Demographic Trends, the Dividend-Price Ratio and the Predictability of Long-Run Stock Market Returns

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Abstract

This paper documents the existence of a slowly evolving trend in the dividend-price ratio, dp_t, determined by a demographic variable, MY: the middle-aged to young ratio. Deviations of dp_t from this long-run component explain transitory but persistent fluctuations in stock market returns. The relation between MY and dp_t is a prediction of an overlapping generation model. The joint significance of MY and dp_t in long-horizon forecasting regressions for market returns explain the mixed evidence on the ability of dp_t to predict stock returns and provide a model-based interpretation of statistical corrections for breaks in the mean of this financial ratio.

KEYWORDS: dynamic dividend growth model, long run returns predictability, demographics.

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I Introduction

This paper characterizes the relationship between the dividend price ratio and stock market returns in a model where a demographic variable, MY, the middle aged to young ratio, captures the slowly evolving component in the dividend-price ratio. Interest in this model is partly motivated by the very high persistence of the dividend-price ratio that makes long-horizon regressions hard to interpret. MY allows to extract from the log dividend-price a stationary variable capturing time-variation in the investment opportunity set and to specify a more reliable forecasting model for long-horizon stock market returns. Demographics are a very natural input into a forecasting model of long-horizon returns, and, consequently, into the optimal asset allocation decision of a long-horizon investor. We interpret MY as the information component that drives long-horizon stock market fluctuations, after the noise in short-horizon stock market fluctuations subsides.

The empirical relevance of the dividend-price ratio for predicting long-run stock market returns is one of the most debated issues in financial econometrics. In fact, this variable regularly plays an important role in recent empirical literature that has replaced the long tradition of the efficient market hypothesis (Fama, 1970) with a view of predictability of returns (see, for example, Cochrane, 2007). However, there is an ongoing debate on the robustness of return predictability and its potential use from a portfolio allocation perspective (Boudoukh et al. (2008), Goyal and Welch (2008)).

Most of the available evidence on predictability can be framed within the dynamic dividend growth model proposed by Campbell and Shiller (1988). This model relies on a log-linearized version of one-period returns on the stock portfolio. Under the assumption of its stationarity and of the validity of a standard transversality condition,the log of the price-dividend ratio, dp_t, is expressed as a linear function of the future discounted dividend growth,

Δd_{t+j} and of future returns, h^s_{t+j} :

$$dp_t = \overline{dp} + \sum_{j=1}^{\infty} \rho^{j-1} E_t[(h^s_{t+j} - \bar{h}) - (\Delta d_{t+j} - \bar{d})] \tag{1}$$

where $\overline{dp}$, the mean of the dividend-price ratio, $\bar{d}$, the mean of dividend growth rate, $\bar{h}$, the mean of log return and ρ are constants.

Under the maintained hypothesis that stock market returns, and dividend-growth are covariance-stationary, Eq. (1) says that the log dividend-price ratio is stationary, i.e. the (log) price and the (log) dividend are cointegrated with a (-1,1) cointegrating vector, and that deviations of (log) prices from the common trend in (log) dividends summarize expectations of either stock market returns, or dividend growth or some combination of the two.

The empirical investigation of the dynamic dividend growth model has established a few empirical results :

(i) dp_t is a very persistent time-series and forecasts stock market returns and excess returns over horizons of many years (Fama and French (1988), Campbell and Shiller (1988), Cochrane (2005, 2007).

(ii) dp_t does not have important long-horizon forecasting power for future discounted dividend-growth (Campbell (1991), Campbell, Lo and McKinlay (1997) and Cochrane (2001)).

(iii) the very high persistence of dp_t has led some researchers to question the evidence of its forecasting power for returns, especially at short-horizons. Careful statistical analysis that takes full account of the persistence in dp_t provides little evidence in favour of the stock-market return predictability based on this financial ratio (Nelson and Kim (1993); Stambaugh (1999); Ang and Bekaert (2007); Valkanov (2003); Goyal and Welch (2003) and Goyal and Welch (2008)). Structural breaks have also been found in the relation between dp_t and future returns (Neely and Weller (2000), Paye and Timmermann (2006) and Rapach and Wohar (2006)).

(iv) More recently, Lettau and Ludvigson (2001, 2005) have found that dividend growth

and stock returns are predictable by long-run equilibrium relationships derived from a linearized version of the consumer's intertemporal budget constraint. The excess consumption with respect to its long run equilibrium value is defined by the authors alternatively as a linear combination of labour income and financial wealth, cay_t, or as a linear combination of aggregate dividend payments on human and non-human wealth, cdy. cay_t and cdy_t are much less persistent than dp_t, they are predictors of stock market returns and dividend-growth, and, when included in a predictive regression relating stock market returns to dp_t, they swamp the significance of this variable. Lettau and Ludvigson (2005) interpret this evidence in the light of the presence of a common component in dividend growth and stock market returns. This component cancels out from (1) , cay_t and cdy_t are instead able to capture it as the linearized intertemporal consumer budget constraint delivers a relationship between excess consumption and expected dividend growth or future stock market returns that is independent from their difference.

A recent strand of the empirical literature has related the contradictory evidence on the dynamic dividend growth model to the potential weakness of its fundamental hypothesis that the log dividend-price ratio is a stationary process (Lettau and Van Nieuwerburgh (2008), LVN henceforth). LVN use a century of US data to show evidence on the breaks in the constant mean $\overline{dp}$. We report the time series of US data on dp_t over the last century in Figure 1. As a matter of fact, the evidence from univariate test for non-stationarity and bivariate cointegration tests does not lead to the rejection of the null of the presence of a unit-root in dp_t[1].

Insert here Figure 1

[1]The Dickey-Fuller test for the null of non-stationarity delivers an observed statistics of -2.34 when computed over the full sample 1911-2008 and a value of -1.72 when computed over the sample 1955-2008. This evidence is confirmed by the implementation of the Johansen (1991) test on a bivariate VAR for p_t and d_t, that does not reject the null hypothesis of at most zero cointegrating vectors over the full-sample and the post-war subsample.

As shown in Figure 1, LVN identify two statistically significant breaks in the mean of dp_t in 1954 and 1991. They then provide evidence that deviations of dp_t from its time-varying mean have a much stronger forecasting power for stock market returns than deviations of dp_t from a constant mean[2]. This evidence for time-variation in the mean of the dividend-price ratio has been also confirmed by Johannes et al. (2008), who estimate the process for log dividend-price ratio within a particle filtering framework.

So far the evidence towards a slowly evolving mean in dp_t has been reported as a pure statistical fact. LVN give some hints on possible causes for the breaks arising from economic fundamentals due to technological innovation, changes in expected return, etc. but do not explore the possible effects of fundamentals any further. The idea of correcting dp_t to reduce its persistence has been also pursued by an alternative strand of research that relates the apparent non-stationarity of this variable to a shift in corporate payout policies. Boudoukh et al. (2007) provide a new measure of the cash flow based net payout yield (dividends plus repurchases minus issuances are used instead of dividends to construct the relevant ratio) that is more quickly mean reverting than the dividend-price ratio. Yet, this suggested measure is unlikely to explain the full decrease in this financial ratio as argued by LVN. Moreover other financial ratios such as earning-price ratio witness similar declines.

The aim of our paper is to investigate the possibility that the slowly evolving mean in the log dividend-price is related to demographic trends. We first illustrate how the theoretical model by Geanakoplos, Magill and Quinzii (2004, henceforth, GMQ) implies that a specific demographic variable, MY_t, the proportion of middle-aged to young population, explains fluctuations in the dividend yield.

GMQ consider an overlapping generation model where the demographic structure mimics the pattern of live births in the US, which have featured alternating twenty-year periods of

[2]These results are confirmed by the search for possible structural breaks in the cointegrating relationship based on the application of the recursive test based on the non zero-eigenvalues suggested in Hansen and Johansen (1999). The eigenvalue shows a remarkable amount of variability over the examination period with indication of three break points around 1950, 1980, 2000.

boom and busts. They conjecture that the life-cycle portfolio behavior (Bakshi and Chen (1994)), which suggests that agents should borrow when young, invest for retirement when middle-aged, and live off from their investment once they are retired, plays an important role in determining equilibrium asset prices. In their model, the demographic structure requires that when the MY ratio is small (large), there will be excess demand for consumption (saving) by a large cohort of retirees (middle-aged). For the market to clear, equilibrium prices of financial assets and therefore the dividend-price ratio should adjust, i.e. decrease (increase), so that saving (consumption) is encouraged for the middle-aged. We take the GMQ model to the data via the conjecture that fluctuations in MY_t could capture a slowly evolving mean in dp_t within the dynamic dividend growth model. Demographic trends should capture the slowly evolving mean in dp_t and then, deviations of dp_t from $\overline{dp_t}$ could be used as a potential predictor for long-term stock market returns and dividend growth. Our empirical strategy has the potential for identifying separately the importance of demographic variables for high-frequency and low-frequency fluctuations in asset prices. Investigations conducted in the literature on the interaction between asset prices and demographic variables have traditionally concentrated either on high-frequency or low frequency fluctuations but have never considered an empirical framework based on the dynamic dividend growth model, capable of accommodating both of them, with a different role (see Erb et al. (1996), Poterba (2001), Goyal (2004), Ang and Maddaloni (2005) and DellaVigna and Pollet (2006)).

We first use long-run predictive regressions and cointegration analysis to assess the statistical significance of MY_t in a dynamic dividend growth model. The robustness of our results is evaluated by comparing the predictive power of the dividend-price ratio corrected for demographics with that of the dividend-price ratio, the dividend-price ratio corrected for breaks in mean (LVN) and the cash flow based net payout yield (Boudoukh et al. (2007)). The role of MY_t is then further investigated against different alternative specifications, in particular those based on cay_t and cdy_t. Finally, the availability of long-run projections for MY_t is exploited to derive predictions of long-run equity returns up to 2050.

II Demography and the Dividend-Price Ratio: The GMQ Model

GMQ analyze an overlapping generation model in which the demographic structure mimics the pattern of live births in the US. Live births in the US have featured alternating twenty-year periods of boom and busts. They consider an OLG exchange economy with a single good (income) and three periods; young, middle-aged, retired. Each agent (except retirees) has an endowment and labor income w= (w^y,w^m,0). There are two types of financial instruments, a riskless bond and a risky asset, which allow agents to redistribute income over time. In the simplest version of the model, dividends and wages are deterministic, hence the bond and the risky asset are perfect substitutes. GMQ assume that in *odd (even)* periods a large (small) cohort N(n) enters the economy, therefore in every odd (even) period there will be $\{N, n, N\}(\{n, N, n\})$ cohorts living. They conjecture that the life-cycle portfolio behavior (Bakshi and Chen (1994)) which suggests that agents should borrow when young, invest for retirement when middle-aged, and live off from their investment once they are retired, plays important role in determining equilibrium asset prices.

