Estimating the Size of the Underground Economy: *A DSGE Approach.*

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Abstract

We study a new approach to estimate underground economy based on a dynamic and stochastic general equilibrium (DSGE) framework. In particular, we generalize an otherwise standard two-sector DSGE model by introducing explicitly underground production and irregular market sectors. In this setup, firms may choose to produce goods on the regular market as well as on the underground sector and households can evade taxes by reallocating worked hours from regular to irregular labor market. Firms can be discovered evading with a given probability and forced to pay a penalty surcharge. Empirical evidence based on Italian data stresses that this phenomenon is relevant in this Country since the estimated level of underground economy is about 22% of the GDP, that is 3 percentage points larger than what reported in the official statistics. Counterfactual analysis suggests that an increase in the probability to be discovered and in the tax surcharge, along with a moderate tax reduction causes either a sensitive reduction in the size of the underground economy and a positive stimulus to the official economy that jointly increases the total fiscal revenues.

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1 Introduction

Underground economy represents a major issue when studying an economic systems because of its huge impact on the public finance and of its distorting effects on production, induced by an unfair competition among firms. Furthermore, social costs may arise in terms of overall tax burden that is shared among a smaller number of citizens, thus increasing economic inequalities, and in terms of labour protection for individuals working in the underground market.

In studying underground economy, then takes on a considerable significance the analysis of conveniences that occur in an irregular mode of production, which involves the complicity of the workers themselves. Such situations allow to start business activities in conditions that let people accept lower incomes and fewer guarantees in the workplace, making possible the birth and development of productive initiatives with a very low investment.

Because of the heterogeneous and non-observable nature of this phenomenon, it is important to precisely define the underground economy, since different definitions imply different aggregates. In this paper, we consider as underground economy, the production of legal commodities and services that are deliberately concealed from public authorities to avoid payment of taxes or social security contributions. In particular, this definition does not include the so-called informal economy, which consists of all services and production yielded within the family by the members of an household, as this does not lead to tax evasion, as well as the criminal economy, since it relies on illegal actions related to crime, robbery, drug dealing and so on. In other words, in our definition, the irregularities related to the underground economy rely on the ways regular economic activities are carried out.

At different extents, irregular economy is a relevant issue for most of the countries throughout the world. In fact, underground economy is universally widespread, present in economies with low growth rates as well as in advanced ones. In particular, there is evidence of growing trends on irregular economies, due to the combined effects of international competition and of high fragmentation of working organization. Furthermore, the enhancement of skills and knowledge, the emergence of new professions, the consistent use of immigrant labor, are also opportunities to disobey obligations and rules. In a recent paper, Schneider *et al.* (2010) report estimates of the shadow economies for 162 countries, including developing Eastern European, Central Asian, and high-income countries over the period 1999 to 2007. According to the reported results, the average size of the shadow economy, measured as a percentage of *official* gross domestic product, in 2006 in 98 developing countries is 38.7 percent; in 21 Eastern European and Central Asian countries, it is 38.1 percent, and in 25 high-income countries, it is 18.7 percent. The authors find also that the main driving forces of the shadow economy can be identified in the increased burden of taxation, combined with labor market regulations and the quality of public goods and services, as well as the state of the *official* economy. Legal activities conducted underground to escape taxation appear to be the faster growing component of the irregular economy, largely because of the structure of the tax system.

Because of its nature, the size of the underground economy is difficult to measure and to study empirically. Law enforcement and taxation officials readily admit that underground economy is a widespread phenomenon, but it is difficult to agree on its size. Having a look at the literature on the underground economy, we recognize that there has been a good deal of progress on ascertaining data and developing techniques for quantifying its size and importance, even if the discussion regarding the most appropriate methodology to quantify the importance of this phenomena has not come to an end yet.

In this paper, we tackle the issue of measuring the underground economy using a structural econometric approach. Following a Bayesian procedure, we build and estimate a DSGE model that explicitly accounts for irregular transactions. With respect to standard methodologies, we believe our approach has three main advantages. First, being theory-based, the DSGE methodology provides a deeper understanding into the causes of underground economy. Second, the inferential procedure based on Markov chain Monte Carlo methods (MCMC) allows to estimate the dynamics of the unobservable underground output together with the parameters of the model. Third, by means of counterfactual analysis, a general equilibrium framework allows to derive policy implications.

The model developed in this paper is a variant of the standard Neoclassical stochastic growth model. As in Busato and Chiarini (2004) and Conesa Roca *et al.* (2001), we consider a two-sector economy in which each firm may decide to produce their own goods on the regular market as well as on the underground, to evade taxation. In doing so, firms face a risk to be fined by fiscal authorities with a given probability. These two sectors are characterized by different technologies, and in particular, the regular sector production function is capital intensive whereas the irregular market assume a production function that is labour intensive. On the other side, households might evade income taxes by reallocating labor services from regular to underground labor markets.

Our empirical analysis is based on Italian quarterly data in the interval 1982:Q1 to 2006:Q4. We think the Italian case is peculiar in this framework, since the relevance of the underground economy phenomenon appear to be larger with respect to other developed countries. Furthermore, the severe sovereign debt crisis in this country is requiring policy makers to propose effective policies to fight against tax evasion. However, our method is general in enough to be easily adapted to other countries. Our main result highlights that the estimated size of the underground economy is on average 3 percentage GDP points larger than the official statistics available, even though the dynamics of the two series are closely related. In light of these findings, our model appears to be a reliable tool for policy evaluations. In fact, we find evidence that a reduction of the tax rates and an increase of the monitoring on the irregular activities by the government has a strong impact on the total fiscal revenues, due by the increase on the regular activity and on a reduction of the irregular one. Interestingly, albeit fiscal shocks represents a relevant factor on explaining the underground dynamics, we also find that the main driving force of fluctuations for this sector is the technology shock of the underground production, which mostly captures exogenous movements in the labor forces. As a side but relevant results, we also find that the model predicts that the cyclical component of the underground economy is negatively correlated with the cyclical component of the official output, thus providing evidence of a double business cycle in the Italian economy.

This paper is organized as follows. Section 2 reviews the literature on underground economy, emphasizing the methodologies implemented to measure this phenomenon. The DSGE model is presented in Section 3 whereas inferential methodologies and description of the data are described in Section 4. Empirical results are provided in Section 5 and policy implications are analyzed in Section 6. Finally Section 7 concludes.

2 Current approaches in estimating the underground economy

As is evident the underground economy cannot be directly observed, and the one question that arises is: how does one measures the seemingly unmeasurable? Information about underground economic activities, their magnitude and the different manners with which these activities occur is difficult to obtain, because these activities don't belong to official economy and the individuals involved don't like to be identified. Thus the only possibility to get some quantification of its size is through estimation. Many attempts have been made and several different methods are employed for this purpose, as evidenced by a vast literature (Schneider, 2005 and Schneider and Enste, 2002, among others) even if a certain disagreement still exists about the definition as well as the *best* approach for estimating the underground economy. To analyze and estimate the amount of the underground economy, first of all it is necessary to clarify what is meant by underground economy. Is precisely for this purpose the Organization for Economic Cooperation and Development (OECD) almost a decade ago published a handbook that addresses this question, and an harmonization of terms has been provided by integrating the underground economy into the gross national product (GNP). This is done starting from 2000 by adopting a global definition of underground economy that recognizes a list of activities as related to the underground economy. The objective to achieve is to provide an indirect measurement of economic activities which are not included in the official statistics, but which are relevant for the economy of a country. As far as a uniform definition of underground economy is concerned, starting from the 90's the statistical offices of OECD countries adopted the international definition established according to SNA93 and SEC95 accounting systems, which represents the point of reference for the national accounts estimates and assures homogeneity in the statistical evaluation of GDP figures. In order to provide a definition that makes comparable and fairly uniform the concept of underground economy for the countries belonging to the European community, the European Union's statistical office (Eurostat) has provided details of how to account for the *non observed economy* and monitors compliance with these directives in the definition of national accounts of the member countries.

All the economic activities for which we encounter difficulties concerning their statistical observability contribute to define what is named *non (directly) observed economy* which is composed of:

- *underground economy*, which regards legal production which is not official and is not recorded because of tax and contributory evasion, tax labor regulations avoidance and the non observance of the administrative rules;

- *informal economy*, which includes all legal activities carried out by individuals, small or home enterprises (part-time secondary work, moonlighting, baby-sitting and so on), goods and services produced and consumed within the household, for which is very difficult, or even impossible, to rely on statistical observation and measurement, even if these activities are not directed to tax evasion, and so they are not included in the genuine underground economy as defined above;

- *illegal economy or criminal economy*, which regards all economic activities which violate penal norms, like drugs business, prostitution, criminal activities

To come to the point, the definition of underground economy adopted by European countries which is harmonized and is commonly recognized by the countries members, rules out the illegal economy, mainly because of the unhomogeneity in defining an illegal activities in the different countries, and includes the underground economy as defined above along with the informal economy. The definition we adopt in our paper, and figures reporting underground economy for empirical purposes are composed by the *underground economy* as define above.

2.1 An overview of the main methods

Measuring the size of an underground economy is difficult and, in a certain sense, is a challenging task. In literature many different methods are proposed for this purpose. The common feature that characterizes the different methods is to rely on adequate information that allows for an assessment, albeit indirect, of the phenomenon. So, for example, surveys can be used on household expenditure and income held by the National Statistical Institutes, in examining the discrepancies that may indicate some form of unreported income. Or conduct special surveys to provide guidance on cash payments and possibly on below the counter income. At a macro level, some inference can be made from income-expenditure accounts, precisely looking at differences between national income and national expenditure, on the assumption that if expenditure is greater than income, this discrepancy can be attributed to some underground activity. The most popular methods in the literature are based on macroeconomic models of either a monetary approach, according to which a demand for currency is analyzed with the aim of using movements in narrow money to track movements in the underground economy, or the aggregate consumption of some standard commodities like electricity or energy.
It is usual in the literature, to classify the existing approaches in four main classes.

- *Direct methods.* These methods mainly deal with microeconomic approaches which make use either of survey data and samples based on voluntary participation, or tax auditing and other compliance methods. The main limit of these approaches is due to the fact that the reliability of the results obtained through these methods depends upon the collaboration of individuals, since interviewees are hesitant to affirm the truth and the answers, therefore, are not always reliable. Nevertheless direct methods based on surveys give some help in providing a useful information about the structure of the underground economy at one particular period of time.

- *Indirect methods.* Contrary to direct methods, these approaches are mostly macroeconomic grounded, since they make use of various indicators proxying the size of the underground economy over time, which are referred to macroeconomic figures. The indicators belonging to this class can basically be classified in three different typologies, since they are based on: (i) discrepancies between national accounts and income statistics, by assuming that a discrepancy between the expenditure and the income measure can be used as an indicator of the underground economy, (ii) discrepancies between the official and actual labor force, by assuming that the total labor force participation, in a country, is almost constant over a short time horizon, and a downturn in labor force participation may be interpreted as an indicator of increased activity in the underground economy, and (iii) monetary methods based on the assumption that underground economy activities are largely settled by cash payments so that no traces of transactions are left, meaning that a rise in the demand for cash may be interpreted as a signal of an increase in the underground economic activities. In literature there are two different proposal belonging to this category, the Currency Demand Approach, proposed originally by Cagan (1958), successively improved by Gutmann (1977) and Tanzi (1980, 1983), and the Transaction Approach, proposed by Feige (1979). According to the Currency Demand Approach, since the hidden transactions occur mainly in cash, an increase in currency demand signals an increase in the underground economy. This method is one of the most commonly used in empirical analysis, even if it has been frequently criticized.

- *Electric power consumption methods.* These methods exploit the relationship existing between the electric power consumption and the overall economic activity. The underlying assumption is that overall economic activity and electricity consumption move with an elasticity of electricity consumption to GDP close to one. So, according to this approach, the discrepancy between the gross rate of official GDP and the gross rate of total electricity consumption may be interpreted as a measure of the growth of the underground economy.

- *Model or structural approach.* In the different methods considered up to now the underground economy is commonly ascribed to just one main cause and related to a single effect. According the model approach, the underground economy is considered as a latent variable which is caused by a multiplicity of factors. The empirical analysis is based on a factor analysis model which provide a measurement of the underground economy as a latent variable over time. The most commonly used model in empirical analysis is the Multiple Indicator, Multiple Causes (MIMIC) proposed by Frey and Weck-Hannemann (1984) and Giles (1999). The idea is to represent the underground economy as a latent variable which has causes and effects that are observables but which cannot itself be directly measured. The observed variables in the model (causal variables and indicator variables) are connected by a single unobserved variable, the underground economy. This method has been criticized because of its instability in face of minor changes of the observed variables and some inconsistency regarding the obtained results.

The approach we intend to propose in this paper does not belong specifically to any of these categories, although, at least conceptually, we can consider it of the type *model or structural approach*. In fact, we formulate a dynamic stochastic general equilibrium model by assuming that firms can choose to produce for the regular market as well as for the non-regular one. Similarly, workers can decide whether to enter the official labor market or adhere to the offers coming from the unofficial market. Solving the model, with all the appropriate caveats, we get the size of the underground economy as a latent variable.

3 Model

In this section we lay out a simple DSGE model that captures the main features of underground economic activities. We consider an economy that consists of a continuum of homogenous goods indexed by $i \in [0, 1]$, each produced by a perfectly competitive producer. Goods are sold by firms to a continuum of measure 1 of identical households for consumption and investment purposes and to the government, which collects taxes from households and firms to finance public spending. To introduce underground transactions in this environment, we assume that the economy is divided into a regular and unofficial sector. All of the transactions that the occur in the underground sector are not recorded by government authorities. Firms therefore use factors from underground markets to hide part of their production in order to evade taxation. In each period of time, however, firms face non negligible probability of being inspected by fiscal authorities, convicted of tax evasion and forced to pay taxes augmented by a penalty surcharge. Households might also evade personal income taxation by reallocating labor services from the regular to the underground sector. All of the interactions among firms, households and the government occur in a stochastic environment where the short-run dynamics of the economy are driven by productivity, demand, and fiscal shocks.

