



Università Commerciale
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An Introduction to Structured Financial Products

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20263– Advanced Tools for Risk Management and
Pricing

Spring 2016

Outline and objectives

- The Nature of Investment Certificates
- Market statistics for investment certificates
- Key ideas of structuring: from linear to non-linear payoffs
- Exotic options: digital, Asian, and barriers
- Reverse convertibles and Bonus Cap certificates, certificates with (limited) capital protection
- The autocallability feature: Express certificates
- Certificates without capital protection: Benchmarks, Outperformance, and Discounts
- Leverage and Turbo certificates

The nature of investment certificates

Investment certificates are securitized derivatives, i.e. portfolios of derivatives , that originate non-linear synthetic payoffs

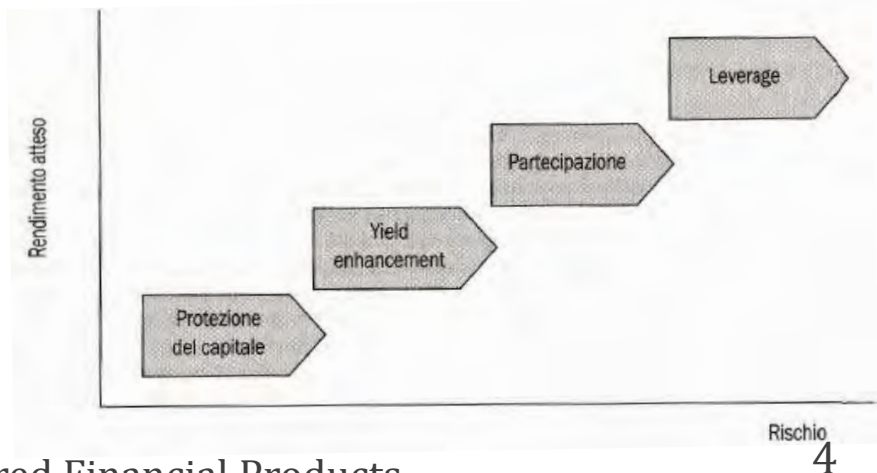
- Investment certificates (ICs) are structured products that are (portfolios of exotic) derivatives that have been **securitized** and that usually trade on regulated markets (e.g., SEDEX)
- They are best classified as vehicles to **passively** implement **alternative investment** choices
- This derives from a number of their features:
 - Wide variety of available risk/return profiles that make them alternative to traditional, linear payoffs
 - They are sometimes written on alternative asset classes (real estate, commodities, currencies, etc.)
 - Ability to neutralize the risk profiles from traditional asset classes
 - They often provide considerable **leverage**
- For instance, a portfolio of a long position in the Euro STOXX 50 plus a long put option at a given strike

The nature of investment certificates

ICs expose to market, liquidity, and issuing counterparty risks

ICs may provide equity protection, yield enhancement, diversification, and leverage

- ICs expose to three types of risk:
 - **Market risk**, due to variation in the price of the derivatives that enter the IC and reflecting interest rate, forex, underlying price, etc. risks
 - **Liquidity risk**, although the presence of dedicated market makers within official (electronic) trading venues tends to moderate it
 - **Issuing counterparty risk**, as IC are structured/assembled by private names with a specific merit of credit, unless collateral has been posted (e.g., Dws Go Safe, by DB) or SP accounts are created (e.g., Dws Go)
- Four main kinds of ICs based on the customer's need they fulfill:
 - **Capital protection** (e.g., Equity Protection and Butterflies), total or partial, in spite of the limited participation to gains or losses by the underlying asset



The nature of investment certificates

- **Yield enhancement/stable market strategies** (e.g., Bonus, Cash Collect, and Express), providing high coupons on (seemingly) fixed income products through a (limited) participation to downside risk
- **Participation/diversification**, providing direct (linear, as in the case of Benchmarks or magnified to the upside as in the case of Outperformance CIs) exposures to large indices of (usually alternative, sophisticated) asset classes
- **Leverage** (e.g., Turbo and Short) that provide exposure with leverage, to the up- or the down-side
- ICs are related to but different from **covered warrants**, that are European plain vanilla calls and puts that have been securitized and require no marginal activity
 - Some ICs can be seen as covered warrants with a zero strike which makes them “very deep” in-the-money instruments the exercise of which remains profitable until the underlying is worthless
- They are also different from structured bonds, in which the principal must be refunded at maturity to avoid default

Market Statistics for ICs

ICs have appeared in Germany and Switzerland in the 1990s and the corresponding markets have been rapidly growing in terms of turnover and listings also in Italy, France, and the Netherlands

- In a IC, refund may be partial or even zero (covered warrant) without any default occurring
- However also structured bonds contain derivative components that will be priced in an economic sense in ways similar to ICs
- The market has recorded an impressive growth rate over time
 - The first IC has been issued in Germany, in 1989
 - The Swiss and German markets have been growing steadily over time since the 1990s
 - Also the Italian, French, and Dutch markets have recorded a boom in the 00s, while growth has been more difficult in the UK and Spain
 - ICs are currently listed in at least 14 European exchanges even though Stuttgart, Frankfurt, Zurich, Milan, and NYSE Euronext are main ones
 - The German market remains the largest and is propelled by individual investors

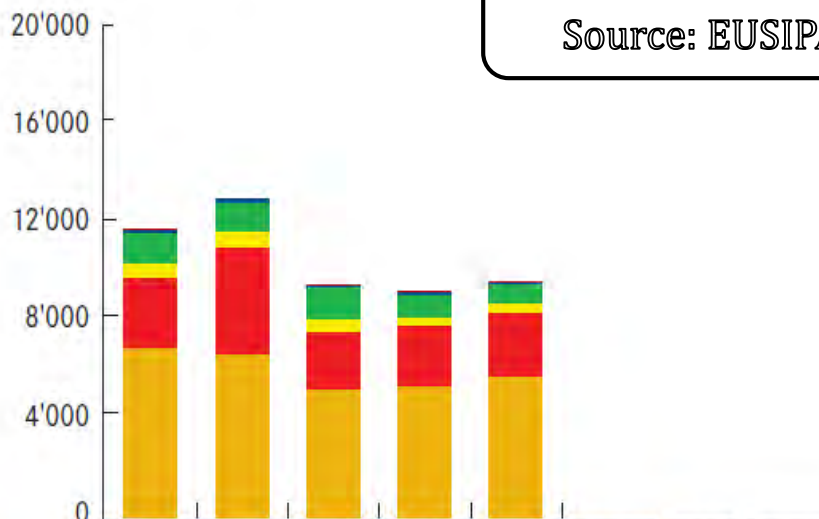
Market Statistics for ICs: Exchange Turnover

Leveraged ICs are strongly growing everywhere

Germany, Switzerland and Italy have the lion's share, Italy in leveraged

Recently, growth has been strong in Austria and Sweden

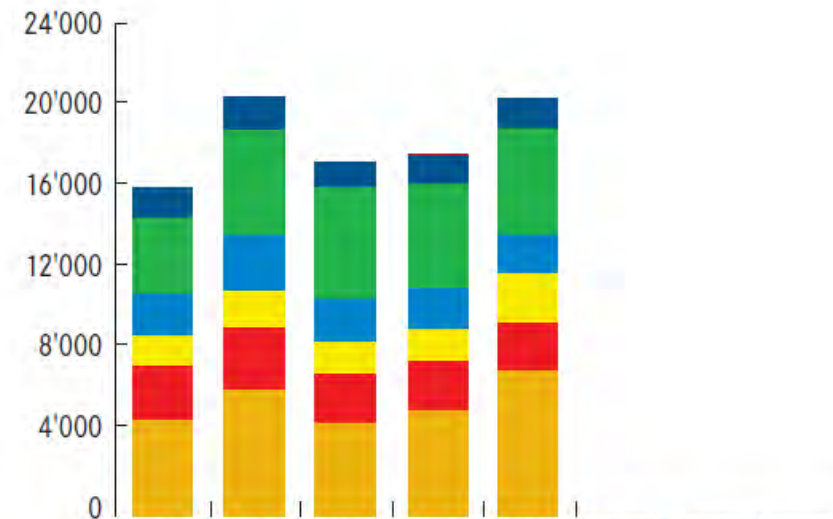
Investment Products
in Mio. EUR



2014 vs. 2013
Source: EUSIPA

	2013 Q4	2014 Q1	2014 Q2	2014 Q3	2014 Q4	Total 2014	Δ% 4/13	Δ% 3/14
Austria	52	61	47	47	65	220	25%	38%
France	115	135	88	108	79	410	-31%	-27%
Italy	1'185	1'162	1'241	900	839	4'142	-29%	-7%
Netherlands	9	10	6	9	2	27	-78%	-78%
Sweden	545	685	516	335	499	2'035	-8%	49%
Switzerland	2'745	4'272	2'318	2'444	2'846	11'880	4%	16%
Germany	6'519	6'559	5'167	5'302	5'321	22'348	-18%	0%
Total	11'169	12'884	9'383	9'145	9'651	41'062	-14%	6%

Leverage Products
in Mio. EUR



	2013 Q4	2014 Q1	2014 Q2	2014 Q3	2014 Q4	Total 2014	Δ% 4/13	Δ% 3/14
Austria	4	6	4	3	5	18	38%	72%
France	1'459	1'616	1'231	1'360	1'811	6'018	24%	33%
Italy	3'557	5'185	5'384	4'925	5'026	20'520	41%	2%
Netherlands	1'947	2'682	2'082	1'947	2'282	8'993	17%	17%
Sweden	1'451	1'841	1'497	1'493	1'866	6'698	29%	25%
Switzerland	2'593	3'015	2'425	2'402	3'034	10'876	17%	26%
Germany	4'465	6'180	4'400	4'930	6'394	21'904	43%	30%
Total	15'476	20'526	17'023	17'059	20'418	75'025	32%	20%

