Università Commerciale
Luigi Bocconi

Lecture 3: Forecasting interest rates (Part 2)

Prof. Massimo Guidolin

Advanced Financial Econometrics III

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Overview

- A dynamic latent factor model of the yield curve and macroeconomic variables
- QML estimation via the EM algorithm
- Model selection
- In- and Out-of-sample results
- Monetary policy under regimes and the yield curve

The Yield Curve and Macroeconomic Factors

- A literature has established close ties btw. macro- and yield curve factors, see Diebold, Rudebusch, and Aruoba (2006, JoE)
  - The short end of the yield curve moves closely with the policy instrument under the direct control of the central bank
  - The average level of the curve is associated with the inflation rate
  - The spread between long and short rates with temporary business cycles conditions
- Some debate as to whether macroeconomic information helps forecasting future interest rates and excess bond returns
- Coroneo, Giannone and Modugno (2016, JBES) focus on such a debate with an emphasis on macroeconomic information that is not spanned by the traditional yield curve factors
- Use a dynamic factor model (DFM) for Treasury zero-coupon yields and a representative set of macroeconomic variables with restrictions on the factor loadings
- Advantage: capture all effects of macro variables on Treasuries

The Dynamic Latent Factor Model

- Cost: need to also treat macroeconomic factors as latent
  - In the model, the level, slope, and curvature Nelson-Siegel factors are spanned by both the bond yields and macroeconomic variables
  - The additional macro factors, instead, are contemporaneously loaded only by the macroeconomic variables and, thus, are unspanned by the cross-section of the yields

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} 0 \\ a_x \end{pmatrix} + \begin{bmatrix} \Gamma_{yy} & \Gamma_{yx} \\ \Gamma_{xy} & \Gamma_{xx} \end{bmatrix} \begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix} + \begin{pmatrix} v_t^y \\ v_t^x \end{pmatrix}$$

$$\Gamma_{yy} = \Gamma_{\text{NS}}, \;\; \Gamma_{yx} = 0,$$

Row of the matrix of loadings for maturity τ

$$\Gamma_{yy}^{(\tau)} = \left[1 \quad \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \quad \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right] \equiv \Gamma_{\text{NS}}^{(\tau)}$$

  - Diebold and Li (2006, JoE) show that this functional form of the factor loadings implies that the three yield curve factors can be interpreted as the level, slope, and curvature of the yield curve
  - The decay parameter λ is calibrated to maximize the loading on the curvature factor for yields with maturity 30 months
  - Joint dynamics of yield and macroeconomic factors follow a VAR(1)

The Dynamic Latent Factor Model

$$\begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix} = \begin{pmatrix} \mu_y \\ \mu_x \end{pmatrix} + \begin{bmatrix} A_{yy} & A_{yx} \\ A_{xy} & A_{xx} \end{bmatrix} \begin{pmatrix} F_{t-1}^y \\ F_{t-1}^x \end{pmatrix} + \begin{pmatrix} u_t^y \\ u_t^x \end{pmatrix}$$

$$\begin{pmatrix} u_t^y \\ u_t^x \end{pmatrix} \sim N\left(0, \begin{bmatrix} Q_{yy} & Q_{yx} \\ Q_{xy} & Q_{xx} \end{bmatrix}\right).$$

- The idiosyncratic components in the measurement equation are modeled to follow independent autoregressive processes

$$v_t = B v_{t-1} + \xi_t, \quad \xi_t \sim N(0, R)$$

- where **B** and **R** are diagonal matrices, implying that the common factors fully account for the joint correlation of the observationss
- $\Gamma_{xy} \neq O$ crucial to ensure that the macroeconomic factors capture only those source of co-movement in the macro variables that are not already spanned by the yield curve factors
- However, the paper imposes $\Gamma_{yx} = O$ which restricts the macro factors to be unspanned not only by the yield factors but also by the entire cross-section of yields
- This restriction is expected to be immaterial because the yield factors are notoriously effective at fitting the entire yield curve

Estimation

- Cost: need to also treat macroeconomic factors as latent
- This a restricted state-space model with autocorrelated idiosyncratic components for which MLEs are not in closed form
- The model is estimated by QMLE, i.e., by maximizing the likelihood of a potentially misspecified model
  - Yet, conditionally on factors, the model reduces to a set of regressions $\Rightarrow$ MLEs can be computed by expectation-maximization algorithm
  - Likelihood computed assuming that the DFM is Gaussian and exact (idiosyncratic errors are assumed to be cross-sectionally orthogonal)
  - Doz, Giannone, and Reichlin (2012, JoE) showed that, when estimation is carried out with a large number of highly correlated variables, the estimator is consistent and robust to non-Gaussianity
- Sktech of EM algorithm:
  - Initialize the yield curve factors with the NS factors using the two-step OLS procedure introduced by Diebold and Li (2006, JoE)
  - Then project the macro variables on the NS factors and use the PCs of the residuals to initialize the unspanned macroeconomic factors

