

# INTERREGIONAL COMPETITION IN TAX ADMINISTRATION

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## ABSTRACT

Tax auditing parameters have been largely overlooked by the literature as policy-making instruments of any relevance; however, enforcement strategies are critical elements of the tax burden. In this paper we show that, in a federal framework, tax auditing policies can serve as additional tools for regional interaction. We examine the presence of this interaction by adopting a spatial econometric approach. We employ a *time-space recursive model* that accounts for sluggish adjustment in auditing policies and obtain results that are congruent with standard theory, corroborating the presence of horizontal competition between regions in their tax auditing policies. We also find that once regional governments acquire legal power, the opaque competition in enforcement policies disappears apparently switching to a more transparent competition in statutory tax parameters.

Keywords: tax administration and auditing, fiscal competition, fiscal federalism

JEL Classification: H71, H77, H83

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## 1. Introduction

Enforcement strategies are crucial elements in the tax management process since they help determine the level and distribution of effective tax rates (e.g. Johns & Slemrod, 2010; Traxler, 2011) and, hence, the total amount of tax revenues collected. Moreover, these strategies are of particular interest to federal countries, as auditing policies can represent a second, additional, tax instrument in the hands of sub-central authorities (Besfamille *et al.*, 2012) – along with the setting of statutory tax rates – on which they can interact. Yet, the possibility of tax enforcement interdependence has received limited attention in the literature (with notable exceptions being Janeba & Peters, 1999; Cremer & Gahvari, 2000 and, Stöwhase & Traxler, 2005) and, to the best of our knowledge, there are no empirical studies investigating the presence of these interactions, which might be due to an absence of data on auditing policies and/or the difficulties in finding an adequate measure to represent the level of “tax enforcement”.

We aim to fill this gap in the literature by analyzing the presence of horizontal tax interdependence between sub-central administrations in a federal context. In Spain, regional governments, the so-called “Comunidades Autónomas” (henceforth CAs), have had the power to administer several wealth taxes since the mid-eighties, albeit without any legal authority to modify the rule (though note that following reforms in 1997 and 2002 they did obtain the legislative power to modify significant tax parameters)<sup>1</sup>. Here, we focus specifically on the Inheritance and Gift Tax (IGT), the main decentralized tax on wealth, which has recently become the subject of considerable debate both in Spain and in other countries<sup>2</sup>. There is evidence that the decentralization of the IGT in federal countries can induce a race to the bottom in statutory tax parameters (see, for example, Bird, 1991, Conway & Rork, 2004; Brulhart & Parchet, 2011)<sup>3</sup>. The origin of this process is the mobility of tax bases<sup>4</sup>. A similar effect has been documented for the Spanish case (see Durán-Cabré & Esteller-Moré 2010; López Casasnovas & Durán-Sindreu, 2008), provoking an academic and a more general debate. The Spanish press headlines on these issues are symptomatic: “Cheaper Gifts and Inheritances”; “Regional Tax Competition”; “The Fiscal War among Regions Threatens the IGT”; “Regional

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<sup>1</sup> More specifically, following the 1997 reform, CAs were permitted to modify their tax rate schedules in line with national schedules. Following the 2002 reform, CAs were granted complete legislative control over the tax rates ceded to them by the central government.

<sup>2</sup> Taxing wealth and wealth transfers is generally unpopular and has become the subject of debate in several OECD countries, including United States and Canada. In Europe, the UK case is highly illustrative: the IGT is popularly ostracized because it raises relatively little revenue, but it is characterized by an excessively high flat rate (40%). Likewise, it raises issues about double taxation as well as about the absence of effects on wealth distribution (Boadway *et al.*, 2010).

<sup>3</sup> Recently, the European Commission has shown interest in such issues and even though they might arise under different circumstances – cross-border discrimination and double taxation, it would seem to confirm that questions surrounding the inheritance tax are of growing concern to European citizens (European Commission, 2011).

<sup>4</sup> In a decentralized framework, when the principle of residence is applied, an individual finds it profitable to move his fiscal residence to the region with the lowest IGT rate so as to reduce the bequest tax burden.

Taxation and Voting with Feet”<sup>5</sup>. These articles corroborate the presence of mobility-based competition in the regional IGT statutory tax parameters. Similarly, we hypothesize that the same type of competition between regions occurred even before the decentralization of legal power, in the form of opaque competition on tax enforcement since it is the effective tax rate that conditions mobility.

The objective of our paper, therefore, is to determine the form and degree of interaction between decentralized administrations when setting their policies. To achieve this, we develop a model of horizontal competition with two tax instruments, i.e., the tax rate and the auditing rate, and empirically test its findings. The results of the theoretical framework are in line with the literature on tax rate competition: the mobility threat tames the revenue maximizing jurisdictions that compete in a race to the bottom over both tax instruments so as not to lose their tax bases. We derive the slope of the administration’s reaction function and obtain a positive sign. We proceed to test this result using a spatial econometric approach. We estimate a *time-space recursive model* to account for possible sluggish adjustments in the setting of auditing policies (see Anselin *et al.*, 2008). Our results corroborate the presence of horizontal interdependence between the regions and generate credible results that are coherent with the tax competition model. Moreover, we obtain an additional result: following the decentralization of legislative power on statutory tax parameters we observe a disappearance of competition in enforcement policies at the regional level. It seems that a substitution of instruments occurs with positive implications at the normative level: somewhat paradoxically in this perspective the tax decentralization process seems welcome since an opaque source of tax competition is substituted by a transparent source.

The rest of the paper is organized as follows. In the next section, we provide a summary of the relevant literature, then the theoretical framework is developed and the empirical analysis performed. Finally, we conclude.

## **2. Relevant Literature**

This study is closely related to the vast literature on taxation policy interactions between governments and, in particular, to that research line that deals with horizontal tax competition (see Brennan & Buchanan, 1980; Zodrow & Mieszkosky, 1986; and Wilson, 1986). This approach analyzes a decentralized framework in which local governments compete in a race to the bottom when fixing tax rates in order to gain or, at least not to lose, their tax bases. The

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<sup>5</sup> The articles quoted are ABC (2008); El Periodico (2007); El País (2007) and Expansión (2011). Among other articles see El Mundo, 2007; El Pais, 2006; El Periódico, 2007a and Expansión, 2007.

mobility or simply the threat of mobility of capital and people reduces government discretion to set tax rates at an optimal level with the effect of tax revenue cuts<sup>6</sup>.

This literature has offered limited attention to enforcement policies although they represent critical elements in the tax management process. The most relevant theoretical contribution in this sense is that of Cremer & Gahvari (2000). Using a welfare maximizing framework, they examine the implications of tax evasion for fiscal competition and tax harmonization policies in an economic union. The countries have the power to set both tax rates and tax auditing policies. In a closed economy framework, allowing for tax evasion increases the marginal cost of public funds and reduces the level of public good provision. From our perspective the most interesting result of the paper concerns the economic union of two tax-evading countries. In this setting, the states engage in mobility-based competition that produces less than optimal equilibrium values of both tax and audit rates. Harmonization policies can theoretically circumvent this problem but, according to the authors, coordinating audit strategies may be problematic because it is difficult for the government of one country to observe and verify the enforcement efforts of the other. For this reason, although a harmonization policy on tax rates is effective in circumventing tax rate sub-optimality, it is not sufficient for avoiding the inefficient outcome of the auditing rate: since member states are no longer allowed to compete over tax rates, they lower their effective rates by cutting their auditing probabilities.