Source: Country Associations, Derivative Partners

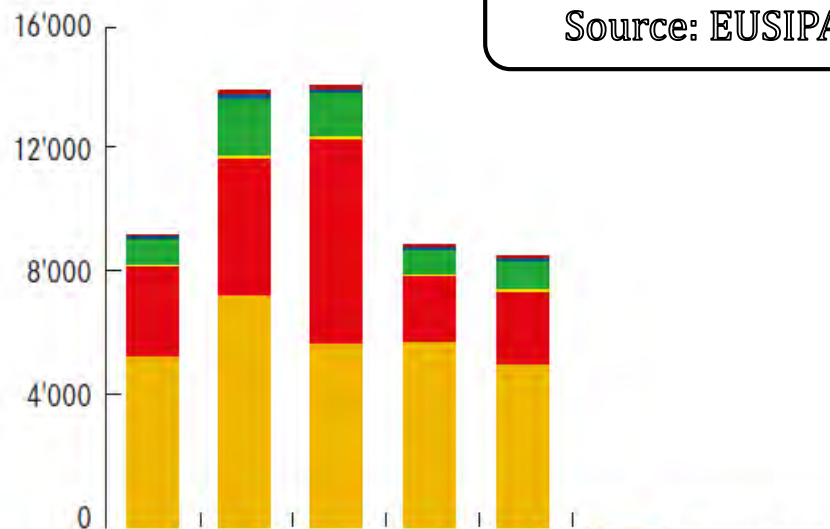
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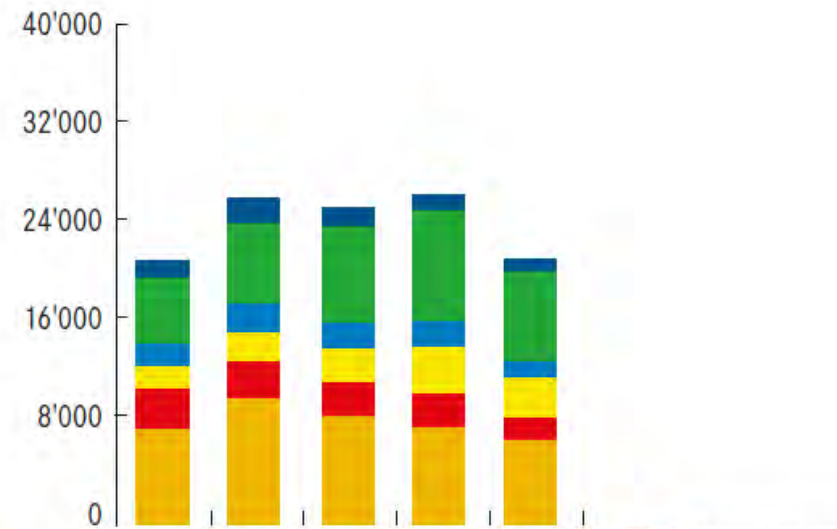
Investment Products
in Mio. EUR



2015 vs. 2014
Source: EUSIPA

	2014 Q4	2015 Q1	2015 Q2	2015 Q3	2015 Q4	Total 2015	Δ% 4/14	Δ% 3/15
Austria	65	172	130	93	88	482	36%	-5%
France	79	84	68	63	59	274	-25%	-6%
Italy	839	1'818	1'420	772	910	4'919	8%	18%
Netherlands	2	28	19	21	15	83	709%	-26%
Sweden	11	20	17	6	10	53	-12%	59%
Switzerland	2'846	4'306	6'446	2'307	2'307	15'366	-19%	0%
Germany	5'321	7'236	5'717	5'644	5'077	23'674	-5%	-10%
Total	9'651	13'722	13'875	9'005	8'466	45'069	-12%	-6%

Leverage Products
in Mio. EUR



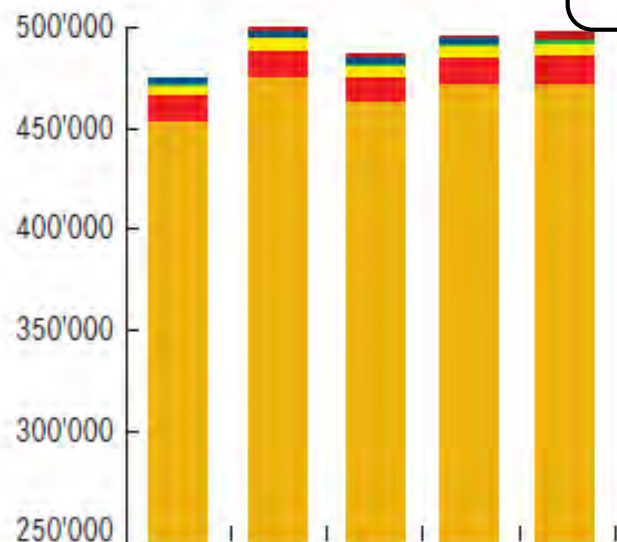
	2014 Q4	2015 Q1	2015 Q2	2015 Q3	2015 Q4	Total 2015	Δ% 4/14	Δ% 3/15
Austria	5	14	9	14	6	43	34%	-55%
France	1'811	1'956	2'031	1'725	1'340	7'053	-26%	-22%
Italy	5'026	6'412	7'152	8'264	6'887	28'716	37%	-17%
Netherlands	2'282	2'566	2'313	2'354	1'854	9'087	-19%	-21%
Sweden	1'866	2'956	3'225	4'124	3'098	13'403	66%	-25%
Switzerland	3'034	2'992	2'532	2'546	1'589	9'659	-48%	-38%
Germany	6'394	8'600	7'450	6'640	5'703	28'393	-11%	-14%
Total	20'418	25'497	24'713	25'666	20'478	96'353	0.3%	-20%

Source: Country Associations, Derivative Partner

Market Statistics for ICs: Number of Listed Products

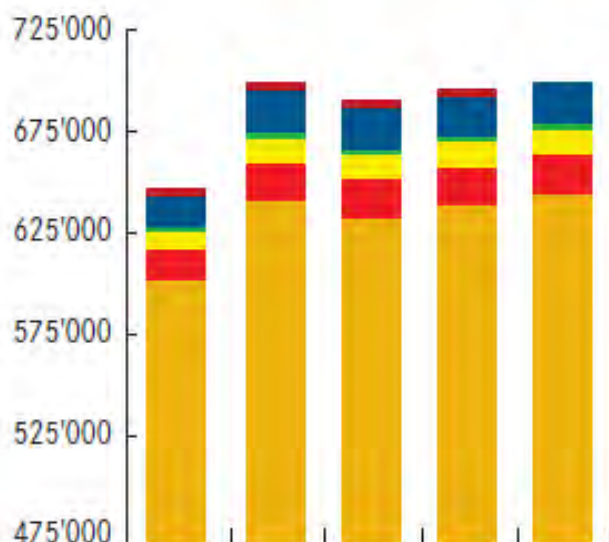
The stock is massive in the case of Germany, for all types of products
 The differential btw. turnover and listing stats implies differences in the way ICs are used in different countries

Investment Products



2014 vs. 2013
 Source: EUSIPA

Leverage Products



	2013 Q4	2014 Q1	2014 Q2	2014 Q3	2014 Q4	Δ% 4/13	Δ% 3/14
Austria	1'968	2'085	2'126	2'048	1'992	1%	-3%
France	1'438	1'299	1'270	1'250	1'569	9%	26%
Italy	941	1'011	943	994	866	-8%	-13%
Sweden	3'988	4'209	4'434	4'518	4'651	17%	3%
Switzerland	11'567	11'465	11'597	11'422	11'579	0%	1%
Germany	451'706	483'097	464'770	472'521	474'765	5%	0%
Total	471'608	503'166	485'140	492'753	495'422	5%	1%

	2013 Q4	2014 Q1	2014 Q2	2014 Q3	2014 Q4	Δ% 4/13	Δ% 3/14
Austria	3'953	4'410	4'515	4'095	3'782	-4%	-9%
France	7'137	27'589	21'700	24'999	27'705	62%	15%
Italy	4'199	4'186	3'686	4'118	3'346	-20%	12%
Sweden	10'215	10'609	9'522	8'109	8'028	-21%	-15%
Switzerland	22'739	23'376	22'081	22'475	20'410	-10%	2%
Germany	591'255	636'015	631'309	638'420	646'683	9%	1%
Total	649'498	706'185	692'813	702'216	709'954	9%	1%

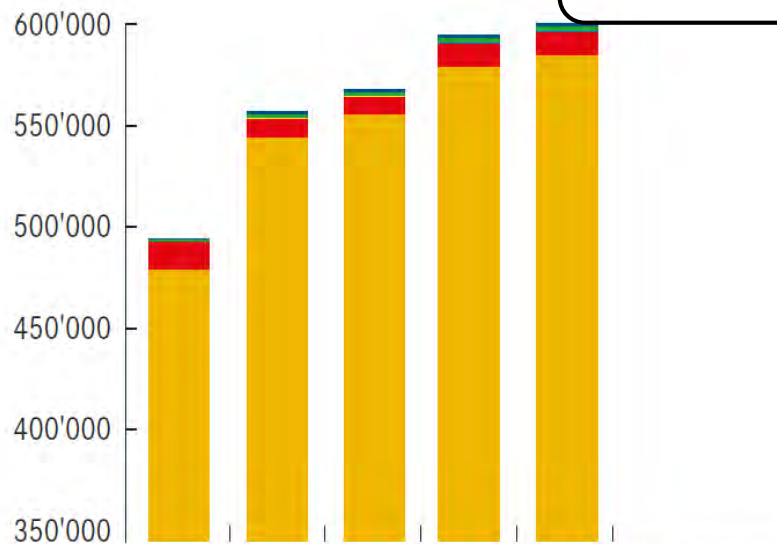
Source: Country Associations, Positive Partners

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 The differential btw. turnover and listing stats implies differences in the way ICs are used in different countries

