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Mean-Variance Portfolio Choice in Excel

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Mean-Variance Portfolio Choice in Excel

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Overview

- Fundamentals: dealing with matrices in Excel
- Problem one: the efficient frontier with two risky assets (and no risk free asset)
- Problem two: asset allocation with many assets, including the risk-free one

Fundamentals: the use of matrices in Excel (1/4)

Matrix notation is essential to deal with multi-asset allocation problems in a compact and efficient way

We now review the most common and helpful functions that Excel provides to you to deal with matrices:

- Let's suppose that **A** is 2-by-2 matrix and that you want to perform the transpose (**A'**)

- TRANSPOSE (C5:D6)**

	A	B	C	D	E	F	G	H	I
1									
2									
3									
4									
5			5	8			5	7	
6		A=	7	3		A'=	8	3	
7									
8									
9									

REMEMBER, WHEN YOU DEAL WITH MATRICES YOU ALWAYS HAVE TO PRESS **CTRL + SHIFT + ENTER**

Fundamentals: the use of matrices in Excel (2/4)

- Let's suppose that **A** and **B** are two 2-by-2 matrix and that you want to perform their multiplication
 - **MMULT (C5:D6,C9:D10)**

	A	B	C	D	E	F	G	H	I
1									
2									
3									
4									
5		A=	5	8		A'=	5	7	
6			7	3			8	3	
7									
8									
9		B=	9	5		AB=	181	57	
10			17	4			114	47	
11									
12									
13									

- You need to select the result area G9:H10 (matrix is 2 x 2 so the result will be 2 x 2 too) and press CTRL + SHIFT + ENTER

Fundamentals: the use of matrices in Excel (3/4)

- Let's suppose that **A** is a 3-by-3 matrix and that you want to invert it

- MINVERSE (C13:E15)**

	A	B	C	D	E	F	G	H	I	J
1										
2										
3										
4						TRANSPOSE OF MATRIX A				
5		A=	5	8		A'	5	7		
6			7	3			8	3		
7										
8						MULTIPLICATION OF A AND B				
9		B=	9	5		AB=	181	57		
10			17	4			114	47		
11										
12										
13		C=	8	5	11		C⁽⁻¹⁾=	-0.002409639	0.231325301	-0.040963855
14			6	2	1			-0.05060241	-0.142168675	0.139759036
15			9	11	5			0.115662651	-0.103614458	-0.03373494
16										
17										

- You need to select the result area H13:J15 (matrix is 3 x 3 so the result will be 3 x 3 too) and press CTRL + SHIFT + ENTER

Fundamentals: the use of matrices in Excel (4/4)

- In addition, remember that the formula SUMPRODUCT allows you to multiply two vectors
 - **SUMPRODUCT (C18:C20, F18:F20)**

	A	B	C	D	E	F	G	H	I	J
1										
2										
3										
4						TRANSPOSE OF MATRIX A				
5		A=	5	8		A'=	5	7		
6			7	3			8	3		
7										
8						MULTIPLICATION OF A AND B				
9		B=	9	5		AB=	181	57		
10			17	4			114	47		
11										
12								INVERSE OF C		
13		C=	8	5	11		C ⁽⁻¹⁾ =	-0.002409639	0.231325301	-0.040963855
14			6	2	1			-0.05060241	-0.142168675	0.139759036
15			9	11	5			0.115662651	-0.103614458	-0.03373494
16										
17										
18			3			E=	6	MULTIPLICATION OF D' AND E		
19		D=	4				7	86		
20			5				8			
21										

SUMPRODUCT
MEANS:
 $3 \times 6 + 4 \times 7 + 5 \times 8$

Problem one: the stock-bond asset allocation

- Let's suppose you can only invest in two assets:
 - a (US) **stock** index (here represented by the value-weighted CRSP index)
 - a (US) long-term (Treasury) **bond** index (here represented by the Ibbotson 10-year government bond index)
- You have available the monthly **log-returns** of the two indices
- First of all, you need to compute the statistics of the two series: the mean and the standard deviation of each series and the pair-wise correlation between them

