Were Jevons, Menger, and Walras Really Cardinalists? On the Notion of Measurement in Utility Theory, Psychology, Mathematics, and Other Disciplines, 1870–1910

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1. Utility and Measurement: An Overview

The canonical narrative of the history of utility theory is largely concerned with the contrast between ordinal and cardinal views of utility. According to this narrative, in a first phase, lasting roughly from 1870 to 1910, William Stanley Jevons, Carl Menger, Léon Walras, and other early marginalists treated individual utility as cardinally measurable. In a second phase, inaugurated by Vilfredo Pareto ([1898] 1966, [1900] 2008) and concluded by J. R. Hicks's *Value and Capital* (1939), utility theorists moved away from cardinalism and embraced an ordinal approach to utility.[1]

This standard story overlooks the fact that throughout this entire period discussions about cardinal and ordinal utility were deeply interrelated

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1. See, e.g., Schumpeter 1954, Jaffé 1977, Niehans 1990, Ingrao and Israel 1990, Mandler 1999, Giocoli 2003, and Moscati 2007.

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with the ways in which utility theorists conceived of measurement, that is, with their understanding of what it means to measure a thing, and more specifically, what it means to measure a tricky thing such as the utility a commodity gives to an individual. In this article I focus on Jevons, Menger, and Walras and show that they endorsed what I call, following the terminology introduced by Joel Michell (1993, 1999), the "classical concept of measurement." The concept is labeled "classical" because it dates back to Aristotle and, from the time of Galileo until the early decades of the twentieth century, dominated both the natural and the human sciences. According to the classical view, measuring the property of an object (e.g., the length of a table) consists of comparing it with some other object that displays the same property and is taken as a unit (e.g., a meter-long ruler) and then assessing the numerical ratio between the unit and the object to be measured (if the ratio is two to one, the table is two meters long). When applied to the measurement of utility, this classical concept requires the identification of a unit of utility and the capacity to state that one utility is, for example, two or three times greater than another. In other words, it requires the capacity of assessing utility ratios.[2]

The notion of measurement underlying current cardinal utility is different and, indeed, less demanding. Rather than relying on the ascertainment of units and ratios, the current cardinal utility approach is associated with the economic agent's ability to rank the utility of alternatives (as in the ordinal utility approach) and, in addition, with her capacity to rank differences of utility and so state that, for example, the utility difference between alternatives A and B is larger than the difference between alternatives C and D. If utility is measurable in the classical sense, the existence of a unit of utility permits the ranking of utility differences and so warrants the measurability of utility also in the cardinal utility sense. The reverse, however, is not true: the cardinal measurability of utility does not entail its classical measurability because the ranking of utility differences does not allow for the assessment of utility ratios.[3]

2. The measurement of individual utility is related to the comparability of the utilities of different individuals, which in turn is linked to the measurement of welfare. Since Jevons denied that interpersonal comparisons of utilities are possible, while Menger and Walras neither used interpersonal comparisons in their theories nor discussed them, I concentrate on the measurement of individual utility. For more on interpersonal comparisons by our three marginalists, see Howey 1960.

3. Ordinal utility, cardinal utility, and classically measurable utility are associated with different classes of functional transformations that do not modify the economic information provided by the utility function $u(x)$. Ordinal utility is associated with increasing transformations of $u(x)$,

In this article I argue that Jevons, Menger, and Walras applied their classical conception of measurement to the measurement of utility and, accordingly, focused on the possibility of ascertaining a unit of utility and assessing utility ratios rather than on the mere ranking of utility differences as in current cardinal utility analysis. Therefore, and contrary to the canonical narrative of the history of utility theory, they were not cardinalists in the current sense of the term. To put it differently, I argue that a third form of utility consistent with the classical conception of measurement, namely, classically measurable utility, should be added to the traditional dichotomy between cardinal utility and ordinal utility, and that the utility theories of Jevons, Menger, and Walras belong in what I call the classically measurable utility camp rather than the cardinal utility camp.

At issue here is not merely a point of classification. The substantial point is that the traditional cardinal-ordinal dichotomy is conceptually too threadbare and barren to clothe an accurate narrative of the history of utility theory and, more specifically, to illuminate the problem situation that Jevons and Walras in particular were facing. Both of these two economists clearly perceived, on the one hand, that the measurability of utility would have made their economic theories scientifically sounder and more defensible against the attacks of their critics; on the other hand, however, they thought they knew what measurement was: classical measurement. Because of their classical understanding of measurement, and given the apparent impossibility of identifying a unit of utility and assessing utility ratios, Jevons and Walras believed that the utility featuring in their theories was not measurable. In fact, discussions by Jevons and Walras of the measurability of utility or the extent to which their theories actually relied on such measurability largely originated from the tension between their classical understanding of measurement and the fact that their scientific practices did not square with it.

That tension can be seen as arising from what the historian of mathematics Leo Corry (1996) has called the body of knowledge of a discipline and the image of knowledge prevailing in that discipline. In economics, the body of knowledge includes the objects, assumptions, theorems, results, and empirical implications of economic theory. The economic

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i.e., with any $f[u(x)]$ such that $f' > 0$. Cardinal utility is associated with increasing linear transformations of $u(x)$, i.e., with transformations of the form $\alpha u(x) + \beta$, where $\alpha > 0$. Classically measurable utility is associated with proportional transformations of $u(x)$ that do not modify the zero point of utility, i.e., with transformations of the form $\alpha u(x)$, where $\alpha > 0$. See Fishburn 1970.

image of knowledge shapes the research practices of economists by determining, either tacitly or explicitly, what they consider a relevant economic question, which methods they judge scientifically legitimate to address such a question, what the standards for accepting proofs are, and when an economic theory should be abandoned. I argue that the way in which utility theorists conceived of measurement is a key part of their image of knowledge, and that the tension between their classical understanding of measurement and the fact that their scientific practices did not square with it is a tension between their image of measurement and the body of their utility analysis. And it is precisely that tension that the twofold categorization in terms of cardinal utility and ordinal utility cannot properly illuminate—and that I hope to illuminate here.

As the seminal works of Jevons and Menger that initiated the so-called marginal revolution appeared in 1871, and as Walras published his last economic writing in 1909 (Walras [1909] 1990), the time span covered by their respective writings, approximately 1870–1910, seems an appropriate period on which to focus. In this article I also review the understanding of measurement in a variety of other disciplines in this period: philosophy, which has played a major role in articulating the dominant conception of measurement; physics, which has long provided a model of measurement; psychology, with its discussion of the measurement of sensations; and mathematics, where the distinction between cardinal and ordinal numbers first arose. I also examine the concept of measurement in other areas of economics. This review shows that in the period under consideration the classical understanding of measurement dominated not only economics but also other disciplines and, further, that the conflict between the classical notion of measurement and scientific practices hardly compatible with it was not exclusive to utility theory but also concerned other disciplines, most notably, psychology.

1.1. After 1910: An Outline

The resolution of the conflict between the classical notion of measurement and the new scientific practices involved a reconceptualization of measurement that was worked out only after 1910 and gave rise to what is usually labeled the "representational" view of measurement. This view was elaborated by, among others, Norman Robert Campbell (1920), Ernest Nagel (1931), and Stanley Smith Stevens (1946). According to the representational view, measurement consists of assigning numbers to objects in such a way that the relations among the assigned numbers represent the relevant relations among the objects. Moreover, since objects may entertain different types of relations, the representational view admits different forms of measurement, usually labeled as "scales." For instance, the "nominal scale" applies when objects can be classified but not ranked. In this scale, objects belonging in the same class are associated with the same number (e.g., male = 0, female = 1), and numbers serve in fact only as labels. When objects can also be ranked, the "ordinal scale" applies. In it, the order of numbers mirrors the ranking of objects: if painting A is judged to be more beautiful than painting B, the former will be associated with a number larger than the one associated with the latter. Without describing all possible scales of measurement, I mention that in the representational approach classical measurement expresses a quite demanding form of measurement called "ratio-scale measurement" in which the ratios of numbers mirror the ratios of objects: if the length of table A is twice the length of table B, the number associated with the former could indicate centimeters or inches, but must be twice the number associated with the latter. From the 1950s onward, the representational view has replaced the classical one as the dominant conception of measurement in most disciplines (see Michell 1993, 1999). When applied to utility theory, the representational view associates ordinal utility, cardinal utility, and classically measurable utility with specific scales of measurement, respectively, the ordinal scale, the interval scale, and the ratio scale.

In utility theory, the reconceptualization of the notion of measurement is associated with the ordinal revolution. The nature of this association, however, calls for a modification of the canonical picture of this important passage in the history of economics. On the one hand, while Pareto and the ordinalists of the 1930s showed that the main results of utility theory were largely independent of utility measurement, by *measurement* they still intended classical measurement and, accordingly, considered ordinal utility as unmeasurable. Therefore, their adoption of ordinal utility should not be viewed as a soft passage from cardinally measurable utility to ordinally measurable utility, but rather as a more radical jump into the domain of unmeasurable magnitudes involving the abandonment of the scientific ideal of making utility measurable. On the other hand, however, it was in the course of the ordinal revolution, and more specifically in a debate—overlooked in the standard narrative—on the "determinateness of the utility function," which took place in the *Review of Economic Studies* between 1934 and 1938, that the first clear signs of a representational understanding of measurement appeared in utility theory. In that debate the expression *cardinal utility* acquired its current economic meaning, a meaning that couples it with the ranking of utility differences (see Moscati 2013).

1.2. Objections and Replies

Let me address some possible objections to my main thesis. First, since the classical measurability of utility entails its cardinal measurability, the classical viewpoint of Jevons, Menger, and Walras is consistent with cardinalism: they were "more-than-cardinalists" or "hyper-cardinalists"; so, the standard narrative of the history of utility theory is fundamentally correct. But, I maintain, this interpretation fails to appreciate the specifically "classical" (in the measurement-theoretic sense of the term) character of their utility theories and prevents us from properly understanding their search for utility units and ratios rather than utility differences. It also hinders our appreciation of the main problem they faced: reconciling a classical understanding of measurement with their scientific practices.

Second, the three founders of marginal utility theory assumed that the marginal utility of each commodity is decreasing. To modern eyes, this assumption entails that individuals are able to rank variations of their total utility and state that the increment of total utility obtained from the consumption of the n-th unit of the good is larger than the increment obtained from the $(n + 1)$-th unit; this ranking, however, is none other than the ranking of utility differences that delivers cardinal utility, and thus Jevons, Menger, and Walras were cardinalists. But the three marginalists did not associate decreasing marginal utility with the ranking of variations of total utility. In discussing the measurability of utility, they all referred to the classical, unit-based measurement, and they did so with respect to *both marginal and total utility*. Furthermore, and contrary to contemporary microeconomic analysis, for them the key notion of utility theory was not total but marginal utility. Accordingly, they conceived of the challenge of utility measurement as related to the search for a unit to measure marginal utility, which is a search that goes well beyond current cardinal utility analysis.