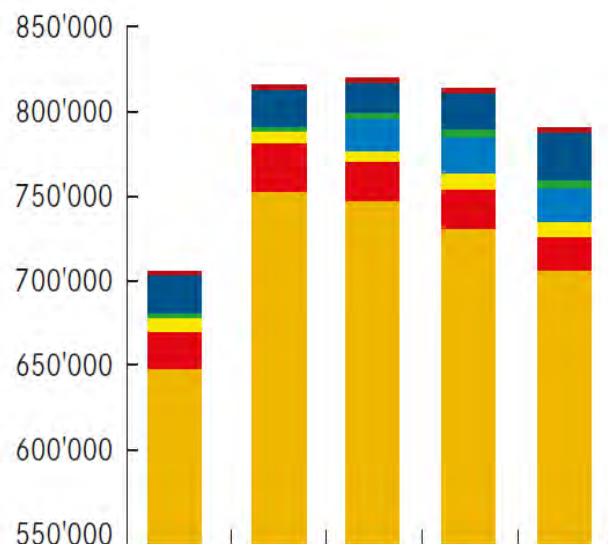
Investment Products

2015 vs. 2014
 Source: EUSIPA



	2014 Q4	2015 Q1	2015 Q2	2015 Q3	2015 Q4	Δ% 4/14*	Δ% 3/15**
Austria	1'992	2'232	2'169	1'995	1'918	-4%	-4%
France	1'569	1'400	1'532	1'496	1'322	-16%	-12%
Italy	866	1'063	1'250	1'226	1'143	32%	-7%
Netherlands	n/a	n/a	170	129	162	n/a	26%
Sweden	543	524	509	465	455	-16%	-2%
Switzerland	11'579	12'140	11'981	12'019	11'478	-1%	-5%
Germany	474'765	540'990	557'551	573'275	582'648	23%	2%
Total	495'422	562'601	579'600	595'225	599'126	21%	1%

Leverage Products



	2014 Q4	2015 Q1	2015 Q2	2015 Q3	2015 Q4	Δ% 4/14*	Δ% 3/15**
Austria	3'782	4'448	4'400	4'217	4'016	6%	-5%
France	27'705	29'120	28'590	26'752	28'841	4%	8%
Italy	3'346	4'314	4'709	4'554	4'466	33%	-2%
Netherlands	n/a	n/a	12'670	14'422	13'501	n/a	-6%
Sweden	8'028	7'922	8'586	9'646	10'865	35%	13%
Switzerland	20'410	23'194	23'227	22'206	20'359	0%	-8%
Germany	646'683	753'805	747'634	735'351	709'620	10%	-3%
Total	710'620	822'755	829'633	816'957	791'668	11%	-3%

Market Statistics for ICs: Italian Market

The search for participation to the underlying assets' upside under some degree of capital protection seems to be the main driver of new issues
Credit linked ICs are partly disappearing for regulatory issues

2014
Source: ACEPI

2014	1° trim	2° trim	3° trim	4° trim	TOTALE
collocato (in MLN €)	2.584,18	2.493,13	1.915,30	1171,10	8163,71
numero di ISIN emessi	127	122	86	94	429
<i>Top 4 collocato per tipologia di prodotto</i>					
Express (Capitale Cond. Protetto)	33%	18%	29%	31%	
Equity Prot. (Capitale Protetto)	19%	47%	60%	33%	
Credit Linked (Capitale Cond. Protetto)	14%	-	2%	-	
Bonus (Capitale Cond. Protetto)	-	15%	4%	8%	

Market Statistics for ICs: Italian Market

The search for participation to the underlying assets' upside under some degree of capital protection seems to be the main driver of new issues
Digital products gain a fair share and credit linked ICs are back

2015	1° trim	2° trim	3° trim	4° trim	TOTALE
collocato (in MLN €)	2.646,02	3.583,52	1.750,92	2.666,84	10.646,80
numero di ISIN emessi	113	156	134	132	535
<i>Top collocato per tipologia di prodotto</i>	<div style="border: 1px solid black; border-radius: 15px; padding: 10px; display: inline-block;"> <p>2015 Source: ACEPI</p> </div>				
1	23% Express (Capitale Cond. Protetto)	33% Express (Capitale Cond. Protetto)	31% Express (Capitale Cond. Protetto)	33% Equity Prot. (Capitale Protetto)	29% Express (Capitale Cond. Protetto)
2	24% Bonus (Capitale Cond. Protetto)	18% Bonus (Capitale Cond. Protetto)	14% Equity Prot. (Capitale Protetto)	31% Express (Capitale Cond. Protetto)	22% Digital (Capitale Protetto)
3	18% Equity Prot. (Capitale Protetto)	6% Equity Prot. (Capitale Protetto)	2% Bonus (Capitale Cond. Protetto)	18% Credit Linked (Capitale Cond. Protetto)	14% Bonus (Capitale Cond. Protetto)
4	1% Credit Linked (Capitale Cond. Protetto)	-	-	-	14% Equity Protection (Capitale Protetto)

Key European Venues

Most markets are officially regulated and not just self regulated
 Order-driven and continuous trading are the dominant microstructural set ups; market makers provide liquidity

	SeDeX	Euwax	Scoach Francoforte	Scoach Svizzera	NYSE Euronext
<i>Status normativo</i>	Regolamentato	Non regolamentato	Non regolamentato	Regolamentato	Regolamentato
<i>Autorità di vigilanza</i>	Consob	BaFin	BaFin	FINMA	AMF (F), AFM (NL), CMVM (P), CBFA (B)
<i>Sistema esecuzione ordini</i>	Order driven con specialista	Ibrido con elementi di brokered market, quote driven, order driven	Ibrido con elementi di brokered market, quote driven, order driven	Order driven con specialista	Order driven con specialista
<i>Modalità di negoziazione principale</i>	Negoziazione continua	Aste continue	Aste continue	Negoziazione continua	Negoziazione continua con request for execution
<i>Piattaforma di trading</i>	TradElect	Stuttgart trading system	XETRA	SWXess ^a	UTP (Universal Trading Platform)
<i>Primi tre emittenti^b</i>	1. Société Générale 2. UniCredit 3. BNP Paribas	1. Deutsche Bank 2. Commerzbank 3. Royal Bank of Scotland	1. Deutsche Bank 2. Dresdner Bank 3. Commerzbank	1. Vontobel 2. ZKB 3. UBS	1. Royal Bank of Scotland 2. BNP Paribas 3. Société Générale

Structuring: from Linear to Asymmetric (Nonlinear) Payoffs

Structured products are **portfolios of zero coupon bonds and options**; they allow one to build highly customized risk-return profiles, potentially beneficial to most investors

- They are obtained by combining a Zero Coupon Bond (ZCB) with plain vanilla and/or exotic options
 - A simple example is an **Equity Linked Bond (ELB)**, (or an Equity Protection **Certificate**, which is similar in terms of economics and structuring, but differs in terms of legal/taxation matters)
 - In its simplest form this bond pays back at maturity the invested capital plus the performance of the underlying asset
 - The underlying may be a share, an index, a basket of shares, a basket of indexes, possibly also mutual funds or ETFs

Structuring: from Linear to Asymmetric (Nonlinear) Payoffs

Let's try with some numbers... How do we replicate a 5-year bond that pays the principal plus the performance (if positive) of the Euro STOXX 50 at maturity ?

- Buy a ZCB that will pay back the principal at maturity (suppose the cost is 92 Euros)
- You have 8 Euros (8% of the notional) to spend in the option
- Would you afford to buy a call option on the entire notional to get the performance of Euro STOXX 50 at maturity? If the cost of the option is 16% you would only afford to pay 50% of the performance of the Euro STOXX 50 at maturity:

$$100 \times [1 + 0.5 \times \max \{ S_5 / S_0 - 1; 0 \}]$$

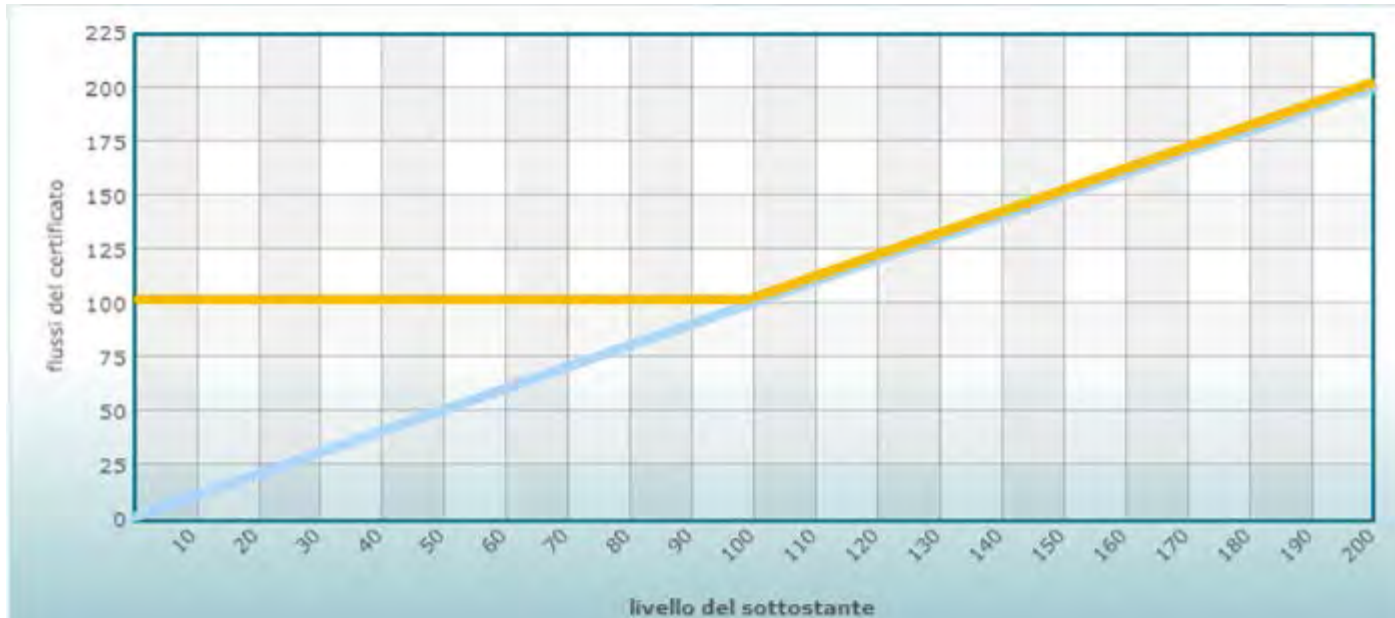
Underlying price



- Otherwise you can go for something cheaper...(e.g. exotic options)

