Università Commerciale
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Lecture 3: Multivariate GARCH and Conditional Correlation Models

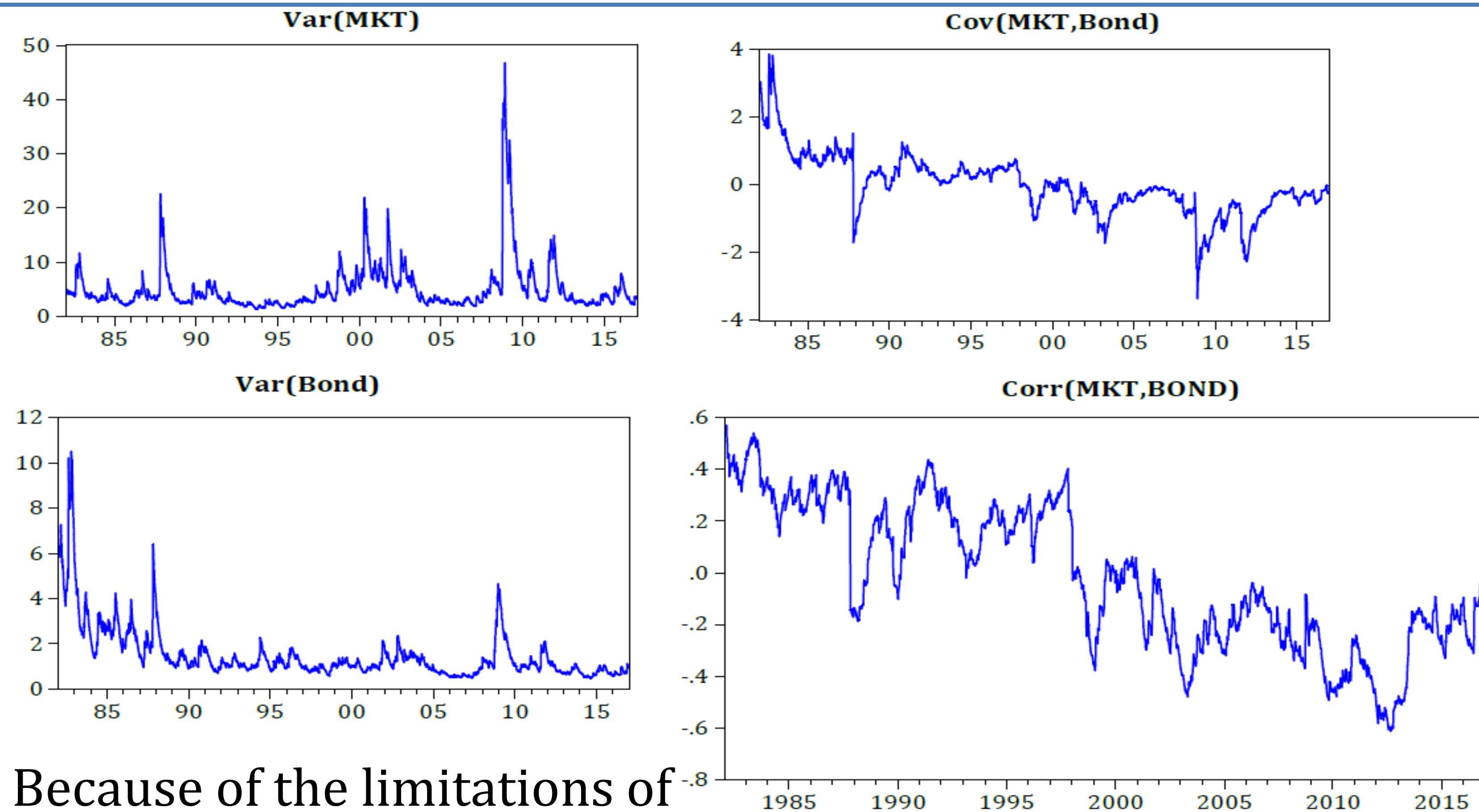
20541– Advanced Quantitative Methods for Asset Pricing and Structuring

Spring 2020

Overview

- Three issues in multivariate modelling of CH covariances
- Naïve models that extend univariate models to covariances
- Full Multivariate GARCH, part 1: VECH GARCH
- Full Multivariate GARCH, part 2: BEKK GARCH
- Constant Conditional Correlations (CCC) models
- Dynamic Conditional Correlations (DCC) models
- Hints to Estimation and Inference

BEKK (Baba-Engle-Kraft-Kroner) GARCH



Stock-bond corr under VECH GARCH

- Because of the limitations of VECH, during the 1990s, one multivariate GARCH model surged to popularity, Engle and Kroner's (1995) BEKK (p, q):

$$\Sigma_{t+1|t} = \mathbf{CC}' + \sum_{i=1}^{p} \mathbf{A}_i(\varepsilon_{t+1-i}\,\varepsilon'_{t+1-i})\mathbf{A}_i' + \sum_{j=1}^{q} \mathbf{B}_j \Sigma_{t+1-j|t-j} \mathbf{B}_j'$$

Non-negative NxN matrices

BEKK (Baba-Engle-Kraft-Kroner) GARCH

- The special "sandwich" structure of the coefficient matrices guarantees that $\boldsymbol{\Sigma}_{t+1|t}$ is (semi-)PD without imposing other restrictions
  - The popular BEKK that many empiricists have come to appreciate is a simpler (1,1) diagonal BEKK that restricts the matrices **A** and **B**
- BEKK models possess three attractive properties:

① When symmetry of **A** and **B** is imposed, a BEKK is a truncated, low-dimensional application of a theorem by which all nonnegative, symmetric *N*x*N* matrices (say, **M**) can be decomposed as:

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \sum_{k=1}^{2N} \begin{bmatrix} \mathbf{m}'_{k,1}\mathbf{m}_{k,1} & \mathbf{m}'_{k,1}\mathbf{m}_{k,2} \\ \mathbf{m}'_{k,2}\mathbf{m}_{k,1} & \mathbf{m}'_{k,2}\mathbf{m}_{k,2} \end{bmatrix}$$

for appropriately selected vectors $\mathbf{m}_{k,j}$

② BEKK ensures (S)PD-ness of $\boldsymbol{\Sigma}_{t+1|t}$, because by construction, the sandwich form and outer vector products have this property

③ BEKK is invariant to linear combinations, i.e., if $\mathbf{R}_{t+1}$ follows a BEKK GARCH(*p*, *q*), then any ptf. of the *N* assets in $\mathbf{R}_{t+1}$ will also follow a BEKK, see lecture notes for examples and counterexamples under VECH ARCH

- However, the number of parameters in BEKK remains rather large

Stock-Bond Correlation under BEKK

- In the application to US stock and bond returns, BIC leads to select a t-Student diagonal BEKK(1,1):

t-Student parameter v

$$MKT_{t+1} = \underset{(0.000)}{0.582} + \underset{(0.001)}{0.103} Bond_t + \varepsilon_{MKT,t+1}$$

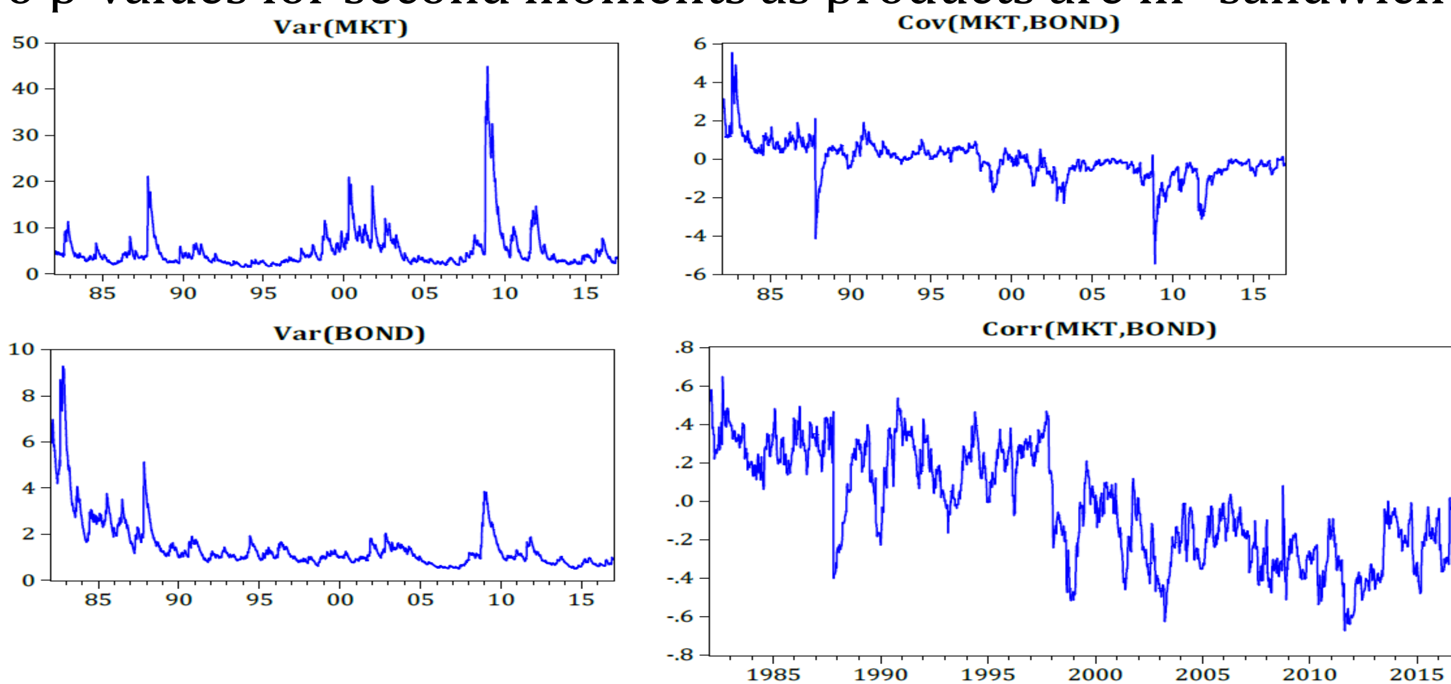
$$\sigma^2_{MKT,t+1|t} = 0.080 + 0.068\varepsilon^2_{MKT,t} + 0.915\sigma^2_{MKT,t|t-1}$$

$$Bond_{t+1} = -\underset{(0.935)}{0.002} + \underset{(0.000)}{0.242} Bond_t + \varepsilon_{Bond,t+1} \quad [\varepsilon_{MKT,t+1} \;\; \varepsilon_{Bond,t+1}]' \text{ IID } Mt(0, \mathbf{\Sigma}_{t+1|t}; \underset{(0.000)}{9.148})$$