Estimation

- Γ_{yy} is restricted to equal to the NS loadings so this is also a DNS paper
- Given the initial parameters, a new set of factors is obtained using the Kalman smoother.
- MLEs are obtained by iterating these two steps until convergence provided that OLS regressions are modified to take into account the fact that the common factors are estimated

- Benchmark is the only-yields model, $Q_{yx} = A_{yx} = \Gamma_{xy} = 0$
- The yield curve factors are identified as the NS factors that have a clear interpretation as level, slope, and curvature
- However, true number of unspanned macro factors is unknown $\Rightarrow$ use information criteria approach in Bai and Ng (2002, ECTA)

$$IC^*(s) = \log(V(s, \hat{F}_{(s)})) + s\,\frac{\log C_{NT}^{*2}}{C_{NT}^{*2}}$$

Number of estimated factors

Sum of squared idiosyncratic components (divided by NT)

Convergence rate of the estimator

$$C_{NT}^{*2} = \min\left\{\sqrt{T}, \frac{N}{\log N}\right\}$$

- The information criterion needs to be minimized over a range of selections for *s*, between 3 and 8 in the paper

Data

- Using monthly U.S. data from Jan. 1970 to Dec. 2008, they find that a significant portion of macroeconomic information is not captured by the yield curve factors and is unspanned by the yield curve
- Unspanned == it does not affect contemporaneously the cross-section of yields; unspanned macroeconomic information is driven by two factors proxied by economic growth and real interest rates
- These factors have substantial predictive information for bond yields and excess bond returns
- The model explains up to 55% of the variation in excess bond returns and outperforms all existing models in forecasting
- The data are on U.S. Treasury zero-coupon yields from the Fama-Bliss dataset, with obs on 3 month through 5-year bond yields

Series N.	Mnemonic	Description	Transformation
1	AHE	Average Hourly Earnings: Total Private	1
2	CPI	Consumer Price Index: All Items	1
3	INC	Real Disposable Personal Income	1
4	FFR	Effective Federal Funds Rate	0
5	HSal	House Sales—New One Family Houses	1
6	IP	Industrial Production Index	1
7	M1	M1 Money Stock	1
8	Manf	ISM Manufacturing: PMI Composite Index (NAPM)	0
9	Paym	All Employees: Total Nonfarm	1
10	PCE	Personal Consumption Expenditures	1
11	PPIc	Producer Price Index: Crude Materials	1
12	PPIf	Producer Price Index: Finished Goods	1
13	CU	Capacity Utilization: Total Industry	0
14	Unem	Civilian Unemployment Rate	0

NOTE: This table lists the 14 macro variables used to estimate the macro-yields. Most series have been transformed prior to the estimation, as reported in the last column of the table. The transformation codes are: 0 = no transformation, and 1 = annual growth rate.

Number of Factors

- The information criterion selects the model with 5 factors, 3 DNS factors plus 2 unspanned macro factors
- I will ask you to do robustness checks on these results

Number of factors	IC*	V
3	0.02	0.44
4	−0.03	0.31
5	−0.11	0.22
6	0.01	0.18
7	0.23	0.17
8	0.43	0.16

- The 1st unspanned macro factor captures the dynamics of IP and real variables, while the 2nd unspanned factor is mainly inflation

Table 3. Cumulative variance of yields and macro variables explained by the macro-yields factors

