t+4}$
	(1)	(2)	(3)	(4)	(5)
$E_t(rx^{(8)}_{t+4})$	0.808***				
	(0.150)				
$E_t(rx^{(12)}_{t+4})$		0.788***			
		(0.165)			
$E_t(rx^{(20)}_{t+4})$			0.682***		
			(0.159)		
$E_t(rx^{(28)}_{t+4})$				0.698***	
				(0.162)	
$E_t(rx^{(40)}_{t+4})$					0.614***
					(0.155)
Observations	156	156	156	156	156
R^2	0.360	0.362	0.329	0.360	0.319

predict quarterly holding period bond returns (i.e., non-overlapping returns). Overall, this evidence suggests that the adjustment of bond prices towards their drift is a key economic mechanism for understanding bond returns predictability.

Finally, Appendix Table C.1 shows that the US cyclical component $u_t^{(n)}$ predict UK and Canadian bond returns, even after controlling for the local slope of the term structure.[29] This finding resonates with the evidence in Dahlquist and Hasseltoft (2013). Despite this similarity, Dahlquist and Hasseltoft (2013) attributes the international comovement in bond returns to a global (admittedly, mostly US) bond risk premium; on the other hand, we have not imposed no-arbitrage restrictions so that our cyclical component is also compatible with investors overreacting to deviations of policy rates from its trend leading to overestimation of future short rates (and lower bond returns).[30]

4.2 The Information Content of Yield Cycles

Several bond returns predictors have been proposed in the literature since the seminal papers by Fama and Bliss (1987) and Campbell and Shiller (1991). It is then natural to ask to what extent the yield cycles $u_t^{(n)}$ capture new information not already conveyed by other variables.

Specifically, we compare the predictive power of our yield cycles to two well known return-predicting factors that are both constructed from the yield curve:[31] (1) the Cochrane and

[29] In the spirit of our model, we employ the local slope of the term structure as a proxy for the deviations of non-US yields from their drifts. Controlling for the local cyclical component does not change our conclusion. However, it is worth to emphasize that the lack of an exogenous potential output series, Δx_t^{pot}, and of a perceived target inflation rate, π_t^*, may be responsible for the poor performance of the local cycle in Canada and UK. Further investigation on this topic is on our agenda for future research.

[30] Our findings are also consistent with the idea that the Fed is the leader among central banks in setting monetary policy (Brusa, Savor and Wilson, 2019). See also `One Policy to Rule Them All: Why Central Bank Divergence Is So Slow` (Wall Street Journal, 2016) for a recent discussion on the topic.

[31] Several papers have found that the state of the economy also conveys information about future bond returns. E.g., Cooper and Priestley (2008) propose the output gap, whereas Ludvigson and Ng (2009) propose to extract information from a large set of macrofinancial variables. Related, Bansal and Shaliastovich (2013)

Table 5: Dissecting Predictive Regressions

This table reports OLS estimates for the regression $rx_{t+4}^{(40)} = \alpha + \beta' X_t + \epsilon_t$ where $rx_{t+4}^{(40)}$ is the realized one-year holding period excess return of a bond with maturity 10-year and X_t contains different return predictors. Column (1) exploits equation (19) reported here for reader's convenience:

$$E_t(rx_{t+4}^{(40)}) = y_t^{(40)} - y_t^{(4)} - (40-4)\left(E_t(y_{t+4}^{(40-4)}) - y_t^{(40-4)}\right) - (40-4)\left(y_t^{(40-4)} - y_t^{(40)}\right).$$

Column (2) shows that the slope is a significant predictor of excess bond returns when considered in isolation. Columns (3) and (4) exploit the decomposition of expected price changes per equation (20) reported here for reader's convenience:

$$E_t(y_{t+4}^{(40-4)}) - y_t^{(40-4)} = E_t\left(y_{t+4}^{(40-4),*} - y_t^{(40-4),*}\right) + \left(\rho_{(40-4)} - 1\right) u_t^{(40-4)}.$$

In columns (3) and (4) we neglect the roll-down term which empirically is found to be insignificant. Values in parenthesis are conservative standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

	$rx_{t+4}^{(40)}$			
	(1)	(2)	(3)	(4)
$y_t^{(40)} - y_t^{(4)}$	3.257	2.320	1.139	1.010
	(1.995)	(1.548)	(1.933)	(1.390)
$-(40-4)\left(E_t(y_{t+4}^{(36)}) - y_t^{(36)}\right)$	0.538***			
	(0.151)			
$-(40-4)(y_t^{(36)} - y_t^{(40)})$	−4.359			
	(3.871)			
$-(40-4)\left(E_t(y_{t+4}^{(36),*}) - y_t^{(36),*}\right)$			0.204	
			(2.071)	
$-(40-4)\ u_t^{(36)}$			−0.607***	−0.615***
			(0.182)	(0.160)
Adjusted R^2	0.341	0.060	0.317	0.321

Piazzesi (2005, CP) factor which is based on a linear combination of forward rates; (2) the Cieslak and Povala (2015, CPo) factor which relies on information contained in yields that have been detrended using a long-term moving average of inflation.[32]

Rather than using a specific cycle for each maturity n we construct a common yield cycle using a procedure akin to Cochrane and Piazzesi (2005). Specifically, we run regressions of the average (across maturity) excess return on all cycles,

$$\frac{1}{9}\sum_{n=2}^{10} rx_{t+4}^{(n)} = \gamma_0 + \gamma_1 u_t^{(1)} + \ldots + \gamma_{40} u_t^{(40)} + \varepsilon_{t+1}.$$

Our yield-based cycle factor is given by $\widetilde{u}_t = \widehat{\boldsymbol{\gamma}}' \mathbf{u}_t$.