Equity Protection Investment Certificates

In the world of ICs, ELBs are structured as equity protection certificates, that are in fact rather popular

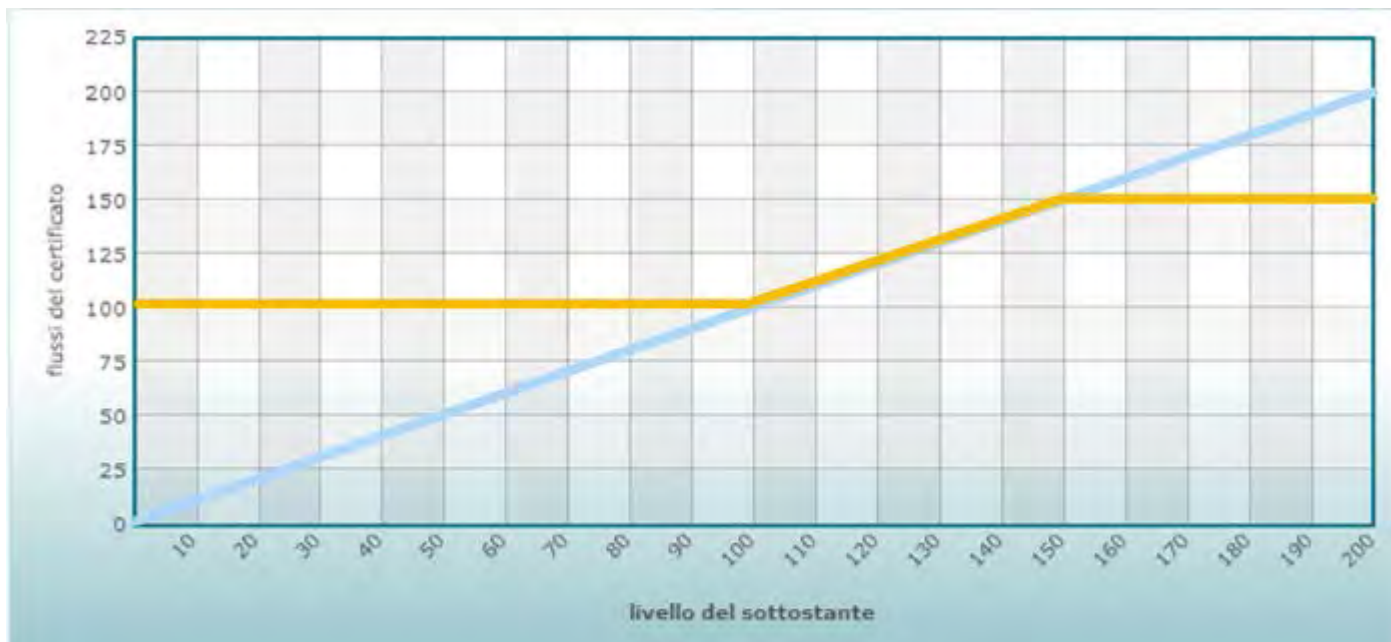


Variabili	Emissione	Barriera toccata**	Scadenza (prossimità)***
(↑) Sottostante	↑	↑	↑
(↑) Volatilità	↑	↑	↑
(↑) Tempo*	↓	↓	↓
(↑) Tassi d'interesse	↓	↓	↓
(↑) Dividendi	↓	↓	=

Close to strike price

Equity Protection Caps (Collar ICs)

Version of EP in which one adds the sale of a call out of the money to cap the payoff and increase interim participation rate

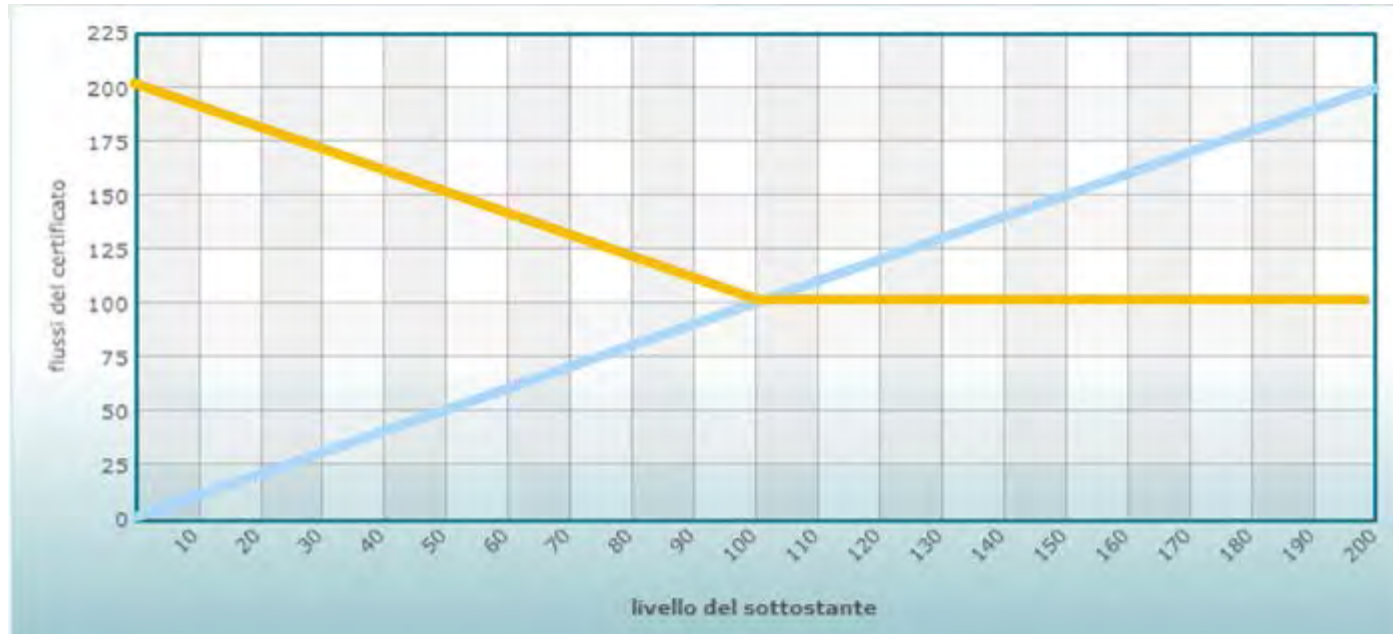


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(↑) Tempo*	↓	↓	↓
(↑) Tassi d'interesse	↓	↓	↓
(↑) Dividendi	↓	↓	=

Close to strike price

Equity Protection Short

In this case the ZCB is supplemented by the purchase of an ATM put option to allow participation in losses



Variabili	Emissione	Barriera toccata**	Scadenza (prossimità)***
(↑) Sottostante	↓	↓	↓
(↑) Volatilità	↑	↑	↑
(↑) Tempo*	↓	↓	↓
(↑) Tassi d'interesse	↓	↓	↓
(↑) Dividendi	↑	↑	

Close to strike price

Structuring: from Linear to Asymmetric (Nonlinear) Payoffs

ZCB can be combined not only with vanilla (European and/or American) call and put options but also with **exotics**, to obtain, peculiar (**often asymmetric/non-linear**) risk-return profiles

- Examples of exotic options are:
 - **Digital Options** (which pay a fixed amount if the underlying is above a certain level);
 - **Asian Options** (which pay the average performance of the underlying);
 - **Barrier Options** (which come into life / expiry if the knock – in / knock – out event happens, i.e. if the barrier is touched/crossed)

Exotic Options: Digital

Under Black-Scholes, the pricing of **digital options only depends on the (risk-neutral) probability that the underlying will be above the strike at maturity**

- **Under GBM**, the pricing of the digital options is straightforward, because they only depend on the (risk-neutral) probability that the underlying will be above the strike at maturity:

$$P = \text{Fixed Amount} \times N(d_2) \times DF$$

where $N(d_2)$ is the probability that the Euro STOXX 50 will exceed 3,656 at the expiry of the option and DF is the discount factor

- From BS formula you know that:

$$d_2 = \frac{\ln\left(\frac{St}{K}\right) + r\frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} - \sigma\sqrt{T-t}$$

Exotic Options: Digital

Suppose that, instead of paying out the performance of the Euro STOXX 50 at maturity, our ELB pays a digital coupon every year, if the value of Euro STOXX 50 is above the strike

- Suppose that at issuance of ELB, the value of Euro STOXX 50 is 3,656 points
- We structure the ELB so that at the end of each year the bond pays a fixed amount if the value of the Euro STOXX 50 is above 3,656 points
- Instead of buying a 5-year ATM call, we invest the 8 Eur to buy 5 European digitals (a 1-year digital, a 2-year digital option, etc.)

Exotic Options: Digital

How do we calculate the fixed amount that we can afford to pay?