Finally, one may claim that in the canonical narrative the expression *cardinal utility* is not used in its current meaning as connected with rankings of utility differences but in a much vaguer sense that encompasses any form of utility stronger than ordinal utility and hence includes even classically measurable utility. But if in the canonical story the term *cardinal utility* is used in a vague and possibly misleading sense, this is in itself sufficient reason to introduce, for the first time, an appropriate conceptual framework that distinguishes between cardinal utility and classically measurable utility, and to build upon it a more accurate narrative of the history of utility theory.

2. Measurement in Philosophy

The classical understanding of measurement dates back to Aristotle. He strictly separated the notions of quality and quantity and associated the notion of measurement with those of quantity, unit, and number. Aristotle argued that a number is either one unit or a plurality of units (*Metaphysics*, bk. X, chap. 1) and defined *quantity* as that which is divisible into two or more constituent parts of which each is by nature a unit (V, 13). Measuring, in this philosophy, means assessing the number of units constituting a quantity, and measurement is in fact the specific way in which a quantity qua quantity is known (X, 1). Similar ideas can be found in Euclid, who defined measurement as a sort of relationship in respect of size between two magnitudes of the same kind (*Elements*, bk. V, definition 3), in which the lesser magnitude measures the greater, while the greater is said to be a multiple of the lesser.

Ancient and medieval scholars recognized that some qualities, although not measurable in the classical sense, admit of predication by the qualifiers *more* and *less*. Aristotle himself observed that a thing may be whiter, more beautiful, or warmer than another and that the same thing may exhibit a quality in different degrees at different moments (*Categories*, chap. 5), but he did not investigate measurement issues with respect to these kinds of qualities. In the Middle Ages these issues were taken up in the context of theological discussions (e.g., can the virtue of charity increase or decrease?), but these medieval debates did not modify the classical understanding of measurement. Thus, in his *Rules for the Direction of the Mind*, René Descartes ([1628] 1999, 64) continued to contrast ordering with measuring: “All the relations which may possibly obtain between entities of the same kind should be placed under one or other of two categories, *viz*. order or measure.”

The distinction between properly measurable quantities and entities only admitting predication by the qualifiers *more* and *less* returned also in the distinction between extensive and intensive magnitudes introduced by Kant in his *Critique of Pure Reason* ([1787] 1997, 288–92).

For Kant, extensive magnitudes refer to spatial and temporal dimensions, are made of homogeneous parts, and thus can be measured in units. Intensive magnitudes, by contrast, are related to perception, cannot be conceived as composed of parts, and have only degrees, that is, one intensive magnitude can be larger or smaller than another. As instances of intensive magnitudes, Kant mentioned the same entities Aristotle referred to as qualities admitting predication by the qualifiers *more* and *less*, namely, color and warmth (292). Kant's distinction between extensive and intensive magnitudes lived on in philosophy well into the twentieth century, whereby extensive magnitude generally meant a magnitude measurable in the classical sense, while intensive magnitudes were thought of as things that can be ranked but not measured.

3. Measurement in Physics

Although the quantitative natural philosophy that arose in the scientific revolution was in many ways opposed to Aristotle's qualitative physics, Galileo, Newton, and the other main natural philosophers of the period endorsed Aristotle's view of measurement. For example, in his *Arithmetica Universalis* (1707), Isaac Newton defined number as "the abstracted Ratio of any Quantity to another Quantity of the same kind, which we take for Unity" (quoted in Michell 2007, 20). More than 150 years later, at the very beginning of his *Treatise on Electricity and Magnetism*, the eminent Scottish physicist James Clerk Maxwell (1873, 1) expressed the same view: "Every expression of a Quantity consists of two . . . components. One of these is . . . a certain known quantity . . . , which is taken as a standard of reference. The other component is the number of times the standard is to be taken in order to make up the required quantity."

Between 1870 and 1910 the classical understanding of measurement remained fundamentally undisputed in physics, probably because most of the entities physicists dealt with in the period were measurable in the classical sense, and thus they were not motivated to commence any fundamental reconsideration of the notion of measurement. Rather than questioning what measurement means, physicists focused on more practical issues, such as how to attain precision and reproducibility in classical measurement (see Darrigol 2003).

An important physical magnitude that for a long time had eluded classical measurement was heat. We saw that Aristotle considered the heat of a body as a quality admitting the more and less, and this view remained the standard one for centuries. From the seventeenth century onward a gradual move toward the quantification of heat commenced: beginning with Galileo, several types of thermometers were introduced and continuously improved; different temperature scales were proposed, such as the Fahrenheit (1724) and the Celsius (1742) scales; various thermal units were put forward, such as Clemént's calorie (1824), which is the quantity of heat required to raise the temperature of a kilogram of water by one degree. Yet this process of quantification notwithstanding, at the end of the eighteenth century Kant still considered heat an intensive magnitude that could not be measured in the classical sense. Around 1850, however, William Thomson (Lord Kelvin) and James Prescott Joule showed how the thermal properties of gases could be used to measure temperature in an absolute, that is, classical, way. From that point on, heat was conceived of as a classically measurable magnitude (see Chang 2004).

4. Measurement in Psychology

During the period 1870–1910, a fundamental discussion about measurement took place in psychology in the context of the emergence of psychophysics, which is generally considered the immediate ancestor of twentieth-century quantitative and experimental psychology.

4.1. Fechner's Psychophysics

Gustav Fechner was a German philosopher and scientist who attempted to overcome the dualism between mental and physical phenomena by constructing an exact science of the relations between body and mind, which he called "psychophysics" (Fechner [1860] 1966, xxvii). For this purpose, Fechner sought to measure sensations, a goal he understood in terms of classical measurement: "Generally the measurement of a magnitude consists of assessing how many times another magnitude of the same kind taken as a unit is contained in the former" (38).[4] As the unit of sensation, he took the "just-perceivable difference" of sensation, that is, the minimal discernible difference of sensation generated by a change in a physical stimulus. Fechner acknowledged that the change of physical stimulus

4. Here and in a few other places, I modified the translation of the cited work. In this case, in the original translation, H. E. Adler translated Fechner's *Grösse* as "quantity" while I translated it as "magnitude"; he also used "how often" where I use "how many times."

necessary to produce a just-perceivable difference of sensation varies with the magnitude of the stimulus. Thus, if a subject carries an object weighing forty grams, an increment of one gram may be sufficient to produce the just-perceivable sensation of a heavier object, while if the object weighs eighty grams it might be necessary to add two grams to produce the just-perceivable sensation of a heavier object. However, Fechner assumed that all just-perceivable increments of sensation are equal, that is, that they are independent of the magnitude of the generating stimulus. Based on this assumption, he argued that a given sensation can be split up into equal, just-perceivable increments and is measured—"as if by the bits of a yardstick" (50)[5]—by the number of increments necessary to generate that sensation starting from the point where no sensation is perceived. Thus, if the sensation corresponding to carrying an object of 130 grams can be reached from the zero point of sensation through ten just-perceivable increments of sensation, the measure of the sensation associated with 130 grams is ten. If the sensation corresponding to carrying another object can be reached through twenty just-perceivable increments of sensation, the latter sensation is twice the former. In particular, Fechner arrived at a logarithmic formula connecting a physical stimulus to the corresponding sensation, a formula that came to be known as the Weber-Fechner law.

4.2. The Debate on Psychophysical Measurement

Psychophysics was subsequently developed by, among others, Wilhelm Wundt (1873–74) and Joseph Delboeuf (1873). In the 1870s and 1880s, however, Fechner's basic claim that sensations can be measured was attacked from many quarters, a few of which shall be considered below.[6]

The French mathematician Jules Tannery argued that the only measurable magnitudes are those for which we can conceive addition and difference, which is not the case for sensations: "I can conceive neither the sum of two sensations nor their difference: when a sensation increases, it becomes another" (Tannery 1875, 1020; my translation). Therefore, sensations cannot be treated with mathematical tools such as differentiation and integration that presuppose the measurability of the objects to which

5. The original translation reads "like inches on a yardstick."

6. For more on the debate over the measurability of sensations, see Titchener 1905 and Heidelberger 2004.

they apply: "Since we do not know at all what the difference between two sensations means, how can we speak of the differential of a sensation? . . . Since we do not know what the sum . . . of two sensations is, what does integration, i.e. the sum of differentials of sensation, mean?" (877; my translation). Based on these considerations, Tannery attacked the supposed differentiation and integration of sensations that Fechner had employed to arrive at his logarithmic formula.

The German physiologist Johannes von Kries ([1882] 1995) criticized Fechner's assumption that all just-perceivable differences of sensation are equal. For Kries, the increments of sensations occurring at different levels of stimuli are not comparable and thus the claim that they are equal is meaningless: "If a location on the skin is subjected first to a two-pound and then to a three-pound load, and subsequently to a ten-pound and then a fifteen-pound load, . . . the one increment is something quite different from the other; at first they admit of no comparison. The claim that they may be equal makes absolutely no sense" (291). Kries went on to criticize Fechner's claim that a sensation can be divided into equal, just-perceivable differences of sensation: "A loud tone does not conceal within itself this or that many faint tones, in the same sense that a foot contains twelve inches or a minute sixty seconds" (292).

Echoing some of Tannery's arguments, the French philosopher Henri Bergson ([1889] 1960) argued that sensations are not even intensive magnitudes admitting the more and the less. Different sensations are qualitatively different, and the interpretation of the transition from sensation S to sensation S' as an increase or a decrease is a symbolic but arbitrary "interpretation of quality as quantity" (69). For Bergson, because sensations are qualities, they are unmeasurable.

Fechner replied to some of these criticisms but basically did not modify his stance. By contrast, in their effort to defend Fechner and psychophysics from these and other attacks, Delboeuf (1875, 1878, 1883), Wundt (1880), and other psychologists began to argue that psychophysics does not measure the absolute magnitude of sensations, as in Fechner's theory, but rather the magnitude of difference between sensations. Significantly, however, the revised measurement of sensation differences was still of the classical, unit-based kind, as the unit by which these differences were to be measured was still the equal, just-perceivable increment of sensation. This modified solution left many problems open and failed to satisfy critics of psychophysics such as Tannery (1884, 1888).