Table 6 shows the results. In Panel A we investigate the predictive content of our cycle relative to the CP factor, whereas in Panel B we compare it to the CPo factor. The odd columns confirm that both CP and CPo forecasts excess returns of all bonds. Importantly, Panel A shows that our yield cycle drives away the CP factor, and delivers R^2 that are about tree times those obtained by the CP regressions. Panel B tells a similar story. Despite the large R^2 obtained by the CPo factor, our yield cycle continues to be a significant predictor of bond returns at all maturities ranging from 2- to 10-years. In fact, comparing the R^2 from the multiple regression in Panel A to those in Panel B, we see that replacing CP with CPo does not alter the predictive content of our yield-based cycle.

document that real growth and inflation uncertainties predict, respectively, lower and higher bond risk premia, and propose a long-run risk type model for rationalizing this finding. Since our yield cycles are obtained by removing the stochastic trend (due to the equilibrium rate) in interest rates, we restrict our attention only to yield-based predicting factors.

[32] To construct the CP and CPo factors we follow the procedure described in the original papers. E.g., to construct the CP factors we use only one- through five-year zero coupon bond prices and estimate the loadings by running a regression of the equally-weighted average (across maturity) excess return on the forward rates. To construct the CPo factor instead we employ duration standardized returns. To be consistent with the overall empirical analysis, unlike in the original papers, both factors are constructed using quarterly observations.

Table 6: Predictive Regressions: Horse race against other bond predictors

This table reports OLS estimates for the regression $rx_{t+4}^{(n)} = \alpha + \beta_1 F_t + \beta_2 \tilde{u}_t + \epsilon_t$ where $rx_{t+4}^{(n)}$ is the realized one-year holding period excess return of a bond with maturity n-period, F_t is the Cochrane and Piazzesi (2005) factor (CP_t) in Panel A and the Cieslak and Povala (2015) factor (CPo_t) in Panel B, and $\tilde{u}_t$ is the single-return forecasting factor implied by our model with drifting equilbrium rates. The Cochrane-Piazzesi factor is constructed as in Cochrane and Piazzesi (2005) using quarterly zero-coupon Treasury yields from Gürkaynak et al. (2007) with maturities from 1 to 5 years. The Cieslak-Povala factor is constructed as in Cieslak and Povala (2015) using quarterly zero-coupon Treasury yields from Gürkaynak et al. (2007) with maturities from 1 to 10 years. $\tilde{u}_t$ is the fitted value from regressing the average one-year holding-period excess returns on a n-periods Treasury bond for $n = 4, 8, \ldots, 40$ on our cyclical components $u_t^{(n)}$ $n = 1, \ldots, 40$ (see Eq. (20)). Values in parenthesis are conservative standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

Panel A: Cochrane-Piazzesi (2005).

	$rx_{t+4}^{(8)}$		$rx_{t+4}^{(12)}$		$rx_{t+4}^{(20)}$		$rx_{t+4}^{(28)}$		$rx_{t+4}^{(40)}$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
CP_t	0.434***	0.104	0.821***	0.168	1.554***	0.340	2.272***	0.589	3.307***	1.069
	(0.139)	(0.148)	(0.305)	(0.331)	(0.580)	(0.633)	(0.810)	(0.887)	(1.141)	(1.263)
$\tilde{u}_t$		0.219***		0.434***		0.807***		1.118***		1.488***
		(0.035)		(0.079)		(0.164)		(0.240)		(0.344)
Observations	156	156	156	156	156	156	156	156	156	156
Adjusted R^2	0.130	0.390	0.132	0.421	0.145	0.451	0.159	0.459	0.174	0.446

Panel B: Cieslak-Povala (2015).

