

# Welfare and public debt

C.A. Favero and F.Giavazzi

## 1 Introduction

## 2 Certainty, Dynamic Efficiency and Debt

## 3 Debt and welfare under certainty

- The Phelps argument
- The Diamond Model
- The Limit to Debt in the Diamond Model

## 4 Debt-and-welfare-under-uncertainty

Why Debt ? (?) discusses a traditional argument pro-debt

- under certainty, when there is dynamic inefficiency, i.e.  $(r - g) < 0$ , Public debt is welfare improving.
- Uncertainty is then introduced and its effects are discussed

Public Debt can be welfare improving under certainty and dynamic inefficiency

- Under certainty when  $(r - g) < 0$ , less capital accumulation increases welfare
- issuing debt decreases capital accumulation and increases welfare

- In 1961, ? argued the following: "A market economy could accumulate too much capital. Such over-accumulation would be reflected in a simple inequality, namely  $(r - g) < 0$ , where  $r$  was the net marginal product of capital
- because Phelps was working in the context of a model with no uncertainty,  $r$  was also the safe rate of interest.
- in this scenario decreasing capital would actually be welfare improving.

# The Phelps argument

Consider a simplified economy in which the demand side is described as follows:

$$C = Y - I \quad (1)$$

output,  $Y$ , is determined by a production function

$$Y = F(K, \dots) \quad (2)$$

where  $K$  is capital and the three dots denote other factors of production such as labor and an index of the state of technology.

The economy is on a balanced growth path, so that  $C, Y, I$  are all growing at some rate  $g$ . The stock of capital is determined by the following relation:

$$\dot{K} = -\delta K + I \quad (3)$$

Capital depreciates at rate  $\delta$

# The Phelps argument

For capital to grow at rate  $g$ , investment must cover both depreciation and the growth of the capital stock:

$$\frac{\dot{K}}{K} = \frac{I}{K} - \delta \quad (4)$$

If  $\frac{\dot{K}}{K} = g$ , then

$$I = (\delta + g)K$$

# The Phelps argument

Replacing Investment in equation (1) gives

$$C = F(K, \dots) - (\delta + g)K$$

The effect of additional capital on consumption is thus given by:

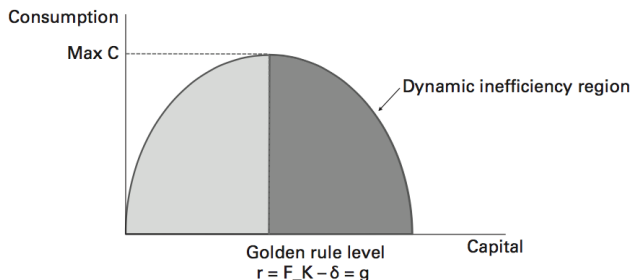
$$\frac{dC}{dK} = F_K(K, \dots) - (\delta + g) = (F_K(K, \dots) - \delta) - g$$

Taking the interest rate to be equal to the net marginal product of capital,  $r \equiv F_K(K, \dots) - \delta$ , the equation above becomes

$$dC/dK = r - g$$

# The Phelps argument

The relation between capital and consumption at any point along the growth path is represented in Figure 5.1.



**Figure 5.1**

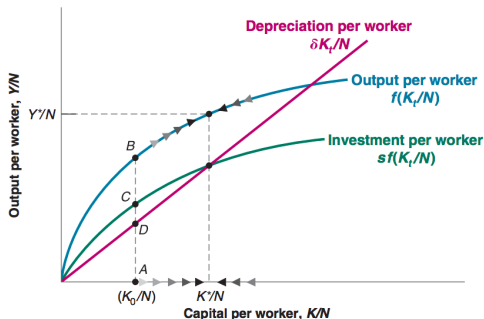
Consumption as a function of capital, golden rule, and dynamic inefficiency

Consumption is an increasing function of capital, until  $(r - g) = 0$ . That level of capital is called the **golden rule** level of capital. As we look at levels of capital higher than the golden rule level,  $(r - g)$  becomes negative and consumption becomes a decreasing function of capital.

# The Intuition

The intuition is that

- as the capital stock increases, depreciation increases linearly with capital,
- but the gross marginal product of capital increases at a slower pace, so that the net marginal product of capital becomes negative.
- Although output is higher, so much has to be put aside for investment that what is left for consumption is lower.



**Figure: Capital and output dynamics**

Source: Figure 12.2 - Macroeconomics a European perspective, Alessia Amighini, Olivier Blanchard and Francesco Giavazzi,

# Why Debt helps ?

Can there really be capital overaccumulation? And why should public debt help in this case? ? using a two-period overlapping generations model, gave the answers

- Even if people are fully rational and take individually optimal saving decisions, there can indeed be capital overaccumulation.
- If this is the case, then anything that decreases saving can, if distribution effects do not stand in the way, increase everybody's consumption and welfare, now and in the future. Intergenerational transfers, or public debt, can play that role.

# The Diamond Model: setup

The setup of the overlapping generation model goes as follows:

- Agents live for two periods, working in the first, retiring in the second.
- They receive a wage in the first period, save by investing in capital (so there is no separate saving/investment decision), and consume the capital and the returns from capital in the second period.
- There no depreciation of capital, thus, the saving of the young determines the capital stock of the economy in the next period.
- The economy grows on its balanced growth path, with all aggregate variables growing at rate  $g$ , equal to the sum of the growth rate of population (or equivalently, assuming a fixed ratio of employment to population, the rate of growth of employment),  $n$ , and the growth rate of productivity,  $x$ .
- The saving rate of the young determines the level of capital along the growth path.

