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Austrian Debates on Utility Measurement from Menger to Hayek

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1 Introduction

This chapter examines how some of the main exponents of the Austrian school of economics addressed the issues related to the measurability of utility. The first part of the chapter (Sections 2–9) is devoted to the period between the publication of Carl Menger's *Principles of Economics* in 1871 and World War I, and studies the approaches to utility measurement of Carl Menger, Friedrich von Wieser, Eugen von Böhm-Bawerk, František Čuhel, Joseph Schumpeter, and Ludwig von Mises. In the pre-1914 period, two main views on the measurability of utility clashed. According to the first one, defended in particular by Böhm-Bawerk, the utility of goods can be measured and expressed as a multiple of a unit. According to the second view, advocated by Čuhel and Mises, utilities can only be compared and ranked but not measured. I will argue that by World War I the latter view, that is the ordinal understanding of utility, had become the dominant position among Austrian economists.

The second part of the chapter (Sections 10–12) briefly considers the interwar years. During this period, the consensus around ordinal utility was enlivened by discussion of whether individuals are not only able to rank the utility of goods (as in the ordinal utility approach) but are also capable of ranking differences of utility. The chapter examines some of the contributions to this discussion, particularly those of Leo Schönfeld, Paul Rosenstein-Rodan, Oskar Morgenstern and Franz Alt, and concludes by illustrating Friedrich von Hayek's ordinal view of utility.

The reconstruction of the Austrian approaches to utility measurement I propose in this chapter has some features that distinguish it from other

reconstructions (see, for example, Schumpeter 1954; Rothbard 1956; Kauder 1965; McCulloch 1977; High and Bloch 1989). It is useful to illustrate these features here.

The classical understanding of measurement

Existing reconstructions tend to overlook the fact that the Austrian discussions about the measurability of utility were deeply intertwined with the way in which the Austrian utility theorists conceived of measurement, that is, with their understanding of what it means to measure something. In contrast, in this chapter I connect the Austrian economists' approach to utility measurement with their more general understanding of measurement. In particular, I show that the Austrians endorsed what I call, following the terminology introduced by Joel Michell (1999), the 'classical concept of measurement'.

The concept is labelled 'classical' because it dates back to Aristotle. According to the classical view, measuring the property of an object (such as the length of a table) consists of comparing it with some other object that displays the same property and is taken as a unit (such as a metre-long ruler) and then assessing the numerical ratio between the unit and the object to be measured (so if the ratio is 2:1, the table is two metres long). When applied to the measurement of utility, this classical concept requires the identification of a unit of utility and the capacity of assessing utility ratios, that is, of stating that one utility is, for example, two times greater than another.[1]

Because the Austrian utility theorists adhered to the classical concept of measurement, they associated the possibility or impossibility of measuring utility with the possibility or impossibility of ascertaining a unit of utility and assessing utility ratios.

Beyond the cardinal–ordinal dichotomy

Existing narratives of the history of Austrian utility theories are typically concerned with the contrast between ordinal and cardinal views of utility. While the notion of ordinal utility is univocal – utility is ordinal if it expresses only the ranking of preferences – the concept of cardinal utility may indicate (at least) two different forms of utility: 'cardinal utility' in the specific sense this term has in current economic theory, and what I call 'classically measurable utility'.

In current economic theory, a utility function is called 'cardinal' if it is unique up to positive linear (or affine positive) transformations.[2] Different assumptions generate cardinal utility in this specific sense. However, in the period under consideration in this chapter, cardinal utility had been associated with the hypothesis that individuals are

normally capable of stating that the utility difference between two goods is larger than the utility difference between two other goods.

In contrast, 'classically measurable utility' is the form of utility that corresponds to the classical concept of measurement: utility is classically measurable if it is possible to identify a unit of utility and assess utility ratios.[3] If utility is measurable in the classical sense, the existence of a unit of utility permits the ranking of utility differences and so warrants the measurability of utility also in the cardinal utility sense. The reverse, however, is not true; the cardinal measurability of utility does not entail its classical measurability, because the ranking of utility differences does not allow for the assessment of utility ratios.

As I have argued elsewhere (Moscati 2013a), the fact that these two different forms of utility have been often conflated under the single label of 'cardinal utility' has generated a number of misunderstandings in the history of utility theory. In this chapter, therefore, I move beyond the traditional cardinal–ordinal dichotomy, and base my reconstruction of the Austrian discussions of utility measurement on a threefold conceptual framework rooted in the categories of: (i) ordinal utility, (ii) classically measurable utility, and (iii) cardinal utility in the current sense of the term.

In particular, the relevant divide until World War I was that between classically measurable utility and ordinal utility. Cardinal utility in the current sense of the term began to play some role only in the 1910s. From a terminological viewpoint, the very expression 'cardinal utility' began to be employed even later, namely in the mid-1930s.

The analogy of quantity

This chapter emphasizes the fallacies generated by what the English economist Henry Phelps Brown called the 'analogy of quantity'. These are fallacies induced by representing psychological phenomena through numbers without keeping clearly in mind that not all properties of numbers extend to the psychological phenomena at issue:

> The analogy of quantity [...] though permissible, is dangerous, because quantities have properties which we cannot easily banish from our thoughts, and some of these properties have no part in the [...] analogy. (Phelps Brown 1934, 68–69)

As we will see, in dealing with issues related to the measurability of utility most Austrian economists did incur fallacies related to the analogy of quantity. Basically, utilities were represented through numbers, and then the properties of these numbers – such as the capacity of expressing ratios, or the possibility of being summed, subtracted and multiplied – were

extended to utilities, without any discussion of whether this extension was warranted.

The Austrians and the Ordinal Revolution

In the history of utility theory, the economists' adoption of the idea that utility is ordinal in nature – and, more importantly, the demonstration that the main results of demand and equilibrium analysis can be based on ordinal utility only – is usually called the 'Ordinal Revolution'. According to a well-established reconstruction, the ordinal approach to utility was inaugurated by Irving Fisher (1892) and significantly developed by Vilfredo Pareto ([1909] 1971); then, after some developments in the 1910s and 1920s, the ordinal revolution underwent a sudden and decisive acceleration in the mid-1930s: in 1934 an influential article co-written by two scholars based at the LSE, namely John Hicks and Roy Allen (1934), began the conclusive phase of the ordinal revolution, which was virtually completed by Hicks's *Value and Capital* (1939).

Austrian economists and historians of economics writing during or after the completion of the ordinal revolution have sometimes claimed that the Austrians developed an ordinal approach to utility well before, and independently of, Fisher, Pareto and the other 'ordinal revolutionaries'.[4] In this chapter, I show that this claim, although correct to some extent, requires substantial qualification.

It is true that in Wieser's first book, published in 1884, we already find explicit declarations that utilities can be ranked but not be measured. However, in practice Wieser and other early Austrians treated utilities as if they were classically measurable magnitudes. Moreover, it should be kept in mind that while Fisher, Pareto or Hicks built their theories on *total* ordinal utility, the early Austrians focused on *marginal* utility, which they declared to be ordinal. The differences between total ordinal utility and marginal ordinal utility make comparison between the Fisher–Pareto–Hicks theory and the Austrian theory tricky, and render problematic any claim concerning the alleged priority of one group over the other in discovering the 'holy grail' of ordinal utility.[5] In any case, in this chapter, I will focus on the Austrian part of the history of ordinal utility, and bring into the picture the Fisher–Pareto–Hicks part only when necessary to better contextualize and understand the Austrian approaches to utility measurement.

Menger, 1871[6]

Carl Menger (1840–1921) avoided taking an open stance on issues surrounding the measurability of utility. Neither in Hayek's edition of

Menger's collected works (1934–1936), nor in the second edition of the *Principles of Economics* (1923), was I able to trace any explicit discussions of the topic. However, based on Menger's analysis of money as a possible measure of exchange value and his use of numbers to express the marginal utility of goods, we can draw some conclusions about his understanding of measurement and his stance on the measurability of utility.

The measurement of exchange value

In the *Principles* ([1871] 1981, 272–280), as well in other works, Menger discussed the issue of whether money measures the exchange value of goods. His most thorough discussion of the issue can be found in a dictionary entry on money published in 1909. Here, he asked whether 'the valuation of goods in money [should] be regarded as measurement of their exchange value by the monetary unit' ([1909] 2002, 60).[7] In answering, Menger first described measurement as 'a procedure by which we determine the as yet unknown magnitude of an object by comparison with a known magnitude of the same kind taken as a unit' (60); then he claimed that money cannot measure the exchange value of goods because, unlike the fixed and invariable units used in physical measurement, the exchange value of the monetary unit changes.

Utility numbers

In the *Principles*, Menger argued that the value to an individual of a given quantity of a good is measured by the importance of the need-satisfaction assured by the last unit of the good, that is, by what today we would call the marginal utility of that unit ([1871] 1981, 132).

In particular, Menger considered the marginal utility of ten different goods, assumed that the marginal utility of each good is decreasing, and associated the decreasing marginal utilities with a decreasing series of numbers. For instance, he imagined that for a particular individual the first unit of the first good, identified as food, has a marginal utility of 10, the second unit has a marginal utility of 9, the units following have a marginal utility of 8, 7, 6, etc., respectively, while the eleventh unit of that first good has zero marginal utility. The marginal utilities of the remaining nine goods display a similar trend.

Menger represented this situation by a numerical table, which is reproduced in Figure 4.1. In the table, each good corresponds to a column and a Roman numeral (for instance, food is associated with the first column and the Roman numeral I). Each row indicates which unit of the good is considered (the first unit, the second, etc). The Arabic numbers in the cells express the marginal utility of each unit of the

	I	II	III	IV	V	VI	VII	VIII	IX	X
1st unit	10	9	8	7	6	5	4	3	2	1
2nd	9	8	7	6	5	4	3	2	1	0
3rd	8	7	6	5	4	3	2	1	0	
4th	7	6	5	4	3	2	1	0		
5th	6	5	4	3	2	1	0			
6th	5	4	3	2	1	0				
7th	4	3	2	1	0					
8th	3	2	1	0						
9th	2	1	0							
10th	1	0								
11th	0									

Figure 4.1 Menger's utility numbers, in Menger ([1871] 1981, 127)

good under consideration (for instance, the marginal utility of the sixth unit of good IV is 2).

