Human Capital, Background Risks, and Optimal Portfolio Decisions

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Outline and objectives

- A formal mean-variance framework
  - Guidolin-Pedio, chapter 6, sec. 1

- Extensions to constant relative risk aversion preferences
  - Guidolin-Pedio, chapter 6, sec. 2

Figure 7: Hypothetical Depiction of the Relative Weight of Different Assets over an Individual's Lifetime
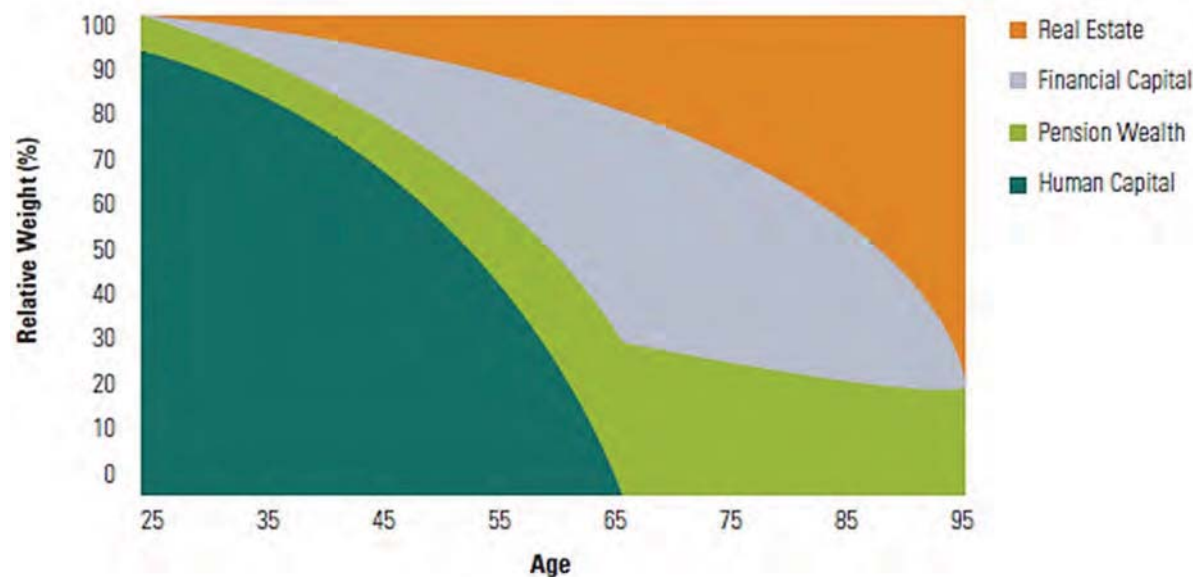
"Generally speaking, investing in yourself is the best thing you can do. Anything that improves your own talents; nobody can tax it or take it away from you. They can run up huge deficits and the dollar can become worth far less. You can have all kinds of things happen. But if you've got talent yourself, and you've maximized your talent, you've got a tremendous asset that can return tenfold. (Warren Buffet in an ABC news interview in July 2009)

Effects of human capital on portfolio choice

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Introduction

- So far we have modeled portfolio choices made by investors who derive "happiness" from capital income only

- Realistic to ask how and whether our earlier results are affected by the fact that most investors are in fact either individuals or households that receive a flow of labor or entrepreneurial income

- We generally speak of human capital, the present discounted value of the expectation of life-time incomes deriving from labor or other services rendered

  - As other assets, human capital is risky, because earnings can fluctuate—and in the most unfortunate cases fall to and stay at zero

  - The return on human capital is often computed as wage growth

- More generally, background risk: any uncertain source of income or terminal wealth that is indepen-dent of portfolio decisions that cannot be (fully) insured

UK & Greece Come Bottom For Wage Growth
% change in real wages from Q4 2007 to Q4 2015