	Level	Slope	Curv	UM1	UM2
Government bond yield with maturity 3 months	0.59	0.94	1.00	1.00	1.00
Government bond yield with maturity 1 year	0.61	0.83	0.99	0.99	0.99
Government bond yield with maturity 2 years	0.65	0.78	0.99	0.99	0.99
Government bond yield with maturity 3 years	0.70	0.79	1.00	1.00	1.00
Government bond yield with maturity 4 years	0.74	0.80	0.99	0.99	0.99
Government bond yield with maturity 5 years	0.78	0.82	0.99	0.99	0.99
Average Hourly Earnings: Total Private	0.07	0.29	0.33	0.33	0.67
Consumer Price Index: All Items	0.19	0.48	0.48	0.50	0.85
Real Disposable Personal Income	0.00	0.02	0.03	0.34	0.36
Effective Federal Funds Rate	0.53	0.93	0.96	0.96	0.97
House Sales—New One Family Houses	0.00	0.19	0.19	0.23	0.23
Industrial Production Index	0.02	0.02	0.03	0.69	0.69
M1 Money Stock	0.17	0.25	0.25	0.25	0.31
ISM Manufacturing: PMI Composite Index (NAPM)	0.03	0.05	0.05	0.61	0.65
Payments All Employees: Total Nonfarm	0.00	0.02	0.10	0.70	0.70
Personal Consumption Expenditures	0.16	0.23	0.33	0.46	0.78
Producer Price Index: Crude Materials	0.03	0.14	0.14	0.20	0.43
Producer Price Index: Finished Goods	0.03	0.32	0.32	0.33	0.80
Capacity Utilization: Total Industry	0.02	0.16	0.21	0.63	0.64
Civilian Unemployment Rate	0.44	0.54	0.55	0.65	0.68

In-Sample Results

Maturity	MY	OY	CP	LN	LN+CP
2y	0.55	0.12	0.22	0.33	0.41
3y	0.53	0.12	0.24	0.33	0.43
4y	0.50	0.14	0.27	0.32	0.43
5y	0.46	0.15	0.24	0.30	0.40

NOTE: This table reports the R^2 for 1-year ahead 1-year holding period excess bond returns from different models. The columns MY and OY refer to the model-implied expected excess bond returns from the macro-yields model (MY) and the only-yields model (OY), respectively. The columns CP, LN, and CP+LN refer to the predictive regression using the CP factor, the LN factor, and both the CP and the LN factors jointly.

- The macro-yields model explains 46%–55% of the variation of 1-year excess returns, while the only-yields model can explain only the 12%–15% of the variation of the 1-year ahead excess returns
- The model outperforms all benchmarks, Cochrane-Piazzesi tent factor and Ludvigson-Ng (2009, RFS) PC factors, even combined
- Unspanning restrictions are also tested by testing the restrictions $\Gamma_{yx} = O$ (do not explain) and $A_{yx} \neq O$ (predict) using a LR test:

$$\text{LR} = 2 \times (L_u - L_r)$$

Imposing unspanning restrictions

In-Sample Results

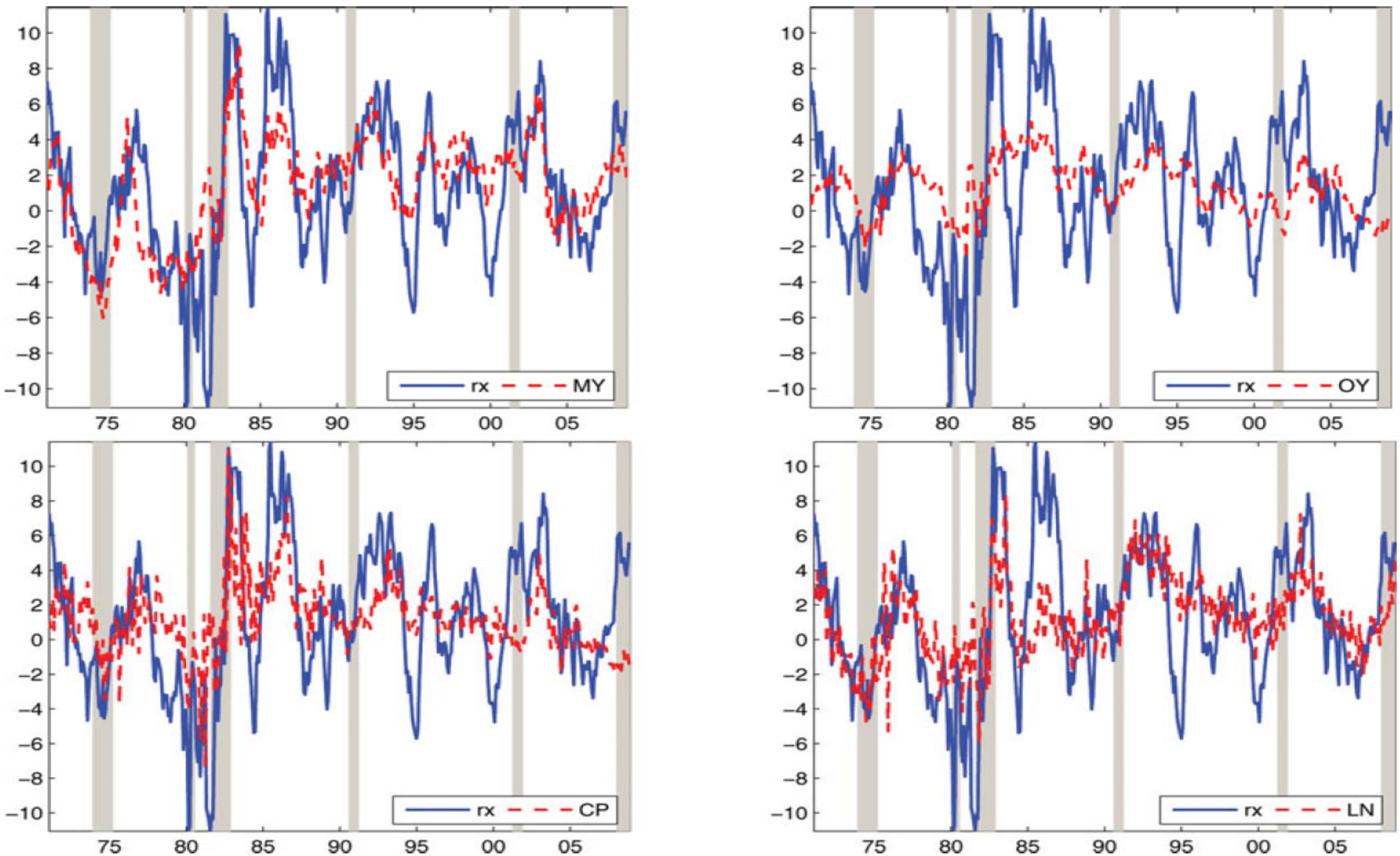


Figure 3. Average 1-year holding period excess return: realized and predicted. This figure displays the average excess return $\overline{rx}_{t+12}$ (blue continuous line) and the corresponding predicted values from different models (dashed red line). The dashed red line in the top plots refer to the model-implied predicted values from the macro-yields MY model (top right) and only-yields OY model (top left). The dashed red line in the bottom plots refer to the predicted values from the predictive regressions using the CP factor (bottom left) and the LN factor (bottom right). The gray-shaded areas indicate the recessions as defined by the NBER.

In-Sample Results

H_0	Test statistic	p-value
$\Gamma_{yx} = 0$	12.85	0.38
$A_{yx} = 0$	79.03	0.00

NOTE: This table reports the likelihood ratio test statistic for the unspanning restrictions and the corresponding p-values, computed using a chi-squared distribution with degrees of freedom equal to the number of restrictions tested. The first line refers to the null hypotheses $\Gamma_{yx} = 0$ in Equation (4) while the second line refers to the null hypotheses $A_{yx} = 0$ in Equation (5).

- Cannot reject the null of factor loadings of the yields on the macro factors equal to zero
- Reject the null of no Granger causality from the macro factors to the yield curve factors
- The result of the LR tests show that the macro factors identified by the model do not explain the cross-section of yields but have predictive ability for the future evolution of the yield curve
- Next, the paper performs recursive yield and excess return forecasting on a sample January 1990 - December 2008
- Prediction accuracy is assessed using MSFE relative to random walk:

$$\mathrm{rMSFE}_{t_0}^{t_1}(\tau, h, M) = \frac{\mathrm{MSFE}_{t_0}^{t_1}(\tau, h, M)}{\mathrm{MSFE}_{t_0}^{t_1}(\tau, h, RW)}$$

- The macro-yields model outperforms the only-yields model for all but 1-month horizon

Out-of-Sample Results: Yields

- The macro-yields model outperforms the RWat 3-, 6-, 12-, and 24-month ahead for all the maturities, with a significant outperforman-ce, according to the White (2000) reality check test, for the 12- and 24-month ahead forecasts.

