Università Commerciale
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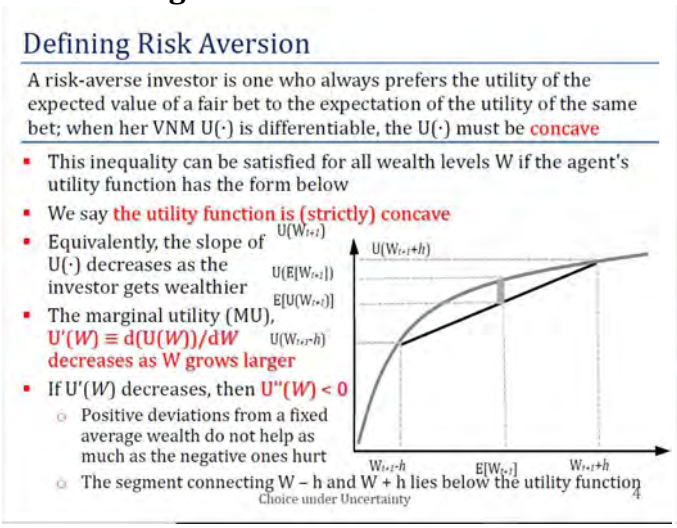
MSc. Finance/CLEFIN
2019/2020 Edition

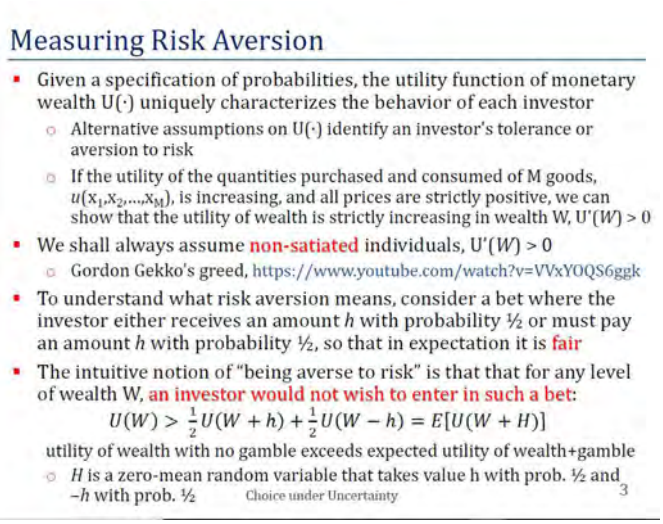
20135 Theory of Finance – Part 1
February 2020 Exam
Time Advised 40 minutes

*Please answer all the questions by writing your answers in the spaces provided. No additional papers will be collected and therefore they will not be marked. USE A BLACK OR BLUE INK PEN, no pencils allowed. You always need to carefully justify your answers and show your work. If **you think you need** to make assumptions in order to answer a question, state them and proceed: the necessity and validity of the assumption(s) will be assessed along with your answer to the question. The exam is closed book, closed notes. Calculators are permitted. The length of this part of the exam should be 7 pages: if this were not the case, it is your responsibility to immediately report any deviations to one of the proctors assigned to your exam room.*

Question 1.A (3.75 points)
Define the concepts of non-satiation and risk aversion and discuss their implications for the shape of Von Neumann-Morgenstern utility functions. Define the concepts and formulas for local absolute and relative risk aversion. In what sense are these measures local ones? Provide one example of a VNM utility function that is characterized by a constant coefficient of relative risk aversion and show that such a function is also characterized by a risk tolerance that depends on wealth in a linear affine fashion.

Debriefing.