If you recall log-returns properties (i.e. return over two periods is just the sum of the returns of each period) you can compute the annual mean return: it is simply equal to the monthly mean return multiplied by 12

	A	B	C	D
1				
2		Mean	Variance	Weights
3	Equity	9.06%		
4	Tbond	6.01%		
5				

B3 contains the formula => $\text{MEAN}(\text{Equity}) * 12$
(where Equity is the name we gave to C2:C253, i.e. the monthly equity log-returns)

B4 contains the formula=> $\text{MEAN}(\text{Tbond}) * 12$
(where bond are monthly bond log-returns)

Problem one: the stock-bond asset allocation

Similarly, the annual standard deviation of log-returns is obtained by multiplying by $\sqrt{12}$

C3 contains the formula =>
STDEV(Equity)*SQRT(12)

C4 contains the formula =>
STDEV(bond)*SQRT(12)

Finally, we compute the correlation between the two series with the function "CORREL"=>
CORREL(Equity, Tbond)

	A	B	C	D
1				
2		Mean	Standard Deviation	Weights
3	Equity	9.06%	15.57%	
4	Tbond	6.01%	7.047%	
5				
6				
7				
8				

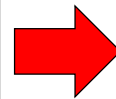
5				
6	CORRELATION MATRIX			
7		Equity	Tbond	
8	Equity	1	-0.18	
9	Tbond	-0.18	1	
10				
11				
12				
13				
14				
15				

Problem one: the stock-bond asset allocation

- Finally, we construct the variance-covariance matrix (let's call it **V**); as you know this is a symmetric matrix contains the variances of each asset in the main diagonal and the pair-wise covariances out of the main diagonal
- Recall that the formula for co-variance is:
$$\text{Cov}(i,j) = \rho \sigma_i \sigma_j$$

In order to compute the variance-covariance matrix we can use COVAR formula of excel. HINT: you can use "INDIRECT" function plus the names that we have given to the data

Why would you bother to do such a formula when you know that cell B13 is just the variance of equity returns (i.e. the square of cell C3)?



	A	B	C	D
1	TASK 1: compute the statistics for stocks, bonds and for an equally weighted portf			
2				
3		Mean	Standard Deviation	equal weights
4	Equity	9.056%	15.568%	50%
5	Tbond	6.015%	7.047%	50%
6				
7	CORRELATION MATRIX			
8		Equity	Tbond	CORRELATION HAVE BEEN CO CORRELATION FORMULA, THE FORMULA
9	Equity	1.00	-0.18	
10	Tbond	-0.18	1.00	
11				
12	VARCOV			
13		Equity	Tbond	
14	Equity	0.024235338	-0.001921698	
15	Tbond	-0.001921698	0.004966527	
16				
17				
18				
19	Statistics equally weighted portfolio			
20	Portfolio Mean	7.54%		THE GLOBAL MI ONE THAT HAS 7 COMPUTE IT IN PORTFOLIO VAR
21	Portfolio Variance			
22	Standard Deviation			
23				

Excel makes your life **easier** when you deal with a LARGE amount of data (e.g. 5 assets imply a 5-by-5 **V** matrix!)
Now you can just drag and drop!

