Università Commerciale Luigi Bocconi

Intro Lecture

Professors Massimo Guidolin/Daniele Bianchi

20135 – Theory of Finance, Part I

(Sept. –October)

Fall 2019

Outline and objectives

- An introduction to quantitative financial economics, with the goal of building a common basis for all the first-year students in view of the specialized courses of the following semesters
- Part I of the course specializes in the theory and practice of optimal portfolio choice
  - Standard knowledge of mathematics and statistics is assumed
- Two goals: build concepts and tools for part II (less exciting) and to give an overview of issues and topics in asset management, possibly with interviews in sight
- Statistics and Quant/Math Prep courses were offered between the end of August and early September 2019 and that the material covered in those 40 hours represent essential background, see

  http://didattica.unibocconi.eu/mypage/map.php?IdUte=135242&idr=14063&lingua=eng

  http://didattica.unibocconi.eu/mypage/doc.php?idDoc=15646&IdUte=48622&idr=7083&Tipo=m&lingua=eng
- The goal is to build concepts and an overall framework with applications to cases: no memorizing of proofs is expected

Assessment (the exams)

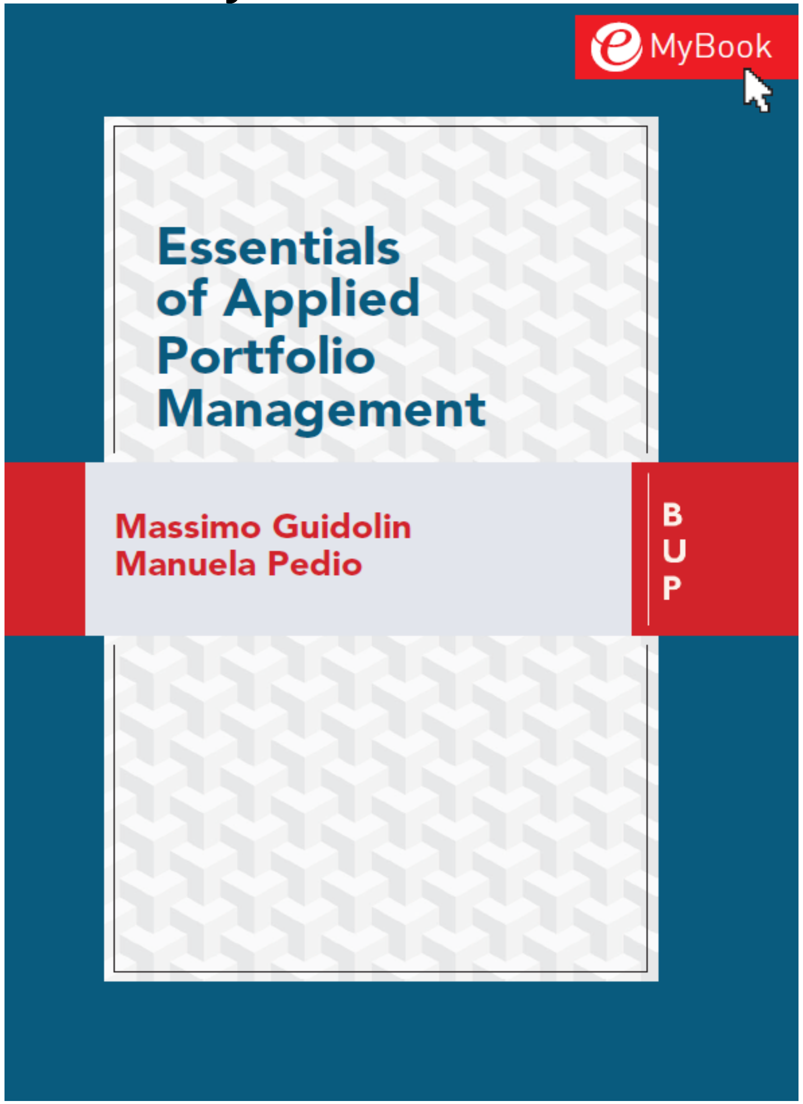
- The exam is written and closed-book
- The weight of this part of the exam is up to 11 points out of 32
- The structure and length of the exam is decided by the course director: I will provide two question with the structure below and he will decide whether to assign them and other details

Question 1A (3.5 points): essay-type **Question 1B (1.5 points): case/problem**	**Question 2A (3.5 points): essay-type** **Question 2B (1.75 points): case/problem** **Question 2C (0.75 points): essay-type on one reading (not in slides)**