# The Diamond Model: results

- The first result the model delivers is that, while individual saving decisions are rational, there is no guarantee that these decisions imply that  $r = (F_K - \delta) > g$ :  $(r - g)$  can be negative and there can indeed be capital overaccumulation and thus be dynamic inefficiency.
- If this is the case, transfers from the young to the old can increase welfare for all generations, current and future

# The role of transfers in the Diamond Model

- When the young save one unit, they get  $(1 + r)$  units when old. Suppose that the government puts in place a transfer scheme, taking  $D_t$  from each of the young, and giving  $(1 + n)D_t$  to each of the old within the same period (as there are  $(1 + n)$  young for each old), with  $D$  increasing at rate  $x$  over time.
- Think of it as a pay-as-you-go retirement system, in which the contributions from the young finance the benefits for the old, and per capita retirement contributions and benefits increase with productivity over time.
- When young, people lose  $D_t$  in income. When old they receive  $D_{t+1}(1 + n) = D_t(1 + x)(1 + n) = D_t(1 + g)$ .
- If  $(r - g) > 0$ , the transfer scheme delivers less than saving, and thus decreases their welfare. But if  $(r - g) < 0$  however, the transfer scheme is more attractive than saving, and increases the welfare of each generation.
- In this case, a pay-as-you-go retirement system can make all generations better off.

Debt also generates intergenerational transfers, in a slightly different way.

- Think of the government issuing one-period debt every period, with debt issuance increasing at rate  $g$ .
- The young who buy the debt receive  $D(1+r)$  when old, and are indifferent between investing in capital or buying the debt: Both pay  $r$ .
- The issuance of debt next period is equal to  $D(+1) = D(1+g)$ . Thus, each period, the government gets the difference between revenues from debt issuance  $D(1+g)$  and payments on debt  $D(1+r)$ . If  $r < g$ , this difference, equal to  $D(g-r)$ , is positive and can be redistributed to a combination of the young and the old, making them better off.

# The Limit to Debt in the Diamond Model

There is a limit to debt:

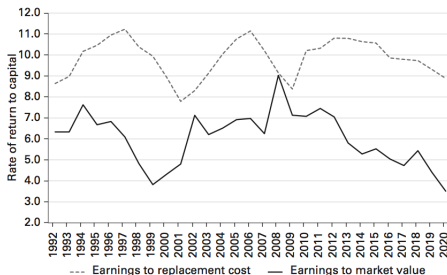
- As debt decreases capital accumulation, it has general equilibrium effects. The marginal product of capital and thus the interest rate increases.
- When the interest rate becomes equal to the growth rate, the economy is at the golden rule.
- Further debt leads to  $(r - g) > 0$ , and debt no longer improves the welfare of all generations. The initial old gain, the others however lose.
- The government then has to think about the trade-off between the current old, who benefit from debt, and future generations, who face lower consumption and lose from debt. But until this threshold is reached, public debt can improve welfare for all.

# Debt-and-welfare-under-uncertainty

- In a world of uncertainty there are many different interest rates, reflecting compensations for different risk premia
- debt sustainability depends on the average cost of financing the debt which is the safe rate or close to it in most advanced economies
- in the welfare discussion what is important is instead the marginal product of capital, net of depreciation.
- On average, this rate appears substantially higher than the real safe rate and the average growth rate.

# Two Measures of the MPC in the US

The following figure shows the evolution of two measures of rates of return on capital, for the United States since 1992. Both use the same measure of earnings in the numerator, namely the pre-tax earnings of U.S. non-financial corporations, with different denominators, namely the capital stock measured at replacement cost to and the capital stock measured at market value. In absence of rents, the ratio of earnings to replacement cost would be the natural measure.



**Figure 5.2**

Rates of return to capital.

Source: Blanchard 2019b, fig. 15, with data extended to 2020.

# The evolution of the safe rate



**Figure 3. 1**  
US, Euro, Japan 10-year real rates, 1992-2020

Either measure of the marginal product is substantially higher than the real safe rate, and, more importantly, substantially higher than the growth rate.

# Which Rate should be used to assess the welfare effect of Debt ?

- To answer, think again of the Diamond economy with an underlying constant rate of growth  $g$ , but with fluctuations in the marginal product of capital  $F_K$ , leading to fluctuations around the growth path in both the marginal product of capital and in output.
- When the young save one unit, they get  $(1 + F_K - \delta)$  units next period.
- under the transfer scheme the young give  $D_t$  in period  $t$  to receive  $D_{t+1}(1 + n) = D_t(1 + x)(1 + n) = D_t(1 + g)$
- As we have seen, the average value of  $(F_K - \delta)$  appears substantially higher than  $g$ , so it looks as if the transfer scheme (and by implication, the use of public debt) decreases the welfare of the young.
- But this is not right.  $(F_K - \delta)$  is risky, while the transfer is riskless. Thus, we must adjust the rate of return on capital for risk. But the risk-adjusted rate of return on capital is precisely the riskless rate,  $r$ .
- Thus, the comparison must be between  $r$  and  $g$ .

The conclusion that, even under uncertainty, whether the effect of debt on welfare is positive still depends on a comparison between the riskless interest rate and the growth rate, requires some qualifying remarks.

- The argument leaves aside the indirect effects of public debt. As debt is issued and displaces capital in the portfolio of the young, lower capital decreases returns to labor and increases returns to capital, and these in turn affect welfare. The implication of these indirect effects turns out to be complicated, but the conclusion is that, in general, both the safe rate and the average rate of return on capital will matter.
- The argument assumes that the difference between the safe rate and the expected rate of return on capital reflects investors' rational decisions, based on their degree of risk aversion and the degree of aggregate risk associated with capital. There is however substantial controversy about whether this is the case. The issue is known as the *equity premium puzzle*

# Bibliography I