Problem one: the stock-bond asset allocation

- Now, let's suppose for a minute that we have an **equally weighted** portfolio and compute portfolio mean and variance (the two asset case is very simple and you do not necessarily need to use matrices... however we want to create a general set up that will be valid also when we add other assets)

PORTFOLIO MEAN

$$E(r_p) = w^T e$$

MEAN RETURNS

PORTFOLIO MEAN WEIGHTS

SUMPRODUCT(B3:B4, D3:D4)

PORTFOLIO VARIANCE

VARIANCE - COVARIANCE MATRIX

$$Var(r_p) = w^T V w$$

PORTFOLIO VARIANCE WEIGHTS WEIGHTS

MMULT(TRANSPPOSE(B3:B4),MMULT(B14:C15, B3:B4))

Problem one: the stock-bond asset allocation

- Now, we can compute also the Global Minimum Variance Portfolio, i.e., the portfolio with the minimum possible variance.
- This is an optimization problem that can be solved by using the **solver**
- To find the GMVP we ask to the solver to find the combination of weights that minimize the variance
- The only constraint is that the sum of weights should be equal to 100%

The screenshot shows the Excel Solver dialog box with the following settings:

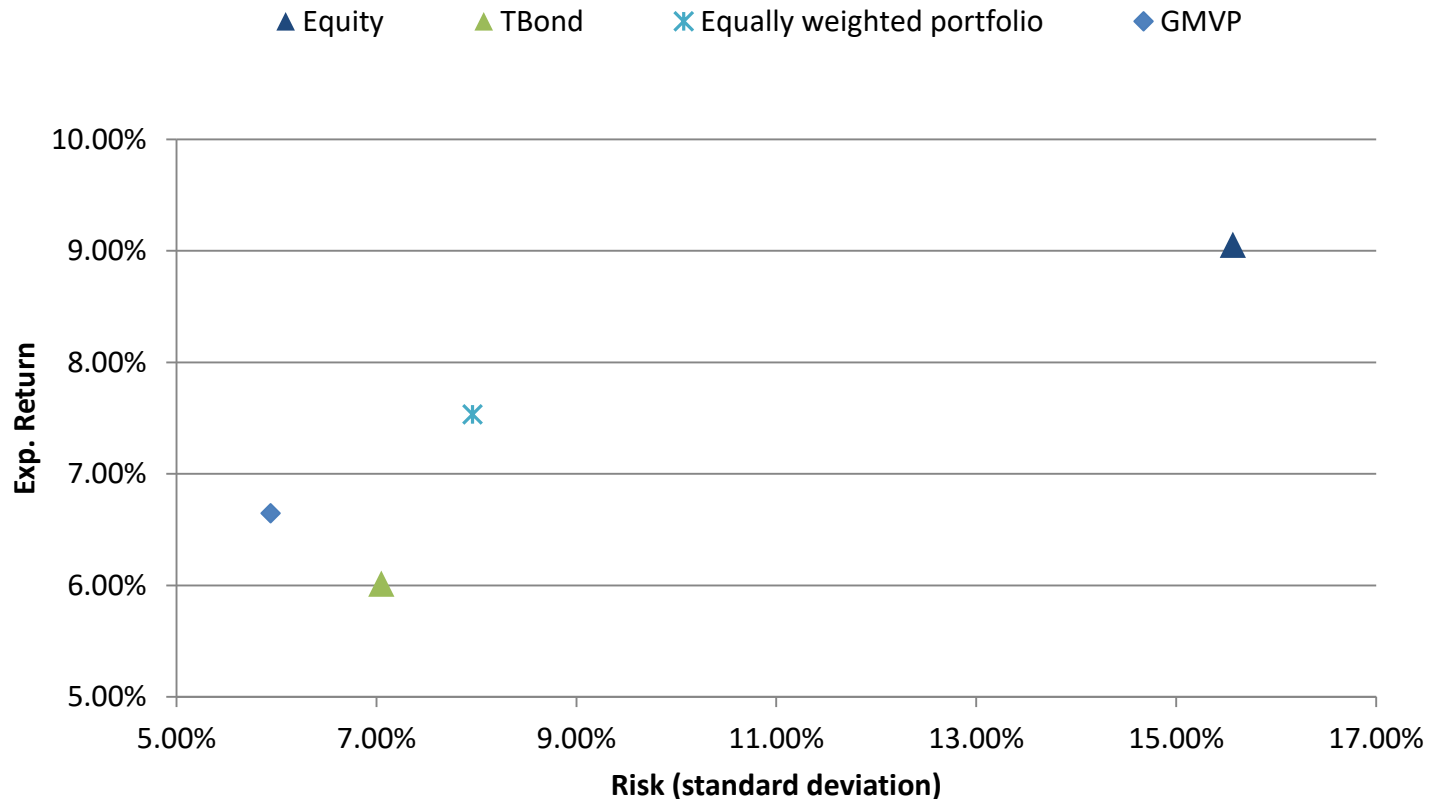
- THE VARIANCE** (indicated by a callout bubble above the dialog)
- Imposta obiettivo:** \$B\$27
- A:** Max **Min** Valore di: 0
- Modificando le celle variabili:** \$E\$4:\$E\$5
- Soggette ai vincoli:** \$E\$6 = 1
- Aggiungi** button