3.1 Firms

Each firm $i \in [0, 1]$ uses regular labor $h^m_{i,t}$, and capital $k_{i,t}$ to produce regular output via a Cobb-Douglass production function

$$y^m_{i,t} = A_t \left(\Gamma_t h^m_{i,t}\right)^{\alpha} \left(k_{i,t}\right)^{1-\alpha} \tag{1}$$

where $\alpha \in (0, 1)$, A_t is a purely transitory technological shock,[1] while Γ_t is the labor augmenting technological progress, which follows a deterministic trend of the form $\Gamma_t = \gamma\Gamma_{t-1}$ with $\gamma > 1$.

Every unit of output produced is taxed at the stochastic corporate tax rate $\tau^c_t < 1$,[2] but compliance is only partial and firms can hide part of their production in order to evade taxes. In particular, we assume that firms may hire labor from the unofficial market $h^u_{i,t}$ to produce underground output via a labor-insensitive Cobb-Douglass technology:[3]

$$y^u_{i,t} = B_t \Gamma_t \left(h^u_{i,t}\right)^{\alpha_u} \tag{2}$$

where B_t is a purely transitory sector-specific technological shock, and $\alpha_u \in (0, 1]$.

[1]The process governing the evolution of stochastic shocks will be introduced shortly.

[2]To keep the analysis relatively simple, our model abstracts from endogenous fiscal policies determining the evolution of marginal tax rates. We thus treated the latter as aggregate stochastic disturbances.

[3]This specification is adapted from Busato and Chiarini (2004) and Conesa Roca *et al.* (2001). As for our model, both papers abstract from capital in the production of irregular output to take into account that underground production activities are typically more labor-intensive than the regular ones. One way to think about this assumption is to suppose that in the short-run underground firms are endowed by a fixed stock of capital and thus, in response to transitory shocks, they only adjust their demands of irregular workers.

Let $p^m_{i,t}$ and $p^u_{i,t}$ denote the price of the $i-th$ good in the regular and unofficial markets respectively. Following Busato and Chiarini (2004), we assume that the good produced in the underground sector is indistinguishable from the regular one, and therefore in equilibrium their prices must be the same.[4] Hence, without loss of generality, we will impose hereafter that $p^m_{i,t} = p^u_{i,t} = P_t \ \forall \ i \in [0, 1]$, where P_t is the market price of each good that perfectly competitive firms take as given. Additionally, because of goods produced in the two markets are homogenous, total final output produced by a firm i at date t, namely $y_{i,t}$, can be simply defined as

$$y_{i,t} = y^m_{i,t} + y^u_{i,t} \tag{3}$$

We believe that the assumptions made on the production side of the economy are sufficiently general to capture several features of underground economic activities. Firstly, according to equation (3), a firm is always allowed to produce total output $y_{i,t}$ by using only the regular technology. In this perspective, unofficial productive factors are not strictly necessary to produce final output. As a result, underground production takes place in our model mainly because firms aim at taking advantages from tax evasion. Secondly, the assumption of sector-specific technological shocks incorporates into the model potentially important differences in productivity between regular and irregular labor forces.[5] This model's feature is consistent with the available empirical evidence that documents a clear association between level of education and participation in the irregular labor market. For example, Marcelli *et al.* (1999) and Gallaway and Bernasek (2002) show that in urban settings, high skilled individuals are more likely to work in the regular sector, whereas those with the lowest level of education have the higher probability of working in the irregular sector. Finally, the idiosyncratic shock B_t may also be interpreted as capturing exogenous changes in the overall labor force that mostly affect irregular workers productivity. For example, several empirical papers have documented that most workers hired under irregular work arrangements are immigrants (see e.g. Leonard, 1998). As noted by Busato and Chiarini (2004), it is reasonable to believe that those individuals have strong incentives to be very productive in order to increase the probability of being hired as regular workers. In most of western European countries, in fact, immigrants enter with temporary visas that are converted to permeant ones when they prove to be regular workers. In this perspective, an increase in legal immigration might result in temporary boost in the underground sector productivity. Moreover, the idiosyncratic cost B_t also capture movements in the labor force that, by definition, are specific to the irregular sector, such as those implied by illegal immigration or by workers that are officially inactive or retired

[4]This assumption embodies the idea that costumers in the regular markets are not able to detect those products that are manufactured with irregular workers. While realistic for commodities, this hypothesis is somewhat too restrictive in the case of some specific services where underground transactions often result from direct agreements between customers and producers. In such a circumstances, customers' costs for regular and underground service cannot be equal in equilibrium. One way to incorporate this feature into our framework is to specify a service sector where customers face a different demand for underground and regular services, and pay valued added taxes only on the latter. We however decided to abstract from this possibility because tax evasion of this nature is likely to be quantitatively negligible.

[5]An alternative way to introduce this property into the model is to assume that goods produced in the two sectors are manufactured with the same technology but firms have to pay an additional cost for every unit of underground output produced. We prefer to rely on sector-specific production functions in order to take into account the labor-intensive nature of underground production activities.

In order to discourage tax evasion, the government enforces a monitoring process. Following Allingham and Sadmo (1974), we assume that in each date t firms face a not zero probability $p \in (0,1)$ of being inspected, and forced to pay the tax rate τ_t^c on the concealed production, augmented by a penalty surcharge factor $s > 1$. As a result, for a given market price P_t, total expected net revenues from an amount of final output $y_{i,t}$ at time t are given by:

$$\begin{aligned} E_t\left\{NR(y_{i,t})\right\} &= P_t(1-\tau_t^c)y_{i,t}^m + E_t\left\{P_t y_{i,t}^u\right\} \\ &= P_t\left[(1-\tau_t^c)y_{i,t}^m + (1-ps\tau_t^c)y_{i,t}^u\right] \end{aligned} \tag{4}$$

where E_t denotes the mathematical expectations operator conditional on information available at time t. Equation (4) indicates that as long as $(1-ps\tau_t^c) > 0$, firms have an incentive to produce underground output as revenues from such an activity are expected to be positive.

Capital and labor markets are perfectly competitive, and thus firms take factor prices in these markets as given. We assume that the cost of renting capital is equal to the nominal rental rate R_t paid per unit of capital. The total cost of labor instead depends on whether firms hire workers in the regular or in the underground sector. More precisely, we assume that the cost of labor in the regular market is represented by the nominal wage paid for unit of labor services W_t^m, augmented by a stochastic social security tax rate $\tau_t^s < 1$. On the other hand, the cost of labor hired in the underground market is given by the nominal wage, W_t^u. Accordingly, total costs for a firm i, namely TC, are defined as follows:

$$TC(h_{i,t}^m, h_{i,t}^u, k_{i,t}) = (1+\tau_t^s)W_t^m h_{i,t}^m + R_t k_{i,t} + W_t^u h_{i,t}^u \tag{5}$$

Given equations (4) and (5), the optimal amount of final output produced by a firm i at date t is the solution of the following static problem:

$$\begin{cases} \max\limits_{h_{i,t}^m, h_{i,t}^u, k_{i,t}} \; E_t\left\{NR(y_{i,t})\right\} - TC(h_{i,t}^m, h_{i,t}^u, k_{i,t}) \\ s.t. \\ y_{i,t}^m = A_t\left(\Gamma_t h_{i,t}^m\right)^{\alpha}\left(k_{i,t}\right)^{1-\alpha} \\ y_{i,t}^u = B_t\Gamma_t\left(h_{i,t}^u\right)^{\alpha_u} \end{cases}$$

where the vector of prices $\{P_t, W_t^u, W_t^m, R_t\}$ is taken as given. The associated optimal planning satisfies the following three conditions:

$$(1-\alpha)\frac{y_{i,t}^m}{k_{i,t}} = \frac{r_t}{1-\tau_t^c} \tag{6}$$

$$\alpha\frac{y_{i,t}^m}{h_{i,t}^m} = \frac{w_t^m(1+\tau_t^s)}{1-\tau_t^c} \tag{7}$$

$$\begin{cases} \alpha_u\frac{y_{i,t}^u}{h_{i,t}^u} = \frac{w_t^u}{1-ps\tau_t^c}, & \text{if } 1-ps\tau_t^c > 0 \\ h_{i,t}^u = 0, & otherwise. \end{cases} \tag{8}$$

where $r_t = R_t/P_t$, $w_t^m = W_t^m/P_t$ and $w_t^u = W_t^u/P_t$ respectively denote the real rental rate, the real wage paid in the regular labor market and the real wage paid in the underground sector.

Equation (6) and (7) describe the optimal demand of capital and regular labor respectively. These equations highlight that in equilibrium, because of tax wedges, both factors are not paid at their marginal productivity. Moreover, for given factor prices, the optimal demand of both capital and regular-market labor decreases with larger tax rates. Equation (8) instead describes the optimal demand of underground labor services. Accordingly, as long as $1 - ps\tau_t^c > 0$, a firm demands irregular labor until its marginal productivity equates its marginal cost, where the latter is given by the real wage w_t^u, discounted by the expected real revenue from an additional unit of underground output, $1 - ps\tau_t^c$. Conversely, when $1 - ps\tau_t^c < 0$, firms have no incentives to hire irregular workers to produce final output as real revenues from the underground sector are expected to be negative. In this case, total output is entirely produced with the regular technology (i.e. $h_{i,t}^u = 0$) and therefore firms do not evade taxation.

3.2 The representative Household

The representative household has preferences in period 0 given by:

$$U_t^h = \sum_{t=0}^{\infty} \beta^t E_0 \left\{ \frac{(c_t/\Gamma_t)^{(1-\sigma)} - 1}{1-\sigma} - \xi_t^h B_0 \frac{(h_t^m + h_t^u)^{1+\xi}}{1+\xi} - B_1 \frac{(h_t^u)^{1+\phi}}{1+\phi} \right\} \quad (9)$$

where $\sigma > 0$ is the inverse of the inter-temporal elasticity of substitution, $\beta \in (0,1)$ is the subjective discount factor, $B_0 \geq 0$ and $B_1 \geq 0$ are preference parameters controlling for the disutility of working activities, $\xi > 0$ and $\phi > 0$ respectively denote the inverse labor supply elasticities of aggregate and underground labor supplies. ξ_t^h stands for a purely transitory demand shock that affects the marginal rate of substitution between consumption and leisure.

The inter-temporal utility function (9) embodies a number of properties that are worth emphasizing. Firstly, to ensure that the economy evolves along a balanced growth path, we assume that households take utility from consumption relative to the rate of technology Γ_t. As in An and Schorfheide (2007), we interpret Γ_t as an exogenous habit component. Secondly, we assume that there exists perfect substitutability across sectors, in the sense that households do not face additional costs while transferring labor supply from one sector to another. This feature is captured by the second term in equation (9) that describes the household disutility of total working activities. Third, the last term in equation (9) reflects an idiosyncratic cost of working in the underground sector. It might be interpreted as capturing the cost associated with the lack on any social and health insurance in the underground sector.[6] Finally, the households' disutility of total working activities is stochastic, depending on the realization of the shock ξ_t^h. This assumption has been introduced mainly because, according to the available empirical evidence, a shock on the disutility of labor of this form, turns out to be particularly important to capture actual dynamics of worked hours in estimated DSGE models (see e.g., Smets and Wouters, 2007).

Households supply labor services per unit of time and rents whatever capital they own to firms. We assume that the capital stock, k_t, held by households evolves over time according

[6] An alternative interpretation is that parameter B_1 measures the degree of households' tax morality (see e.g., Gordon, 1989).

to the following law of motion

$$k_{t+1} = \xi_t^x x_t + (1 - \delta_k)\, k_t \tag{10}$$

where x_t denotes the investment at date t, $\delta_k \in [0,1]$ is the capital depreciation rate. Following Justiniano *et al.* (2010), we assume that the efficiency through which the final good can be transformed into physical capital is random, and determined by the purely transitory exogenous shock ξ_t^x. As shown in Greenwood *et al.* (1988), a stochastic disturbance of this type is equivalent to a sector-specific technological shock affecting the production of investment goods in a simple two sectors model. As such, this assumption is useful to capture potentially different sources of fluctuations between consumption and investment.[7]

Households might evade income taxes by reallocating labor services from regular to irregular labor markets. Underground-produced income flows, $w_t^u h_{h,t}^u$ are, therefore, not subject to the stochastic income tax rate $\tau_t^h < 1$. Under these assumptions, the household's period-by-period real budget constraint can be written as:

$$c_t + x_t = (1 - \tau_t^h)(w_t^m h_t^m + r_t k_t) + w_t^u h_t^u \tag{11}$$

The utility maximization problem for the representative household can be stated as a matter of choosing the processes c_t, h_t^u and h_t^m that maximize the inter-temporal utility function (9) subject to the law of motion of capital (10) and and to the budget constraint (11). An optimal consumption, labor supply, and saving plan for the representative household must satisfy the following conditions:

$$\Gamma_t^{(1-\sigma)} c_t^{-\sigma} = \lambda_t$$

$$\frac{\lambda_t}{\xi_t^x} = \beta E_t \left\{ \lambda_{t+1} \left[\frac{(1-\delta_k)}{\xi_{t+1}^x} + (1 - \tau_{t+1}^h) r_{t+1} \right] \right\} \tag{12}$$

$$B_0 \left(h_t^m + h_t^u\right)^{\xi} \xi_t^h = (1 - \tau_t^h) w_t^m \lambda_t \tag{13}$$

$$B_0 \left(h_t^m + h_t^u\right)^{\xi} \xi_t^h + B_1 \left(h_t^u\right)^{\phi} = w_t^u \lambda_t \tag{14}$$

where λ_t is the Lagrange multiplier for the constraint (11). Equation (12) is the usual Euler equation that gives the intertemporal optimality condition, whereas equation (13) describes the (total) labor supply schedule. Equation (14) instead describes the optimal allocation of time for working activities in the underground sector. To gain intuitions on the determinants of the irregular labor supply, it is useful combining (13) with (14), and solving the resulting equation with respect to h_t^u to obtain

$$h_t^u = \begin{cases} \lambda_t^{\frac{1}{\phi}} \left[\frac{w_t^u - (1-\tau_t^h) w_t^m}{B_1} \right]^{\frac{1}{\phi}} & if\ w_t^u - (1-\tau_t^h) w_t^m \ \geq\ 0 \\ 0 & otherwise \end{cases} \tag{15}$$

This equation states that households supply labor services in the underground sector as long as the wage they earn from such an activity exceeds the net real wage they earn

[7] This is particularly important as both consumption and investment aggregates are treated as observable variables in the model's estimate. See section 4 for further details.

by working in the regular labor market. In this perspective, $1/\phi$ stands for the Frisch elasticity of irregular-labor supply with respect to the net-of-taxes wage differential between the underground and the regular labor market. Additionally, for a given wage differential, the supply of irregular-labor shifts to the left when parameter B_1 increases. Intuitively, to keep the same amount of irregular labor supplied, households require an higher wage gap to compensate for the increased disutility they derive by working in the irregular sector.