- Greece -10.4%
- United Kingdom -10.4%
- Portugal -3.7%
- Italy 0.9%
- Ireland 1.6%
- Spain 2.8%
- Netherlands 3.4%
- Japan 4.7%
- United States 6.0%
- Sweden 7.2%
- Canada 9.4%
- France 10.5%
- Switzerland 11.3%
- Germany 13.9%
- Poland 23.0%

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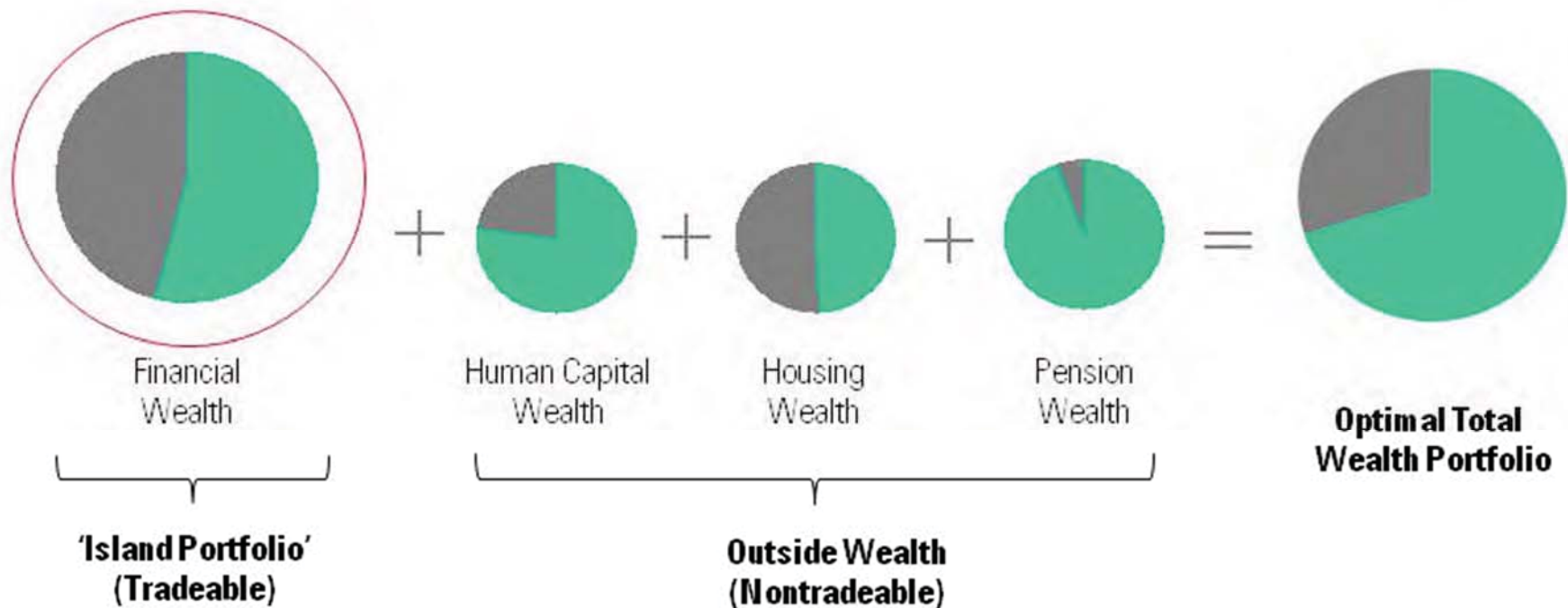
Key Qualitative Findings

- We assume that investors make the choices leading to their labor income "profile" independently of their capital income: rarely do individuals select jobs on the basis of their portfolios

- Intuitively, human capital has two effects on optimal portfolios:

① It alters the total size of the initial (discounted) value of lifetime wealth and this may affect (absolute or relative) risk aversion…

…. hence the elasticity of her investments to wealth

  - This effect depends entirely on an investor's preferences
  - Little can be said in advance apart that when investors find or lose a job, or are promoted/demoted within their workplace, one expects their portfolio decisions to be affected

② It may alter the composition of the optimal portfolio if and only if the labor income process displays non-zero correlations with the returns of the assets otherwise in the investor's menu

  - An investor should optimally choose securities to hedge the risks of such income process

Key Qualitative Findings

- o This can be accomplished by tilting her portfolio towards assets that pay out well when labor income is below average (or zero!)