	$rx_{t+4}^{(8)}$		$rx_{t+4}^{(12)}$		$rx_{t+4}^{(20)}$		$rx_{t+4}^{(28)}$		$rx_{t+4}^{(40)}$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
CPo_t	1.366***	0.428	2.720***	0.959	5.230***	2.177	7.544***	3.535	10.650***	5.700*
	(0.247)	(0.366)	(0.516)	(0.800)	(1.011)	(1.572)	(1.467)	(2.259)	(2.119)	(3.224)
$\tilde{u}_t$		0.184***		0.346***		0.599**		0.787**		0.972**
		(0.050)		(0.118)		(0.240)		(0.344)		(0.478)
Observations	156	156	156	156	156	156	156	156	156	156
Adjusted R^2	0.305	0.396	0.342	0.433	0.389	0.472	0.413	0.487	0.424	0.480

4.3 Out-Of-Sample Predictability

As a final robustness test we consider out-of-sample predictability as measured by R^2_{OOS} computed as follows:

$$R^2_{OOS} = 1 - \frac{\sum_{t=1}^{T}\left(rx^{(n)}_{t+4} - \widehat{rx}^{(n)}_{t+4}\right)^2}{\sum_{t=1}^{T}\left(rx^{(n)}_{t+4} - \bar{rx}^{(n)}_{t+4}\right)^2}$$

where $\widehat{rx}^{(n)}_{t+4}$ is the fitted value from our predictive regression estimated through period $t-1$ and $\bar{rx}^{(n)}_{t+4}$ is the historical average return estimated thorough period $t-1$. If the R^2_{OOS} is positive, then the predictive regression has lower average mean squared prediction error than the historical average return. This is always the case for all regressions reported in Table 7.

Table 7: Out-Of-Sample Tests

This table reports R^2_{OOS} for the predictive regression $rx^{(n)}_{t+4} = \alpha + \beta'\tilde{u}_t + \epsilon_t$ where $rx^{(n)}_{t+4}$ is the realized one-year holding period excess return of a bond with maturity n-period and $\tilde{u}_t$ is the single-return forecasting factor implied by our model with drifting equilibrium rates. $\tilde{u}_t$ is the fitted value from regressing the average one-year holding-period excess returns on a n-periods Treasury bond for $n = 4, 8, \ldots, 40$ on our cyclical components $u^{(n)}_t$ $n = 1, \ldots, 40$ (see Eq. (20)). We use a rolling window for estimating the predictive regressions. The R^2_{OOS} is computed as in Campbell and Thompson (2008); p-values for R^2_{OOS} are computed as in Clark and West (2007). In Panel A the out-of-sample period starts in 1990; in Panel B the out-of-sample period starts in 2000. Quarterly observations.

Panel A: Out-of-sample period: 1990-2019.

	$rx^{(8)}_{t+4}$	$rx^{(12)}_{t+4}$	$rx^{(20)}_{t+4}$	$rx^{(28)}_{t+4}$	$rx^{(40)}_{t+4}$
	(1)	(2)	(3)	(4)	(5)
R^2_{OOS}	20.9***	27.18***	31.63***	31.65***	27.29***

Panel B: Out-of-sample period: 2000-2019.

	$rx^{(8)}_{t+4}$	$rx^{(12)}_{t+4}$	$rx^{(20)}_{t+4}$	$rx^{(28)}_{t+4}$	$rx^{(40)}_{t+4}$
	(1)	(2)	(3)	(4)	(5)
R^2_{OOS}	0.99***	3.38***	10.23***	14.26***	13.65***

5 Conclusions

This paper proposed a general framework to model a common drift in bond prices, and studied its implications for monetary policy, interest rates and bond returns predictability.

First, we have shown that there is a drift in monetary policy rates which can be successfully modeled by fluctuations in productivity, demographics and long-term inflation expectations. This produces monetary policy residuals that are substantially less persistent than those implied by standard policy rules. Thus, through the lens of our modeling approach, monetary inertia is just the manifestation of omitted factors in the estimated rule.

The drift in bond prices is then described by the average of expected monetary policy (drifting) rates over the residual life of the bond. Appropriate modeling of the drift in monetary policy must deliver stationary deviation of yields to maturity from their drift. These stationary deviations of bond prices from their drift could be explained by the presence of term premia in a no-arbitrage framework or by temporary deviations from rational expectations in a behavioral framework. When the deviations of bond prices from their drift are interpreted as term premia, our finding implies that models that mispecify (or, worse, do not model) the drift in monetary policy and in bond prices will fail to generate *stationary* term premia.

Our empirical evidence shows that deviations from rational expectations in the form of Diagnostic Expectations (DE) can account for up to 30% of the yield cycle variability at 2- and 3-year maturities. However, the importance of DE decreases at longer maturities leaving an important role for term premia.

Finally, we have shown that persistent but stationary deviations of US Treasury bond prices from their drift predict excess returns in- and out-of-sample, as well as outside the US. Next period returns from holding long-term bonds are negative in times when bond prices are higher than those implied by their drift. Once again this predictability could be related either to predictable term premia or to the reversion of temporary overreaction about future monetary policy. Future research should investigate the origins of bond price deviations from their drift and of the associated returns predictability documented in this paper.

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Appendix

A Data

We employ quarterly data in our empirical analysis; thus, we proxy for the 1-period bond yields using the end-of-quarter 3-month Treasury bill rates from the Federal Reserve's H.15 release. Our sample period starts with Paul Volckers appointment as Fed chairman, because of evidence that monetary and macroeconomic dynamics changed at that time (e.g., Gertler et al., 1999).