- The material covered in the course is outlined in lecture slides at http://didattica.unibocconi.eu/mypage/map.php?IdUte=135242&idr=14063&lingua=eng

The textbook

- The course is based on selected chapters from:
  - Guidolin, M, and M., Pedio, 2016, *Essentials of Applied Portfolio Management*, EGEA and Bocconi University Press
- The book contains most of what is needed to excel in the exam
- A few sample questions with the same structure/style as the exam questions will be made available on the course web site
- The classes are organized to be self-cointained, unless exceptions are explicitly announced
- Classes give you two types of added value: (i) know what topics are important, (ii) learn in real time
- It may be a good idea to print and read slides in advance



The syllabus (see handout)

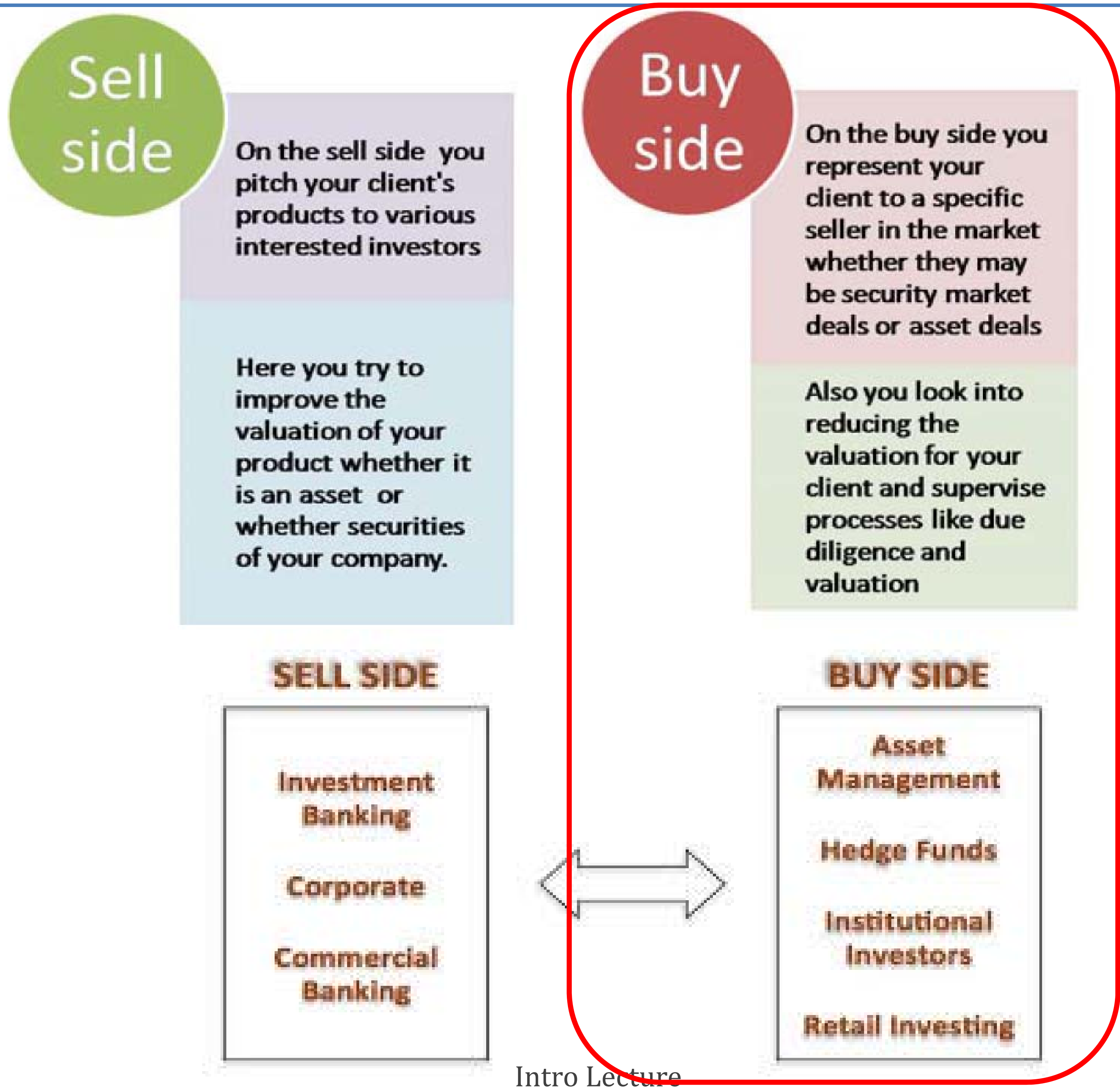
- The syllabus wants to provide elements on six key questions/topics:
  - The workhorse of quant ptf. management: mean-variance framework (D. Bianchi)
  - How do you incorporate customers' preferences for risk in asset allocation decisions (aka wealth management/1)? (D. Bianchi)
  - How do optimal portfolio decisions depend on labor/entrepreneural income (background risk), aka wealth management/2
  - Using sentiment indicators in asset management: big data, deep machine learning, and artificial intelligence
  - Does it pay out to pick the «good guys»? ESG criteria and constraints in asset management
  - Dynamic, sophisticated asset allocation in action—hedge funds
- A number of interesting questions are left out, alas:
  - Tactical vs. strategic asset allocation under predictable investment opportunities (needs advanced econometrics)
  - How to separate skilled from unskilled ptf. managers (needs asset pricing, part 2 of 20135)

