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Mean-Variance Portfolio Choice in Excel

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20550 – Quantitative Methods for Finance

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One Quiz on Constrained (Portfolio) Optimization

- John is a quant strategist at "SackingAlpha & Co."; until last week he had selected the optimal weights of the fund he manages by solving:

$$\max_{\boldsymbol{\omega}} E\left[r_t^p\right] - \frac{\lambda}{2} Var\left[r_t^p\right] \; (mean - variance\ objective)$$

$$\begin{aligned} s.t. \qquad \boldsymbol{\omega}'\mathbf{1} &= 1 \; (definition\ of\ weights) \\ r_t^p &= \boldsymbol{\omega}'\boldsymbol{r}_t \; (definition\ of\ ptf.return) \\ \boldsymbol{\omega} &\geq \mathbf{0} \; (no\ short\ sales) \end{aligned}$$

- Starting on Monday, John has changed strategy and now solves:

$$\max_{\boldsymbol{\omega}} E\left[r_t^p\right] - \frac{\lambda}{2} Var\left[r_t^p\right] \; (mean - variance\ objective)$$

$$\begin{aligned} s.t. \qquad \boldsymbol{\omega}'\mathbf{1} &= 1 \; (definition\ of\ weights) \\ r_t^p &= \boldsymbol{\omega}'\boldsymbol{r}_t \; (definition\ of\ ptf.return) \\ \mathbf{C}\boldsymbol{\omega} &\geq \boldsymbol{k} \; (some\ weird\ no - arb\ constraint) \end{aligned}$$

- Will the optimized mean-variance objective (risk-adjusted performance) achieved by John increase or decrease as a result of the switch?

One Quiz on Constrained (Portfolio) Optimization

- Mary is another analyst at “Iforgotmyalphaathome” and she happens to solve the second of John’s problem with $\mathbf{C} = \mathbf{I}_n$ and $\mathbf{k} = \mathbf{0}$; will the optimized mean-variance objective (risk-adjusted performance) achieved by Mary be more or less vs. John, after Monday?

One Quiz on Constrained (Portfolio) Optimization

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Answers

- The 2nd problem is a version of the 1st in which $\boldsymbol{\omega} \geq \mathbf{0}$ has been generalized to $\mathbf{C}\boldsymbol{\omega} \geq \boldsymbol{k}$; one may argue that the 1st problem is identical to the 2nd but solved under specific constraints: if specific means tighter, you know that tighter the constraints, the lower the maximized objective function will be
- However, note that $\mathbf{I}_n$ is a rather special case—in which all columns and rows are linearly independent—vs. letting $\mathbf{C}$ be any mxn matrix; for instance $\mathbf{C}$ may contain linerarly dependent rows/columns and therefore constrain the portfolio weights LESS than $\mathbf{C} = \mathbf{I}_n$ does
- Therefore without knowing specificaly what $\mathbf{C}$ and $\boldsymbol{k}$ are and actually solving the problem, one cannot really say, since constraints play a first-order role
- Mary is solving the first, older of John's problem: we are asking the same question ahahhahhahhhaha

Problem one: the stock-bond asset allocation

- Let's suppose you can only invest in two assets:
  - a (US) stock index (here represented by the value-weighted CRSP index)
  - a (US) long-term (Treasury) bond index (here represented by the Ibbotson 10-year government bond index)
- You have available the monthly log-returns of the two indices
- First of all, you need to compute the statistics of the two series: the mean and the standard deviation of each series and the pair-wise correlation between them

	A	B	C	D
1				
2		**Mean**	**Variance**	**Weights**
3	**Equity**	9.06%		
4	**Tbond**	6.01%		
5				

If you recall log-returns properties (i.e. return over two periods is just the sum of the returns of each period) you can compute the annual mean return: it is simply equal to the monthly mean return over the full sample multiplied by 12

B3 contains the formula => MEAN(Equity)*12 (where Equity is the name we gave to C2:C253, i.e. the monthly equity log-returns)