4.3. Mental Measurement in the 1900s

From the end of the nineteenth century on, quantitative psychologists began to extend their mental measurements from sensations to intellectual abilities. For our purposes, the important point is that, while the object of attempted measurement may have changed, the understanding of measurement remained the classical one. For instance, in his entry on measurement for the *Dictionary of Philosophy and Psychology*, the American James Cattell (1902, 57), who was one of the pioneers of mental measurement, asserted, "[Measurement] is the determination of a magnitude in terms of a standard unit. . . . A ratio is the basis of all measurement."[7]

4.4. Measurement in Psychology: Summing Up

In the period 1870–1910, the emergence of psychophysics stimulated a discussion on measurement in psychology. Nevertheless, both advocates and critics of psychological measurement consistently referred to the classical understanding of measurement.

The entire debate on psychological measurement may be viewed as originating from a conflict very similar to that faced by Jevons, Menger, and Walras. On the one hand, psychologists pursued the scientific goal of making sensations and other mental variables measurable and elaborated different measurement practices to achieve this goal. But on the other hand, they consistently adhered to the classical understanding of measurement, according to which these practices did not deliver true measurement. As in utility theory, however, in the period 1870–1910 this conflict did not prompt a reconceptualization of the notion of measurement within psychology.

5. Measurement in Mathematics

Our period witnessed major developments in mathematics, with mathematicians addressing explicitly the question of what conditions make a magnitude measurable in the classical sense and introducing the distinction between cardinal and ordinal numbers that later passed into economics.

7. For more on the early history of the measurement of intellectual abilities, see Boring 1957 and Michell 1999.

5.1. Conditions for Classical Measurability

In 1882, and in the context of the debate on psychophysics and the measurability of sensations, the German mathematician Paul Du Bois-Reymond attempted to define the properties that make a magnitude measurable. By measurement, Du Bois-Reymond (1882, 30–31, 47–48) intended the assessment of the ratio between the magnitude and a unit, that is, classical measurement. He took length as the archetypal form of a measurable magnitude, labeled "linear magnitudes" (my translation) those magnitudes analogous to length, and proceeded to indicate six formal properties that would characterize them (23–47). Among other characteristics, he argued that any linear magnitude can be obtained from the sum of smaller linear magnitudes and can also be divided arbitrarily with the result always a linear magnitude. Du Bois-Reymond praised Fechner and claimed that warmness and other tactile sensations are linear magnitudes and hence measurable (29–33).

In 1887, in "Numbering and Measuring from an Epistemological Viewpoint" (Helmholtz [1887] 1999), the German physicist, physiologist, and philosopher Hermann von Helmholtz developed some of Du Bois-Reymond's ideas. In accord with the classical understanding of measurement, Helmholtz defined a magnitude as measurable if it can be expressed as the sum of some kind of unit (740–41). He then put forward a set of features identifying measurable magnitudes, but, in contrast to Du Bois-Reymond, Helmholtz intended these features as empirical conditions to be verified practically or experimentally, rather than as formal properties. In particular, for Helmholtz there must be some practical procedure to compare two magnitudes and ascertain whether they are alike, and a further procedure analogous to mathematical addition to connect them. For instance, physical magnitudes such as the weights of two bodies are compared by placing the bodies on the two pans of a balance, and they are connected by putting the two bodies in the same pan.

The notion of measurement was also discussed by Henri Poincaré, a prominent French mathematician, physicist, and philosopher of science. In an article on the mathematical continuum, Poincaré (1893, 26–27) pointed out that order alone is insufficient to make continuum magnitudes measurable. A first condition of attaining measurability is the transitivity of the equality relation between magnitudes, which means that if magnitude A equals magnitude B, and magnitude B equals magnitude C, then magnitude A must equal magnitude C. Poincaré argued that sensations do not satisfy this transitivity condition and thus are not measurable (29). As a second condition for the measurability of magnitudes, Poincaré mentioned their divisibility into *n* equal parts. He did not go into the subject, however, simply referring approvingly to Helmholtz's 1887 article for a thorough treatment of the conditions required for the measurability of magnitudes. His reference to Helmholtz indicates that Poincaré, like the German scientist, understood measurement in the classical sense.

5.2. Cardinal and Ordinal Numbers

The distinction between cardinal numbers and ordinal numbers appears to have been introduced by the German mathematician Ernst Schröder in 1873. Schröder's cardinal numbers match the classical understanding of number and measurement as stated by Aristotle: cardinal numbers express the total number of units constituting a given quantity, for example, five, and thus are the relevant ones for the unit-based measurement of it. Ordinal numbers, by contrast, come to the fore in the process of counting the units belonging to a quantity and express the position of a specific unit of the quantity, for instance, the fifth unit (Schröder 1873, 13–14).

Helmholtz ([1887] 1999, 729) referred approvingly to Schröder's discussion of cardinal and ordinal numbers; but while Schröder treated cardinals and ordinals as associated with two different but equally original functions of number, Helmholtz suggested that ordinals are more fundamental than cardinals. For Helmholtz, in fact, numbers derive from the psychological capacity of "retaining, in our memory, the sequence in which acts of consciousness successively occurred in time" (730) and thus are originally ordinal in nature. Cardinal numbers come into play only when we apply the numbers generated by the internal intuition of time to external experience, typically in the attempt to determine the amount of objects belonging to a given set.

Although on different theoretical grounds, two other eminent German mathematicians, Leopold Kronecker ([1887] 1999) and Richard Dedekind ([1888] 1999), also argued that ordinals are more fundamental than cardinals.[8]

8. Georg Cantor ([1887] 1932), another major German mathematician, did not agree with this idea. His stance was related to the specific meaning he attributed to the terms *cardinal number* and *ordinal number*. Although in mathematics these terms have generally been used according to Cantor's sense, in economics his terminology has had no significant influence.

5.3. Cardinals and Ordinals from Mathematics to Economics

In an article published in 1893 in the political-economy journal *Zeitschrift für die Gesamte Staatswissenschaft*, the German mathematician and economist Andreas Voigt cursorily mentioned the distinction between ordinal and cardinal numbers as presented by Helmholtz, Kroneker, and Dedekind, and he endorsed their view that ordinals come first: "In accordance with the fundamental conceptions of the nature of numbers which mathematics has developed in recent times, we see the primary form of the number concept in the ordinal number and not in the cardinal number" (Voigt [1893] 2008, 502).[9] As Torsten Schmidt and Christian Weber (2008, 498) have noted, this passage would seem to mark the first appearance of the terms *ordinal* and *cardinal* in an economics paper. However, his reference to ordinal numbers notwithstanding, Voigt thought of measurement in the classical sense (Voigt [1893] 2008, 503–4). Moreover, Voigt's notion of cardinal was not the current economic one associated with the ranking of differences, but the mathematical notion expressing the total number of units constituting a given quantity and, as such, was thus associated with classical measurement (Voigt 1893, 606).

Among the early marginalists, only Edgeworth seems to have noticed Voigt's paper.[10] Edgeworth (1894, 1900, 1907, 1915) referred to Voigt and his distinction between cardinal and ordinal numbers in four articles in the *Economic Journal*, although always in a cursory way. The point to be stressed here is that, like Voigt, Edgeworth associated cardinal numbers with the availability of a unit and hence with classical measurement, and he did so without mentioning utility differences. At any rate, until the mid-1930s the cardinal-ordinal terminology did not catch on in economics.

5.4. Measurement in Mathematics: Summing Up

Like the contemporaneous debate in psychology about the measurability of sensations, the mathematical discussion concerning the conditions

9. In the original translation, Torsten Schmidt used the plural form "ordinal numbers" and "cardinal numbers," while I follow Voigt and use the singular.

10. In the writings of Menger and Walras I found no mention of Voigt (Jevons had died in 1882). For the time span 1893–1910, the JSTOR database contains no references to Voigt's article apart from those made by Edgeworth. In the literature in German, Voigt's article was apparently referred to only by František Čuhel (1907).

that make a magnitude measurable remained confined within the boundaries of the classical understanding of measurement. Furthermore, the mathematical notion of "cardinal," associated with the counting of objects and the decomposition of a number into units, was related to the classical understanding of measurement and thus had nothing to do with the ranking of differences between objects. When the cardinal-ordinal terminology passed into economics through Voigt and Edgeworth, the notion of "cardinal" maintained its classical connotation.

6. Measurement in Economics, before and beyond Marginal Utility Theory

At least since Sir William Petty's *Political Arithmetick* (1690), economists have attempted to measure many things aside from utility, such as the national income, the general price level, and the quantity and velocity of money. Like utility theorists, economists working in other areas of economics have also faced a number of theoretical and practical problems in their measurement attempts. But again as in the case with utility theory, in the period 1870–1910 these problems did not prompt any reconceptualization of the notion of measurement.[11]

An area of economics in which measurement issues were discussed thoroughly, even before 1870, was the theory of exchange value. Classical economists such as David Ricardo, as well as later marginalists, attempted to measure the exchange value of commodities. By *measurement*, all these authors intended the assessment of the ratio, which in this case was an exchange ratio, between a given commodity and another commodity, be it money or some other commodity, taken as a unit. In other words, by measurement these authors understood classical measurement. Generally speaking, the problem they faced was that the unit available to measure exchange value was not fixed and invariable as are the units used to measure physical magnitudes.

Even scholars who addressed the issue of the measurability of pleasure or utility prior to the emergence of marginal utility theory understood the measurement of these feelings in the classical sense. Daniel Bernoulli ([1738] 1954) argued that the value of a risky proposition for an individual depends on the average utility (*emolumentum medium*) that the proposi-

11. On economic measurement beyond utility theory, see Porter 1994; Rima 1995; Klein and Morgan 2001; and Boumans 2005, 2007.

tion's outcomes give to him. However, Bernoulli did not discuss the possible issues related to the measuring and averaging of utilities, and he identified without further ado the utility of outcomes with that archetypal classically measurable magnitude, the length of segments (26). Moreover, Bernoulli talked of a utility that is "twice as much" (*duplum emolumentorum*) as another utility (25), and so implicitly took for granted the possibility of assessing utility ratios. Finally, nowhere in his essay did Bernoulli mention the ranking of utility differences.[12]

In the late eighteenth century Jeremy Bentham claimed that pleasures can be measured directly by taking as a unit "that pleasure which is the faintest of any that can be distinguished to be pleasure" (quoted in Halevy 1901, 398). Bentham's faintest pleasure evidently resembles the just-perceivable sensation of later psychophysics. The founder of utilitarianism also suggested an indirect way to measure a pleasure, namely, by the quantity of money paid to obtain it (410). The soundness of his arguments aside, the point to note is that both of Bentham's measures of pleasure in principle allow for the measurement of utility in the classical sense: in both cases there is a unit, that is, the "faintest pleasure" or the monetary unit, that makes the assessment of utility ratios possible.