A further contribution to this literature is provided by Stöwhase & Traxler (2005) who analyze the implications of different equalization systems on regional enforcement policies in a federal framework taking the statutory tax rates as being exogenously fixed at the central level. The benchmark framework presents no equalization scheme and is congruent with the results of Cremer & Gahvari (2000). Their most interesting result suggests that one way of partially circumventing the inefficient outcome of enforcement is to use a particular equalization scheme. By introducing a gross revenue sharing scheme, under which tax revenues are shared but auditing costs are borne fully by each region, an even more inefficient enforcement policy outcome is obtained. By considering instead a net revenue sharing scheme, under which both tax revenues and auditing costs are shared, the outcome is more efficient than both under the benchmark and the gross revenue sharing schemes.

Janeba & Peters (1999) analyze the taxation of interest income in an economic union of two countries in the presence of tax evasion. In their setting, the enforcement effort is proxied by the treatment of the nonresidents' tax base. In fact, any state can decide whether to discriminate against the mobile tax base when setting the tax rate. The result is analogous to the prisoners' dilemma. The authors show that if a sequential structure of the game is considered and any

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<sup>6</sup> The applied literature that tests these theoretical models from an empirical point of view is vast and takes a spatial econometric approach (among others, see, for example, Devereux et al., 2008; Esteller-Moré & Rizzo, 2011; Figlio et al., 1999; Overesch & Rincke, 2011).

country has initially to decide whether or not to discriminate and then to set the level of the tax rates, an equilibrium will always exist: both countries discriminate by offering a lower tax rate to nonresident's income with respect to that of the residents. In equilibrium this strategy will allow the mobile bases to evade taxation successfully. In this sense, a discrimination strategy is assimilable to mobility-based competition in both enforcement policies and tax rates. If, by contrast, all countries harmonize their policies and decide not to discriminate, tax competition will lead to a lower level of tax evasion. This strategy is dominated by the one in which both countries discriminate and so cannot be reached in equilibrium.

The literature on tax enforcement mobility-based competition, therefore, agrees on the impossibility of overcoming the inefficient outcome produced by auditing policies by implanting a harmonization policy, and, although some alternative strategies have been proposed, further research is needed in this field. In particular no empirical study has been conducted to test these models. Seen from this perspective, the case of wealth taxes seems to be particularly appropriate for investigation. Indeed the literature suggests that the cost of levying these taxes in federal systems is significantly increased by both vertical and horizontal tax competition (Bird, 1991). In Australia and Canada, for instance, the coexistence of a federal and a sub-central IGT led to the abolishment of the former (in 1978 and 1972 respectively). This favored the disappearance of the local IGT too which succumbed (in 1983 in Australia and in 1986 in Canada) to the pressures of horizontal tax competition (Duff, 2005). In the U.S. the IGT has been repealed in 33 of the 48 contiguous states and its elimination is under discussion in the remaining 15. Conway & Rork (2004), drawing on historical elderly migration data, show that this is the result of a mobility-based competition process. The same process has occurred in the majority of Swiss cantons since the early 1990s. Tax competition was the main argument in the political debate regarding these reforms. In particular, Brulhart & Parchet (2011) find that a change in the IGT burden has a statistically significant effect on the very wealthy retirees' tax base but not on the tax base of the retirees considered as a whole. This suggests that the incentive to move comes solely from the upper tail of the income distribution among retirees.

The empirical evidence on wealth taxes corroborates the presence of mobility-based competition in statutory tax parameters but the possibility that these interactions may also occur at the enforcement level has yet to be investigated. From this perspective, it is also useful to relate our analysis to the literature examining the determinants of tax administration. Although there is no agreement as to the objective function of a tax administration, the dominant approach sees it as a public agency that maximizes tax revenues (e.g. Shaw *et al.*, 2009; Slemrod & Yitzhaki, 2002, 1987). However, recent empirical papers suggest that political as well as budgetary variables play a role in determining a tax administration's enforcement effort (see, for example, Young *et al.*, 2001; Barette *et al.*, 2002; Esteller-Moré, 2005, 2011).

The literature recognizes the possibility that sub-central governments might interact on tax administration parameters and it identifies the mobility of tax bases as the source of this interdependence. However, further research is needed in this field if we are to gain a better understanding of sub-central administration behaviour. We aim to fulfil this objective by conducting an empirical analysis of the case of wealth taxes.

### **3. The theoretical framework: “mobility-based” competition in presence of tax evasion**

Here, we consider mobility-based competition as a potential source of interdependence between sub-central tax administrations: we present a simple model of tax competition in the presence of tax evasion<sup>7</sup>. This permits us to set up the basic hypotheses for empirical testing. The framework is modelled as a federal state comprising two regions ( $i = 1, 2$ ) of equal size in which the total population is normalized to one. At the regional level there are two institutional agents: the government that sets the tax rate  $t_i \in (1, 0)$  and the tax administration that controls the auditing probability  $\beta_i \in (1, 0)$ . Following the most common approach in the literature, we assume that both institutions have aligned incentives and act as Leviathans: they respectively set tax rates and auditing policies, both maximizing total tax revenues<sup>8</sup>. Since we are not interested in statutory tax parameter interactions we do not solve the government’s problem and take tax rates as given. Taxpayers decide the share  $\alpha \in (1, 0)$  of wealth  $B$  to declare minimizing their tax payment. To ensure an interior solution, tax evasion is assumed to be costly for the individual. Moreover, taxpayers are neutral risk-averse in order to avoid any income effects. For the sake of simplicity, we do not develop the individual’s problem but the results are in line with the standard literature (see Allingham & Sandmo 1972; Cremer & Gahvari, 2000).

The model is developed in two *stages*:

1. Regional tax administrations set tax auditing policies.
2. Individuals decide in which region of the federation to locate by comparing their indirect utility function (based on their current tax burden) in the two regions. This *stage* is solved exploiting the concept of “home attachment” (see Mansoorian & Myers, 1993 and 1997, for the original framework and Wellisch, 2000, for a recent formulation).

The solution is provided by backward induction.

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<sup>7</sup> The model is based on Cremer & Gahvari (2000).

<sup>8</sup> This would seem reasonable in the sense that the regional government sets only the statutory tax rate directly while it only indirectly controls the auditing policy implemented by the administrative office. A possible extension might involve supposing that a bargaining process between the two institutions takes place (Fuest, 2000).