Menger never declared explicitly that the numbers he used have only an ordinal meaning, but he might have intended them in this way. However, even if this was the case he fell victim to the analogy of quantity, because in several passages of the *Principles* he treated the numbers in the table as if they measured *marginal* utilities in the classical sense of the term.

First, Menger took for granted that there is a zero point of marginal utility (126–127, 135, 183–186), although in a purely ordinal approach, where the individual can only state whether a marginal utility is larger or smaller than another, the proper way to identify a zero point of utility is far from obvious. Moreover, on two occasions Menger claimed that the ratio of his utility numbers expresses the ratio of marginal utilities. In a footnote contained in the chapter of the *Principles* devoted to the theory of exchange, Menger first made a statement that looks ordinal in spirit:

> I need hardly point out that the figures in the text are not intended to express numerically the *absolute* but merely the *relative* magnitudes of importance of the satisfactions in question. (183)

However, by 'relative magnitudes of importance' he did not mean the ranking of marginal utilities, but their ratio. In fact he continued:

> When I designate the importance of two need-satisfactions with 40 and 20 for example, I am merely saying that the first of the two satisfactions has twice the importance of the second to the economizing individual concerned. (183)

In another passage Menger argued that if the marginal utility of a cow to an individual is 10 while the marginal utility of an additional horse is 30, then the horse has 'three times the value of a cow' (184). All of this indicates that Menger's utility numbers express marginal utility in terms of some (unspecified) unit of satisfaction, that is, that they measure marginal utilities in the classical sense of the term.

To sum up the above discussion, we can say that Menger understood measurement in the classical fashion and, possibly because he committed fallacies in the analogy of quantity, treated marginal utility as if it were classically measurable.

Wieser, 1884 and 1889

For more than a decade after its publication there was no sign of public interest in Menger's *Principles*.[8] But two fellow students at Vienna University read the book, were struck by the theories expounded in it, and became Menger's first disciples. The two fellow students (and future brothers-in-law) were Wieser (1851–1926) and Böhm-Bawerk (1851–1914).

In 1884, Wieser published the second important contribution to Austrian marginal utility theory after Menger's *Principles*, namely a treatise entitled *Über den Ursprung und die Hauptgesetze des wirtschaftlichen Werthes* (On the Origin and Fundamental Laws of Economic Value). This work is usually remembered in the history of economics because in it Wieser extended Menger's explanation of the value of production factors and introduced into German the term *Grenznutzen* (1884, 128).

Measurable value, unmeasurable utility

With respect to issues concerning the measurability of utility, in the *Ursprung* Wieser took a stance that is ordinal in character. He contrasted the unmeasurability of the psychical phenomena that are at the origin of economic value, such as wants, desires and interests, with the meas-

urability of economic value. By measurement, he understood classical measurement:

> [Economic] value is not only evaluated, but also measured [...], that is, value is not only ascribed a magnitude, but the ascribed magnitude is also reduced to a unit, a yardstick, and expressed as multiple of it. (180)

In practice, for Wieser economic value is measured by money.

Wieser argued that the psychical phenomena at the origin of economic value also have a magnitude and, more specifically, an intensity. However, this intensity cannot be measured. We can compare the intensities of psychical phenomena, such as interest in goods, and possibly even compare the difference between these intensities, but we are not able to measure an intensity, that is, to express it as a multiple of a unit:

> We are able to state that the intensities of certain interests are of equal or different magnitude; we can even state whether the difference between the perceived intensity levels is larger or smaller [...]. But we are not capable of [...] reducing them to a unit; we are not even able to specify how many times one level of interest is stronger than, i.e. a multiple of, another level. (180)

In this passage, Wieser talked of 'interest' rather than 'utility', but from the context of his discussion it is clear that the intensity of the interest in a good coincides with the marginal utility of the good. Thus, if we forget for a moment the very brief reference to the comparison of intensity differences, the passage suggests that Wieser adopted a fundamentally ordinal conception of marginal utility. The solution Wieser gave to the problem of reconciling the unmeasurability of marginal utility with the measurability of economic value shows that, in effect, his ordinal conception was far from consistent.

Summing and multiplying marginal utilities

Wieser argued that the economic value W (W stands for *Werth*, value) of n items of a given good is equal to the number of items, n, multiplied by the marginal utility of the least useful item, indicated as I_1 (I stands for *Intensität*, intensity). Wieser even expressed this idea in a mathematical formula: $W=n\times I_1$ or $= I_1+ I_1+ I_1+ I_1+ \ldots n$ times (196).

Wieser's formula is questionable in a number of respects, and became in effect the subject of an intense discussion among Austrian economists. With regard to the specific issue of utility measurement, the formula is problematic because it requires that the *I*-figures associated with marginal utilities have a more-than-ordinal meaning. If these were in fact purely ordinal, the economic value of a given quantity of a good could be larger or smaller than the economic value of another quantity, depending on the ordinal magnitude chosen for the *I*-figures.[9] More generally, and as will become clear much later in the Austrian debates on the measurability of utility, if figures have only an ordinal meaning their summation and multiplication is meaningless (see Section 'Multiplication is summation' below).

Wieser's second book is entitled *Natural Value* ([1889] 1893). While Wieser now extended and systematized many of the theories presented in the first work, his approach to utility measurement remained fundamentally the same. On the one hand, he explicitly introduced the notion of the 'scale of satiation' to indicate the series of decreasing numbers that represent the series of decreasing marginal utilities, and stressed that marginal utilities cannot be measured (10–11); on the other hand, he continued to calculate the economic value of goods by summing and multiplying the marginal utility numbers in a way that contradicts the ordinal conception of them (27–32).

Conclusions on Wieser

Unlike Menger, Wieser declared explicitly that the marginal utilities of goods can be compared but not measured. However, *de facto* Wieser treated marginal utilities just like Menger; he associated them with numbers, and then summed and multiplied these numbers as if they measured utilities in terms of some unit of satisfaction. I surmise that Wieser also fell victim to the analogy of quantity; when the numbers were there, the temptation to sum, subtract and multiply them was just too strong to be resisted.

Böhm-Bawerk, 1886

The other early disciple of Menger was Böhm-Bawerk. In his first two books, Böhm-Bawerk ([1881] 1962, [1884] 1890) dealt only tangentially with marginal utility theory, and did not discuss issues related to utility measurement. His first important contribution to marginalism was a long, two-part article entitled '*Grundzüge der Theorie des wirtschaftlichen*

Guterwerthes' (Fundamental Elements of the Theory of the Economic Value of Goods), that was published in 1886 in the *Jahrbucher für Nationalökonomie und Statistik*, the leading German economic journal ([1886] 1932). In contrast to Wieser, Böhm-Bawerk argued in favour of the measurability of utility.

The case for the measurability of utility

In the first two sections of the 'Grundzüge', Böhm-Bawerk presented the theory of marginal utility very much along the lines of Menger, and reproduced Menger's table of marginal utilities (25). Then, in Section 3, he explicitly addressed the criticism raised by the German economist Friedrich Julius Neumann against the claim of the measurability of sensations and desires.[10] In particular, Neumann (1882) had argued that:

> It is impossible for me to say that this picture of my father [...] is worth to me 1¼ [...] times as much as the picture of my brother [...]. The totality of the sensations, desires, interests, etc. that are here in question cannot be at all reduced to units, and therefore is not subject to measurement. (quoted in Böhm-Bawerk [1886] 1932, 46)

To Neumann's criticisms, Böhm-Bawerk replied that in the first place we are at least undoubtedly able to compare different sensations of pleasure and state whether one sensation is stronger or weaker than another. Like Wieser, he added in passing that we are also able to compare differences of sensations, and judge 'whether one sensation of pleasure is considerably or only negligibly stronger than another' (48). Then Böhm-Bawerk asked whether we could do even more, and judge whether one sensation of pleasure is, for example, three times as large as another, and boldly answered that: 'I definitely believe we can do that.' (48)

To make his case, Böhm-Bawerk argued that when we face the alternative between one greatest pleasure on the one hand and a multiplicity of lesser pleasures on the other, in order to make a decision it is not sufficient to judge that the first kind of pleasure is greater than the second kind. Not even judging that the first kind of pleasure is *considerably* greater than the second kind would do. For Böhm-Bawerk, the decision between those two alternatives requires us to judge 'how many times greater the one pleasure is than the other' (48). For example, if a boy has to choose between one apple and six plums, he must judge 'whether the pleasure of eating an apple is *more or less six times greater* than the pleasure of eating a plum' (48). Böhm-Bawerk's line of thought has an

obvious corollary: if the boy considers as equal the pleasure of eating an apple and the pleasure of eating six plums, than the pleasure of eating the apple is *exactly* six times greater than the pleasure of eating a plum (49–50, footnote 2).[11]

From the 'Grundzüge' to *The Positive Theory of Capital*

We postpone the appraisal of Böhm-Bawerk's argument to the next section, where we examine the views on utility measurement of Čuhel (1907), and illustrate his critique of Böhm-Bawerk's case for the measurability of utility. Here, we only mention that large parts of the 'Grundzüge' passed with only minor modifications into *The Positive Theory of Capital*, Part 2 of *Capital and Interest*, the first edition of which appeared in 1889 (second, unaltered edition, 1902; third edition 1909–12).[12] However, while Böhm-Bawerk dropped from the first two editions of the *Positive Theory* the case for the measurability of utility based on the apple–plum example, he re-inserted the case in the third edition, principally in response to Čuhel's criticism of the measurability of utility.