B4 contains the formula=> MEAN(Tbond)*12 (where Tbond are monthly bond log-returns)

Problem one: the stock-bond asset allocation

Similarly, the annual standard deviation of log-returns is obtained by multiplying by $\sqrt{12}$

C3 contains the formula => STDEV(Equity)*SQRT(12)

C4 contains the formula => STDEV(Tbond)*SQRT(12)

	A	B	C	D
1				
2		**Mean**	**Standard Deviation**	**Weights**
3	**Equity**	9.06%	15.57%	
4	**Tbond**	6.01%	7.047%	
5				
6				
7				
8				

Finally, we compute the correlation between the two series with the function "CORREL"=> CORREL(Equity, Tbond)

5				
6	CORRELATION MATRIX			
7		Equity	Tbond	
8	**Equity**	1	-0.18	
9	**Tbond**	-0.18	1	
10				
11				
12				
13				
14				
15				

Problem one: the stock-bond asset allocation

- Finally, we construct the variance-covariance matrix (let's call it **V**); as you know this is a symmetric matrix contains the variances of each asset in the main diagonal and the pair-wise covariances out of the main diagonal
- Recall that the formula for co- variance is: $\mathrm{Cov}(\mathrm{i,j}) = \varrho \sigma_i \sigma_j$

	A	B	C	D
1				
2		**Mean**	**Standard Deviation**	**Weights**
3	**Equity**	9.06%	15.57%	
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6	CORRELATION MATRIX			
7		Equity	Tbond	
8	**Equity**	1	-0.18	
9	**Tbond**	-0.18	1	
10				
11	VARCOV			
12		Equity	Tbond	
13	**Equity**	0.024235338	-0.001921698	
14	**Tbond**	-0.001921698	0.004966527	
15				
16				
17				
18				
19				
20				
21				

Cell B13 contains the formula:
B8*VLOOKUP($A13, $A$2:$C$4, 3, FALSE)*VLOOKUP(B$12, $A$2:$C$4, 3, FALSE)

Why would you bother to do such a formula when you know that cell B13 is just the variance of equity returns (i.e. the square of cell C3)?

Excel makes your life **easier** when you deal with a LARGE amount of data (e.g. 5 assets imply a 5-by-5 **V** matrix!)
Now you can just drag and drop!

Problem one: the stock-bond asset allocation

- Now, let's suppose for a minute that we have an equally weighted portfolio and compute portfolio mean and variance (the two asset case is very simple and you do not necessarily need to use matrices… however we want to create a general set up that will be valid also when we add other assets)

PORTFOLIO MEAN

$$\underbrace{E(r_p)}_{\text{PORTFOLIO MEAN}} = \underbrace{w^T}_{\text{WEIGHTS}} \overbrace{e}^{\text{MEAN RETURNS}}$$

SUMPRODUCT(D3:D4, B3:B4)

Here we do not formally transpose because the function works without transposing in a single product
(we avoid formally treating cells as vectors in Excel because that forces you to use combinations of keys)