Measuring Risk Aversion

- Given a specification of probabilities, the utility function of monetary wealth U(·) uniquely characterizes the behavior of each investor
  - Alternative assumptions on U(·) identify an investor's tolerance or aversion to risk
  - If the utility of the quantities purchased and consumed of M goods, $u(x_1,x_2,\ldots,x_M)$, is increasing, and all prices are strictly positive, we can show that the utility of wealth is strictly increasing in wealth W, $U'(W) > 0$
- We shall always assume non-satiated individuals, $U'(W) > 0$
  - Gordon Gekko's greed, https://www.youtube.com/watch?v=VVxYOQS6ggk
- To understand what risk aversion means, consider a bet where the investor either receives an amount h with probability ½ or must pay an amount h with probability ½, so that in expectation it is fair
- The intuitive notion of "being averse to risk" is that that for any level of wealth W, an investor would not wish to enter in such a bet:

$$U(W) > \tfrac{1}{2}U(W+h) + \tfrac{1}{2}U(W-h) = E[U(W+H)]$$

utility of wealth with no gamble exceeds expected utility of wealth+gamble

  - H is a zero-mean random variable that takes value h with prob. ½ and $-h$ with prob. ½

Absolute and Relative Risk Aversion Coefficients

- How can we manage to measure risk aversion and compare the risk aversion of different decision makers?
- Given that under mild conditions, risk aversion is equivalent to $U''(W)<0$ for all wealth levels, one simplistic idea is to measure risk aversion on the basis of the second derivative of U(·)
  - E.g., John is more risk averse than Mary is iff $|U_{John}''(W)| > |U_{Mary}''(W)|$
- Unfortunately, this approach leads to an inconsistency because when $U_{John}(W) = a + bU_{Mary}(W)$ with $b > 0$ and $b \neq 1$, clearly $U''_{John}(W) = bU_{Mary}''(W) \neq U_{Mary}''(W) > 0$
- But we know that by construction, John and Mary have the same preferences for risky gambles and therefore that it makes no sense to state the John is more risk averse than Mary
- Two famous measures that escape these drawbacks are the coefficients of absolute/relative risk aversion:

$$ARA(W) \equiv -\frac{U''(W)}{U'(W)} \qquad RRA(W) \equiv -\frac{U''(W)}{U'(W)}W = ARA(W)\cdot W$$

  - Because MU(W) is a function of wealth, ARA(W) and RRA(W) are too

Absolute and Relative Risk Aversion Coefficients

Both ARA(W) and RRA(W) are invariant to linear monotonic transforms; this occurs because both are "scaled" at the denominator $U'(W)$

- If nonzero, the reciprocal of the measure of absolute risk aversion, $T(W) \equiv 1/ARA(W)$ can be used as a measure of risk tolerance
- When ARA is constant, RRA(W) must be a linear (increasing) function of wealth; when RRA is constant, then it must be the case that ARA(W) = RRA/W, a simple inverse function of wealth
- ARA and RRA are invariant to linear monotonic transformations; e.g.,

$$ARA_{John}(W) \equiv -\frac{U''_{John}(W)}{U'_{John}(W)} = -\frac{bU''_{Mary}(W)}{bU'_{Mary}(W)} = -\frac{U''_{Mary}(W)}{U'_{Mary}(W)} = ARA_{Mary}(W)$$

- To rank John and Mary's risk aversion, we need to verify whether $ARA_{John}(W) > ARA_{Mary}(W)$ (or the opposite) for all wealth levels
  - Same applies to their coefficient of relative risk aversion for all wealth
  - Possible that for some intervals of wealth it may be $(R)ARA_{John}(W) > (R)ARA_{Mary}(W)$ but for other levels/intervals the inequality be reversed
- Both measures are local as they characterize the behavior of investors only when the risks (lotteries) considered are small

Introducing a Few Common Utility of Wealth Functions

- All these utility functions are strictly increasing and concave, have risk tolerance $T(W)$ that depends of wealth in a linear affine fashion:

$$T_{exp}(W) = \frac{1}{\theta} \qquad T_{power}(W) = \frac{1}{\gamma}W \qquad T_{quadr}(W) = \frac{1}{\kappa} - W$$

- These functions are called linear risk tolerance (LRT) utility functions (alternatively, HARA utility functions, where HARA stands for hyperbolic absolute risk aversion, since ARA(W) defines a hyperbola)
- LRT utility functions have many attractive properties:

$$U(W) = \frac{\gamma}{1-\gamma}\left(\frac{\theta W}{\gamma} + \beta\right)^{1-\gamma} \quad \text{with } \gamma \neq 1, \frac{\theta W}{\gamma} + \beta > 0, \theta > 0$$