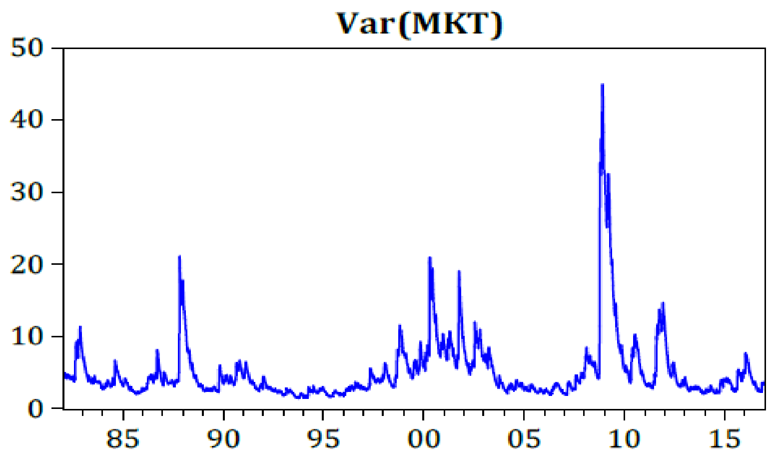
$$\sigma^2_{Bond,t+1|t} = 0.014 + 0.044\varepsilon^2_{Bond,t} + 0.945\sigma^2_{Bond,t|t-1}$$

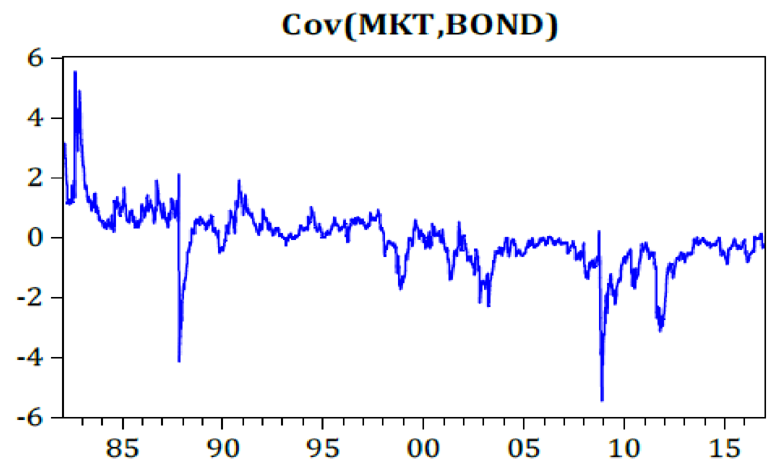
$$\sigma_{MKT-Bond,t+1|t} = -0.002 + 0.055\varepsilon_{MKT,t}\varepsilon_{Bond,t} + 0.930\sigma_{MKT-Bond,t|t-1}.$$

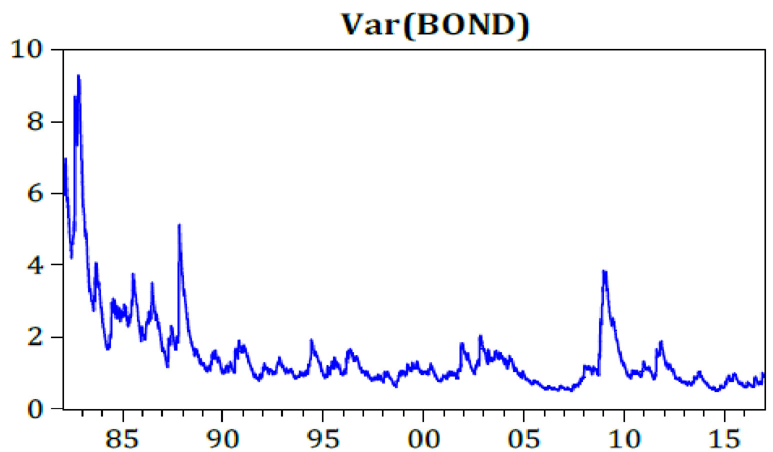
- No p-values for second moments as products are in "sandwich form"

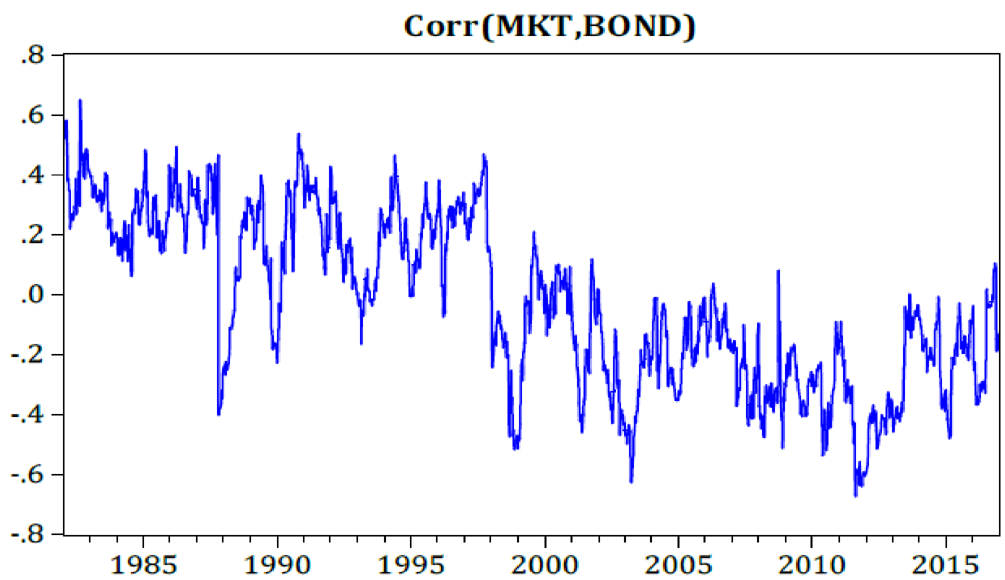


Stock-Bond Correlation under BEKK









Conditional Correlation (DCC and CCC) Models

- Stationarity and moment convergence criteria for multi-GARCH are complex and explicit results are only available for a few special cases