- o To maximize the possibility of achieving this goal, an investor shall also be ready to under-weight the assets in her portfolio that pay out when she cares the least, i.e., when labor income is above average

- o This is of course an application of old adage that "you do not put all eggs in the same basket."

  - E.g., if you are an analyst in the financial/banking sectors, you will under-weight equity indices that invest in financials or banks
  - Unless you are forced into such participation schemes (often consisting in assigning shares of stock and warrants at prices below the market fair value) do not invest in stocks and bonds issued by your own employer or parent company!

- All assets, including human capital and housing/real estate, should be explicitly considered as components of the investor's overall portfolio

  - o The correlation structure of cash flows from these income sources that play a crucial role in determining the optimal portfolio weights

Key Qualitative Findings

Using Financial Wealth as a Completion Portfolio

Financial Wealth

+ Human Capital Wealth + Housing Wealth + Pension Wealth = Optimal Total Wealth Portfolio

'Island Portfolio' (Tradeable)

Outside Wealth (Nontradeable)

Source: "No Portfolio is an Island." by David Blanchett and Philip Straehl in the *Financial Analysts Journal*

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Deterministic Tradable Income

- Consider the simple case in which labor income is riskless so that human capital is simply the present discounted value of such deterministic sums to be received in the future

  - When labor income is riskless, then by construction it will display zero correlation with the returns on any risky assets

- Consider an investor that maximizes power utility of terminal wealth with CRRA coefficient γ and she can only invest in the risky vs. the riskless asset, i.e. a canonical portfolio problem

- Suppose in a completely counterfactual way, that human capital were totally tradable with a value of H_t

- Investor's total wealth is then $W_t + H_t$ and the expression of the optimal asset allocation for this investor when all wealth is tradable is to sell claims against her human capital and invest $\widehat{\omega}_t(W_t + H_t)$ dollars in stocks, and the remaining $(1 - \widehat{\omega}_t)(W_t + H_t)$ dollars in the riskless asset, where

$$\widehat{\omega}_t = \frac{E_t[(r_{t+1} - r^f)] + \frac{1}{2}\sigma^2}{\gamma\sigma^2}$$

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Deterministic Non-Tradable Income

- o The $0.5\sigma^2$ correction at the numerator is a Jensen's inequality factor that derives from the continuously compounded definition of returns

- Under **non-tradable labor income**, because labor is riskless and the investor has implicit holdings of H_t in the riskless asset, she should adjust her financial portfolio so that her total dollar holdings of each asset equal the optimal unconstrained holdings

- The share of risky assets in proportion to financial wealth is then

$$\widehat{\omega}_t^{H-adj} = \frac{\widehat{\omega}_t(W_t + H_t)}{W_t} = \frac{E_t[(r_{t+1} - r^f)] + \frac{1}{2}\sigma^2}{\gamma\sigma^2}\left(1 + \frac{H_t}{W_t}\right)$$

- This implies that $\widehat{\omega}_t^{H-adj} > \widehat{\omega}_t$: **an investor endowed with riskless, non-tradable human wealth should tilt her financial portfolio toward risky assets relative to an investor who owns only tradable assets**

- o When H_t is large relative to W_t, the investor may want to hold a leveraged position in stocks by borrowing at the riskless rate: the investor is trying to "undo" the endowment of riskless labor income

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Deterministic Non-Tradable Income

- The share of risky assets over financial wealth is increasing in the ratio of human to financial wealth (H_t/W_t)

- This ratio changes over the investor's life cycle: at retirement, the ratio tends to zero; early in adult life the ratio is typically large

- Therefore, a young, employed investor should invest more in risky assets (say, stocks) than an older, retiring investor with identical risk aversion and large financial wealth

    o The ratio of human to financial wealth changes with financial asset performances: If the stock market performs well, the investor's financial wealth grows relative to his human capital

    o This should lead to a reduction in the share invested in risky securities: this model predicts a "contrarian" investment strategy

- This simple model ignores some characteristics of wealth: future labor earnings are uncertain, making human capital a risky rather than a safe, non-tradable asset; investors can influence the value of human capital by varying how much they work

- The risk of human wealth is likely affect asset allocation: but how?