Let q_o (q_e) be the bond price and $\{c_y^o, c_m^o, c_r^o\}$ ($\{c_y^e, c_m^e, c_r^e\}$) the consumption stream in the odd (even) period. The agent born in an odd period then faces the following budget constraint

$$c_y^o + q_o c_m^o + q_o q_e c_r^o = w^y + q_o w^m \tag{2}$$

and in an even period

$$c_y^e + q_e c_m^e + q_o q_e c_r^e = w^y + q_e w^m \tag{3}$$

Moreover, in equilibrium the following resource constraint must be satisfied

$$Nc_y^o + nc_m^o + Nc_r^o = Nw^y + nw^m + D \tag{4}$$

$$nc_y^e + Nc_m^e + nc_r^e = nw^y + Nw^m + D \tag{5}$$

where D is the aggregate dividend for the investment in financial markets. If q_o were equal to q_e, the agents would choose to smooth their consumption, i.e. $c_y^i = c_m^i = c_r^i$ for $i = o, e$, but then for values of wages and aggregate dividend calibrated from US data the equilibrium condition above would be violated leading to excess demand either for consumption or saving. To illustrate this point we refer to the calibration provided by GMQ; take $N = 79, n = 69$ as the size (in millions) of Baby Boom (1945-64) and Baby Bust (1965-84) generations (thus, we obtain in an even period a high MY ratio of $MY_t = \frac{N}{n} = 1.15$, and in odd period $MY_t = \frac{n}{N} = 0.87$ (see Figure 3a)) and $w^y = 2, w^m = 3$ to match the ratio (middle to young cohort) of the average annual real income in US. We can calculate the total wage in even and odd periods using $Nw^y + nw^m$ for odd periods and $nw^y + Nw^m$ for even periods, and then given the average ratio (0.19) of dividend to wages we compute the aggregate dividends. Assuming an annual discount factor of 0.97, which translates to a discount of 0.5 in the model of 20-year periods, if $q_o = q_e = 0.5$ were to hold and agents smooth their consumption, from the budget constraint (eq. 6-7) we obtain $c_y^i = c_m^i = c_r^i = \bar{c}$ $= 2$, but then the resource constraint (eq. 8-9) above would have been violated. For instance, an agent from the Baby Bust generation would enter in an even period in the model, i.e. (n, N, n) and high MY ratio, and faces the following aggregate resource constraint: $n(c_y^e - w^y) + N(c_m^e - w^m) + nc_r^e - D = 69 \times (2 - 2) + 79(2 - 3) + 69 \times 2 - 70 = -11$, where $D = 0.19(\frac{375+365}{2}) = 70$. This leads to excess saving in the economy. For equilibrium conditions to hold, the model implies that asset prices should increase and hence discourage saving in the economy (the experience we observed during the 90's in US). When the MY ratio is small (large), i.e. an odd (even) period, there will be excess demand for consumption (saving) by a large cohort of retirees (middle-aged) and for the market to clear, equilibrium prices of financial assets should adjust, i.e. decrease (increase), so that saving (consumption) is encouraged for the middle-aged. Thus, letting q_t^b be the price of the bond at time t, in a stationary equilibrium, the following holds

$$q_t^b = q_o \text{ when period odd}$$
$$q_t^b = q_e \text{ when period even}$$

together with the condition $q_o < q_e$. Moreover the model predicts a positive correlation between MY and market prices, consequently a negative correlation with the dividend yield.

So, since the bond prices alternate between q_o and q_e, then the price of equity must also alternate between q_t^e and q_t^e as follows

$$\begin{aligned} q_o^{eq} &= Dq_o + Dq_oq_eq_o + Dq_oq_eq_oq_o + ... \\ q_e^{eq} &= Dq_e + Dq_eq_oq_e + Dq_eq_oq_eq_e + ... \end{aligned}$$

which implies

$$\begin{aligned} DP_o &= \frac{D}{q_o^{eq}} = \frac{1 - q_oq_e}{q_oq_e + q_o} \\ DP_e &= \frac{D}{q_e^{eq}} = \frac{1 - q_oq_e}{q_oq_e + q_e} \end{aligned}$$

where DP_o (DP_e) is the dividend-price ratio implied by low (high) MY in the model for odd (even) periods.

III The Empirical Evidence

The GMQ model provides a foundation for a long-run negative relationship between the dividend-price ratio and demography. GMQ define the empirical counterpart of the MY_t ratio as the proportion of the number of agents aged 40-49 to the number of agents aged 20-29, which serves as a sufficient statistic for the whole population pyramid. We report the MY_t ratio in Figure 2. Interestingly, this variable shows highly persistent dynamics and a twin peaked behavior, with peaks and troughs around 1950, 1980, 2000, around the break points in dp_t.

Insert here Figure 2

To combine the GMQ model with the dynamic dividend growth we consider the derivation of LVN, who allow for a time varying mean in the linearization and consider MY_t as the potential determinant of this slowly evolving process.

$$\begin{aligned} dp_t &= \overline{dp_t} + \sum_{j=1}^{\infty} \rho^{j-1} E_t[(h^s_{t+j} - \bar{h}) - (\Delta d_{t+j} - \bar{d})] \\ \overline{dp_t} &= \beta_0 + \beta_1 MY_t + u_t \end{aligned} \tag{6}$$

Inserting GMQ into the dynamic dividend growth model leads to the prediction that the (log) dividend-price adjusted for demographics should be significant in the long-horizon forecasting regression for real stock market returns, the real dividend growth, and their difference. MY_t should also be significant in explaining the persistence of the dividend-price, and the variable predicted to be stationary in this extended model is not the dividend-price but a combination between price, dividends and MY_t. We investigate the hypothetical cointegrating relation between dividend, prices and MY_t, by running the Johansen (1988) procedure on a cointegrating system based on the vector of variables $\mathbf{y}'_t = \begin{bmatrix} d_t & p_t & MY_t \end{bmatrix}$.

A Long-Horizon Forecasting Regressions

We report in Table 1 the evidence from the long-horizon forecasting regression. To make our evidence directly comparable with that reported in Lettau and Ludvigson (2005) we consider predictive regressions for annual data with horizons ranging from one to six years. We consider the annual data for the S&P 500 index from 1909 to 2008 taken from Robert Shiller's website, dividends are twelve-month moving sums of dividends paid on the S&P 500 index. These series coincide with those used in Goyal and Welch (2008), and made available at Amit Goyal's website. A full description of all data used in our empirical analysis is provided in the Data Appendix.

Table 1.1-1.3 reports the evidence for forecasting returns, dividend growth, returns ad-

justed for dividend growth, based on the following three models:

$$\begin{aligned}
\sum_{j=1}^{k}(h_{t+j}^{s}) &= \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j} \\
\sum_{j=1}^{k}(\Delta d_{t+j}) &= \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j} \\
\sum_{j=1}^{k}(h_{t+j}^{s} - \Delta d_{t+j}) &= \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j} \\
k &= 1, ..., 6
\end{aligned}$$

Insert here Table 1.1-1.3

In each forecasting regression MY_t is measured at the end of the forecasting period. We report heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) to account for overlapping observations where the bandwidth has been selected following the procedure described in Newey and West (1994). Alternatively, we also conduct a (wild) bootstrap exercise (Davidson and Flachaire (2008)) to compute p-values. To take care of the potential effect on statistical inference in finite sample of the use of overlapping data we also report the rescaled t-statistic recommended by Valkanov (2003) for the hypothesis that the regression coefficient on the dividend-price adjusted for the effect of demographics is zero. We report test of predictability at each horizon but we also compute joint tests across horizons based on SUR estimation and report in the last row the relevant χ^2 statistics with associated p-values.

The evidence can be summarized as follows:

i) MY_t is always significant along with p_t and d_t in all the forecasting regressions for real stock market returns (Panel A). The adjusted R^2 of the predictive regression increases with the horizon from 0.09 at the 1-year horizon to 0.54 at the 6-year horizon. Consistently with

the prediction of the GMQ model, the effect of MY is negative on the slowly evolving mean of the dividend-price and hence positive for expected returns at all horizons.

ii) MY_t is never significant in the forecasting regressions for real dividend growth (Panel B). The adjusted R^2 of the predictive regression declines with the horizon from 0.15 at the 1-year horizon to 0.06 at the 6-year horizon.

iii) MY_t is always significant along with p_t and d_t in all the forecasting regressions for real stock market returns adjusted for real dividend growth (Panel C). The adjusted R^2 of the predictive regression increases with the horizon from 0.26 at the 1-year horizon to 0.67 at the 6-year horizon. The evidence of the strongest predictability of $\sum_{j=1}^{k}(h^s_{t+j} - \Delta d_{t+j})$ is fully consistent with the dynamic dividend growth model. Such evidence, paired with that on the different forecastability of the two components of stock market returns adjusted for dividend growth, rules out the dominance of a common stochastic component for the determination of the dynamics of dividend growth and stock market returns.

iv) MY_t dominates alternative approaches proposed in the literature to capture an evolving mean in the dividend-price ratio. In the last rows of each panel of the Tables 1.1-1.3 we report the results of augmenting the long-run forecasting regressions based on the GMQ model with alternative filtered dividend-price series. In particular we consider, dp_t^{LVN}, the (log) dividend-price corrected for breaks in LVN and dp_t^{BMRR}, the cash flow based net payout yield (dividends plus repurchases minus issuances) proposed by Boudoukh et al. (2007).

Overall the long-run forecasting regressions lend strong support to the inclusion of MY_t in the traditional dynamic dividend growth model. The dividend-price corrected for a slowly long-run mean, determined by MY_t , predicts long-run stock market returns and long-run stock market returns adjusted for dividend growth, but it does not predict long-run dividend growth. The R^2 associated to the relevant predictive regressions increases with the horizon. This evidence of a positive relation between predictability and the forecasting horizon is interesting, in that both the dynamic dividend growth model and the GMQ model establish a predictive relation for long-run returns. In fact, the most natural horizon for the GMQ

model is one generation, i.e. about twenty years. Of course, it is difficult to establish some evidence via predictive regressions for twenty years returns, as we have only one century of data. To give the reader a visual impression on the relationship between real stock market returns and MY_t at a frequency as close as possible to that implied by the relevant models, we report in Figure 3 MY_t and 20-year real stock market returns. We find the graphical evidence interesting and fully consistent with the statistical evidence from the long-run regressions at higher frequencies.