In "On the Measurement of the Utility of Public Works," the French engineer Jules Dupuit ([1844] 1952) took, as the measure of the utility of an object, the maximum price a consumer is willing to pay for it. Although Dupuit's measure of utility is questionable (see section 9.3 below), it allows in principle for a classical measurement of utility: there is a definite unit, that is, the monetary unit, and utility ratios between commodities can be easily assessed by comparing the maximum prices the consumer is willing to pay for them.[13]

This concludes our review of the notion of measurement in various disciplines—philosophy, physics, psychology, mathematics, and economics—before and beyond marginal utility theory. The review has offered a picture of the broad intellectual context within which Jevons's, Menger's, and Walras's discussions of utility measurement took place

12. Today, Bernoulli's contribution is usually seen as an early formulation of the expected utility theory presented by John von Neumann and Oskar Morgenstern (1944), and since this theory features a cardinal utility function, Bernoulli is often pictured as a cardinalist *ante litteram*. However, the "classical" way Bernoulli treated utility in his theory suggests that this was not the case. For more on Bernoulli's theory, see Giocoli 1998.

13. Heinrich Gossen ([1854] 1983) also argued that pleasures are measurable in the classical sense. On Gossen's understanding of utility measurement, see Stigler 1950.

and has shown that the classical understanding of measurement had reigned unrivaled within all of those disciplines since their earliest origins. Given this background, the fact that Jevons, Menger, and Walras identified measurement with classical measurement and remained committed to the classical notion despite its conflict with their economic practices should hardly appear surprising.

7. Jevons

William Stanley Jevons (1835–1882) contributed to disciplines as diverse as formal logic, meteorology, political economy, statistics, and the philosophy of scientific method. In *The Principles of Science*, his main work as a philosopher of science, Jevons (1874) assigned great importance to the measurement of phenomena, which he discussed at length in chapters 13 and 14, "The Exact Measurement of Phenomena" and "Units and Standards of Measurements" (1:313–86). Here Jevons associated the advance of scientific knowledge with "the invention of suitable instruments of measurement" (313). By measurement, he intended classical measurement: "The result of every measurement is to make known the purely numerical ratio existing between the magnitude to be measured, and a certain other magnitude, which should, when possible, be a fixed unit or standard magnitude" (331).

In his work as a scientist Jevons also assigned great importance to measurement and engaged in numerous enterprises in empirical measurement. In these, some unit of measure was readily available or could be constructed—be it a weight unit, a temporal unit, or a monetary unit—and thus Jevons's scientific efforts fitted smoothly into the classical approach to measurement.[14]

In dealing with utility in *The Theory of Political Economy* (first edition, 1871; second edition, 1879), Jevons faced a conflict between the importance he assigned to measurement, his classical conception of it, and the apparent lack of an appropriate unit to measure utility in the classical way.

7.1. Economic Calculus and Utility Measurement

Inspired by Bentham, Jevons (1871, vii) famously defined economics "as a Calculus of Pleasure and Pain." This calculus is possible, Jevons claimed,

14. For more on Jevons's empirical measurements, see Peart 1995, 1996, 2001; and Maas 2005.

because pleasure and pain are "quantities," where by *quantity* he intended that which is "capable of being *more or less* in magnitude" (4), that is, Kant's intensive magnitudes. However, being a quantity in this sense does not warrant its measurability, and Jevons frankly acknowledged that pleasure and pain cannot be measured in the unit-based sense:

> We have no means of defining and measuring quantities of feeling, like we can measure a mile, or a right angle, or any other physical quantity. . . . We can hardly form the conception of a unit of pleasure or pain, so that the numerical expression of quantities of feeling seems to be out of the question. (19)

Jevons's problem was that at least some parts of his economic calculus, such as the sum and integration of utility increments in order to obtain total utility (54–56) or the differentiation of total utility to derive the final degree of utility, that is, marginal utility (59–60), appeared to depend indeed upon "the numerical expression of quantities of feeling." Jevons therefore felt obliged to defend the soundness of his economic calculus, which he did with four main arguments.

First, Jevons speculated that, although pleasure and pain could not then be measured, they might become measurable in some near future (9–11). In the history of science, he noted, there are several entities such as probability, electricity, and heat that had for a long time appeared unmeasurable but had recently become amenable to exact measurement:

> We cannot weigh, or gauge, or test the feelings of the mind; there is no unit of . . . suffering, or enjoyment. . . . If we trace the history of other sciences, we gather no lessons of discouragement. In the case of almost everything which is now exactly measured, we can go back to the time when the vaguest notions prevailed. (8–9)

Here we observe only that Jevons's hope that utility would soon become measurable was overly optimistic and note his insistence on the inextricable connection between the possibility of measurement and the availability of a unit.

Second, Jevons claimed that his theory was largely independent of the measurability of utility because it mainly relied upon the direct comparison of different pleasures in order to determine which is the greater, and it "seldom or never affirm[s] that one pleasure is a multiple of another in quantity" (20). For Jevons, the mind of an individual is able to directly compare different pleasures just as the balance compares different weights, and insofar as this mental comparison is feasible, units to measure pleasures are unnecessary:

> We only employ units of measurement . . . to facilitate the comparison of quantities; and if we can compare the quantities directly, we do not need the units. Now the mind of an individual is the balance which makes its own comparisons. (19)

In this argument one may detect the seeds of an ordinal approach to utility, which, however, Jevons did not develop. Moreover, the argument confirms, by negation, Jevons's understanding of measurement: if true measurement were possible, it would employ units and consist of assessing how many times a pleasure is a multiple of another; luckily, economic calculus relies on true measurement only seldom, for the mere ranking of pleasures usually suffices.

Third, Jevons argued that, although not directly measurable, pleasure and pain can be measured through their indirect but observable market effects, notably through prices. In this respect, he claimed, feelings are analogous to gravity, for both are unobservable but both can be measured through their effects on, respectively, market prices and physical bodies:

> We can no more . . . measure gravity in its own nature than we can measure a feeling, but just as we measure gravity by its effects in the motion of a pendulum, so we may estimate . . . feelings by the varying decisions of the human mind. The will is our pendulum, and its oscillations are minutely registered in all the price lists of the market. (14)

Here Jevons again refers to measurement in relation to a physical entity, namely, gravity, which is measurable in the classical sense.

Although in 1871 he did not know Dupuit's 1844 article, in his fourth argument Jevons put forward an idea similar to the one proposed by the French engineer.[15] Jevons suggested in fact that the final degree of utility that a commodity gives to an individual can be measured by the money she is willing to pay to purchase an additional unit. As observed in section 6, money is appealing as a measuring rod since it allows for classical measurement. Jevons, however, specified that money measures utility only if "the general utility of a person's income is not affected by the changes of the price of the commodity" (140). However, this condition is violated for commodities that absorb a significant fraction of the person's income,

15. For more on Jevons and Dupuit's article, see section 9.3 below.

as it was in the case of bread for the nineteenth century's poor: "When the price of bread rises much, the resources of poor persons are strained, money becomes scarcer . . . , and . . . the utility of money . . . rise[s]" (142). As a consequence, the willingness to purchase an additional unit of bread and possibly other commodities decreases, although the final degree of utility of that unit may not have changed.

To summarize, Jevons put forward different arguments to avoid, or at least soften, the conflict between the acknowledged unmeasurability of utility in the classical sense and the fact that at least certain parts of his utility theory appeared to depend on the classical measurement of utility. From the viewpoint of present-day measurement theory, one possible way out of this conflict could have been the abandonment of the classical notion of measurement. However, like psychologists of the period 1870–1910, Jevons did not take this route. Indeed, each of his arguments shows that he consistently stuck to the classical understanding of measurement. Another possible route would have been to show that the economically relevant outcomes of utility analysis were in effect independent of the classical measurability of utility. This is the solution reached at the end of the ordinal revolution, which Jevons foreshadowed in his second argument but did not carry through.

7.2. Utility Measurement and Decreasing Marginal Utility

As with the other early marginalists, Jevons (1871, 61–68) assumed that the marginal utility of each commodity will eventually decrease. Jevons's economic theory relies significantly on this assumption, but as noted in section 1.2, this does not imply that he was reasoning in terms of interval-scale measurement of total utility or cardinal utility analysis. In fact, for him, marginal utility, and not total utility, was the key notion "upon which the whole Theory of Economy will be found to turn" (61–62). Marginal utility determines how commodities are distributed among different uses, the laws of exchange between them, and the offer of labor. Total utility is of secondary economic importance and, generally, it is more difficult to assess than marginal utility (61).

Accordingly, Jevons's discussion of the measurability of pleasure summarized in section 7.1 refers primarily to marginal utility, not total utility. It is marginal utility that is, at present, indirectly measured by prices (third argument), or by money, at least in some special cases (fourth argument).

It is marginal utility that in the future might even be measured in a direct way (first argument) and whose measurement, in the end, is seldom employed in the theory of political economy (second argument). Thus Jevons consistently thought in terms of a classical, unit-based measurement of marginal utility, not a modern, interval-scale measurement of total utility.

7.3. Utility Measurement for Jevons's Critics

Anticipating the arguments of Fechner's critics, a number of reviewers of the *Theory* argued that Jevons's analysis relied on the assumption that utility is measurable but that he was unable to substantiate this assumption. Again, as with Fechner's critics, when Jevons's critics wrote of measurability they too meant classical measurability.

The anonymous reviewer of the *Saturday Review* argued that the mathematical treatment of utility, and specifically its differentiation, presupposed the measurability of utility in terms of a unit, but that Jevons had not established his own presuppositions: "When a mathematician wishes to calculate the variations of a force, he begins by telling us distinctly what is the measure of force. . . . But what is the measure of utility? To this we can discover no answer in Mr. Jevons's book" ("Jevons on the Theory of Political Economy" [1871] 1981, 153). Another anonymous reviewer, whose comments appeared in the *Glasgow Daily Herald*, criticized Jevons's assumption that utilities can be added or subtracted on the ground that this kind of manipulation would require the possession of "some unit of pleasure . . . by which [pleasures] might be measured" (quoted in Howey 1960, 62). For this reviewer such a unit was simply not available.

The classical economist John Elliott Cairnes ([1872] 1981, 150) maintained that Jevons's argument, that utility although not directly measurable may nevertheless be measured indirectly by prices, is circular: "How are we to measure pleasure? . . . We may take exchange-value as the criterion of utility; and this is the test that Mr. Jevons ultimately adopts. . . . So that what we come to is this—exchange-value depends upon utility, and utility is measured and can only be known by exchange value." In a letter to Cairnes of 1872, Jevons replied to this criticism by repeating a point already made, namely, that the method of measuring utility is indirect but analogous to that used to measure gravity, and that insofar as the latter is appropriate, so too is the former (Black 1972–81, 3:246).

The opinions Jevons expressed in the second edition of the *Theory* (1879) on the measurability of utility are almost identical to those presented in the first edition, and this suggests that the critical remarks of Cairnes and others did not change Jevons's ideas on utility measurement.