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Deterministic Non-Tradable Income

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A Formal One-Period Mean-Variance Framework

- Let's adopt a simple but formal mean-variance framework to investigates these issues

  - As in lecture 3, suppose you have MV preferences defined over your terminal wealth, with risk aversion coefficient κ

  $$MV(E[W_{t+1}], Var[W_{t+1}]) = E[W_{t+1}] - \frac{1}{2}\kappa Var[W_{t+1}]$$

  - For simplicity, assume unit initial wealth

  - The asset menu is composed of N risky assets with vector of returns $\mathbf{R}_{t+1}$ and one riskless asset with return r^f_t

  - You have labor income measured by the random variable Y_{t+1}

  - This variable potentially correlated with returns from securities in the asset menu

- The portfolio optimization problem may be written as:

$$\max_{\boldsymbol{\omega}_t} E[W_{t+1}] - \frac{1}{2}\kappa Var[W_{t+1}]$$

$$s.t. \quad W_{t+1} = Y_{t+1} + (1 + R^f)(1 - \boldsymbol{\iota}'\boldsymbol{\omega}_t) + (\boldsymbol{\iota} + \mathbf{R}_{t+1})'\boldsymbol{\omega}_t$$
$$= Y_{t+1} + (1 + R^f) + (\mathbf{R}_{t+1} - R^f\boldsymbol{\iota})'\boldsymbol{\omega}_t$$

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A Formal One-Period Mean-Variance Framework

- Plugging in the budget constraint as usual, the problem becomes:

$$\max_{\omega_t} E[Y_{t+1}] + \text{E}[\boldsymbol{R}_{t+1} - R^f \iota]' \boldsymbol{\omega}_t - \frac{1}{2} \kappa Var[Y_{t+1}] - \frac{1}{2} \kappa \boldsymbol{\omega}_t' \Sigma \boldsymbol{\omega}_t$$
$$- \kappa Cov[Y_{t+1}, \boldsymbol{R}_{t+1}]' \boldsymbol{\omega}_t,$$

  - This maximization program is now unconstrained and it is quadratic and globally concave
  - Hence the FOCs will be necessary but also sufficient
  - The variance term comes from the fact that

$$Var_t[W_{t+1}] = Var_t[Y_{t+1} + (1 + r_t^f) + \mathbf{w}_t'(\mathbf{R}_{t+1} - r_t^f \iota_N)]$$

$$= \underbrace{Var_t[Y_{t+1}]}_{\sigma_Y^2} + Var_t[(1 + r_t^f) + \mathbf{w}_t'(\mathbf{R}_{t+1} - r_t^f \iota_N)] +$$

$$+ 2Cov_t[Y_{t+1}, (1 + r_t^f) + \mathbf{w}_t'(\mathbf{R}_{t+1} - r_t^f \iota_N)]$$

$$= \sigma_Y^2 + \mathbf{w}_t' \Sigma_t \mathbf{w}_t + 2\mathbf{w}_t' \boldsymbol{\sigma}_{Y,\mathbf{R}} \qquad \boldsymbol{\sigma}_{Y,\mathbf{R}} \equiv Cov_t[Y_{t+1}, \mathbf{R}_{t+1}] \text{ (a } N \times 1 \text{ vector)}$$

  - At this point, the FOCs are:

$$\text{E}[\boldsymbol{R}_{t+1} - R^f \iota] - \kappa \Sigma \widehat{\boldsymbol{\omega}}_t - \kappa Cov[Y_{t+1}, \boldsymbol{R}_{t+1}] = 0$$