Insert here Figure 3

B Cointegration

The evidence of forecasting power of a linear combination of dividend, prices and MY_t for forecasting long-run returns and long-run returns adjusted for dividend growth, provides indirect evidence of stationarity of such a combination. The validity of this hypothesis can be further investigated by running the Johansen (1988, 1991) procedure on a cointegrating system based on the vector of variables $\mathbf{y}_t' = \left[\begin{array}{ccc} d_t & p_t & MY_t \end{array} \right]$. We then test for cointegration within a three-variate VAR[3].

Insert here Table 2.1-2.2

We report in Table 2.1-2.2 the evidence for the full-sample and for the sub-sample 1955-2008. The results lead to the rejection of the null of at most zero cointegrating vectors, while the null of at most one cointegration vector cannot be rejected. The evidence in favour of one cointegrating vector in which all variables are always significant confirms that the high

[3]See Appendix A for the details of the specification of our statistical model. In a previous version of this paper we allow for a presence of a technology-driven trend (GMQ, p.6), proxied by Total Factor Productivity, in the long-run equilibium relationship. We have decided to exclude TFP from the cointegrating relationship on the basis of two arguments i) the presence of a technology driven trend in the dividend pice ratio is very hard to justify theoretically ii) the TFP trend does not attract any significance when included in the long-run forecasting regressions discussed in the previous section. We are grateful to an anonymos referee for attracting our attention on this point.

persistence of the dividend-price is matched by the high persistence of MY_t. Using the augmented Dickey-Fuller test, the null of a unit root in MY cannot be rejected. The coefficients determining the adjustment in presence of disequilibrium in the Vector Error Correction model confirm the evidence from the forecasting regressions reported in the previous section: stock market returns adjust in presence of disequilibrium. The significance of MY_t increases in the second sub-sample, where LVN found the two breaks in dp_t[4].

Insert here 4.1-4.2

Figures 4.1-4.2 provides a graphical assessment of the capability of MY_t of capturing the slowly evolving mean of dp_t. Figure 4.1 reports the residuals from our cointegrating vector, along with dp_t, and the deviations of dp_t from $\overline{dp}^{LVN}$, the shifting mean identified by LVN. Figure 4.2 reports residuals from our cointegrating vector with the cycle of dp_t, obtained by applying an Hodrick-Prescott filter to the original series. The graphical evidence illustrates how the cointegration based correction matches the break-based correction in LVN (2008) and the cycle obtained by applying the HP filter. It is important to note that while the cointegration based analysis can be promptly used for forecasting, the same does not apply to both the HP filter and the correction for breaks.

Overall we take the evidence of long-run forecasting regressions and cointegration analysis as consistently supportive of the GMQ model. Two more remarks are in order before we move forward.

First, in the GMQ model, bond and stock are perfect substitutes, therefore the evaluation of the performance of MY_t in forecasting yields to maturity of long-term bonds seems a natural extension of our empirical investigation. In fact, the debate on the so-called FED

[4] We have also investigated the stability of the cointegrating relationship by using the recursively calculated eigenvalues and the tests for constancy of the parameters in the cointegrating space proposed by Nyblom (1989), Hansen and Johansen (1999) and Warne et al. (2003). The results, available upon request, show no evidence of instability.

model (Lander et al. (1997)) of the stock market, based on a long-run relation between the price-earning ratio and the long-term bond yield, brings some interesting evidence on this issue. The FED model is based on the equalization, up to a constant, between long-run stock and bond market returns. This feature is shared by the GMQ framework, and it requires a constant relation between the risk premium on long-term bonds and stocks. It has been shown that, although the FED model performs well in periods where the stock and bond market risk premia are strongly correlated, some measure of the fluctuations in their relative premium is necessary to model periods in which volatilities in the two markets have been different (see, for example, Asness (2003)). As a consequence, to put MY_t at work to explain bond yields, some modelling of the relative bond/stock risk premia is also in order. We consider this as an interesting extension beyond the scope of this paper which is on our agenda for future work.

Second, although MY_t is the GMQ model consistent measure of demographics, there are a number of different potential measures for demographic trends. We have therefore conducted robustness analysis of our cointegration results to the introduction of different measures of demographic structure of the population and productivity trends. The results, discussed in Appendix B, are supportive of our preferred specification.

IV MY, CAY and CDY

In the light of the evidence reported in the previous section it is interesting to reconsider point iv) in the introduction and evaluate the significance of the introduction of MY_t in the dynamic dividend growth model against cay_t and cdy_t. As stated in the introduction, Lettau and Ludvigson (2001, 2005) have found that dividend growth and stock returns are predictable by long-run equilibrium relationships derived from a linearized version of the consumer's intertemporal budget constraint. The excess consumption with respect to its long run equilibrium value is defined by Lettau and Ludvigson (2001) alternatively as a

linear combination of labour income and financial wealth, cay_t, or as a linear combination of aggregate dividend payments on human and non-human wealth, cdy_t. cay_t and cdy_t are much less persistent time-series than dp_t, they are predictors of both stock returns and dividend-growth, and when included in a predictive regression relating stock market returns to dp_t, they swamp the significance of this variable.

Evaluating the effect of the inclusion of cay_t and cdy_t in the long-run forecasting regressions that also include MY_t is important for a number of reasons. First, it is a parsimonious way of evaluating the model with MY_t against all financial ratios traditionally adopted to predict returns. In fact, Lettau and Ludvigson (2001,2005) show the superior performance in predicting long-run returns of cay_t and cdy_t with respect to all the traditionally adopted financial ratios, such as the detrended short term interest rate (Campbell (1991), Hodrick (1992)), the log dividend earnings ratio and the log price earning ratio (Lamont (1998)), the spread of long term bond yield (10Y) over 3M Treasury bill, and the spread between the BAA and the AAA corporate bond rates. Second, it would allow further investigation on the presence of a common component in dividend and stock market returns suggested by Lettau and Ludvigson (2005) but not consistent with our findings in Table 1.3, that witness the significance of MY_t for predicting long-run returns and long-run returns adjusted for dividend growth. Third, it could shed further light on the relative importance of cay_t and cdy_t and MY_t for predicting returns and dividend growth in the dynamic dividend growth model. Note that a joint significance of cay_t or cdy_t and MY_t in long-run forecasting regressions for real stock market returns is fully consistent with the GMQ model if the significance of MY_t is interpreted in the light of its role as a predictor for $\overline{dp_t}$ while cay_t or cdy_t are taken as predictors of long term expectations of real returns and dividend growth.

Insert here Table 3.1-3.3

We report the relevant evidence in Tables 3.1-3.3. cay_t and cdy_t are estimated by Lettau

and Ludvigson (2001,2005) as cointegrating residuals for the systems (c_t, a_t, y_t) and (c_t, d_t, y_t), where c_t is log consumption, y_t is log labor income, a_t is log asset wealth (net worth), d_t is log stock market dividends. We have taken the cointegrating relationship directly from Lettau and Ludvigson (2005): $cay_t = c_t - 0.33a_t - 0.57y_t$, $cdy_t = c_t - 0.13d_t - 0.68y_t$. The evidence clearly indicates that the significance of dp_t corrected for MY_t in the long-horizon regressions is not reduced by the augmentation of the model with cay_t and cdy_t. These two variables, and in particular cdy_t, have strong predictive power for dividend growth. Therefore, the evidence that the best predictive model for long-horizon stock returns is the one combining dividend-price with the demographic variable and cdy_t is indeed fully consistent with an interpretation based on the Dynamic Dividend Growth model where MY_t explains the slowly evolving component of the mean of the dividend-price and cdy_t acts as a predictor of dividend-growth. Such an interpretation is supported by the long-horizon regressions for stock returns adjusted for dividend growth, in which both MY_t and cdy_t enter with highly significant coefficients of the opposite sign, positive for MY_t and negative for cdy_t.

In Figure 4.3 and 4.4 we plot $dp_t - \overline{dp_t}$ against cay_t and cdy_t, respectively. We derive $\overline{dp_t}$ by using the coefficients from Table.1.1 (k=3)[5], while cay_t and cdy_t series are taken from Lettau and Ludvigson (2005). The graph shows positive but not too strong correlation between $dp_t - \overline{dp_t}$ and cay_t (cdy_t), of 0.57 (0.18). This evidence is consistent with our inclusion of MY_t in the dynamic dividend growth model and the derivation of cay_t and cdy_t from the consumer's intertemporal budget constraint. Consider for example $dp_t - \overline{dp_t}$ and cay_t, they have a common component, which is the weighted sum of future returns, but they are also determined by idiosyncratic components: future dividend growth and future consumption growth, respectively.

[5] We take 3-year horizon as a representative example, results for other horizons remain qualitatively similar (available upon request). We use the coefficients from table Table 1.1, the results are very similar when Table 3.1 is used instead.

A Out-of-Sample Evidence

In this section we follow Goyal and Welch (2008), and analyze the performance of cdy_t and MY_t adjusted dividend-price ratio from the perspective of a real-time investor. We therefore consider out-of-sample evidence, for the 1-year, 2-year, and 3-year horizons, and we compare the performance of the bivariate model based on the combination of the two predictors with that of the two univariate models based on each predictor and the univariate models based on dp_t and dp_t^{LVN}.

We run rolling forecasting regressions for the one, three and five years ahead horizon by using 1955-1981 as an initialization sample. The forecasting period beginning in 1982 includes the anomalous period of late 90's where the sharp increase in the stock market index weakens the forecasting power of financial ratios. In particular, we consider both the univariate models and the bivariate encompassing model; we compare the forecasting performance with the historical mean benchmark. In the first two columns of Table 4 we report the adjusted $\bar{\mathrm{R}}^2$ and t-statistics using the full sample 1955-2008. Then we report mean absolute error (MAE) and root mean square error (RMSE) calculated based on the residuals in the forecasting period, namely 1982-2008. The first column of the out-of- sample panel reports the out-of-sample R^2 statistics (Campbell and Thomson (2008)) which is computed as

$$R^2_{OS} = 1 - \frac{\sum_{t=t_0}^{T}(r_t - \hat{r}_t)^2}{\sum_{t=t_0}^{T}(r_t - \bar{r}_t)^2}$$

where $\hat{r}_t$ is the forecast at $t-1$ and $\bar{r}_t$ is the historical average estimated until $t-1$. In our exercise, $t_0 = 1982$ and $T = 2008$. If R^2_{OS} is positive, it means that the predictive regression has a lower mean square error than the prevailing historical mean. In the last column, we report the Diebold-Mariano (DM) t-test for checking equal-forecast accuracy from two nested models for forecasting h-step ahead excess returns.

$$DM = \sqrt{\frac{(T+1-2*h+h*(h-1))}{T}} * \left[\frac{\bar{d}}{\widehat{se}(\bar{d})}\right]$$

where we define e_{1t}^2 as the squared forecasting error of prevailing mean, and e_{2t}^2 as the squared forecasting error of the predictive variables, $d_t = e_{1t}^2 - e_{2t}^2$, i.e. the difference between the two forecast errors, $\bar{d} = \frac{1}{T}\sum_{t=t_0}^{T} d_t$ and $\widehat{se}(\bar{d}) = \frac{1}{T}\sum_{\tau=-(h-1)}^{h-1}\sum_{t=|\tau|+1}^{T}(d_t - \bar{d}) * (d_{t-|\tau|} - \bar{d})$. A positive DM t-test statistic indicates that the predictive regression model performs better than the historical mean.