What is asset management? An Overview

Asset management involves a wide range of functions and tasks

- This question is relevant both to the objectives of this course as well as to the career choices of a number of people
- Asset management is the scientific study of strategies and methods useful to the **buy-side** of the financial system
- The buy-side is composed of institutions and professionals that systematically **collect and invest the wealth** (positive financial imbalances) of households and firms
- What are the heterogeneous functions and processes that take place within the "asset management box"
  - Qualitative portfolio construction and rotation, "stock picking"
    - Traditionally this has to do more with valutation issues than portfolio management strictly defined
  - Quantitative portfolio construction, aka **asset allocation**
  - Marketing & placement of investment vehicles, how to collect wealth
  - Risk management and performance measurement and reporting

What is asset management? An Overview



This mini-course (most of what I teach really) is buy-side oriented

What is asset management? An Overview

Even though quant-based AM techniques are spreading, stock picking remains alive and well

- This course is mostly about formal, quant-based asset allocation models
  - The familiar techniques you may have heard about are tactical vs. strategic asset allocation
  - Often these techniques require the manager to be active and to apply market timing strategies
- In fact, it turns out that in the perspective of formal asset allocation, stock picking often makes little sense
  - What types of institutions apply quant methods? Top banks with AM divisions, hedge funds, large-scale mutual fund family suppliers, and insurance companies (also engaged in asset-liability management)

Dynamic asset allocation: what is it?

An asset allocation problem maps an asset menu, a set of preferences, and a sequence of subjective discount factors, into a sequence of optimal portfolio weights, possibly subject to constraints