3.3 Government

In each period t, government raises taxes in order to finance a given amount of government consumption, g_t. For simplicity, we abstract for public debt and assume that in each period public expenditures are decided on a balanced basis. The period-by-period government budget constraint can then be written as follows

$$g_t = \tau_t^h (w_t^m h_{h,t}^m + r_t k_{h,t}) + \tau_t^c \int_0^1 (psy_{i,t}^u + y_{i,t}^m) di + \tau_t^s w_t^m \int_0^1 h_{i,t}^m di \tag{16}$$

where the first term in the right end side of (16) is the total fiscal revenues from personal income taxation, G_t^h; the second is the total fiscal revenues from corporate taxation, G_t^c; and the last term is the total fiscal revenues from social security contributions, G_t^s.

Total tax evasion at date t, namely TE_t, takes the following form

$$TE_t = (\tau_t^s + \tau_t^h) w_t^u \int_0^1 h_{i,t}^u di + (1-p)\tau_t^c \int_0^1 y_{i,t}^u di$$

3.4 Stochastic Processes

To close the model, we formulate productivity, demand and tax rates disturbances as a stationary VAR(1) process

$$\mathbf{z}_t = (I - \Phi)\mathbf{z} + \Phi \mathbf{z}_{t-1} + \varepsilon_t \tag{17}$$

where $\mathbf{z}_t = \left\{ log(A_t), log(B_t), log(\tau_t^c), log(\tau_t^s), log(\tau_t^h), log(\xi_t^i), log(\xi_t^h) \right\}'$, $\mathbf{z}$ is a vector containing the mean values the exogenous state variables, $\Phi = diag[\rho_a, \rho_b, \rho_c, \rho_s, \rho_h, \rho_H, \rho_I]$, and $\varepsilon_t = \left\{ \varepsilon_t^a, \varepsilon_t^b, \varepsilon_t^c, \varepsilon_t^s, \varepsilon_t^h, \varepsilon_t^I, \varepsilon_t^H \right\}'$ is the vector of zero-mean normal random innovations with diagonal variance-covariance matrix $\Omega = diag[\sigma_a^2, \sigma_b^2, \sigma_c^2, \sigma_s^2, \sigma_h^2, \sigma_H^2, \sigma_I^2]$.

3.5 Symmetric equilibrium

We restrict the analysis to symmetric equilibria where all firms produce the same quantity of their respective good, using the same amount of official and irregular productive factors. In addition, we normalize the price P_t to be 1 in each period of time t. The symmetric equilibrium of the model is then formally derived by imposing the following clearing conditions in goods and labor markets:

$$c_t + x_t + g_t = \int_0^1 y_{i,t} di$$

$$h_t = \int_0^1 (h_{i,t}^m + h_{i,t}^u) di$$

where h_t denotes the total amount of time for working activities supplied by households at date t.

Given the assumptions made on the production functions and on preferences, the model economy features a balanced growth path equilibrium in which all the variables growth at a constat rate. It is therefore convenient to express the model in terms of detrended variables, for which there exists a deterministic steady state.[8] Thus, denoting with $\widehat{S}_t = S_t/\Lambda_t$ the original variable S_t detrended by means of its trend Λ_t, and letting $\mathbf{x}_t = (\hat{r}_t, \hat{w}_t^m, \hat{w}_t^u, \hat{w}_t^h, \hat{y}_t^m, \hat{y}_t^u, \hat{h}_t^m, \hat{h}_t^u, \hat{G}_t^c, \hat{G}_t^s, \hat{G}_t^h, \hat{c}_t, \hat{k}_t, \hat{x}_t, \hat{y}_t, \hat{h}_t)$ the vector of all endogenous variables, then a symmetric equilibrium for the economy can be formally defined as an initial condition $\widehat{k}_0 \in \mathbb{R}_+$ and a process $\{\mathbf{x}_t\}_{t=0}^{\infty}$ that, given the exogenous stochastic process $\{\mathbf{z}_t\}_{t=0}^{\infty}$, satisfies the following system of equations:

$$\hat{y}_t^m = A_t \left(\hat{h}_t^m\right)^{\alpha} \left(\hat{k}_t\right)^{1-\alpha} \tag{18}$$

$$\hat{y}_t^u = B_t \left(\hat{h}_t^u\right)^{\alpha_u} \tag{19}$$

$$\hat{y}_t = \hat{y}_t^m + \hat{y}_t^u \tag{20}$$

$$(1-\alpha)\frac{\hat{y}_t^m}{\hat{k}_t} = \frac{\hat{r}_t}{1-\tau_t^c} \tag{21}$$

$$\alpha \frac{\hat{y^m}_t}{\hat{h}_t^m} = \frac{\hat{w}_t^m (1+\tau_t^s)}{1-\tau_t^c} \tag{22}$$

$$\begin{cases} \alpha_u \frac{\hat{y}_t^u}{\hat{h}_t^u} = \frac{\hat{w}_t^u}{1-ps\tau_t^c}, & \text{if } 1 - ps\tau_t^c \geq 0 \\ \hat{h}_t^u = 0, & otherwise. \end{cases} \tag{23}$$

$$\gamma \hat{k}_{t+1} = \xi_t^x \hat{x}_t + (1-\delta_k)\hat{k}_t \tag{24}$$

$$\hat{c}_t + \hat{x}_t = (1-\tau_t^h)(\hat{w}_t^m \hat{h}_t^m + \hat{r}_t \hat{k}_t) + \hat{w}_t^u \hat{h}_t^u \tag{25}$$

$$\frac{\hat{c}_t^{-\sigma}}{\xi_t^x} = \frac{\beta}{\gamma} E_t \left\{ \hat{c}_{t+1}^{-\sigma} \left[\frac{(1-\delta_k)}{\xi_{t+1}^x} + (1-\tau_{t+1}^h)\hat{r}_{t+1} \right] \right\} \tag{26}$$

$$B_0 \left(\hat{h}_t\right)^{\xi} \xi_t^h = (1-\tau_t^h)\hat{w}_t^m \hat{c}_t^{-\sigma} \tag{27}$$

[8]The perfect foresight equilibrium (or non-stochastic steady state) of the model is derived by setting the shocks $\mathbf{z}_t$ to their mean values in every period and assuming that the vector of endogenous variables $\mathbf{x}_t$ is constant over time.

$$\hat{h^u}_t = \begin{cases} \lambda_t^{\frac{1}{\phi}} \left[\frac{\hat{w}_t^u - (1-\tau_t^h)\hat{w}_t^m}{B_1}\right]^{\frac{1}{\phi}} & if\ \hat{w}_t^u - (1-\tau_t^h)\hat{w}_t^m \geq 0 \\ 0 & otherwise \end{cases} \tag{28}$$

$$\hat{h}_t = \hat{h}_t^u + \hat{h}_t^m \tag{29}$$

$$\hat{G}_t^h = \tau_t^h(\hat{w}_t^m \hat{h}_t^m + r_t \hat{k}_t) \tag{30}$$

$$\hat{G}_t^c = \tau_t^c(ps\hat{y}_t^u + \hat{y}_t^m) \tag{31}$$

$$\hat{G}_t^s = \tau_t^s \hat{w}_t^m \hat{h}_t^m \tag{32}$$

4 Parameter Estimates

4.1 Method

Estimation and inference are major issues when dealing with DSGE models. A common solution in the empirical literature is to recur to Bayesian methods and in particular to Markov chain Monte Carlo algorithms (MCMC). The model, defined through equations (18)-(32) is in fact an highly nonlinear system that cannot be estimated straightforwardly. For this reason the system is linearized and solved in order to derive an easy to handle reduced form. To tackle these nonlinearity issues, it is common practice to linearize the system through a first order Taylor expansion around its steady state. This approximation leads to the following representation of the dynamic system

$$\Gamma_0 \mathbf{x}_t = c_x + \Gamma_1 \mathbf{x}_{t-1} + \Gamma_z \mathbf{z}_t + \Pi \eta_t \tag{33}$$

in which $\mathbf{x}_t$ is the endogenous state vector, $\mathbf{z}_t$ is a zero mean autoregressive exogenous process with shocks ϵ_t, whereas η_t are the forecasting errors. Γ_0, Γ_1, Γ_z, Π and c_x are matrices whose entries are functions of the structural parameters and of the steady states of the model. Even if (33) is an approximate version of the model, we stress that it is still a structural representation of the system, that has to be solved to derive its reduced form. There are many strategies available in the literature to overcome this problem, (see An and Schorfheide, 2007 for instance). In this paper we use the algorithm implemented in Sims (2002), that leads to

$$\mathbf{x}_t = \Theta_c + \Theta_x \mathbf{x}_{t-1} + \Theta_z \mathbf{z}_t \tag{34}$$

in which the system's matrices still depends on the structural parameters $\boldsymbol{\theta}$ and on the steady states. The second relevant issue for inference is that the system cannot be estimated through standard methods, since $\mathbf{x}_t$ is partially non observable and then the likelihood cannot be computed. To cope with this problem, the vector $\mathbf{x}_t$ is linked to a set of observable variables, indicated through the vector $\mathbf{y}$. Using matrix notation, the observables are related to the state vector through the following relation

$$\mathbf{y}_t = S\tilde{\mathbf{x}}_t \tag{35}$$

in which S is a selection matrix, whereas $\tilde{\mathbf{x}}_t = (\mathbf{x}_t, \mathbf{x}_{t-1})$ is the augmented state vector that includes eventual lagged observations. Equations (34) and (35) define a linear and gaussian state space system that can be handled through the Kalman filter, a recursive algorithm that allows to exactly evaluate the likelihood function $L(\mathbf{y}|\boldsymbol{\theta})$ even in presence of latent processes. Here, we base our inference on the Bayesian paradigm, that has proved to be successful in the empirical macroeconomic literature. In particular, Bayesian methods allow to incorporate additional information into the parameter estimation procedure through prior distributions, that eventually reduce the risks of non identification troubles for the parameters by adding curvature to the likelihood function. The choice of these prior distributions will be extensively described in Section 4.3. Our goal is to estimate jointly the parameter vector together with the latent process $\mathbf{x}_t$. In particular we aim at evaluating the magnitude of the underground economy in Italy. This task can be easily handled through an MCMC algorithm.

The basic idea behind MCMC is to build a Markov chain transition kernel starting from a given initial point and with limiting invariant distribution equal to the posterior distribution of the quantities of interest. Under suitable conditions (see Robert and Casella, 1999, chap. 6-7), such a transition kernel converges in distribution to the target posterior density $p(\boldsymbol{\theta}|\mathbf{y})$. This Markov chain trajectories are obtained through simulations, following a two steps procedure. First, a new movement is proposed by simulating the new position from a proposal distribution, and second, this move is accepted or rejected according to some suitable probabilities that depend on the likelihood function and on the prior distribution of the parameters $p(\boldsymbol{\theta})$. In a nutshell, given a starting value for the parameter's vector $\boldsymbol{\theta}^{(0)}$, we simulate trajectories of the Markov chain $\{\boldsymbol{\theta}^{(j)},\ j = 1, \ldots, n\}$ whose draws converge to the posterior distribution. Once convergence is achieved, inference can be based on the generated serially dependent sample simulated from the posterior. More precisely, estimates of the posterior means $\mathrm{E}_{p(\boldsymbol{\theta}|\mathbf{y})}[\boldsymbol{\theta}]$ are obtained by averaging over the realization of the chains, i.e., $\hat{\boldsymbol{\theta}} = n^{-1} \sum_{j=1}^{n} \boldsymbol{\theta}^{(j)}$. To account for serial correlation induced by the markovian nature of this procedure, we estimate the numerical standard error of the sample posterior mean using the approach implemented, for instance, in Kim *et al.* (1998).