7.4. Jevons and Measurement in Other Disciplines

As we have seen above, in looking for a solution to the problem of utility measurement Jevons often referred to measurements performed in other disciplines. His main models were physical measurements such as that of gravity, heat, and electricity, but he also referred to the geometrical measurement of miles and angles, as well as to the measurement of probability. As already noted, at the time at which Jevons was writing, all these magnitudes were measurable in the classical sense.

Contrary to the suggestions of some popular histories of economic thought, such as those by Mark Blaug (1997, 318) or Robert Ekelund and Robert Hébert (1990, 357), and also contrary to a potentially beautiful fit with the narrative developed in the present article, Jevons appears to have been influenced by neither Fechner's psychophysics nor the related Continental discussion on psychophysical measurement. Rather, Jevons drew significantly upon a different and specifically British approach to psychological research known as physiological psychology, an approach that predated Fechner's psychophysics and aimed at explaining complex mental phenomena like thoughts, will, and emotions as the physiological product of certain parts of the nervous system. While Michael White (1994) and Harro Maas (2005) have shown the profound influence of physiological psychology on Jevons's political economy, the same cannot be said of Fechner and psychophysics. The only reference to psychophysics existing in Jevons's works and correspondence appears to be his cursory allusion to "Fechner's law, Wundt's curve of pleasure and pain, or Delboeuf's formulae" (Jevons 1881, 581) in his review of Edgeworth's *Mathematical Psychics* (1881). Moreover, this was only an indirect reference, for Jevons was merely mentioning Edgeworth's sources. As a matter of fact, Jevons was probably aware of Fechner and psychophysical research before 1881, since he was a friend of the psychologist James Sully, who had studied psychology in Germany and played an important role in introducing psychophysics into England around the mid-1870s (see Boring 1957). Yet even so, no traces of an appreciation of psychophysics can be found in Jevons's writings. Later psychological discussions concerning the measurement of sensation or mathematical discussions concerning number and measurement could not influence Jevons, for he died in 1882.

8. Menger

Unlike Jevons, Carl Menger (1840–1921) avoided taking an explicit stance on issues surrounding the measurability of utility. Therefore, in order to appraise his views we need to take a roundabout path and start from Menger's discussion of money as a possible measure of the exchange value of goods. From here we pass to Menger's *Principles of Economics* and the numbers he used there to express the marginal utility of goods.

8.1. The Measurement of Exchange Value

Menger discussed the issue of whether money measures the exchange value of goods in his *Principles* ([1871] 1981, 272–80), in his article "Money as Measure of Value" ([1892] 2005), and in the entry on money published in the *Handwörterbuch der Staatswissenschaften* ([1909] 2002). His most thorough discussion of the issue can be found in this entry, but the ideas it expresses are substantially identical to those presented in the *Principles*.

For Menger ([1909] 2002, 60), the precise question was whether "the valuation of goods in money [should] be regarded as measurement [*Messung*] of their exchange value by the monetary unit." In answering, Menger first described measurement as "a procedure by which we determine the as yet unknown magnitude of an object by comparison with a known magnitude of the same kind taken as a unit" (60) and then claimed that money does not measure the exchange value of goods. In fact the exchange ratios between money and goods, that is, prices, vary from time to time and from place to place, and it is not only absolute but also relative prices that vary. This variation may be due, not only to modifications in the exchange value of goods, but also to modifications in the exchange value of money: unlike the fixed and invariable units used in physical measurement, the monetary unit changes. Nevertheless, and although for Menger money does not measure exchange value, it does provide "estimates" (*Schätzungen*) or "valuations" (*Bewertungs*) of exchange value that are useful for many practical purposes ([1871] 1981, 276, and [1909] 2002, 61, respectively).

Menger's definition of measurement, his rejection of the idea that money measures the exchange value of goods, and the fact that he contrasted measurement with estimates and valuations show that his general understanding of measurement was the classical one. We turn now to the question of whether Menger applied this understanding to utility measurement.

8.2. From the Measurement of Value to the Measurement of Marginal Utility

In his *Principles* Menger defined *value* as the importance of a commodity for an individual, and he argued, cursorily but nevertheless explicitly, that value is "a magnitude that can be measured" ([1871] 1981, 293).[16] For him, the value to an individual of a given quantity of a good is measured by the importance of the need-satisfaction (*Bedürfnissbefriedigung*) assured by the last unit of the good, that is, by what today we would call the marginal utility of that unit (132). This assertion shifts the issue of measurement from the measurement of value to the measurement of marginal utility.

8.3. Menger's Utility Numbers

In investigating the value of consumption goods and productive factors, as well as in analyzing how bilateral exchange works, Menger expressed the marginal utility of goods by numbers and assumed that marginal utility is decreasing. For instance, he imagined that for a particular individual the first unit of food has a marginal utility of 10, the second unit has a marginal utility of 9, the units following have a marginal utility of 8, 7, 6, etc., respectively, while the eleventh unit has zero marginal utility (125–27).

During the heyday of the ordinal revolution, Friedrich Hayek (1934) and George Stigler (1937) rushed to enroll Menger in the ordinal camp, claiming that his utility figures represent only the ranking of the importance of need-satisfactions. As with Jevons, one may find some ordinal insights in Menger; but an ordinalist interpretation clashes with the fact that Menger's utility numbers express decreasing marginal utility. As noted in section 1, decreasing marginal utility entails that total utility is cardinally measurable rather than only ordinally measurable.

16. Menger's stance on value measurement in the *Principles* differs from that expressed in a notebook from 1867–68, in which he wrote "value can be merely estimated, not measured" (quoted in Yagi 1993, 706).

As a matter of fact, Menger's utility figures entail even more than cardinal utility. For he assumed a zero point of marginal utility ([1871] 1981, 126–27, 135, 183–86) and on two occasions also claimed that his utility figures express the ratio of marginal utilities. In a footnote in the *Principles*, Menger in fact specified the following: "When I designate the importance of two need-satisfactions with 40 and 20 for example, I am merely saying that the first of the two satisfactions has twice the importance of the second to the economizing individual concerned" (183). Furthermore, if the marginal utility of a cow to an individual is 10, while the marginal utility of an additional horse is 30, then, Menger argued, the horse has "three times the value of a cow" (184). All of this entails that Menger's utility numbers measure marginal utility in the classical sense in terms of some (unspecified) unit of satisfaction.[17]

8.4. Conclusions on Menger

His elusiveness notwithstanding, Menger understood measurement in the classical fashion, treated marginal utility as if it were classically measurable, and never associated utility measurement with the ranking of utility differences. In contrast to Jevons, Menger never addressed explicitly the tension between his understanding of measurement and his utility analysis. One can speculate why this was the case: maybe Menger understood that the problem of utility measurement could undermine his economic theory and hence intentionally decided to avoid the issue. Alternatively, the absence of criticisms of the kind received by Jevons as to the measurability of utility, or Menger's fundamentally unmathematical approach to utility theory, might have prevented the Austrian economist from having properly appreciated that tension. Arguably, some reference by Menger to the discussions on measurement taking place in other disciplines in the period 1870–1910, or some passage in his correspondence with Walras, might help us better understand Menger's position.[18] Unfortunately, neither in Hayek's edition of Menger's collected works (Menger 1934–36)

17. This is also the interpretation of James Dingwall and Bert Hoselitz, the English translators of Menger's *Principles*; see Menger [1871] 1981, 126–27. In the handwritten notes to his copy of the *Principles*, as transcribed by Emil Kauder (1961), Menger made no comment on the two above-quoted passages. Furthermore, in the second edition of the *Principles*, posthumously published by Menger's son, the passages remained unchanged (Menger 1923, 176–77).

18. There exists no Menger-Jevons correspondence, for the English economist died in 1882, unaware of Menger and his work.

nor in the second edition of the *Principles* (which is not included in the collected works) was I able to trace any reference to extra-economic discussions on measurement. Indeed, the Menger-Walras correspondence is also disappointing in this respect, for the two economists never discussed the issue of utility measurement.

9. Walras

In many respects, when grappling with the issue of utility measurement, Léon Walras's problem situation was very similar to that faced by Jevons. On the one hand, Walras (1834–1910) always maintained an unequivocally classical understanding of measurement, evident in both his dealing with the measurement of utility and of exchange value. Based on this understanding, Walras had to acknowledge that utility was not directly measurable. On the other hand, Walras believed that his economic analysis depended on the measurability of utility. Again, as with Jevons, a number of commentators criticized Walras for relying on the false assumption that utility was measurable. Like Jevons, therefore, Walras put forward a number of arguments in order to reconcile his economic analysis with the apparent unmeasurability of utility.

9.1. As-If Measurement of Utility at the Académie and in the *Elements*

In two 1873 meetings of the Académie des sciences morales et politiques of Paris, Walras read a paper titled "Principe d'une théorie mathématique de l'échange" ("Principle of a Mathematical Theory of Exchange," henceforth "Principle"), which outlined the theory of utility and exchange later expounded in the *Elements of Pure Economics*. The paper was subsequently published in the *Journal des Économistes* (Walras [1874] 1987).

In this paper, Walras discussed how to derive the demand curves of commodities from their "intensive utility," that is, their marginal utility (273–77). In this context, he addressed the measurability of utility and assumed the stance that he was to maintain for the next forty years on the subject: although utility is not directly measurable, we can nevertheless derive demand curves from it and thus carry out a suitable analysis of demand by treating marginal utility *as if* it were measurable. It is to be emphasized that Walras was concerned with the measurement of intensive or marginal utility, not of total utility. Moreover, he associated the measurability of marginal utility, however fictitious, with the availability of a unit, not with the ranking of utility differences:

> Let us suppose, for a moment, that utility is directly measurable, and we shall find ourselves in a position to give an exact, mathematical account of the influence it exerts . . . on demand curves and hence on prices. I shall, therefore, assume the existence of a standard of measure [*étalon de mesure*] of intensive utility. (274; my translation)

Walras briefly justified this as-if procedure by claiming that it is the same as that employed in physics and mechanics, where masses enter into scientific calculations despite the fact that they are not directly measurable (274). Although in 1873 he did not know Jevons's *Theory*, his justification is similar to Jevons's argument that the method of measuring utility is indirect but analogous to that used to measure gravity and thus is scientifically appropriate (see section 7.1).

Walras's paper was not well received at the Académie, and one of the objections raised bore upon the measurability of utility. Emile Levasseur, a professor of economic history, geography, and statistics at the Collège de France, argued that Walras's analysis was misleading because it relied on the false assumption that utility is measurable: "It would be very good if desire and need were measurable in an exact way . . . but it is not like that, quite the opposite. . . . [Walras's] data are, so to speak, incommensurable; it follows that his [intensive utility] curves are without foundation; . . . they are false" (Levasseur [1874] 1987, 530; my translation). Levasseur denied that intensive utility is measurable, but he did not object to Walras's association of measurement with the availability of a unit.

