# Out-of-Sample Equity Premium Prediction: Combination Forecasts and Links to the Real Economy

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# Overview

- Introduction
- Model
- Main Results
- Connection with Real Economy
- Conclusion

# Introduction

## Background & Problems

- Numerous economic variables have been proposed as predictors of stock returns, and in-sample tests shows that there is significant evidence to return predictability.
- However, Welch and Goyal (2008) show that a long list of predictors from the literature is unable to deliver consistent superior out-of-sample forecast relative to historical average.

## Solution

- A combination approach to the out-of-sample equity premium forecasting problem.

# Model

## Predictive Regression Model

$$r_{t+1} = \alpha_i + \beta_i x_{i,t} + \epsilon_{t+1}$$

- As in Welch and Goyal (2008), we generate out-of-sample forecasts using a recursive (expanding) estimation window.
- Followed by Campbell and Thompson (2008) and Welch and Goyal (2008), the historical average equity premium serves as a benchmark forecasting model.

# Forecast Combination

## Forecast Combination

$$\hat{r}_{c,t+1} = \sum_{i=1}^{N} \omega_{i,t} \hat{r}_{i,t+1}$$

- We use two classes of simple averaging schemes:
  - Simple average scheme: mean, median, trimmed mean.
  - Stock and Watson (2004): DMSPE.

# DMSPE

## Weights of DMSPE

$$\omega_{i,t} = \phi_{i,t}^{-1} \Big/ \sum_{j=1}^{N} \phi_{j,t}^{-1}$$

$$\phi_{i,t} = \sum_{s=m}^{t-1} \theta^{t-1-s} (r_{s+1} - \hat{r}_{i,s+1})^2$$

- Assigns greater weights to model forecasts that have lower MSPE.
- When $\theta < 1$, we attach greater weights to the recent forecast accuracy.
- We consider 1.0 and 0.9 for $\theta$.

# Forecast Evaluation

- Two methods are used: Out-of-Sample $R^2$, realized utility gains.

**Out-of-sample $R^2$: $R^2_{OS}$, by Campbell and Thompson (2008)**

$$R^2_{OS} = 1 - \frac{\sum_{k=q_0+1}^{q}(r_{m+k} - \hat{r}_{m+k})^2}{\sum_{k=q_0+1}^{q}(r_{m+k} - \bar{r}_{m+k})^2}$$

$$f_{t+1} = (r_{t+1} - \bar{r}_{t+1})^2 - [(r_{t+1} - \hat{r}_{t+1})^2 - (\bar{r}_{t+1} - \hat{r}_{t+1})^2]$$

- When $R^2_{OS} > 0$, forecast outperforms the historical average forecast.
- $f_{t+1}$ is MSPE-adjusted statistic, by regressing it on a constant and compute the t-statisics, we can obtain p-value, which is used to test whether $R^2_{OS} > 0$ is significant.

# Forecast Evaluation

- $R^2_{OS}$ measure has one limitation: it does not explicitly account for the risk borne by an investor over the out-of-sample period.
- **Solution:** Realized utility gains for a mean-variance investor.