- It is possible to check that $ARA(W) = \left(\frac{W}{\gamma} + \frac{\beta}{\theta}\right)^{-1}$
  - When $\gamma \to +\infty$ and $\beta=1$, ARA(W) $\to \theta$ (the CARA case), and when $\beta=0$, RRA(W)=γ/W (the CRRA case)
  - Correspondingly, the risk tolerance function is $T_{HARA}(W) = \frac{\beta}{\theta} + \frac{W}{\gamma}$
  - It is clearly linear affine and increasing in wealth
  - This nests all cases reported above

The power utility function, $U(W) = \frac{W^{1-\gamma}}{1-\gamma}$ for $\gamma \neq 1$, is characterized by a constant coefficient of relative risk aversion. Indeed,

$$ARA = -\frac{U''(W)}{U'(W)} = \gamma\frac{W^{-1-\gamma}}{W^{-\gamma}} = \gamma\frac{1}{W}$$

and

$$RRA = -ARA(W) \times W = \gamma.$$

Because the risk tolerance is the inverse of the ARA coefficient, it is easy to show that $T(W) = W\frac{1}{\gamma}$, which is clearly linear in wealth.

Question 1.B (1.5 points)

John is characterized by a negative exponential utility function of terminal wealth, $U_{Joh}(W) = 1 - e^{-\theta W}$ with $\theta > 0$ and has an initial wealth $W = 1{,}000$ euros. Mary offers John the following risky (but small) bet; they will flip a (fair) coin: if head comes out, John will pay 2 euros to Mary; if tail comes out, Mary will pay 2 euros to John. John states that he would be ready to pay 0.60 euros to Mary to avoid such a bet. Based on this information, can you compute John's risk aversion coefficient? If so, please proceed making sure to show all your work. Otherwise, carefully explain why not and which additional information you would require to accomplish the task. Next, you are told that Mary, when offered to receive a sure amount of 1.2 euros, would still prefer to take the bet. What can you say about Mary's preference for risk? Carefully justify your answer.

Debriefing.

Yes, we can! In particular, because the bet is small, we can use the approximation

$$\Pi_{john}(100,2) \cong \frac{1}{2}\theta h^2 = \frac{1}{2}\theta 2^2.$$

Because we know that $\Pi_{john}(100,2) = 0.6$, we can simply compute that $\theta = 0.3$.

Mary is a risk-lover! Indeed, she would prefer to take the bet even when she is offered a sure amount that is larger than the expected value of the bet (which is obviously 1 euro).

Question 2.A (3.5 points)

Define the meaning of ESG, making sure to describe what each of E, S and G stand for in the acronym ESG. Describe the typical behavior of a SR investors and the most prevalent form of SR investing adopted. What are the main alternative hypotheses that describe the possible differences in relative returns between the stocks of SRI vs. conventional companies? Briefly summarize Statman and Glushkov's (2009) conclusions relating them with the ones in Amel-Zadeh and Serafeim (2019) as discussed in the lectures.

Debriefing.

What is ESG/SRI?

- Exponential growth in the number of companies that report
  ① environmental data (e.g., carbon emissions, water consumption, waste generation)
  ② social data (e.g., employee composition, product information, customer-related information)
  ③ governance data (e.g., political lobbying, anticorruption programs, board diversity)—that is, ESG data. Whereas fewer than
- In short, ESG data
  - From 20 companies that disclosed ESG data in the early 1990s, the number of companies issuing sustainability or integrated reports has increased to nearly 9,000 by 2016
  - As of 2016, the 2006 UN Principles for Responsible Investment had been subscribed by firms with an AUM of about $60 trillion, https://www.unpri.org/signatories/signatories/
- A literature has shown that ESG has deep economic effects
  - ESG disclosures are associated with lower capital constraints, lower costs of capital, and large price movements around mandatory ESG disclosures

What is ESG/SRI?