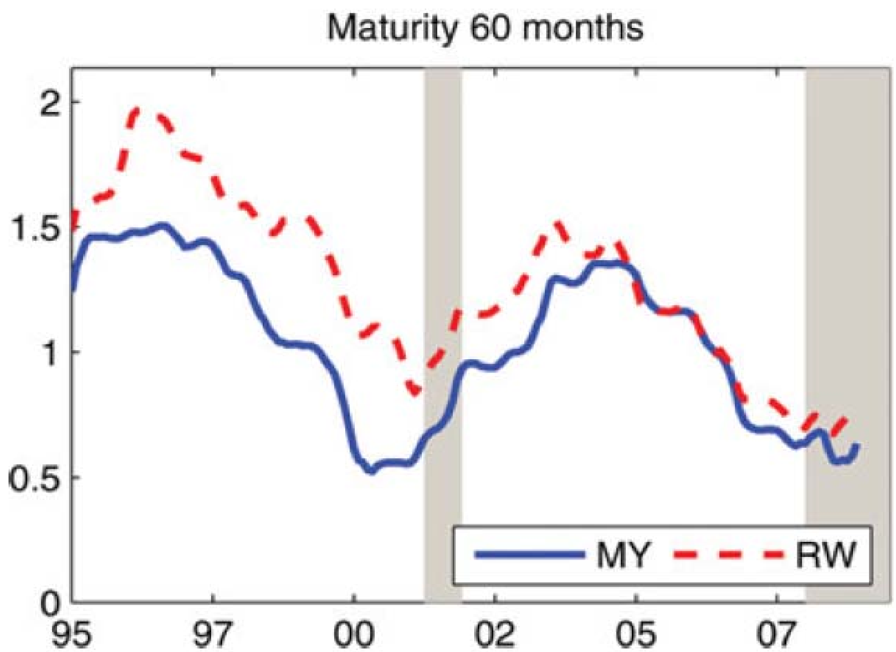


Table 6. Out-of-sample performance for yields

Macro-yields						
Maturity	3m	1y	2y	3y	4y	5y
$h = 1$	1.17	1.05	1.06	1.00	1.05	1.14
$h = 3$	0.79*	0.93	0.99	0.96	0.99	1.02
$h = 6$	0.78**	0.89	0.94	0.93	0.93	0.94
$h = 12$	0.69**	0.74**	0.79**	0.80***	0.80***	0.80***
$h = 24$	0.62***	0.66***	0.74**	0.82**	0.88*	0.97
Only-yields						
Maturity	3m	1y	2y	3y	4y	5y
$h = 1$	0.93	1.09	1.17	1.11	1.07	1.11
$h = 3$	0.96	1.13	1.20	1.14	1.10	1.13
$h = 6$	0.99	1.18	1.25	1.21	1.15	1.16
$h = 12$	1.04	1.16	1.26	1.27	1.25	1.26
$h = 24$	1.06	1.12	1.27	1.39	1.49	1.62

NOTE: This table reports the relative MSFE of the macro-yields model and the only-yields model over the MSFE of the random walk for multi-step predictions of the yields. The first column reports the forecast horizon h. The sample starts on January 1970 and the evaluation period is January 1990 to December 2008. *, **, and *** denote significant outperformance at 10%, 5%, and 1% level with respect to the random walk according to the White (2000) reality check test with 1000 bootstrap replications using an average block size of 12 observations.

⇦ 12-months ahead smoothed squared forecast errors for yields.

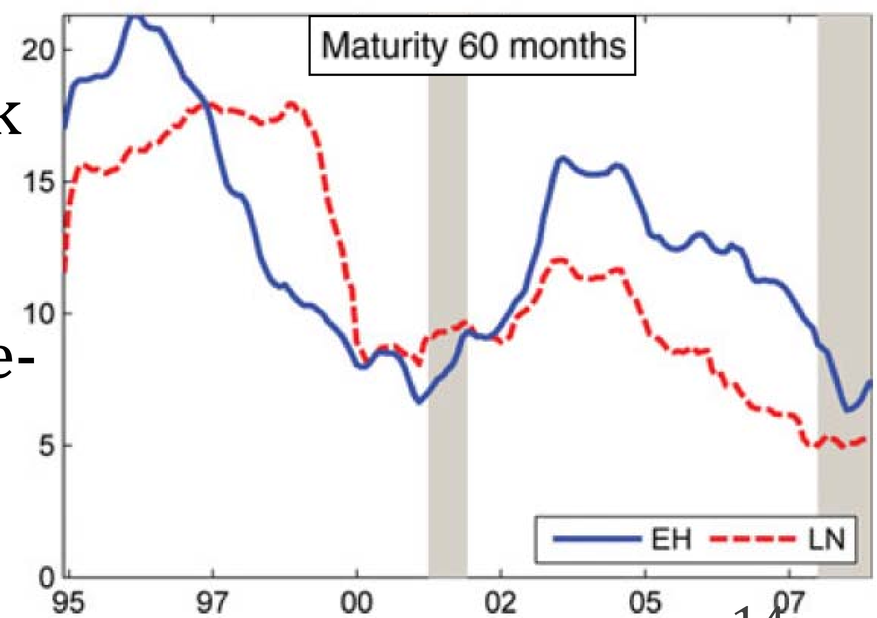
Out-of-Sample Results

- Unspanned macro factors, while not important for explaining the contemporaneous variation of the yields curve, contain useful information to predict the future yields
- In the case of excess return forecasts, the benchmark is a constant ⇒ excess returns are unpredictable, as under the expectations hypothesis