### 3.1 Stage 2: The decision as to which region to reside in

To model the concept of “home” we assume that taxpayers are indexed by  $n \in (1, 0)$  and are uniformly distributed between 0 and 1<sup>9</sup>. The preferences of taxpayer  $n$  with respect to his location are given by:

$$V(n) = \begin{cases} U_1^* + a \times (1 - n) & \text{if } n \text{ lives in region 1} \\ U_2^* + a \times n & \text{if } n \text{ lives in region 2} \end{cases} \quad (1)$$

Where  $U_i^* = U_i^*(1 - \alpha^*(t_i, \beta_i))$  for  $i = 1, 2$ , represents the (pecuniary) indirect utility function<sup>10</sup> and where  $n \in (1, 0)$  measures the non-pecuniary (psychic) benefit the individual derives from living in region 2 and  $(1 - n)$  the benefit from living in region 1. Thus, taxpayers indexed by  $n \in (0, \frac{1}{2})$  reside in region 1 while those identified by  $n \in (\frac{1}{2}, 1)$  reside in region 2. The parameter  $a \in (0, +\infty)$  measures the degree of individual mobility. The interpretation of  $a$  is crucial. We assume  $a$  to represent the cost incurred when moving from the home region<sup>11</sup>. The taxpayer’s utility from living in his own region increases with the cost of mobility: if the costs are low (high) then the relative importance that the taxpayer assigns to the psychic part of the utility function, with respect to the pecuniary function, is low (high)<sup>12</sup>.

The mobility equilibrium is characterized as:

$$\begin{aligned} U_1^* + a \times (1 - n_1) &= U_2^* + a \times n_1 \\ U_1^* + a \times (1 - n) &> U_2^* + a \times n \quad \forall n < n_1 \\ U_1^* + a \times (1 - n) &< U_2^* + a \times n \quad \forall n > n_1 \end{aligned} \quad (2)$$

where  $n = n_1$  represents the marginal individual indifferent between living in region 1 and region 2 and, since  $\int_0^{n_1} dn = n_1$ , it also represents the population in region 1 in the migration equilibrium:

$$n_1 = n_1(t_1, \beta_1, t_2, \beta_2; a) = \frac{1}{2} + \frac{U_1^* - U_2^*}{2a} = \frac{1}{2} + \frac{B \times [\theta_2 - \theta_1 + g_2 - g_1]}{2a} \quad (3)$$

<sup>9</sup> See Appendix 1 for a generalisation of the model that makes this assumption about the population distribution.

<sup>10</sup> The direct utility function is defined as  $U = B \times [1 - t_i \times [\alpha + (1 - \alpha) \times \tau \times \beta_i] - g(1 - \alpha)]$ . Where  $(\tau - 1) > 0$  is the exogenous tax penalty per unit of tax evaded and the function  $g(1 - \alpha)$  represents the cost of tax evasion  $(1 - \alpha)$ , such that  $g'(1 - \alpha) > 0$ ,  $g''(1 - \alpha) > 0$ ,  $g(0) = 0$ ,  $g(1) \rightarrow +\infty$ .

<sup>11</sup> Since mobility could be either real or fictitious, this could be interpreted as the cost of actual mobility or the cost of an expected move.

<sup>12</sup> When the mobility cost is null ( $a = 0$ ) the tax bases become perfectly mobile: only the pecuniary part of the utility function matters in the taxpayer’s migration decision. By contrast, when the mobility costs are extremely high ( $a \rightarrow +\infty$ ) the taxpayers are perfectly immobile. This can be interpreted as a centralized economy case in which tax policies are set by a sole federal planner. These two limit cases are excluded to allow for imperfect mobility of individuals.

where  $\theta_i \equiv t_i \times [\alpha + (1 - \alpha) \times \tau \times \beta_i]$  is defined as the optimal effective tax rate for the region  $i = 1, 2$ . For the sake of simplicity, the superscripts on the variables are omitted. The population in region 2 in the migration equilibrium is:

$$n_2 = \int_{n_1}^1 dn = 1 - n_1 \quad (4)$$

### 3.2 Stage 1: Regional administrations set tax auditing policies

The problem is symmetric: the two administrations compete “à la Cournot” simultaneously setting their tax policies. We develop the problem of administration 1. Formally, administration 1 faces the following problem given the governments’ decisions regarding tax rates and anticipating the results of the last *stage*:

$$\underset{\beta_1}{Max} R_1(\beta_1, \beta_2; t_1, t_2, a) = n_1 \times r_1 = \left( \frac{1}{2} + \frac{B \times [\theta_2 - \theta_1 + g_2 - g_1]}{2a} \right) \times [B \times \theta_1 - d(\beta_1)] \quad (5)$$

where  $d(\beta_i)$  represents the tax administration cost such that  $d'(\beta_1) > 0, d(\beta_1)'' > 0$  and  $r_i \equiv \frac{R_i}{n_i} = [B \times \theta_i - d(\beta_i)]$  is the unitary tax revenue.

Since the two regions are symmetric, we can show that a symmetric Nash equilibrium exists, satisfying the following condition obtained from the first order condition (FOC) of the administrations and from the analogous FOC of the governments imposing  $t_1 = t_2 = t$ ,  $\beta_1 = \beta_2 = \beta$ :

$$\beta: \quad r'_\beta = -2n'_\beta \times r > 0 \quad (6)$$

The factor  $-2n'_\beta$  represents the expected loss in the number of taxpayers due to an increase in  $\beta$ . So the right-hand side of equation (6) corresponds to the marginal mobility costs for the regional governments in terms of tax revenue losses due to an increase in  $\beta$ . The left-hand side represents the net marginal revenue due to an increase in  $\beta$ .

By developing condition (6) we find that  $B \times \frac{\partial \theta}{\partial \beta} - d'(\beta) = r \times \frac{B \times \left( \frac{\partial \theta}{\partial \beta} + \frac{\partial g}{\partial \beta} \right)}{a}$ . This shows us immediately that in the limit case of centralization ( $a \rightarrow +\infty$ ), the marginal mobility costs are null and that  $r'_\beta = 0$ : we are at the bliss point of the Laffer curve. Since the marginal mobility costs are positive, under decentralization ( $a \in (0, +\infty)$ ) the tax auditing implementation is more costly. In fact, the net marginal tax revenue is positive ( $r'_\beta > 0$ ) and tax enforcement is less severe than under centralisation: the threat of the mobility of the tax base tames the administration. This result replicates that reported by Cremer and Ghavari (2000).



### 3.3 The slope of the reaction function and other comparative statics

Since the purpose of this paper is to test empirically the presence of regional interdependence in the setting of tax auditing policies, we wish to examine the process by which regional administrations reach the equilibrium level of auditing probability. In other words, we are interested in evaluating the slope of the reaction function  $\beta_i(\beta_j)$ . A non-null sign would highlight the presence of some kind of interaction between regions. In particular, we expect to find a positive sign, i.e. following a game-theory definition, we guess  $\beta_i$  and  $\beta_j$  are strategic complements. If this were the case, we could conclude that in a decentralized framework, the lower equilibrium level in  $\beta$  is the consequence of a competition process due to the potential tax base mobility.