Insert here Table 4

We report in Figure 5 the cumulative squared prediction errors of the historical mean minus the cumulative squared prediction error of our best forecasting model

Insert here Figure 5

We use all available data from 1910 until 1954 for initial estimation and then recursively calculate the cumulative squared prediction errors until the sample end, namely 2008.

Overall, the results reported in Table 4 and Figure 5 confirm the evidence from the forecasting regressions, with a clear indication that the model combining cdy_t and MY_t adjusted dividend-price ratio dominates all alternative specifications, both within-sample and out-of-sample.

V Long-Run Equity Premium Projections

An interesting feature of MY_t is that long-run forecasts for this variable are readily available. In fact, the Bureau of Census (BoC) provide projections up to 2050 for MY_t. In this section we combine a long-run horizon regression with the cointegrating system estimated in section 2 to construct a model that can be simulated to generate long-run equity premium projections.

We concentrate on 5-year excess returns and estimate the following model:

$$\sum_{j=1}^{5}(h^{s}_{t+j} - r_{f,t+H}) = c_1 + c_2\left(p_t - c_3 d_t - c_4 MY_t\right) + u_{1t} \tag{7}$$

$$\begin{bmatrix} \Delta p_{t+1} \\ \Delta d_{t+1} \end{bmatrix} = \begin{bmatrix} c_5 \\ c_{10} \end{bmatrix} + \begin{bmatrix} c_6 \\ c_{11} \end{bmatrix} \begin{bmatrix} 1 & -c_3 & -c_4 \end{bmatrix} \begin{bmatrix} p_t \\ d_t \\ MY_t \end{bmatrix} + \begin{bmatrix} c_7 & c_8 & c_9 \\ c_{12} & c_{13} & c_{14} \end{bmatrix} \begin{bmatrix} \Delta p_t \\ \Delta d_t \\ \Delta MY_t \end{bmatrix} + \begin{bmatrix} u_{2t} \\ u_{2t} \end{bmatrix},$$

(7) Combines a long-run forecasting regression for five year excess returns, defined as the difference between returns on the S&P500 and the risk-free rate[6], with the equations for Δp_{t+1}, Δd_{t+1} in the cointegrated VAR estimated in Section 2. Equity premium projections are obtained by forward simulation of the first equation. This requires projections for the three right-hand side variables. We obtain them directly from the BoC for MY_t and by forward simulation of the CVAR estimated in Section 2 for p_t and d_t.Three comments are in order on the specification of (7). First, omitting an equation for MY_t from the model used for projections requires (strong) exogeneity of this variable: we believe in the validity of such an assumption. Second, we impose cross-equation restrictions in order to have the same estimates of the coefficients determining the long-run equilibrium of the system in the equation for excess-returns and in the equation for 1-year returns and dividend growth. Third, we did not report the results based on the inclusion of cdy_t in our forecasting model. In fact, the long-horizon forecast for this variable do rapidly converge to its historical mean to leave the variability of projections of the risk-premium to be dominated by projections for MY_t. Moreover, as pointed out by Goyal and Welch (2008), this variable might suffer from look-ahead bias, as the cointegrating coefficients are computed using full-sample estimates.

[6]See the Data Appendix for a detailed description of the construction of our risk-free rate.

The estimates are fully consistent with those reported in Table 1.1 and 2.1[7]. Figure 6 illustrates the results from the projection of the model.

Insert here Figure 6

Over the sample up to 2008 we report (pseudo) out-of-sample 5-year annualized equity premium forecasts and its realizations. The model consistently performs very well with only two exceptions: the 1929 crisis and the boom market at the end of the millennium. We then conduct the out-of-sample exercise by estimating the model with data up to 2008, and then by solving it forward stochastically to obtain out-of-sample forecasts until 2050. Our simulation predicts a rapid stock market recovery for the next two years followed by fluctuations of the risk premium around a mean of 5.02 per cent, just below the historical average. The width of the 95 per cent confidence intervals points to the existence of a sizeable amount of uncertainty around point estimates. Interestingly, the model does not foresee a dramatic market meltdown, a "doomsday" scenario, due to a collective exit from the stock market by the retired baby boomers. This evidence is a natural outcome of the GMQ model which relies on the cyclicality of U.S. age structure.

VI Conclusions

This paper has documented the existence of a slowly evolving trend in the mean dividend-price ratio determined by a demographic variable, MY_t, the proportion of middle-age to young population. We have shown that MY_t captures well a slowly evolving component in the mean dividend-price ratio and it is strongly significant in long-horizon regressions for real stock market returns.

[7] All the evidence reported for the long-run forecasting regressions are based on real equity returns, the dependent variable consistent with the dynamic dividend growth model. Results are robust when excess returns are used as a dependent variable instead of real returns.

A model including MY_t overperforms all alternative models for forecasting returns. The best forecasting model for real stock market returns found in our work is the one combining MY_t with cdy_t, a variable constructed by Lettau and Ludvigson (2005) to capture excess consumption with respect to its long run equilibrium value. We take this evidence as strongly supportive of the Dynamic Dividend Growth model with an evolving mean, determined by MY_t. In fact, the model predicts that long-horizon returns should depend on the deviations of the dividend-price ratio from its mean and on long-run dividend growth. We show that MY_t models the mean of the dividend-price ratio while cdy_t is a predictor of long-horizon dividend-growth, confirming the evidence in Lettau and Ludvigson (2005). We provide evidence that an important component of time-varying expected returns is captured by allowing the mean of the dividend-price ratio to fluctuate MY_t. The importance of such a component increases with the forecasting horizon.

The empirical results we have reported should be of special relevance to the strategic asset allocation literature (e.g. Campbell and Viceira (2002)), in which the log dividend-price ratio is often used in VAR models as a stationary variable capturing time-variation in the investment opportunity set, and as an input into the optimal asset allocation decision of a long-horizon investor. In a companion paper (Favero and Tamoni (2010)) we show that allowing for the presence of MY_t in the VAR models that are used to estimate the time profile of stock market return and its volatility does cast new light on the hot debate on the safety of stock market investment for the long-run (Pastor and Stambaugh (2009)).

Finally, by exploiting the exogeneity and the predictability of MY_t, we have also provided projections for equity risk premia up to 2050. Our simulations point to an average equity risk premium of about five per cent for the period 2010-2050.

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Table 1.1: Long-horizon regression. Sample 1910 -2008. Annual data.

j-period regressions for real stock returns

$$\sum_{j=1}^{k}(h^{s}_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j}$$

χ^2 15.63 (0.000) horizon k in years

		1	2	3	4	5	6
β_1		−0.27	−0.28	−0.23	−0.21	−0.19	−0.16
	(t−stat)	(−4.000)	(−6.287)	(−6.563)	(−9.077)	(−9.352)	(−7.824)
β_2		0.30	0.32	0.26	0.24	0.21	0.17
	(t−stat)	(3.679)	(5.748)	(5.801)	(7.974)	(8.291)	(7.065)
β_3		0.44	0.45	0.37	0.34	0.29	0.26
	(t−stat)	(4.065)	(5.875)	(6.509)	(7.627)	(7.865)	(7.710)
$\beta_1 = -\beta_2$		−0.19	−0.20	−0.16	−0.16	−0.14	−0.13
	(t−stat / w.b p−values)	(−3.645 / 0.002)	(−5.439 / 0.000)	(−6.016 / 0.000)	(−7.652 / 0.000)	(−8.584 / 0.002)	(−7.740 / 0.008)
$t/\sqrt{T} - test$		{0.30*}	{0.49**}	{0.59**}	{0.75**}	{0.89**}	{0.95**}
$\beta_3 \mid (\beta_1 = -\beta_2)$		0.42	0.44	0.38	0.36	0.33	0.29
	(t−stat)	(3.447)	(4.636)	(5.059)	(6.006)	(6.578)	(6.915)
$adjR^2$		0.09	0.25	0.33	0.44	0.52	0.54
$adjR^2(\beta_1 = -\beta_2)$		0.07	0.20	0.27	0.37	0.46	0.50
F-statistic		4.10	12.18	16.74	26.62	36.74	39.48

Testing MY against alternative models:

$$\sum_{j=1}^{k}(h^{s}_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 z_t + \varepsilon_{t,t+j}$$

$z_t =$		1	2	3	4	5	6
dp_t^{LVN}	β_3	0.44	0.45	0.39	0.36	0.32	0.28
	(t−stat)	(4.149)	(6.138)	(7.242)	(9.009)	(10.383)	(10.449)
	β_4	0.07	0.09	0.05	0.03	0.01	−0.03
	(t−stat)	(0.665)	(1.128)	(0.665)	(0.626)	(0.140)	(−0.636)
dp_t^{BMRR}	β_3	0.44	0.46	0.39	0.35	0.31	0.27
	(t−stat)	(2.850)	(4.849)	(6.229)	(8.561)	(10.286)	(10.925)
	β_4	0.51	0.37	0.21	0.09	0.02	−0.03
	(t−stat)	(3.602)	(2.945)	(2.443)	(1.496)	(0.407)	(−1.103)

The Table reports in parentheses t-statistic based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) where the bandwidth has been selected following the procedure described in Newey and West (1994). For the univariate model with the restriction $(\beta_1 = -\beta_2)$, $\sum_{j=1}^{k}(h^{s}_{t+j}) = \beta_0 + \beta_1(p_t - d_t + \frac{\beta_3}{\beta_1}MY_t) + \varepsilon_{t,t+j}$, we also conduct a (wild) bootstrap exercise (Davidson and Flachaire (2008)) to compute p-values and report for β_1 $t/\sqrt{T}$-test suggested in Valkanov (2003) in curly brackets. Significance at the 5% and 1% level of the $t/\sqrt{T}$ test using Valkanov's (2003) critical values is indicated by * and **, respectively. The χ^2 statistics with associated p-values is for the joint tests of significance across all different horizon within a SUR estimation framework.