Čuhel, 1907

No new significant contribution to the issue of utility measurement appeared in the Austrian literature during the 1890s and early 1900s. As mentioned in the Introduction, however, outside Austria in this period the Ordinal Revolution began. In his *Mathematical Investigations in the Theory of Value and Prices* (1892), Fisher began to carry out an ordinal approach to utility based on indifference curves. This approach was significantly developed by Pareto in a series of writings ([1898] 1966, [1900] 2008, 1901 [1966], 1906) that culminated in the French edition of the *Manual of Political Economy* ([1909] 1971).[13] Within Austrian economics, a clear case for ordinal utility was made by a scholar little known in the history of economics, the Czech František (or Franz, as he called himself when publishing in German) Čuhel.

Introducing Čuhel

Čuhel (1862–1914) studied law in Vienna and Prague in the 1880s, and after graduation became a clerk in the Prague Chamber of Commerce.[14] In 1903 he retired from his job and devoted himself to research. He moved back to Vienna, and participated in Böhm-Bawerk's famous economics seminar; in 1905, Böhm-Bawerk, after more than 15 years spent as a civil servant in the Austrian administration and government, had returned to academic life as professor at Vienna University (previously

he had been professor at Innsbruck University) and began running an economics seminar that attracted many gifted students, such as Mises and Schumpeter, and quickly became a focal point for the elaboration of Austrian economic theory.[15] It is not irrelevant to mention here that since 1903 Wieser had also been a professor at Vienna, as in that year he had taken up Menger's chair when the latter retired.

In 1907 Čuhel published his only book, *Zur Lehre von den Bedürfnissen* (*On the Theory of Needs*), which bore the subtitle 'Investigations in the boundary region between economics and psychology'. In his introduction, Čuhel thanked Böhm-Bawerk for having supported the publication of the book, and Menger for helpful suggestions. Unfortunately, the book was not sufficient to secure Čuhel an academic position, and in 1908 he returned to administrative work in Vienna.

The starting point of Čuhel's work was the observation that the concept of need, although fundamental to economics and particularly to marginal utility theory, had not been sufficiently investigated in the literature. Čuhel therefore undertook a painstaking analysis of this concept that led him to distinguish no less than 29 different types of need. To name them, Čuhel introduced a rich new terminology, and also proposed substituting the term 'utility' with the allegedly more precise technical term *Egenz*.

As Čuhel himself acknowledged, his investigations remained fundamentally in the region of psychology and belonged to economics only to a minor degree. For our purposes, the relevant part of Čuhel's *Lehre* is its Chapter 6, where he addressed the issue of the measurability of *Egenzen*, i.e. utilities.[16]

Decreasing marginal utility prevents utility measurement

Čuhel distinguished, in the first place, between 'comparing' and 'measuring' magnitudes. In comparing two magnitudes, he argued, it is sufficient to state which of them is the larger, 'but the amount by which it is larger is not ascertained in the comparison' ([1907] 1994, 313). For measuring, more is required, namely finding 'a number which indicates how many times a magnitude accepted as a unit is contained in the magnitude to be measured' (313). Thus, Čuhel also adhered to the classical concept of measurement.

Having made this preliminary distinction, Čuhel moved to the question of whether the utilities of goods can be not only compared – which he took as self-evident – but also measured. He answered that in principle an ideal unit of utility could be conceived of, and that utilities could therefore be expressed as multiples of this ideal unit. In practice,

however, the measurement of utility is impossible. The main obstacle to the possibility of measuring utilities is constituted by what was, especially for the Austrian marginalists, the fundamental law of marginal utility theory, namely the law of decreasing marginal utility.

To illustrate the issue, Čuhel considered Böhm-Bawerk's example involving apples and plums, and imagined that for a particular individual the utility of one apple is equal to the utility of six plums. If the utilities of all plums were identical, one could take the utility of any plum as a unit, and state that the utility of one apple is six times the utility of a plum. But because of the law of decreasing marginal utility, the utilities of the six plums are *not* identical, but precisely decreasing: U(1st plum)>U(2nd plum)>…>U(6th plum).[17] Moreover, we have no idea about how much larger the utility of each plum is compared to the utility of the following plum in the series. Therefore, according to Čuhel, the only thing we can state is that the utility of one apple is equal to the *sum* of the utility of the first plum, the utility of the second plum, etc.: U(1 apple)=U(1st plum)+… +U(6th plum). By this, however, we are not expressing the utility of the apple as a multiple of a certain unit; that is, we are not measuring the apple's utility (315–316). In other words, Čuhel noticed that because of the law of decreasing marginal utility

> *there is no unit of egence [utility] which would remain unchanged if one were to form multiples of any size of it.* (italics in original, 317)

The variability of the utility unit makes the economist who tries to measure utility similar to a physicist who attempts to measure the weight of a body using a balance in a situation in which

> the attraction the earth exerts on the balance weights were paralyzed to an extent, not precisely determinable, by the mutual attraction of the weights if there were more than one of them on the pan at once. (317)[18]

Numerals and letters

In conclusion, for Čuhel utilities cannot be measured, but only compared. This does not mean that we cannot associate figures with utilities. However, these figures can serve only as 'ordinal numerals' (*Ordnungszahlwörten*) that express the ranking of utilities, and should not be taken as 'basic numerals' (*Grunzahlwörten*), that is, standard numbers capable of expressing proportionality with respect to a unit (323).[19]

As an example of *Ordnungszahlwörten*, Čuhel considered the figures used in the Mohs scale of mineral hardness. This is the scale introduced by the German geologist Friedrich Mohs to order minerals according to their ability to scratch each other, that is, according to their 'hardness': if one mineral can scratch another, the former is harder than the latter and is assigned a higher numeral in the scale. But the numerals in the Mohs scale, stressed Čuhel, do not identify a unit, and therefore do not allow us to express the hardness of a mineral as a multiple of the unit:

> Saying that talc has the first degree of hardness and gypsum the second degree of hardness certainly does not mean that gypsum is twice as hard as talc, but only that it is harder to a certain extent. (322–323)

For Čuhel, the figures associated with utilities should be intended in the same sense as the figures of the Mohs scale.

Čuhel even pointed out that when figures are aimed only at representing the utility ranking, they could well be substituted by letters, whereby higher utilities could be associated with letters that come later in the alphabetical order (325). Thus if the plum's utility is smaller than the apple's utility, this ranking can be represented by associating the letters *a* and *b* respectively with these utilities.

Attacking Böhm-Bawerk

Based on the above illustration of Čuhel's views on the measurability of utility, it should be easy to see why he criticized Böhm-Bawerk's stance on the subject. Böhm-Bawerk had argued that in order to decide between one apple and six plums an individual would need to judge how many times greater the pleasure of eating the apple is than the pleasure of eating one plum. Čuhel replied that in fact the individual only needs to judge whether the first kind of pleasure is larger or smaller than the *sum* of the six smaller pleasures. And to make this latter judgment, knowledge of how many times larger is the pleasure of eating the apple than the pleasure of eating the n^{th} plum is 'completely superfluous.' (320)

Analogously, if the individual is indifferent between one apple and six plums this does not mean that the pleasure of eating one apple is exactly six times greater than the pleasure of eating a plum. In fact, the law of decreasing marginal utility modifies the pleasure we obtain from eating the successive plum-units, and therefore prevent us from expressing the pleasure of eating the apple as a multiple of the pleasure of eating a plum.[20]

Conclusions on Čuhel

Of the Austrian economists, Čuhel was apparently the first to make an explicit and extensive case for ordinal utility. Outside Austria and before Čuhel, Fisher and Pareto had also advocated an ordinal approach to utility. In his book, Čuhel occasionally cited Fisher's 1892 *Investigations*, but did not mention Pareto.

One main difference between Čuhel's approach to ordinal utility and the approach of Fisher and Pareto is that while the latter two economists connected ordinal utility to demand and equilibrium analysis Čuhel did not. Thus, whereas Fisher and, more systematically, Pareto attempted to show that the main results of demand and equilibrium theory do not depend on the measurability of utility, Čuhel's discussion of utility measurement remained fundamentally in the field of psychology. In effect, and as we will see, this lack of interest in exploring in a systematic way the implications of an ordinal conception of utility on demand and equilibrium analysis is a characteristic not only of Čuhel's ordinalism but of Austrian ordinalism in general.

Another important difference between Čuhel's approach and the Fisher–Pareto one is that for Čuhel, as for the other early Austrian economists, the basic concept of utility theory was that of marginal utility, while Fisher and Pareto started directly with total utility. As a consequence, in the Fisher–Pareto approach the very problem discussed by Böhm-Bawerk and Čuhel – 'How can the boy decide between one apple and six plums?' – is not a problem at all, as the boy can directly compare the total utilities of the two alternatives without multiplying or summing anything.

In contrast, in Čuhel's Austrian approach the total utility of a set of goods is not directly given but it is derived from the marginal utilities of the items in the set. Therefore, the boy first needs to find out the total utility of the six plums by multiplying (Böhm-Bawerk) or summing (Čuhel) their marginal utilities, and can only then compare the total utility of the six plums with the total utility of the apple (as the apple is a single item, its total and marginal utilities coincide). The fact that Čuhel stuck to marginal utility as the primary notion of utility analysis created a number of inconsistencies in his ordinal approach. As we will see in Section 'Multiplication is summation', these inconsistencies were spotted by Böhm-Bawerk, who exploited them to argue that he and Čuhel ultimately conceived utility in the same way.

Finally, Čuhel's idea of using letters instead of numbers to represent utility rankings suggests that Čuhel was somehow aware of the perils concealed in the analogy of quantity. However, even Čuhel fell victim

to the analogy's attraction. In fact, as we have seen above, he *summed* the figures representing utilities, although if these figures have a purely ordinal character it is meaningless to sum them. If we indeed use letters to represent the utility ranking, this becomes apparent: if the plum's utility is associated with the letter *a* and the apple's utility with the letter *b*, what is the meaning of *a*+*b*?

Schumpeter, 1908

Before examining Böhm-Bawerk's response to Čuhel's criticisms, we consider Schumpeter's juvenile stance on the measurability of utility. Together with Hans Mayer and Ludwig von Mises, Schumpeter (1883–1950) belongs to the so-called third generation of the Austrian school. He had both Wieser and Böhm-Bawerk among his teachers, but was more influenced by the latter.[21] In his first book, entitled *The Nature and Essence of Economic Theory* and published in 1908, Schumpeter ([1908] 2010, 72–74) devoted a brief section to the problem of the measurability of utility without mentioning Čuhel.