Table 7. Out-of-sample predictive performance for excess returns

Maturity	MY	OY	CP	LN	LN+CP
2y	0.76**	1.20	1.17	0.80	0.80
3y	0.75**	1.20	1.21	0.79	0.83
4y	0.74**	1.18	1.21	0.78	0.83
5y	0.75**	1.18	1.18	0.81	0.83

- The macro-yields model outperforms the constant excess return benchmark for all maturities and the outperformance is significant
- Among the benchmarks, the Cochrane-Piazzesi factors fails to forecast OOS, while LN's factors do, but they fail to outperform recursive sample mean



Monetary policy under regimes and the yield curve

- Sims and Zha (2006, AER) model regime changes in monetary policy (MP) and in the volatility of its shocks and find that regimes affected the economy via the changing shocks to private sector
- Bibkov and Chernov (2013, JoE) argue that MP regimes may not be estimated precisely unless we use information from the cross-section of yields, which by its nature is forward-looking
- In a no-arbitrage linear affine model, BC find that US MP can be characterized as switching between active and passive regimes, judging by differential response of short rate to expected inflation
- Their model has structural, New-Keynesian features, they explicitly posit a MP reaction function and the dynamics of the economy
  - However, their specification is silent about investors' preferences

$$g_t = m_g + (1-\mu_g)g_{t-1} + \mu_g E_t g_{t+1} - \phi(r_t - E_t \pi_{t+1}) + \sigma_g(s_t^e)\varepsilon_t^g \qquad \text{(IS)}$$

$$\pi_t = m_\pi + (1-\mu_\pi)\pi_{t-1} + \mu_\pi E_t \pi_{t+1} + \delta g_t + \sigma_\pi(s_t^e)\varepsilon_t^\pi \qquad \text{(AD)}$$

$$r_t = m_r(s_t^m) + \left(1-\rho(s_t^m)\right)\left[\alpha(s_t^m)E_t\pi_{t+1} + \beta(s_t^m)g_t\right] + \rho(s_t^m)r_{t-1} + \sigma_r(s_t^d)\varepsilon_t^r . \qquad \text{(forward-looking Taylor rule)}$$

Monetary policy under regimes and the yield curve

- Model avoids latent factors and rely only on three observable variables: inflation, output, and the short interest rate
- All regimes and diffusive shocks are assumed to be independent, also to achieve identification; they command no risk premium
- Unfortunately, regimes are assumed to be observable, although they remain only partially predictable
- They follow a standard Markov chain process with constant transition probabilities
- MP responds to expected future inflation and current output
- The private sector parameters are not regime-dependent
  - Because private sector expectations of future state variables are conditional upon the realization of the regime, the reduced-form representation of dynamics of the private sector will be regime-dependent
- They numerically characterize the rational expectations solution in which the parameters are nonlinear function of the primitive ones:

$$x_t = \mu(S_t) + \Phi(S_t)x_{t-1} + \Sigma(S_t)\varepsilon_t \qquad \varepsilon_t = (\varepsilon_t^g, \varepsilon_t^\pi, \varepsilon_t^r)'$$

Monetary policy under regimes and the yield curve

- The SDF is exogeneously specified as:

Function of deep parameters

$$\log M_{t,t+1} = -r_t - \frac{1}{2}\Lambda'_{t,t+1}\Lambda_{t,t+1} - \Lambda'_{t,t+1}\varepsilon_{t+1} \quad \Lambda_{t,t+1} = \Sigma'(S_{t+1})\Pi(x_t)$$