- In the practice of risk and asset management, most chattering is about time-varying correlations and not covariances
  - E.g., to debate evidence that correlations increase during crises
- On the one hand, as already exploited in many examples, obvious that given any type of model to forecast covariances and variances, one can compute the implied dynamic (prediction of) correlation:

$$\hat{\rho}_{ij,t+1|t} = \hat{\sigma}_{ij,t+1|t} / (\hat{\sigma}_{i,t+1|t} \hat{\sigma}_{j,t+1|t}), \forall i, j = 1, \ldots, N$$

- On the other hand, fruitful to directly model correlations although there is one obvious problem, to forecast $\rho_{ij,t+1|t} \in [-1,\ 1]$
- Any dynamic estimator implies a need to constrain parameters
- Engle (2002) offers a nifty trick: appealing to model an appropriate auxiliary variable, $q_{ij,t+1}$, than correlations directly
  - An auxiliary variable is a by-product of modeling and estimation that has no direct meaning but that can be used in subsequent steps

Conditional Correlation (DCC and CCC) Models

- DCC approach is based on a generalization of the standard result that $\sigma_{ij,t+1|t} = \sigma_{i,t+1|t}\rho_{ij,t+1|t}\sigma_{j,t+1|t}$ to matrices:

$$\mathbf{\Sigma}_{t+1|t} \equiv \mathbf{D}_{t+1}\mathbf{\Gamma}_{t+1|t}\mathbf{D}_{t+1}$$

- Here $\boldsymbol{D}_{t+1}$ is an NxN matrix of predicted standard deviations, $\sigma_{i,t+1}$, the ith element of the diagonal and 0 everywhere else
- $\mathbf{\Gamma}_{t+1|t}$ is a matrix of predicted correlations, $\rho_{ij,t+1/t}$ with 1s on its main diagonal, for instance:

$$\mathbf{\Sigma}_{t+1|t} \equiv \begin{bmatrix} \sigma^2_{1,t+1|t} & \sigma_{12,t+1|t} \\ \sigma_{12,t+1|t} & \sigma^2_{2,t+1|t} \end{bmatrix} = \begin{bmatrix} \sigma_{1,t+1} & 0 \\ 0 & \sigma_{2,t+1} \end{bmatrix} \begin{bmatrix} 1 & \rho_{12,t+1|t} \\ \rho_{12,t+1|t} & 1 \end{bmatrix} \begin{bmatrix} \sigma_{1,t+1} & 0 \\ 0 & \sigma_{2,t+1} \end{bmatrix}$$

- The key step of the DCC approach is based on the ability to **disentangle the estimation and prediction of $\boldsymbol{D}_{t+1}$ from the estimation and prediction of $\mathbf{\Gamma}_{t+1|t}$**; we proceed in two steps:

① The volatility of each asset is estimated/predicted through a GARCH or one of the other methods considered in lectures 6-7

- E.g., NAGARCH(p, q) for some assets and GARCH(1,1) for others

Conditional Correlation (DCC and CCC) Models

- Homoskedasticity (constant variance is not ruled out)

② Model the conditional covariances of the resulting standardized residuals, $z_{i,t+1}$ ⍰ ⍰$_{i,t+1}/\sigma_{i,t+1|t}$, derived from the first-step GARCH

- We exploit the fact that the conditional covariance of the standardized residuals equals the conditional correlation of original residuals:

$$Cov_t\left[z_{i,t+1},\ z_{j,t+1}\right] = Cov_t\left[\frac{\varepsilon_{i,t+1}}{\sigma_{i,t+1|t}},\ \frac{\varepsilon_{j,t+1}}{\sigma_{j,t+1|t}}\right] = \frac{Cov_t[\varepsilon_{i,t+1},\ \varepsilon_{j,t+1}]}{\sigma_{i,t+1|t}\sigma_{j,t+1|t}} = \rho_{ij,t+1|t}$$

- GARCH-type modeling in the second step will not directly concern the covariance of stdzed residuals, but an auxiliary variable $q_{ij,\mathrm{t+1}}$

Conditional Correlation (DCC and CCC) Models

- Typically, the most popular model used in this second DCC step is:

$$q_{ij,t+1} = \bar{\rho}_{ij} + \alpha(z_{i,t}z_{j,t} - \bar{\rho}_{ij}) + \beta(q_{ij,t} - \bar{\rho}_{ij}) \quad \forall i,j$$

a GARCH(1,1) for the auxiliary variable, written in deviations from the unconditional, long-run mean correlation, $\bar{\rho}_{ij} = \omega_{ij}/(1-\alpha-\beta)$

Common across assets

- An alternative is a RiskMetrics-type model:

$$q_{ij,t+1} = (1-\lambda)z_{i,t}z_{j,t} + \lambda q_{ij,t} \quad \forall i,\ j \qquad \lambda \in (0,1)$$

  - Complex, asymmetric or nonlinear (e.g., power) GARCH are possible
  - These models apply to all pairs of assets even when $i = j$
- To go from a forecast of auxiliary variable $q_{ij,t+1}$ to correlations use:

$$\rho_{ij,t+1|t} = \frac{q_{ij,t+1}}{\sqrt{q_{ii,t+1}}\sqrt{q_{jj,t+1}}}$$

- This transformation guarantees that $\rho_{ij,t+1|t} \in [-1,\ 1]$

Conditional Correlation (DCC and CCC) Models

- The RiskMetrics/DCC and GARCH/DCC models can be written in matrix form as:

$$\mathbf{Q}_{t+1} = (1-\lambda)\mathbf{z}_t\mathbf{z}_t' + \lambda\mathbf{Q}_t$$
$$\mathbf{Q}_{t+1} = \mathbf{\Pi} + \alpha\mathbf{z}_t\mathbf{z}_t' + \beta\mathbf{Q}_t.$$

$$\mathbf{z}_t \equiv \begin{bmatrix} z_{1,t+1} & z_{2,t+1} & \cdots & z_{N,t+1} \end{bmatrix}'$$