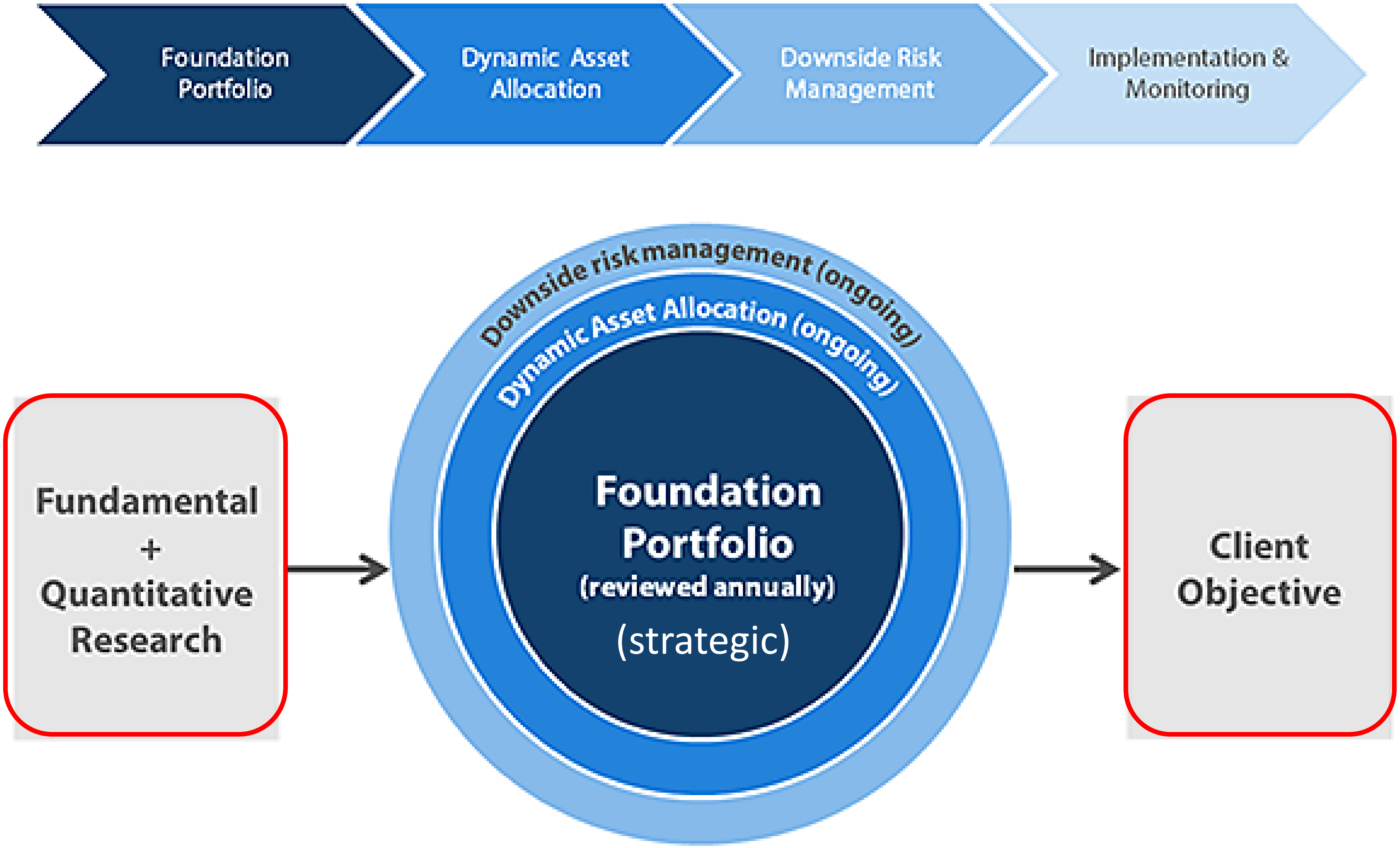
- Let's examine a first, sample portfolio choice problem written as:

$$\max_{\{\boldsymbol{\omega}_t\}_{t=0}^{T-1}} \sum_{t=0}^{T} \beta^t U(P_t(\boldsymbol{\omega}_0, \boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_t))$$

$$\text{s.t.} \quad \boldsymbol{\omega}_t \in \mathbb{C}$$

  - It is a maximization problem
  - W.r.t. a N x 1 vector of portfolio weights ω, where N is the number of alternative assets (possibly, securities, we may choose from)
    - ✓ N and which securities we can choose from define the asset menu
  - We maximize the present discounted sum of the realized values of some function U(•) that measures our level of happiness and therefore reflects preferences
    - ✓ We shall often write of a (Von Neumann-Morgenstern) utility function

Dynamic asset allocation: an overview



Dynamic asset allocation: an overview

An asset allocation problem maps an asset menu, a set of preferences, and a sequence of subjective discount factors, into a sequence of optimal portfolio weights, possibly subject to constraints

$$\max_{\{\boldsymbol{\omega}_t\}_{t=0}^{T-1}} \sum_{t=0}^{T} \beta^t U(P_t(\boldsymbol{\omega}_0, \boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_t))$$
$$\text{s.t.} \quad \boldsymbol{\omega}_t \in \mathbb{C}$$

- The sum of "felicity levels" is discounted at a subjective rate β
  - ✓ Time "0" means today, now
  - ✓ Subjective because β does not have to be an interest rate
  - ✓ Because β performs a discounting operation, we normally assume that $\beta \in (0,1]$
- $P_t(\omega_0, \omega_1, \ldots, \omega_t)$ is a performance criterion, for instance, realized portfolio return:

$$P_t(\boldsymbol{\omega}_0, \boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_t) = R_t^p = \sum_{n=1}^{N} \sum_{\tau=1}^{t} \omega_{n,\tau-1} r_{n,\tau}$$

Dynamic asset allocation: an overview

An asset allocation problem maps an asset menu, a set of preferences, a sequence of subjective discount factors into a sequence of optimal portfolio weights, possibly subject to constraints

$$\max_{\{\omega_t\}_{t=0}^{T-1}} \sum_{t=0}^{T} \beta^t U(P_t(\omega_0, \omega_1, ..., \omega_t))$$
$$\text{s.t.} \quad \omega_t \in \mathbb{C}$$