Schumpeter asserted that it is indeed possible to measure the utilities of goods by taking as a unit the utility of an arbitrarily chosen quantity of an arbitrarily chosen good, and then expressing the other utilities as a multiple of this unit (73). However, instead of discussing in detail how all this could be done, Schumpeter redirected the reader to Irving Fisher's *Investigations* (1892) for more details.[22] Since Fisher's method of measuring utility will be analyzed in detail by Mises, we postpone the discussion of it to Section 'Mises, 1912'.

To conclude this brief section on Schumpeter, we should mention that he changed his views on utility measurement. Later he became a supporter of the ordinal approach to utility, and in his *History of Economic Analysis* he described the development of utility theory as a progressive emancipation from the erroneous idea that the measurability of utility is necessary to demand and equilibrium analysis (1954, 1053–1069). In particular, Schumpeter attributed to Pareto the merit of developing the idea of ordinal utility and working out the fundamentals of the modern theory of value.

Böhm-Bawerk reloaded, 1912

In the third and final edition of *The Positive Theory of Capital* (Part I, 1909; Part II, 1912), Böhm-Bawerk inserted new notes, passages and one excursus on utility measurement, designed mainly to address Čuhel's

objections.[23] Although Böhm-Bawerk made some concessions to Čuhel and the ordinal approach to utility, he fundamentally maintained his original stance according to which utility is measurable. In particular, he re-inserted into the *Positive Theory* the passages of the 'Grundzüge' that had been excluded from the first two editions of the book, and in which he claimed that we can indeed judge how much larger one sensation of pleasure is than another. Čuhel's objections are addressed in detail in Excursus X. Here Böhm-Bawerk begins by declaring, in a conciliatory fashion, that that 'the kernel of this objection is correct' ([1912] 1959, vol. III, 124). But he then counterattacked with two arguments.

Precise vs. imprecise measurement

Böhm-Bawerk's first argument against Čuhel is based on the distinction between 'precise' and 'imprecise' measurement. Precise measurement requires the perfect equality of the employed units. However, observed Böhm-Bawerk, we do measure things even when this perfect equality is lacking:

> Nothing is more commonplace than to measure distances by steps, whereby the complete equality of each single step [...] cannot be guaranteed. (128)

When the units are not perfectly equal, we obtain imprecise measures. However, imprecision does not transform measurement into ordinal ranking:

> The lack of accuracy of a procedure changes measuring into ranking, no less than its accuracy changes ranking into measurement. (128)

In relation to utility measurement, Böhm-Bawerk claimed that by making the utility units unequal the law of decreasing marginal utilities does not render utility measurements practically impossible, as Čuhel had argued, but only imprecise. For Böhm-Bawerk, these imprecise utility measurements are sufficient for most practical purposes, just as for most practical purposes it is sufficient to measure distances by steps (130–131).

Multiplication is summation

With his second argument, Böhm-Bawerk pointed out two inconsistencies in Čuhel's analysis of the measurability of utility that render Čuhel's position on the subject untenable.

As we have seen, Čuhel denied the possibility of expressing one utility as a *multiple* of another, but considered it meaningful to *sum* the figures representing the utilities. Böhm-Bawerk noticed that if these figures represent only a ranking it would be meaningless to sum them. To illustrate this point he reconsidered Čuhel's example of the Mohs scale of mineral hardness. Böhm-Bawerk acknowledged that these numerals do not allow us to state that, for example, a mineral of the 8th degree of hardness is four times as hard as a mineral of the 2nd degree. But then, he added, the Mohs numbers do not even allow us to express the hardness of one mineral as the sum of the hardness of other minerals:

> We can by no means maintain that a mineral of the 8th degree is as hard as three minerals of the 5th, 2nd, and 1st degree together. (131)

Similarly, if utility numerals represent only a ranking, we can by no means sum them.

To illustrate the second inconsistency of Čuhel's analysis, Böhm-Bawerk began by observing that multiplication is only a special case of summation, namely summation of equal quantities. But then, Böhm-Bawerk went on, if Čuhel allows for the possibility of summing utilities, then he should allow also for the possibility of expressing one utility as a multiple of another:

> In the case of intensities of sensations and 'egences', according to Čuhel's own concession, summation of unequal quantities is feasible. Therefore summation of equal quantities or, what is the same, the determination of a multiple of a quantity, cannot be unfeasible. (132)

In other words, for Böhm-Bawerk the fact that Čuhel had summed the numerals representing utility showed that even for Čuhel the numerical determination of utilities was different from the numerical determination of mineral hardness. More precisely, Böhm-Bawerk concluded that as multiplication is only a special case of summation, his and Čuhel's numerical determination of utilities are in fact of one and the same kind.

Conclusions on Böhm-Bawerk

Böhm-Bawerk's arguments expound the inconsistencies in Čuhel's ordinal utility approach that are related to the fact that Čuhel maintained

his conception of the comparison of total utilities as dependent on the sum of marginal utilities. But if one allows for the possibility that the individual can directly compare total utilities, such arguments lose their force. In fact, if the individual can directly compare the total utility of the apple with the total utility of the six plums, he does not need to *measure*, not even in an imprecise way, the total utility of the six plums in order to make a choice. The individual does not even need to *sum* the marginal utilities of the plums.

Building on Čuhel's analysis, however, Mises was able to free himself from the traditional Austrian idea that the comparison of total utilities requires the summation of marginal utilities and thus to elaborate an ordinal view of utility impervious to arguments like those used by Böhm-Bawerk.

Mises, 1912

Mises (1881–1973) enrolled in the University of Vienna in 1900 and when in 1905 Böhm-Bawerk began running his seminar together with Čuhel, Schumpeter and others, Mises became a regular participant. In 1906 he graduated and in 1912 he published *The Theory of Money and Credit*, a systematic and comprehensive treatise on monetary topics ([1912] 1953). The book was intensively discussed in Böhm-Bawerk's seminar, and became a key reference for the Austrian theory of money and credit.

In Chapter 2 of this book, Mises broadened the discussion from monetary theory to value theory. Without mentioning Pareto, he argued that subjective value, that is utility, cannot be measured but only ordered, and he criticized the stances of Böhm-Bawerk, Fisher, Schumpeter and Wieser on utility measurement. As we will see, Mises' arguments were in fact to a large extent a restatement of the arguments Čuhel had used against Böhm-Bawerk.

Against Böhm-Bawerk

Mises clearly stated that subjective evaluations concerning the significance of goods do not measure this significance, that is, they do not assess the significance of a good as a multiple of some unit or as a fraction of the significance of another good. Subjective evaluations only arrange goods in order of their significance. Among the many passages in which Mises made this point, the following is particularly telling:

> It is impossible to measure subjective use-value [...]. We may say, the value of this commodity is greater than the value of that; but it is not

> permissible for us to assert, this commodity is worth *so much*. Such a way of speaking necessarily implies a definite unit. It really amounts to stating how many times a given unit is contained in the quantity to be defined. But this kind of calculation is quite inapplicable to processes of valuation. (45)

Mises then examined Böhm-Bawerk's 1886 argument in favour of the measurability of utility – namely, that if eating an apple is preferred to eating six plums this means that the pleasure of eating the apple is at least six times greater than the pleasure of eating a plum. Mises dismissed the argument by repeating Čuhel's point that the law of decreasing marginal utility modifies the pleasure obtained from eating the successive plum-units, and thus prevents us from concluding that the pleasures of eating an apple is larger than the pleasure of eating a plum multiplied by six. Mises paid full credit to Čuhel for this argument:

> The credit of having exposed the error contained in [Böhm-Bawerk's argument] belongs to Čuhel. (41)

It is important to note at this point that the first edition of Mises' book appeared in 1912, the same year as the volume of Böhm-Bawerk's *Positive Theory of Capital* containing the response to Čuhel was published. Thus the first edition of *Theory of Money* does not contain any reference to Böhm-Bawerk's response. As we will see, in the second edition of the book, published in 1924, Mises added a footnote on Böhm-Bawerk's rejoinder to Čuhel.

Against Fisher

After rejecting Böhm-Bawerk's case for the measurability of utility, Mises moved on to criticizing the method proposed by Fisher for measuring utility. As already mentioned, in his *Investigations* of 1892 Fisher outlined an ordinal approach to demand analysis based only on utility ranking. In Chapter 1 of this book, Fisher stressed that the ranking of utilities is different from their measurement, for measurement requires expressing one utility as a multiple of another utility taken as a unit. Fisher called this utility unit a 'util' (1892, 18). However, he added that if the marginal utility of each good depends only on the consumption of that good and is therefore independent of the consumption of other goods, then marginal utilities can indeed be measured.

To illustrate how this could be done, Fisher (14–18) supposed that an individual consumes 100 loaves of bread and B gallons of oil per

year, and that for him the marginal utility of the 100th loaf is equal to the marginal utility of an increment of β gallons over B. In symbols, U(100th loaf)=U(β). Moreover, Fisher imagined that for the individual the marginal utility of the 150th loaf equals the marginal utility of an increment of β/2 gallons over B: U(150th loaf)=U(β/2). Based on the assumption that the marginal utility of β is twice the marginal utility of β/2, that is, that U(β)=2×U(β/2), Fisher concluded that 'the utility of the 150th loaf is said to be half the utility of the 100th' (15). In particular, U(β/2) could be taken as the utility unit, and in this case U(150th loaf)=1 and U(100th loaf)=2.

Now, Mises ([1912] 1953, 43–44) pointed out that the key passage of Fisher's reasoning – namely, that the marginal utility of β is twice the marginal utility of β/2 – is undermined by the law of decreasing marginal utility. Mises' point against Fisher follows Čuhel's point against Böhm-Bawerk quite closely: just as the marginal utility of six plums taken together is not equal to the marginal utility of one plum multiplied by six, so the marginal utility of β is not equal to the marginal utility of β/2 multiplied by two. Rather, because of decreasing marginal utility, the marginal utility of β is smaller – but to an extent not precisely determinable – than the marginal utility of β/2 multiplied by two.