- $\mathbf{Q}_{t+1}$ is an NxN symmetric (S)PD matrix that collects the values/predictions of the auxiliary variables $q_{ij,t+1}$:
- $\mathbf{Q}_{t+1}$ is (S)PD because it is a weighted average of (S)PD and positive definite matrices

$$\mathbf{Q}_{t+1} \equiv \begin{bmatrix} q_{11,t+1} & q_{12,t+1} & \cdots & q_{1N,t+1} \\ q_{12,t+1} & q_{22,t+1} & \cdots & q_{2N,t+1} \\ \vdots & \vdots & \ddots & \vdots \\ q_{1N,t+1} & q_{2N,t+1} & \cdots & q_{NN,t+1} \end{bmatrix}$$

- This will ensure that the correlation matrix $\mathbf{\Gamma}_{t+1|t}$ and the covariance matrix, $\mathbf{\Sigma}_{t+1|t}$ will be (S)PD
- When $i = j$, $q_{ij,t+1}$ in general differs from $\sigma^2_{i,t+1}$ from the first step—this represents the "approximation burden" of two-step DCC estimation

▪ Covariance stationarity of all the GARCH processes that "populate" $\boldsymbol{D}_{t+1}$ along with covariance stationarity of the (matrix) process for $\boldsymbol{Q}_{t+1}$ are sufficient for a DCC model to be weakly stationary and for unconditional variances, covariances, and correlations to exist

Stock-Bond Correlation under RiskMetrics DCC

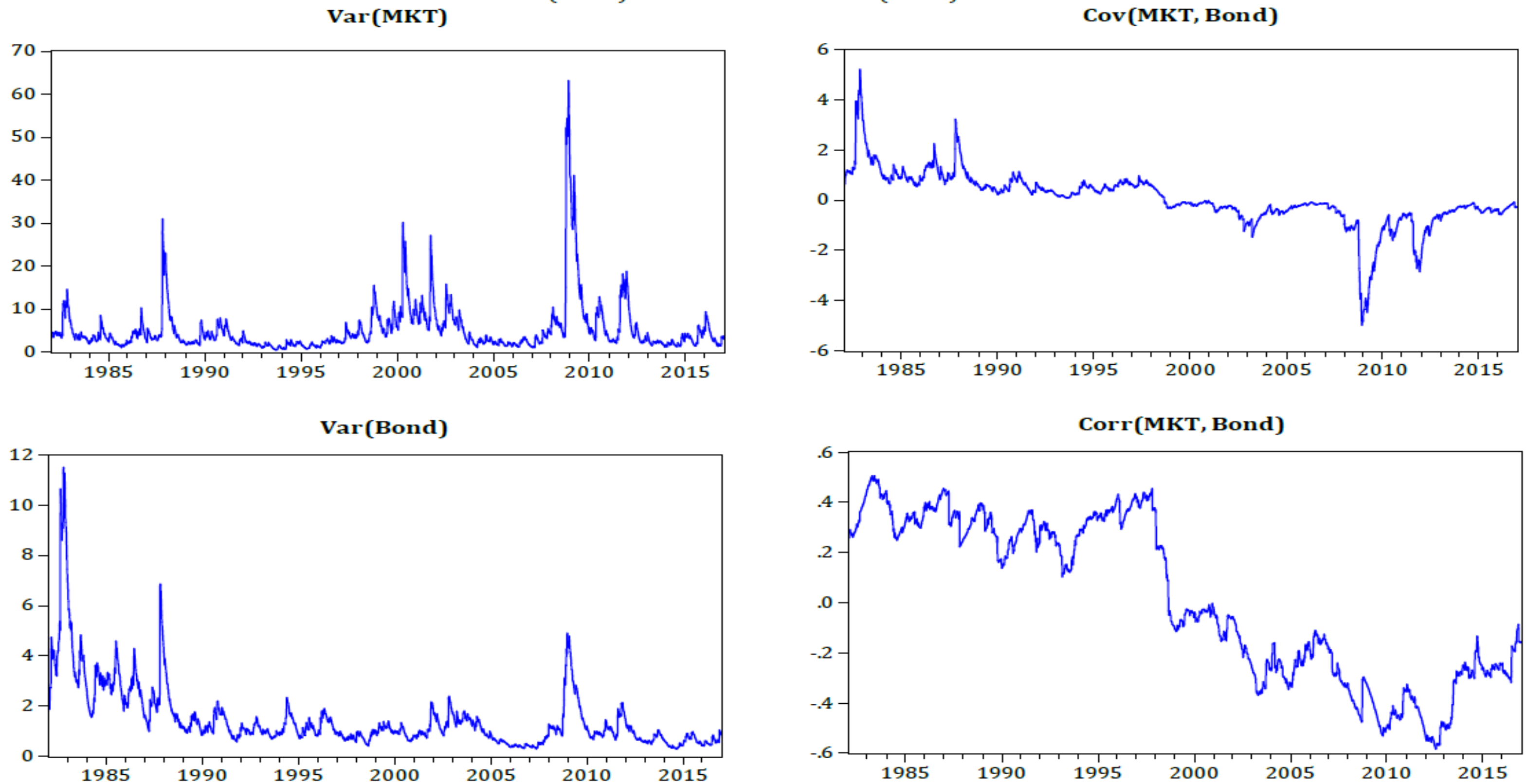
$$MKT_{t+1} = \underset{(0.000)}{0.582} + \underset{(0.008)}{0.094} Bond_t + \varepsilon_{MKT,t+1}$$