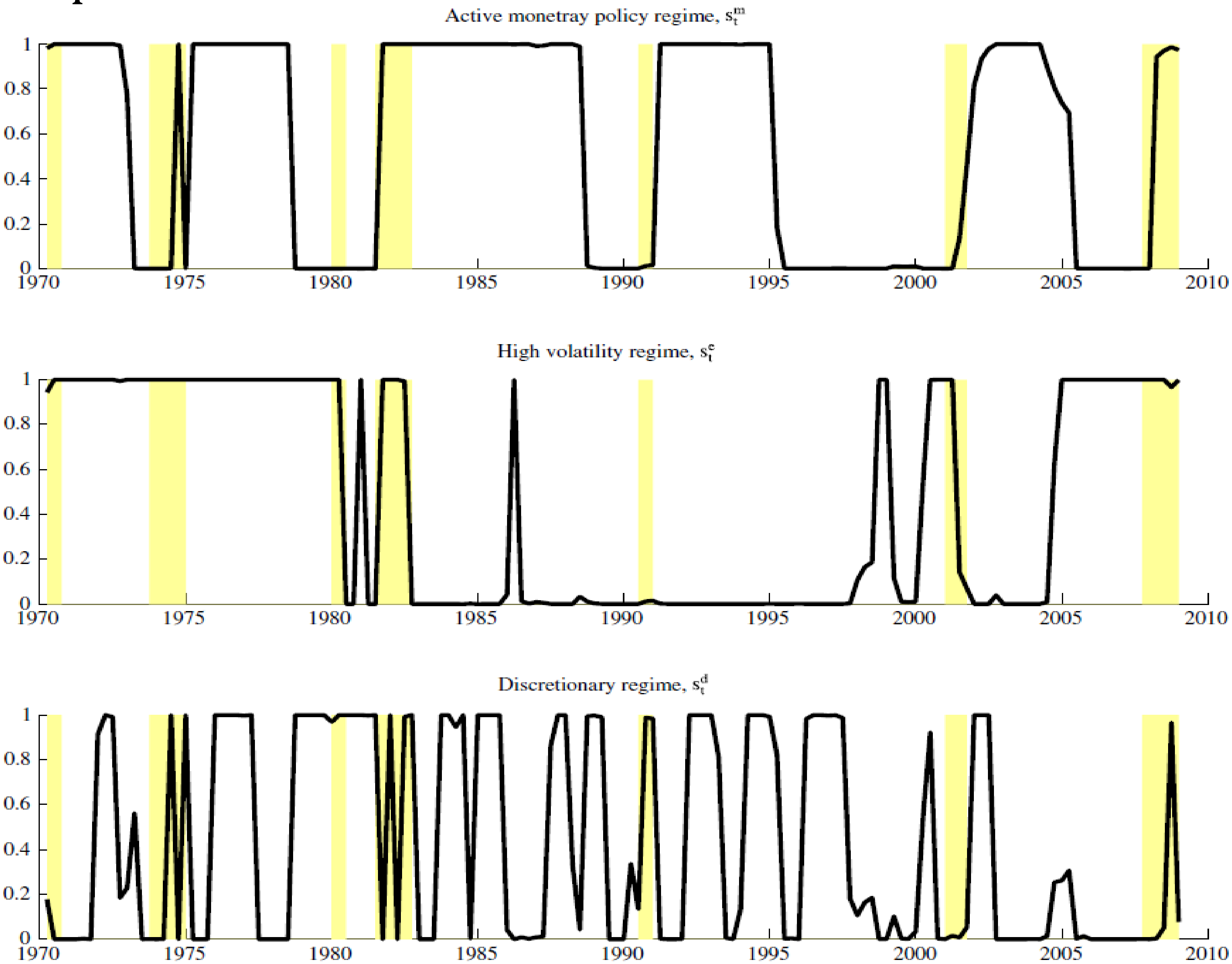
- Following Duffee (2002, JF) and Dai et al. (2007, RFS), they assume an essentially affine structure of "preferences" $\Pi(x_t)$:

$$\Pi(x_t) = \Pi_0 + \Pi_x x_t$$

- They allow for 3 regime variables, for a minimum of 8 states:
  - The first shifts volatilities of exogenous inflation and output shocks
  - The second switches the parameters in the MP reaction function modeled as a forward-looking interest rate rule
  - The third affects the volatility of the MP shock
- The model is estimated by MLE, with concerns for identification
- Using quarterly 1970-2008 US data, results indicate the presence of 2 regimes for volatilities of inflation and output shocks, two for MP policy, and 2 for volatility of MP shock, for a total of $2^3 = 8$
  - The two MP regimes are distinguished by how the Fed reacts to expected inflation, aggressive (the Volcker's period) vs. "passive"