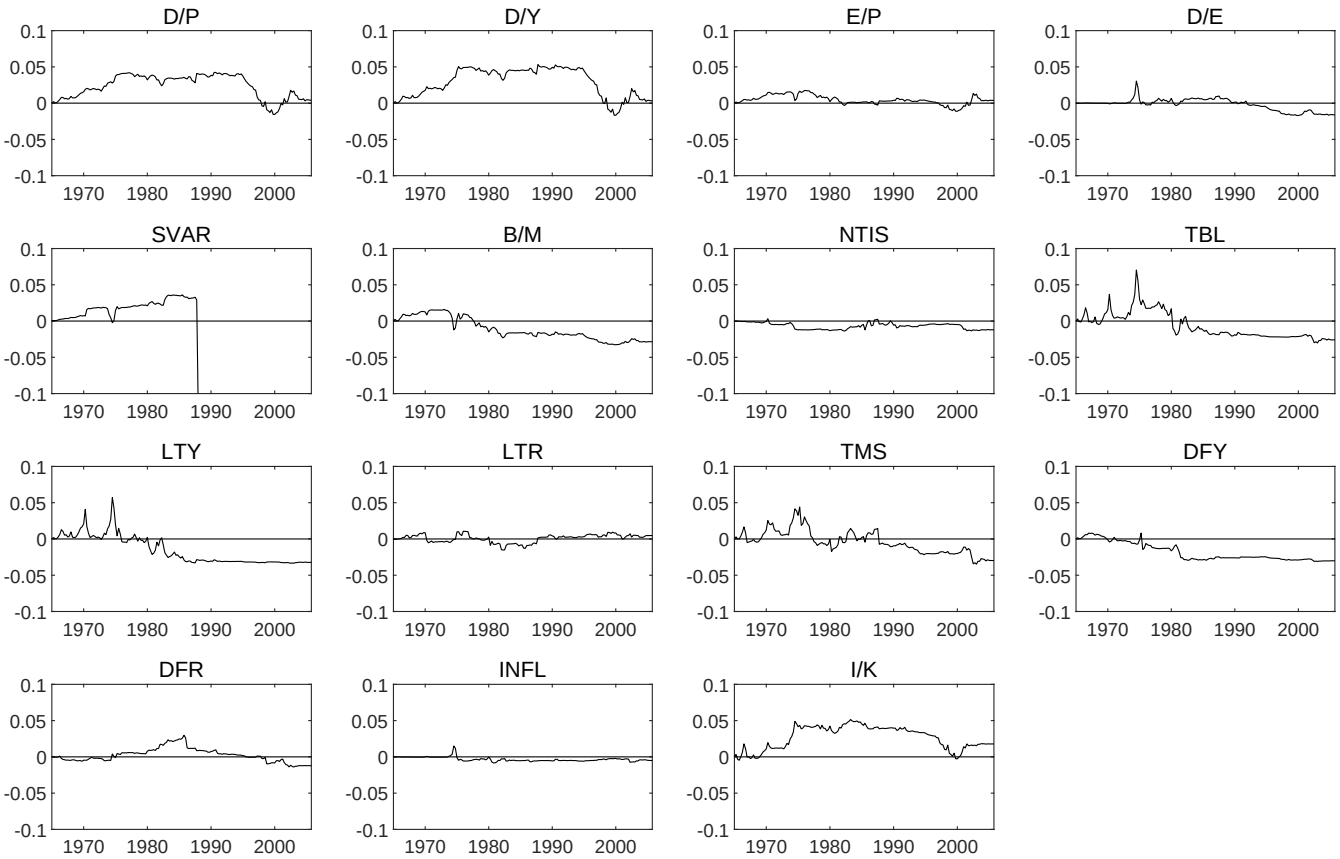
## Utility Function

$$w_{0,t} = \frac{1}{\gamma}\left(\frac{\bar{r}_{t+1}}{\hat{\sigma}^2_{t+1}}\right) \qquad w_{j,t} = \frac{1}{\gamma}\left(\frac{\hat{r}_{t+1}}{\hat{\sigma}^2_{t+1}}\right)$$

$$\hat{v}_0 = \hat{\mu}_0 - \left(\frac{1}{2}\right)\gamma\hat{\sigma}^2_0 \qquad \hat{v}_j = \hat{\mu}_j - \left(\frac{1}{2}\right)\gamma\hat{\sigma}^2_j$$