Levasseur's objection did not modify Walras's stance on the measurability of utility. In the *Elements* he repeated almost identically the argument already made in the "Principle" the previous year: marginal utility is not measurable, but by assuming that it is, that is, by assuming the existence of a unit, we can analyze individual demand. Therefore, declared Walras ([1874] 1954, 117), "I shall . . . assume the existence of a standard of measure of intensive utility."[19] The main difference between the arguments of the "Principle" and the *Elements* is that in the latter Walras did not attempt to justify his as-if approach by analogy with the methodology

19. In the original translation, William Jaffé translated *étalon de mesure* as "standard measure," while I find "standard of measure" more accurate.

of physics and mechanics. Possibly, Levasseur's objections suggested to him that it would be safer to remove this claim.[20]

We are not here concerned with the soundness of Walras's as-if argument for utility measurement. What does concern us is Walras's classical association of measurement with the availability of a unit, which is confirmed by his discussion of the measurement of exchange value in lesson 25 of the *Elements*.

9.2. The Measurement of Exchange Value in the *Elements*

Lesson 25 is titled "Choosing an Instrument of Measure and a Medium of Exchange." Here Walras addresses a question analogous to that discussed by Menger in his entry on money, namely, how to measure the exchange value of commodities. Walras examined and rebutted the popular opinion that exchange value can be measured by a monetary unit (e.g., the franc), just as a meter can measure length. His decisive argument was that, contrary to the measurement of length, the measurement of exchange value does not consist in the assessment of the numerical ratio between a given unit and a given object. "When I measure any given length, for example the length of a façade," observed Walras ([1874] 1954, 187), "I have to take three things into account: the length of the façade, the length of a ten-millionth part of a quarter of the earth's meridian [i.e., the length of a meter] and the ratio of the first length to the second, which is the measure of the façade." Walras argued that in the case of value measurement two of the three components of length measurement do not exist: there is neither an intrinsic value of the commodity to be measured nor an intrinsic value of the franc that can be used as a unit for measurement: "The word *franc* [as a unit of value] is the name of a thing that does not exist" (188).

According to Walras, this does not entail that we cannot measure exchange value in a proper sense. All that follows is that to measure exchange value we must choose as a unit "a certain quantity of a given commodity [what Walras called a *numéraire*], and not the value of this

20. The as-if-utility-were-measurable case presented in the first edition of the *Elements* remained in identical form in the following editions of the book (Walras 1988, 105–7). The argument was also repeated in an article in the *Giornale degli Economisti* (Walras [1876] 1987), in an appendix to the third edition of the *Elements* (Walras 1988, 694), in *Théorie de la monnaie* (Walras 1886), and in a number of letters (Jaffé 1965, letters 232, 268, 789).

quantity of the given commodity" (188). The exchange value of a commodity is then measured by the number of units of the *numéraire* exchanged for one unit of the commodity in question.

Again, our concern is not the soundness of Walras's stance on the measurement of exchange value but simply that his discussion of the topic confirms his classical understanding of measurement.[21]

9.3. Walras on Willingness to Pay as a Measure of Utility

As we have seen in sections 6 and 7.1, Dupuit and Jevons had independently suggested that the consumer's willingness to pay can be used as a measure of marginal utility. Walras's correspondence with Jevons illustrates his conviction that this idea is ill founded.

In 1874 Walras sent Jevons an article from the French journal *Le Temps* that attributed the discovery of marginal utility theory to the two of them, together with a subsequent letter to the journal claiming that Dupuit had already arrived at the theory in 1844. In his letter to Jevons, Walras acknowledged that Dupuit "had in fact addressed the problem of the mathematical expression of utility" but argued that "he did not solve it at all" (Jaffé 1965, 1:456; my translation). In 1877 Jevons wrote to Walras that he had finally read Dupuit's 1844 article and affirmed that Dupuit "had anticipated us [Jevons and Walras] as regards the fundamental ideas of utility" (533). In his reply, Walras disagreed, insisting that Dupuit had confused the marginal utility and the demand functions and had failed to see that the maximum price an individual is willing to pay for a commodity depends not only on the utility of the commodity itself but "also, in part, on the utility of other commodities; and it depends also, in part, on the quantity of wealth . . . the consumer possesses" (535; my translation). Therefore, the willingness to pay cannot be taken as a proper measure of marginal utility. In the second part of the first edition of the *Elements*, which was published in September 1877, Walras (1988, 670–71) repeated almost literally the same criticism of Dupuit's utility measure.

Apparently, Jevons was unimpressed by Walras's argument. He wrote Walras that his "remarks upon the Memoirs of M. Dupuit, shall have my best attention" (Jaffé 1965, 1:538), but he made no specific comment on

21. For instance, from a Mengerian perspective the switching from money to some other commodity does not solve the problem of measuring exchange value.

Walras's points. In the second edition of the *Theory*, Jevons (1879, xxx) credited Dupuit with "the earliest perfect comprehension of the theory of utility" and continued to argue that, at least in some circumstances, marginal utility is measured by the individual's willingness to pay (158–60).

This exchange between Jevons and Walras shows that their argumentative strategies for addressing the problem of the apparent unmeasurability of utility were different. While Walras univocally insisted on his epistemological as-if-utility-were-measurable argument, Jevons put forward a wider and more diverse array of arguments. For our purposes, it is also notable that the discussion of willingness to pay as a measure of utility is the only part of the entire correspondence that touches upon the issue of utility measurement. The fact that the two founders of marginal utility analysis did not exchange more ideas about this crucial issue corroborates the claim that they looked at it within a shared conceptual framework, namely, the one offered by the classical conception of measurement.

9.4. The Discussion with Laurent and Poincaré on Utility Measurement

In 1900, when he was sixty-six years old and had almost completed the fourth and final edition of the *Elements*, Walras was brought back to the issue of the measurability of utility by criticisms leveled at him by Hermann Laurent, a distinguished French mathematician. In an exchange of letters in May 1900 (Jaffé 1965, letters 1452–56), and in a communication at a 1900 meeting of the Institut des actuaires français, Laurent objected that Walras's analysis of demand and exchange was based on the assumption that utility is measurable, an assumption that Laurent denied: "How can one accept that satisfaction is capable of being measured? Never will a mathematician agree to that" (Jaffé 1965, 3:113).[22] In reply, Walras stood by his earlier stance that his analysis was in fact independent of any actual measurement of utility: "I skip completely the standard of utility and *rareté*" (119; my translation).

The issue of the measurability of utility cropped up again the following year in an exchange of letters between Walras and Henri Poincaré. As we saw in section 5.1, Poincaré (1893, 1902) stressed order as the basic property of continuum magnitudes, but he also argued that for magnitudes to

22. The original translation, which was by Jaffé and appears on p. 304 of Jaffé 1977, reads "believe" where I use "accept."

be measurable, further conditions such as those indicated by Helmholtz ([1887] 1999) are necessary.

In September 1901 Walras sent to Poincaré, whom he did not know, a copy of the fourth edition of the *Elements*. A brief correspondence between the two followed, in the course of which Walras solicited Poincaré's opinion on his as-if treatment of utility measurement. Walras here brought back into play the alleged analogy with the physical sciences put forward in the "Principle": "I open Poinsot's *Statics* . . . ; I see that he defines the mass of a body as 'the number of molecules composing it.' . . . I notice that, by so doing, he too regards as appreciable a magnitude which is not, given that no one has ever counted the molecules in a body" (Jaffé 1965, 3:161).[23]

In his reply, Poincaré observed that satisfactions can be ordered but not measured, and he agreed with Walras that the immeasurability of satisfactions does not preclude their mathematical treatment:

> Can satisfaction be measured? I can say that one satisfaction is greater than another . . . , but I cannot say that the first satisfaction is two or three times greater than the other. . . . Satisfaction is therefore a magnitude but not a measurable magnitude. Now, is a non-measurable magnitude *ipso facto* excluded from all mathematical speculation? By no means. (162–63)[24]

Poincaré also pointed out that, as satisfactions can only be ordered, the mathematical function expressing them is not unique, and in particular any increasing transformation of the function represents the same sensations equally well. Finally, Poincaré warned that the lack of uniqueness of the function representing satisfactions limits the significance of the results obtained by the mathematical treatment of them.

Two brief comments on Poincaré's reply are in order. First, his remark as to the invariance of the utility function to any increasing transformation recalls the ordinal approach that Pareto ([1898] 1966, [1900] 2008) had recently put forward. There is no evidence, however, that in September 1901 Poincaré was aware of Pareto's ordinal approach, and his remark

23. The original translation, which was by Bruna Ingrao and Giorgio Israel (1990, 156–57), omitted "I notice that," and it used "seeing that no one has ever counted" where I use "given that no one has ever counted." Louis Poinsot (1777–1859) was a French mathematician and physicist; Walras referred here to Poinsot 1842. On Poinsot's influence on Walras's thought, see Jaffé 1965, 3:148–50.

24. Jaffé's translation, with no modifications.

appears rather to be related to the German debates about the conditions for measurement and to his own work on the mathematical continuum. More importantly, in order to argue that satisfactions are not susceptible to measurement, Poincaré claimed that it is impossible to identify their ratios ("I cannot say that the first satisfaction is two or three times greater than the other"). This argument confirms that, as already suggested in section 5.1, Poincaré maintained a classical understanding of measurement and thus was, at least in this respect, completely in accord with Walras.

In the final letter of the correspondence, Walras cursorily acknowledged the partial arbitrariness of the utility function, but he passed over Poincaré's warnings about the significance of the results obtained in an ordinal utility framework and self-servingly interpreted Poincaré's comments as an unreserved statement of support of his own as-if-measurable treatment of marginal utility (Jaffé 1965, 3:167).

In conclusion, his exchanges with Laurent and Poincaré toward the end of his scientific career did not modify Walras's understanding of measurement or his stance on the measurability of marginal utility as expressed in the "Principle" almost thirty years earlier.

9.5. Utility Measurement in "Economics and Mechanics"

In 1909, in his last economic article, "Economics and Mechanics," Walras returned to the issue of utility measurement. Walras here compared economics with mechanics and claimed that both investigate quantitative facts using mathematics and arrive at equilibrium conditions that are formally identical. The nature of the facts investigated is, however, different: mechanics studies exterior or physical facts; economics deals with interior or psychological facts, such as need. The main dissimilarity between physical and psychical facts is, in turn, that units and instruments may be used to measure the former, but not the latter:

> There are meters and centimeters to establish the length of the arms of the steelyard balance, grams and kilograms to ascertain the weight supported by these arms. . . . There are no [instruments] to measure the intensities of need of traders. (Walras [1909] 1990, 212–13)[25]

25. In the original translation, Philip Mirowski and Pamela Cook used "steelyard" where I use "steelyard balance."

Just as before, Walras claimed that the lack of instruments to measure psychological facts entails no difficulty for economics, but he now developed this argument along three different lines.