- The Social Investment Forum (2006) describes socially responsible investing as "an investment process that considers the social and environmental consequences of investments, both positive and negative, within the context of rigorous financial analysis"
- Typical socially responsible investors (SRIs) tilt their portfolios toward stocks with high scores on SR characteristics and shun companies associated with tobacco, alcohol, gambling, firearms, and the military
- Screening is the most prevalent form of SR investing
  - Negative screening excludes/reduces the portfolio weights of companies with weak environmental, social, or governance records
  - Positive screening includes/increases weights of strong records
- A literature compares the returns of SRIs and aggregate indices, such as the S&P 500, but informativeness is limited as indices overlap
  - E.g., the Domini 400 Social Index and the S&P 500 share approximately 250 companies
  - SRI criteria and their relative weights vary among indices, e.g., the Calvert Social Index excludes all tobacco companies, the Dow Jones Sustainability Index (DJSI) does not

What is ESG/SRI?

- Three hypotheses address the relative returns of the stocks of SRI vs. conventional companies
- "Doing Good but Not Well": the returns of SR stocks are lower than those of conventional stocks as the benefits of company actions that tilt it toward ESG fall short of the costs
  - Barnea and Rubin (2006) suggest that managers engage in SR actions whose costs exceed the benefits to shareholders because they reap private benefits, such as awards and other expressions of appreciation
  - Insiders in companies that rank high on SR hold few shares of their company and thus bear little of the cost of the accolades they receive
- "Doing Good While Doing Well": the returns of SR stocks are higher than those of conventional stocks as managers and investors underestimate the benefits of being SR or overestimate its costs
- "No Effect": the expected returns of SR stocks are equal to conventional stocks, as actions are costless, such as when actions amount to no more than words
  - Hypothesis might also be true if costly company actions increase benefits by as much as they increase costs

Statman and Glushkov (2009)

- The abnormal excess returns for the human rights and governance characteristics are negative, but not significant
- They find no statistically significant relationship between governance and stock returns
- The generally higher returns of stocks of companies with high social responsibility scores are especially evident in a long–short portfolio of top-overall and bottom-overall companies
  - A top-overall company is one in the top third of companies by two or more SR characteristics and not in the bottom third by any characteristic
  - A bottom-overall company is one in the bottom third of companies by two or more SR characteristics and not in the top third by any characteristic
- The annualized excess return of the "top-overall minus bottom-overall" portfolio is 5.54%, with a 0.00 p-value, by a 4-factor model
- The portfolio is tilted toward growth stocks and stocks with high momentum, with no significant tilt toward large- or small-caps
- These findings are consistent with the "doing good while doing well" hypothesis: ESG is on average a good investment idea!

Amel-Zadeh and Serafeim (2019)

- Negative screening is considered to be the least financially beneficial ESG investment method, albeit with a neutral impact on returns
  - The results for this strategy contrast with results in Table 4, which found negative screening to rank as the third most used investment style
- Investors in Europe are generally more optimistic about the financial impact of the various ESG strategies than are US investors
- Ethical motivations are associated with a higher likelihood of negative and positive screening and with a significantly lower probability of thematic investment or integration
- The survey contained a question about how important the ESG investment strategies will be for investors in the next five years
- Overall, investors ranked positive screening as the most important strategy in the future, although its rating is not statistically higher than the ratings for active ownership (the second ranked), negative screening (the third ranked), and full integration (the fourth)
- Thematic investment, relative screening, risk factor, and portfolio tilt are considered to be less important in the next five years

Question 2.B (1.5 points)