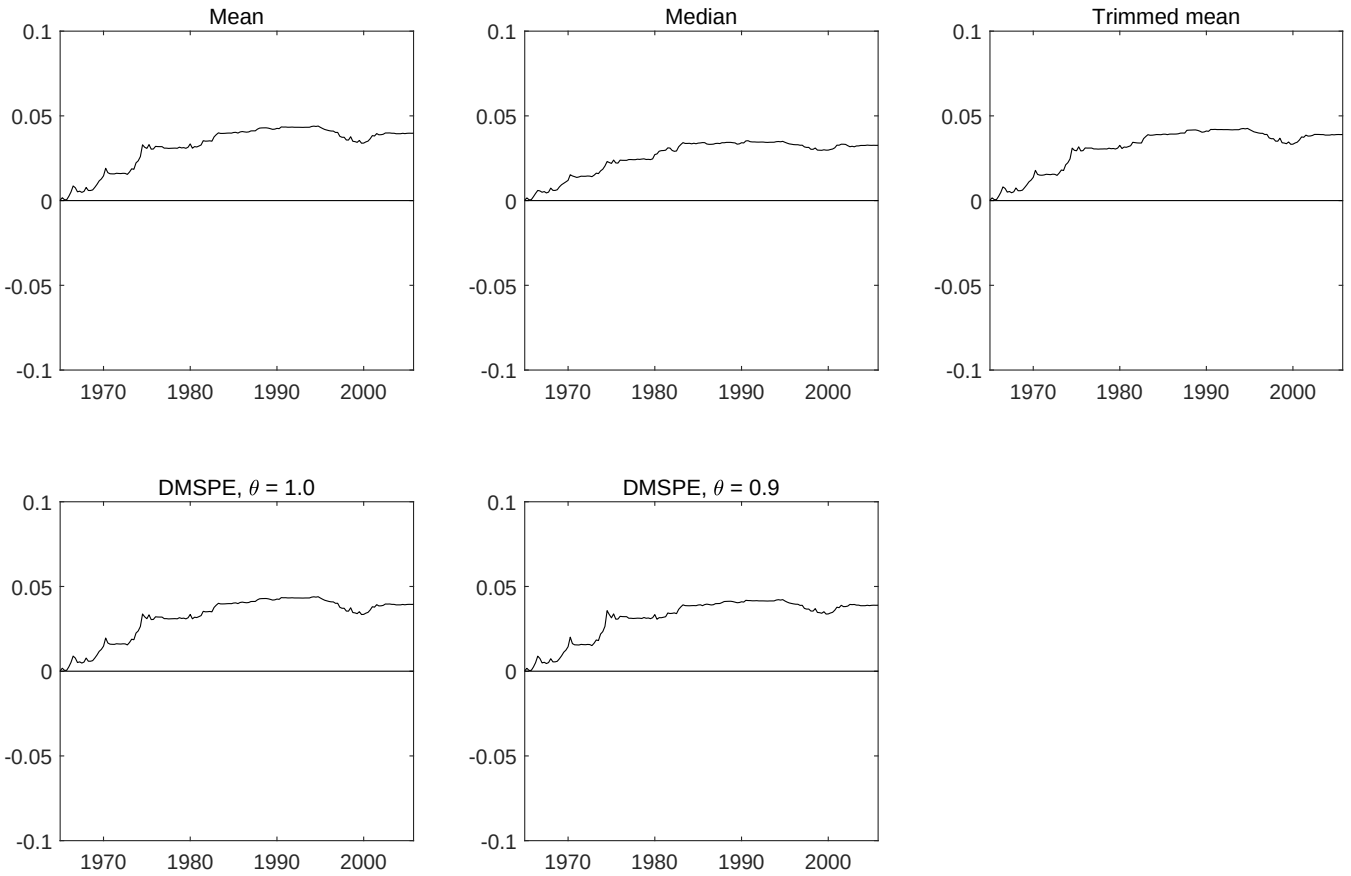
- We report the difference in utility to express the percentage return.

# Out-of-Sample Forecasting Results




- If the curve increases, the regressive model outperforms.
- If the curve is higher at the end, our model has a lower MSPE.
- **Difficult to identify a predictor that reliably outperforms.**