In the MCMC literature, there are many different ways to propose a move for the Markov chain. Our inferential procedure is based on a Random Walk Metropolis-Hastings algorithm, as suggested for instance in An and Schorfheide (2007), in which the proposal distribution depends uniquely on the current state of the chain at time j, i.e., $q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(j)})$. The procedure can be summarized as follows

`MCMC algorithm`

- Initialize the chain at $\boldsymbol{\theta}^{(0)}$
- At step $j = 1, \ldots, n$
  - Update $\boldsymbol{\theta}$ in block through a random walk Metropolis-Hastings scheme

$$\boldsymbol{\theta}^* \sim q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(j-1)});$$

  - Compute the acceptance probability $\alpha(\boldsymbol{\theta}^{(j-1)}, \boldsymbol{\theta}^*)$ defined as

$$\alpha(\boldsymbol{\theta}^{(j-1)}, \boldsymbol{\theta}^*) = \frac{p(\boldsymbol{\theta}^*)L(\mathbf{y}|\boldsymbol{\theta}^*)q(\boldsymbol{\theta}^{(j-1)}|\boldsymbol{\theta}^*)}{p(\boldsymbol{\theta}^{(j-1)})L(\mathbf{y}|\boldsymbol{\theta}^{(j-1)})q(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{(j-1)})}$$

- Draw u from an $U(0,1)$ random variable. If $\alpha(\boldsymbol{\theta}^{(j-1)}, \boldsymbol{\theta}^*) \leq u$
    - Then $\boldsymbol{\theta}^{(j)} = \boldsymbol{\theta}^*$;
    - Else $\boldsymbol{\theta}^{(j)} = \boldsymbol{\theta}^{(j-1)}$;

- $j = j + 1$

In this paper, all the calculations are based on software written using the Ox©5.00 language of Doornik (2001), combined with the state space library `ssfpack` of Koopman *et al.* (1999) and the `LiRE` library to solve rational expectation models of Mavroeidis and Zwols (2007). Moreover, the initial value $\boldsymbol{\theta}^{(0)}$ has been set by maximizing the posterior mode $p(\boldsymbol{\theta})L(\mathbf{y}|\boldsymbol{\theta})$. Once the initial value has been set, we built a multi-chain MCMC procedure based on 4 chains of size 200,000. As stated before, the movement of the chain is characterized by a random walk dynamics, i.e., $\boldsymbol{\theta}^* = \boldsymbol{\theta}^{(j-1)} + \tilde{\eta}_j$ in which $\tilde{\eta}_j \sim N(0, \Sigma)$. A rule of thumb to define an *optimal* scaling factor Σ that allows for reasonable convergence properties of the algorithm, is to guarantee an acceptance rate ranging between 25% to 35%. In our empirical application we found a rate of about 28 percent.

4.2 Data

In this paper we consider the following set of *measurement* equations to link our theoretical model to the real world economy

$$\mathbf{y}_t \equiv \begin{bmatrix} \Delta c_t \\ \Delta x_t \\ \Delta G_t^c \\ \Delta G_t^s \\ \Delta G_t^h \\ \Delta w_t^h \end{bmatrix} = \begin{bmatrix} \gamma^{(Q)} \\ \gamma^{(Q)} \\ \gamma^{(Q)} \\ \gamma^{(Q)} \\ \gamma^{(Q)} \\ \gamma^{(Q)} \end{bmatrix} + 100 \begin{bmatrix} \hat{c}_t - \hat{c}_{t-1} \\ \hat{x}_t - \hat{x}_{t-1} \\ \hat{G}_t^c - \hat{G}_{t-1}^c \\ \hat{G}_t^s - \hat{G}_{t-1}^s \\ \hat{G}_t^h - \hat{G}_{t-1}^h \\ \hat{w}_t^h - \hat{w}_{t-1}^h \end{bmatrix} \tag{36}$$

in which Δc_t is the consumption growth expressed in percentage terms, Δx_t is the investment growth, Δw_t^h is the change in the gross real total earnings paid in the regular market (i.e. $w_t^h = w_t^m h_t^h$), $\Delta G_t^i,\ i = c, s, h$ are the growth rates of fiscal revenues from respectively corporate taxation, social security contributions and personal income taxation, and finally $\gamma^{(Q)} = 100log(\gamma)$ is the common quarterly trend growth rate.

The model is estimated by using quarterly figures provided by the Italian National Institution of Statistics (ISTAT) over the full sample period 1982:1 to 2006:4.[9] All the data are in real terms (base year 2000) and divided by total population aged 15-64. The choice of the observable variables is directly guide by the theory. More precisely, given that our ultimate goal is to estimate the size and trend of the underground economy, we choose as observables those aggregates that, according to our model, are particularly informative on the magnitude of underground economic activities. In this respect, data on aggregate consumption and investment proxy the general level of economic activity in Italy; fiscal revenues data capture the incentives of firms and households to engage in underground transactions; and finally official labor earnings data are informative on the households' opportunity cost to supply labor services in the underground sector.

[9]ISTAT provides data on fiscal revenues and labor earnings at yearly frequencies. Quarterly figures for these series are made available by Associazione Prometeia of Bologna.

Table 1: DSGE with Underground Economy - **Prior distributions**

	Mean	S.E.	Type	Domain
α	0.650	0.020	Beta	$[0,1]$
δ_k	0.025	0.005	Beta	$[0,1]$
α_u	0.710	0.010	Beta	$[0,1]$
ρ_a	0.800	0.100	Beta	$[0,1]$
ρ_b	0.500	0.100	Beta	$[0,1]$
ρ_c	0.600	0.100	Beta	$[0,1]$
ρ_h	0.500	0.100	Beta	$[0,1]$
ρ_s	0.600	0.100	Beta	$[0,1]$
ρ_H	0.600	0.100	Beta	$[0,1]$
ρ_I	0.800	0.100	Beta	$[0,1]$
σ	1.000	0.050	Gamma	$\mathbb{R}^+$
ϕ	0.060	0.010	Gamma	$\mathbb{R}^+$
ξ	1.000	0.100	Gamma	$\mathbb{R}^+$
B_1	16.000	0.400	Gamma	$\mathbb{R}^+$
$100\sigma_a$	0.600	0.160	Inverse Gamma	$\mathbb{R}^+$
$100\sigma_b$	0.600	0.160	Inverse Gamma	$\mathbb{R}^+$
$100\sigma_c$	0.600	0.160	Inverse Gamma	$\mathbb{R}^+$
$100\sigma_h$	0.600	0.160	Inverse Gamma	$\mathbb{R}^+$
$100\sigma_s$	0.600	0.160	Inverse Gamma	$\mathbb{R}^+$
$100\sigma_H$	0.600	0.160	Inverse Gamma	$\mathbb{R}^+$
$100\sigma_I$	0.600	0.160	Inverse Gamma	$\mathbb{R}^+$
$\gamma^{(Q)}$	0.230	0.1	Normal	$\mathbb{R}$

4.3 Prior distributions and calibrated parameters

In a Bayesian framework, one of the crucial tasks is to define the prior distribution of the parameters that summarizes our prior beliefs about the state of the economy we aim at modeling. Our priors are summarized in Table 1. Overall, we considered prior densities that match the domain of the structural parameters. Starting with the underground economy-related parameters, our prior choice is mostly based on some previous analysis provided by ISTAT. More specifically, the elasticity of labor in the underground production function, α_u, is assumed to be a Beta random variable with mean 0.71 and standard deviation 0.02 while the disutility of working activities in the underground economy, B_1, is assumed to follow a Gamma distribution with mean 16.00 and standard deviation 0.4. Conditional to all the other prior parameter expected values, the prior means of α_u and B_1 imply a steady-state size of underground economy (Y^u/Y) and a steady-state share of total worked hours ascribed to the underground sector (H^u/H) of respectively 19% and 13%. These numbers match the estimates of underground output-to-GDP ratio and the irregular labor share provided by ISTAT over the period 1982-2006. This prior choice is motivated as we firmly believe that these estimates are the most reliable available on the underground economy in Italy. For this reason, these information have been set as the starting point for our analysis. The inverse of the Frisch elasticity of underground labor supply ϕ is instead described by a Gamma distribution with mean 0.06 and standard deviation 0.01. The prior mean is chosen consistently with the calibration reported in Busato *et al.* (2005).

For the parameters that are commonly used in the DSGE literature, our prior choice is consistent with previous studies (An and Schorfheide, 2007, Smets and Wouters, 2007,

Table 2: DSGE with Underground Economy - **Posterior computation (MCMC)**

	MCMC output (posterior distribution)			prior information		
	Mean	Mode	95% Conf. Int.	Mean	S.E.	Type
$p(\alpha\|\mathbf{y})$	0.5660	0.5689	[0.504,0.610]	0.650	0.020	Beta
$p(\delta_k\|\mathbf{y})$	0.0405	0.0403	[0.029,0.052]	0.025	0.005	Beta
$p(\alpha_u\|\mathbf{y})$	0.6880	0.6880	[0.669,0.706]	0.710	0.010	Beta
$p(\rho_a\|\mathbf{y})$	0.7278	0.7393	[0.534,0.862]	0.800	0.100	Beta
$p(\rho_b\|\mathbf{y})$	0.5459	0.5521	[0.312,0.741]	0.500	0.100	Beta
$p(\rho_c\|\mathbf{y})$	0.5911	0.6038	[0.352,0.753]	0.600	0.100	Beta
$p(\rho_h\|\mathbf{y})$	0.4878	0.4892	[0.282,0.664]	0.500	0.100	Beta
$p(\rho_s\|\mathbf{y})$	0.5881	0.5923	[0.394,0.760]	0.600	0.100	Beta
$p(\rho_H\|\mathbf{y})$	0.6200	0.6221	[0.444,0.787]	0.600	0.100	Beta
$p(\rho_I\|\mathbf{y})$	0.9967	0.9971	[0.991,0.999]	0.800	0.100	Beta
$p(\sigma\|\mathbf{y})$	1.0702	1.0692	[0.981,1.168]	1.000	0.050	Gamma
$p(\phi\|\mathbf{y})$	0.0694	0.0690	[0.049,0.091]	0.060	0.010	Gamma
$p(\xi\|\mathbf{y})$	1.2914	1.2894	[1.080,1.516]	1.000	0.100	Gamma
$p(B_1\|\mathbf{y})$	16.3094	16.3031	[15.537,17.090]	16.000	0.400	Gamma
$p(100\sigma_a\|\mathbf{y})$	0.1814	0.1807	[0.156,0.210]	0.600	0.160	IG
$p(100\sigma_b\|\mathbf{y})$	0.1704	0.1697	[0.147,0.197]	0.600	0.160	IG
$p(100\sigma_c\|\mathbf{y})$	0.1652	0.1645	[0.144,0.191]	0.600	0.160	IG
$p(100\sigma_h\|\mathbf{y})$	0.1656	0.1651	[0.144,0.189]	0.600	0.160	IG
$p(100\sigma_s\|\mathbf{y})$	0.1647	0.1641	[0.145,0.187]	0.600	0.160	IG
$p(100\sigma_H\|\mathbf{y})$	0.3032	0.3010	[0.246,0.373]	0.600	0.160	IG
$p(100\sigma_I\|\mathbf{y})$	0.1696	0.1690	[0.147,0.195]	0.600	0.160	IG
$p(\gamma^{(Q)}\|\mathbf{y})$	0.1595	0.1579	[0.040,0.288]	0.230	0.1	Normal

Iacoviello and Neri, 2010 among the others). More precisely, we assume that the inverse of the inter-temporal elasticity of substitution σ and the inverse of the elasticity of total labor supply ξ are distributed according to a Gamma random variable both with mean 1 and standard deviation respectively set to 0.05 and 0.1. The elasticity of labor in the regular production function α is assumed to follow a Beta distribution with mean 0.65 and standard deviation 0.02. The capital depreciation rate δ_k is assumed to be a Beta random variable centered at a quarterly rate of 2.5 percent, i.e. $\mathrm{E}[\delta_k] = 0.025$, and with standard deviation 0.005. Finally, the common quarterly trend growth rate, γ^Q, is assumed to follow a Gaussian prior with mean 0.23 and standard deviation 0.1. The prior mean is chosen to match the average growth rate of actual per-capita GDP over the period 1982-2006. This choice is consistent with the balanced growth path hypothesis.

Regarding the exogenous processes, we assume that the standard error of the innovations follow a rather dispersed Inverse Gamma distribution, to summarize a lack of a priori information about these quantities. The persistence of the AR(1) processes (i.e., parameters ρ_a, ρ_b, ρ_c, ρ_h, ρ_s, ρ_H and ρ_I) are instead described by Beta distributions with means ranging between 0.5 and 0.8 to allow for moderate to high persistence on the mechanism of propagation of the exogenous shocks. In particular, the variances of these priors are relatively high, to account for a wide range of possible posterior values for these parameters. This hypothesis is consistent with Smets and Wouters (2007).

Finally, the remaining parameters are fixed, either because they reflect some characteristics that are regulated ex-ante by law, or because they are difficult to be identified.

More specifically, the steady state parameters τ_c, τ_h and τ_s, that represent respectively the average tax rates on corporate profits and personal income, and the rate of social security contributions are fixed at 41.55%, 34.26% and 21%. These numbers are consistent with the average tax rates imposed in Italy over the period 1982-2006.[10] Furthermore, the penalty payed by a firm once detected, is set to 30% of the tax rate on corporate. The surcharge factor is thus $s = 1.30$, in line with the current Italian Tax Law (**?**). The probability p for a company to be inspected is set to 3%, corresponding to the estimate found by **?** using data on the number of inspected firms released by the Italian Ministry of Labour. The subjective discount factor β is set to 0.9840, implying a steady-state gross interest rate of 1.0186. Finally, the parameter controlling for the disutility of total labor supply, B_0, has been set to the value implying a steady-state share of hours for working activities of 19%.[11] This value corresponds to the average hours worked in a quarter as a fraction of the total quarterly hours for the period 1982-2006.

4.4 Posterior Distributions

Table 2 shows the posterior mean, mode, an 95 percent probability interval for the structural parameters, together with mean and standard deviation of the prior distributions. A practical way usually employed to assess identification of the parameters, is to compare prior to posterior distributions, to check if observable variables are informative for inferential purposes. These results are displayed in Figure 14.

On closer inspection at the parameters governing production, i.e., α and α_u, suggests that the contribution of the observed data (likelihood) is relevant. In particular, likelihood provides a sensitive negative shift with respect to the prior information. Specifically, the elasticity of regular production to labor α has a posterior mean of .57 while irregular labor elasticity α_u has a posterior mean of about .68. The observed difference between the estimates of these two parameters might be interpreted as an evidence in favor of a higher output sensitivity to labor in the irregular market.