Annotations with arrows pointing to the dialog:

- SHOULD BE MINIMIZED** points to the **Min** radio button.
- BY CHANGING THE WEIGHTS....** points to the **Modificando le celle variabili:** field.
- WEIGHTS MUST SUM TO ONE** points to the **\$E\$6 = 1** constraint.

Problem one: the stock-bond asset allocation

- Notably, we can see from the picture that, as the “equity only” portfolio is below the GMVP, holding only equity is **NOT EFFICIENT**



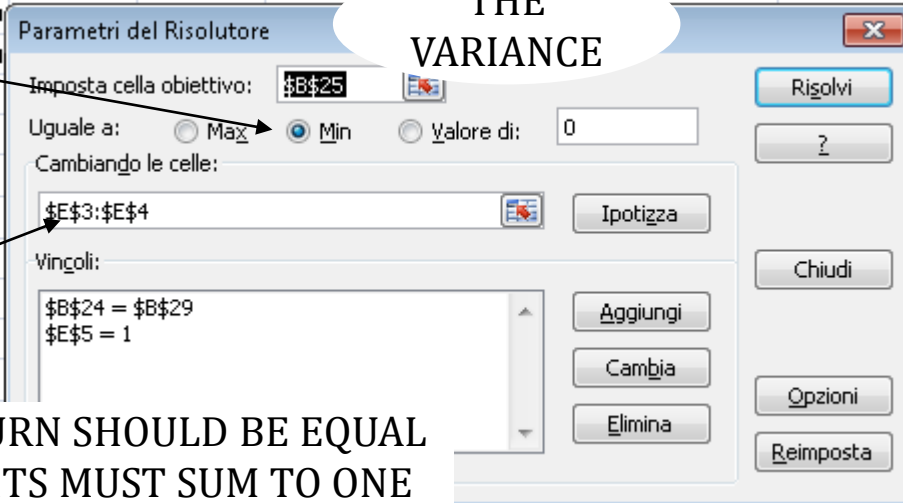
Problem one: the stock-bond asset allocation

- We can compute any point of the efficient frontier, using the **solver**
- Compared to what we did to find the GMVP, we ask to the solver to find the combination of weights that minimize the variance given a certain **target return**
- The only constraint is that the sum of weights should be equal to 100%
- If we want, we can also restrict the weights to be only positive (i.e., no-short selling allowed)

SHOULD BE MINIMIZED

BY CHANGING THE WEIGHTS....