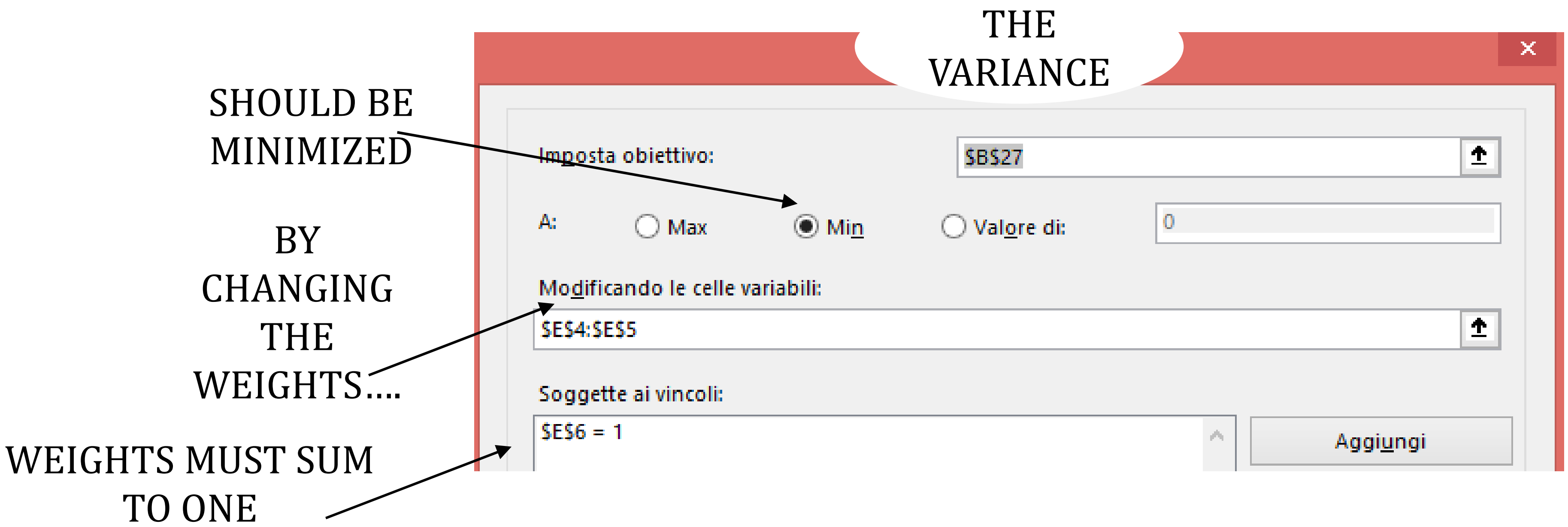
PORTFOLIO VARIANCE

$$\underbrace{Var(r_p)}_{\text{PORTFOLIO VARIANCE}} = \underbrace{w^T}_{\text{WEIGHTS}} \overbrace{V}^{\text{VARIANCE – COVARIANCE MATRIX}} \underbrace{w}_{\text{WEIGHTS}}$$

MMULT(TRANSPOSE(D3:D4),MMULT (B13:C14, D3:D4))