With regard to the households, the posterior estimate of the inverse of the inter-temporal elasticity of substitution σ is larger than its prior counterpart. Data therefore suggests that consumption is somewhat less sensitive to movements in the real interest rate than what implied by the prior distribution. We also find that the posterior estimate of the labor supply elasticity in the regular market ξ is substantially higher than the a priori hypothesis, while its counterpart in the irregular market ϕ is only slightly larger compared to the a priori assumption. In particular, the posterior mean of ϕ is small (0.07), suggesting that the labor supply in the underground sector is highly sensitive to movements in the net wage differential. The estimate of δ_k (0.045) implies an half-life of the capital stock of about 4 years. Furthermore, the posterior mean of the parameter controlling for the disutility of

[10] More precisely, the values of average tax rate on personal income and the rate of social security contributions are taken from **?**, while the value of τ_h corresponds to the average Italian statutory corporate tax rates over the period 1982-2006. Values for statutory tax rates are taken from the OECD Tax Database. For the period 1998-2006 the OECD data has been augmented by 4.25% to account for the newly introduced regional corporate taxation (IRAP).

[11] In the estimation procedure, this parameter is updated at any iteration by using equation (27) evaluated at the steady-state.

irregular labor, B_1 is equal to 16.31, that is slightly larger than its prior mean. This number, together with the estimate of α_u, implies a steady-state size of the underground economy of about 22 percent, that is 4 percentage points larger with respect of the official statistics provided by ISTAT.

Turning to the exogenous processes, the autoregressive coefficients provide information about the persistence of the shocks. Some lack of identification may be suspected for the parameters describing the mechanism of propagation of the exogenous shocks, such as ρ_c, ρ_h and ρ_s, since the information provided by data does not change much the posterior distribution with respect to the prior. However, it is worth nothing that the persistence level of the investments shock process, that is ρ_I, dramatically increases, providing strong evidence of shock persistence for the investments held by consumers. On the other side, the variances of exogenous shocks are clearly identified.

Finally, some convergence diagnostics are presented in Figure 14, where recursive averages of the sampler have been reported. Specifically, the plot reports the evolution of $\frac{1}{i}\sum_{j=1}^{i}\boldsymbol{\theta}^{(j)}, \forall i$. Of course, it is impossible to assess convergence properties of an MCMC algorithm through the study of just few realization of the chains. However a common practice is to check for the convergence of the empirical averages of the draws (see Robert and Casella, 1999, chap. 8 for a survey on this topic.) We can say that a chain converges rapidly if evolution of its empirical averages stabilizes after few iterations. As Figure 14 illustrates, it is quite evident that the running averages for the algorithm stabilize soon with the only exception of ρ_a, showing evidence of convergence of the algorithm. According to our experience, it seems that the random walk algorithm adopted needs about 100,000 iterations to converge to its correct expected value, and, according to these results, we discarded the first 100,000 draws from each chain to remove the dependence from the initial condition $\boldsymbol{\theta}^{(0)}$.

5 The underground economy in Italy and its sources

Having estimated the model, we now use it to address the main questions of the paper. First, how big is the size of the underground economy in Italy? Second, how does the underground sector respond to exogenous shocks? Finally, what are the main driving forces of fluctuations in the underground output?

5.1 The size and trend of the underground economy

The top panel of Figure 1 depicts the smoothed estimate of the ratio of underground production to total GDP along with the 95 percent confidence bands.[12] This figure summarizes how our model predicts the size and the trend of the underground economy in Italy over the period 1982-2006. The series started the sample around its estimated mean, slightly over 21%, and then grew slowly during the 1980s, until reaching its peak in late 1991. After

[12] In practice, we picked at random a subset of 1,000 posterior draws from the MCMC algorithm and, for each set of these parameters, we computed an estimate of the endogenous variables through the simulation smoother algorithm of de Jong and Shephard (1995). Our posterior estimate is thus the average of all the trajectories obtained, whereas confidence bands have been computed as the 2.5 and the 97.5 percentiles of the empirical distribution. This procedure allows to take into account also parameter uncertainty.

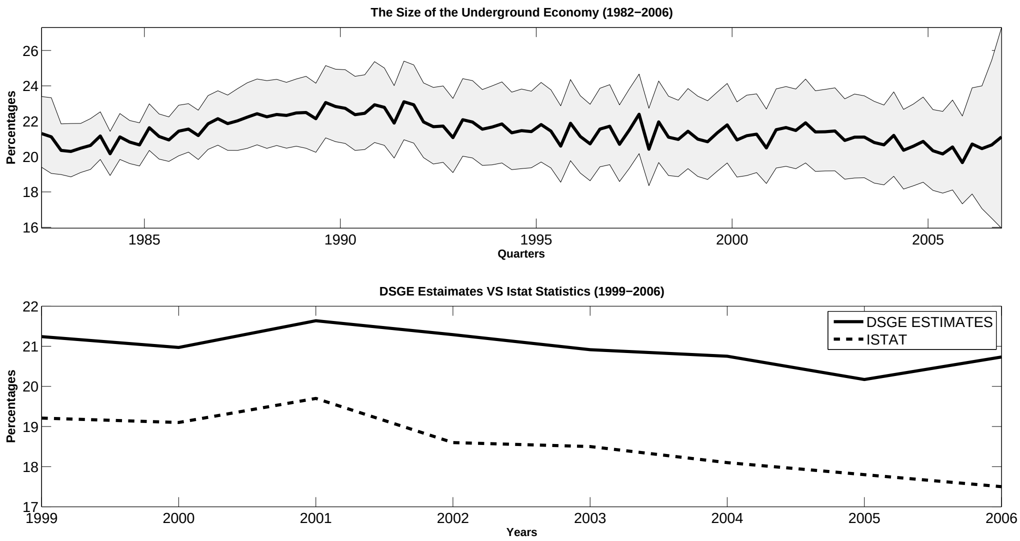

Figure 1: **The Estimated size of the underground economy.** The top panel depicts the smoothed estimate of the quarterly ratio of underground production to total output. The series is depicted along with the 95 percent confidence bands. The bottom panel compares our model's predictions on the size of the underground economy (continous line) with the ISTAT estimates over the period 1999-2006. Model's predictions have been annualized by taking the average over the four quarters in each year.

that year, the series decreases slowly up to early 1994, and then fluctuates around its mean up to the end of the sample, with a peak in the year 2001. The reported confidence bands show that our estimates are quite precise. Excluding the last two quarters of 2006 where the length of the confidence interval substantially increases, the two bands rang from a minimum of 17% to a maximum of 25% over the whole sample, a range of values that is quite tight around the smoothed estimates.

In order to assess whether our results represent a reliable estimate of the underground economy in Italy, we now compare our model's predictions with the ISTAT's estimates. The official data are released in the form of intervals, and thus, in order to compare these data with ours, we take the upper bound of the reported interval in each year. This corresponds to the maximum hypothesis assumed by ISTAT on the size of the underground economy, and involves an aggregate which, relative to the minimum hypothesis (the lower bound of the released interval), is more consistent with our definition of underground transactions. The results of this comparison is provided in the bottom panel of Figure 1, where we graph our series (continuous line) along with the ISTAT estimates (dashed lines) over the period 1999-2006.[13] As the official data are released at yearly frequencies, we annualized

[13]ISTAT has released two different vintages of data on the underground economy. One in the year 2004 covering the period of time 1992-2000, and another one in the year 2010 for the period 2000-2008. The two series have been constructed by using different methods that rely to different macro aggregates. These data are therefore not fully compatible. For this reason, in order compare our estimates with the official data,

our series by taking the average over the four quarters of each year. Two main results are worth emphasizing. First, although our priors specification is designed to match the official estimate, we note that at posterior parameters values our model predicts percentages for the share of the GDP ascribed to the underground economy which are systematically higher (on average, about 3 percentage points) than the official ones. This result is somewhat consistent with other available estimates which, relative to the ISTAT's official data, also find an higher size of the underground economy (see Schneider *et al.* (2010) among others). Second, the dynamic profile of the two series is strikingly similar, with a coefficient of contemporaneous correlation of 0.83.[14] Notice, for instance, that both series peak in the year 2001, and then slowly decline. The most important discrepancy occurs in the year 2006, where our model predicts an increase in the size of the underground economy while according to the official data it was instead declining. Most likely, this result is related to the fact that in the year 2006 the precision of our estimates substantially deteriorates, as evidence in the first panel of figure 1. Overall, we believe that the strikingly similar dynamics between our estimates and the official data is among the most interesting findings of our study. This is particularly true taking into account that our approach, which is essentially model based, is deeply different from the method followed by the Central Statistical Institute in Italy which is basically an accounting method. As such, we interpreted this result as reviling that the information we processed is relevant and entirely pertinent to tackle the issue of measuring the underground economy.

5.2 Impulse response

The response of the underground economy model to the estimated exogenous shocks can be assessed through the impulse-response functions. This is done in figures 7-13 where we graph the impulse-response functions of regular and underground production, total output (GDP), consumption, investment, and total worked hours along with the 95 per cent confidence bands.[15]

To begin with, we note that consumption, investment and total hours worked all increase in response of an exogenous boost in the official sector productivity, A_t (see figure 7). This is the well known effect of a positive technology shock that characterizes any standard real business cycle model (see,e.g. King and Rebelo, 1999). The presence of underground economy, however, implies an additional resource reallocation effect. As an increase in the rate of technology A_t makes official output relatively more productive, firms find more convenient to produce final output with regular workers rather than with the irregular ones. Consequently, in response of a temporary boost in A_t, total official output increases while

we only focused on period of time covered by the last vintage of data as they are constructed in a way that satisfies the Eurostat's requirements. The data set released in the year 2004 has been only used to recover the size of underground economy for the year 1999.

[14]Results do not change if we use the minimum hypothesis as official estimates of the underground economy In this case, the coefficient of contemporaneous correlation between this series and our estimates is equal to 0.81.

[15]As for the estimated size of the underground economy, these quantities have been computed as the posterior average of the impulse response functions obtained for each draw of the MCMC algorithm. Confidence bands have been computed as the 2.5 and 97.5 percentiles of the empirical distributions obtained.

the underground one declines. This effect partially dampens the response of total GDP, which in fact, relative to the official output, increases by a lower rate.

Results are reverted when the economy is hit by a temporary boost in the rate of irregular sector productivity, B_t.[16] In this case, the underground level of output increases while the official one declines, as clearly appears in figure 8. At the posterior parameter values, the effect of this shock on the underground production is strong enough to overcompensate for the decline in official output, so that total GDP also increases. Interesting, this shock also affects the short-run intertemporal elasticity of substitution, making the households less willing to substitute present with future consumption. In fact, with a shock of this type, our model predicts that the response of investment is negative for about the first four quarters after the shock, while the response of consumption is always positive and monotonic.

The impact of fiscal shocks are summarized in figures 9-11, where we report the effects of temporary increases in tax rates. Unsurprisingly, as taxation is distortive in our framework, the estimated model suggests that increasing taxes implies a negative response of consumption, investment, total worked hours and GDP. Furthermore, movements in taxes also imply a resource reallocation effect: underground production increases while the official one declines. However, while in the case of corporate taxation (figure 9) and social security contributions (figure 10) this effect results from a higher (net) expected returns from underground production, with taxes on personal income (figure 11) the effect operates instead through a labor-supply channel. All else being equal, an increase of τ_t^h induces on impact a larger net wage-gap differential, thus pushing households to reallocate labor services from the regular to the irregular labor market. This effect provides downward pressures on the irregular labor wage and, at the same time, upward pressures on the official one. As a result, firms find it more convenient to produce a larger part of their outputs with irregular workers.

Finally, the effects of demand shocks are provided in figures 12 and 13 where we graph the response of the economy to respectively a temporary increase in the rate of transformation of investment in capital and a temporary boost in disutility of total hours worked. As the pictures illustrate, these two demand shocks have a rather different impact on the economy. In particular, an unexpected increase in ξ_t^h has a depressive effect on the economy, leading to a decrease in the equilibrium level of all the main aggregates. This is due to the lower demand of both investment and consumption goods that results from a shock of this type. With larger values of ξ_t^h, in fact, households experience an increase in the disutility of labor that pushes them to substitute consumption with leisure over the time. This effect results in lower demand of both consumption and investment goods, and thus in a decline of total production. On the contrary, an unexpected increase in ξ_t^I stimulates current investment (at the cost of a lower present consumption), and thus results in a net increase of aggregate demand to which firms respond by increasing both underground and official production. This is illustrated in figure 12 which shows that, with the exception of consumption, all the main economic aggregates increase in response to this shock.

Summarizing, the analysis of the estimated impulse-response functions shows that the presence of the underground economy gives rise to an additional inter-sector resources reallocation effect. Basically, our analysis suggests that agents engage in concealed transactions

[16]For instance, the effect provided by an unexpected increase in regular migration.

Table 3: Cyclical properties and variance decomposition

Business Cycle Statistics				
		$(Y_t^u; Y_t)$	$(Y_t^u; Y_t^m)$	$(Y_t^m; Y_t)$
Correlation		0.17	-0.20	0.93
Relative standard deviation		1.77	1.26	1.40
Variance Decomposition (Percentages)				
Variable	*A*	*B*	Fiscal	Demand
Y_t^u	2.73	84.71	1.40	11.16
Y_t^m	79.51	1.03	4.14	15.32
Y_t	70.44	5.93	2.62	20.99
X_t	32.99	13.67	16.37	39.97
C_t	2.74	5.95	1.74	89.56
H_t	21.62	0.11	10.89	67.38

in order to further insure themselves against exogenous shocks. This mechanism is particularly important in a political economy perspective. When we explicitly consider the underground economy, in fact, the efficiency of fiscal policy interventions also depends on how important is the resulting inter-sectorial resources reallocation effect.