- I calculate the value of the 5 digital options (a 1-year digital option, a 2-year digital option etc.) paying a fixed amount equal to 1 Euro;
- I then sum them and divide 8 Euros (money to be spent) by that number (cost of a strip of 5 digitals paying 1 Euro at maturity)
- E.g. if the 5 digitals paying 1 Euro cost 2.40 Euros I can afford to pay a fixed amount of 3.33 Euros

<i>Maturity</i>	<i>DF</i>	<i>N(d2)</i>	<i>Premium</i>
<i>1</i>	<i>0.9949</i>	<i>0.52</i>	<i>0.5174</i>
<i>2</i>	<i>0.9816</i>	<i>0.51</i>	<i>0.5006</i>
<i>3</i>	<i>0.9623</i>	<i>0.50</i>	<i>0.4811</i>
<i>4</i>	<i>0.9403</i>	<i>0.49</i>	<i>0.4608</i>
<i>5</i>	<i>0.9178</i>	<i>0.48</i>	<i>0.4406</i>
<i>Sum</i>			<i>2.4000</i>

Exotic Options: Asians

An Asian option has a payoff that depends **on the average value of the underlying** at some predetermined dates (or during the whole life of the option)

- An **Asian option** has a payoff that depends on the average value of the underlying at some predetermined dates (or during the whole life of the option)
 - E.g., the payoff of an Asian call option with n observation dates is:

$$\text{Payoff } f_n = NA \times \frac{1}{K} \times \text{Max} \left\{ \frac{\sum_{t=1}^n S_t}{t} - K, 0 \right\} = NA \times \text{Max} \left\{ \frac{S_{\text{Average}}}{K} - 1, 0 \right\}$$

Strike price

Average of the underlying price at the n observation dates

Exotic Options: Asians

Suppose that your structured bond will pay, instead of the performance of the underlying, the average performance computed at the end of each year

- Consider an Asian option on the Euro STOXX 50 with strike equal to 3,656 and 5 yearly observation dates
- The payoff of the Asian option at maturity will be:

t	S_t
1	3,700
2	3,860
3	3,850
4	3,800
5	3,750
Average	3,792

$$NA \times \text{Max} \left\{ \frac{S_{\text{Average}}}{K} - 1, 0 \right\} = NA \times \text{Max} \left\{ \frac{3,792}{3,656} - 1, 0 \right\} = NA \times 3.72\%$$

- If the premium of the Asian option is equal to 10% and we have 8% to spend, we can afford 80% participation to any positive returns on the underlying so that at maturity, the ELB will pay:

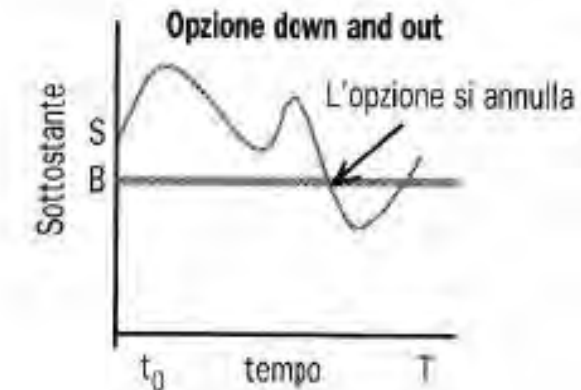
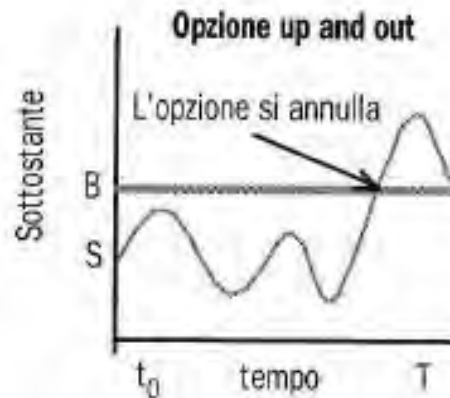
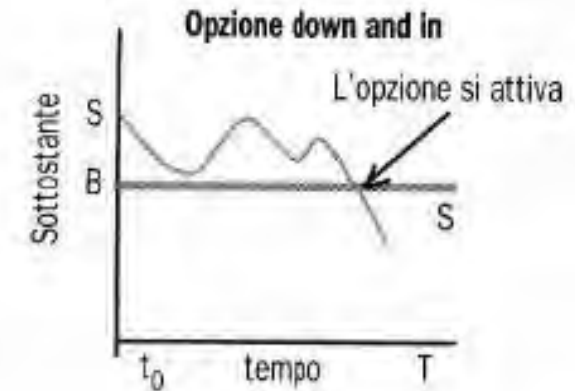
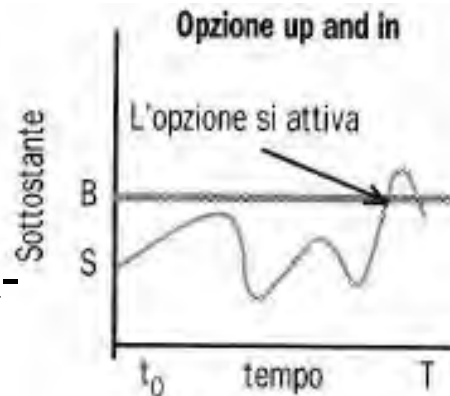
$$ELB = 100 \times [1 + 0.8 \times 3.72\%] = 103$$

- Obviously, being path-dependent, an Asian call will always be **cheaper** than the equivalent European call option

Exotic Options: Barriers

A Knock-in barrier option is **an option that comes into existence if the price of the underlying crosses a predetermined barrier**

- Obviously, being path-dependent, an Asian call will always be cheaper than the equivalent European call option
- There are many types of barrier options, but the most common ones are Knock-In and Knock-Out options
- A **Knock-In option** is an option that comes into existence if the price of the underlying crosses the barrier



Exotic Options: Barriers

A barrier option comes into life/ disappears if a certain barrier is touched

- A barrier option may be knock-in (if it comes into life when the barrier is touched) or knock-out if it vanishes when the barrier is touched)
 - According to level of barrier vs. initial underlying price, we distinguish up-and-out, down-and-out, up-and-in, and up-and-out options

Type	Barrier Location	Description
Down&In	Below	Price has to decrease below the barrier for the option to come into life
Up&In	Above	Price has to increase above the barrier for the option to come into life
Down&Out	Below	Price has not to decrease below the barrier for the option to stay alive
Up&Out	Above	Price has not to increase above the barrier for the option to stay alive

- The barrier can be observed only at maturity (European barrier) or during the whole life of the option (American barrier)

Exotic Options: Barriers

- Let's make an example on how a barrier option works:
 - Consider an ATM call option on Fiat with spot price = Strike = Eur 15
 - A European Up&Out barrier at Eur 19 is written: if at maturity the price of Fiat is equal to Eur 19 the option will expire and nothing will be paid
 - Instead, if the price at maturity is equal to Eur 17, the pay-out of the option will be equal to 2
 - From the payoff table below, a barrier option with a barrier $K = 19$, should be cheaper than a standard, plain vanilla option (even if $K = 15$ for both)

	Payoff	
S_t	Knock-Out _{B=19} Call	Standard Call
14	0	0
15	0	0
16	1	1
17	2	2
18	3	3
19	0	4
20	0	5

Exotic Options: Barriers

- This difference in the prices of a standard option vs. a barrier option with the same strike is exactly the reason why someone may wish to buy a barrier
 - Suppose, as an example, that you believe that Fiat will slightly increase in the next three months to a target price of 17 Eur
 - You have three options to bet on this increase of Fiat
 - (a) buy Fiat at 15 Eur and sell it in three-month's time; if you have 1,500 Eur to invest you will buy 100 shares and then sell them in three months
 - (b) buy an ATM option with three-month maturity; if the premium of the option is 1.5 Eur and you have 1,500 Eur to invest then he can underwrite an option on a notional of 15,000 Eur (1,000 shares)
 - (c) buy an ATM option with Up&Out barrier (barrier equal to 18 Eur) with a three-month maturity; if the premium of the option is 1 Eur and you have 1,500 Eur to invest you can underwrite an option on a notional of 22,500 Eur (1,500 shares)

Exotic Options: Barriers

Barrier options may (ex-post) maximize the profits from strategies based on (ex-post accurate) range-level forecasts of the underlying

- The table below considers the possible scenarios at maturity:

S_t	P&L(a)	P&L(b)	P&L(c)
13	-200	-1500	-1500
14	-100	-1500	-1500
15	0	-1500	-1500
16	100	-500	0
17	200	500	1500
18	300	1500	0
19	400	2500	0

- In essence, if you are not interested in the upside of Fiat above 18 Eur, the Up&Out option allows you not to pay for it
- Conversely, an Up&In option will be bought by a client that believes that the price increase will be higher than a certain level
- If you believe that Fiat will quote higher than 18 Eur in three months, buying an ATM Up&In call option with barrier equal to 18 Eur will be cheaper than buying a plain vanilla ATM option

Reverse Convertible Products

A reverse convertible is a structured product that pays a fixed coupon but **refunds a portion of the notional principal that depends on the behavior of the price of one underlying** security

- A **standard Reverse Convertible** is a note that pays an unconditional coupon at maturity (e.g., 10%) regardless of the behavior of the underlying
- In addition, if the price of the underlying has not declined, the note also pays back its notional
- On the contrary, if the underlying has depreciated, the investor obtains a number of shares equal to the notional divided by the Strike Price (also known as Conversion Price)

- As an example, consider a 1-year Reverse Convertible note on Fiat:

Reverse Convertible Termsheet/Features	
Tenor	1 year
Underlying	Fiat
Strike Price (conversion price)	16 Euro
Coupon	10%
Notional	100 Euro (equal to 6.25 Fiat shares)
Payoff in case Fiat \geq 16 Euro	110 Euro
Payoff in case Fiat $<$ 16 Euro	6.25 shares plus 10 Euro

Reverse Convertible Products

In a reverse convertible, **an investor absorbs downside market risk in exchange of a coupon rate higher than the risk-free**

- This payoff is replicated with a purchase of a ZCB plus the sale of a put option with 100% strike
 - Indeed, the payoff of the ZCB at maturity is equal to 100 Euro
 - If the value of Fiat is below 16 Euro the put option will be exercised and the payoff is: $-(K - S)$
 - Consequently the total payoff will be: $100 - 100 + S = S$
 - Instead, if Fiat is above 16 Euro, the put option will be OTM, so it will not be exercised
- The proceeds from the sale of the put are invested at the risk free rate to delivery a fixed coupon at maturity which is higher than the one that could be simply obtained from investing in a ZCB
- The investor is buying downside risk to obtain a higher coupon
- Note that instead of receiving a coupon at maturity, the investor may prefer to buy the note at a discounted price, the case of **Discount Certificates**