- The sequence of portfolio weights may be restricted to belong to some set $\mathbb{C}$
  - ✓ E.g., short sales may be restricted or impossible outright
  - ✓ Almost all institutional investors must fulfill maximum investment rules/criteria (positive diversification constraints)
  - ✓ Occasionally, more technical constraints may be imposed: e.g., a portfolio that maximizes utility subject to a value-at-risk constraint
- Even though the entire sequence $\{\omega_0, \omega_1, ..., \omega_t\}$ gets selected, many problems do allow that future weight revisions be taken into account (rebalancing case)

Review of Basic Concepts (Prep Course)/1

Under the assumptions of the EUT, one ranks assets/securities on the basis of the expectation of the utility of their payoffs across states

- Under the six axioms detailed in Lecture 4 of the Prep Course, there exists a cardinal, continuous, time-invariant, real-valued Von Neumann-Morgenstern (VNM) felicity function of money U(·), such that for any two lotteries/gambles/securities (i.e., probability distributions of monetary payoffs) x and y,

$$x \succsim y \quad \text{if and only} \quad \text{if } E[U(x)] \geq E[U(y)]$$

where for a generic lottery z (e.g., one that pays out either x or y),

$$\mathbb{U}(z) \equiv E[U(z)] = \sum_{s=1}^{S} Prob(\,state = s)U(z(s))$$

- The perceived, cardinal happiness of a complex and risky menu of options, is given by the weighted average of the satisfaction derived from each such individual option, weighted by the probabilities
  - In the following example we use a VNM utility function $U(z) = \ln(z)$
  - The ranking by the EU criterion differs from MV: while according the latter only securities B and D are dominated (by A and C), and hence A and C cannot be ranked, according to EU, security A ranks above security C (and B and D)

Review of Basic Concepts (Prep Course)/1

State	Security A		Security B		Security C		Security D	
	Pay-off	Prob.	Pay-off	Prob.	Pay-off	Prob.	Payoff	Prob.
i	20	3/15	18	3/15	18	3/15	5	3/15
ii	18	5/15	18	5/15	16	5/15	14	4/15
iii	14	4/15	10	4/15	12	4/15	14	4/15
iv	10	2/15	5	2/15	12	2/15	18	2/15
v	6	1/15	5	1/15	8	1/15	18	1/15
$E[R_i]$	15.47		13.27		14.27		13.00	
$Stdev[R_i]$	4.10		5.33		2.91		4.29	
$E[lnR_i]$	2.693		2.477		2.635		2.483	

- In words, under some specific conditions (the axioms), ranking lotteries/gambles/securities on the basis of an investor's preferences or on the basis of the
  - Expected value
  - Of the happiness that each payoff from the lottery/gamble/security provides,
  - As measured by some cardinal felicity function U(·),

$$\mathbb{U}(z) \equiv E[U(z)] = \sum_{s=1}^{S} Prob(state = s)U(z(s))$$

lead to the same choice

Review of Basic Concepts (Prep Course)/1

Any linear affine, monotone increasing transformation of a VNM utility function ($V(\cdot) = a + bU(\cdot)$, $b > 0$) represents the same preferences

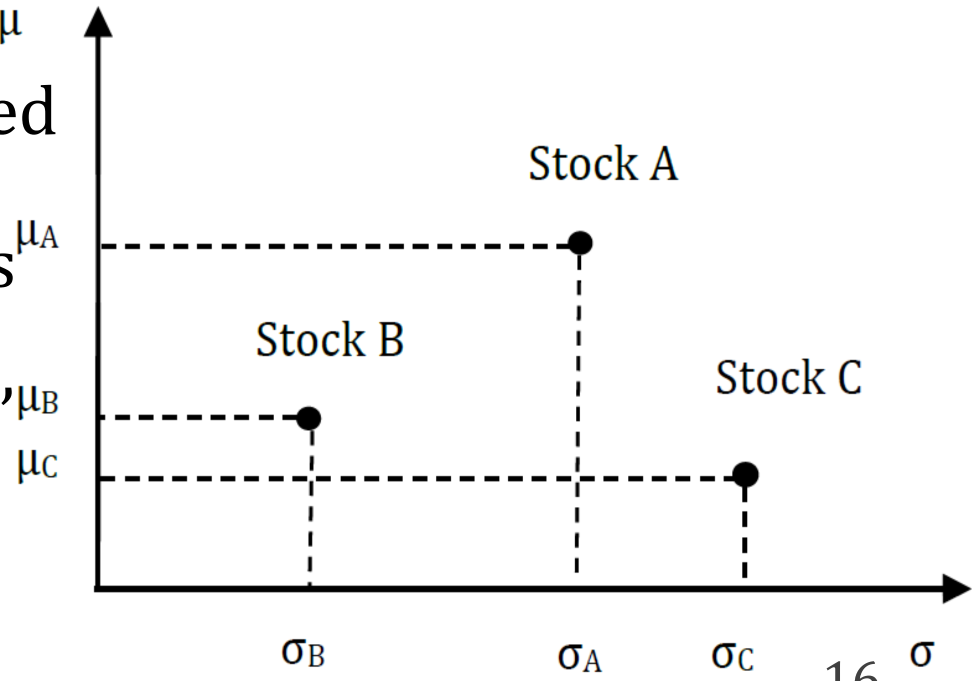
- Arbitrary monotone transformations of cardinal utility functions do not preserve ordering over lotteries
- Are preference defined by the EUT unique up to some kind of transformations as standard $u(\cdot)$ functions were?
- The VNM representation is preserved under linear affine, increasing transformations: if $U(\cdot)$ is a VNM felicity function, then