The first echoes not only Jevons's idea of the mind balancing pleasures but also Poincaré's point in his 1901 letter (published as an appendix to "Economics and Mechanics") and Pareto's ordinal approach to utility.[26] Walras pointed out that at the subjective level each trader compares the utility of different things and determines that which is greater for him. In this sense, although not measurable, utility is at least appreciable: "The *need* which we have for things, or their *utility* for us, is an internal quantitative fact, appreciation of which remains subjective and individual. So be it! Nonetheless it is a magnitude and, I would say, an appreciable magnitude" (207).[27] As in Jevons's mind-as-a-balance argument, one may read in Walras the flavor of ordinal utility theory. However, the seventy-five-year-old Walras did not venture into the ordinal approach, and his distinction between appreciable and measurable magnitudes demonstrates that he did not consider comparison as measurement.

Second, Walras argued that the point of maximum satisfaction for the individual, as well as the point of general equilibrium for the market, is characterized by the equality of marginal utility ratios with price ratios. The circumstance that individuals maximize their utility and that markets are in equilibrium would prove, according to Walras, that traders are capable not only of comparing utilities but also of assessing their ratios and therefore of measuring utility:

> Each trader . . . decides by himself in his internal theatre if his last needs satisfied are proportional to the values of the commodities. The circumstance that the measure is . . . interior . . . does not prevent it from being a *measure*, that is, a comparison of *quantities and quantitative ratios*. (213)[28]

26. Walras had become aware of Pareto's ordinal approach to utility theory by November 1901. Early in that month Pareto sent to Walras the summary of three lessons in which he presented his new ordinal approach (Pareto [1901] 1966). Walras commented on Pareto's summary and noted, among other things, the similarity between Pareto's and Poincaré's ordinal insights. See Jaffé 1965, letters 1501, 1502.

27. In the original translation, Mirowski and Cook used "it is a magnitude and it is even estimable" where I follow the original more closely and use "it is a magnitude and, I would say, an appreciable magnitude."

28. The original translation by Mirowski and Cook reads as follows: "Each trader . . . decides for himself if these last needs satisfied are proportional to the values of the commodities. *Measure*, that is, the comparison of *quantities* and *quantitative relations*, is not prevented by its . . . interior quality."

Scholars such as Levasseur or Laurent may well have objected that this argument begs the question, as the derivation of the conditions for utility maximization and market equilibrium seems to depend on the measurability of utility. The important point for us, however, is simply that Walras's train of reasoning confirms once more that he associated measurement with the classical assessment of quantitative ratios.

The third of Walras's arguments suggests the possibility that even the measurement of physical magnitudes may be problematic. In making this point, Walras quoted from Poinsot's definition of the mass of a body as "the number of molecules contained in it." In meeting this apparent difficulty, Walras appealed again to the authority of Poincaré and mentioned approvingly his instrumentalist definition of mass. According to this definition, "masses are coefficients which are conveniently introduced into calculations" (Poincaré 1902, quoted in Walras [1909] 1990, 213). Walras suggested that utility and *rareté* could be considered in an analogous way, that is, as hypothetical causes to be introduced into the calculations of economics in order to derive the empirical laws of demand, supply, and exchange:

> [I wonder whether] all these concepts, those of *mass* and *force* as well as those of *utility* and *rareté*, might not simply be names given to hypothetical causes which should be necessarily and justifiably introduced into the calculations with a view of linking them to their effects. (213)[29]

If utility and *rareté* are regarded as hypothetical causes of observable effects such as demand and supply, then the burden of measurability shifts to these effects, and the lack of instruments to measure utility and *rareté* ceases to appear problematic. Although now presented in more sophisticated epistemological fashion, the argument is very much the same as that of the "Principle."

9.6. Conclusions on Walras

From the first public presentation of his economic theories in 1873, Walras attempted to reconcile his utility analysis with the circumstance that, according to the classical understanding of measurement he shared

29. In the original translation, Mirowski and Cook used "it would be essential and valid for the hypothetical causes to be incorporated in the calculations" where I use "hypothetical causes . . . should be necessarily and justifiably introduced into the calculations."

with his critics, marginal utility appeared unmeasurable. Abstractly, one possible way out of this difficulty would have been to abandon the classical notion of measurement. Yet Walras, like his contemporaries, consistently adhered to the classical understanding of measurement. In particular, he never associated measurement with the ranking of utility differences. Another possible way out might have been the development of an ordinal approach to utility along the lines suggested by Poincaré and Pareto. However, Walras never explored this path. Walras's chosen way out was to deny the existence of an actual conflict between his utility analysis and the unmeasurability of marginal utility by claiming that if one treats marginal utility as if it were measurable, one can derive the empirical laws of demand, supply, and exchange. Basically, he maintained this stance from 1873 to 1909.

In addressing the issue of the measurability of utility, Walras took as a model the measurement of physical and mechanical magnitudes such as length, weight, mass, and force, all of which are measurable in the classical sense. Turning from physics to psychology, however, we find that, despite the fact that Walras and the early researchers in psychophysics shared a similar problem situation with respect to, respectively, the measurement of utility and the measurement of sensations, neither in his complete economic works (Walras and Walras 1987–2005) nor in his correspondence (Jaffé 1965) are there any references to Fechner or psychophysics. With regard to Walras's possible connections with the mathematical discussions about number and measurement, these seem to be limited to his correspondence with Poincaré, which, as we have seen, did not lead him to modify his earlier views.

10. Summary and Conclusions

In the present article I have argued that the canonical dichotomy between cardinal utility and ordinal utility is conceptually too threadbare and barren to tell an accurate story of utility theory, and that a third form of utility consistent with the classical understanding of measurement should be added to the traditional picture. In particular, the article has shown that Jevons, Menger, and Walras understood measurement in the classical sense and applied this understanding to utility measurement, and therefore they were not cardinalists in the current sense of the term associated with the ranking of utility differences.

I have also analyzed the argumentative strategies adopted by Jevons and Walras to address the conflict between the scientific importance they attributed to measurement, their understanding of it, and the apparent unmeasurability of the utility featuring in their economic theories. Possible solutions to that conflict could have been the abandonment of the classical notion of measurement or the pursuit of an ordinal approach to utility theory. However, Jevons and Walras did not go down this road. They rather provided various economic, epistemological, and historical arguments to argue that the conflict at issue was, if not completely fictitious, then at least less severe than some critics of their theories claimed.

Finally, in order to appreciate the broad intellectual context within which Jevons's, Menger's, and Walras's discussions on utility measurement took place, I have reviewed the understanding of measurement in other disciplines somehow related to utility theory. In particular, I focused on physical measurements, to which Jevons and Walras often referred; on the psychological measurement of sensations, which shares many features with the economic measurement of utility; and on the mathematical distinction between cardinal and ordinal numbers that later passed into economics. The article has shown that in the years from 1870 to 1910, the period in which Jevons, Menger, and Walras were active, the classical understanding of measurement dominated not only utility theory but also the other disciplines considered. This circumstance helps to explain why the three founders of marginal utility theory remained committed to the classical understanding of measurement in the face of its conflict with their economic practices.

References

Bergson, H. (1889) 1960. *Time and Free Will*. Translated by F. L. Pogson. New York: Harper.

Bernoulli, D. (1738) 1954. "Exposition of a New Theory on the Measurement of Risk." Translated by L. Sommer. *Econometrica* 22:23–36.

Black, R. D. C., ed. 1972–81. *Papers and Correspondence of William Stanley Jevons*. 7 vols. London: Macmillan.

Blaug, M. 1997. *Economic Theory in Retrospect*. 5th ed. Cambridge: Cambridge University Press.

Boring, E. G. 1957. *A History of Experimental Psychology*. New York: Appleton-Century-Crofts.

Boumans, M. 2005. *How Economists Model the World into Numbers*. London: Routledge.

———, ed. 2007. *Measurement in Economics: A Handbook*. London: Academic Press.
Cairnes, J. E. (1872) 1981. "New Theories in Political Economy." In Black 1972–81, 7:146–52.
Campbell, N. R. 1920. *Physics: The Elements*. Cambridge: Cambridge University Press.
Cantor, G. (1887) 1932. "Mitteilungen zur Lehre vom Transfiniten." In *Gesammelte Abhandlungen*, edited by E. Zermelo, 378–439. Berlin: Springer.
Cattell, J. M. 1902. "Measurement." In *Dictionary of Philosophy and Psychology*, edited by J. M. Baldwin, 2:57. London: Macmillan.
Chang, H. 2004. *Inventing Temperature*. Oxford: Oxford University Press.
Corry, L. 1996. *Modern Algebra and the Rise of Mathematical Structures*. Basel: Birkhäuser.
Čuhel, F. 1907. *Zur Lehre von den Bedürfnissen*. Innsbruck: Wagnerschen Universität Buchhandlung.
Darrigol, O. 2003. "Number and Measure: Hermann von Helmholtz at the Crossroads of Mathematics, Physics, and Psychology." *Studies in the History and Philosophy of Science* 34:515–73.
Dedekind, R. (1888) 1999. *Was sind und was sollen die Zahlen?* In Ewald 1999, 787–833.
Delboeuf, J. 1873. *Etude psychologique*. Brussels: Hayez.
———. 1875. "La mesure des sensations." *La Revue Scientifique* 46:1014–17.
———. 1878. "La loi psychophysique et le nouveau livre de Fechner." *Revue Philosophique* 5:34–63, 127–57.
———. 1883. *Examen critique de la loi psychophysique*. Paris: Baillere.
Descartes, R. (1628) 1999. *Rules for the Direction of the Mind*. Translated by D. Murdoch. In *The Philosophical Writings of Descartes*, 1:7–77. Cambridge: Cambridge University Press.
Du Bois-Reymond, P. 1882. *Die allgemeine Functionentheorie*. Tübingen: Laupp.
Dupuit, J. (1844) 1952. "On the Measurement of the Utility of Public Works." Translated by R. H. Barback. *International Economic Papers* 2:83–110.
Edgeworth, F. Y. 1881. *Mathematical Psychics*. London: Kegan Paul.
———. 1894. "Professor J. S. Nicholson on 'Consumers' Rent.'" *Economic Journal* 4:151–58.
———. 1900. "The Incidence of Urban Rates." [Pt. 1.] *Economic Journal* 10:172–93.
———. 1907. "Appreciations of Mathematical Theories." *Economic Journal* 17:221–31.
———. 1915. "Recent Contributions to Mathematical Economics, I." *Economic Journal* 25:36–63.
Ekelund, R. B., and R. F. Hébert. 1990. *A History of Economic Theory and Method*. 3rd ed. New York: McGraw-Hill.
Ewald, W. 1999. *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*. Vol. 2. Oxford: Clarendon Press.
Fechner, G. T. (1860) 1966. *Elements of Psychophysics*. Vol. 1. Translated by H. E. Adler. New York: Holt, Rinehart and Winston.