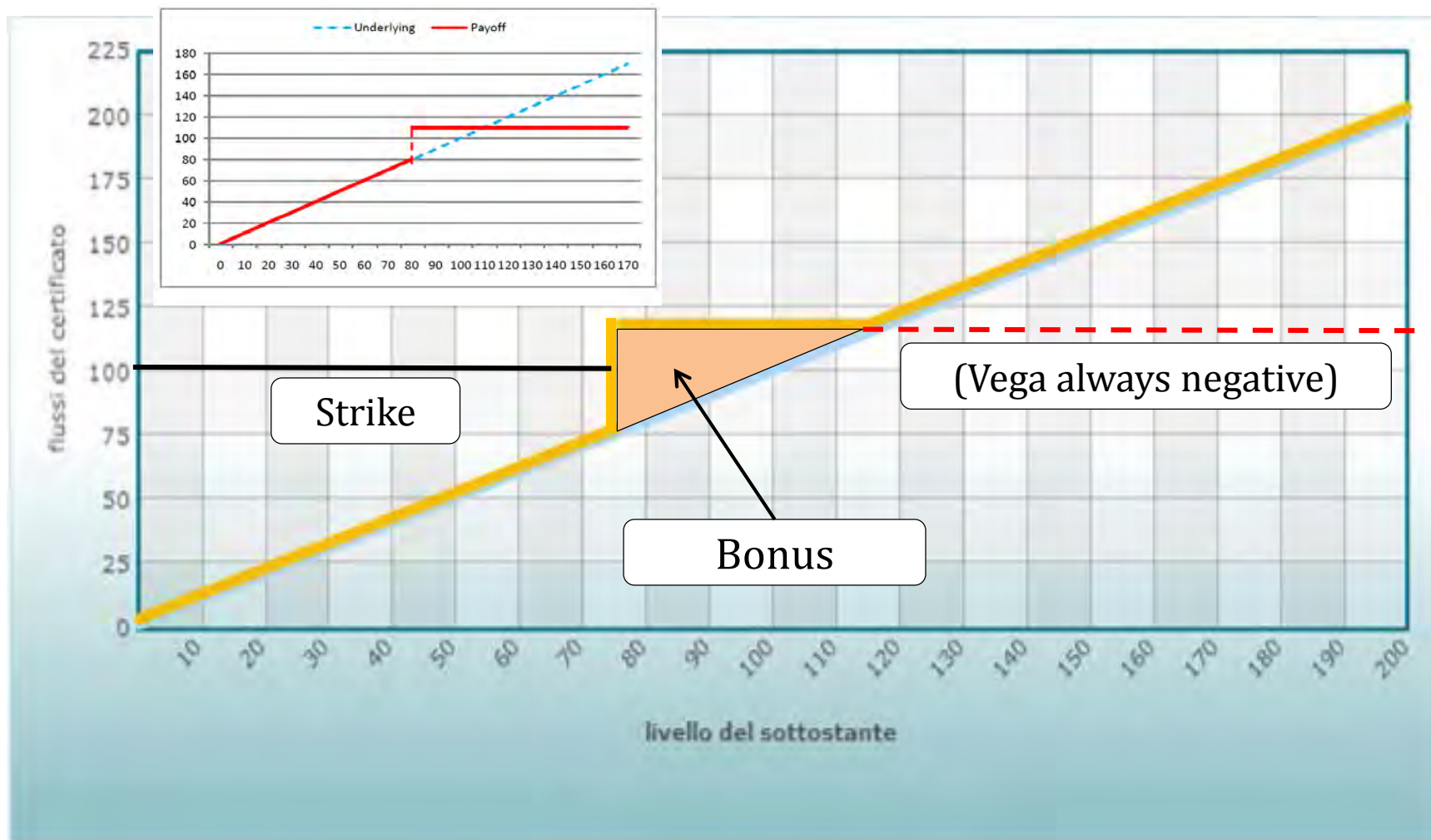
Bonus Cap ICs

A reverse convertible **that includes a barrier (e.g., down-and-in) option** instead a plain vanilla put is a Bonus Cap IC

- As the premium of an option...
 - increases with volatility and
 - increases with the dividend yield (this is for put options).... to maximize the coupon a structurer should look for underlying assets which have high volatility and high dividend yield
 - The classical version of a Reverse Convertible does not imply the use of exotic options
- A possible variation of the Reverse Convertibles does: Bonus Cap IC
- A Bonus Cap **IC offers a “conditional protection” to the downside**
- The investor will receive back the capital plus a coupon **unless the barrier is touched/crossed**
 - The barrier can be European (observed only at maturity) or American (observed in continuous time)

Bonus Cap ICs

A reverse convertible **that includes a barrier (e.g., down-and-in) option** instead a plain vanilla put is a Bonus Cap IC



Bonus Cap ICs

Let's discuss one specific case of a Bonus Cap with a European barrier

Bonus Certificate Termsheet/Features	
Tenor	1 year
Strike	100% (15 Euro)
Underlying	Fiat
Barrier	80% (12 Euro)
Notional	100 Euro
Bonus Amount	110 Euro
Cap Amount	110 Euro

- If at maturity Fiat is above 12 Euro, the investor receives a “Bonus Amount” equal to 110 Euros (which is also the maximum amount that the investor can get, i.e., the “Cap”)
- Otherwise the investor gets an amount proportional to the performance of Fiat, i.e., $100 \times \min[\text{Cap}, S_t/S_0]$
- **A Bonus Cap is replicated with a ZCB plus a short Down&In put option**

Bonus Cap ICs

Let's focus on the case in which the 1-year discount factor is equal to 99% and a Down&In put with 80% barrier and $K = 110\%$ has a premium equal to 9.9%

- Today we can invest 108.9 Euro in ZC bonds with 1-year to expiry to get 110 Euro at maturity
- At maturity we can have the two scenarios:

	Fiat \geq 12 Euro	Fiat $<$ 12 Euro
ZC	110 Euro	110 Euro
Put D&I	Not triggered	$-(K - S) = -110 + S$
Total	110 Euro	S

- In the case of an American Barrier, the payoff will be the same, but the barrier of the Down&In put will be observed during the whole life of the product
- At this point pricing the Bonus Cap just implies an ability to price the down-and-in put option with strike 110% and 80% barrier
- We shall speak about this operation later

Bonus Cap ICs: The Greeks

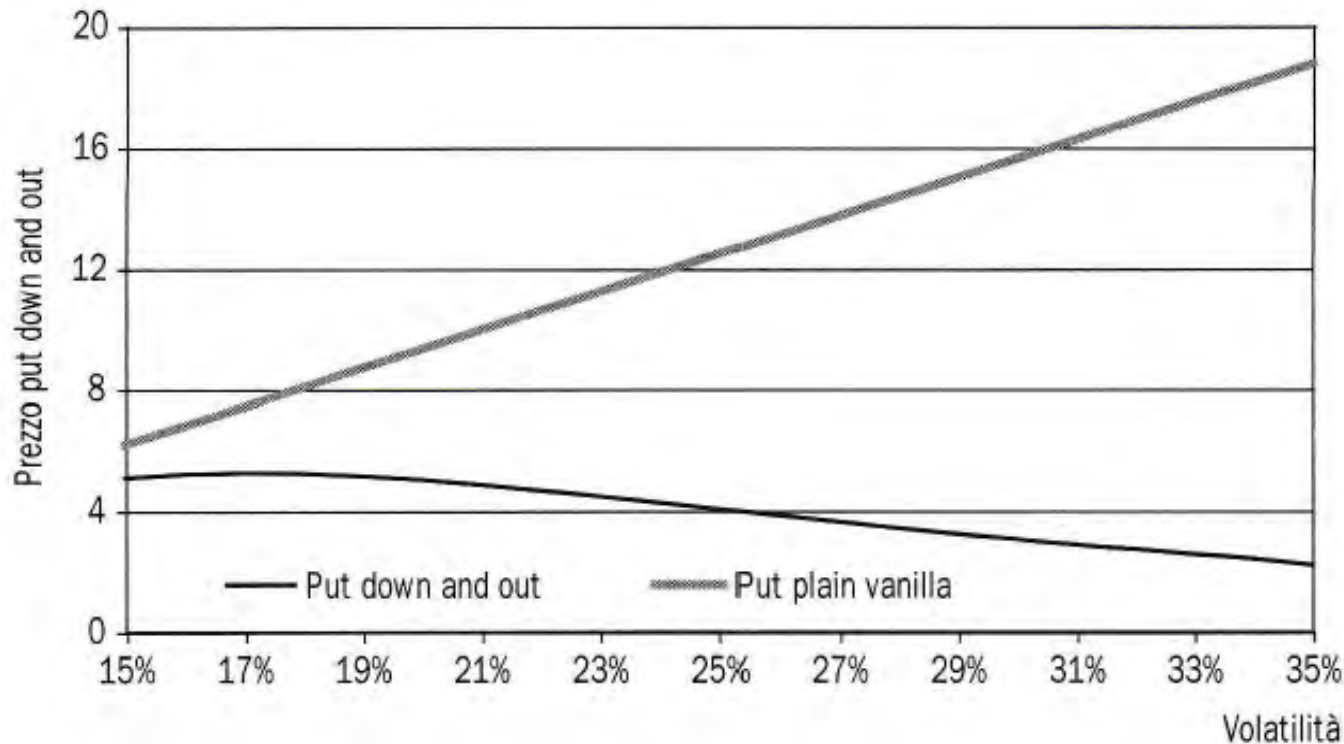
- Vega is negative for long-term Bonus Caps because as volatility increases, the chances that the barrier is hit increase
- When the Bonus Cap is ATM and close to maturity, on net volatility has a weak positive effect on price because of the linear upward segment
- Theta is positive because it makes less likely to hit the barrier
- Dividends reduce the value of the underlying and such they penalize the performance of a Bonus Cap