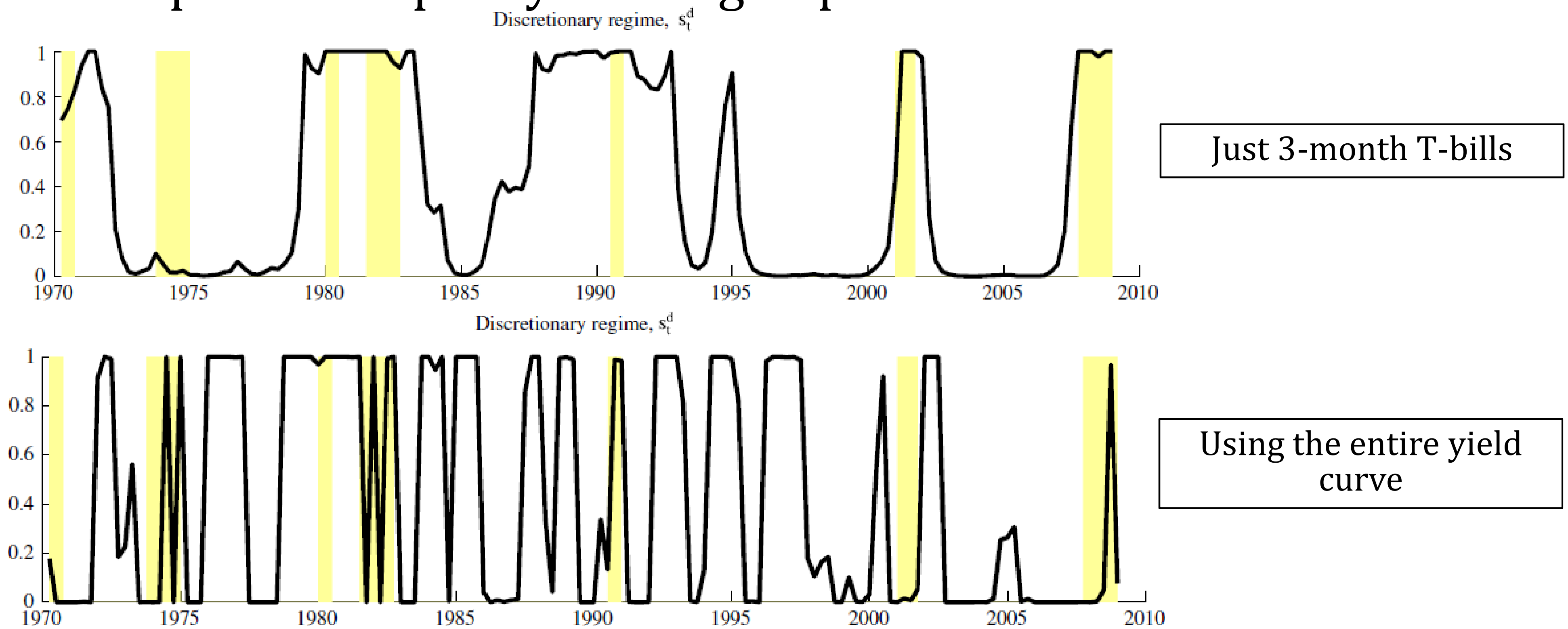
Monetary policy under regimes and the yield curve

- The high and low regimes for the volatility of monetary shock are interpreted as “discretion” and “commitment”



Monetary policy under regimes and the yield curve

- Intuitively, the yield curve contains information about expected future interest rates, which, in particular, reflect the probabilities that a particular policy is being implemented



- A simulation study suggests that using the yield curve reduces the bias of the estimated monetary policy regime by a factor of 20
- By simulating counterfactual economies, they show that a permanent transition from high to low volatility of exogenous shocks to output and inflation made a large contribution to great moderation

Monetary policy under regimes and the yield curve

- However, this is an incomplete explanation of the real economy's improvement over the last two decades
- As the inflation realized in the post-1982 sample is, on average, lower and less volatile than inflation in any of the individual regimes, the changing MP policy contributed to the great moderation in addition to the "lucky" low-volatility exogenous shocks
  - Different from literature, Ang, Boivin, Dong, and Loo-Kung (2011, QJE) investigate time-varying policy in the context of a fully fledged term structure model but do not allow for changing volatilities of shocks
  - Amisano and Tristani (2010) developed a DSGE term-structure model that contains regime shifts, allows for changes in volatility of shocks, but not in the policy response to expected inflation and output
- However the forecasting performance of the models is not investigated, while for yields it would be natural
- The point of the model seems to be that estimation of macro regimes is helped by asset prices, but the opposite remains an open issue