Table 1.2: Long-horizon regressions. Sample 1910 - 2008. Annual data.

j-period regressions for real dividend-growth

$$\sum_{j=1}^{k}(\Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j}$$

χ^2 (t−stat) 2.66 (0.015) horizon k in years

	1	2	3	4	5	6
β_1 (t−stat)	0.19 (2.097)	0.10 (1.827)	0.05 (1.400)	0.03 (1.222)	0.02 (1.006)	0.01 (0.803)
β_2 (t−stat)	−0.23 (−2.196)	−0.13 (−1.996)	−0.07 (−1.585)	−0.04 (−1.441)	−0.03 (−1.308)	−0.02 (−1.153)
β_3 (t−stat)	−0.15 (−1.250)	−0.03 (−0.371)	0.04 (0.780)	0.07 (1.863)	0.08 (2.428)	0.08 (2.755)
$\beta_1 = -\beta_2$ (t−stat / w.b.p−values)	0.10 (1.580 / 0.240)	0.05 (1.118 / 0.433)	0.02 (0.638 / 0.353)	0.01 (0.298 / 0.171)	−0.00 (−0.018 / 0.096)	−0.00 (−0.234 / 0.106)
$\beta_3 \mid (\beta_1 = -\beta_2)$ (t−stat)	−0.12 (−1.118)	−0.03 (−0.366)	0.03 (0.714)	0.06 (1.700)	0.06 (2.067)	0.06 (2.134)
$adjR^2$	0.15	0.09	0.05	0.04	0.05	0.06
$adjR^2(\beta_1 = -\beta_2)$	0.06	0.03	0.02	0.02	0.03	0.03
F-statistic	6.87	4.05	2.73	2.52	2.66	3.04

Testing MY against alternative models:

$$\sum_{j=1}^{k}(\Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 z_t + \varepsilon_{t,t+j}$$

$z_t =$		1	2	3	4	5	6
dp_t^{LVN}	β_3 (t−stat)	−0.09 (−0.877)	−0.00 (−0.033)	0.05 (0.993)	0.07 (1.886)	0.08 (2.511)	0.09 (2.886)
	β_4 (t−stat)	−0.24 (−2.412)	−0.14 (−2.094)	−0.08 (−1.520)	−0.05 (−0.942)	−0.02 (−0.321)	0.00 (0.024)
dp_t^{BMRR}	β_3 (t−stat)	−0.18 (−1.304)	−0.05 (−0.697)	−0.00 (−0.001)	0.03 (0.772)	0.04 (1.752)	0.06 (2.509)
	β_4 (t−stat)	0.09 (0.879)	0.14 (1.981)	0.16 (2.122)	0.16 (2.206)	0.15 (2.977)	0.08 (2.304)

The Table reports in parentheses t-statistic based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) where the bandwidth has been selected following the procedure described in Newey and West (1994). For the univariate model with the restriction $(\beta_1 = -\beta_2)$, $\sum_{j=1}^{k}(\Delta d_{t+j}) = \beta_0 + \beta_1(p_t - d_t + \frac{\beta_3}{\beta_1} MY_t) + \varepsilon_{t,t+j}$, we also conduct a (wild) bootstrap exercise (Davidson and Flachaire (2008)) to compute p-values. The χ^2 statistics with associated p-values is for the joint tests of significance across all different horizon within a SUR estimation framework.

Table 1.3: Long-horizon regressions. Sample 1910 - 2008. Annual data.

j-period regressions for real stock returns adjusted for dividend growth

$$\sum_{j=1}^{k}(h^s_{t+j} - \Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j}$$

χ^2 27.94 (0.000) horizon k in years

	1	2	3	4	5	6
β_1 ($t-stat$)	−0.46 (−6.159)	−0.39 (−8.285)	−0.28 (−9.393)	−0.24 (−10.344)	−0.20 (−14.944)	−0.17 (−12.523)
β_2 ($t-stat$)	0.53 (6.127)	0.45 (8.234)	0.33 (9.157)	0.28 (10.022)	0.24 (13.150)	0.20 (10.940)
β_3 ($t-stat$)	0.59 (4.631)	0.47 (5.313)	0.33 (5.297)	0.27 (5.697)	0.22 (6.202)	0.18 (6.361)
$\beta_1 = -\beta_2$ ($t-stat$ / $w.b$ $p-values$)	−0.28 (−4.961/ 0.001)	−0.24 (−6.603 / 0.000)	−0.18 (−7.345 / 0.000)	−0.16 (−8.516 / 0.000)	−0.14 (−13.217 / 0.000)	−0.12 (−12.976 / 0.002)
$t/\sqrt{T} - test$	{0.47**}	{0.67**}	{0.77**}	{0.91**}	{1.04**}	{1.12**}
$\beta_3 \mid (\beta_1 = -\beta_2)$ ($t-stat$)	0.55 (3.610)	0.47 (4.206)	0.35 (4.204)	0.31 (4.646)	0.27 (5.160)	0.23 (5.406)
$adjR^2$	0.24	0.45	0.51	0.60	0.66	0.67
$adjR^2(\beta_1 = -\beta_2)$	0.16	0.29	0.35	0.43	0.50	0.53
F-statistic	11.49	27.74	35.07	50.25	65.08	67.56

Testing MY against alternative models:

$$\sum_{j=1}^{k}(h^s_{t+j} - \Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 z_t + \varepsilon_{t,t+j}$$

$z_t =$		1	2	3	4	5	6
dp_t^{LVN}	β_3 ($t-stat$)	0.53 (4.111)	0.45 (5.041)	0.34 (5.453)	0.29 (6.172)	0.24 (7.314)	0.19 (8.584)
	β_4 ($t-stat$)	0.31 (2.618)	0.23 (2.739)	0.13 (1.915)	0.08 (1.512)	0.02 (0.513)	−0.03 (−0.719)
dp_t^{BMRR}	β_3 ($t-stat$)	0.62 (4.100)	0.51 (6.105)	0.39 (6.490)	0.32 (7.313)	0.27 (8.837)	0.21 (9.256)
	β_4 ($t-stat$)	0.42 (2.294)	0.22 (1.881)	0.05 (0.704)	−0.07 (−1.734)	−0.13 (−4.349)	−0.11 (−4.035)

The Table reports in parentheses t-statistic based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey and West (1987) where the bandwidth has been selected following the procedure described in Newey and West (1994). For the univariate model with the restriction $(\beta_1 = -\beta_2)$, $\sum_{j=1}^{k}(h^s_{t+j} - \Delta d_{t+j})$ $= \beta_0 + \beta_1(p_t - d_t + \frac{\beta_3}{\beta_1} MY_t) + \varepsilon_{t,t+j}$, we also conduct a (wild) bootstrap exercise (Davidson and Flachaire (2008)) to compute p-values and report for β_1 $t/\sqrt{T} - test$ suggested in Valkanov (2003) in curly brackets. Significance at the 5% and 1% level of the $t/\sqrt{T}$ test using Valkanov's (2003) critical values is indicated by * and **, respectively. The χ^2 statistics with associated p-values is for the joint tests of significance across all different horizon within a SUR estimation framework.

Table 2.1: Cointegrated VAR Estimates. Sample 1910-2008. Annual Data.

Cointegrating vector	p_t	d_t	MY_t	C
$\boldsymbol{\beta}$	-1.00	1.21	1.107	2.16
$(s.e.)$		(0.035)	(0.25)	
Error Correction Model	Δp_t	Δd_t	ΔMY_t	
$\boldsymbol{\alpha}$	0.29	-0.12	0.007	
$(s.e.)$	(0.096)	(0.046)	(0.007)	
Adj. R^2	0.126	0.43	0.63	
Cointegration Test	Trace	p-value	Max eigen	p-value
Hypothesized No of CE(s)				
None	29.68	0.05	22.86	0.028
At Most 1	6.82	0.59	6.75	0.51

Table 2.2: Cointegrated VAR Estimates. Sample 1955-2008. Annual Data

Cointegrating vector	p_t	d_t	MY_t	C
$\boldsymbol{\beta}$	-1.00	1.248	1.14	2.07
$(s.e.)$		(0.035)	(0.14)	
Error Correction Model	Δp_t	Δd_t	ΔMY_t	
$\boldsymbol{\alpha}$	0.63	-0.02	0.03	
$(s.e.)$	(0.16)	(0.035)	(0.015)	
Adj. R^2	0.22	0.40	0.75	
Cointegration Test	Trace	p-value.	Max eigen	p-value
Hypothesized No of CE(s)				
None	28.71	0.06	19.48	0.08
At Most 1	9.22	0.34	8.81	0.30

The Table reports the trace and max eigenvalue statistics obtained from Johansen cointegration test with linear trend in the data. We report the coefficients of the cointegrating vector , β and the weights α (see Appendix A). The reported p-values for the relevant null to test for cointegration are McKinnon-Haugh-Michelis (1999) p-values. The lag length in the VAR model is chosen according to optimal information criteria, i.e. sequential LR test, Akaike (AIC), Schwarz (SIC), Hannan-Quinn (HQ) information criterion.

Table 3.1: CAY and CDY. Sample 1948-2008. Annual Data.

k-period regressions for real stock returns

$$\sum_{j=1}^{k}(h^s_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 z_t + \varepsilon_{t,t+j}$$

		horizon k in years					
	z_t	1	2	3	4	5	6
β_1	cay_t	−0.41	−0.29	−0.21	−0.20	−0.19	−0.15
$(t-stat)$		(−4.050)	(−4.694)	(−5.570)	(−7.222)	(−8.257)	(−8.790)
	cdy_t	−0.51	−0.40	−0.29	−0.24	−0.21	0.19
		(−6.487)	(−6.442)	(−6.840)	(−8.592)	(−11.549)	(−12.403)
β_2	cay_t	0.50	0.36	0.25	0.24	0.22	0.17
$(t-stat)$		(3.994)	(4.492)	(5.230)	(6.373)	(7.067)	(7.384)
	cdy_t	0.63	0.48	0.34	0.28	0.25	0.22
		(6.210)	(6.134)	(6.287)	(7.143)	(9.064)	(9.832)
β_3	cay_t	0.66	0.48	0.36	0.32	0.29	0.24
$(t-stat)$		(4.194)	(5.077)	(6.327)	(8.823)	(10.967)	(13.000)
	cdy_t	0.781	0.57	0.42	0.35	0.31	0.26
		(5.467)	(5.774)	(6.146)	(7.756)	(9.394)	(10.067)
β_4	cay_t	2.06	2.41	1.94	0.87	0.71	1.15
$(t-stat)$		(1.336)	(3.329)	(3.111)	(1.192)	(1.471)	(3.739)
	cdy_t	−0.51	0.25	0.27	0.04	0.06	0.44
		(−0.718)	(0.490)	(1.102)	(0.225)	(0.498)	(3.246)
$adjR^2$	cay_t	0.35	0.61	0.70	0.75	0.82	0.87
	cdy_t	0.34	0.56	0.65	0.74	0.81	0.86

The table reports the OLS estimates from k-period regressions for real stock returns. Each column reports a different horizon, odd (even) rows refer to $z_t = cay_t(cdy_t)$. The reported t-statistics are based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey & West (1987) where the bandwidth has been selected following the procedure described in Newey & West (1994).