$$\sigma^2_{MKT,t+1|t} = \underset{(0.000)}{0.112} \varepsilon^2_{MKT,t} + \underset{(0.000)}{0.888} \sigma^2_{MKT,t|t-1}$$

$$Bond_{t+1} = \underset{(0.169)}{0.032} + \underset{(0.000)}{0.230} Bond_t + \varepsilon_{Bond,t+1} \quad [\varepsilon_{MKT,t+1} \;\; \varepsilon_{Bond,t+1}]' \text{ IID } N(\mathbf{0}, \mathbf{I}_2)$$

$$\sigma^2_{Bond,t+1|t} = \underset{(0.000)}{0.066} \varepsilon^2_{Bond,t} + \underset{(0.000)}{0.934} \sigma^2_{Bond,t|t-1}.$$

$$q_{MKT-Bond,t+1} = \underset{(0.000)}{0.014} \hat{z}^{RiskM}_{MKT,t} \hat{z}^{RiskM}_{Bond,t} + \underset{(0.000)}{0.986} q_{MKT-Bond,t}$$



Stock-Bond Correlation under TARCH(1,1,1) DCC

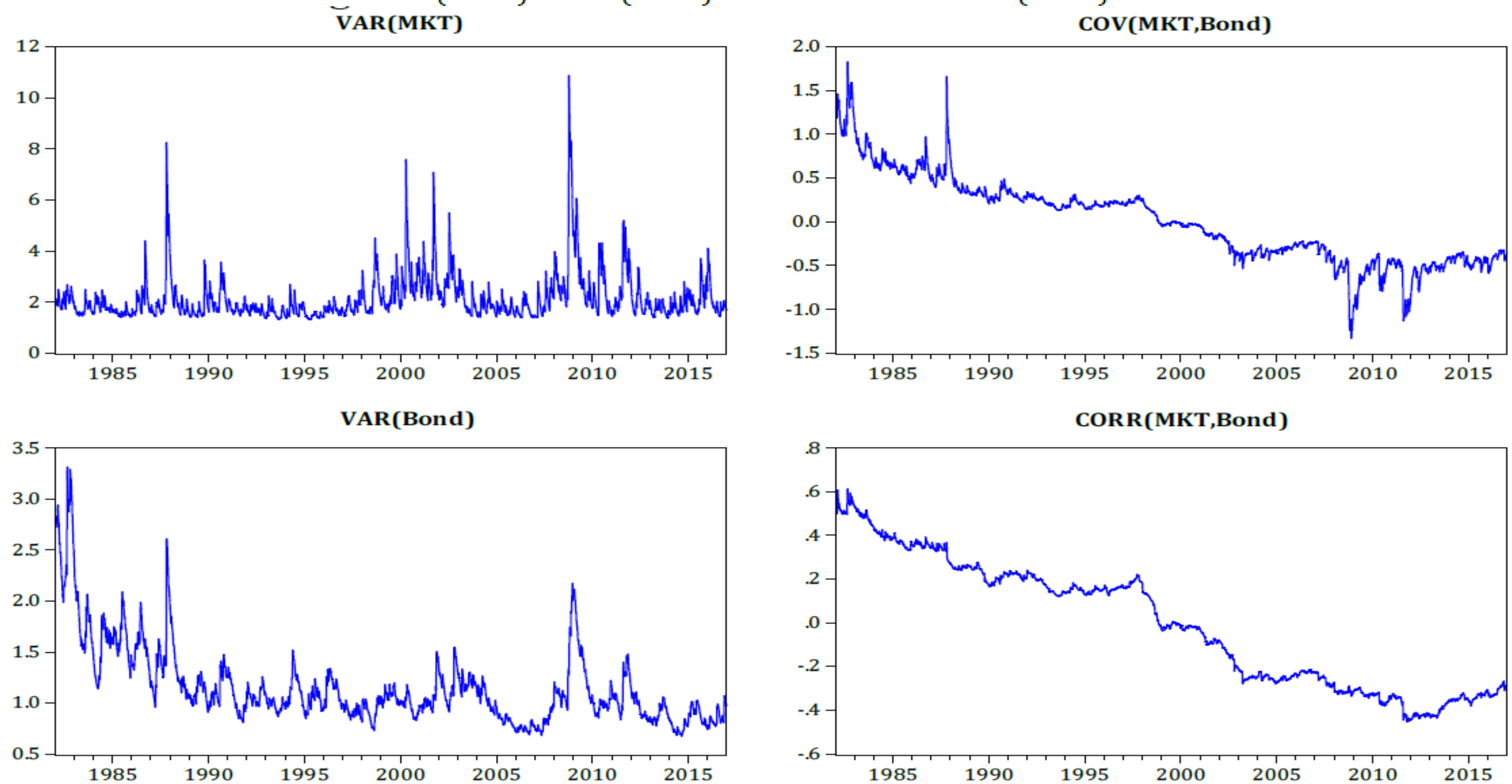
$$MKT_{t+1} = \underset{(0.000)}{0.507} + \underset{(0.000)}{0.125} Bond_t + \varepsilon_{MKT,t+1}$$

$$\sigma^2_{MKT,t+1|t} = \underset{(0.000)}{0.382} + \underset{(0.009)}{0.043} \varepsilon^2_{MKT,t} + \underset{(0.000)}{0.229} I_{\{\varepsilon_{MKT,t}<0\}} \varepsilon^2_{MKT,t} + \underset{(0.000)}{0.764} \sigma^2_{MKT,t|t-1}$$

$$Bond_{t+1} = -\underset{(0.707)}{0.000} + \underset{(0.000)}{0.226} Bond_t + \underset{(0.018)}{0.053} Bond_{t-1} + \varepsilon_{Bond,t+1} \quad [\varepsilon_{MKT,t+1} \varepsilon_{Bond,t+1}]' \text{IID } N(\mathbf{0}, \mathbf{I}_2)$$

$$\sigma^2_{Bond,t+1|t} = \underset{(0.003)}{0.020} + \underset{(0.000)}{0.065} \varepsilon^2_{Bond,t} + \underset{(0.000)}{0.919} \sigma^2_{Bond,t|t-1}.$$

$$q_{MKT-Bond,t+1} = \underset{(0.016)}{0.0001} + \underset{(0.000)}{0.009} \hat{z}^{GARCH}_{MKT,t} \hat{z}^{GARCH}_{Bond,t} + \underset{(0.000)}{0.986} q_{MKT-Bond,t}$$