Mises also considered the defence of Fisher's method based on the argument that, since β and β/2 are infinitesimal quantities, the law of decreasing marginal utility does not apply to them. Mises objected that, in the first place, if β and β/2 are infinitesimal quantities, then they 'remain imperceptible to the valuer and cannot therefore affect his judgment' (44). As a consequence, the valuer would judge the marginal utility of β equal to, and not twice as large as, the marginal utility of β/2. Moreover, if β and β/2 are infinitesimal quantities it is impossible to equate their infinitesimal marginal utilities with the *finite* marginal utilities of the 100th and 150th loaves, and thus impossible to express the latter as a ratio of the former.

Against Schumpeter and Wieser

Mises concluded his attack on the non-ordinal approaches to utility by taking issue with Schumpeter and Wieser. As we have seen, in his 1908 book Schumpeter had claimed that utility is measurable by using Fisher's measurement method. Insofar as the latter is untenable, argued Mises, so also is Schumpeter's claim.

Wieser was criticized by Mises for the method he had suggested for calculating the economic value of a stock of goods, namely by multiplying the number of items constituting the stock by the marginal utility

of the least useful item (see above, Section 'Summing and multiplying marginal utilities'). Mises maintained that Wieser's method, in addition to other flaws,

> has the defect of assuming that it is possible to measure marginal utility, i.e., the 'intensity' of value. The fact that such measurement is impossible renders [Wieser's method] impracticable. (45)

Conclusions on Mises

As already mentioned, the first edition of Mises' book does not contain any reference to Böhm-Bawerk's response to Čuhel's criticisms, because both the *Theory of Money* and the last edition of the *Positive Theory of Capital* appeared in the same year (1912). At any rate, Böhm-Bawerk's most effective point against Čuhel – namely that by summing utilities Čuhel was treating them as classically measurable – does not apply to Mises' position. For Mises, in order to compare the utility of one apple and the utility of six plums neither multiplication nor the summation of utilities is needed. All that is needed is direct and immediate comparison between the two utilities:

> There is no value outside the process of valuation. There is no such thing as abstract value. [...] The person making the choice does not have to make use of notions about the value of units of the commodity. His process of valuation [...] is an immediate inference from considerations of the utilities at stake. [...] Like every other act of valuation, this is complete in itself. (47)

In part because Böhm-Bawerk's argument does not apply to his position, Mises, in the second edition of the *Theory of Money*, published in 1924, limited his comments on Böhm-Bawerk's rejoinder to Čuhel to a single sentence in a footnote. Here he dismissed Böhm-Bawerk's rejoinder by arguing that

> Böhm-Bawerk endeavoured to refute Čuhel's criticism, but did not succeed in putting forward any new considerations that could help towards a solution of the problem [of utility measurement]. (41)

Another, and possibly more important reason why in 1924 Mises did not feel it necessary to discuss Böhm-Bawerk's rejoinder in more detail is that, as we will see, by that time the ordinal approach to utility had become the dominant view among Austrian economists.

Two final brief comments on Mises and Čuhel are in order. First, unlike Čuhel, Mises eluded the pitfalls of the analogy of quantity. One possible reason for this elusion is that Mises focused on the direct comparison of total utilities and was therefore not tempted to sum marginal utilities to obtain total utility. Another possible reason is that Mises simply did not employ numbers to represent utilities. In accord with his general aversion to the use of mathematics in economics, in fact, Mises discussed utility comparisons in a purely discursive way.[24] Second, like Čuhel, and in contrast to Fisher and Pareto, Mises did not attempt to investigate in detail if and how the main results of demand and equilibrium analysis are affected by the unmeasurability of utility.

Wieser 1914, and summary of the pre-World War I period

Social economics

The last important economics book published in Austria before World War I was Wieser's *Social Economics*, a treatise intended as a handbook for students at Vienna University ([1914] 1927). Wieser's method of dealing with issues concerning utility measurement in this book is very similar to his puzzling handling of the subject in the *Ursprung* (1884) and *Natural Value* (1889).

Thus, Wieser first argued that marginal utilities 'cannot be computed or reduced to a common standard in multiples of which they may be represented' ([1914] 1927, 124). Rather, marginal utilities 'may be compared, but only in relative terms: i.e., greater, less or equal' (124). As in his 1884 book, Wieser added that in effect individuals can also compare the differences between marginal utilities, for there is

> a sense of distance (*Distanzgefühl*) that there is a greater spread between one pair of magnitudes than between another. (124)

A few lines further on, however, Wieser is to be found talking of 'units of utility' and claiming that the total utility of a given stock of a good 'can be considered [...] as a sum of units each of which is computed by the marginal utility' (124). He even declares that that 'the units of mass are at the same time units of utility' (124, 126). In his discussion of utility measurement, Wieser did not refer at all to what had been said on the subject by Böhm-Bawerk, Čuhel, Schumpeter or Mises in the 30 years that had elapsed since the publication of his (Wieser's) first book in 1884.

The 1871–1914 period: summing up

The fact that Wieser's stances on utility measurement did not evolve significantly between 1884 and 1914, and his failure to refer to the debates on the topic that had occurred in the meantime, may suggest that since Menger there had been little progress in the Austrian discussions on utility measurement. But this is not the case.

We have seen that during this period two main positions confronted each other: the view that, at least in principle, utility can be *measured* in the classical sense of the term, and the view that utility is ordinal in nature so that utilities can only be *ranked*. Among the explicit supporters of the first view were Böhm-Bawerk and the young Schumpeter, while Menger and Wieser treated marginal utilities as if they were *de facto* classically measurable magnitudes. The ordinal view was advocated by Čuhel and Mises.

In the course of the confrontation, Austrian ordinalism became progressively more explicit and clear-cut. This occurred by way of the overcoming of a series of internal contradictions resulting primarily from misuse of the analogy of quantity, such as the idea that numbers representing marginal utilities can be meaningfully summed, divided or multiplied. After World War I, and despite the fact that Wieser's *Social Economics* became the standard textbook for students enrolled at Vienna University, the ordinal approach to utility became the dominant one among Austrian economists.

Both Wieser and Böhm-Bawerk suggested very briefly that individuals can also compare the differences between marginal utilities. However, we have seen that before World War I the ranking of utility differences, and therefore cardinal utility in the sense of current economic theory, played almost no role in the Austrian discussions on utility measurement. After the war, however, this changed.

The interwar period: setting the stage

The rise of ordinalism

I was not able to find, in the Austrian publications on utility theory that appeared after World War I, arguments in favour of classically measurable utility similar to those put forward by Böhm-Bawerk or Schumpeter before the war. All post-war writers argued that utilities can be ranked but not measured, and did this without Wieser's inconsistencies. In my opinion, two main factors contributed to the rise of the ordinal approach to utility in Austrian economics

The first factor was Mises. The arguments in favour of the ordinal approach that he put forward in the *Theory of Money* are compelling and, as mentioned above, the book was widely read and discussed. In the 1920s and early 1930s, he reiterated his criticism of measurable utility and restated his case for a purely ordinal conception of utility in a number of other publications, such as his highly influential book on *Socialism* ([1922] 1951), an essay on the development of the subjective theory of value ([1931] 1978), and an article on the controversies in value theory ([1932] 1978). Besides, in the 1920s Mises' views on economics, and arguably on utility theory, rose to prominence through the fortnightly seminar he held in his office at the Chamber of Commerce. Mises' *Privatseminar* became for the fourth generation of Austrian economists – that is, those like Hayek who entered Vienna University after World War I – what Böhm-Bawerk's seminar had been for the economists of the third generation.[25]

The second factor is that Mises' ordinalism was very much in agreement with the ordinal approach of Pareto, which was gaining increasing acceptance after World War I outside Austria.[26] Therefore, in following Mises' ordinal approach to utility, the Austrians were very much 'joining the stream', and in the process the focus of their utility analysis shifted from marginal utility to total utility. This convergence of interwar Austrian utility theory with mainstream utility theory was part of a more general process of assimilation between the theories of the Austrian School, the Lausanne School and the Anglo-American School, which took place between the wars.[27]

Utility differences

The post-war Austrian consensus around ordinal utility was somewhat enlivened by a discussion of whether individuals are not only able to state whether one utility is larger or smaller than another, but are also capable of stating whether the difference between two utilities is larger or smaller than the difference between two other utilities. As we have seen, albeit in passing, both Wieser (1884, 180, 1914, 124) and Böhm-Bawerk (1886, 48, [1912] 1959, vol. II, 198) had suggested that the ranking of marginal utility differences is possible.

Apparently independently of Wieser and Böhm-Bawerk, Pareto also had briefly proposed a similar idea in his *Manual* ([1909] 1971, 192). In particular, Pareto had affirmed that, if individuals are capable of ranking *transitions* from one combination of goods to another, that is, of stating that in passing from combination A to combination B they experience more pleasure than in passing from combination B to combination C,

then the difference between the total utility indices associated with A and B is greater than the difference between the total utility indices associated with B and C; that is, U(A)–U(B)>U(B)–U(C).[28] However, Pareto took for granted that the ranking of transitions and the ranking of utility differences are one and the same thing, while, as became clear much later, this is not in fact the case.

In the 1920s and early 1930s, the issue of utility differences was discussed also by Austrian economists.