5.3 Cyclical Properties

The top panel of table 3 reports second order moments for total GDP (Y_t), official production (Y_t^m) and underground output (Y_t^u). These statistics are useful to asses the cyclical properties of the underground economy at the business cycle frequencies. As the table illustrates, the estimated model predicts that the underground production is a weakly procyclical and highly volatile variable over the course of the business cycle. The contemporaneous correlation of this variable with total GDP is in fact equal to 0.17, while its standard deviation turns out to be 1.77 times larger than that of GDP. Interestingly, our model predicts that the cyclical component of the underground economy is negatively correlated with the cyclical component of the official output (-0.20). In line with the findings reported in **?**, our results provide evidence of a *double business cycle* in the Italian economy, with peaks of the official economy associated with troughs of the underground and vice versa.

The second panel of table 3 presents results from the variance decomposition. Accordingly, the technology shock in the irregular sector (B_t) is predicted to be the main driving force of fluctuations in underground output. As illustrated in the table, this shock alone explains around 85% of the variance in underground production at business cycle frequencies. The demand component (the sum of investment and preference shocks) is also quantitatively important, explaining around 11%, while contributions of the other shocks are rather small. For instance, the fiscal component (the sum of the three fiscal shocks) explains only 1.40 percent of the variance in the underground economy. Regarding the other aggregates, we

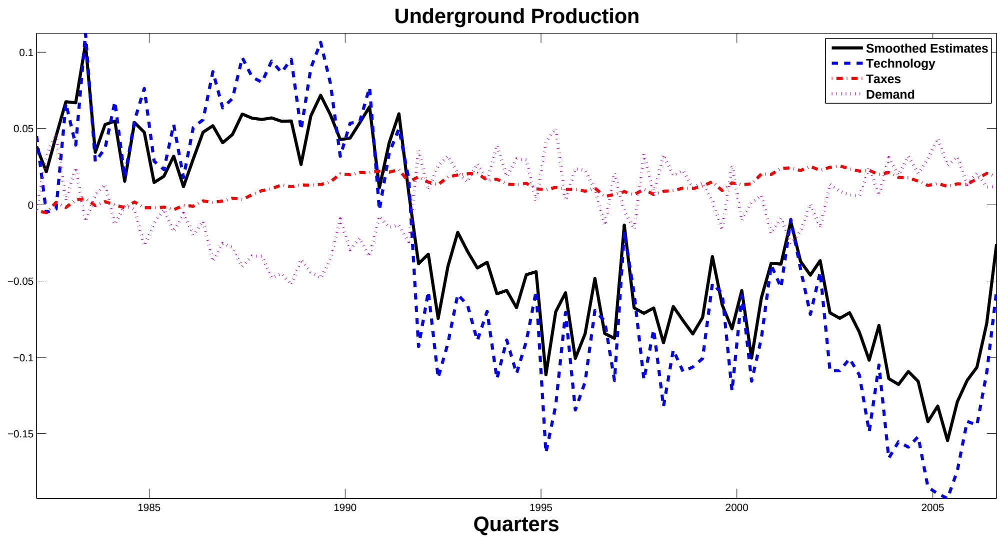

Figure 2: **Historical decomposition.**

see that the variance in consumption (C_t) is mostly explained by the demand component (89.56%) while investment (X_t) are predicted to be particularly sensitive to fiscal shocks (16%). Additionally, we note that the fiscal component also explains an important part (10%) of the variance in total worked hours (H_t). This result reflects the intra-temporal reallocation effect that occurs in our model in correspondence of tax changes.

Figure 2 displays the smoothed estimates of underground output (in log-deviation from the steady-state) along with the historical contribution of technological, demand and fiscal factors. This picture gives an immediate visual representation of the relative contribution of each shock to specific cycles of the underground economy in Italy. According to figure 2, the estimated model predicts two major contractions in the underground economy over the period 1982-2006: one from 1992 to 1995, and the second from 2002 to 2005. In the first drop, the underground production decreased (relative to its estimated steady-state) by 11 percent. Between 2002 and 2005, the total amount of output produced in the underground sector was about 15 percent lower than its steady-state. The major expansion instead occurs in the period 1982-1984 where the underground rose by 10 percent. As the picture illustrates, fluctuations in the underground production are mostly explained by the technological component. The contributions of the the other shocks is instead asymmetric, being somewhat important during contractions and negligible during the expansion.

6 Policy Implications

The recent Italian sovereign debt crisis has strengthened the urge among Italian policy makers to design suitable policies for fighting tax evasion. This issue is nowadays perceived as a priority in Italy, not only to increase fiscal revenues in order to prevent risk of national

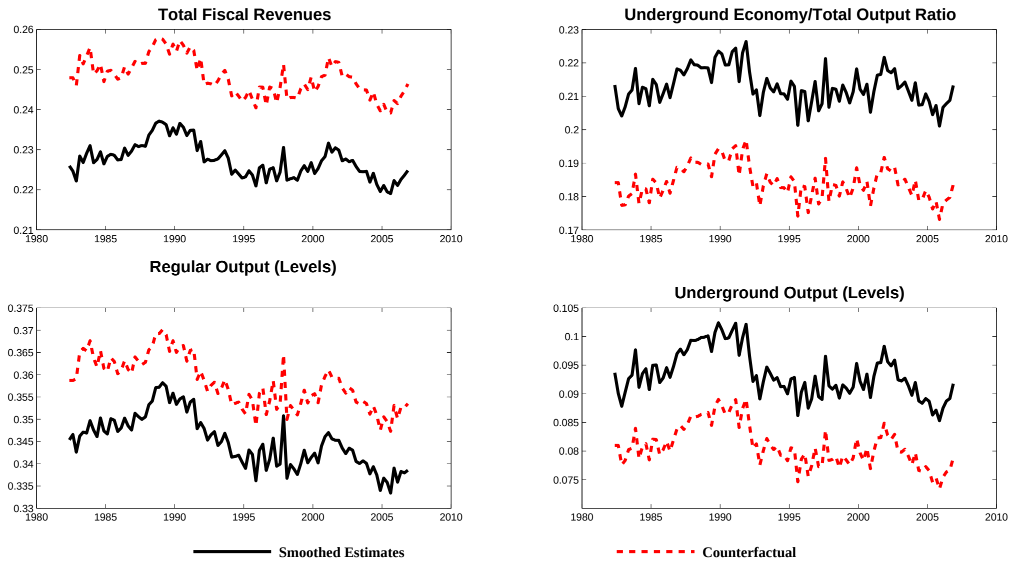

Figure 3: Effect of a change of p and s on the total fiscal revenues (upper left panel), on the underground economy expressed in percentage terms (upper right panel), and on the regular and the underground levels of the economy (lower panels)

default[17] but also to improve fiscal equity among individuals. Clearly, to achieve these targets in an efficient way, it is of fundamental importance to know how the underground economy reacts to different fiscal and institutional stimuli. In this perspective, one of the main advantages of our approach with respect to more traditional methodologies, is that it provides a natural *laboratory* to derive policy implications. The estimated model allows us to assess in a general equilibrium perspective how macro aggregates, such as underground production or total fiscal revenues, react to specific fiscal policies. As such, although the primary focus of this paper is to estimate the size of the underground economy, we believe our methodology might be useful to derive and evaluate policy implications.

To this end, we present the results of several counterfactual experiments. Specifically, in the following exercises, we aim at measuring the dynamic path of some selected aggregates in correspondence to a tax reduction and to an increase of sanctions for tax evasion.[18] In particular, for each alternative scenario, we compare the predictions of some selected endogenous variables based on the estimated parameters versus the counterfactual ones. This enables us to measure the reaction of our economic system to different fiscal interventions.

To begin with, in the first exercise we aim at checking the effects of an increase on the efforts to tackle tax evasion in combination with an embitterment of economic sanctions. In practice, we move the parameter p from 3% to 20% and we increase the penalty parameter

[17] In this respect, one of the main political economy objective of the new Monti's government in Italy is to balance the government budget by the year 2013.

[18] To implement our counterfactual analysis, we first set the parameters of the model at their posterior mean and then we computed the smoothed estimate of the shocks $\hat{\varepsilon}_t$. Then, after changing some parameters of interest, we finally simulate the endogenous variables using these updated parameters, while keeping fixed the smoothed shocks at $\hat{\varepsilon}_t$.

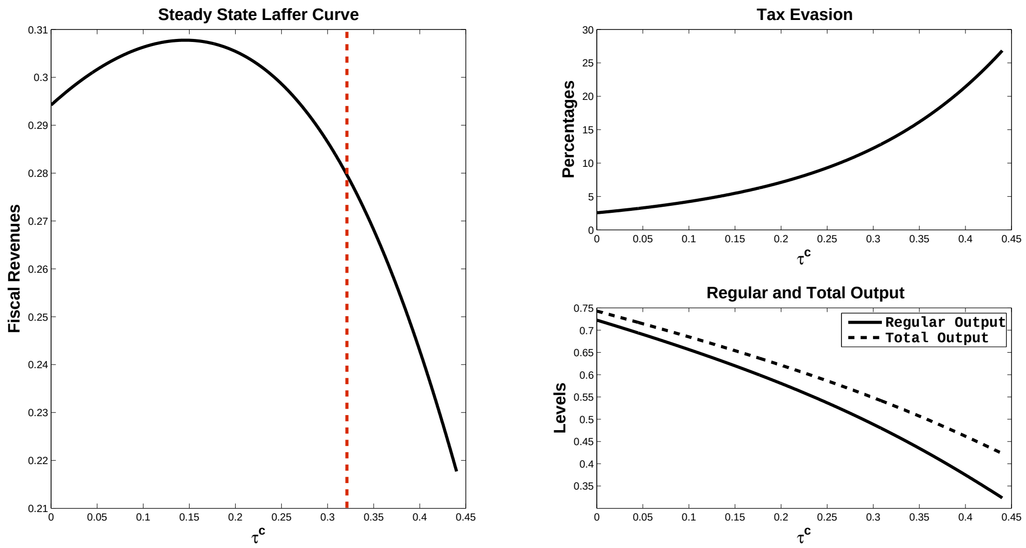

Figure 4: **Steady-state effects of corporate taxation.** This picture depicts the steady-state Laffer curve (panel A), total tax evasion (panel B) and regular and total output (panel C) as a function of the steady-state corporate tax rate τ^c. All the other parameters are kept fixed to their posterior mean values. Total tax evasion is expressed as a share of total taxes due.

s from 1.2 to 2. In our model higher values of these parameters result in a lower expected returns from underground production. This experiment therefore assesses the effects of a policy specifically designed to discourage tax evasion by weakening firms' incentives to engage in underground transactions. Results are depicted in Figure 3 where we graph the smoothed estimates (continuous lines) of total fiscal revenues (left top panel), of the size of underground economy (right top panel), of regular output (left bottom panel) and of underground output (right bottom panel) along with the corresponding counterfactual series (dashed lines). Unsurprisingly, our simulation results suggests that this policy is effective in discouraging concealed transactions. We see that the counterfactual level of underground output substantially decreases with respect to the smoothed one, being on average about 15% lower. Additionally, this policy also drives a reallocation of resources from the unofficial to the official sector, leading to a larger level of regular output which, on average, increases by 3.82%. This effect has two related implications. First, it partially compensates the negative impact of irregular production on total output, so that the size of the underground economy eventually decreases (on average by 2.88 percentage points). Second, although tax rates are left unchanged, the larger amount of regular production implies that also total fiscal revenues increase (on average by 9.08%).

As a second exercise, we aim at assessing the impact of a tax cut. One of the most distinctive feature of Italian public finances is the very high fiscal pressure. According to the OECD tax revenue statistics, in the year 2009 the taxes to GDP ratio was 43.4% for Italy, implying a fiscal pressure that was higher than both the OECD and the European averages (respectively, 33.8% and 37.1%). Because of this evidence, there exists a certain

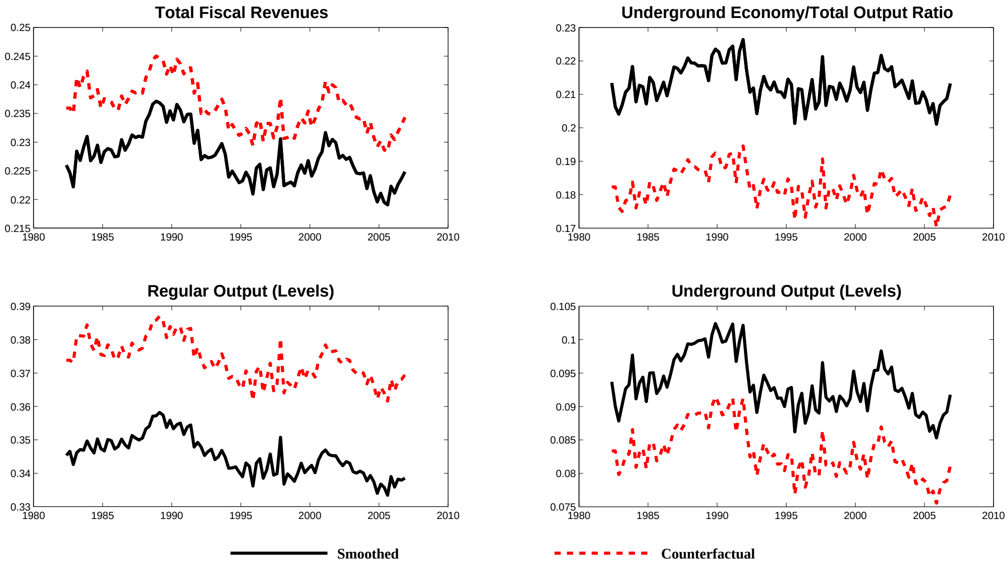

Figure 5: Effect of a reduction of a tax burden of 2 percentage points on the total fiscal revenues (upper left panel), on the underground economy expressed in percentage terms (upper right panel), and on the regular and the underground levels of the economy (lower panels)

agreement among Italian economists in saying that the fiscal burden is responsible not only for tax evasion, but also in discouraging foreign direct investment and weakening the competitiveness of Italian firms. All of these issues are likely the main driving forces responsible of the poor performance of the Italian economy in the last fifteen years.[19] In this respect, a widespread idea is that a general reduction in the tax burden would benefit the Italian economy.