Variabili	Emissione	Barriera toccata**	Scadenza (prossimità)***
(↑) Sottostante	↑	↑	↑
(↑) Volatilità	↓	=	↑
(↑) Tempo*	↑	=	↑
(↑) Tassi d'interesse	↓	↓	↓
(↑) Dividendi	↓	↓	=

Close to strike price

Bonus Cap ICs: Vega

- The plot shows the different sign of vega for plain vanilla puts vs. a down and out put
- Calculations are performed with reference to $S_t = K = 100$, volatility = 25%, $T = 3$ years, Barrier = 60, $r = 3\%$
- Calculations are performed using the model by Rubinstein and Reiner (1991)



The Autocallability Feature (aka Express)

Express certificates are characterized by the possibility of early redemption if the underlying is higher than a certain level; they yield profitable **“lateral” strategies hurt by volatility**

- Autocallable Certificates, also known as “Express” are certificates which are **characterized by the possibility of early redemption if the underlying is higher than a certain level** (the **strike**)
- In case they are redeemed early, they pay the notional amount plus a coupon, multiplied by the number of observation periods elapsed
- In case they are not redeemed early, at maturity the capital is protected if the underlying is above some threshold level (**barrier**)
 - E.g., this certificate can be redeemed early at the end of year 1 and at the end of year 2, if the price of Fiat exceeds the trigger level (15 Euro)
 - If redeemed at the end of year 1 the certificate will pay 106

Autocallable Certificate Term Sheet/Features	
Maturity	3 years
Underlying	Fiat
Strike	15 Euro
Observation Dates	End of each year
Trigger	100% of the Strike
Barrier	70% of the Strike
Coupon	6%

The Autocallability Feature (aka Express)

Autocallable certificates are path-dependent as their expiry depends on the price of the underlying at the observation dates

- If redeemed at the end of year 2 the certificate will pay 112 Euro
- If instead the product reaches maturity, the certificate will pay 118 Euros, if the underlying is above the trigger level, 100 Euro if the underlying is below the trigger level, but above the barrier, and Eur $100 \times (S_T/S_0)$ otherwise
- Because autocallable certificates are path-dependent, given that their maturity depends on the underlying at some future dates (observation dates), **they are priced using Monte Carlo simulations**
- A stochastic volatility model is in general needed to capture the probability of kickout, and to assess the remaining value of the structure if no kickout has occurred, e.g.,

$$S_{t+1} = S_t + (r - d - 0.5 \times \nu_t) \Delta t + \sqrt{\nu_t \Delta t} \varepsilon_{1t}$$
$$\nu_{t+1} = \nu_t + \lambda(\bar{\nu} - \nu) \Delta t + \eta \sqrt{\nu_t \Delta t} \varepsilon_{2t} + \frac{\eta^2}{4} + \Delta t(\varepsilon_{2t}^2 - 1)$$

The Autocallability Feature (aka Express)



Variabili	Emissione	Barriera toccata**	Scadenza (prossimità)***
(↑) Sottostante	↑	↑	↑
(↑) Volatilità	↑	↑	↑
(↑) Tempo*	↓	↓	↓
(↑) Tassi d'interesse	↓	↓	↓
(↑) Dividendi	↓	↓	

Close to strike price

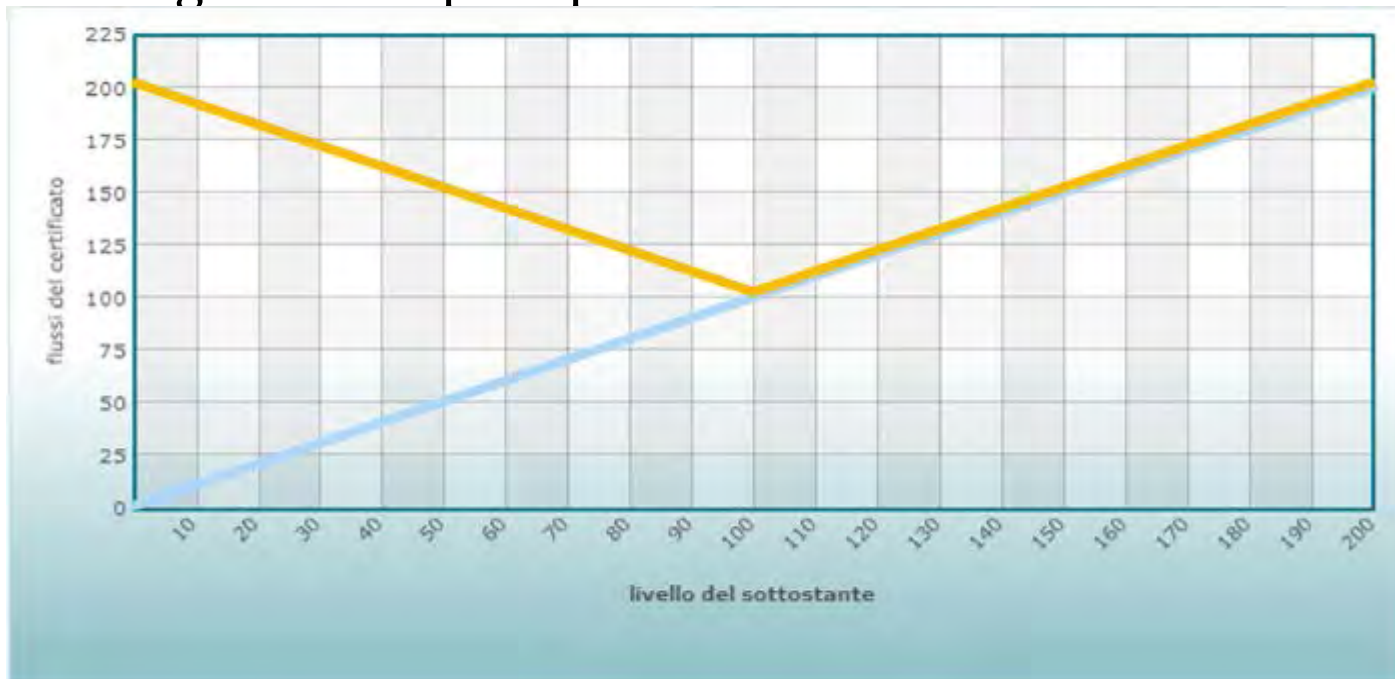
The Autocallability Feature (aka Express)

- An Express IC is replicated by:
 - Buying a call at strike zero, **equivalent to buying the underlying**
 - Buying a barrier option put down-and-out; if the price goes below the barrier, then the express makes an investor fully participate in the losses of the underlying; otherwise it compensates any losses between strike and barrier
 - Selling a call with strike equal to the express strike; this cancels any profits from increases in the prices of the underlying
 - Buying a series of digital/barrier calls of knock-out type with strike = express strike, maturities equal to the liquidation dates, in number equal to the coupons paid in case the option stays alive; the knock-out feature kills them when express goes below strike
- The number of knock-out digitals increases with the difference in price between the call sold and the put down-and-out barrier
- An Express IC without the autocallability features goes under the name of **Cash Collect**

Double Win Certificates

Double win ICs securitize long-volatility positions under a (partial) capital protection

- Double Win ICs allow to participate to both positive and negative returns on the underlying under (partial) capital protection
 - Therefore they are equivalent to bets on volatility
 - They may include a leverage factor that makes them a combination of leveraged and capital protection ICs



Double Win Certificates

- Double wins are replicated by the following plain vanilla, European option strategy
 - Purchase a ZCB
 - Purchase a call with strike == guaranteed capital level
 - Purchase a put with strike == guaranteed capital level
- When capital protection is partial, then the amount to be spent in purchasing puts and calls exceeds $100\exp(-rT)$ and participation rates $> 100\%$ to the upside and/or downside become possible
 - In case a limited protection double win acquires leveraged features

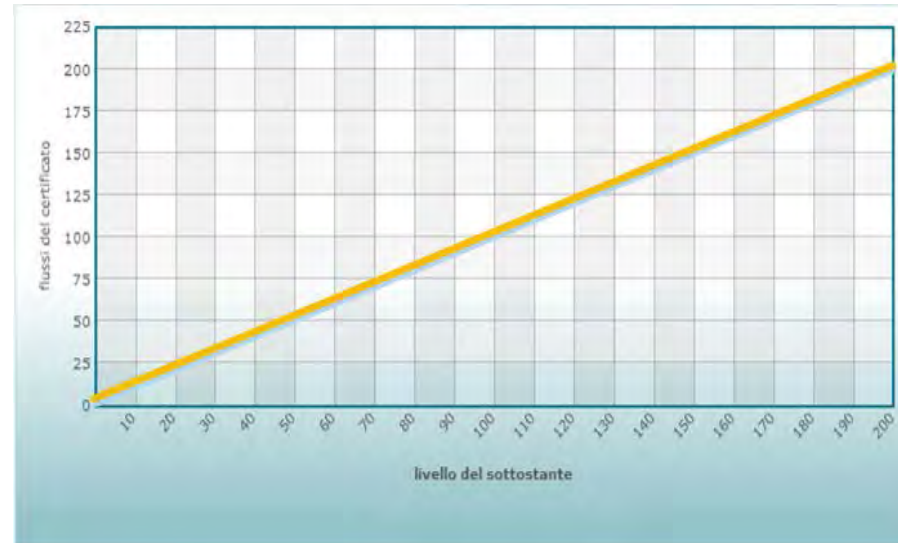
Variabili	Emissione	Barriera toccata**	Scadenza (prossimità)***
(↑) Sottostante	↑	↑	↑
(↑) Volatilità	↑	↑	↑
(↑) Tempo*	↓	↓	↓
(↑) Tassi d'interesse	↓	↓	↓
(↑) Dividendi	↓	↓	↓

