20135 Theory of Finance – Part 1

Mock Question 7 (5 points)
Time Advised: 20-22 minutes (for this questions)
Difficulty Level: MEDIUM-EASY

Question 7.A (3.75 points)
What are the three key models of the effects of the existence of human capital (more generally, background risks) on optimal portfolio decisions? Make sure to list and discuss the assumptions under which the three models are developed and their implications for optimal weights.

Question 7.B (0.75 points)
Professor Max Fifty is a risk-averse investor that maximizes power utility of terminal wealth with a given CRRA coefficient, γ. When he was unemployed, he used to invest 50% of his wealth in the market portfolio and the rest in the riskless asset. However, he has recently found a new job in a top five university that will pay him an essentially riskless, non-tradable salary. Without any additional information, how would you expect Prof. Fifty to change his optimal weights after he starts serving as a professor? Do you think that the age of Prof. Fifty may be related to the percentage of wealth that he will now invest in the riskless asset, and why? Make sure to carefully justify your answer in the light of one or more analytical frameworks of optimal portfolio decision.

Question 7.C (0.5 points)
Consider the monthly statistics concerning the following two equity indices:
$$E\left[R_{t+1}^{banks} \; R_{t+1}^{industrials}\right]' = [2.7\% \; 0.5\%]'$$
$$\mathbf{\Sigma} \equiv Var[\boldsymbol{R}_{t+1}] = \begin{bmatrix} (4.0)^2 & 0 \\ 0 & (3.0)^2 \end{bmatrix}$$
The risk-free rate is 0.2%. Mr. Baldtree is a risk averse, mean-variance optimizer. He was originally unemployed and characterized by a coefficient of risk aversion $\kappa = 0.1$. Compute Mr. Baldtree's optimal risky portfolio. At this point, Mr. Baldtree finds a high-ranking job with a bank that will pay an income characterized by a rate of growth of 0.3% per month and a variance of 2.5% per month; furthermore, Mr. Baldtree's labor income implies the following covariances with the equity returns:
$$Cov[\tilde{Y}_{t+1}^{Bald}, R_{t+1}^{banks}]=5 \quad Cov[\tilde{Y}_{t+1}^{Bald}, R_{t+1}^{ind}]=2.$$
Compute his new optimal portfolio that takes into account the effect of labor income. Will Mr. Baldtree tilt his portfolio towards or away from bank stocks? (*Hint*: note that the inverse of a diagonal matrix is the matrix of the inverses of the elements on the diagonal).