We start out once more performing simpler, bi-variate tests. If the variables are I(1), because *N*=2, in this case there can be at most one cointegrating relationship between pairs of variables. In the case of real stock prices and dividends, $\boldsymbol{y}_t \equiv [P_t \ \mathrm{D}_t]'$, we perform the $\lambda_{trace}(0)$ and the $\lambda_{\max}(0,1)$ tests while specifying an intercept in both the testing VAR and in the cointegrating vector. An intercept in the VAR accommodates a likely drift in real dividends, while an intercept in the cointegrating vector also allows the stationary relationships between variables to trend over time, as in $\boldsymbol{\Pi y}_t + \boldsymbol{Dx}_{t+1} = \boldsymbol{\Lambda}(\boldsymbol{K}'\boldsymbol{y}_t + \boldsymbol{\kappa}_0)$, where $\boldsymbol{Dx}_{t+1}$ captures determinist regressors (such as drifts, polynomial trends, and seasonal factors) in an extended version of (4.58). The number of lags in (4.58) is set to 12. Because the estimated eigenvalues are rather small, $\hat{\lambda}_1 = 0.0073$ and $\hat{\lambda}_1 = 0.0008$, we obtain that $\lambda_{trace}(0) = 14.173$ (with *p*-value 0.078) and $\lambda_{\max}(0,1) = 12.762$ (*p*-value 0.085) which do not allow us to reject the null hypothesis of no cointegration, i.e., that *rank*($\boldsymbol{\Pi}$)=0. This conclusion echoes the one reached in Example 4.5: even though in this case, with *p*-values between 5 and 10%, the failure to reject is not empirically overwhelming, there is enough evidence to conclude that our 1872-2016 sample is inconsistent with a dividend growth model for aggregate equity valuations.

We repeat these tests with reference to $\boldsymbol{y}_t \equiv [P_t \ \mathrm{E}_t]'$ using identical options in terms of deterministic regressors and selection of the number of lags. The estimated eigenvalues from $\hat{\boldsymbol{\Pi}}$ are 0.0115 and 0.0002 and the first eigenvalue appears to be large enough to give evidence that *rank*($\boldsymbol{\Pi}$)=1. In fact, $\lambda_{trace}(0) = 20.345$ (with *p*-value 0.009) and $\lambda_{\max}(0,1) = 20.020$ (*p*-value 0.006) which lead us to reject the null of no cointegration in favor of *r*=1. However, the variables are not stationary either, as we have reported in Section 3: $\lambda_{trace}(1) = 0.325$ (*p*-value 0.569) and $\lambda_{\max}(1,2) = 0.325$ (*p*-value 0.569), so that *r*=2 is rejected. Once more, real stock prices and earnings are cointegrated and this is evidence consistent with a discounted fundamentals model of equity valuations. When the cointegrating vector is normalized to have $\kappa_1 = 1$, we obtain (standard errors are in parenthesis):

$$\hat{\boldsymbol{\Lambda}} = [\underset{(0.0025)}{-0.0062} \ \ \underset{(0.0001)}{0.0002}]' \quad \hat{\boldsymbol{K}} = [1 \ \ \underset{(2.343)}{-25.785}] \quad \hat{\boldsymbol{\kappa}}_0 = 264.27$$

These estimates imply a long-run equilibrium cointegrating equation that, plugging in the point estimates, we can write as $P_t = 264.27 + 25.785 E_t$ i.e., a price-earnings ratio that asymptotically converges to 25.8. This value is substantially larger than what we had found using univariate cointegration tests, and the discrepancy between 18.1 and 25.8 will be of considerable importance to investor, asset managers, and academics debating whether and when a bubble in stock valuations may be forming. The estimates of the adjustment coefficients suggest once more that the error correction is rather slow— given that $P_t - 264.27 - 25.785 E_t = 1$, only 0.0062 of it is corrected by the price declining between two consecutive months—and that the correction is almost entirely supported by equity price changes given that the correction coefficient of earnings has the right sign but it is essentially zero. This is confirmed by a plot of the ECM deviations, which we may interpret as mispricings, in Figure 4.13. For instance, a major over-pricing of S&P stocks has occurred between 1997 and 2001, which somebody has then dubbed the "dot-com tech bubble". Interestingly, while Figure 4.11 seemed to suggest some evidence of equity over-pricing between 2015 and 2016, this is not the case here.

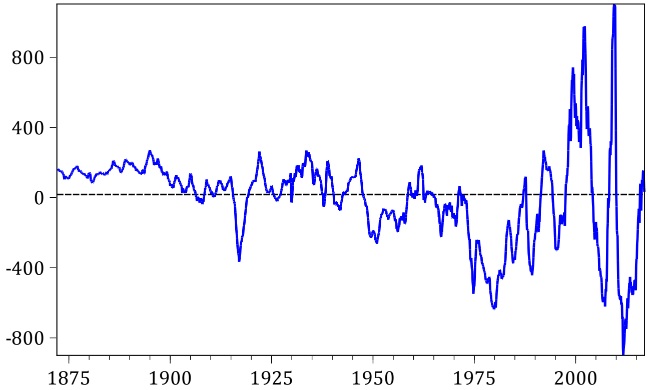

Figure –Deviations from Long-Run Equilibrium Cointegrating Rela-tion Between Real S&P Equity Index Prices and Aggregate Earnings