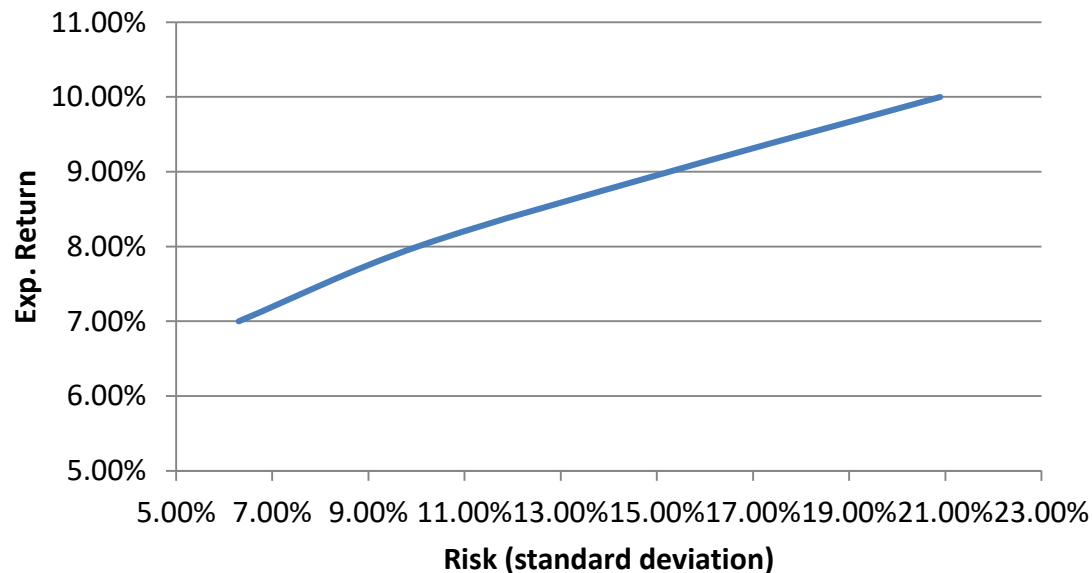
THE VARIANCE



GIVEN THAT EXPECTED RETURN SHOULD BE EQUAL TO THE TARGET AND WEIGHTS MUST SUM TO ONE

Problem one: the stock-bond asset allocation

- We can generate enough points on the efficient frontier such that we can draw (approximate by interpolation) with the excel scattered plot
- We start from the minimum-variance portfolio (as you know, it is non-sense to invest in anything that gives lower returns than the minimum-variance portfolio)
- We then generate other points on the frontier by setting higher target returns (than the return of the minimum variance portfolio)



Problem two: asset allocation with many assets

- We now consider a more general set up where:
 - we have 4 risky assets: equity, Treasury bonds, corporate bonds, and real estate
 - the investor can borrow and lend at the risk free rate (R_f)
 - we can consider lending at the riskless rate as investing in an asset with a safe outcome (e.g., T-bill) and borrowing at the riskless rate as selling such security short
 - therefore, we consider R_f equal to 2.64% (the average return of the T-bill)
 - by definition, the variance of the risk free asset is equal to zero
 - the formula for the expected return of a combination of a risky portfolio (A) and a risk-free asset is:

$$\bar{R}_C = R_F + \left(\frac{\bar{R}_A - R_F}{\sigma_A} \right) \sigma_C \quad (\text{CML})$$