Table 3.2: CAY and CDY. Sample 1948-2008. Annual data.

k-period regressions for real dividend-growth

$$\sum_{j=1}^{k}(\Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 z_t + \varepsilon_{t,t+j}$$

		horizon k in years					
		1	2	3	4	5	6
β_1	cay_t	0.10	0.06	0.03	0.01	0.01	0.01
$(t-stat)$		(1.649)	(1.481)	(0.977)	(0.451)	(0.268)	(0.251)
	cdy_t	−0.03	−0.04	−0.03	−0.02	−0.01	−0.01
		(−0.528)	(−0.981)	(−1.392)	(−1.353)	(−1.353)	(−0.608)
β_2	cay_t	−0.14	−0.09	−0.04	−0.02	−0.01	−0.01
$(t-stat)$		(−1.733)	(−1.574)	(−1.109)	(−0.626)	(−0.466)	(−0.463)
	cdy_t	0.01	0.03	0.03	0.02	0.01	0.01
		(0.233)	(0.698)	(1.071)	(0.940)	(0.586)	(0.284)
β_3	cay_t	−0.07	−0.02	0.02	0.04	0.05	0.05
$(t-stat)$		(−0.838)	(−0.363)	(0.542)	(1.286)	(1.806)	(2.406)
	cdy_t	0.02	0.04	0.05	0.05	0.04	0.04
		(0.188)	(0.669)	(1.479)	(2.033)	(2.238)	(2.416)
β_4	cay_t	2.92	2.49	1.62	0.99	0.61	0.39
$(t-stat)$		(2.503)	(3.176)	(4.377)	(3.359)	(2.084)	(1.244)
	cdy_t	1.16	1.00	0.69	0.59	0.56	0.52
		(1.561)	(2.115)	(4.500)	(4.853)	(4.272)	(3.670)
$adjR^2$	cay_t	0.33	0.42	0.40	0.28	0.20	0.19
	cdy_t	0.20	0.27	0.30	0.31	0.33	0.35

The table reports the OLS estimates from k-period regressions for real dividend-growth. Each column reports a different horizon, odd (even) rows refer to $z_t = cay_t(cdy_t)$. The Table reports in parentheses t-statistic based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey & West (1987) where the bandwidth has been selected following the procedure described in Newey & West (1994).

Table 3.3: CAY and CDY. Sample 1948-2008. Annual data.

k-period regressions for real stock returns adjusted for dividend growth

$$\sum_{j=1}^{k}(h^s_{t+j} - \Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 z_t + \varepsilon_{t,t+j}$$

		horizon k in years					
		1	2	3	4	5	6
β_1	cay_t	−0.51	−0.36	−0.24	−0.22	−0.19	−0.15
$(t-stat)$		(−4.796)	(−5.417)	(−7.799)	(−9.769)	(−9.506)	(−10.203)
	cdy_t	−0.49	−0.36	−0.25	−0.21	−0.20	−0.18
		(−6.547)	(−8.500)	(−9.022)	(−9.930)	(−13.824)	(−15.141)
β_2	cay_t	0.65	0.45	0.29	0.26	0.23	0.18
$(t-stat)$		(4.713)	(5.209)	(7.635)	(9.596)	(8.871)	(9.070)
	cdy_t	0.62	0.45	0.31	0.26	0.24	0.21
		(6.220)	(7.653)	(7.887)	(8.109)	(10.452)	(11.669)
β_3	cay_t	0.72	0.50	0.34	0.29	0.24	0.19
$(t-stat)$		(4.487)	(5.255)	(6.670)	(9.758)	(10.237)	(10.029)
	cdy_t	0.77	0.53	0.37	0.31	0.27	0.21
		(6.020)	(6.801)	(6.419)	(7.152)	(8.580)	(8.883)
β_4	cay_t	−0.86	−0.08	0.329	−0.12	0.10	0.757
$(t-stat)$		(−0.508)	(−0.079)	(0.501)	(−0.158)	(0.169)	(2.278)
	cdy_t	−1.67	−0.74	−0.422	−0.55	−0.50	−0.08
		(−2.428)	(−1.295)	(−2.141)	(−3.636)	(3.818)	−0.722)
$adjR^2$	cay_t	0.28	0.50	0.63	0.72	0.79	0.87
	cdy_t	0.32	0.52	0.64	0.74	0.81	0.85

The table reports the OLS estimates from k-period regressions for real stock returns adjusted for dividend-growth. Each column reports a different horizon, odd (even) rows refer to $z_t = cay_t(cdy_t)$. The Table reports in parentheses t-statistic based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey & West (1987) where the bandwidth has been selected following the procedure described in Newey & West (1994).

Table 4: Predictive Performance. Sample 1948-2008. Annual data.

	In-Sample				Out-of-Sample			
Panel A (k=1)	R^2	t-stat	MAE	RMSE	R^2_{OS}	MAE	RMSE	DM
dp_t	3.03	1.64	12.92	16.17	−11.22	14.54	18.60	−17.43
dp_t^{LVN}	6.36	2.64	11.93	16.20	−5.25	13.58	18.09	−8.09
cdy_t	−1.47	0.48	12.03	14.00	−16.11	13.24	15.12	−19.97
dp_t^{DT}	19.48	4.37	10.08	15.32	11.20	10.91	16.27	4.68
$dp_t^{DT} + cdy_t$	38.64	5.97/ − 0.32	8.71	10.97	28.61	9.83	11.86	14.36
Hist.Mean	–	–	12.92	16.70	–	13.40	17.63	–
Panel B (k= 2)	R^2	t-stat	MAE	RMSE	R^2_{OS}	MAE	RMSE	DM
dp_t	5.46	1.70	15.72	20.71	−55.77	24.21	30.94	−4.01
dp_t^{LVN}	15.74	2.45	14.20	19.91	−11.72	19.99	26.20	−3.30
cdy_t	2.92	1.13	16.00	21.93	−55.19	20.04	27.45	−1.33
dp_t^{DT}	48.32	7.88	12.20	17.52	35.52	14.39	19.90	3.11
$dp_t^{DT} + cdy_t$	62.63	6.32/1.40	10.58	14.17	40.85	13.16	16.94	6.08
Hist. Mean	–	–	16.19	21.91	–	18.65	24.79	–
Panel C (k= 3)	R^2	t-stat	MAE	RMSE	R^2_{OS}	MAE	RMSE	DM
dp_t	6.34	1.99	18.26	24.87	−88.56	33.98	43.59	1.32
dp_t^{LVN}	10.76	1.70	17.91	24.81	−25.89	28.40	35.61	−4.41
cdy_t	5.91	1.41	19.45	26.94	−41.43	26.64	33.70	−1.97
dp_t^{DT}	49.73	6.70	13.22	19.00	43.95	18.15	23.32	2.90
$dp_t^{DT} + cdy_t$	64.89	7.24/2.99	13.67	16.99	48.54	17.06	20.33	2.50
Hist. Mean	–	–	19.38	26.65	–	25.26	31.74	–

This table presents statistics on k-year ahead forecast errors (in-sample and out-of-sample) for stock returns. The first column lists the regressors in both univariate and bivariate predictive regressions; dp_t, log dividend-price ratio, dp_t^{LVN}, dp_t corrected for breaks in mean (LVN), dp_t^{DT}, dp_t adjusted for MY_t and cdy_t, cointegrated vector suggested by Lettau and Ludvigson (2005). The sample starts in 1948 and we construct first forecast in 1982. All numbers are in percent. RMSE is the root mean square error, MAE is the mean absolute error. DM is the Diebold and Mariano (1995) t-statistic for difference in MSE of the unconditional forecast and the conditional forecast. The out-of-sample R^2_{OS} compares the forecast error of the historical mean with the forecast from predictive regressions.

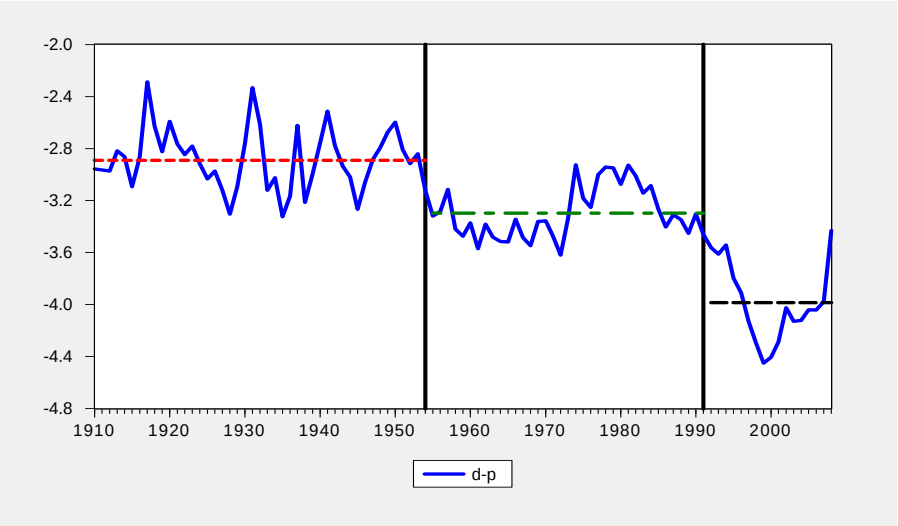

Figure 1: The time series of log dividend- price ratio (dp_t). Two vertical lines indicate the breaks in 1954 and 1991 identified by LVN. Sample 1909 - 2008. Annual data.