It is easy to show that:

$$\frac{\partial \beta_1}{\partial \beta_2} = - \frac{R_{1\beta_1\beta_2}(\beta_1, \beta_2; t_1, t_2, a)}{R_{1\beta_1\beta_1}(\beta_1, \beta_2; t_1, t_2, a)} = - \frac{n_{1\beta_2} \times r_{1\beta_1}}{R_{1\beta_1\beta_1}(\beta_1, \beta_2; t_1, t_2, a)} > 0 \quad (7)$$

The first term in the numerator of equation (7) represents the derivative of the population in region 1 with respect to the enforcement of region 2 and is positive: once region 2 begins to increase its auditing probability, some residents in region 2 will start to move to region 1. The second factor in the numerator represents the marginal unitary tax revenue that is positive under the FOC. According to the second order condition (SOC) of the administration's problem the denominator in equation (7) should be negative. The slope of the reaction function is then positive: the regional administrations set their auditing strategies in a complementary fashion and so they are competing over this instrument in order to attract (or at least not to lose) their tax base. We test this result by means of econometric techniques. Our main research question can therefore be stated as follows: to what extent does the auditing policy of each region depend

on the strategies adopted by the other regions? Moreover, it is possible to show that  $\frac{\partial(\frac{\partial \beta_1}{\partial \beta_2})}{\partial a} < 0$  (see Appendix 2 for details). This means that the competition between regions weakens as the mobility costs rise. Since it seems reasonable to assume that mobility costs will be positively correlated with the distance between regions, two distant regions will compete less than two regions that lie closer together. We explicitly take this into consideration when choosing the econometric strategy.

A further result we find and test is that  $\beta_1$  and  $t_2$  are strategic complements, in fact:

$$\frac{\partial \beta_1}{\partial t_2} = - \frac{R_{1\beta_1 t_2}(\beta_1, \beta_2; t_1, t_2, a)}{R_{1\beta_1\beta_1}(\beta_1, \beta_2; t_1, t_2, a)} = - \frac{n_{1t_2} \times r_{1\beta_1}}{R_{1\beta_1\beta_1}(\beta_1, \beta_2; t_1, t_2, a)} > 0 \quad (8)$$

This result is reasonable: if the government in one region increases competition in tax rates (i.e. it reduces  $t_j$ ), *ceteris paribus*, the administration in the other region will unambiguously react by setting a more tolerant auditing rate (i.e. it reduces  $\beta_i$ ) in order not to lose any tax base<sup>13</sup>.

## 4. Empirical Analysis

In this section we test the main hypothesis by means of an econometric model, we provide a description of the data base and we present and comment on the main results emerging from the analysis.

### 4.1 The empirical framework

The theoretical framework presented in the previous section offers interesting insights that require empirical testing: the horizontal tax competition model suggests that revenue-maximizing administrations set their auditing policies in a complementary fashion, interacting so as not to lose tax bases. This result can be derived from equation (7). To test it we estimate a *time-space recursive model* that adopts a spatial econometric specification (see Anselin *et al.* 2008).

#### 4.1.1 Time-space recursive model

The recent literature on horizontal tax interdependence acknowledges the possibility that policy reactions are not immediate and the need for inertia to be considered when setting statutory tax parameters (Overesch & Rincke, 2011; Esteller-Moré & Rizzo, 2011). Moreover, since tax enforcement policy is not expected to change radically from one year to the next, we consider the possibility that a sluggish adjustment in auditing policies might play an even stronger role in our case. Thus, we build a *dynamic time-space recursive model* (Anselin *et al.* 2008), introducing the time-lagged endogenous variable as a regressor, in addition to the lagged spatial regressor:

$$\begin{aligned} \beta_{it} = & \vartheta_i + \tau_t + \alpha\beta_{it-1} + \gamma\beta_{-it-1} + \mu TR_{it} + \delta EL_{it} + \pi Left_{it} + \rho GDPpc_{it} + \sigma Defgdp_{it} \\ & + \varphi Transfexp_{it} + \omega Prft_{it} + \xi Ded_{it} + \psi Ded_{-it-1} + \varepsilon_{it} \end{aligned} \quad (9)$$

So the coefficient  $\alpha$  accounts for the presence of inertia and is expected to be positive (i.e. the higher the value, the stronger the inertia) and less than one to be congruent with the concept of Nash equilibrium.  $\beta_{it}$  represents the total number of audits performed by region  $i$  during the year  $t$ , while the term  $\beta_{-it-1} \equiv \sum_{j=1}^N w_{ij} \beta_{jt-1}$  and  $w_{ij}$  is the spatial weight that describes the

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<sup>13</sup> Unfortunately it is not possible to unambiguously determine the sign of the slope of  $\beta_1(t_1)$ .

relative interdependence of regions  $i$  and  $j$  in such a way that  $w_{ij} \geq 0$  if  $i \neq j$  and  $w_{ij} = 0$  if  $i = j$ <sup>14</sup>. Since the number of audits performed depends on the total number of auditable tax forms received in any one year, we introduce the number of tax returns received by region  $i$  during the year  $t$  ( $TR_{it}$ ) in the regression, in this way we control for the size of the tax administration. The specification includes fixed effects for regions  $\vartheta_i$  and years  $\tau_t$ , while  $\varepsilon_{it}$  is the error term.

So the term  $\beta_{-it-1}$  accounts for potential strategic competition in auditing policies and it is introduced with a time lag because in practice the tax auditing policies of the other regions are not simultaneously observable by a regional administration (Fredriksson & Millimet, 2002b; Fredriksson *et al.*, 2004; Millimet & Rangaprasad, 2007). If coefficient  $\gamma$  differs significantly from zero, the model will predict the presence of regional interactions in the setting of tax auditing policies. More specifically, according to the theoretical framework, eq. (7), we expect  $\gamma$  to be positive.

We assume the size of the tax administration to be optimal when the increase in the number of tax returns corresponds to an exactly proportional increase in the number of audits, in the sense that the audited share of the tax return remains unchanged, which corresponds to  $\mu$  being equal to one. Tax administration policies might be sensitive to “budgetary” and “political” effects (see e.g. Esteller-Moré, 2005, 2011), as well as to other elements for which we control.  $El_{it}$ , a dummy variable equal to one if there is an election in region  $i$  during the year  $t$ , is introduced to control for the electoral cycle. Its coefficient is expected to be negative because regional governments might deem it profitable to reduce the effective tax rate through enforcement effort cuts to ensure re-election.  $Left_{it}$  is another dummy equal to one if the party in office in a specific region and year is to the left of the political spectrum. We use per capita GDP ( $gdppc_{it}$ ) to control for the regional economic cycle and regional size. The deficit-GDP ratio ( $defgdp_{it}$ ) and the total amount of transfers received from the central government divided by total regional expenditure ( $transfexp_{it}$ ) are introduced to account for further relevant budgetary factors. We also control for a proxy of profitability ( $prft_{\beta_{it}}$ ) defined as the mean revenue per audit collected by a region in a specific year. To account for possible normative modifications to the statutory tax parameters, we include a dummy ( $ded_{it}$ ) equal to one if the regional government  $i$  makes a marked deduction in favour of the most common heirs during