# Out-of-Sample Forecasting Results




- **The combination forecasts delivers more consistent out-of-sample gains.**

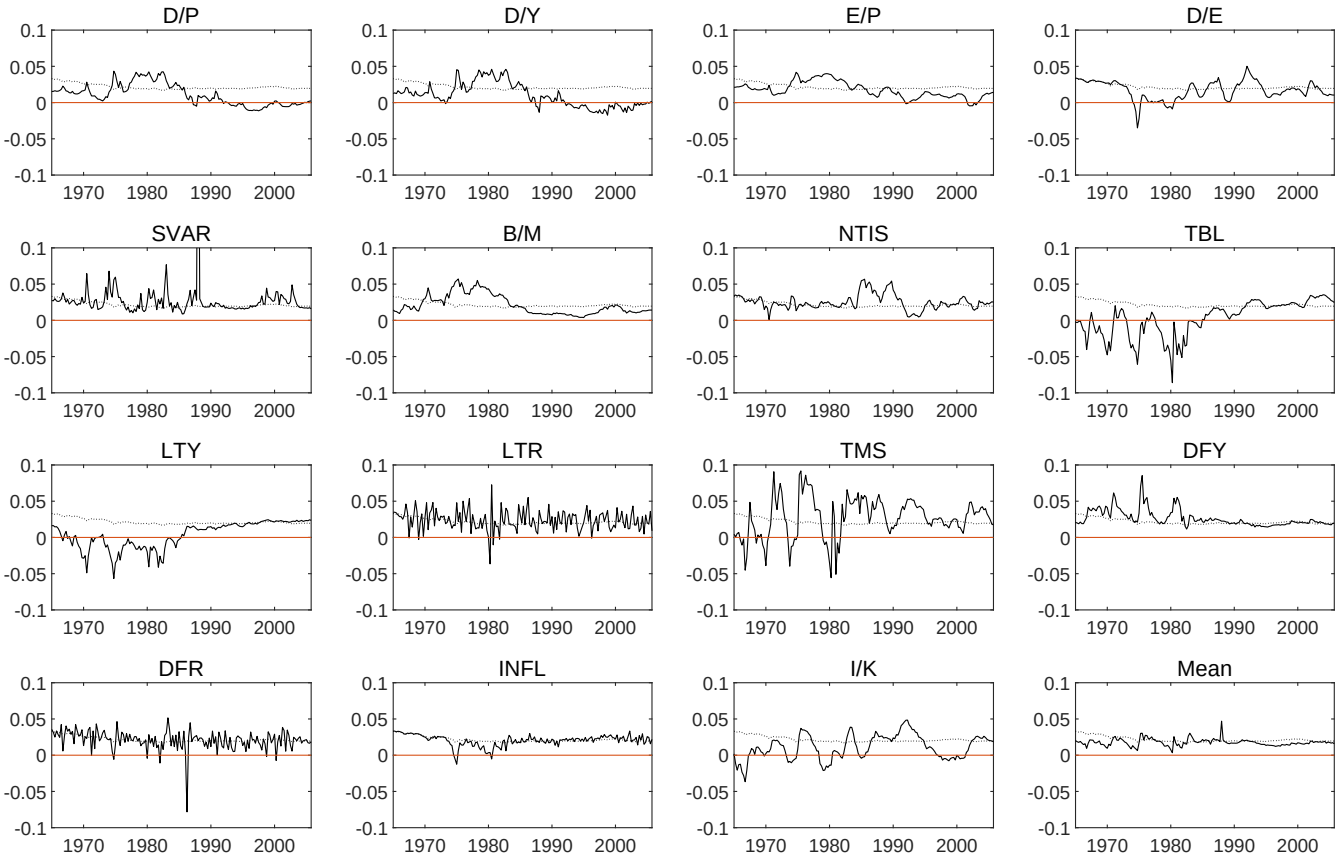
# Out-of-Sample Forecasting Results

**Table 1**
**Equity premium out-of-sample forecasting results for individual forecasts and combining methods**

| Individual predictive regression model forecasts | | | | | | Combination forecasts | | |
|---|---|---|---|---|---|---|---|---|
| Predictor | $R^2_{OS}$ (%) | Δ (%) | Predictor | $R^2_{OS}$ (%) | Δ (%) | Combining method | $R^2_{OS}$ (%) | Δ (%) |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| | | | Panel A. 1965:1–2005:4 out-of-sample period | | | | | |
| *D/P* | 0.34* | 0.55 | *LTY* | −3.09 | 2.29 | Mean | 3.58*** | 2.34 |
| *D/Y* | 0.25* | 1.41 | *LTR* | 0.33 | 1.30 | Median | 3.04*** | 1.03 |
| *E/P* | 0.36 | 0.64 | *TMS* | −2.96 | 5.14 | Trimmed mean | 3.51*** | 2.11 |
| *D/E* | −1.42 | 0.58 | *DFY* | −2.72 | −0.83 | DMSPE, θ = 1.0 | 3.54*** | 2.41 |
| *SVAR* | −12.97 | 0.13 | *DFR* | −1.10 | 0.57 | DMSPE, θ = 0.9 | 3.49*** | 2.59 |
| *B/M* | −2.60 | −0.58 | *INFL* | −0.84 | 1.39 | | | |
| *NTIS* | −0.91 | 0.08 | *I/K* | 1.44** | 2.80 | Mean, CT | 3.23*** | 1.25 |
| *TBL* | −2.78 | 2.60 | | | | | | |
| | | | Panel B. 1976:1–2005:4 out-of-sample period | | | | | |
| *D/P* | −5.08 | −0.70 | *LTY* | −5.59 | −0.89 | Mean | 1.19* | 0.57 |
| *D/Y* | −6.22 | −0.54 | *LTR* | −0.27 | 1.43 | Median | 1.51** | 0.53 |
| *E/P* | −1.70 | 0.75 | *TMS* | −7.24 | 2.08 | Trimmed mean | 1.23* | 0.59 |
| *D/E* | −2.26 | −1.65 | *DFY* | −2.48 | −1.18 | DMSPE, θ = 1.0 | 1.11* | 0.54 |
| *SVAR* | −22.47 | 0.06 | *DFR* | −2.14 | −0.64 | DMSPE, θ = 0.9 | 1.01* | 0.46 |
| *B/M* | −4.72 | −1.27 | *INFL* | −0.08 | 0.45 | | | |
| *NTIS* | 0.10 | 0.60 | *I/K* | −3.47 | −0.85 | Mean, CT | 1.20* | 0.55 |
| *TBL* | −7.31 | −0.82 | | | | | | |
| | | | Panel C. 2000:1–2005:4 out-of-sample period | | | | | |
| *D/P* | 10.32* | 12.96 | *LTY* | −0.32 | 0.24 | Mean | 3.04** | 2.31 |
| *D/Y* | 10.40* | 12.98 | *LTR* | −1.72 | 2.57 | Median | 1.56* | 0.28 |
| *E/P* | 8.02* | 9.53 | *TMS* | −4.98 | 4.23 | Trimmed mean | 2.98** | 2.12 |
| *D/E* | 0.56 | 0.50 | *DFY* | −0.53 | −1.52 | DMSPE, θ = 1.0 | 2.56** | 1.65 |
| *SVAR* | −5.62 | −1.64 | *DFR* | −2.10 | 1.76 | DMSPE, θ = 0.9 | 2.66** | 1.97 |
| *B/M* | 2.32 | 3.09 | *INFL* | −1.42 | 0.57 | | | |
| *NTIS* | −4.09 | 1.33 | *I/K* | 8.96** | 9.13 | Mean, CT | 2.43** | 1.32 |
| *TBL* | −2.50 | −0.20 | | | | | | |