  $V(\cdot) = a + bU(\cdot) \quad b > 0$ is also a VNM felicity

  - This is because $V((x,y;\pi)) = a + bU((x,y;\pi))$
    $= a + b[\pi U(x) + (1-\pi)U(y)]$
    $= \pi[a + bU(x)] + (1-\pi)[a + bU(y)] = \pi V(x) + (1-\pi)V(y)$
  - E.g., if John's felicity function is $U_{John}(R_i) = \ln(R_i)$ and Mary's felicity is instead $U_{Mary}(R_i) = -2 + 4\ln(R_i)$, Mary and John will share the same preferences
  - However, when $U_{Mary}(R_i) = +1000 - \ln(R_i)$ or $U_{Mary}(R_i) = (\ln(R_i))^3$, this will not be the case

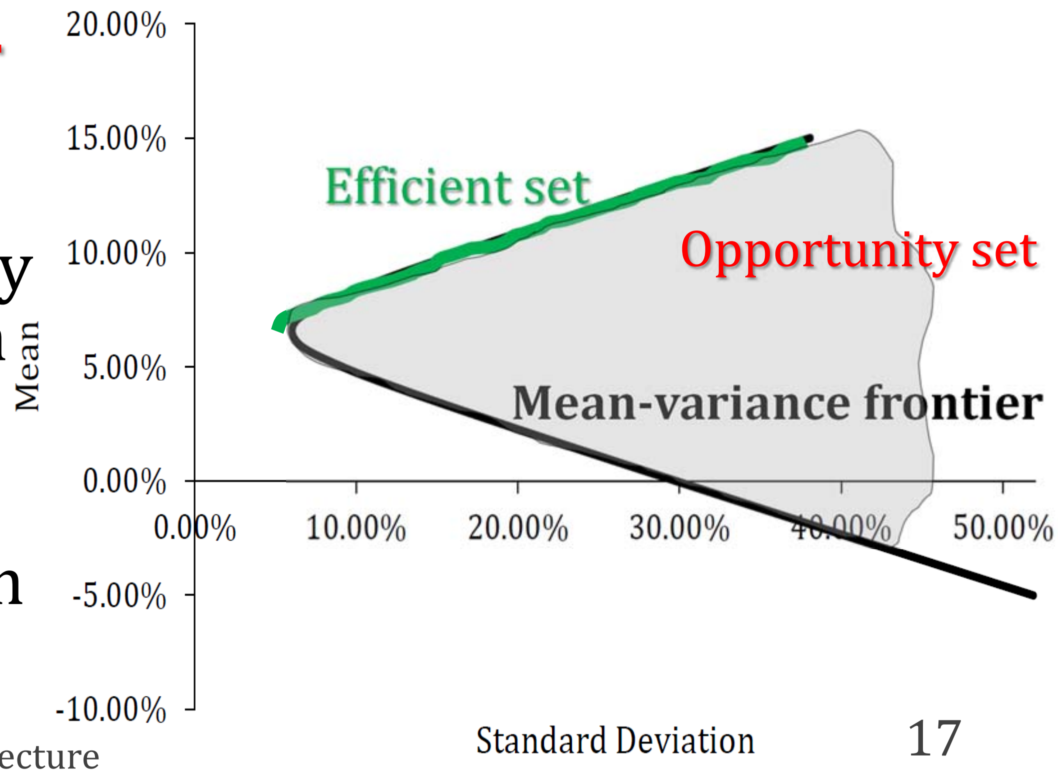
Review of Basic Concepts (Prep Course)/2