Zero-coupon Treasury yields with 1- to 10-year maturities are from Gürkaynak et al. (2007).

The Federal Reserve's perceived target rate (PTR) for inflation is a survey-based measure of long-run inflation expectations; PTR is used in the Fed's FRB/US model and available at https://www.federalreserve.gov/econres/us-models-package.htm.

MY is available until 2050 and is hand-collected from various past Census reports available at https://www.census.gov/data.html. Potential output is available until 2030 and can be downloaded at https://fred.stlouisfed.org/series/GDPPOT. See also Figure A.1.

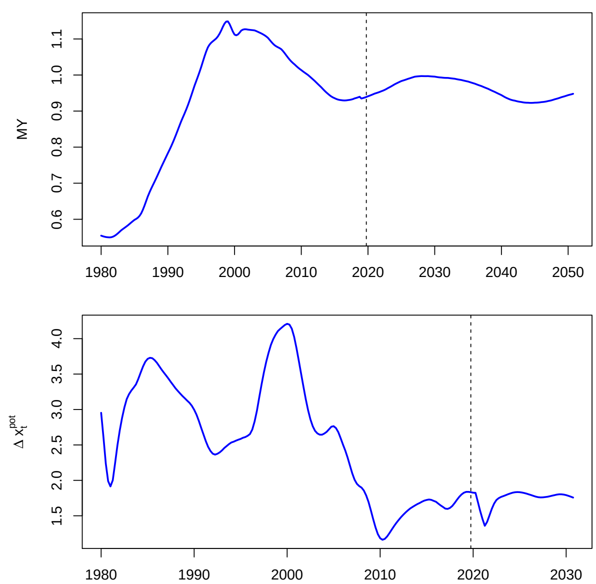

Figure A.1: Demographics and Potential Output Growth. This figure shows the dynamics for the ratio of middle-aged (40-49) to young (20-29) population, MY, and for potential output growth, Δx_t^{pot}. MY is available until 2050 and is hand-collected from various past Census reports available at https://www.census.gov/data.html. Potential output is available until 2030 and can be downloaded at https://fred.stlouisfed.org/series/GDPPOT. Dotted vertical lines denote the end of our sample, i.e., 2019:Q4. Quarterly observations.

B Additional Results

Table B.1: Testing Parametric Restriction on the Cointegrating Relationship between Yields and Drifting Equilibrium Rates

This table reports OLS estimates for the regression $y_t^{(n)} = \alpha + \beta y_t^{(n),*} + \varepsilon_t$, where $y_t^{(n)}$ is the observed yield at time t of a bond with maturity n-period and $y_t^{(n),*} = \left(\frac{1}{n}\right)\sum_{i=0}^{n-1} E[y_{t+i}^{(1)} \mid I_t]$. Values in parethesis are 95% confidence interval. Costant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

	$y_t^{(8)}$	$y_t^{(12)}$	$y_t^{(20)}$	$y_t^{(28)}$	$y_t^{(40)}$
	(1)	(2)	(3)	(4)	(5)
$y_t^{(8),*}$	1.077***				
	(0.940, 1.214)				
$y_t^{(12),*}$		1.056***			
		(0.913, 1.199)			
$y_t^{(20),*}$			1.013***		
			(0.855, 1.171)		
$y_t^{(28),*}$				0.985***	
				(0.815, 1.155)	
$y_t^{(40),*}$					0.962***
					(0.766, 1.158)
Observations	160	160	160	160	160
R^2	0.921	0.913	0.903	0.894	0.882

Table B.2: Hidden Factor Regression for Yields

This table reports OLS estimates for the regression $y_t^{(n)} = \alpha + \beta f_t + \epsilon_t^{(n)}$, where $y_t^{(n)}$ is the yield with maturity n and f_t are different factors to be tested. Panel A reports results for the deviation of inflation from its long-run target $(\pi_t - \pi_t^*)$. Panel B reports results for the output gap x_t. Panel C reports results for the monetary policy shocks $u_t^{(1)}$. Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors with automatic bandwidth selection procedure as described in Newey and West (1994). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

Panel A: The inflation gap.

	$y_t^{(8)}$	$y_t^{(12)}$	$y_t^{(20)}$	$y_t^{(28)}$	$y_t^{(40)}$
	(1)	(2)	(3)	(4)	(5)
$(\pi_t - \pi_t^*)$	0.364	0.205	0.022	−0.057	−0.091
	(1.524)	(1.603)	(1.546)	(1.438)	(1.426)
Observations	159	159	159	159	159
R^2	0.005	0.002	0.000	0.000	0.000

Panel B: The Output Gap.

	$y_t^{(8)}$	$y_t^{(12)}$	$y_t^{(20)}$	$y_t^{(28)}$	$y_t^{(40)}$
	(1)	(2)	(3)	(4)	(5)
x_t	0.063	−0.082	−0.313	−0.468	−0.608
	(2.552)	(2.629)	(2.538)	(2.350)	(2.008)
Observations	159	159	159	159	159
R^2	0.000	0.000	0.004	0.011	0.020

Panel C: The Monetary Policy Shocks.