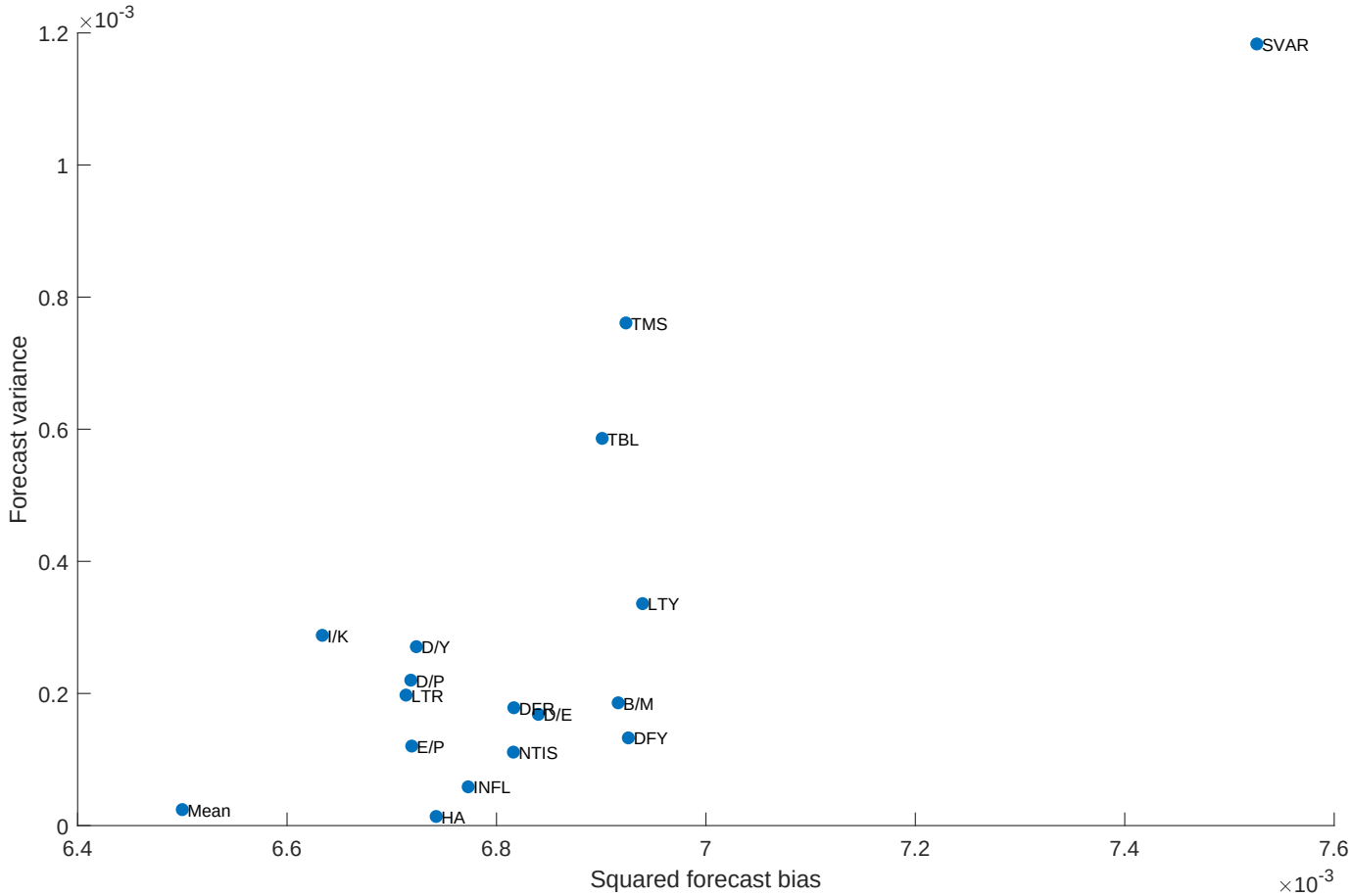
- Out-of-sample predictive ability deteriorates in 1976 - 2005.
- **DMSPE combination selects weights relatively close to** $1/N$**.**