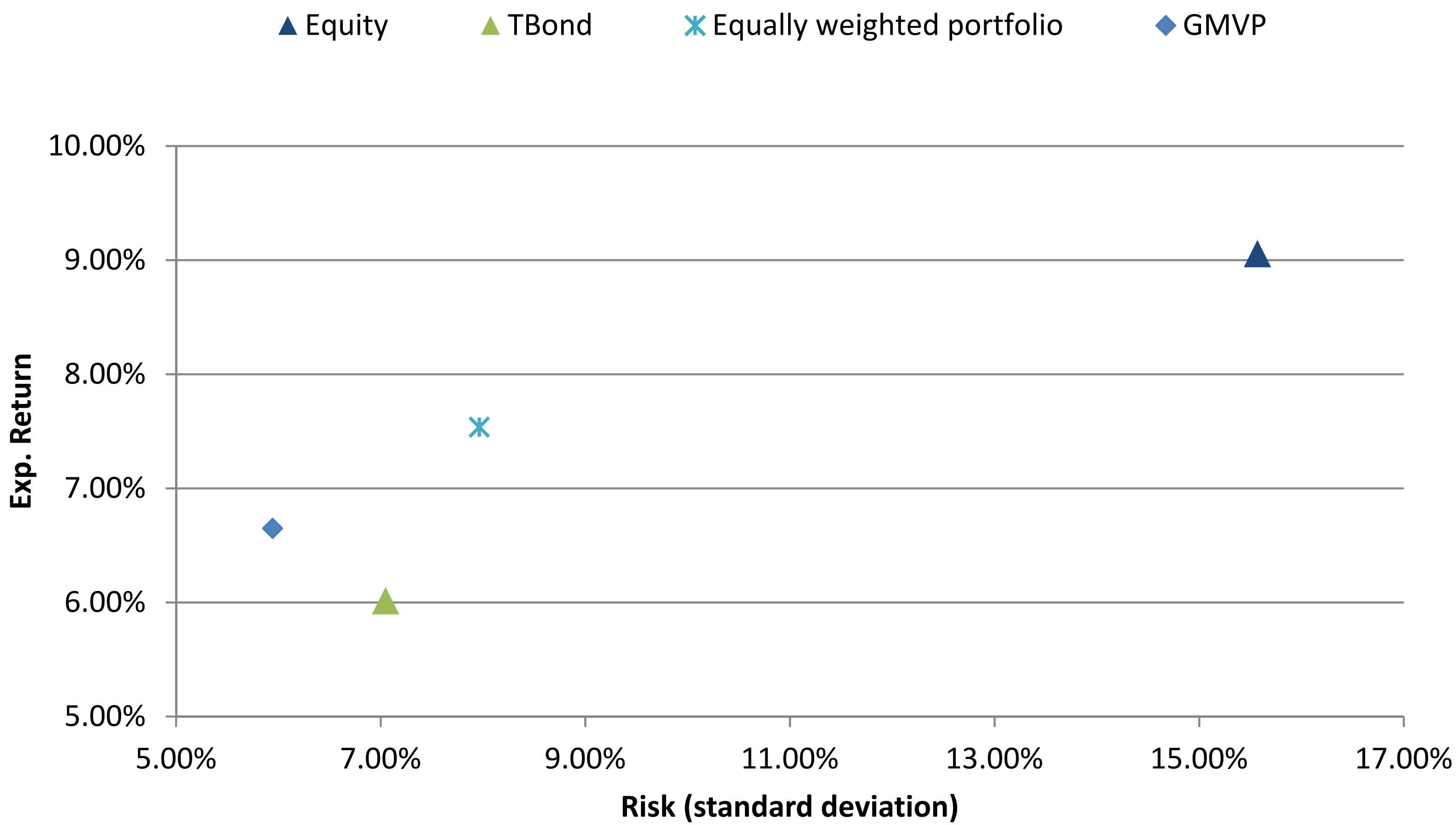
Problem one: the stock-bond asset allocation

- Now, we can compute also the Global Minimum Variance Portfolio, i.e., the portfolio with the minimum possible variance.
- This is an optimization problem that can be solved by using the solver
- To find the GMVP we ask to the solver to find the combination of weights that minimize the variance
- The only constraint is that the sum of weights should be equal to 100%



Problem one: the stock-bond asset allocation

- Notably, we can see from the picture that, as the "T bond only" portfolio is below the GMVP, holding only bonds is **NOT EFFICIENT**