Problem two: the tangency portfolio

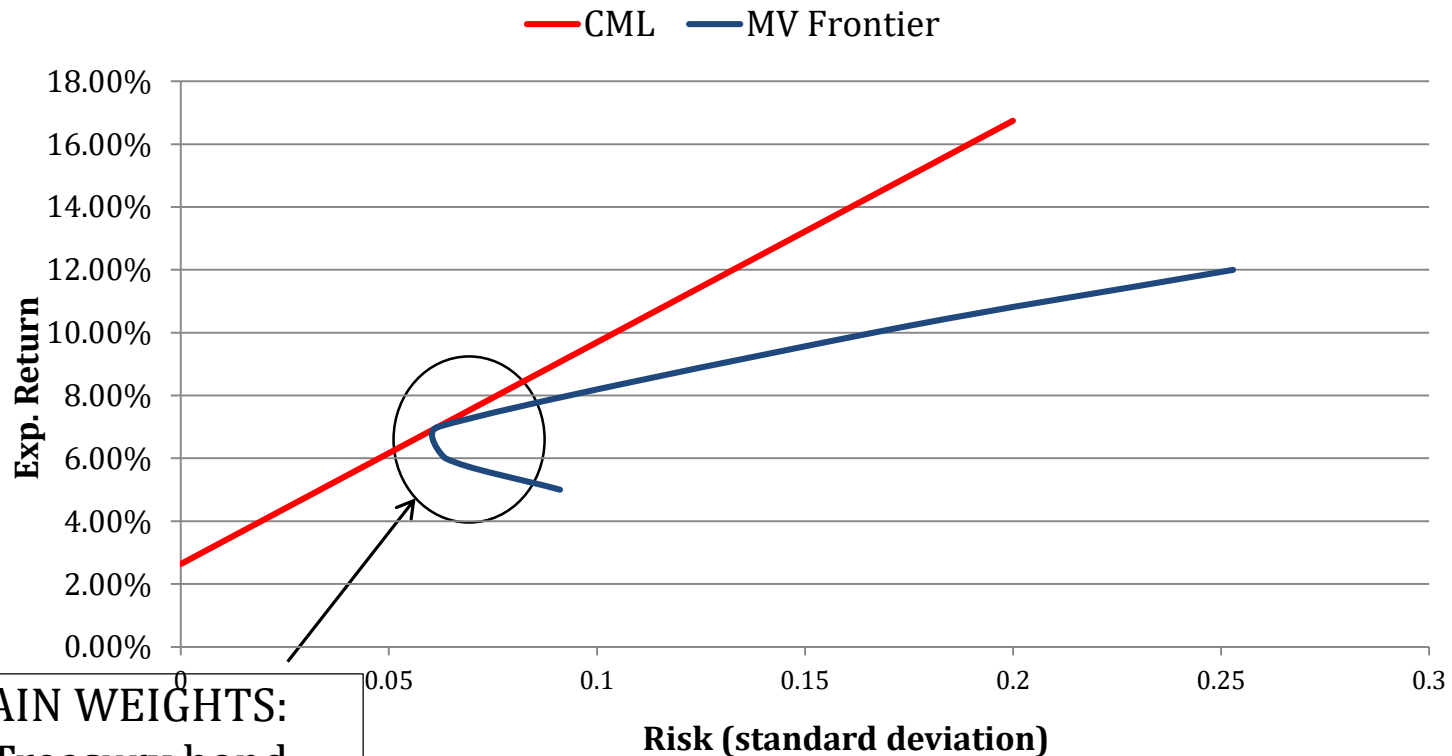
- As you already know, in this framework (with unlimited borrowing and lending at the risk free rate) we can split the allocation problem into two parts:
 - We now focus on determining the tangency portfolio (G)
=> NO NEED TO KNOW INVESTOR'S RISK AVERSION COEFFICIENT
 - To solve this problem we need to maximize:

$$\tan \alpha = (R_A - R_f) / \sigma_A \quad \text{subject to} \quad \sum w_i = 1$$

The image shows the 'Solver Parameters' dialog box in Excel. The 'Set Objective' field is set to '\$B\$34'. The 'To:' section has 'Max' selected. The 'By Changing Variable Cells:' field is '\$D\$3:\$D\$6'. The 'Subject to the Constraints:' field is '\$D\$7 = 1'. The 'Method:' is set to 'GRG Nonlinear'. There are buttons for 'Change', 'Delete', 'Reset All', 'Load/Save', 'Options', 'Solve', and 'Close'. Two text boxes are overlaid on the dialog: one on the right says 'THE OBJ IS TO MAX THE SLOPE COEFFICIENT' and one on the bottom left says 'UNDER THE ASSUMPTION OF WEIGHTS SUMMING TO 1'.

Problem two: (a) the tangency portfolio

- The tangency portfolio (or market portfolio) is unique, does not depend on the preferences of the investor



WE OBTAIN WEIGHTS:

- 0.71% Treasury bond
- -0.01% Corporate bond
- 0.25% Equity
- 0.06% Real Estate



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