Austrian discussions of utility differences

When Wieser retired in 1923, Hans Mayer (1879–1955) was appointed to his chair at the University of Vienna. A circle of economists formed around Mayer, the leading figures of which were Leo Schönfeld (1888–1952), a veteran of Böhm-Bawerk's seminar and a link between the third and fourth generation of Austrian economists, and two young brilliant economists of the fourth generation, namely Paul Rosenstein-Rodan (1902–1985) and Oskar Morgenstern (1902–1977). The relationship between Mayer and Mises was one of open hostility.[29]

Schönfeld, 1924

In 1924, Schönfeld published a treatise on *Grenznutzen und Wirtschaftsrechnung* (Marginal Utility and Economic Calculation). He took total utility (*Gesamtwirtschaftsnutzen*) as the basic concept of his analysis, rather than a given system of needs of decreasing marginal importance as in the traditional Austrian approach (1924, 2). Accordingly, he conceived the economic problem of the individual as one of maximizing total utility, rather than allocating the available resources to independent uses displaying decreasing marginal utility.

In his work, Schönfeld also addressed the issues related to the measurability of total utility. He argued that it is impossible to state that 'one good is two or three times more useful than another', and therefore denied that utility is measurable (12). For him, utilities can only be compared, in the sense that it is only possible to state whether one utility is larger or smaller than, or equal to, another. In a footnote, he graciously conceded that Pareto and other mathematical economists had also acknowledged the unmeasurability of utility, and therefore had attempted to make their theories independent of measurability assumptions (13).

Schönfeld then claimed that the ordinal character of utility is not modified by the fact that individuals often attempt to judge whether

'one utility is "much" or "little" larger or smaller than another' (16). For Schönfeld, this kind of judgment remains a comparative evaluation and therefore does not imply that the intervals (*Spannungsräume*) between one utility and another are measurable.

Rosenstein-Rodan, 1927

Rosenstein-Rodan studied under Mayer, and after graduation became Mayer's research assistant. Unlike Morgenstern, Rosenstein-Rodan did not take part in Mises' *Privatseminar*.[30] In 1927, Rosenstein-Rodan published an important entry on 'Marginal Utility' in the German encyclopedia *Handwörterbuch der Staatswissenschaften*, in which he argued that 'utilities are not susceptible of exact quantitative measurement' ([1927] 1960, 75). However, he added, the measurement of utility is not necessary for economic calculation, since 'all we have to do is to decide whether one utility is larger or smaller than the other or whether they are equal' (75). Rosenstein-Rodan also considered it possible that individuals are able to rank utility differences, and cited Wieser in support of this hypothesis. However, Rosenstein-Rodan denied that individuals are also capable of stating how much larger or smaller the utility difference is:

> It may be possible [...] to state that the difference between two utilities is larger or smaller than, or equal to, the differences between two other utilities $(U_1 - U_2 \gtreqless U_3 - U_4)$, but [...] it is not possible to state how much larger or smaller the utility difference is. We have only 'a sense of distance', as regards the size of the divergence, as Wieser says in his *Social Economics*. (75)

Rosenstein-Rodan ended his discussion with a classification of the different opinions put forward on the measurability of utility. According to him, Menger, Böhm-Bawerk, Schumpeter and, among the non-Austrian economists, 'Pareto until 1900', belonged to the group of those who considered utility as (classically) measurable. In contrast, Čuhel, 'Wieser (to some extent)', Mayer, Schönfeld, together with 'Pareto after 1900' and other non-Austrian economists, believed that 'utilities are not measurable, but can be compared' (75).[31] Rosenstein-Rodan's classification is very much in accord with the one proposed in this chapter. However, Mises is strikingly missing from Rosenstein-Rodan's list of ordinalists. This omission could be connected with the fact that Rosenstein-Rodan did not participate in Mises' seminar, and with the acrimony between

Mises and Mayer: possibly Rosenstein-Rodan did not mention Mises to please Mayer, his academic boss.

For our purposes, it is important to notice that, in dealing with utility differences, Rosenstein-Rodan also fell victim to the pitfalls of the analogy of quantity. In fact, he failed to notice that, if the numbers U_1, U_2, U_3, and U_4 that represent utilities are ordinal in nature, then it makes little sense to subtract one number from the other and compare their differences.

Morgenstern, 1931

Morgenstern was another of Mayer's research assistants. He completed his degree in economics in 1925, and became a *Privatdozent* at Vienna University in 1928. At that time, Morgenstern also joined Rosenstein-Rodan as managing editor of the *Zeitschrift für Nationalökonomie*, a new economic journal that was edited by Mayer, and in the 1930s enjoyed a significant international standing.

In 1931, Morgenstern published an article on 'Die drei Grundtypen der Theorie des subjektiven Wertes' (The Three Fundamental Types of the Theory of Subjective Value), in a volume entitled *Probleme del Wertlehre* (*Problems of Value Theory*) that was co-edited by Mises and the German economist Arthur Spiethoff. In his article, Morgenstern outlined the Austrian version of utility theory and presented the Lausanne and the Anglo-American versions of the theory as imperfect variations of the Austrian doctrine. Morgenstern also touched upon measurability issues, basically repeating what Rosenstein-Rodan had written on the subject in 1927.

Morgenstern argued that economic subjects cannot measure utilities, but only rank them. In addition, they can 'compare the differences [...] between total economic utilities by comparing them two at time' (1931, 13). These two abilities, claimed Morgenstern, are all that the subjects need in order to behave rationally in the economy. He concluded his brief discussion by describing how the views on the measurability of utility had evolved: while 'Menger, Böhm-Bawerk, and others' had judged utility measurable, 'Wieser, Mayer, Mises' and others had considered it as unmeasurable (14). This time Mises, who was one of the editors of the volume in which Morgestern's article appeared, was included in the list of ordinalists.

Two final considerations on Morgenstern are in order. First, not even he noticed the tension between the ordinal character of the numbers representing utility and the possibility of comparing the differences between these numbers. Second, over the course of time Morgenstern became

sceptical about the possibility of defending the assumption that individuals are able to compare utility differences. In *Theory of Games and Economic Behavior* (1944), the book that Morgenstern wrote with John von Neumann, the authors noticed that this assumption is open to objections of artificiality, lack of empirical verification, and loss of simplicity (1944, 24–29). However, they showed that the main implication of the assumption – that is, that the utility function is unique up to positive linear transformations – can also be obtained on the basis of a set of different and apparently less problematic axioms concerning preferences over risky alternatives. But if the utility function turns out to be unique up to positive linear transformations, then it does make sense to compare utility differences. Therefore, albeit in a roundabout way, Morgenstern ultimately stuck to his 1931 endorsement of comparisons of utility differences.

Alt, 1936

Outside Austria, the discussion about the ranking of transitions and utility differences was revived by the Polish economist Oskar Lange. Lange (1934) claimed that the assumption that individuals are able to rank transitions from one combination of goods to another implies that the utility function representing their preferences is unique up to positive linear transformations. This uniqueness would in turn make it possible to state how many times one increase of utility is greater than another, and therefore would reintroduce measurability into utility theory. Lange's article ignited a debate that led to the definition of the current notion of cardinal utility as utility unique up to positive linear transformations. I have told the story of this debate elsewhere (Moscati 2013b), so in this section I mention only its main passages and the Austrian part of it.

Phelps Brown (1934) pointed out that Lange's claim depended on the identification of the ranking of transitions from one combination to another, with the ranking of the differences between the utility indices associated with the combinations. Phelps Brown showed, however, that the two rankings are different, that the capability of ranking transitions does not imply the capability of ranking utility differences, and therefore that the ranking of transitions does not entail that utility is unique up to positive linear transformations. It is in the context of his criticism of Lange that Phelps Brown talked about the dangers of the analogy of quantity. For Phelps Brown, in fact, Lange's unwarranted conclusions derived ultimately from his having extended the additive properties of numbers to preferences although the latter are ordinal in nature and therefore cannot be summed.

Phelps Brown did not investigate what additional assumptions could warrant the identification of the ranking of transitions with the ranking of utility differences. These conditions were identified by a young Viennese mathematician who was only indirectly connected with the Austrian School of economics, namely Franz Alt.

Alt (1910–2011) graduated in mathematics in 1932 with a dissertation under Karl Menger, the son of the founder of the Austrian School, and became a regular participant in Menger's seminar, the *Mathematische Kolloquium*. As a Jew, Alt had failed to obtain an academic position, but on Menger's recommendation was appointed by Morgenstern as a private tutor in mathematics when the latter decided to improve his mathematical skills. Through participation in the *Kolloquium* and his tutoring of Morgenstern, Alt became interested in the mathematical aspects of economics. In particular, he came to know about Lange's article and the debate it had initiated (Alt and Akera 2006, 7). Alt became interested in the issue, and addressed it in an article that was published in the June 1936 issue of Morgenstern's *Zeitschrift für Nationalökonomie* under the title 'On the Measurability of Utility' ([1936] 1971).

In this chapter, Alt showed that there is a biunivocal relationship between the ranking of transitions and the ranking of utility differences, and that utility is unique up to positive linear transformations if and only if a set of seven postulates is satisfied. The first two postulates require that individuals can rank combinations of goods as well as transitions from one combination to another. The other five postulates impose conditions of transitivity, continuity and additivity on these two rankings, and connect them.

Thus, like Mises, Alt did not fall into the fallacies of the analogy of quantity. But while Mises escaped these fallacies by avoiding the representation of utilities by numbers, Alt explicitly addressed the analogy of quantity and clarified just what specific assumptions on preferences and utilities are needed to sum and subtract the numbers representing them.

Conclusions on the Austrian discussions on utility differences

Wieser, Böhm-Bawerk, Schönfeld, Rosenstein-Rodan and Morgenstern suggested that individuals are capable of stating whether the difference between two utilities is larger or smaller than the difference between two other utilities. However, they did not explore the exact meaning and implications of this supposed capacity. The exact conditions under which the utility differences can be ranked, and utility is unique up to positive linear transformations, were identified by Alt in 1936.