	$y_t^{(8)}$	$y_t^{(12)}$	$y_t^{(20)}$	$y_t^{(28)}$	$y_t^{(40)}$
	(1)	(2)	(3)	(4)	(5)
$u_t^{(1)}$	0.990**	0.921*	0.810*	0.733	0.655
	(0.476)	(0.472)	(0.459)	(0.446)	(0.432)
Observations	159	159	159	159	159
R^2	0.104	0.094	0.080	0.071	0.062

Table B.3: Predictive Regressions (across different maturities): Slope versus Cyclical Component

This table reports OLS estimates for the regression $rx_{t+4}^{(n)} = \alpha + \beta_1(y_t^{(n)} - y_t^{(4)}) + \beta_2(-(n-4)\ u_t^{(n-4)}) + \epsilon_t$, where $rx_{t+4}^{(n)}$ is the realized one-year holding period excess return of a bond with maturity n-period, $y_t^{(n)} - y_t^{(4)}$ is the slope for a n-period bond, and $(-(n-4)\ u_t^{(n-4)})$ is the deviation of a n-period maturity yield from its drift. Values in parenthesis are conservative standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

	$rx_{t+4}^{(8)}$		$rx_{t+4}^{(12)}$		$rx_{t+4}^{(20)}$		$rx_{t+4}^{(28)}$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$y_t^{(8)} - y_t^{(4)}$	1.613***							
	(0.540)							
$-(8-4)\ u_t^{(4)}$	−0.930***	−0.851***						
	(0.219)	(0.217)						
$y_t^{(12)} - y_t^{(4)}$			1.637**					
			(0.765)					
$-(12-4)\ u_t^{(8)}$			−0.792***	−0.759***				
			(0.199)	(0.199)				
$y_t^{(20)} - y_t^{(4)}$					1.563			
					(0.994)			
$-(20-4)\ u_t^{(16)}$					−0.726***	−0.744***		
					(0.178)	(0.179)		
$y_t^{(28)} - y_t^{(4)}$							1.356	
							(1.145)	
$-(28-4)\ u_t^{(24)}$							−0.682***	−0.720***
							(0.168)	(0.171)
Observations	156	156	156	156	156	156	156	156
Adjusted R^2	0.323	0.243	0.339	0.270	0.361	0.313	0.357	0.331

Table B.4: Predictive Regressions (quarterly holding period returns): Slope versus Cyclical component

This table reports OLS estimates for the regression $rx_{t+1}^{(n)} = \alpha + \beta_1(y_t^{(n)} - y_t^{(1)}) + \beta_2(-(n-1)\ u_t^{(n-1)}) + \epsilon_t$, where $rx_{t+1}^{(n)}$ is the realized one-quarter holding period excess return of a bond with maturity n-period, $y_t^{(n)} - y_t^{(1)}$ is the slope for a n-period bond, and $(-(n-1)\ u_t^{(n-1)})$ is the deviation of a n-period maturity yield from its drift. Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors with automatic bandwidth selection procedure as described in Newey and West (1994). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

	$rx_{t+1}^{(8)}$ (1)	$rx_{t+1}^{(12)}$ (2)	$rx_{t+1}^{(20)}$ (3)	$rx_{t+1}^{(28)}$ (4)	$rx_{t+1}^{(40)}$ (5)
$y_t^{(8)} - y_t^{(1)}$	-0.151^{*}				
	(0.084)				
$-(8-1)\ u_t^{(7)}$	-0.146^{***}				
	(0.036)				
$y_t^{(12)} - y_t^{(1)}$		-0.103			
		(0.154)			
$-(12-1)\ u_t^{(11)}$		-0.183^{***}			
		(0.043)			
$y_t^{(20)} - y_t^{(1)}$			-0.026		
			(0.213)		
$-(20-1)\ u_t^{(19)}$			-0.183^{***}		
			(0.046)		
$y_t^{(28)} - y_t^{(1)}$				0.025	
				(0.251)	
$-(28-1)\ u_t^{(27)}$				-0.172^{***}	
				(0.045)	
$y_t^{(40)} - y_t^{(1)}$					0.062
					(0.307)
$-(40-1)\ u_t^{(39)}$					-0.153^{***}
					(0.044)
Observations	159	159	159	159	159
R^2	0.140	0.130	0.116	0.108	0.093