A Formal One-Period Mean-Variance Framework

Labor income modifies the standard MV closed-form result iff the vector of covariances of labor income with asset returns is non-zero

- Solving/manipulating in the usual ways, we have:

$$\hat{\mathbf{w}}_t = \frac{1}{\kappa}\boldsymbol{\Sigma}_t^{-1} \times \underbrace{E_t[\mathbf{R}_{t+1} - r_t^f \boldsymbol{\iota}_N]}_{\text{vector of risk premia}} - \boldsymbol{\Sigma}_t^{-1} \times \underbrace{\boldsymbol{\sigma}_{Y,\mathbf{R}}}_{\text{vector of covariances with asset returns}}$$

- The interpretation is that the presence of labor income modifies the standard MV closed-form result iff the vector of covariances of labor income with asset returns is non zero

  - Does labor income reduce/increase portfolio weights and how? It all depends oh the product $\boldsymbol{\Sigma}^{-1}_t \boldsymbol{\sigma}_{YR}$

  - It is difficult to state in advance how a non-zero $\boldsymbol{\Sigma}^{-1}_t \boldsymbol{\sigma}_{YR}$ will affect portfolio weights

- Yet, notice that per se the variance of labor income, being in the background does not affect portfolio choice

- This case is also special: the scale of wealth does affect RRA

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One Example

o The following example illustrates these features and leads to dramatic conclusions: the same equity index may go from a benign neglect in one investor's portfolio to be in very aggressive demand just because the investor experiences a change in her human capital

$$E\left[R_{t+1}^{banks} \quad R_{t+1}^{industrials} \quad R_{t+1}^{services}\right]' = [1.4\% \quad 1.0\% \quad 0.6\%]'$$

$$\boldsymbol{\Sigma} \equiv Var[\boldsymbol{R}_{t+1}] = \begin{bmatrix} (3.50)^2 & 5.4 & 2.2 \\ 5.4 & (3.23)^2 & 2.0 \\ 2.2 & 2.0 & (2.81)^2 \end{bmatrix}$$

The corresponding Sharpe ratios are:

$$SR^{banks} = \frac{1.4 - 0.1}{3.5} = 0.371 \quad SR^{industrials} = \frac{1.0 - 0.1}{3.23} = 0.279$$

$$SR^{services} = \frac{0.6 - 0.1}{2.81} = 0.178$$

The risk-free rate is 0.1%. The labor income process grows at a rather modest rate of 0.2% per month, it has a small variance of 0.8, and implies the following covariances with equity index returns:

$$Cov\left[Y_{t+1}^{Mary}, R_{t+1}^{banks}\right] = 3, Cov\left[Y_{t+1}^{Mary}, R_{t+1}^{ind}\right] = -0.6, Cov\left[Y_{t+1}^{Mary}, R_{t+1}^{serv}\right] = 1.6.$$

Mary is characterized by a coefficient of risk aversion of $\kappa = 0.25$.

Effects of human capital on portfolio choice

One Example

$$
\begin{bmatrix} \hat{\omega}^{banks}_{t+1} \\ \hat{\omega}^{industrials}_{t+1} \\ \hat{\omega}^{services}_{t+1} \end{bmatrix} = \frac{1}{0.25} \begin{bmatrix} 0.108 & -0.053 & -0.017 \\ -0.053 & 0.126 & -0.117 \\ -0.117 & -0.017 & 0.136 \end{bmatrix} \begin{bmatrix} 1.4 - 0.1 \\ 1.0 - 0.1 \\ 0.6 - 0.1 \end{bmatrix}
$$

$$
- \begin{bmatrix} 0.108 & -0.053 & -0.017 \\ -0.053 & 0.126 & -0.117 \\ -0.117 & -0.017 & 0.136 \end{bmatrix} \begin{bmatrix} 3.0 \\ -0.6 \\ 1.6 \end{bmatrix} = \begin{bmatrix} 0.010 \\ 0.408 \\ -0.055 \end{bmatrix}
$$