# Forecast Stabilization



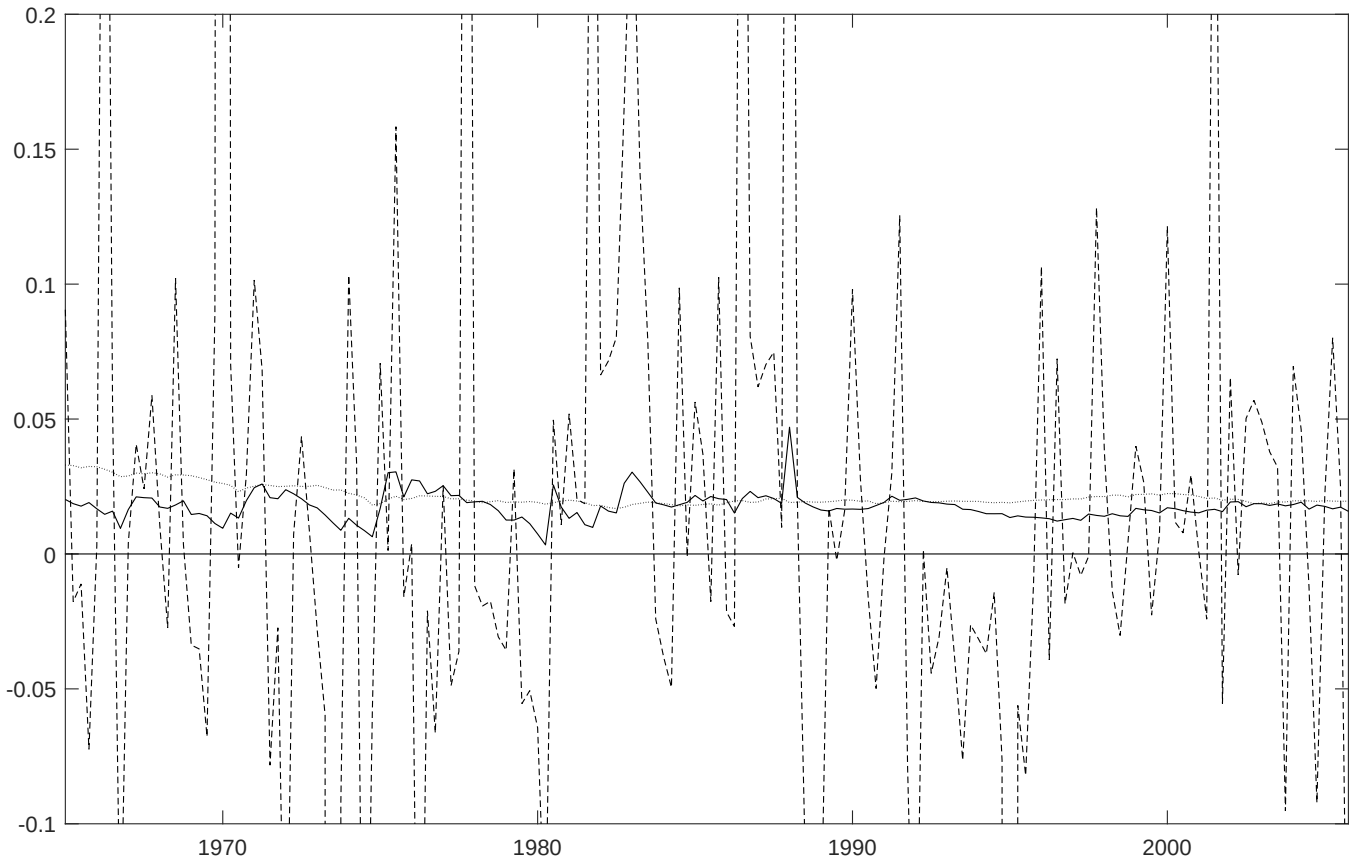

- **The forecast combination reduces forecast variablity.**

# Forecast Stabilization



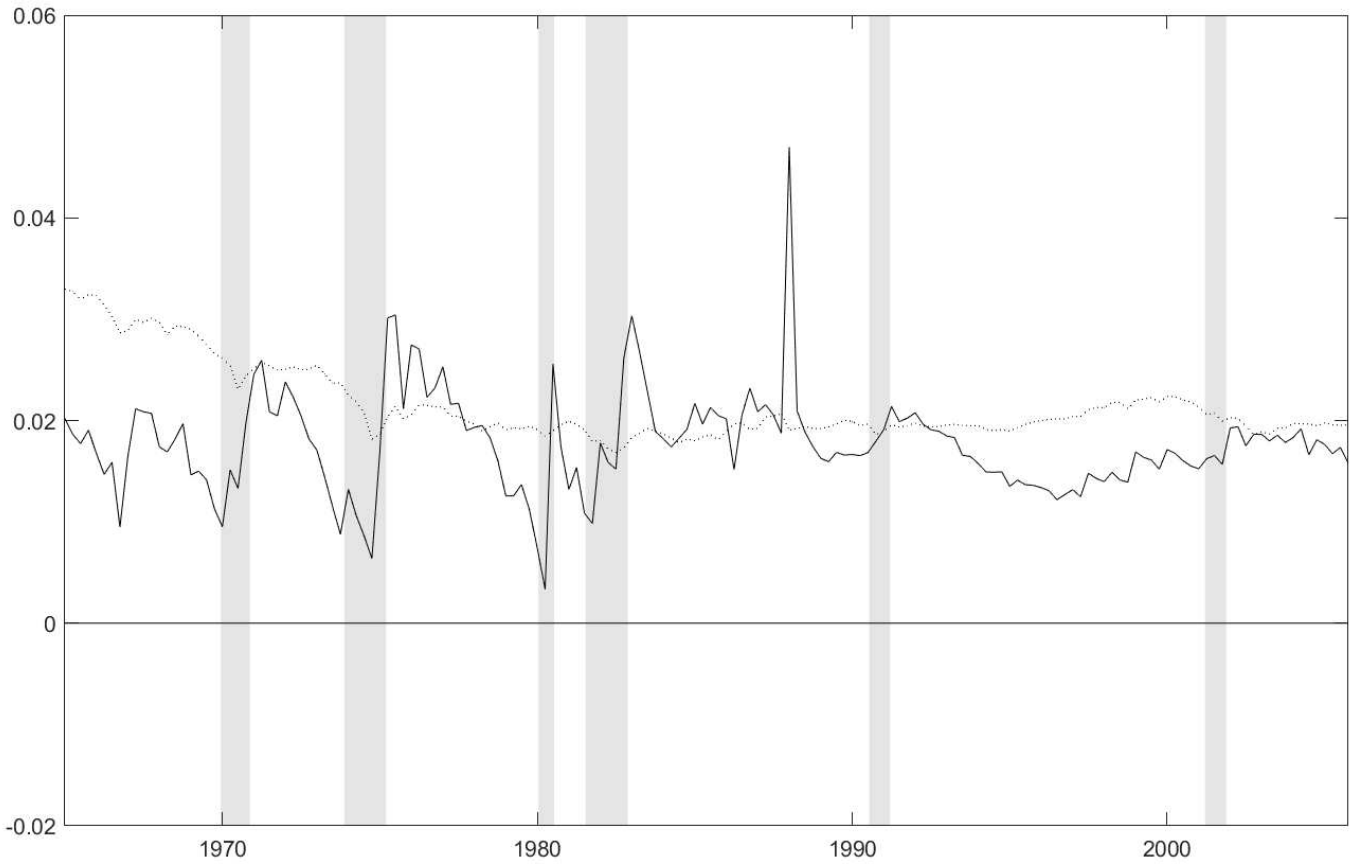

- **The mean combination forecast has a variance moderately higher than that of the historical average forecast, and has a square bias substantially below the historical average.**