Notably, Alt did *not* use the expression 'cardinal utility' to label utility that is unique up to positive linear transformation. In their influential 1934 article (see above, Section 'The Austrians and the Ordinal Revolution'), Hicks and Allen had indeed used the expression '"cardinal" conception of utility', but without clarifying what they meant by the term 'cardinal'. At any rate, they certainly did not associate it with the ranking of utility differences or the uniqueness up to positive linear transformations. The economist who coupled the term 'cardinal utility' with utility unique up to positive transformation was Paul Samuelson, in an article entitled 'The Numerical Representation of Ordered Classifications and the Concept of Utility' (Samuelson 1938).[32]

Hayek, 1925–1936

Hayek (1899–1992) entered Vienna University in late 1918, just after the end of World War I. He was a pupil of Wieser, and graduated under his supervision in law (1921) and in political science (1923). From the very beginning of his scientific career, Hayek's research interests concerned issues that were only indirectly related to utility analysis, such as the theory of money, capital, interest and business cycles. I was not able to find any text in which Hayek addressed the question of the measurability of utility in an extensive way. However, from what he occasionally wrote on the subject it is apparent that he endorsed an ordinal view of utility.

Vienna

At this stage of the story, Hayek's ordinal views on utility should hardly be surprising. During his formative years at Vienna – from 1918, when he entered the university, to 1931, when he left for the London School of Economics – Hayek was exposed to the ordinal view of utility that dominated Austrian economics after World War I. Moreover, the major influence on Hayek's economic thought was Mises who, as we have argued, played a pivotal role in the rise of the ordinal approach to utility in Austria.[33]

The first writing in which Hayek addressed utility theory seems to have been his 1925 review of Schönfeld's 1924 book on marginal utility and economic calculation. Hayek ([1925] 1984, 185) focused on Schönfeld's total utility approach, which he considered 'the most novel and, at first sight, strangest aspect of the entire treatise', and discussed the advantages and disadvantages of comparing total utilities rather than, as in the traditional Austrian approach, marginal utilities. Without entering into

Hayek's discussion, the important point for us is that he took for granted that utilities, may they be total or marginal, can only be compared, and did not ever mention the possibility of measuring utility. Hayek's general assessment of Schönfeld's book was quite favourable.

Two years later, Hayek ([1927] 1991, 364–84) wrote the introduction to a new edition of Hermann Heinrich Gossen's book, *Die Entwickelung der Gesetze des Menschlichen Verkehrs* (The Evolution of the Laws of Human Relations), originally published in 1854. In his introduction, Hayek discussed Gossen's theory of wants, and presented him with a precursor of marginal utility theory, but did not address issues related to the measurability of utility.

At the LSE

In 1931 Hayek left Vienna for the LSE, where he was offered a professorship in the economics department, then directed by Lionel Robbins. After becoming director of the department in 1929, Robbins had managed to form around him a circle of brilliant young economists that included Hicks, Allen, Nicholas Kaldor, Abba Lerner, Ursula Webb, Rosenstein-Rodan (who left Vienna for London in 1930), and others. The research activity of the group centred on the department's weekly seminar.[34] Hayek joined the group and quickly became a leading member.

Among the works discussed in the seminar was the already mentioned paper by Hicks and Allen that was eventually published in the February and May 1934 issues of *Economica*. In a lecture given at the University of Chicago in 1963, Hayek recalled discussions of the Hicks–Allen paper:

> My liveliest recollections [of the Robbins seminar] are of the discussions connected with the work of John Hicks which resulted in the Hicks–Allen article on 'A Reconsideration of the Theory of Value' and later *Value and Capital*. (Hayek [1963] 1995, 57)

Notably, Hayek claimed in this lecture that he himself had in fact suggested to Hicks the application of the indifference-curve approach that would constitute the backbone of the Hicks–Allen article:

> Hicks had come from Oxford to London as a good Marshallian, and I still remember clearly an early discussion when, curiously, I, the Austrian, tried to persuade Hicks of the merits of the indifference-curve approach of which he was soon to become the acknowledged master. (57)[35]

According to Hicks's own account (1984), it was primarily the reading of Pareto and Edgeworth in the late 1920s that had inspired his indifference-

curve analysis. However, Hicks admitted the possibility that Hayek might have suggested to him the idea of applying the idea of elasticity of substitution not only to production isoquants but also to preferences (1981, 3–4). More generally, Hicks acknowledged the Austrian dimension of the paper co-written with Allen: 'The curious title which we gave to our paper betrays that it had an Austrian, as well as Paretian, origin' (4).

Around the same period that Hicks and Allen's paper was discussed in the Robbins seminar, Hayek was working on a comprehensive biographical study of Carl Menger that was published in 1934 both as an introduction to the LSE reprint of Menger's main writings (Menger 1934–36) and as an autonomous article in the November 1934 issue of *Economica*. Since this study was written in the epicentre of the last phase of the ordinal revolution, it is hardly surprising to discover that in it Hayek attempted to portray Menger as an ordinalist *ante litteram*:

> Menger's views are remarkably modern. [...] Of the figures which he uses to represent the scales of utility he says expressly that they are not intended to represent the absolute, but only the relative importance of the wants. (1934, 401)

However, Hayek failed to mention that by 'relative importance', Menger did not mean the ranking of wants but the fact that one want is twice as great as the other (see Section 'Utility numbers' above).

Notably, in discussing the nature of Menger's utility figures, Hayek used the term 'cardinal', which had already been used by Hicks and Allen in their 1934 article: 'He [Menger] thinks of them [the utility figures] not as cardinal but as ordinal figures' (401). Like Hicks and Allen, however, Hayek did not make clear what he meant by the term 'cardinal', and certainly did not associate it with the ranking of utility differences or uniqueness up to positive linear transformations.

While in 1934 Hayek reconstructed Menger as an ordinalist, in a 1936 article on 'Utility Analysis and Interest' Hayek criticized Böhm-Bawerk and Schumpeter for their non-ordinal conception of utility and for the confusions that this concept had generated in their interest theories. In their explanations of the existence of a positive rate of interest, Böhm-Bawerk and Schumpeter had assumed that the tastes of individuals remain constant over time – but according to Hayek they had conceived of utility as 'an absolute magnitude' (1936, 44). When utility is conceived in this way, the constancy of tastes implies that the marginal utility of equal quantities of a commodity is the same at any moment of time, which in turn makes it difficult to explain the existence of interest. If instead, as Hayek illustrated in the paper, we abandon the idea of utility

as an absolute magnitude, adopt the indifference-curve approach, and intend the constancy of tastes in terms of the constancy of marginal rates of substitution between present and future goods, there is no problem in explaining the existence of a positive interest rate. Thus for Hayek, Böhm-Bawerk's and Schumpeter's confusion in explaining the rate of interest are due to pseudo-problems, and more specifically to:

> The sort of pseudo-problems which generally arose out of the idea that the utility of a commodity can be conceived as an absolute magnitude instead of merely as a relation to some other commodity. (44)

Insofar as I am able to judge, the 1936 article on utility and interest is the work in which Hayek expressed his views on utility and in particular his rejection of measurable utility in the clearest way. In all other articles that Hayek published after 1936 and that are listed in JSTOR, the very term 'utility' appears only five times, and always in a cursory way.

Conclusions on Hayek

In joining the Robbins group at the LSE in the early 1930s, Hayek brought from Vienna the idea, then prevalent among Austrian economists, that utilities can only be compared, not measured. At that time, the ordinal conception of utility was also common in the Robbins circle, mainly through the influence of Pareto. In a series of fundamental contributions published between 1934 and 1939, Hicks and Allen systematized the ordinal approach to utility, overcoming some inconsistencies that still affected Pareto's analysis, and showed how to carry out demand and equilibrium analysis in an ordinal utility framework.[36]

Although it seems implausible that Hayek was the person who converted Hicks to ordinalism, Hayek certainly contributed, with his Austrian-style ordinal insights, to the rich discussions that constituted the background of Hicks' and Allen's achievements. These discussions also advanced Hayek's views on utility, because even in the rather unlikely event that the indifference-curve approach was 'somewhere in the air' in Vienna before 1934, certainly no Austrian economists had developed in any detail the indifference-curve analysis. Hayek quickly became a great supporter of indifference-curve theory: he applied it to explain the existence of a positive rate of interest, tended to read it back into Menger's discussions, and even somehow indulged in suggesting that 'he, the Austrian' was the hidden fairy godfather of the entire indifference-curve thing.

Summary and conclusions

By the end of the 1930s most Austrian economists had left Austria, so bringing to an end the Vienna school. Schumpeter had departed as early as 1925, first to Bonn and then to Cambridge, Massachusetts. Rosenstein-Rodan and Hayek had left for London in 1930 and 1931, respectively. Mises went to Geneva in 1934 and later to New York. Morgenstern and Alt fled in 1938, the year of the annexation of Austria by Nazi Germany, and ended up at Princeton and New York respectively. Of the other economists mentioned in the paper, only Mayer and Schönfeld remained in Vienna.[37]

The discussions on the measurability of utility among economists belonging to the Austrian tradition continued after World War II as well, but these developments are outside the scope of the present chapter.[38]

In this chapter, I have reconstructed the Austrian discussions of utility measurement before World War I and during the interwar period using the categories of ordinal utility, classically measurable utility, and cardinal utility in the current sense of the terms. In particular, I have argued that until World War I the relevant divide was between those who considered utility as classically measurable (Menger, Wieser, Böhm-Bawerk and Schumpeter), and those who instead argued that utility is ordinal in nature (Čuhel and Mises). After World War I, and especially through Mises' influence, the latter view became dominant among Austrian economists. In the 1920s and early 1930s Schönfeld, Rosenstein-Rodan and Morgenstern endorsed an ordinal view of utility; however, they admitted that individuals are not only able to rank utilities, but are also capable of ranking the differences of utility. The full implications of this hypothesis were clarified only in 1936 by Alt. Like the other Austrian economists of his generation, Hayek also had an ordinal understanding of utility. In the last part of the chapter I have illustrated how his ordinalism, through his involvement in the discussions of the Robbins' circle, evolved into keen support for the indifference-curve approach systematized by Hicks and Allen.