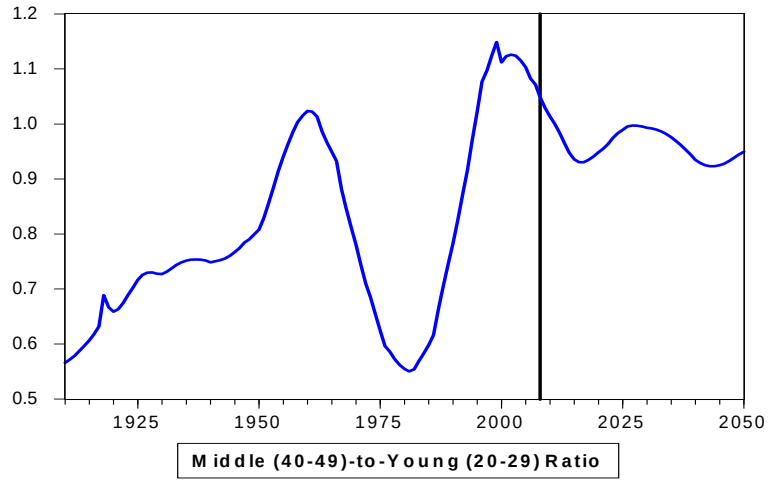

Figure 1:

Figure 2: The time series of Middle-Young (MY) ratio. The vertical line in 2008 indicates the end of in-sample data and the start of Bureau of Census projections. Sample 1909-2050. Annual data.

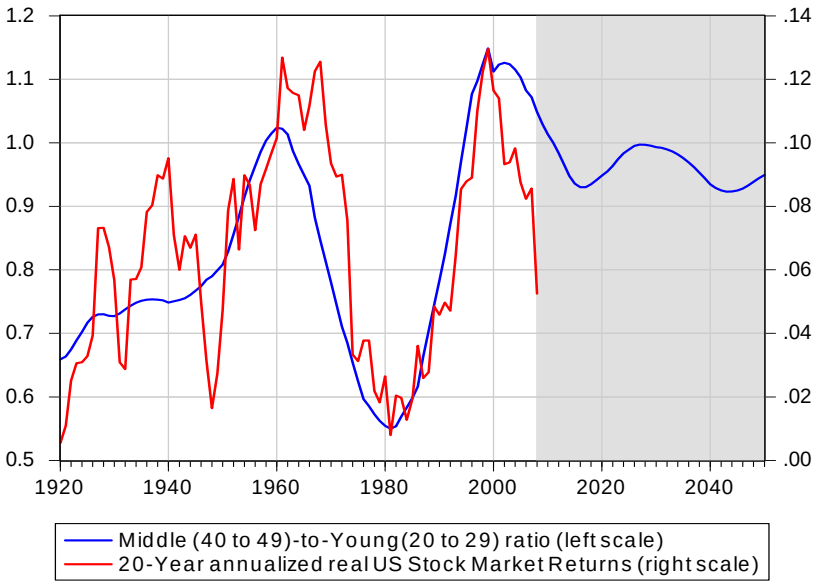

Figure 3: MY and 20-year annualized real US stock market returns. The vertical line in 2008 indicates the end of in-sample data and the start of Bureau of Census projections. Sample 1920-2050. Annual data.

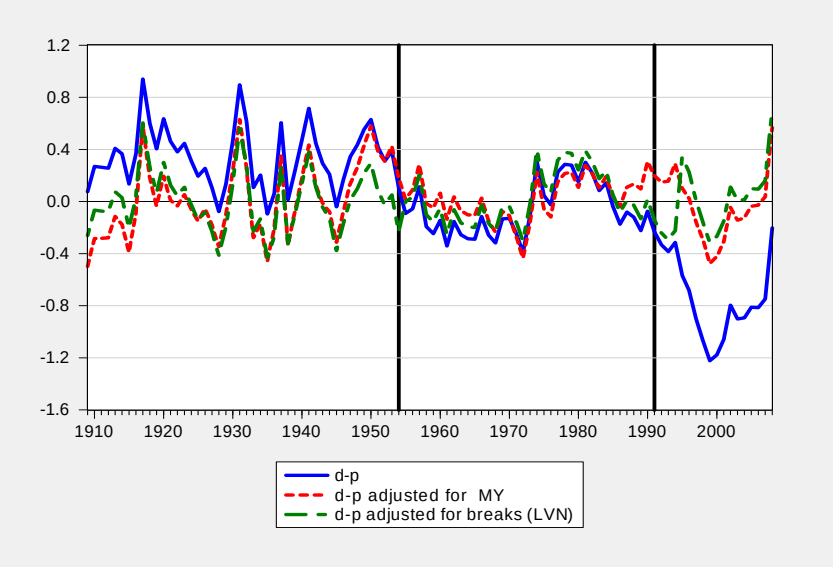

Figure 4.1: dp_t, dp_t adjusted for breaks (LVN) and fluctuations of dp_t around a time-varying mean determined by MY. Two vertical lines indicate the breaks in 1954 and 1991 identified by LVN. Sample 1910-2008. Annual data.

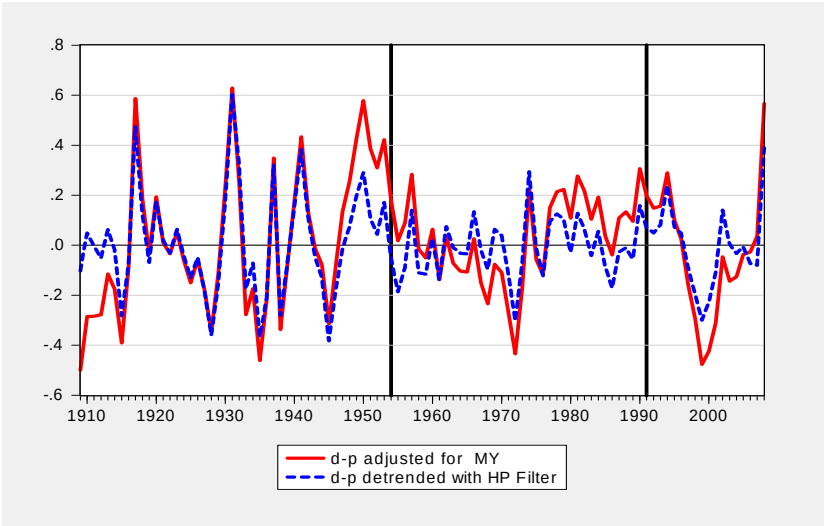

Figure 4.2: Two alternative (and consistent) measures of the cycle in dp_t. Two vertical lines indicate the breaks in 1954 and 1991 identified by LVN. Sample 1910-2008. Annual data.

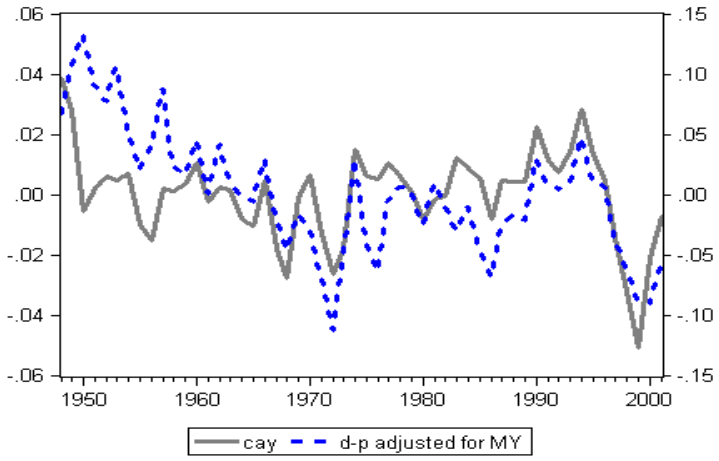

Figure 4.3: cay_t vs. dp_t adjusted for MY_t. cay is the annual series taken from Martin Lettau's website, dp_t is adjusted for MY_t using the coefficients from Table 1.1 (k=3). Sample 1910-2008. Annual data.

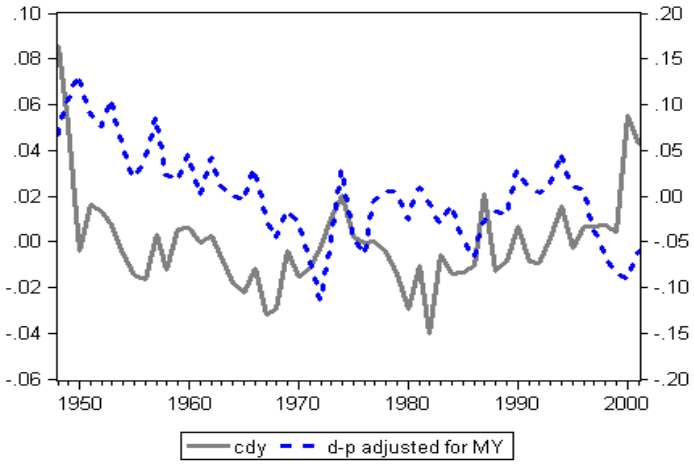

Figure 4.4: cdy_t vs. dp_t adjusted for MY_t. cdy_t is the annual series taken from Martin Lettau's website, dp_t is adjusted for MY_t using the coefficients from Table 1.1 (k=3). Sample 1910-2008. Annual data.

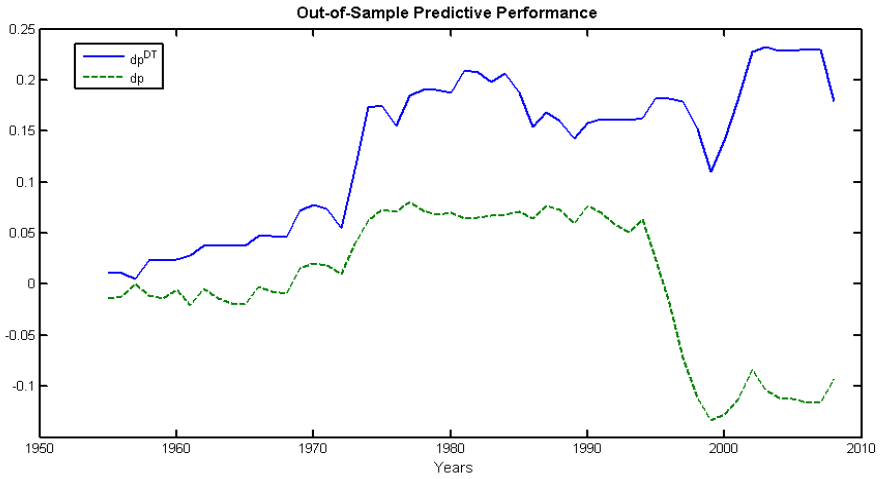

Figure 5: Differences of cumulative RMSE of forecasts based on the historical prevaling mean and RMSE of forecasting models based on dp_t (dashed line) and on dp_t adjusted for MY (solid line). The first forecast is in 1955 up to 2008. Annual data.