Estimation and Inference

- One restricted case of DCC had appeared already in 1990: Bollerslev's constant conditional correlation (CCC) multivariate model, which is a DCC with constant correlation matrix, $\boldsymbol{\Sigma}_{t+1} \equiv \mathbf{D}_{t+1}\boldsymbol{\Gamma}\mathbf{D}_{t+1}$
  - This model simply avoids defining and modeling with GARCH-type processes the $q_{ij,t+1}$ auxiliary variable
- Multivariate GARCH and DCC estimation applies (Q)ML to jointly estimate the parameters of (conditional) mean variance equations
  - E.g., in the multivariate normal case, the log-likelihood contributions (i.e., the PDF values for each of the sample observations) is:

$$\ell(\mathbf{R}_{t+1};\ \boldsymbol{\theta}) \equiv -\frac{1}{2}N\ln(2\pi) - \frac{1}{2}\ln\det\boldsymbol{\Sigma}_{t+1}(\boldsymbol{\theta}) - \frac{1}{2}\boldsymbol{\varepsilon}'_{t+1}\boldsymbol{\Sigma}^{-1}_{t+1}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_{t+1}$$

  - The asymptotic properties of ML (and QML) estimators in multivariate GARCH models are not yet firmly established: while consistency has been proven, asymptotic normality of the QMLE is not established
  - DCC and CCC models are estimated by QMLE by construction: because the model is implemented in three different steps, even though in each of these stages a log-likelihood function is written and maximized
  - QML efficiency loss derives from treating $z_{i,t+1}$ as data and not estimates

Appendix A: Positive Definiteness in VECH ARCH(1)

Consider the case of two assets ($N=2$) and a simple diagonal multivariate ARCH(1) model,

$$vech(\boldsymbol{\Sigma}_{t+1|t}) = (\mathbf{I}_3 - \mathbf{A})vech\left(T^{-1}\sum_{t=1}^{T}\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\right) + \mathbf{A}vech(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'),$$

where the covariance targeting restriction has already been imposed and $\mathbf{A}$ is a diagonal matrix. Because we have set $N=2$, $\boldsymbol{\Sigma}_{t+1|t}$ will be a 2×2 matrix, $\mathbf{A}$ is a 3×3 matrix, $vech(\boldsymbol{\Sigma}_{t+1|t})$ is a 3×1 vector of unique elements from $\boldsymbol{\Sigma}_{t+1|t}$,and $vech\left(T^{-1}\sum_{t=1}^{T}\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\right)$ is a 3×1 vector of unique elements from the sum of cross-product matrices $\sum_{t=1}^{T}\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'$. The number of coefficients to be estimated is of course 3, a^{11}, a^{22}, and a^{33} in the representation:

$$\begin{bmatrix}\sigma^2_{1,t+1|t}\\ \sigma_{12,t+1|t}\\ \sigma^2_{2,t+1|t}\end{bmatrix} = \left(\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} - \begin{bmatrix}a^{11}&0&0\\0&a^{22}&0\\0&0&a^{33}\end{bmatrix}\right)\begin{bmatrix}T^{-1}\sum_{t=1}^{T}\varepsilon^2_{1,t}\\ T^{-1}\sum_{t=1}^{T}\varepsilon_{1,t}\varepsilon_{2,t}\\ T^{-1}\sum_{t=1}^{T}\varepsilon^2_{2,t}\end{bmatrix}.$$

$$+\begin{bmatrix}a^{11}&0&0\\0&a^{22}&0\\0&0&a^{33}\end{bmatrix}\begin{bmatrix}\varepsilon^2_{1,t}\\ \varepsilon_{1,t}\varepsilon_{2,t}\\ \varepsilon^2_{2,t}\end{bmatrix} = \begin{bmatrix}(1-a^{11})T^{-1}\sum_{t=1}^{T}\varepsilon^2_{1,t} + a^{11}\varepsilon^2_{1,t}\\ (1-a^{22})T^{-1}\sum_{t=1}^{T}\varepsilon_{1,t}\varepsilon_{2,t} + a^{22}\varepsilon_{1,t}\varepsilon_{2,t}\\ (1-a^{33})T^{-1}\sum_{t=1}^{T}\varepsilon^2_{2,t} + a^{33}\varepsilon^2_{2,t}\end{bmatrix}$$

As for the conditions that guarantee that both $\sigma^2_{1,t+1|t}$ and $\sigma^2_{2,t+1|t}$ are

Appendix A: Positive Definiteness in VECH ARCH(1)

positive at all times, i.e., that help ensure semi-positive definiteness of $\boldsymbol{\Sigma}_{t+1|t}$, clearly, because under a continuous distribution past squared shocks are unbounded,

$$(1-a^{11})T^{-1}\sum_{t=1}^{T}\varepsilon_{1,t}^{2}+a^{11}\varepsilon_{1,t}^{2}>0 \text{ if and only if } a^{11}\in(0,1)$$

$$(1-a^{33})T^{-1}\sum_{t=1}^{T}\varepsilon_{2,t}^{2}+a^{33}\varepsilon_{2,t}^{2}>0 \text{ if and only if } a^{33}\in(0,1).$$