One important thread running through the chapter has been that related to the fallacies induced by the analogy of quantity. Beginning with Menger, several Austrian economists first represented utilities through numbers, and then extended to utilities some property of numbers without noticing that this extension contradicted their very idea of utility. In particular, many of the Austrian economists who declared utility to be ordinal in nature – such as Wieser, Čuhel, Rosenstein-Rodan, and Morgenstern – summed, subtracted or multiplied the utility

numbers, without realizing that these operations do not apply to ordinal utilities. Among those who did not fall into the pitfalls surrounding the analogy of quantity were Mises, who simply avoided representing utilities by numbers, and Alt, who instead clarified under what assumptions the properties of numbers do extend to utilities.

Notes

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1. A non-classical concept of measurement, usually labelled the 'representational' view of measurement, was progressively elaborated in the 1930s and 1940s in philosophy, psychology and mathematics. More on the classical and representational views of measurement in Michell (1993) and Moscati (2013a).
2. This means that if the utility function U(x) represents the individual's preferences, another utility function obtained by multiplying U(x) by a positive number α and then adding any number β, that is, a transformation of U(x) having the form $\alpha U(x)+\beta$, with $\alpha>0$, also represents the individual's preferences. Ordinal utility is more general than cardinal utility, and is associated with *any* increasing transformation of U(x), i.e., with any f[U(x)] such that $f'>0$.
3. Classically measurable utility is associated with proportional transformations of U(x) that do not modify the zero point of utility, i.e., with transformations of the form $\alpha U(x)$, where $\alpha>0$.
4. See in particular Hayek (1934), Morgenstern (1941), Rothbard (1956), and High and Bloch (1989).
5. On the relationships between marginal and total ordinal utility, see McCulloch (1977).
6. This section largely draws on Moscati (2013a).
7. Since this is the first quotation of the chapter, it is useful to illustrate here how I dealt with the original texts in German and their English translations. Whenever possible, I checked the existing English translations with the original texts in German. If I found any inaccuracies in a translation, I corrected them but maintained the reference to the translation and its page numbers. However, I refrained from specifying each time how and why I modified the translation. When no translation of the original texts was available, the translation is mine.
8. On the reception of Menger's work and the rise of the Austrian school, see Howey (1960).
9. For instance, if the two quantities consist of 9 and 10 units, and the marginal utilities of the 9th and 10th units are associated with figures 6 and 5

respectively, then the economic value of 9 units (9×6=54) is larger than the economic values of 10 units (10×5=50). But if the marginal utility numbers are ordinal in nature, the marginal utilities of the 9th and 10th units could also be associated with figures 12 and 11 respectively. In that case, however, 9 units have a smaller economic value (9×12=108) than 10 units (10×11=110).

10. Neumann was an important member of the Historical School, whose purely descriptive approach to economic analysis was attacked by Menger in his *Investigations into the Method of the Social Sciences* ([1883] 1985). Menger's attack initiated a famous and lasting methodological dispute, the so-called Methodenstreit, between the Historical School, which dominated economic research in Germany at that time, and the rising Austrian School. Böhm-Bawerk's response to Neumann's criticism can also be seen as a minor episode of the Methodenstreit. (On the Methodenstreit, see Bostaph 1978; Caldwell 2004 and Chapter 2 of this book.)
11. We note in passing that Böhm-Bawerk's theory of complementary goods also rests on the assumption that marginal utilities are measurable, and that the numbers representing them can be added and subtracted. See Böhm-Bawerk ([1886] 1932, 56–61, [1912] 1959, vol. II, 161–167).
12. A fourth and posthumous edition of *Capital and Interest*, identical to the third, was published in 1921 under Wieser's editorship.
13. On these developments see, e.g., Moscati (2007, 2013b).
14. These biographical notes on Čuhel are taken from Hudik (2007). It might not be superfluous to mention that in the 19th century and until World War I the Czech territories were part of the Austro-Hungarian Empire.
15. On Böhm-Bawerk's seminar, see Mises ([1978] 2009).
16. For an overview's of Čuhel's book, see Mussey (1909) and the summary written by Čuhel ([1907]2007). Only Chapter 6 of the *Lehre* has been translated into English; see Čuhel ([1907] 1994).
17. Instead of U(1st plum)>U(2nd plum)>…, Čuhel wrote $e_1 > e_2 > \ldots$, whereby the letter 'e' stands for 'egence'. For the sake of clarity, I have preferred to maintain the letter 'U' (for utility) and make explicit the meaning of the subscript numbers.
18. One may imagine avoiding the distortion effects that decreasing marginal utility has on the utility unit by taking as a yardstick the utility of *single units of different goods* (with single units, in fact, decreasing marginal utility does not enter the picture). However, with different goods another distortion effect enters the scene, namely the complementarity or substitutability relationships between goods, which also modify, in a way not precisely determinable, the utility units. For a discussion, see Čuhel ([1907] 1994, 317–318).
19. In the English translation of Chapter 6 of Čuhel's book, the German word *Grunzahlwörten* is rendered as 'cardinal numbers' ([1907] 1994, 323, 326). Insofar as in economics the term 'cardinal' is associated with uniqueness up to a positive linear transformation rather than with proportionality with respect to a unit, this rendering is misleading.
20. Čuhel also briefly criticized the views on utility measurement expressed by the Swedish economist Gustav Cassel (1899). Cassel had argued that the utility of an object for an individual is simply measured by the maximum price he is willing to pay for it. Thus, if the individual is willing to pay at most

10 marks for a certain good, and at most 20 marks for another good, then for him the utility of the second good is twice the utility of the first good. Čuhel ([1907] 1994, 327) stressed that Cassel's inference is unwarranted. In the situation described by Cassel we are in fact entitled to say only that the utility of the first good is equal to the utility of 10 marks, and that the utility of the second good equals the utility of the 20 marks. This, however, tells us nothing about the ratio of those two utilities.

21. Menger, the founder of the school, embodied its first generation, while Wieser and Böhm-Bawerk were the leaders of the second generation.
22. Schumpeter's reference to Fisher's *Investigations* has for some reason been omitted from the English translation of Schumpeter's book. The reference can be found in Schumpeter (1908, 112).
23. A bibliographical note on the *Positive Theory* may be useful here. Part I of the third edition of the work (1909) includes Books I and II and Excurses I–VI, while Part II (1912) includes Books III and IV and Excurses VII–XIV. Böhm-Bawerk's discussion of utility issues is contained in Book III and Excursus X. See Böhm-Bawerk ([1912] 1959, vol. II, 119–204, 421–432, vol. III, 124–136, 232–233).
24. Mises expressed his aversion to the use of mathematics in economics and other social sciences in numerous writings; see, e.g., Mises ([1929] 1978).
25. On Mises' seminar, see Hayek ([1963] 1992), and Haberler (1981).
26. See, among others, Johnson (1913), Slutsky (1915), Amoroso (1921), Bowley (1924), Frisch (1926), de Pietri-Tonelli (1927) and Schultz (1928).
27. On this assimilation process, see Hayek ([1968] 1992) and Boehm (1992).
28. The assumption that individuals are capable of ranking transitions appeared plausible to Pareto because it is in accord with the idea of decreasing marginal utility. However, he failed to notice that decreasing marginal utility is not an ordinal notion. To see this, imagine that an individual prefers three apples to two apples, and two apples to one. The ordinal utility index U representing the individual's preferences should satisfy only the following property: $U(3)>U(2)>U(1)$. If $U(1)=5$, $U(2)=8$, and $U(3)=10$, the marginal utility of the apples appears to be decreasing. If, however, $U(1)=1$, $U(2)=5$, and $U(3)=10$, then the marginal utility of the apples turns out to be increasing. In the event, a number of economists who accepted Pareto's ordinal approach to utility theory, such as Amoroso (1921), Bowley (1924), and de Pietri-Tonelli (1927), admitted the ranking of transitions and utility differences as a way of preserving decreasing marginal utility without returning to pre-Paretian measurable utility; see Moscati (2013b).
29. On Mayer and Mises, see Leonard (2010, chapter 5). So far as I am aware, Mayer did not discuss in print the issues concerning the measurability of utility. In what is probably his best-known work, Mayer ([1932] 1994) criticized the price theories of Jevons, Walras, Pareto and other economists, but without explicitly addressing the issue of utility measurement. The existing accounts of Mayer's views on the measurability of utility seem to conflict. According to Rosenstein-Rodan ([1927] 1960, 75), Mayer believed that utilities can be compared but not measured (see also below, Section 'Rosenstein-Rodan, 1927'). According to Hayek ([1978] 1983, 51), Mayer still adhered to the idea that utility is measurable.
30. See Hayek ([1978] 1983, 52).

31. Among the supporters of the idea that utilities can only be compared, Rosenstein-Rodan also mentions two economists who sympathized with the Austrian views without being members of the Austrian school in a strict sense: the Russian Aleksander Bilimovič (1876–1963), who after the Soviet Revolution had moved to the University of Ljubljana, Slovenia, and the German Oskar Engländer (1876–1936), who taught at the University of Prague, Czechoslovakia. For Bilimovič's and Engländer's views on the unmeasurability of utility see Bilimovič (1929, 1932, 1933, 1934), and Engländer (1932).
32. On Samuelson's pivotal role in the definition of the current notion of cardinal utility, see Moscati (2013b).
33. On Hayek's formative period in Vienna and Mises' influence on him, see Caldwell (2004), Hayek ([1978] 1983, 1983, 17–18, 1992, 126–159).
34. On Hayek, Robbins and the latter's circle, see Hayek ([1963] 1995), Caldwell (2004), and Howson (2011).
35. In a 1978 interview, Hayek made a similar claim: 'I had a curious influence on Hicks. You won't believe it, but I told him about indifference curves. He was a pure Marshallian before. And I remember a conversation after a seminar, when he had been talking in Marshallian terms, when I drew his attention to Pareto. It was the very beginning of the thirties, of course.' ([1978] 1983, 247–248, see also 365)
36. Besides Hicks and Allen (1934) and Hicks (1939), see also Allen (1934a, 1934b, 1936), and Hicks (1937).
37. In Austria's Nazi period, Schönfeld was forced to change his name to Illy; the name Schönfeld was in fact that of his adoptive father, who was Jewish. See Hayek ([1978] 1983, 52).
38. See for instance Rothbard (1956).

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