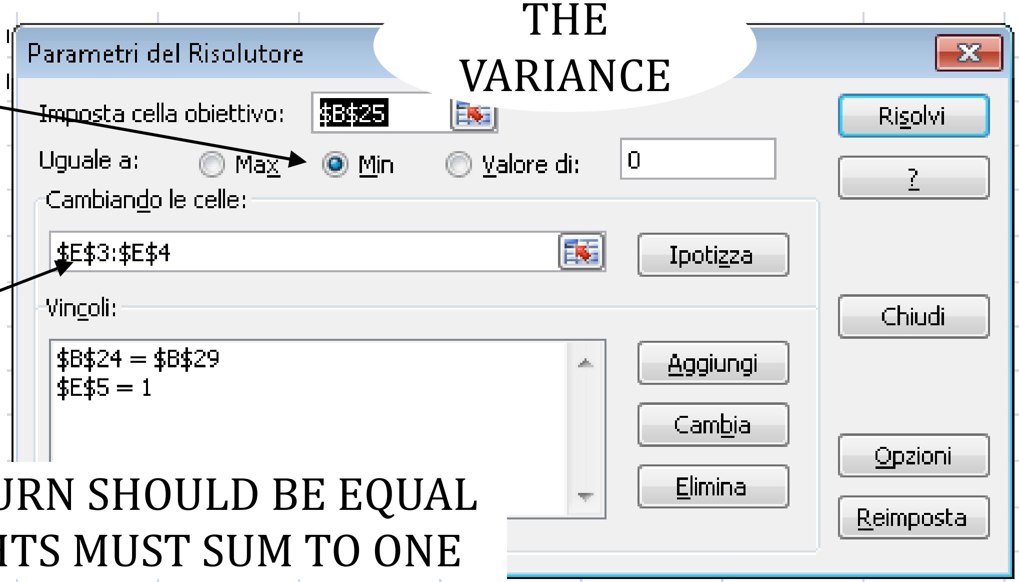


Problem one: the stock-bond asset allocation

- We can compute any point of the efficient frontier, using the solver
- Compared to what we did to find the GMVP, we ask to the solver to find the combination of weights that minimize the variance given a certain target return
- The only constraint is that the sum of weights should be equal to 100%
- If we want, we can also restrict the weights to be only positive (i.e., no-short selling allowed)

SHOULD BE MINIMIZED

THE VARIANCE

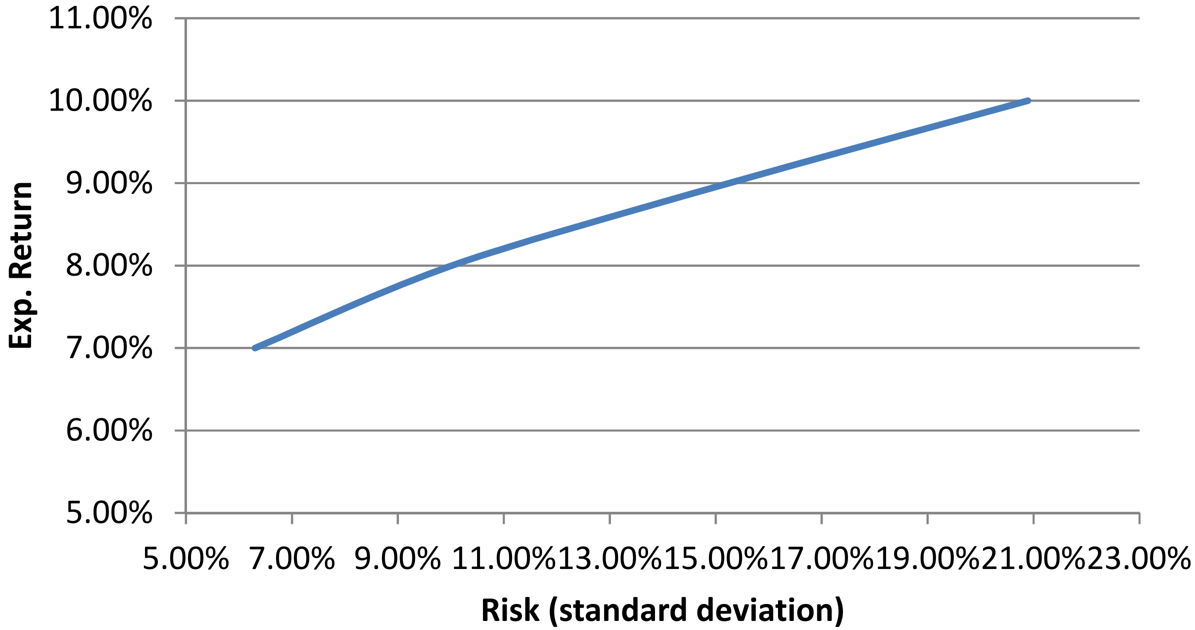


BY CHANGING THE WEIGHTS....