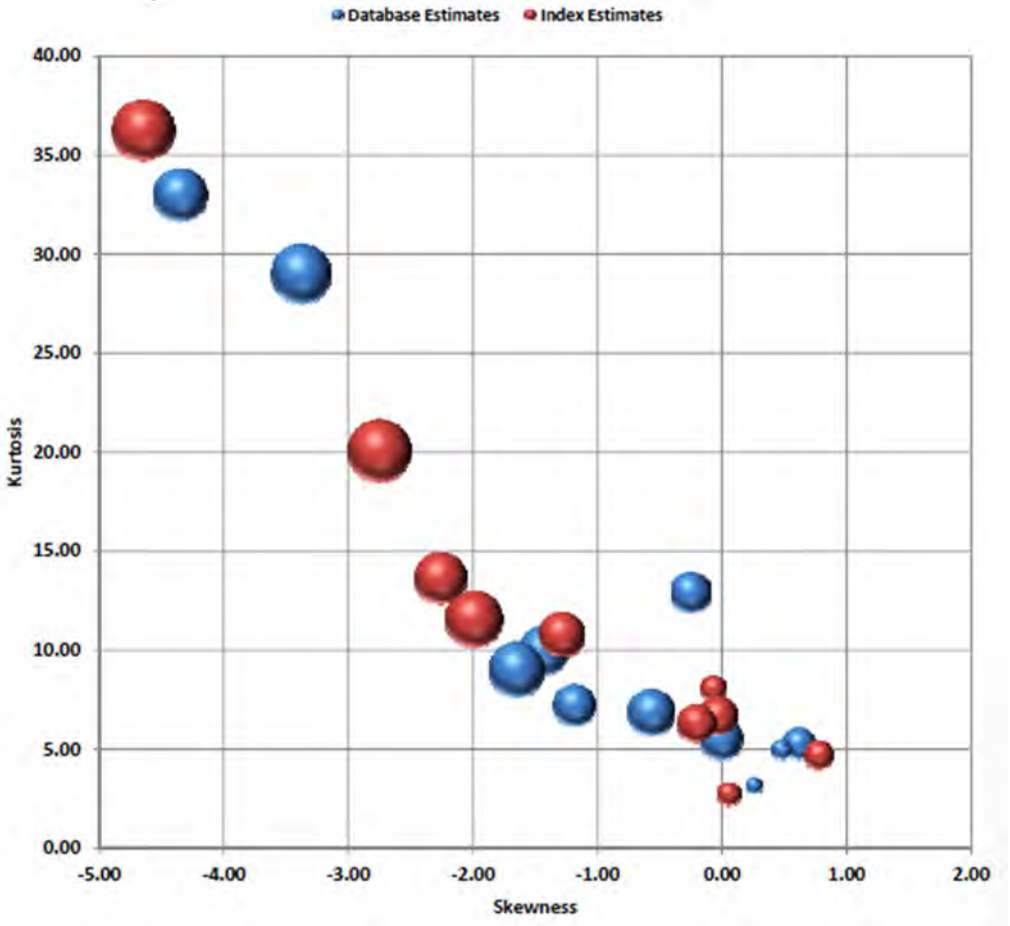
Consider a newly formed hedge fund that aims at implementing long-short strategies founded on sorting stocks on the basis of their ESG score. In practice, the fund will simply buy and hold in a static fashion the stocks in the top decile of the ESG score and sell short the stocks in the bottom decile of the ESG score, without using any derivatives or additional leverage. By being included in the asset menu, will the newly formed hedge fund expand the mean-variance efficient frontier (i.e., move the unconstrained frontier up and/or to the left)? Carefully motivate your answer.

Debriefing.

Although many arguments would prove sensible when applied to this question, the fact is that the newly formed hedge fund is simply using a long-short linear strategy limited to buying and selling the already existing stocks, something that can be freely accomplished in each portfolio on or inside the initial mean-variance frontier. This is further supported by the choice of not investing in derivatives or using complex leveraged schemes. Therefore such a strategy cannot create any payoffs and therefore mean-variance combination that are not already spanned by the portfolios that define the initial mean-variance efficient frontier. In conclusion, we do not expect the mean-variance frontier to move in any ways. Of course, as in the lectures, these answer ignores the role played by frictions and transaction costs (that are in general a limiting factor when considering hedge fund as an asset class).

Question 2.C (0.75 points)

The following plot is taken from Getmansky, Lee, and Lo's article (2015, in Annual Review of Financial Economics).



Please explain its meaning and its significance to the key point made by Getmansky et al. in their paper concerning the economics of the hedge fund industry.

Debriefing.