At this point the filtered (predicted) correlation coefficient has expression

$$\rho_{12,t+1|t}=\frac{c^{22}+a^{22}\varepsilon_{1,t}\varepsilon_{2,t}}{\sqrt{c^{11}+a^{11}\varepsilon_{1,t}^{2}}\sqrt{c^{33}+a^{33}\varepsilon_{2,t}^{2}}},$$

where we have shortened the notation defining $c^{11}\equiv(1\text{-}a^{11})T^{-1}\sum_{t=1}^{T}\varepsilon_{1,t}^{2}$, $c^{33}\equiv(1\text{-}a^{33})T^{-1}\sum_{t=1}^{T}\varepsilon_{2,t}^{2}$, and $c^{22}\equiv(1\text{-}a^{22})T^{-1}\sum_{t=1}^{T}\varepsilon_{1,t}\varepsilon_{2,t}$. Focusing on the upper bound of the interval this means that $(c^{22}+a^{22}\varepsilon_{1,t}\varepsilon_{2,t})^{2}\leq(c^{11}+a^{11}\varepsilon_{1,t}^{2})(c^{33}+a^{33}\varepsilon_{2,t}^{2})$ or

$$(c^{22})^{2}+(a^{22})^{2}\varepsilon_{1,t}^{2}\varepsilon_{2,t}^{2}+2c^{22}a^{22}\varepsilon_{1,t}\varepsilon_{2,t}\leq c^{11}c^{33}+c^{33}a^{11}\varepsilon_{1,t}^{2}+c^{11}a^{33}\varepsilon_{2,t}^{2}+a^{11}a^{33}\varepsilon_{1,t}^{2}\varepsilon_{2,t}^{2}$$

Appendix A: Positive Definiteness in VECH ARCH(1)

which is equivalent to

$$[a^{11}a^{33}-(a^{22})^2]\varepsilon_{1,t}^2\varepsilon_{2,t}^2+[c^{11}c^{33}-(c^{22})^2]+ \\ +c^{33}a^{11}\varepsilon_{1,t}^2+c^{11}a^{33}\varepsilon_{2,t}^2-2c^{22}a^{22}\varepsilon_{1,t}\varepsilon_{2,t}\geq 0,$$

which cannot hold for a continuous distribution for the two return series as, even imposing $[a^{11}a^{33}-(a^{22})^2]\geq 0$ and $[c^{11}c^{33}-(c^{22})^2]\geq 0$,

$$c^{33}a^{11}\varepsilon_{1,t}^2+c^{11}a^{33}\varepsilon_{2,t}^2-2c^{22}a^{22}\varepsilon_{1,t}\varepsilon_{2,t}\geq 0$$

in general does not hold for $a^{22}\neq 0$. However, notice that if one sets $a^{22}=0$, then the inequalities simplify to

$$a^{11}a^{33}\varepsilon_{1,t}^2\varepsilon_{2,t}^2+[c^{11}c^{33}-(c^{22})^2]+c^{33}a^{11}\varepsilon_{1,t}^2+c^{11}a^{33}\varepsilon_{2,t}^2\geq 0,$$

which has a chance to hold if a^{11} and a^{33} are such that

$$\left[(1-a^{11})T^{-1}\sum_{t=1}^{T}\varepsilon_{1,t}^2\right]\left[(1-a^{33})T^{-1}\sum_{t=1}^{T}\varepsilon_{2,t}^2\right]\geq\left[T^{-1}\sum_{t=1}^{T}\varepsilon_{1,t}\varepsilon_{2,t}\right]^2,$$

which also means that

$$\bar{\rho}_{12}=\frac{\bar{\sigma}_{12}}{\bar{\sigma}_{11}\bar{\sigma}_{22}}=\frac{T^{-1}\sum_{t=1}^{T}\varepsilon_{1,t}\varepsilon_{2,t}}{\sqrt{(1-a^{11})T^{-1}\sum_{t=1}^{T}\varepsilon_{1,t}^2}\sqrt{(1-a^{33})T^{-1}\sum_{t=1}^{T}\varepsilon_{2,t}^2}}\leq 1,$$

Appendix A: Positive Definiteness in VECH ARCH(1)

the unconditional correlation implied by the data and the diagonal bivariate ARCH(1) process is well-behaved. Therefore, if $a^{11} \in (0,1)$ and $a^{33} \in (0,1)$, then $a^{22} = 0$ (and possibly some other restrictions on a^{11} and a^{33} such that the condition above holds) must be imposed. This means that it is impossible to model the dynamics of volatilities and covariances simultaneously while satisfying the positivity requirement for the volatilities and keeping the covariance matrix SPD at all times. Equivalently, if one wants to impose that the diagonal VECH-ARCH(1) model delivers a filtered covariance matrix $\boldsymbol{\Sigma}_{t+1|t}$ that is SPD at all times, the diagonal model itself must be turned into a constant covariance multivariate ARCH model, as you understand that $a^{22} = 0$ implies $\sigma_{12,t} = T^{-1}\sum_{t=1}^{T}\varepsilon_{1t}\varepsilon_{2t} = \bar{\sigma}_{12}$ so that

$$\rho_{12,t+1|t} = \frac{\bar{\sigma}_{12}}{\sqrt{c^{11} + a^{11}\varepsilon_{1,t}^{2}}\sqrt{c^{33} + a^{33}\varepsilon_{2,t}^{2}}},$$

and dynamics in conditional correlations will exclusively come from dynamics in volatilities.