GIVEN THAT EXPECTED RETURN SHOULD BE EQUAL TO THE TARGET AND WEIGHTS MUST SUM TO ONE

Problem one: the stock-bond asset allocation

- We can generate enough points on the efficient frontier such that we can draw (approximate by interpolation) with the excel scattered plot
- We start from the minimum-variance portfolio (as you know, it is non-sense to invest in anything that gives lower returns than the minimum-variance portfolio)
- We then generate other points on the frontier by setting higher target returns (than the return of the minimum variance portfolio)



Problem two: asset allocation with many assets

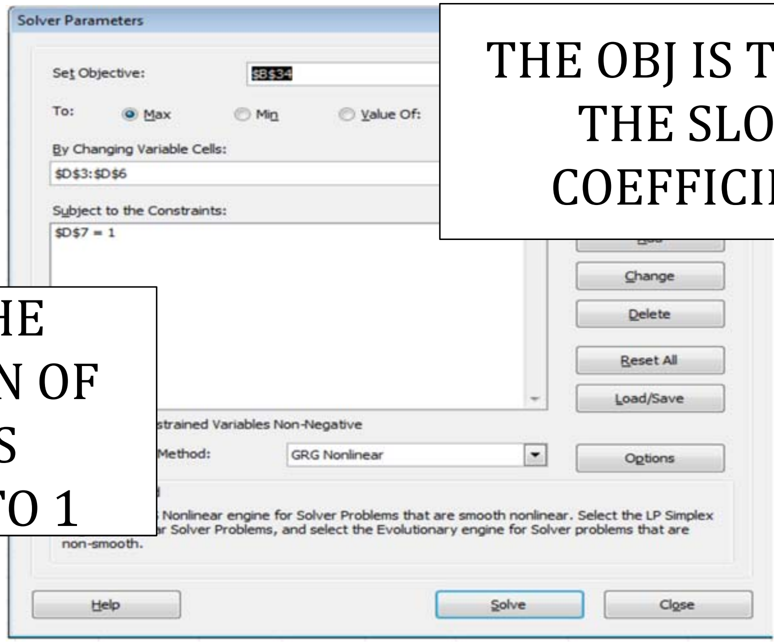
- We now consider a more general set up where:
  - we have 4 risky assets: equity, Treasury bonds, corporate bonds, and real estate
  - the investor can borrow and lend at the risk free rate (R_f)
  - we can consider lending at the riskless rate as investing in an asset with a safe outcome (e.g., T-bill) and borrowing at the riskless rate as selling such security short
  - therefore, we consider R_f equal to 2.64% (the average return of the T-bill)
  - by definition, the variance of the risk free asset is equal to zero
  - the formula for the expected return of a combination of a risky portfolio (A) and a risk-free asset is:

$$\bar{R}_C = R_F + \left(\frac{\bar{R}_A - R_F}{\sigma_A} \right) \sigma_C \quad \text{(CML)}$$

Problem two: the tangency portfolio (1/2)

- As you already know, in this framework (with unlimited borrowing and lending at the risk free rate and of any other security) we can split the allocation problem into two parts:
  - We now focus on determinating the tangency portfolio (G) => NO NEED TO KNOW INVESTOR'S RISK AVERSION COEFFICIENT as everybody wants to hold the same portfolio
    - To solve this problem we need to maximize:

$$\tan\alpha = (R_A - R_f)/\sigma_A \quad \text{subject to} \quad \sum w_i = 1$$

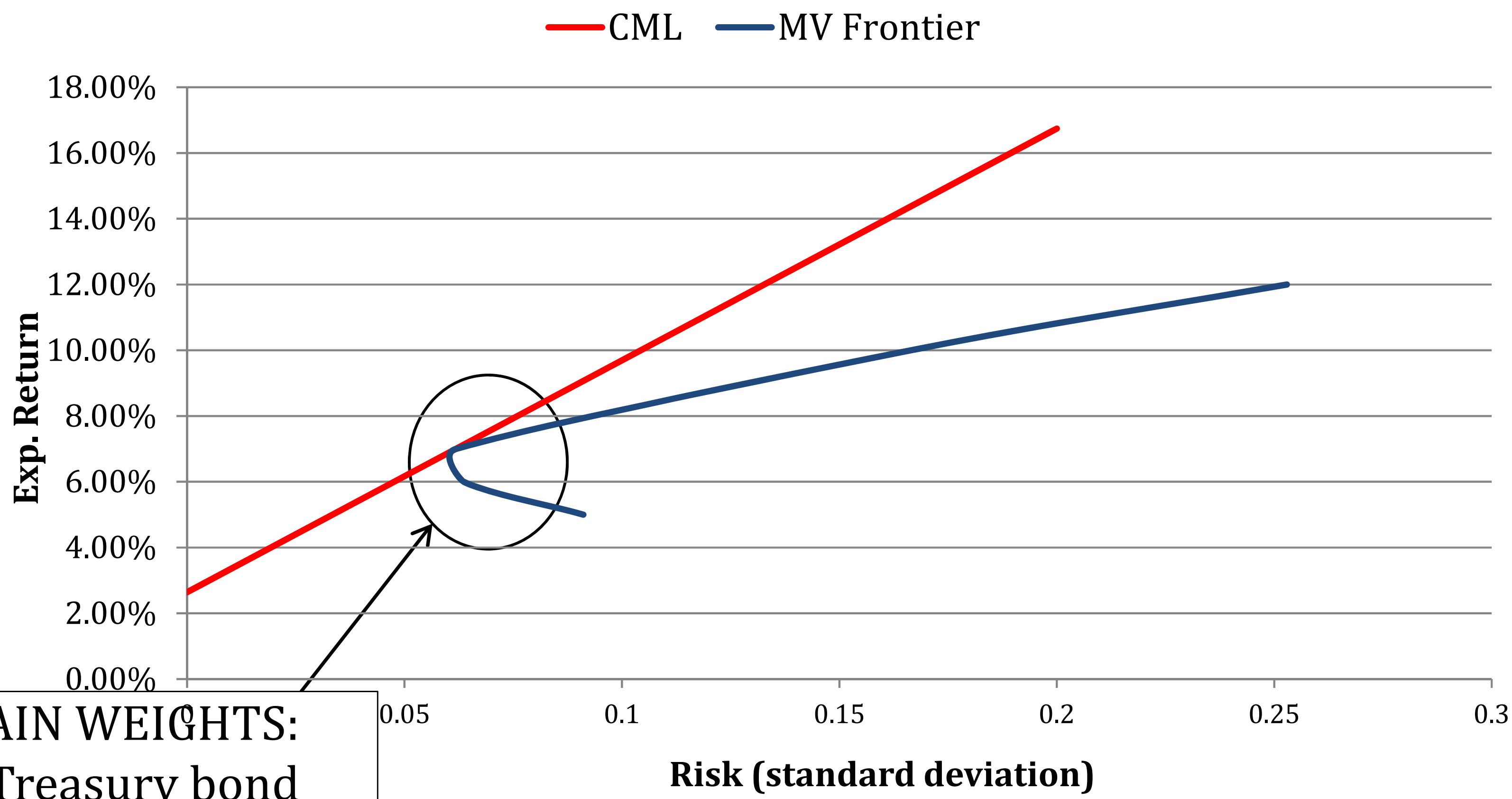


THE OBJ IS TO MAX THE SLOPE COEFFICIENT

UNDER THE ASSUMPTION OF WEIGHTS SUMMING TO 1

Problem two: the tangency portfolio (2/2)

- The tangency portfolio is unique, does not depend on the preferences of the investor



WE OBTAIN WEIGHTS:
- 0.71% Treasury bond
- -0.01% Corporate bond
- 0.25% Equity
- 0.06% Real Estate