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<sup>14</sup> Specifically, we employ a spatial matrix based on the inverse of the distance between regional capitals. The choice is made on the basis of the results of the theoretical model: when the distance between two regions – a proxy of mobility costs – increases we observe a lower level of competition in terms of their auditing policies. While the recent literature suggests that a change in the spatial matrix is not crucial (LeSage & Pace 2010), in our case the model can be assumed to be better specified than one based on a simple natural neighbors matrix because the Spanish state includes a number of islands, the presence of which makes the definition of neighbors arbitrary (see, for example, Costa-Font & Pons-Novell, 2007).

the year  $t^{15}$ . These modifications to the deduction regime substantially reduce the level of the effective tax rate and there is evidence that they induce a convergence process among regions congruent with a race to the bottom (Durán-Cabré & Esteller-Moré 2009, 2010). We can then interpret a  $ded_{it}$  value equal to 1 as a modification to the corresponding regional statutory tax parameters that results in a less severe effective tax rate. Finally, we control for  $ded_{it-1}$ , which represents the weighted average of the neighbours' deduction policies in the previous year. In line with the above reasoning, an increase in this variable is compatible with a decrease in the lagged weighted average of the neighbours' statutory tax parameters. According to the theoretical model (equation (8)), we expect the coefficient of this variable to be negative: a  $ded_{it-1}$  value equal to 1 (i.e. a decrease in  $t_2$  in the equation (8)) would correspond to a decreasing number of audits.

As is well known, the lagged endogenous variable  $\beta_{it-1}$  is by definition positively correlated with the regional fixed effect  $\vartheta_i$ . This implies that estimating  $\alpha$  by means of OLS will lead to inefficient and upward biased estimates. The within-groups estimator eliminates this source of inconsistency by transforming the equation to eliminate  $\vartheta_i$  but it results in an estimator that is downward biased (see Nickell, 1981). As suggested by Bond (2002) “a candidate consistent estimator will lie between the OLS and within-group estimates” and the Generalized Methods of Moments (GMM) provides a convenient framework for obtaining it. Specifically, we employ an augmented version of the Difference GMM (Arellano and Bond, 1991), the so-called System GMM estimator (Arellano and Bover, 1995), which is supposed to be more efficient (Blundell and Bond, 1998). This estimator applies a transformation to the original model taking the first differences in order to eliminate  $\vartheta_i$  and builds a stacked dataset with variables in levels and in differences. The equation in differences is then instrumented with lags 2 and up of the endogenous variables in levels while the equation in levels is instrumented using the same lags of the endogenous variable in differences.

According to Ziliak (1997), instrumenting the lagged endogenous variable with all the available lags is efficient, but in small samples it might cause an over-fitting bias in the GMM estimators. To overcome this, Roodman (2009) suggests reducing the instrument count, seeking to keep the number of instruments below that of the groups. We combine both possible approaches for containing instruments: collapsing instruments and limiting lag depth amounts (see Roodman, 2009, pp. 148-151, for more details on these techniques). Although the number of instruments is significantly reduced it is still larger than the relatively small number of groups, so we are

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<sup>15</sup> The main heirs are the spouse, any descendant younger than 21 years or ascendants. For details on the normative aspect of the exemption regime see Durán-Cabré and Esteller 2009, 2010.

unable to rectify this problem completely. In order to test the instruments validity we perform the Hansen and Sargan tests of overidentifying restrictions, even though their performance is likely to be weakened by the high number of instruments.

To verify the validity of the instruments we also need to check for the absence of serially correlated error terms, which is a required condition for the consistency of the GMM estimator. We perform tests for first (AR(1)) and second order (AR(2)) serial correlation under the null hypothesis of the presence of serial correlation. If there is no serial correlation in  $\varepsilon_{it}$ , the first-differenced residuals should reject the null hypothesis in the AR(1) test but not in the AR(2) test (Arellano and Bond, 1991).

As for the estimation strategy of  $\beta_{-it-1}$ , we employ System GMM using the lags of the spatial lag as instruments (reported as internal), but we also provide a robustness check by adding external instruments<sup>16</sup> to determine the extent to which the estimates are affected by the use of both sets of instruments.

Finally, we enrich the model in order to gain a better understanding of the extent to which the reforms first implemented in Spain in the mid-nineties have affected the horizontal interdependence in tax auditing. More specifically, to test whether  $\beta_{-it-1}$  is affected by the decentralization process that gave greater tax legislative power to the regional governments, we interact  $\beta_{-it-1}$  with  $post97_{it}$ , a dummy equal to one for years posterior to the first IGT reform (1997). In this way we identify the effect of the devolution process as a whole. In order to disentangle the specific role of either one of the two reforms, we interact  $\beta_{-it-1}$  with a dummy that identifies the second reform (2002) and, in the same model, with another dummy associated with the period between the two reforms. Finally, we introduce one last model where  $\beta_{-it-1}$  is interacted with  $ded_{it}$ . The aim in this case is to emphasize the effects of the actual implementation of the second IGT reform on the process of enforcement competition.

If the coefficients of these alternative interaction terms are found to be negative, then it would mean that following the reforms that gradually decentralized legislative power vis-à-vis statutory tax parameters, the regions reduced competition in their auditing policies. In other words: a reduction in opaque competition in the enforcement strategies would be the result of the possibility to compete in statutory tax parameters.

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<sup>16</sup> We use some (lagged) exogenous explanatory variables as instruments, employing the same weighting scheme for the instruments as with  $\beta_{-it-1}$  (see e.g. Kelejian & Robinson, 1993; Kelejian & Prucha, 1998).

#### 4.1.2 Data and Sources

Our panel comprises information about the 15 Spanish “common regime” Autonomous Communities<sup>17</sup> for the period 1987-2009<sup>18</sup>. In Table 1 we present the summary statistics. Information about regional tax enforcement policies is released annually within the tax assignment report, “Informe sobre la cesión de tributos a las Comunidades Autónomas”, published together with the Spanish National Budget, “Proyecto de Presupuestos Generales del Estado”. The report registers the number of audits performed on a yearly basis by each region. We use this information to define our endogenous variable  $\beta_{it}$  together with the number of tax returns ( $TR_{it}$ ) and the proxy of auditing profitability ( $prft\_ \beta_{it}$ ). The other variables are obtained from the following statistical sources. The per capita GDP ( $gdppc_{it}$ ) is provided by the Spanish National Institute of Statistics (INE). The deficit data used to construct the variable  $defgd_{it}$  are calculated as the difference between current availability and current expenditure as provided by the database maintained by the Ministry of Economy and Finance. The  $transfexp_{it}$  is constructed as the ratio between the total amount of transfers received from the central government (extracted from the INE database) and the total regional expenditure (extracted from the Ministry of Economy and Finance database). The information on election years  $el_{it}$  is obtained from the Interior Ministry’s website ([http://www.mir.es/DGPI/Elecciones/Procesos\\_Electorales\\_Celebrados/proceso\\_por\\_tipoyfecha.html](http://www.mir.es/DGPI/Elecciones/Procesos_Electorales_Celebrados/proceso_por_tipoyfecha.html)) while the information about the political colour of each regional government, required to construct the dummy  $left_{it}$ , is obtained from Zarate’s Political Collections website (<http://www.terra.es/personal2/monolith>).