and as a result the weight in the riskless asset is: 1-0.010-0.408+0.055= 0.637. Interestingly, even though bank stocks have by far the highest Sharpe ratio, Mary is demanding th1 n in a very small percentage. This is due to the fact that bank stocks and Mary's labor income have a very high positive correlation of $(3.0/(3.50 \times \sqrt{0.8})) = 0.958$ so that in overall terms, to Mary, buying bank stocks appears to be very risky.

Suppose now that Mary changes job and sector, becoming vice-president in risk management in one industrial corporation. Because of the promotion, Mary's salary doubles, its rate of growth becomes 0.3% per month, its variance declines to 0.6% only. Moreover, Mary's new labor income process implies the following covariances with equity index returns:

$$
Cov\left[\tilde{Y}^{Mary}_{t+1}, R^{banks}_{t+1}\right] = -1, \; Cov\left[\tilde{Y}^{Mary}_{t+1}, R^{ind}_{t+1}\right] = 2, \; Cov\left[\tilde{Y}^{Mary}_{t+1}, R^{serv}_{t+1}\right] = 1.8.
$$

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One Example

Mary's new optimal portfolio will now be:

$$\begin{bmatrix} \widetilde{\omega}_{t+1}^{banks} \\ \widetilde{\omega}_{t+1}^{industrials} \\ \widetilde{\omega}_{t+1}^{services} \end{bmatrix} = \frac{1}{0.25} \begin{bmatrix} 0.108 & -0.053 & -0.017 \\ -0.053 & 0.126 & -0.017 \\ -0.017 & -0.017 & 0.136 \end{bmatrix} \begin{bmatrix} 1.4 - 0.1 \\ 1.0 - 0.1 \\ 0.6 - 0.1 \end{bmatrix}$$

$$- \begin{bmatrix} 0.108 & -0.053 & -0.017 \\ -0.053 & 0.126 & -0.017 \\ -0.017 & -0.017 & 0.136 \end{bmatrix} \begin{bmatrix} -1.0 \\ 2.0 \\ 1.8 \end{bmatrix} = \begin{bmatrix} 0.581 \\ -0.127 \\ -0.104 \end{bmatrix},$$

which implies a percentage investment in the riskless asset of 0.65. Interestingly, the cash investment remains the same and the share in banks greatly increases from 1 to 58 percent, which is obviously caused by the excellent Sharpe ratio of banks; the weight in services moderately declines from -5.5 to -10.4 percent; the share of industrial stocks plunges from 41 to -13%. This is caused by the high correlation between labor incomes and industrial stock returns of $(2.0/(3.23 \times \sqrt{0.6})) = 0.799$. Because everything else is the same, we can quantify the effect of the labor income hedging demand by Mary to be $|0.581 – 0.010| = 0.571$ in the case of bank stocks and $|-0.127-0.408| = 0.535$ in the case of industrial stocks.

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Extension to CRRA Preferences

- Important to generalize results in many directions

  o Multiple period, life-cycle aspects

  o Portfolio re-balancing

  o Consumption and investment problems

  o More realistic preferences

- Here DARA/CRRA utility function, i.e., power utility

- Other generalizations far from trivial and only described in heuristic terms in your textbook

  o Keep assuming the investor has 1-period horizon, investing her wealth to support her consumption from terminal wealth

- Investor receives log-normally distributed labor income:

$$y_{t+1} \equiv lnY_{t+1} \sim N(\bar{y}, \sigma_y^2)$$

  o Assume that labor income is uninsurable (hence, non-tradable): you cannot write claims against your future income

  o The investor uses her portfolio, by changing its structure, in order to (self-) insure any labor income shocks

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Extension to CRRA Preferences

- Moreover $r_{t+1} = E_t[r_{t+1}] + u_{t+1}$ with $u_{t+1} \sim IID\ N(0, \sigma_u^2)$