- Prep course reviews the development of the celebrated mean-variance framework introduced by Markowitz in the 1950s
- Initially at least, risky assets only, no borrowing or lending
- Assume that for some reasons, the joint distribution of asset returns is completely characterized by their means, variances, and covariances
- We represent each asset (and portfolio of assets) in a two-dimensional diagram, where expected portfolio return is plotted on the vertical axis and standard deviation is on the horizontal axis
- Not all securities may be selected, e.g., stock C is dominated by the remaining two stocks in terms of MV dominance



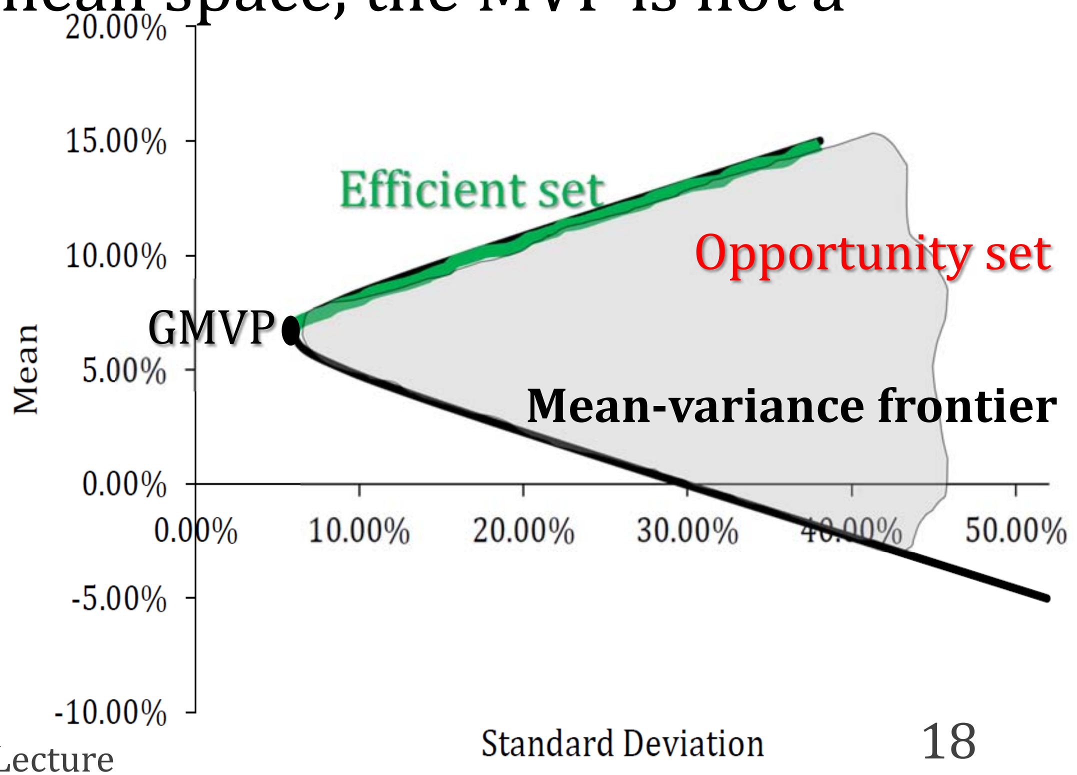
Review of Basic Concepts (Prep Course)/2

- According to MV criterion a portfolio is efficient if and only if there is no other portfolio that allows the investor to achieve the same expected return with a lower level of risk or a higher level of expected return with the same level of risk
- Three key notions: i) the opportunity set (feasible region), which includes all the portfolios (both efficient and inefficient) that the investor is able to build given securities in the asset menu
- (ii) the mean-variance frontier (aka minimum variance frontier, MVF) subset of the opportunity set containing only the portfolio(s) with minimum variance for any target level of expected returns
- (iii) the efficient frontier, which only includes efficient ptfs



Review of Basic Concepts (Prep Course)/2

- Because it is possible that a portfolio exists which has a higher return than another portfolio with the same level of risk, only portfolios that have a higher expected return than the global minimum variance portfolio (GMVP) are efficient
- The preferences of the investor(s) for risk are not relevant to the determination of the efficient frontier
- In the classical standard dev.-mean space, the MVF is not a function, but a «right-rotated hyperbola”
- The GMPV is Global Minimum Variance Ptf. and it is of high interest because «separates» the efficient set from the MVF
- The structure GMVP does not depend on expected returns