Close to strike price

ICs Without Capital Protection

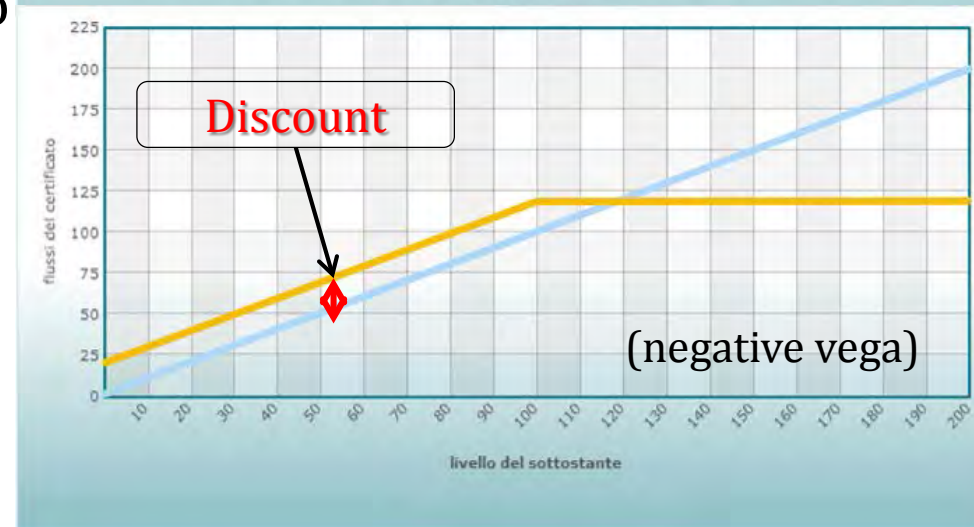
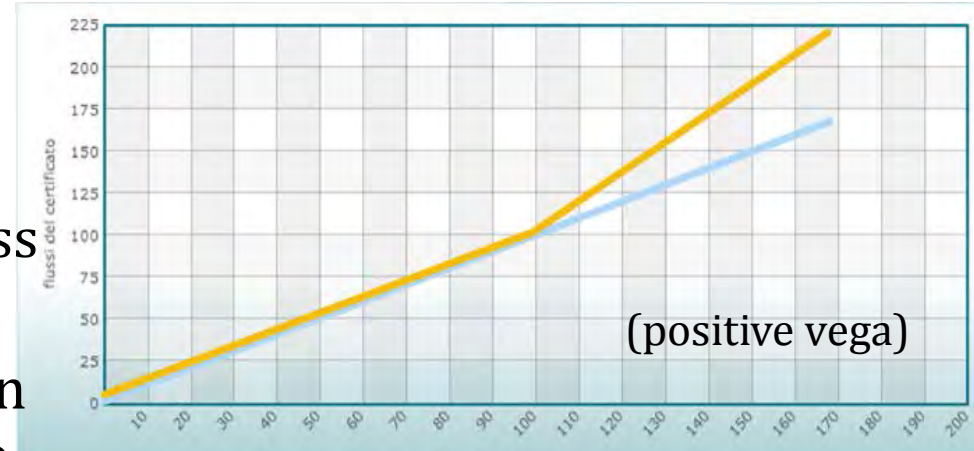
Benchmark and outperformance ICs are examples of symmetric, approximately linear payoffs

- **Benchmarks** replicate some underlying index with a 100% participation rate to profits and losses (linear symmetric payoffs)
- Therefore they are similar to ETFs and index mutual funds
 - However, differently from ETFs they have a maturity date
 - ETF prices fluctuates with dividends and stock splits, while any expected dividends are deducted from IC prices at issuance
 - Their tax treatment is different and generally favorable to ICs
 - Their vega is nil because they are not options
 - Benchmarks on baskets of commodities are popular
 - When written on futures, they pose rollover issues



ICs Without Capital Protection

- **Outperformance** ICs are benchmarks in which participation to positive returns on the underlying are magnified
 - This is replicated by adding ATM European call options in quantity = outperformance rate/100
 - The outperformance feature is financed by charging an initial price of the IC in excess of the underlying index
- **Discount** ICs are benchmarks in which baseline participation to returns on the underlying is magnified by a purchase price below current price, but a cap is imposed on positive returns
 - This is replicated by **selling** appropriate European puts



Leveraged Certificates and Turbos

Leverage certificates allow an investor to participate to profits and losses on the underlying in a more-than-proportional way

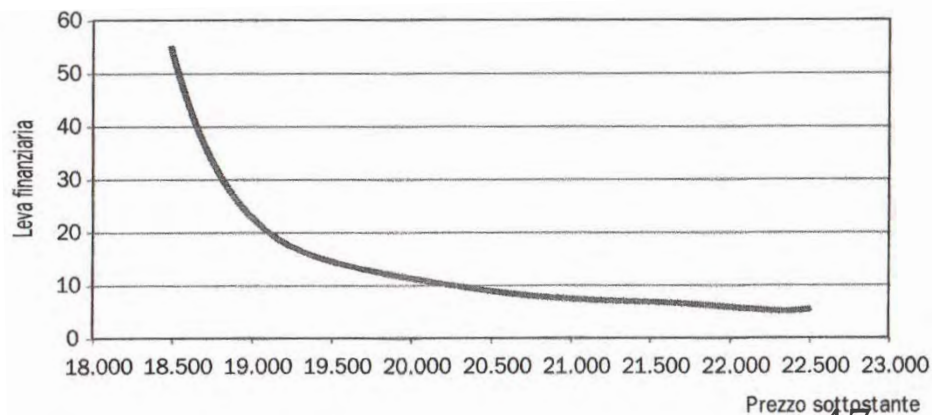
- A **Turbo** certificate allows to participate in profits and losses of the underlying asset on the basis of a multiple and a **stop loss** level determines the underlying price at which the Turbo is extinguished
 - There is an implicit, mechanical auto-callability feature produced by the fact that losses cannot be magnified to go below -100%
 - A key parameter is leverage, the ratio btw. the underlying price at issuance and (underlying price - certificate strike)
- A Turbo can be replicated by a long position in the underlying + the sale of a ZCB with notional = strike price
- They are natural trading tools



Leveraged Certificates and Turbos

While under dynamic leverage the strike is fixed and effective **leverage continuously moves as a (inverse!) function of the underlying price**, under fixed leverage the strike is dynamically adjusted to make the leverage ratio constant over time

- A Turbo implies **dynamic leverage**, i.e., a leverage ratio that is a function of the underlying price, given a fixed strike
- Turbos may often offer abysmal performances that are caused by these dynamic effects, which make them riskier than thought of
 - E.g., Camelia's book reports one example of a Turbo on the FTSE MIB with 10.9 leverage at issuance that, over time and in the face of a +12.5% by the index, makes a 117% return, i.e., $117/12.5 = 9.4$ only
 - In the case of a -5.6% by the underlying, Turbo yields a loss of 70%, i.e., $70/5.6 = 12.5$
- Also **fixed leverage** structured products have a drawback: the **compounding effect**



Leveraged Certificates and Turbos

- When volatility is high, the performance of fixed leverage ICs tends to significantly diverge from the performance of the underlying
 - In principle, under fixed leverage you may record losses even though between t and T on net (averaging) the underlying has not moved...
 - Ideally, this can be avoided by dynamically changing the amount invested in fixed leverage products, at least on every trading day
 - Therefore fixed leverage products are ideal for trading and they tend to imply modest transaction costs
- Fixed leverage is typically packaged as a leveraged benchmark IC
- A few examples of the compounding effect

Favorable Example

	Indice	Performance giornaliera indice	Indice leva 5	Performance giornaliera Indice a leva 5
giorno 1	10,000	5,00%	10,000	25,00%
giorno 2	10,500	1,00%	12,500	5,00%
giorno 3	10,605	3,00%	13,125	15,00%
Complessivo	10,923	9,23%	15,093	50,93%

$$X 5 = 46\% < 51\%$$

Unfavorable Example

	Indice	Performance giornaliera indice	Indice leva 5	Performance giornaliera Indice a leva 5
giorno 1	10,000	2,00%	10,000	10,00%
giorno 2	10,200	-4,00%	11,000	-20,00%
giorno 3	9,792	3,00%	8,800	15,00%
Complessivo	10,08576	0,8576%	10,120	1,20%

$$X 5 = 4.3\% > 1.2\%$$

Leveraged Certificates and Turbos

The compounding effect is the stronger, the higher the volatility of the underlying asset price

Indice	4Q2012	1Q2013	2Q2013	3Q2013
FTSE/MIB	4,83	-9,20	-2,03	12,78
FTMIB 55	-32,31	32,80	-17,81	-53,77
FTMIB 5L	16,45	-46,96	-11,04	68,78
CAC	6,00	-0,07	-1,75	9,98
CAC 5L	26,92	-6,58	-9,15	53,60
CAC 5S	-32,93	-10,15	-16,79	-43,31
CAC 7S	-45,14	-17,80	-27,73	-56,51
CAC 7L	34,25	-13,44	-18,34	76,09
DAX	3,90	0,21	0,19	7,65
DAX 5L	16,37	-3,59	-7,41	36,76
DAX 5S	-22,13	-7,43	-12,22	-36,09
DAX 7L	20,29	-8,02	-15,41	49,50
DAX 7S	-31,49	-12,98	-21,21	-48,53
EUROSTOXX50	6,45	-3,22	-2,88	10,32
STOXX50E 5L	31,11	-19,57	-13,27	56,26
STOXX50E 5S	-33,51	4,14	-11,59	-45,09
STOXX50E 7L	40,94	-29,92	-23,25	79,64
STOXX50E 7S	-45,69	1,09	-21,05	-58,60

Fixed Leverage Benchmarks

nL = n x long
nS = n x short

Fonte: elaborazioni UniCredit su dati pubblici.

Leveraged Certificates and Turbo Short

The bearish-view motivated ICs that accomplish what Turbos do on the long end are the Shorts

- A **Short** certificate allows reverse participation to profits and losses of the underlying on the basis of a multiple and a **stop loss** level determines the underlying price at which the Short is extinguished
 - There is an implicit, mechanical auto-callability feature produced by the fact that losses cannot be magnified to go below -100%
 - A key parameter is leverage, the ratio btw. the underlying price at issuance and strike
- A Short can be replicated by a short position in the underlying + the purchase of a ZCB with notional = strike price
- Expected dividend corrections are necessary to avoid arbitrage