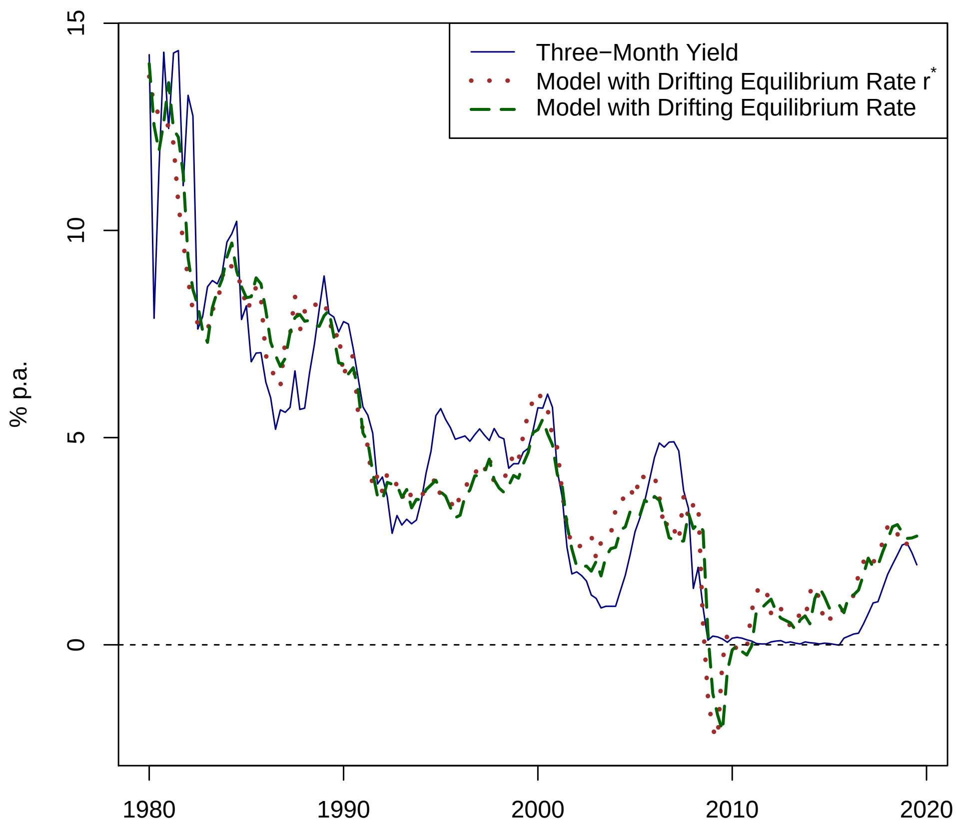

Figure B.1: Actual vs Fitted Short-Term Rate: Additional Results. This figure shows actual three-months yield and fitted values for our baseline (cointegrated) model with drifting equilibrium rates (c.f. equation (2); green dashed line), and for a cointegrated rule with r^* (brown dotted line). The estimated cointegrated policy rule with r^* has the following coefficients (HAC standard errors in parenthesis):

$$y_t^{(1)} = \underset{(0.092)}{0.667}^{***} \; r_t^* + \underset{(0.068)}{1.449}^{***} \; \pi_t^* + \underset{(0.173)}{0.822}^{***} \; E_t(\pi_{t+1} - \pi_{t+1}^*) + \underset{(0.083)}{0.318}^{***} \; E_t(x_{t+1}), \quad R^2 = 94.3\%$$
.

We denote r^* as the equilibrium real rate. We get an estimate for the equilibrium real rate by regressing the real rate $r_t = y_t^{(1)} - E_t(\pi_{t+4})$ on MY and potential output growth. We use as $E_t(\pi_{t+4})$ the expected one-year ahead inflation from the Survey of Professional Forecasters (SPF). The estimates for r^* are:

$$r_t^* = -\underset{(1.397)}{4.040}^{***} \; MY_t + \underset{(0.309)}{1.812}^{***} \; \Delta x_t^{pot}, \; R^2 = 68\% \; .$$

Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

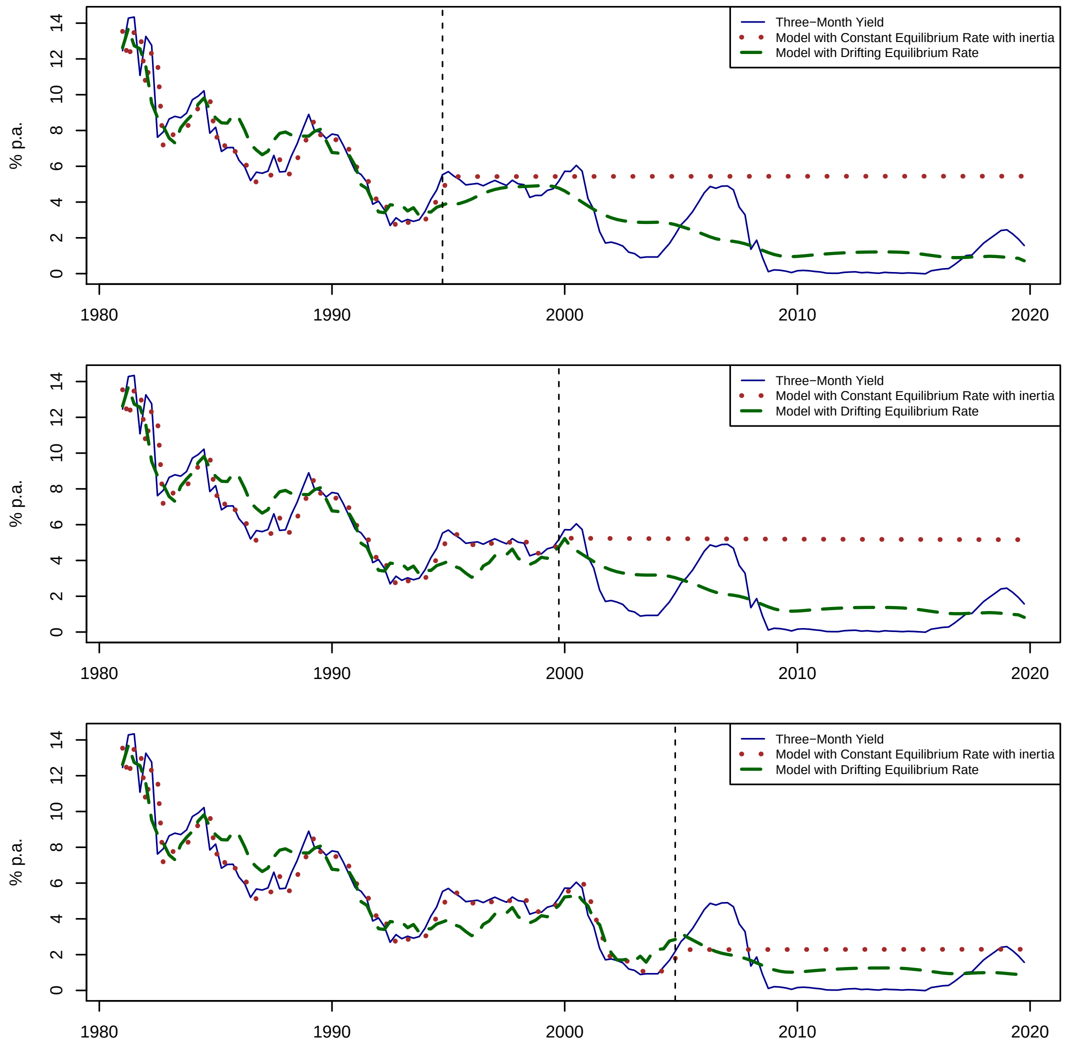

Figure B.2: Short-Term Rate Forecasts: Additional Results. This figure shows actual three-months yield and predicted rates implied by equation (2) in case of the policy rule with constant equilibrium rate and inertia (brown dotted line) or our baseline (cointegrated) model with drifting equilibrium rates (green dashed line). The estimated empirical Taylor rule with inertia has the following coefficients (HAC standard errors in parenthesis):

$$y_t^{(1)} = \underset{(0.108)}{0.310^{***}} + \underset{(0.015)}{0.935^{***}}\ y_{t-1}^{(1)} - \underset{(0.140)}{0.034}\ E_t(\pi_{t+1} - \pi_{t+1}^*) + \underset{(0.028)}{0.070^{**}}\ E_t(x_{t+1}),\ R^2 = 92.7\%.$$

Dotted vertical lines represent the end of in-sample estimation period. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