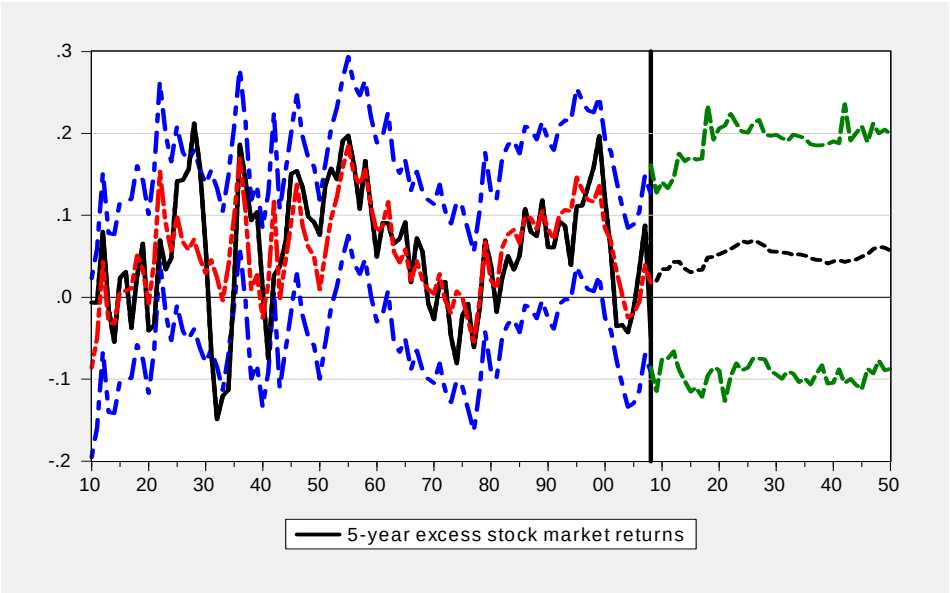

Figure 6: 5-year (ex-post) stock market return (solid black line), within sample prediction (solid red line) and out-of-sample projections for 5-year stock market excess returns (dashed black line) along with 95% confidence intervals(dashed line). The vertical line in 2008 indicates the end of in-sample data and the start of Bureau of Census projections. Sample 1910-2050. Annual data.

APPENDIX A: The Statistical Model for Cointegration Analysis

We consider the following statistical model:

$$\mathbf{y}_t = \sum_{i=1}^{n} \mathbf{A}_i \mathbf{y}_{t-i} + \mathbf{u}_t \tag{8}$$

$$\mathbf{y}_t \text{ is a } m \times 1 \text{ vector of variables} \tag{9}$$

This model can be re-written as follows

$$\begin{aligned} \Delta \mathbf{y}_t &= \mathbf{\Pi}_1 \Delta \mathbf{y}_{t-1} + \mathbf{\Pi}_2 \Delta \mathbf{y}_{t-2} + ... \mathbf{\Pi}_{n-1} \Delta \mathbf{y}_{t-n+1} + \mathbf{\Pi} \mathbf{y}_{t-1} + \mathbf{u}_t \\ &= \sum_{i=1}^{n-1} \mathbf{\Pi}_i \Delta \mathbf{y}_{t-i} + \mathbf{\Pi} \mathbf{y}_{t-1} + \mathbf{u}_t, \end{aligned} \tag{10}$$

where:

$$\begin{aligned} \mathbf{\Pi}_i &= -\left(I - \sum_{j=1}^{i} \mathbf{A}_j \right), \\ \mathbf{\Pi} &= -\left(I - \sum_{i=1}^{n} \mathbf{A}_i \right). \end{aligned}$$

Clearly the long-run properties of the system are described by the properties of the matrix Π. There are three cases of interest:

1. rank (Π) = 0. The system is non-stationary, with no cointegration between the variables considered. This is the only case in which non-stationarity is correctly removed simply by taking the first differences of the variables;

2. rank (Π) = m, full. The system is stationary;

3. rank (Π) = $k < m$. The system is non-stationary but there are k cointegrating relationships among the considered variables. In this case $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where α is an $(m \times k)$ matrix of weights and $\boldsymbol{\beta}$ is an $(k \times m)$ matrix of parameters determining the cointegrating relationships.

Therefore, the rank of Π is crucial in determining the number of cointegrating vectors. The Johansen procedure is based on the fact that the rank of a matrix equals the number of its characteristic roots that differ from zero. The Johansen test for cointegration is based on the estimates of the two characteristic roots of $\mathbf{\Pi}$ matrix. Having obtained estimates for the parameters in the $\mathbf{\Pi}$ matrix, we associate with them estimates for the m characteristic roots and we order them as follows $\lambda_1 > \lambda_2 > ...\lambda_m$. If the variables are not cointegrated, then the rank of Π is zero and all the characteristic roots equal zero. In this case each of the expression $\ln(1-\lambda_i)$ equals zero, too. If, instead, the rank of Π is one, and $0 < \lambda_1 < 1$, then $\ln(1-\lambda_1)$ is negative and $\ln(1-\lambda_2) = \ln(1-\lambda_3) = ... = \ln(1-\lambda_m) = 0$. The Johansen test for cointegration in our bivariate VAR is based on the two following statistics that Johansen derives based on the number of characteristic roots that are different from zero:

$$\begin{aligned} \lambda_{\text{trace}}(k) &= -T\sum_{i=k+1}^{m}\ln\left(1-\widehat{\lambda}_i\right), \\ \lambda_{\max}(k,k+1) &= -T\ln\left(1-\widehat{\lambda}_{k+1}\right), \end{aligned}$$

where T is the number of observations used to estimate the VAR. The first statistic tests the null of at most k cointegrating vectors against a generic alternative. The test should be run in sequence starting from the null of at most zero cointegrating vectors up to the case of at most m cointegrating vectors. The second statistic tests the null of at most k cointegrating vectors against the alternative of at most $k+1$ cointegrating vectors. Both statistics are small under the null hypothesis. Critical values are tabulated by Johansen (1991) and they depend on the number of non-stationary components under the null and on the specification of the deterministic component of the VAR.

APPENDIX B: Robustness Analysis for the Cointegrating Evidence

To assess the robustness of our cointegrating relationship in identifying the low frequency relation between stock market and demographics, we evaluate the effect of augmenting our baseline relation with an alternative demographic factor. Research in demography has recently concentrated on the economic impact of the *demographic dividend* (Bloom et al., 2003; Mason&Lee, 2005). The demographic dividend depends on a peculiar period in the demographic transition phase of modern population in which the lack of synchronicity between the decline in fertility and the decline in mortality typical of advanced economies has an impact on the age structure of population. In particular a high support ratio is generated, i.e. a high ratio between the share of the population in working age and the share of population economically dependent. Empirical evidence has shown that the explicit consideration of the fluctuations in the support ratio delivers significant results in explaining economic performance (see Bloom et al., 2003). The concept of *Support Ratio (SR)* has been precisely defined by Mason and Lee (2005) as the ratio between the number of effective number of producers, L_t, over the effective number of consumers, N_t (Mason&Lee, 2005). In practice we adopt the following empirical proxy:

$$SR = a2064/(a019 + a65ov)$$

where $a2064$: Share of population between age 20-64, $a019$: Share of population between age 0-19, $a65ov$: Share of population age 65+[8].

SR did not attract a significant coefficient when we augmented our cointegrating specification with this variable.

[8] We have checked robustness of our results by shifting the upper limit of the producers to the age of 75. This is consistent with the evidence on the cross-sectional age-wealth profile from Survey of Consumer Finances, provided in Table 1 of Poterba(2001), which shows that the population share between 64-74 still holds considerable amount of common stocks. Results are available upon request.

APPENDIX C: Description of all Time-series used in our Empirical Investigation.

Stock Market Prices: S&P 500 index yearly prices from 1909 to 2008 are from Robert Shiller's website, we take december observations.

Stock Market Dividends: Dividends are twelve-month moving sums of dividends paid on the S&P 500 index. They are from the Robert Shiller website for the period 1900-2008.

These series coincide with those used in Goyal and Welch (2008), and made available at http://www.bus.emory.edu/AGoyal/Research.html.

Stock Market Returns: For S&P 500 index, to construct the continuously compounded return r_t, we take the ex-dividend-price P_t add dividend D_t over P_{t-1} and take the natural logarithm of the ratio.

Risk-free Rate: We download secondary market 3-Month Treasury Bill rate from St.Louis (FRED) from 1934-2008. The risk-free rate for the period 1920 to 1933 is from New York City from NBER's Macrohistory data base. Since there was no risk-free short-term debt prior to the 1920's, we estimate it following Goyal and Welch (2008). We obtain commercial paper rates for New York City from NBER's Macrohistory data base. These are available for the period 1871 to 1970. We estimate a regression for the period 1920 to 1971, which yielded

$$T - billRate = -0.004 + 0.886 \times \ CommercialPaperRate.$$

Therefore, we instrument the risk-free rate for the period 1909 to 1919 with the predicted regression equation.

Hence we build our dependent variable which is the equity premium $(r_{m.t} - r_{f,t})$, i.e., the rate of return on the stock market minus the prevailing short-term interest rate in the year $t-1$ to t.

Second, we construct the independent variables commonly used in the long horizon stock

market prediction literature; namely

Log Dividend-Price Ratio (dp_t): the difference between the log of dividends and the log of prices.

Consumption, wealth, income ratio (cay): The series is taken from Lettau and Ludvigson (2001). Data are available from Martin Lettau's website at annual frequency from 1948 to 2001.

Consumption, dividend, income ratio (cdy): The series is taken from Lettau and Ludvigson (2005). Data are available from Martin Lettau's website at annual frequency from 1948 to 2001.

Demographic Variables

The U.S annual population estimates series are collected from U.S Census Bureau and the sample covers estimates from 1900-2050.

DATA SOURCES

Amit Goyal's Website: http://www.bus.emory.edu/AGoyal/Research.html

Martin Lettau's Website: http://faculty.haas.berkeley.edu/lettau/

Andrew Mason's Website: http://www2.hawaii.edu/~amason/

Michael R. Roberts' Website: http://finance.wharton.upenn.edu/~mrrobert/

Robert Shiller's Website: http://www.econ.yale.edu/~shiller/

Bureau of Labor Statistics Webpage: http://www.bls.gov/data/

FRED: http://research.stlouisfed.org/fred2/

NBER Macrohistory Data Base:

http://www.nber.org/databases/macrohistory/contents/chapter13.html.

US Census Bureau: http://www.census.gov/popest/archives/