The information used to construct the dummy  $ded_{it}$ , which accounts for the introduction of IGT deductions, is taken from Durán-Cabré & Esteller-Moré (2009).

[TABLE 1]

#### 4.2 Basic results

In Table 2 we report the results of the *time-space recursive model* expressed in equation (9). Column (1) is a baseline estimation testing auditing interactions without accounting for inertia (i.e. assuming  $\alpha = 0$  in equation 9)<sup>19</sup>. The autoregressive coefficient is significant and positive:

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<sup>17</sup> The Communities of Navarre and the Basque Country form part of the Foral System, which grants them independence in their laws and institutions. For this reason they are not included in the survey.

<sup>18</sup> Auditing information for the Madrid Community becomes available in 1996, the year in which it was granted this administrative power. We do not have any information about administration policies in 1993 as in 1995 the budget was not approved.

<sup>19</sup> This is a *pure-space recursive model* estimated using a two-stage least square procedure. We employed the external instruments only. We report an endogeneity test of  $\beta_{-it-1}$  (under the null hypothesis that the regressor can actually be treated as exogenous), we perform Hansen’s (1982) overidentifying restrictions test and, finally, we report the first-stage F-statistic to check the performance and the potential weakness of the instruments.

this is congruent with the theoretical model although the coefficient is not credible because it is much higher than one, which is not compatible with the Nash equilibrium. Moreover, even though there are no studies in the literature specifically analyzing these issues with which we can compare our estimates, the literature on spatial interactions in the setting of statutory tax parameters suggests that the slope of the reaction function should be quite a lot lower<sup>20</sup>. Thus, it seems that not taking into account the inertia leads to a misspecification of the model: otherwise, its role is picked up by the spatial lag.

In fact inertia plays an important role in the setting of regional auditing policies: the coefficients are strongly significant and positive<sup>21</sup>. The coefficient of the spatial lag is positive and significant, which confirms that horizontal interactions between regional administrations do take place when setting the auditing policies. This, in turn, is congruent with the hypothesis of tax competition adopted in the theoretical model. In columns (2) to (5) we present the basic model without interactions and the spatial lag coefficient is within the range (0.38 – 0.66) depending on which controls are introduced and which instruments are employed. Even if there is no benchmark in the literature against which to compare these results, the values are much more credible than those obtained without accounting for sluggish adjustments in the endogenous variable. In column (5) we specifically exploit both internal and external instruments and it can be seen that this does not make much difference to the results for the estimates of inertia and horizontal interactions.

If we consider the controls, three robust results can be highlighted. First, the variable “election year” is significant and negatively associated with the setting of auditing policies: in the presence of an election regional administrations reduce their enforcement of the IGT. This suggests that there is a political link between regional governments and regional tax administrations. Politicians in office have an incentive to reduce the effective IGT rate in order to gain votes and re-election and to operate through the tax administration, whose reduction of enforcement policies complies with politicians’ objectives. This supports the hypothesis of the identical incentives of the two institutions adopted in the theoretical model. The second robust result concerns the variable  $ded_{it}$ , which accounts for the introduction of IGT deductions by the regional governments. This variable is significant and positively associated with the enforcement strategies of the regional administrations. This suggests that once the governments are entitled to modify the statutory tax parameters (i.e. after the 2002 reform) and opt to make

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<sup>20</sup> They should be around 0.2 – 0.35 according to Revelli, (2001, 2006).

<sup>21</sup> The coefficients of the lagged endogenous variable are fairly high but lower than those obtained through the OLS estimator (around 0.96) and higher than those obtained through the within group estimator (around 0.83). As such they are within the range indicated by Bond (2002) to contain a consistent estimator. Moreover, note that AR(1) and AR(2) tests detect first order but not second order serial correlation for first-differenced residuals.

generous deductions, the administrations increase the number of audits: statutory tax parameters and auditing policies are strategic substitutes. Finally, the coefficient of the term  $ded_{it-1}$  is negative and significant corroborating the result of the theoretical framework (equation 8): neighbours' statutory tax parameters and auditing policies are strategic complements.

[TABLE 2]

### 4.3 Further results

In Table 3 we perform various interactions. In the first regression we interact  $\beta_{it-1}$  with a dummy that captures the effect of the first major IGT reform introduced in 1997 and obtain a negative and significant coefficient for the interacted term. This reform seems to have a negative effect on the enforcement strategies of the administrations. Moreover, the absolute values of the two coefficients (interacted and uninteracted) are fairly similar and, in fact, if we inspect the linear combination between them we find that the total effect is clearly not significantly different from zero<sup>22</sup>. This result can be interpreted as a corroboration of the hypothesis of horizontal tax competition as a source of auditing interdependence. In particular, this analysis seems to suggest that during the period in which legislative authority over the IGT was decentralized, competition in enforcement policies disappeared and switched to the setting of statutory tax parameters. This result implicitly provides a mechanism for eluding what Cremer and Ghavari (2000) consider the unfeasibility of harmonization policies for enforcement strategies. As discussed in Section 2, their argument is that a coordinated strategy between sub-central administrations is unattainable because of the difficulties in establishing whether a specific region's enforcement effort is adequate or not<sup>23</sup>. This leads to an unavoidable inefficient enforcement outcome. Our result seems to suggest that the devolution process can play a crucial role. Decentralizing the legislative power on statutory tax parameters has the positive and welcoming effect of a switch from opaque tax competition in tax enforcement to more transparent competition in the setting of statutory tax parameters, which makes a harmonization policy more feasible. To improve the specification of this aspect, in column (2) we interact – in the same regression –  $\beta_{it-1}$  with the dummies that identify respectively, the second wave of reform and the period between the two reforms. In this way we can disentangle the specific effects of each reform. We obtain analogous results but in this case, although both interacted terms are negative, the term identifying the second reform presents a lower absolute value that is

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<sup>22</sup> The coefficient is -0.042 and the t-statistic -0.25.

<sup>23</sup> In fact, for instance, a low auditing rate could be identified as inefficient simply because it is low while it is actually low as a result of an improvement process that ensured that enforcement effort was much more precise and efficient.



not significant<sup>24</sup>. This means that the first wave of reform had a stronger negative impact on auditing competition. We interpret this result as evidence that to observe a substantial disappearance of competition in enforcement strategies, it is sufficient for the regions to have the possibility to weakly set (and compete over) statutory tax parameters<sup>25</sup>. This last result is also corroborated by interacting  $\beta_{-it-1}$  with the dummy that identifies the actual implementation of a substantial reduction in the statutory tax parameters ( $ded_{it}$ ): the interacted term is also negative and although it is not significant, the linear combination between the interacted and uninteracted terms is not significantly different from zero.