A first test for this claim is provided in figure 4 where we can see the steady-state effects to changes in the corporate tax rate τ^c. More specifically, keeping all the other parameters fixed to their posterior mean values, the picture depicts total fiscal revenues (Laffer curve), tax evasion (in percentage terms with respect to the overall amount of taxes due) and regular and total output as functions of the corporate tax rate. As can be seen from panel A of the picture, the estimated steady-state Laffer curve has the typical textbook inverted U-shape. Accordingly, total tax revenues reach its maximum when the tax rate is set to approximatively 15%, and then quickly decline. The shape of the Laffer curve is determined by two related effects. First, an increase in the corporate tax rate reduces the equilibrium level of regular output (continuous line panel C). This is the standard effect associated with distortive taxation. Second, the concealed taxes as a share of total tax base (a measure of the strength of tax evasion) is a convex function of the corporate tax rate which increases quickly as τ^c moves from low to high values. This is an additional effect due to the presence of underground economy in our model. However, the expansion in underground production that results from larger corporate tax rates is not strong enough to completely compensate for the negative impact on regular output, so that the equilibrium level of total production

[19]See Orsi and Turino (2010) for a more detailed analysis on this topic.

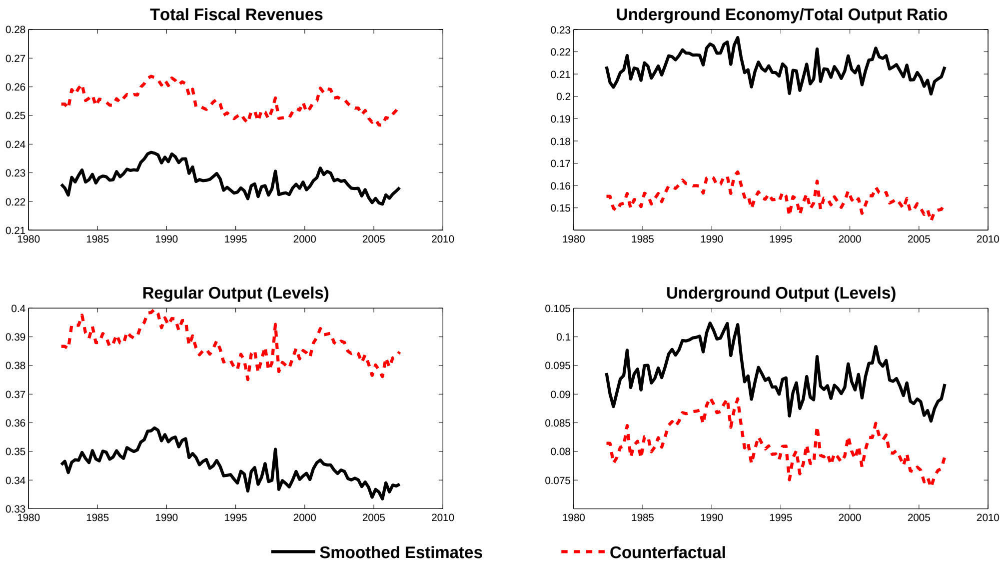

Figure 6: Effect of a reduction of a tax burden of 2 percentage points with $p = 0.2$ and $s = 2$ on the total fiscal revenues (upper left panel), on the underground economy expressed in percentage terms (upper right panel), and on the regular and the underground levels of the economy (lower panels)

also decreases with τ^c (see the dashed line in panel C). Notice furthermore that in the panel A is also depicted the actual tax rate (the vertical dashed line), showing that the level of corporate taxation in Italy is well above the value that would maximize revenues. Our model therefore suggests that, in the long-run, a reduction in the corporate tax rate would effectively benefit the Italian economy in terms of both higher fiscal revenues and higher total production.

To better assess the effects of a reduction in the tax burden, we perform an additional counterfactual experiment by reducing all the tax rates (τ_c, τ_s and τ_h) of 2 percentage points. This exercise assesses the short-run implications of a general reduction of the tax burden in Italy. As depicted in figure 5, the results of the experiment confirm to a large extent the above long-run analysis. Firstly, according to our model, cutting tax rates in Italy would have the effect of discouraging concealed transactions, thereby reducing the size of the underground economy both in levels and as a share of total production (on average, about 3 points in percentage terms). Secondly, in addition to a resources reallocation effect, this policy raises the profitability of regular production which in fact increases strongly (on average, by 9%). Finally, although the policy implies a tax rate reduction, the effect on the regular production sensibly increases total fiscal revenues (on average, by 3%).

Overall, the most remarkable message we learn from the above analysis is that in Italy fiscal pressure might be inefficiently too high. In this perspective, a general reduction of the tax burden would increase both total fiscal revenues and total production. According to our model, however, such a policy would be much more effective if accompanied by a strengthening in the effort to fight tax evasion. This is apparent in figure 6 which displays the results of a counterfactual exercise where, in correspondence of a reduction of the fiscal burden of 2 percentage points, we also augment the probability of being discovered evading ($p = 0.3$) and the penalty surcharge rate ($s = 2$). Two results of this experiment are worth emphasizing. First, the combination between the resource reallocation effect and the higher profitability of regular production induced by this policy, leads to a huge decline in the size of the underground economy. In the counterfactual simulation the ratio of underground production to total output is, on average, 5.82 percentage points lower than the corresponding smoothed estimates. Second, among the alternative scenarios we analyzed, this policy involves the higher increase in total fiscal revenues. For example, in correspondence of the same transitory shocks, we find that mixing the two policies induces an average increase in total fiscal revenues of 11.9%, about 7 percentage points more than the corresponding increase due to the tax cut alone.

7 Some concluding remarks

In this paper, we presented a new method for estimating the underground economy, based on a DSGE model in which the time series of underground output is estimated as a latent variable. The empirical analysis of the model, carried out on Italian data and following a Bayesian approach, produces a time series for the underground economy that is, on average, 3% larger than the official estimates. The dynamic profile of the series is instead quite similar. We also performed policy experiments based on the estimated model. We find that the combined effects of an increase in the probability to be discovered and in the tax surcharge, along with a moderate tax reduction causes both a substantial increase in fiscal revenues and a strong decline in the size of the underground economy. Finally, we believe that the method we propose is quite general and can readily be applied to other countries to perform comparative analysis of the phenomenon. Moreover, the theoretical model can be easily modified to assess labor market implications. We left these issues for future research.

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Appendix

A The complete log-linearized model

In what follows, we describe the log-linearized equilibrium system of equation. Notice that coefficient evaluated at the steady-state are indicated with the sub-index ss. Taxes at the steady-state are indicated just suppressing the temporal index.

$$\hat{r}_t = \hat{y}_t^m - \hat{k}_t - \left(\frac{\tau^c}{1-\tau^c}\right)\hat{\tau}_t^c$$

$$\hat{w}_t^m = \hat{y}_t^m - \hat{h}_t^m - \left(\frac{\tau^c}{1-\tau^c}\right)\hat{\tau}_t^c - \left(\frac{\tau^s}{1+\tau^s}\right)\hat{\tau}_t^s$$

$$\hat{w}_t^u = \hat{y}_t^u - \hat{h}_t^u - \left(\frac{ps\tau^c}{1-ps\tau^c}\right)\hat{\tau}_t^c$$

$$\hat{y}_t^m = \hat{A}_t + (1-\alpha)\hat{k}_t + \alpha\hat{h}_t^m$$

$$\hat{y}_t^u = \hat{B}_t + \alpha_u\hat{h}_t^u$$

$$\hat{y}_t = \left(\frac{y_{ss}^m}{y_{ss}}\right)\hat{y}_t^m + \left(\frac{y_{ss}^u}{y_{ss}}\right)\hat{y}_t^u$$

$$\hat{h}_t = \left(\frac{h_{ss}^m}{h_{ss}}\right)\hat{h}_t^m + \left(\frac{h_{ss}^u}{h_{ss}}\right)\hat{h}_t^u$$

$$\xi\hat{h}_t + \hat{\xi}_t^h = \hat{w}_t^m - \sigma\hat{c}_t - \left(\frac{\tau^h}{1-\tau^h}\right)\hat{\tau}_t^h$$

$$\frac{\Omega_0}{\xi}\hat{\xi}_t^h + \Omega_0\hat{h}_t + \Omega_1\hat{h}_t^u = \hat{w}_t^u - \sigma\hat{c}_t$$

$$\hat{c}_t = E_t\left\{\hat{c}_{t+1}\right\} + \frac{\tau^h\left[\gamma-\beta(1-\delta_k)\right]}{\gamma\sigma\left(1-\tau^h\right)}\rho_h\hat{\tau}_t^h - \frac{\gamma-\beta(1-\delta_k)}{\gamma\sigma}E_t\left\{\hat{r}_{t+1}\right\} - \left(\frac{\gamma-\beta(1-\delta_k)\rho_I}{\sigma\gamma}\right)\hat{\xi}_t^I$$

$$\hat{k}_{t+1} = \left[\frac{(1-\delta_k)}{\gamma}\right]\hat{k}_t + \left[\frac{\gamma-(1-\delta_k)}{\gamma}\right]\hat{x}_t + \left[\frac{\gamma-(1-\delta_k)}{\gamma}\right]\hat{\xi}_t^I$$

$$c_{ss}\hat{c}_t + x_{ss}\hat{x}_t = -\Omega_2\hat{\tau}_t^h + (1-\tau^h)\left[w_{ss}^m h_{ss}^m(\hat{w}_t^m + \hat{h}_t^m) + r_{ss}k_{ss}(\hat{r}_t + \hat{k}_t)\right] + w_{ss}^u h_{ss}^u(\hat{w}_t^u + \hat{h}_t^u)$$

where

$$\Omega_0 = \frac{B_0(h_{ss})^\xi\xi}{B_0(h_{ss})^\xi + B_1(h_{ss}^u)^\phi}$$

$$\Omega_1 = \frac{B_1(h_{ss}^u)^\phi\phi}{B_0(h_{ss})^\xi + B_1(h_{ss}^u)^\phi}$$