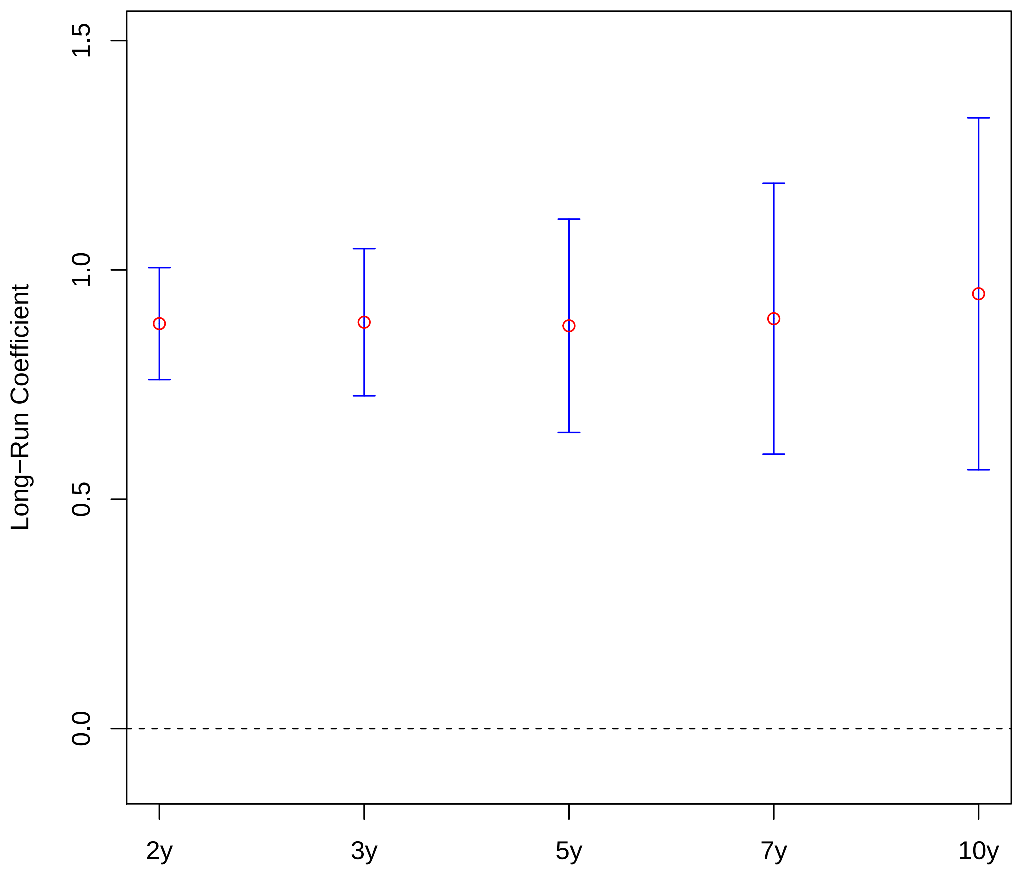

Figure B.3: Long-Run Loadings of Short-Term Cycle. This figure shows the long-run coefficients for the regression $u_t^{(n)} = \phi_1 u_{t-1}^{(n)} + \phi_2 u_t^{(1)} + \varepsilon_t$, where $u_t^{(n)}$ and $u_t^{(1)}$ are defined respectively in equation (7) and (8). Long-run coefficients are computed as $\phi_2/(1-\phi_1)$. The 95% confidence interval is calculated via the delta method. For the 10-year Treasury bond, the estimated regression is (HAC standard errors in parenthesis):

$$u_t^{(40)} = \underset{(0.045)}{0.784^{***}}\ u_{t-1}^{(40)} + \underset{(0.049)}{0.205^{***}}\ u_t^{(1)},\ R^2 = 78.2\%\ .$$

Results are robust to including inflation gap and output gap. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

C International Evidence

Table C.1: Predictive Regressions (across different maturities): Slope versus Cyclical Component

This table reports OLS estimates for the regression $rx^{(n)}_{t+4} = \alpha + \beta_1(y^{(n)}_t - y^{(4)}_t) + \beta_2(-(n-4)\ u^{(n-4)}_t) + \epsilon_t$, where $rx^{(n)}_{t+4}$ is the realized one-year holding period excess return of a bond with maturity n-period, $y^{(n)}_t - y^{(4)}_t$ is the slope for a n-period bond, and $(-(n-4)\ u^{(n-4)}_t)$ is the deviation of a n-period maturity yield from its drift. Values in parenthesis are conservative standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. For UK, zero-coupon bonds data are from the Bank of England (`https://www.bankofengland.co.uk/statistics/yield-curves`); the sample period is 1980:Q1 to 2019:Q4. For Canada, zero-coupon bonds data are from the Bank of Canada (`https://www.bankofcanada.ca/rates/interest-rates/bond-yield-curves/`); the sample period is 1986:Q1 to 2019:Q4.

Panel A: UK.

	$rx^{(12)}_{t+4}$		$rx^{(20)}_{t+4}$		$rx^{(40)}_{t+4}$	
	(1)	(2)	(3)	(4)	(5)	(6)
$y^{(12)}_t - y^{(4)}_t$	1.159*					
	(0.603)					
$-(12-4)\ u^{(8)}_t$	−0.309**	−0.327**				
	(0.150)	(0.149)				
$y^{(20)}_t - y^{(4)}_t$			1.544*			
			(0.883)			
$-(20-4)\ u^{(16)}_t$			−0.321**	−0.327**		
			(0.150)	(0.150)		
$y^{(40)}_t - y^{(4)}_t$					1.979	
					(1.312)	
$-(40-4)\ u^{(36)}_t$					−0.286**	−0.287**
					(0.131)	(0.131)
Observations	156	156	156	156	156	156
Adjusted R^2	0.124	0.069	0.150	0.081	0.160	0.089

Panel B: Canada.

	$rx^{(12)}_{t+4}$		$rx^{(20)}_{t+4}$		$rx^{(40)}_{t+4}$	
	(1)	(2)	(3)	(4)	(5)	(6)
$y^{(12)}_t - y^{(4)}_t$	1.079					
	(0.784)					
$-(12-4)\ u^{(8)}_t$	−0.421**	−0.450***				
	(0.165)	(0.159)				
$y^{(20)}_t - y^{(4)}_t$			1.384			
			(1.054)			
$-(20-4)\ u^{(16)}_t$			−0.397**	−0.428***		
			(0.158)	(0.151)		
$y^{(40)}_t - y^{(4)}_t$					1.906	
					(1.649)	
$-(40-4)\ u^{(36)}_t$					−0.325**	−0.385***
					(0.135)	(0.119)
Observations	132	132	132	132	132	132
Adjusted R^2	0.208	0.152	0.253	0.189	0.263	0.202