[TABLE 3]

## 5. Conclusions

In this paper we have analyzed the presence of another level of tax interdependence that may occur in federal contexts: horizontal competition between regional administrations in their enforcement policies. By applying a theoretical framework we derive a regional auditing reaction function that is positively sloped: regional administrations compete in their auditing policies. This result is tested by means of spatial econometric techniques. The *time-space recursive model* produces outcomes that corroborate the theory. Specifically, allowing for sluggish adjustment we obtain a high degree of inertia in the auditing policy setting and credible coefficients for the spatial lag (around 0.38 – 0.66), which are congruent with the Nash equilibrium condition. This is our main contribution, which is in line with Cremer and Gahvari's results (2000).

These authors suggest that in the presence of horizontal competition, as auditing strategies are not publicly or easily observable, it might prove difficult to set a binding agreement between sub-central governments that is aimed at harmonizing these strategies. This is a reasonable point in the sense that even if the policies are publicly observable (because, for instance, they are recorded in a publicly available report – as is the case in Spain), it might not be readily established whether a specific region's enforcement effort is sufficient or not. As regards this point, here we implicitly find a way of circumventing this problem. Our empirical evidence suggests that if the decentralization process is gradually implemented and administrative responsibility is decentralized before the normative power, enforcement policy competition disappears in the precise moment when it is possible to compete in terms of more powerful instruments, i.e. the statutory tax parameters. Seen from this perspective, the decentralization

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<sup>24</sup> The linear combination of the two effects with the uninteracted term is still not significant: the coefficient is -0.262 and the t-statistic is -0.67.

<sup>25</sup> This could also simply depend on the fact that the observations involved in the evaluation of the second and more recent wave of reform were fewer.

process is welcome: a highly decentralized framework has the advantage of switch from a situation of opaque competition to a situation of transparent competition.

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## TABLES:

**Table 1: Summary statistics**

Variable	Observations	Mean	Median	SD	Max	Min
Audits	307	370.5961	195	486.5621	2550	0
Tax Returns	308	21187	13442	18234.62	88528	1641
Leftish government	322	0.4627329	0	0.4993853	1	0
Election year	322	0.2546584	0	0.4363471	1	0
Deduction	322	0.1335404	0	0.3406872	1	0
Deficit-GDP ratio	308	-0.0028976	-0.0017705	0.0070989	0.0298811	-0.026144
Transfers-GDP ratio	294	0.3977149	0.3853665	0.1348314	1.373906	0.1117062
Per Capita GDP	322	11.52553	11.35349	5.497171	23.01702	2.174576
Auditing Profitability	280	8.936545	4.650814	12.75857	108.2774	0

**Table 2: Tax Auditing interdependence: Time-space recursive model  
(System-GMM/One-step; Fixed effects & Time Effects in all specifications)**

VARIABLES	(1) Audits	(2) Audits	(3) Audits	(4) Audits	(5) Audits
L.audits	-	0.912*** (5.969)	0.888*** (6.265)	0.896*** (6.733)	0.932*** (8.989)
L.Waudits	2.114* (1.718)	0.659** (2.205)	0.521** (2.214)	0.381* (1.685)	0.468** (2.329)
Leftish government	-233.735** (-2.193)	-53.907 (-0.982)	-34.527 (-0.804)	-3.522 (-0.170)	-4.096 (-0.208)
Election year	-101.380 (-1.102)	-34.505** (-2.254)	-37.684** (-2.210)	-33.477** (-2.158)	-33.315** (-2.114)
Deficit/GDP	5740.946 (1.061)	-270.497 (-0.141)	27.531 (0.014)	137.939 (0.067)	-393.251 (-0.180)
Transfers/expenditure	-130.466 (-0.530)	226.583 (1.136)	233.558 (1.168)	225.265 (1.219)	252.761 (1.410)
Tax Return	-0.001 (-0.108)	0.003 (1.421)	0.003 (1.486)	0.003 (1.041)	0.002 (1.027)
Deduction			76.970** (2.307)	59.932*** (2.724)	49.657*** (2.774)
L.WDeduction			-168.552* (-1.726)	-271.726** (-2.226)	-332.964*** (-2.748)
Per capita GDP				4.151 (0.324)	9.031 (1.007)
L.audit_profitability				-0.418 (-0.316)	-0.490 (-0.372)
Tax Returnxd_foral				0.004 (1.147)	0.004 (1.136)
_cons		-211.897* (-1.671)	-175.954 (-1.187)	-195.919 (-0.678)	-279.633 (-1.170)
Observations	237	237	237	237	237
Internal Instruments	NO	YES	YES	YES	YES
External Instruments	YES	NO	NO	NO	YES
# Instruments	5	32	34	37	40
Gmm lag limits	-	(2, 5)	(2, 5)	(2, 5)	(2, 5)
AR(1) ( <i>p</i> -value)	-	0.039	0.040	0.037	0.026
AR(2) ( <i>p</i> -value)	-	0.987	0.951	0.981	0.964
Sargan-test	-	8.673	8.126	9.008	13.533
Sargan-test ( <i>p</i> -value)	-	0.371	0.421	0.342	0.260
Hansen-test	2.105	0.000	0.000	0.000	0.000
Hansen-test ( <i>p</i> -value)	0.716	1.000	1.000	1.000	1.000
Endogeneity-test	10.864	-	-	-	-
Endogeneity-test ( <i>p</i> -value)	0.001	-	-	-	-
First stage F-statistic	54.46	-	-	-	-
First stage F-statistic ( <i>p</i> -value)	0.0000	-	-	-	-

Note: *t* statistics in parentheses; \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ . Model (1) is a *pure-space recursive model* estimated through a 2SLS procedure.

**Table 3: Tax Auditing interdependence: Time-space recursive model – Interactions  
(System-GMM/One-step; Fixed effects & Time Effects in all specifications)**