No, the color was not important, just read the article and you will see. The plot depicts the relationship between the autocorrelation (a common proxy for illiquidity risk of assets) and statistical measures of tail fatness and asymmetry of returns for funds in a specific data base (Lipper TASS) and the Credit Suisse/Dow Jones Hedge-Fund indices. The location of each bubble reflects the skewness (x-axis) and kurtosis (y-axis) of a hedge-fund category. The size of each bubble reflects the corresponding category return autocorrelation. The economic meaning of the plot is that fund categories that exhibit higher autocorrelation generally also exhibit high (excess) kurtosis and negative skewness. This cluster of traits should give investors cause for concern as a few types of fund strategies (the paper lists Convertible Arbitrage, Fixed Income Arbitrage, Event Driven, Emerging Markets, and Multi-Strategy) imply the highest autocorrelations and the most fat-tailed, left-skewed (hence non-normal) return distributions. When investing in a fund with positively auto-correlated returns, and therefore potentially illiquid, investors may want to consider the increased likelihood that a simple mean-variance analysis will understate the actual downside risk and overstate the expected performance of the underlying fund strategy.

Are HFs Just Glorified Mutual Funds?

- HF payoffs are nonlinear due to dynamic option-like strategies
- This can potentially lead to non-normality of fund returns, making Sharpe ratio a less appropriate measure of HF performance

- Standard deviation and historical beta measures can be misleading because the strategies listed frequently display decidedly non-normal return distributions

Index	Mean	St. Dev.	Skewness	Excess Kurtosis	Correl. w/ Russell 1000
Hedge Fund (all)	0.48	2.06	-0.30	2.88	0.59
Convertible Arbitrage	0.37	1.89	-2.64	17.39	0.38
Dedicated Short Bias	-0.60	4.71	0.72	1.59	-0.79
Emerging Markets	0.44	4.07	-0.85	5.94	0.55
Equity Market Neutral	0.22	2.80	-12.50	181.20	0.30
Event Driven	0.53	1.76	-2.24	11.65	0.65
Fixed Income Arbitrage	0.23	1.57	-4.47	32.93	0.34
Global Macro	0.68	2.64	-0.06	4.53	0.24
Long/Short Equity	0.56	2.73	-0.11	3.63	0.70
Managed Futures	0.26	3.31	0.00	-0.01	-0.08

O'Doherty, M. S., Savin, N. E., & Tiwari, A. (2016). Hedge Fund Replication: A Model Combination Approach. *Review of Finance, 21*(4), 1767-1804.

Mean-Variance Allocations for Hedge Funds?

- Historical data show that HFs have not, on average, outperformed traditional ptfs of stocks and bonds after fees
  - On avg., once returns have been adjusted for various sampling biases, HFs do not routinely generate double-digit returns
- However, the ride for HF investors has generally been "smoother"
- There are a number of styles of HF investing: while many are correlated and have much in common, on the whole they are a heterogeneous lot: some are as dissimilar as stocks and bonds
- Guidolin & Orlov (2018,https://www.institutionalinvestor.com/article/b1bk9v1c3Bjk3v/Are-These-the-Best-Hedge-Fund-Strategies) study the optimal allocation to HF strategies allowing for predictability across two alternative allocations
  - Those that exclude HFs from the asset menu
  - Those that abstain from exploiting predictability
- Because there is evidence (see below) that simple quadratic, MV preferences may inappropriate for HFs, they solve for optimal portfolios under expected power utility and assess OOS evidence
  - The predictors are both classical (e.g., dividend yield) and HF-specific

Mean-Variance Allocations for Hedge Funds?

- Not all HFs are likely to benefit a long-term investor already well diversified in stocks, government and corporate bonds, and REITs
- Only strategies whose payoffs are highly nonlinear (relative value, merger arbitrage, distressed restructuring, convertible arbitrage), and therefore not easily replicable, constitute viable options
- HF strategies which are well diversified (e.g., fund of funds) or which invest primarily in stocks (e.g., equity market neutral) may result in lower utility relative to the optimal baseline portfolio
- Medium to highly risk-averse investors benefit the most from this alternative asset class
  - HFs do not increase realized OOS Sharpe ratios
  - However, they create right-skewness and may deflate tails