$$\Omega_2 = \tau^h\left[w_{ss}^m h_{ss}^m + r_{ss}k_{ss}\right]$$

B Impulse-Response Functions

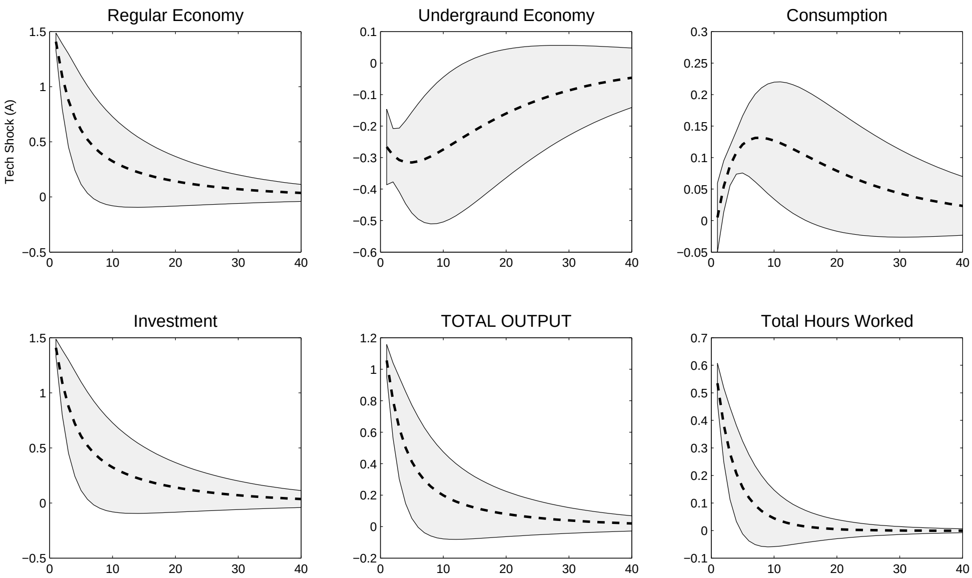

Figure 7: **Technology Shock. Regular Economy.** A_t

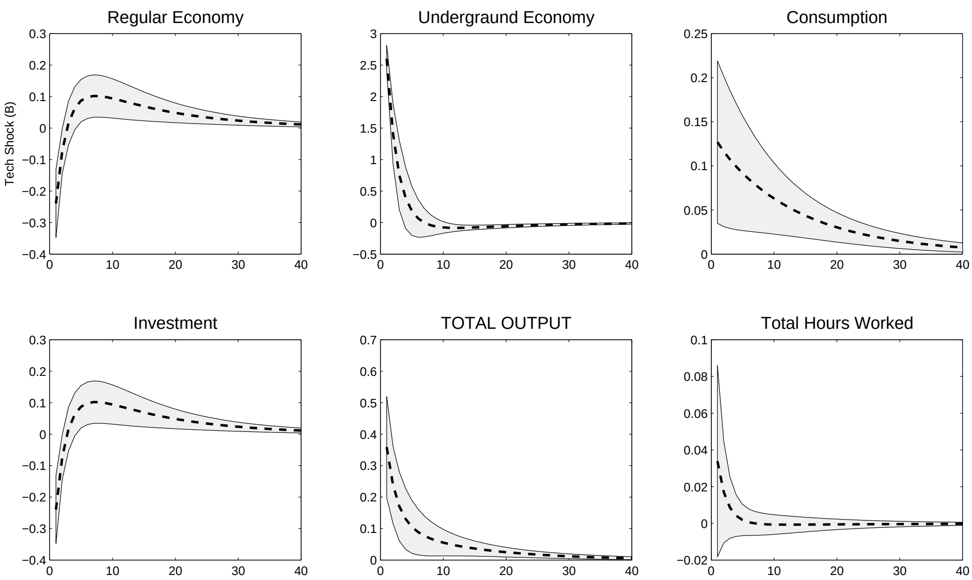

Figure 8: **Technology Shock. Underground Economy.** B_t

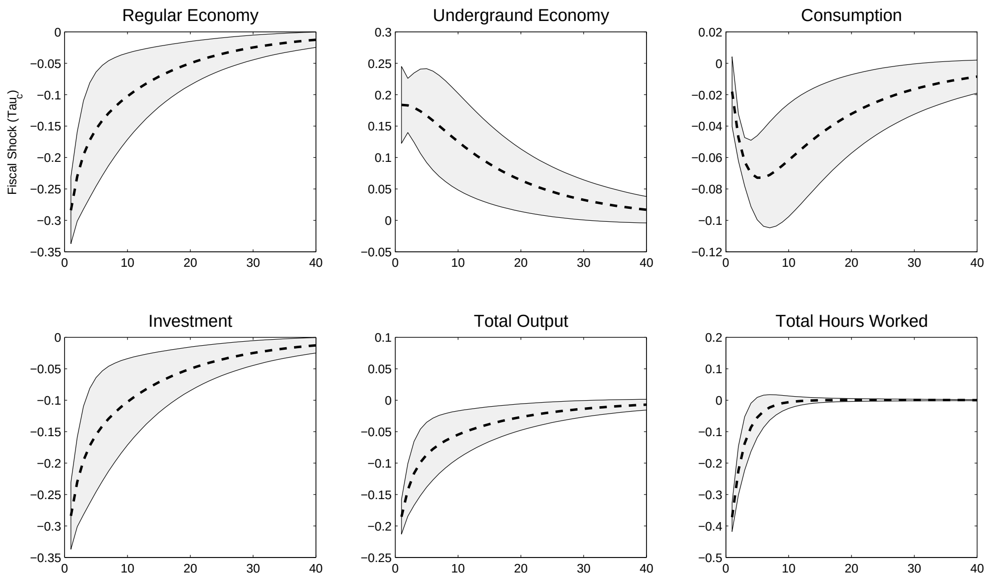

Figure 9: **Fiscal Shock. Corporate Tax Rate.** τ_c

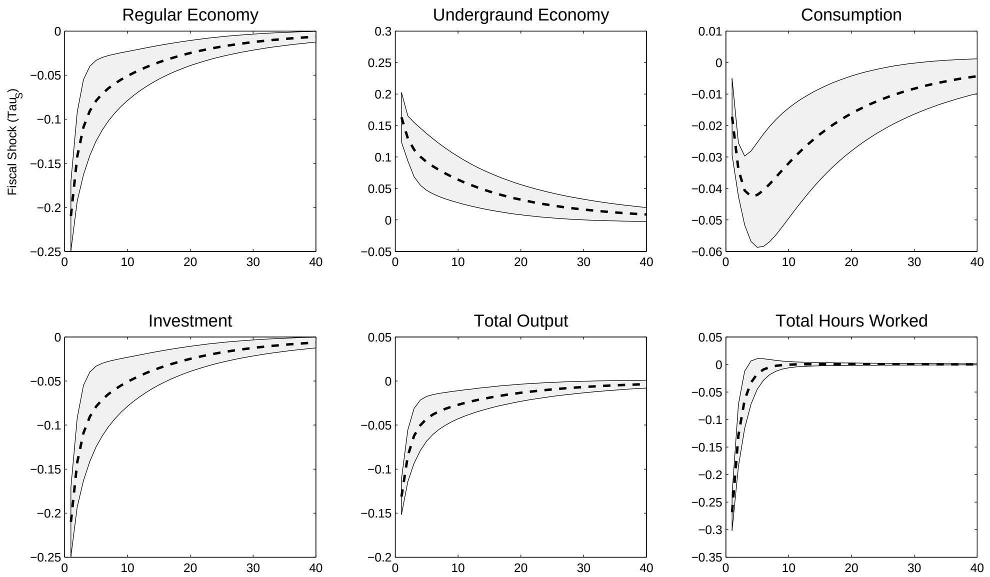

Figure 10: **Fiscal Shock. Social Security Tax Rate.** τ_s

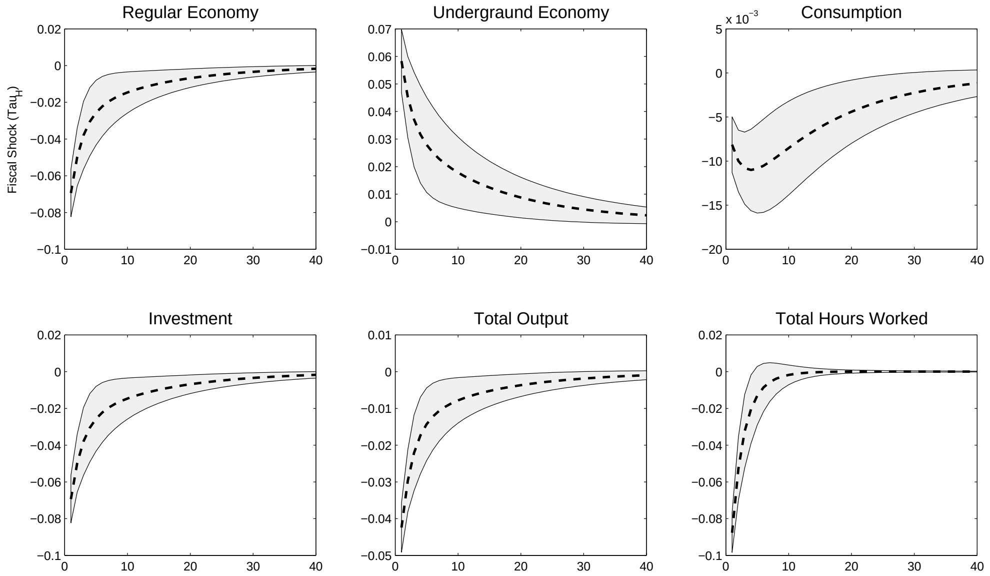

Figure 11: **Fiscal Shock. Personal Income Tax Rate.** τ_h

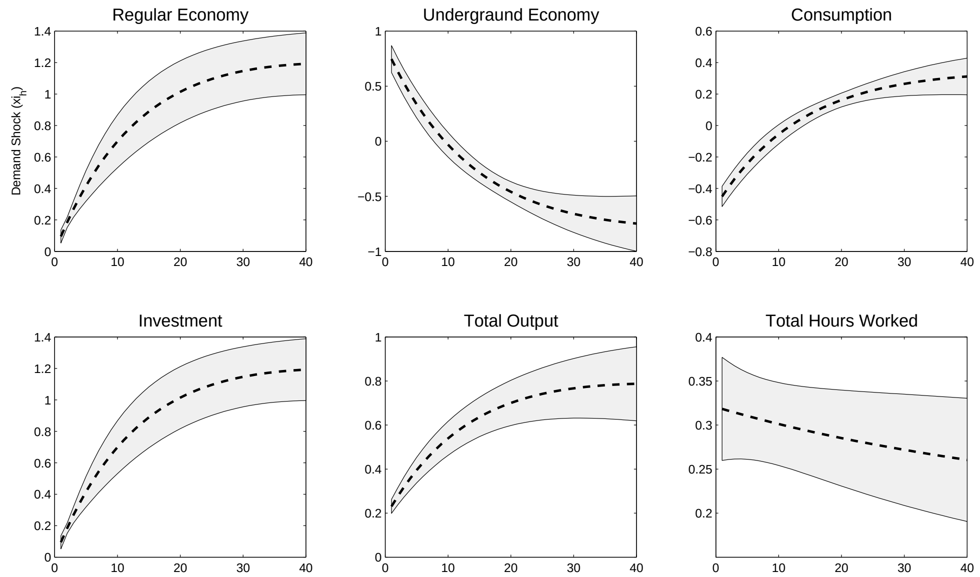

Figure 12: **Demand Shock. Investment Shock.** ξ_I

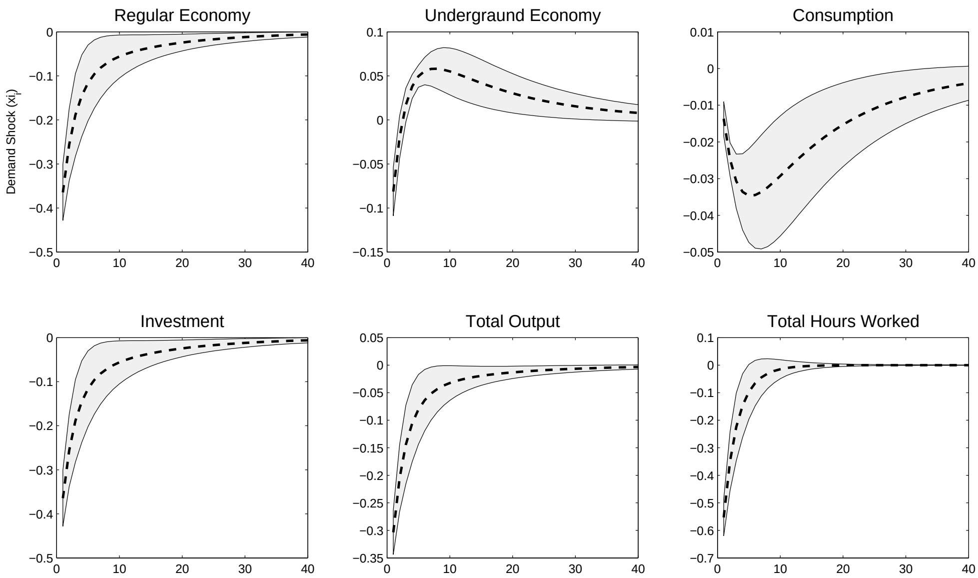

Figure 13: **Demand Shock. Disutility of labor.** ξ_h

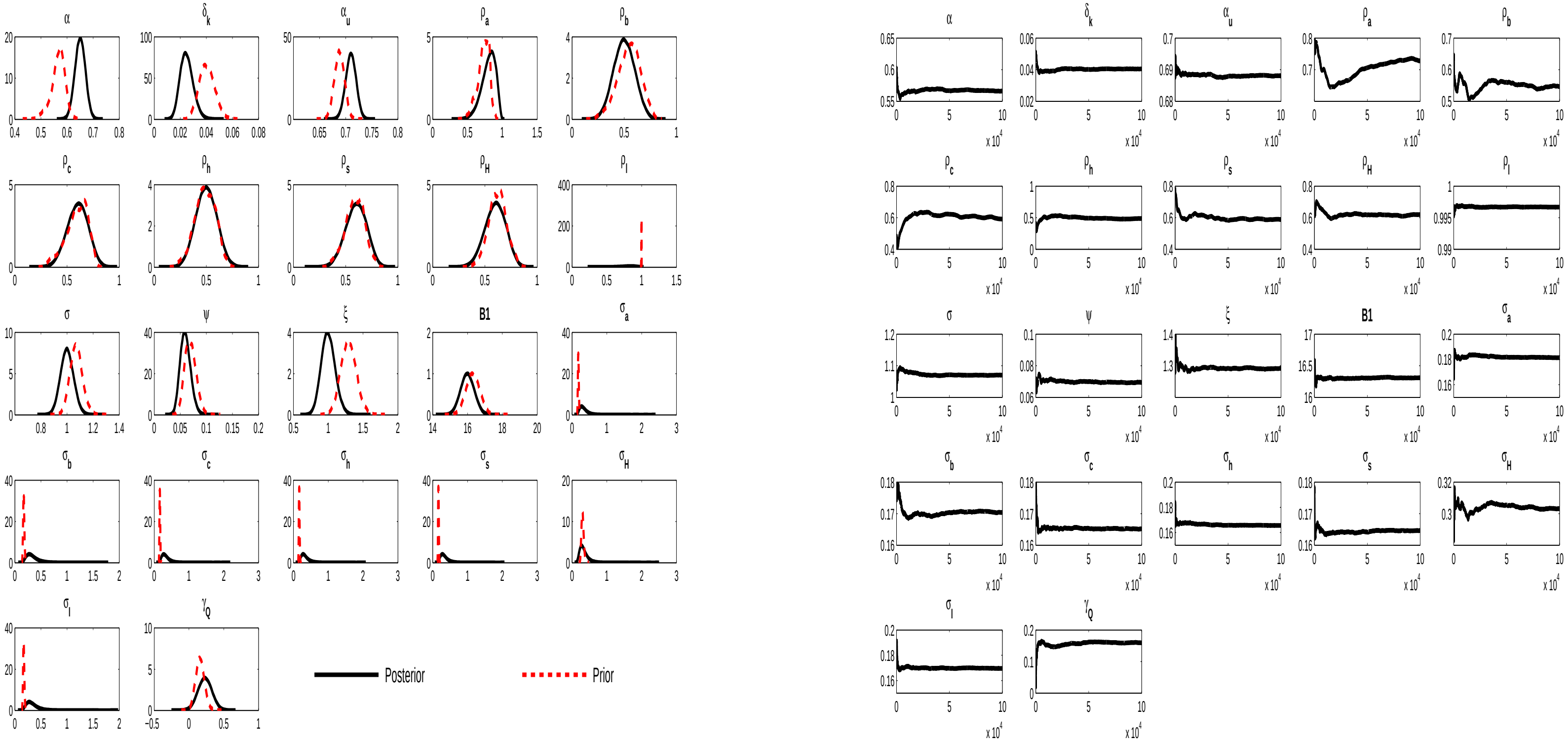

Figure 14: Prior and posterior distribution of estimated parameters