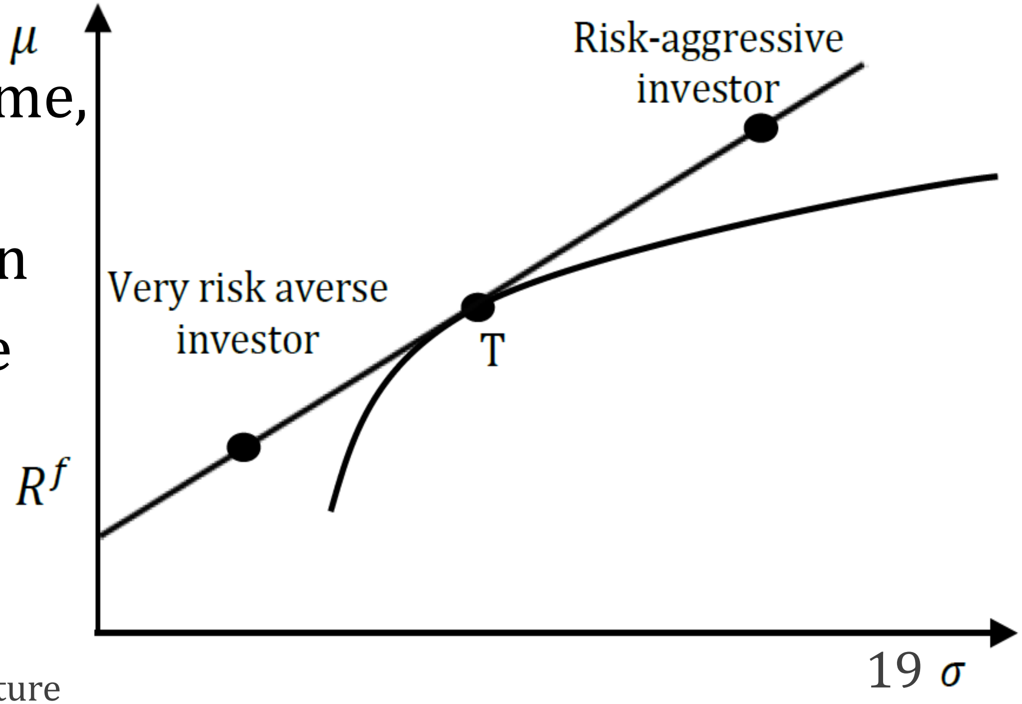
Review of Basic Concepts (Prep Course)/2

Under no frictions and homogeneous beliefs, there exists a tangency ptf. that maximizes the slope of the transformation line

- Assuming beliefs are homogeneous, there are no frictions or taxes, and that individuals face the same R^f and identical asset menus, all rational, non-satiated investors hold the same tangency portfolio
- It is combined with a certain share of risk-free lending or borrowing
- While the share of wealth an investor lends or borrows at R^f depends on the investor's preference for risk, the risky portfolio should be the same for all the investors
- The steepest CTL gets a special name, the Capital Market Line (CML)
- Special case of two-fund separation
- To determine the tangency ptf. one needs to solve:

$$\max_{\{\omega\}} \frac{(\boldsymbol{\omega}'\boldsymbol{\mu} - R^f)}{(\boldsymbol{\omega}'\boldsymbol{\Sigma}\boldsymbol{\omega})^{\frac{1}{2}}}$$

$$subject\ to \quad \boldsymbol{\omega}'\boldsymbol{\iota} = 1$$



Where to Review Basic Concepts

- Elementary choice under uncertainty: dominance and mean-variance
  - Guidolin-Pedio, chapter 1, sec. 2
- Preference representation theorem and its meaning
  - Guidolin-Pedio, chapter 2, sec. 1.1
- Expected utility theorem
  - Guidolin-Pedio, chapter 2, sec. 1.2
- Uniqueness of EU preferences up to monotone increasing linear transformations
  - Guidolin-Pedio, chapter 2, sec. 1.2
- Mean-variance and efficient frontiers: logical meaning
  - Guidolin-Pedio, chapter 3, sec. 1
- The case of no borrowing and lending and two risky assets
  - Guidolin-Pedio, chapter 3, sec. 1.1
- Generalizations to the case of *N* risky assets and two-fund separation
  - Guidolin-Pedio, chapter 3, sec. 1.2
- Extension to unlimited borrowing and lending
  - Guidolin-Pedio, chapter 3, sec. 2