- Because we do not model any labor-leisure choice, this model is implicitly one of fixed labor supply in conjunction with a random wage in which the investor solves the following problem:

$$\max_{\omega_t} \delta E_t \left[\frac{W_{t+1}^{1-\gamma}}{1-\gamma} \right] \text{ subject to } W_{t+1} = W_t[R^f + \omega_t(R_{t+1} - R^f)] + Y_{t+1}$$

  - δ is the subjective discount factor (but with 1 period doesn't matter)
  - When $\gamma > 0$, the problem is a concave programming problem with equality constraints, in which the FOCs are necessary and sufficient:

  $$E_t\left[W_{t+1}^{-\gamma} R_{t+1}\right] = E_t\left[W_{t+1}^{-\gamma} R^f\right] = R^f E_t\left[W_{t+1}^{-\gamma}\right]$$

  - This implies that the MU-weighted expected risky asset return should equal at the optimum an identical MU-weighted riskless rate

- Through a process of log-linearization that follows Campbell and Viceira (2001, 2002), one gets to a quasi-closed form solution

  - To log-linearize means to apply 1st order Taylor expansions to logs

  $$lnE_t\left[W_{t+1}^{-\gamma}(R_{t+1} - R^f)\right] = E_t[-\gamma lnW_{t+1} + ln(R_{t+1} - R^f)]$$

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Extension to CRRA Preferences

The CRRA-based approximation features a hedging component: if ξ < 1, positive correlation btw. ptf. returns and labor income decreases the demand of risky assets

$$= E_t[(r_{t+1} - r^f)] + \frac{1}{2}\sigma_u^2 - \gamma Cov_t[w_{t+1}, r_{t+1}]$$

- Your textbook gives some details to de-mystify the origins of the key result:

$$\widehat{\omega}_t \cong \frac{1}{\xi} \frac{E_t[(r_{t+1} - r^f)] + \frac{1}{2}\sigma_u^2}{\gamma\sigma_u^2} + \left(1 - \frac{1}{\xi}\right)\frac{Cov_t[y_{t+1}, r_{t+1}]}{\sigma_u^2}$$

$$= Myopic\ asset\ demand + Hedging\ background\ risk$$

   o ξ < 1 is the elasticity of terminal wealth with respect to ptf. returns
   o This expression concerns the simple decision between one risky asset (say, some notion of a market portfolio) and the riskless asset
   o $0.5\sigma_u^2$ adjustment is a convexity correction due to log-normality

- Hedging demand arises from the desire to reduce lifetime consumption risk, and here this risk arises from the correlation between returns and labor income

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Extension to CRRA Preferences

- Note that also in the case of zero correlation btw. labor income and risky ptf. returns, when ξ < 1 one obtains:

$$\frac{1}{\xi} \frac{E_t\left[(r_{t+1} - r^f)\right] + \frac{1}{2}\sigma_u^2}{\gamma \sigma_u^2} > \frac{E_t\left[(r_{t+1} - r^f)\right] + \frac{1}{2}\sigma_u^2}{\gamma \sigma_u^2}$$
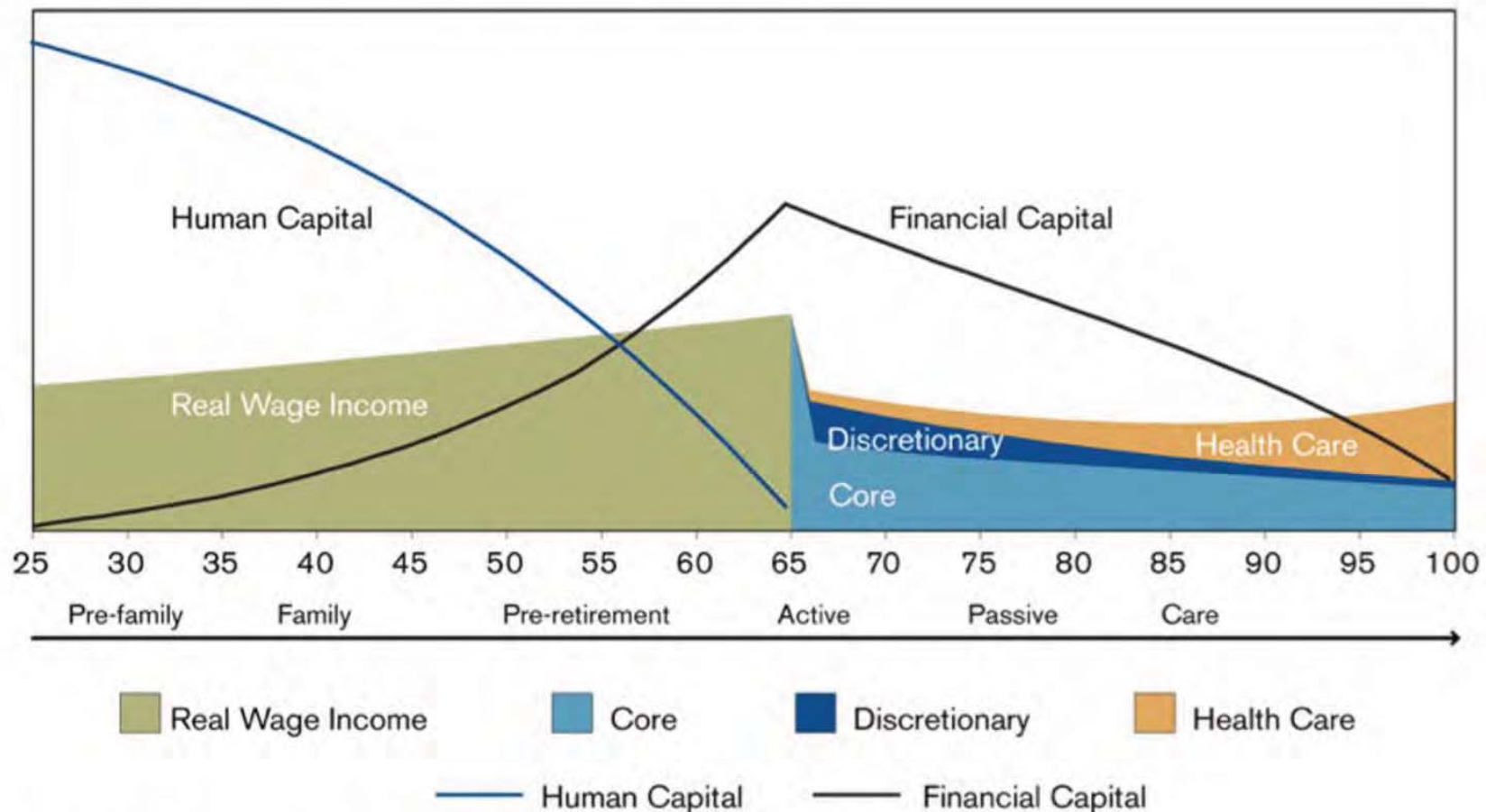
| Formula in this lecture | Formula w/out labor income (see textbook p. 201) |

  - ξ < 1 is due to the presence of labor income in the budget constraint
  - The intuition is that wealth income allows for a good deal of risk reduction, thereby implying a higher optimal risky portfolio weight

- Riskless labor income creates a tilt toward risky equities

- This is not surprising as the labor income stream in this case contributes a risk free asset to the investor's portfolio

- What are plausible correlation values concerning labor income shocks and aggregate risky returns? At an overall level, it is low btw. -0.02 for those with a college degree, 0.01 for those with only a high school, and -0.01 for those not completing their high school studies

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What's Next?

- To integrate the theory of portfolio selection under human capital to the endogeneous choice of: (i) health risk, (ii) reteriment and work intensity choices and, eventually (iii) career path selection

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