Fishburn, P. C. 1970. *Utility Theory for Decision Making*. New York: Wiley.
Giocoli, N. 1998. "The 'True' Hypothesis of Daniel Bernoulli: What Did the Marginalists Really Know?" *History of Economic Ideas* 6:7–43.
———. 2003. *Modeling Rational Agents*. Cheltenham: Elgar.
Gossen, H. H. (1854) 1983. *The Laws of Human Relations*. Translated by R. C. Blitz. Cambridge: MIT Press.
Halevy, E. 1901. *La formation du radicalisme philosophique*. Vol. 1, *La jeunesse de Bentham, 1776–1789*. Paris: Alcan.
Hayek, F. A. von. 1934. "Carl Menger." *Economica* 1:393–420.
Heidelberger, M. 2004. *Nature from Within: Gustav Theodor Fechner and His Psychophysical Worldview*. Translated by Cynthia Klohr. Pittsburgh: University of Pittsburgh Press.
Helmholtz, H. von. (1887) 1999. "Numbering and Measuring from an Epistemological Viewpoint." Translated by M. F. Lowe. In Ewald 1999, 727–52.
Hicks, J. R. 1939. *Value and Capital*. Oxford: Clarendon Press.
Howey, R. S. 1960. *The Rise of the Marginal Utility School, 1870–1889*. Lawrence: University of Kansas Press.
Ingrao, B., and G. Israel. 1990. *The Invisible Hand: Economic Equilibrium in the History of Science*. Cambridge: MIT Press.
Jaffé, W., ed. 1965. *Correspondence of Léon Walras and Related Papers*. 3 vols. Amsterdam: North-Holland.
———. 1977. "The Walras-Poincaré Correspondence on the Cardinal Measurability of Utility." *Canadian Journal of Economics* 10:300–307.
Jevons, W. S. 1871. *The Theory of Political Economy*. London: Macmillan.
———. 1874. *The Principles of Science*. London: Macmillan.
———. 1879. *The Theory of Political Economy*. 2nd ed. London: Macmillan.
———. 1881. Review of *Mathematical Psychics*, by F. Y. Edgeworth. *Mind* 6:581–83.
"Jevons on the Theory of Political Economy." (1871) 1981. In Black 1972–81, 7:152–57.
Kant, I. (1787) 1997. *Critique of Pure Reason*. Translated and edited by Paul Guyer and Allen W. Wood. Cambridge: Cambridge University Press.
Kauder, E. 1961. *Carl Mengers Zusätze zu "Grundsätze der Volkswirtschaftslehre."* Tokyo: Bibliothek der Hitotsubashi Universität.
Klein, J. L., and M. S. Morgan, eds. 2001. *The Age of Economic Measurement*. Supplemental issue to vol. 33 of *HOPE*. Durham, N.C.: Duke University Press.
Kries, J. von. (1882) 1995. "On the Measurement of Intensive Magnitudes and the So-Called Psychophysical Law." Translated by K. K. Niall. In Niall 1995, 282–302.
Kronecker, L. (1887) 1999. "On the Concept of Number." Translated by W. Ewald. In Ewald 1999, 947–55.
Levasseur, P. E. (1874) 1987. "Compte-rendus des séances et travaux de l'Académie des sciences morales et politiques." In Walras 1987, 529–32.
Maas, H. 2005. *William Stanley Jevons and the Making of Modern Economics*. Cambridge: Cambridge University Press.

Mandler, M. 1999. *Dilemmas in Economic Theory*. New York: Oxford University Press.

Maxwell, J. C. 1873. *Treatise on Electricity and Magnetism*. London: Constable and Co.

Menger, C. (1871) 1981. *Principles of Economics*. Translated by J. Dingwall and B. Hoselitz. New York: New York University Press.

———. (1892) 2005. "Carl Menger's 'Money as Measure of Value.'" Translated by G. Campagnolo. *HOPE* 37:245–61.

———. (1909) 2002. "Money." Translated by L. B. Yeager with M. Streissler. In *Carl Menger and the Evolution of Payments Systems*, edited by M. Latzer and S. Schmitz, 25–108. Cheltenham: Elgar.

———. 1923. *Grundsätze der Volkswirtschaftslehre*. 2nd ed. Edited by K. Menger. Vienna: Hölder, Pichler and Tempsky.

———. 1934–36. *Collected Works*. 4 vols. Edited by F. Hayek. London: London School of Economics.

Michell, J. 1993. "The Origins of the Representational Theory of Measurement: Helmholtz, Hölder, and Russell." *Studies in the History and Philosophy of Science* 24:185–206.

———. 1999. *Measurement in Psychology*. Cambridge: Cambridge University Press.

———. 2007. "Representational Theory of Measurement." In Boumans 2007, 19–39.

Mirowski, P., ed. 1994. *Natural Images in Economic Thought: "Markets Read in Tooth and Claw."* Cambridge: Cambridge University Press.

Moscati, I. 2007. "History of Consumer Demand Theory, 1871–1971: A Neo-Kantian Rational Reconstruction." *European Journal of the History of Economic Thought* 14:119–56.

———. 2013. "How Cardinal Utility Entered Economic Analysis, 1909–1944." *European Journal of the History of Economic Thought*, forthcoming.

Nagel, E. 1931. "Measurement." *Erkenntnis* 2:313–33.

Neumann, J. von, and O. Morgenstern. 1944. *Theory of Games and Economic Behavior*. Princeton, N.J.: Princeton University Press.

Niall, K. K. 1995. "Conventions of Measurement in Psychophysics: Von Kries on the So-Called Psychophysical Law." *Spatial Vision* 9:275–305.

Niehans, J. 1990. *A History of Economic Theory*. Baltimore: Johns Hopkins University Press.

Pareto, V. (1898) 1966. "Comment se pose le problème de l'économie pure." In Pareto 1966, 102–9.

———. (1900) 2008. "Summary of Some Chapters of a New Treatise on Pure Economics by Professor Pareto." Translated by J. Cairncross under the editorship of J. Chipman. *Giornale degli Economisti* 67:453–504.

———. (1901) 1966. "L'économie pure." In Pareto 1966, 124–36.

———. 1966. *Œvres complètes*. Vol. 9, *Marxisme et économie pure*. Geneva: Librairie Droz.

Peart, S. 1995. "Measurement in Utility Calculations: The Utilitarian Perspective." In Rima 1995, 63–86.

———. 1996. *The Economics of W. S. Jevons*. London: Routledge.
———. 2001. "'Facts Carefully Marshalled' in the Empirical Studies of William Stanley Jevons." In Klein and Morgan 2001, 252–76.
Petty, W. 1690. *Political Arithmetick*. London.
Poincaré, H. 1893. "Le continu mathématique." *Revue de Métaphisique et de Morale* 1:26–34.
———. 1902. *La science et l'hypothèse*. Paris: Flammarion.
Poinsot, L. 1842. *Eléments de statique*. Paris: Bachelier.
Porter, T. M. 1994. "Rigor and Practicality: Rival Ideas of Quantification in Nineteenth-Century Economics." In Mirowski 1994, 128–70.
Rima, I. H., ed. 1995. *Measurement, Quantification, and Economic Analysis*. London: Routledge.
Schmidt, T., and C. E. Weber. 2008. "On the Origins of Ordinal Utility: Andreas Heinrich Voigt and the Mathematicians." *HOPE* 40:481–510.
Schröder, E. 1873. *Lehrbuch der Arithmetik und Algebra*. Leipzig: Teubner.
Schumpeter, J. A. 1954. *History of Economic Analysis*. New York: Oxford University Press.
Stevens, S. S. 1946. "On the Theory of Scales of Measurement." *Science* 103:677–80.
Stigler, G. J. 1937. "The Economics of Carl Menger." *Journal of Political Economy* 45:229–50.
———. 1950. "The Development of Utility Theory." *Journal of Political Economy* 58:307–27, 373–96.
Tannery, P. 1875. "A propos du logarithme des sensations." *Revue Scientifique* 8:876–77, 1018–20.
———. 1884. "Critique de la loi de Weber." *Revue Scientifique* 17:15–35.
———. 1888. "Psychologie mathématique et psychophysique." *Revue Scientifique*, 25:189–97.
Titchener, E. B. 1905. *Experimental Psychology: A Manual of Laboratory Practice*. Vol. 2, *Quantitative Experiments*; pt. 2, *Instructor's Manual*. New York: Macmillan.
Voigt, A. 1893. "Zahl und Mass in der Ökonomik." *Zeitschrift für die Gesamte Staatswissenschaft* 49:577–609.
———. (1893) 2008. ["Number and Measure in Economics."] Translation of excerpt from Voigt 1893. Translated by T. Schmidt. In Schmidt and Weber 2008, 502–4.
Walras, A., and L. Walras. 1987–2005. *Œuvres économiques complètes*. Paris: Economica.
Walras, L. (1874) 1954. *Elements of Pure Economics*. Translated by William Jaffé. London: Allen and Unwin.
———. (1874) 1987. "Principe d'une théorie mathématique de l'échange." In Walras 1987, 261–81.
———. (1876) 1987. "Une branche nouvelle de la mathématique." In Walras 1987, 290–329.
———. 1886. *Théorie de la monnaie*. Lausanne: Corbaz & C.

———. (1909) 1990. "Economics and Mechanics." Translated by Philip Mirowski and Pamela Cook. In *Economics as Discourse*, edited by W. J. Samuels, 206–14. Dordrecht: Kluwer.

———. 1987. *Mélanges d'économie politique et sociale*. Vol. 7 of Walras and Walras 1987–2005.

———. 1988. *Éléments d'économie politique pure*. Vol. 8 of Walras and Walras 1987–2005.

White, M. V. 1994. "The Moment of Richard Jennings: The Production of Jevons's Marginalist Economic Agent." In Mirowski 1994, 197–230.

Wundt, W. 1873–74. *Grundzüge der physiologischen Psychologie*. Leipzig: Engelmann.

———. 1880. *Grundzüge der physiologischen Psychologie*. 2nd ed. Leipzig: Engelmann.

Yagi, K. 1993. "Carl Menger's *Grundsätze* in the Making." *HOPE* 25:697–724.