# Comparison to Kitchen-Sink Model




- **Unrestricted kitchen sink model forecast is much more volatile, even more volatile than individual predictive regression model forecasts.**

# Business Cycle Phases




- **Figure 3 & Figure 6 shows that combination approach includes relevant macroeconomic information missed by historical**

# Forecast Gains During "Good" and "Bad" Periods

**Table 5**
**$R^2_{OS}$ statistics for out-of-sample equity premium combination forecasts during good, normal, and bad growth periods, 1965:1–2005:4**

| | Forecast horizon: one quarter | | | | Forecast horizon: four quarters | | | |
|---|---|---|---|---|---|---|---|---|
| Combining method | Overall | Good | Normal | Bad | Overall | Good | Normal | Bad |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| **Panel A. Sorting on real GDP growth** | | | | | | | | |
| Mean | 3.58*** | 1.82 | 1.71 | 6.17*** | 8.19*** | 3.07 | 3.63* | 11.58*** |
| Median | 3.04*** | 2.67** | 0.39 | 5.02*** | 6.99*** | 12.74*** | 6.35** | 5.23*** |
| Trimmed mean | 3.51*** | 2.25* | 1.24 | 5.94*** | 8.13*** | 5.41* | 4.01* | 10.63*** |
| DMSPE, $\theta = 1.0$ | 3.54*** | 1.71 | 1.56 | 6.26*** | 7.87*** | 2.32 | 3.15 | 11.46*** |
| DMSPE, $\theta = 0.9$ | 3.49*** | 1.60 | 1.36 | 6.33*** | 5.96*** | 4.71* | 0.27 | 8.27*** |
| **Panel B. Sorting on real profit growth** | | | | | | | | |
| Mean | 3.58*** | 2.87* | −1.03 | 7.94*** | 8.19*** | 0.93 | 4.89* | 14.72*** |
| Median | 3.04*** | 2.56** | 0.21 | 5.74*** | 6.99*** | 1.14 | 8.00** | 10.18*** |
| Trimmed mean | 3.51*** | 2.85* | −0.67 | 7.47*** | 8.13*** | 1.74 | 5.83** | 13.55*** |
| DMSPE, $\theta = 1.0$ | 3.54*** | 2.74* | −1.21 | 8.08*** | 7.87*** | 0.16 | 4.41 | 14.78*** |
| DMSPE, $\theta = 0.9$ | 3.49*** | 2.51 | −1.56 | 8.40*** | 5.96*** | −4.28 | 2.00 | 14.70*** |
| **Panel C. Sorting on real net cash flow growth** | | | | | | | | |
| Mean | 3.58*** | 5.44** | 2.17* | 4.63** | 8.19*** | 3.29* | 8.81*** | 11.42*** |
| Median | 3.04*** | 4.12*** | 1.80** | 4.25** | 6.99*** | 4.99*** | 6.17** | 9.48*** |
| Trimmed mean | 3.51*** | 5.01** | 2.36** | 4.47** | 8.13*** | 4.39** | 9.13*** | 10.04*** |
| DMSPE, $\theta = 1.0$ | 3.54*** | 5.51** | 2.13* | 4.52** | 7.87*** | 2.97* | 8.50*** | 11.09*** |
| DMSPE, $\theta = 0.9$ | 3.49*** | 5.88** | 1.84* | 4.15* | 5.96*** | 0.53 | 6.66** | 9.56*** |

- **Out-of-sample gains for the combination forecasts are concentrated in especially "bad" periods.**

# Conclusion

- Forecast combination appears successful for out-of-sample equity premium prediction, avoiding being too volatile as in individual forecast model, or being too smooth as in historical average model.
- Forecast of equity premium are linked to the real economy.
- Forecasting combining methods stems from highly uncertain, complex and constantly evolving data-generating process underlying expected equity returns.

# Thanks!