VARIABLES	(1) Audits	(2) Audits	(3) Audits
L.audits	0.879*** (10.933)	0.869*** (13.432)	0.904*** (6.721)
L.Waudits	0.641** (2.540)	0.688*** (2.667)	0.398* (1.818)
L.Waudits×post97	-0.683*** (-3.458)		
L.Waudits×D97-01		-0.716*** (-3.337)	
L.Waudits×post01		-0.235 (-0.849)	
L.Waudits×deduction			-0.530 (-1.052)
Leftish government	-4.669 (-0.264)	-4.895 (-0.230)	-3.485 (-0.163)
Election year	-27.053* (-1.655)	-31.638* (-1.815)	-37.983** (-1.996)
Deficit/GDP	-319.061 (-0.152)	-763.501 (-0.354)	26.451 (0.012)
Transfers/expenditure	218.924 (1.156)	208.353 (1.078)	232.217 (1.271)
Tax Return	0.002 (1.232)	0.003 (1.589)	0.002 (0.964)
Deduction	44.531*** (2.960)	72.272*** (3.483)	91.448** (2.185)
L.WDeduction	-198.526 (-1.487)	-278.812* (-1.834)	-341.017*** (-3.060)
Per capita GDP	-4.910 (-0.758)	-1.843 (-0.220)	5.686 (0.446)
L.audit_profitability	-0.113 (-0.095)	-0.158 (-0.134)	-0.425 (-0.307)
Tax Return×d_foral	0.003 (1.227)	0.004* (1.688)	0.004 (1.117)
_cons	36.460 (0.170)	-105.899 (-0.396)	-197.627 (-0.689)
Observations	237	237	237
Internal Instruments	YES	YES	YES
External Instruments	NO	NO	NO
# Instruments	42	45	40
Gmm lag limits	(2, 5)	(2, 5)	(2, 5)
AR(1) ( <i>p</i> -value)	0.017	0.017	0.036
AR(2) ( <i>p</i> -value)	0.985	0.946	0.922
Sargan-test	14.496	14.704	9.126
Sargan-test ( <i>p</i> -value)	0.270	0.399	0.520
Hansen-test	0.000	0.000	0.000
Hansen-test ( <i>p</i> -value)	1.000	1.000	1.000

Note: *t* statistics in parentheses; \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .



### Appendix 1: Generalized results with non uniform distribution of taxpayers

Now we assume that the distribution of taxpayers along the home attachment is not uniform, i.e. we assume that  $n \in (0,1) \sim f(n)$  where  $f(n)$  represents a generic density function.

The value  $n_1(t_1, \beta_1, t_2, \beta_2; a) = \frac{1}{2} + \frac{U_1^* - U_2^*}{2a}$  represents the marginal individual indifferent between living in region 1 and region 2. Below  $n_1$  we have all the taxpayers that settle in region 1, while above  $n_1$  there are all the taxpayers that live in region 2. The respective shares of each group are  $F(n_1) = \int_0^{n_1} f(n)dn$  and  $1 - F(n_1) = \int_{n_1}^1 f(n)dn$ .

At stage 1 the problem of the administration of region 1 becomes:

$$\text{Max}_{\beta_1} R_1 = F(n_1) \times r_1 = F(n_1) \times [B \times \theta_1 - d(\beta_1)]$$

The FOC of this problem is:

$$n_1'_{\beta_1} \times f(n_1) \times r_1 + r_1'_{\beta_1} \times F(n_1) \equiv P(\beta_1, \beta_2; t_1, t_2, a) = 0$$

The SOC is:

$$P_{\beta_1}(\beta_1, \beta_2; t_1, t_2, a) < 0$$

The slope of the reaction function becomes:

$$\frac{\partial \beta_1}{\partial \beta_2} = - \frac{P_{\beta_2}(\beta_1, \beta_2; t_1, t_2, a)}{P_{\beta_1}(\beta_1, \beta_2; t_1, t_2, a)}$$

That is positive as long as  $f'(n_1) \leq 0$ .

### Appendix 2: Comparative statics on $a$

It is possible to express  $\frac{\partial \beta_1}{\partial \beta_2}$  as a function of  $a$  in order to perform a comparative statics analysis:

$$\frac{\partial \beta_1}{\partial \beta_2} = - \frac{N}{A + a \times \frac{\partial^2 r_1}{\partial \beta_1^2}} = -N \times \left( A + a \times \frac{\partial^2 r_1}{\partial \beta_1^2} \right)^{-1}$$

Where:

$$\begin{aligned} A = & -2B \times \left[ \frac{\partial \theta_1}{\partial \beta_1} + \frac{\partial g_1}{\partial \beta_1} \right] \times \left[ B \times \frac{\partial \theta_1}{\partial \beta_1} - d'(\beta_1) \right] + B \times [\theta_2 - \theta_1 + g_2 - g_1] \\ & \times \left[ B \times \frac{\partial^2 \theta_1}{\partial \beta_1^2} - d''(\beta_1) \right] - [B \times \theta_1 - d(\beta_1)] \times B \times \left[ \frac{\partial^2 \theta_2}{\partial \beta_1^2} + \frac{\partial^2 g_2}{\partial \beta_1^2} \right] \end{aligned}$$

and

$$N = B \times \left[ \frac{\partial \theta_2}{\partial \beta_2} + \frac{\partial g_2}{\partial \beta_2} \right] \times \left[ B \times \frac{\partial \theta_1}{\partial \beta_1} - d'(\beta_1) \right]$$

So that under FOC and SOC results  $N > 0$  and:

$$\frac{\partial \left( \frac{\partial \beta_1}{\partial \beta_2} \right)}{\partial a} = \frac{N}{\left( A + a \times \frac{\partial^2 r_1}{\partial \beta_1^2} \right)^2} \times \frac{\partial^2 r_1}{\partial \beta_1^2} < 0$$

### Appendix 3: Interdependence of different instruments

The derivative  $\frac{\partial \beta_1}{\partial \beta_2}$  with respect to  $t_2$  could be written as:

$$\frac{\partial \left( \frac{\partial \beta_1}{\partial \beta_2} \right)}{\partial t_2} = \frac{-R_{1\beta_1\beta_2t_2} \times R_{1\beta_1\beta_1} + R_{1\beta_1\beta_1t_2} \times R_{1\beta_1\beta_2}}{\left( R_{1\beta_1\beta_1} \right)^2}$$

This is positive as long as:

$$M \equiv R_{1\beta_1\beta_1t_2} \times R_{1\beta_1\beta_2} - R_{1\beta_1\beta_2t_2} \times R_{1\beta_1\beta_1} > 0 \Leftrightarrow R_{1\beta_1\beta_1t_2} \times R_{1\beta_1\beta_2} > R_{1\beta_1\beta_2t_2} \times R_{1\beta_1\beta_1}$$

Where:

$$R_{1\beta_1\beta_1t_2} = \frac{\partial n_1}{\partial t_2} \times \frac{\partial^2 r_1}{\partial \beta_1^2} < 0,$$

$$R_{1\beta_1\beta_1} = 2 \frac{\partial n_1}{\partial \beta_1} \times \frac{\partial r_1}{\partial \beta_1} + n_1 \times \frac{\partial^2 r_1}{\partial \beta_1^2} + r_1 \times \frac{\partial^2 n_1}{\partial \beta_1^2} < 0 \text{ under the SOC;}$$

$$R_{1\beta_1\beta_2t_2} = \frac{\partial^2 n_1}{\partial \beta_2 t_2} \times \frac{\partial r_1}{\partial \beta_1} > 0 \text{ under FOC;}$$

$$R_{1\beta_1\beta_2} = \frac{\partial n_1}{\partial t_2} \times \frac{\partial r_1}{\partial \beta_1} > 0 \text{ under FOC.}$$

So, in general, it is not possible to establish an unambiguous relationship between  $\frac{\partial \beta_1}{\partial \beta_2}$  and  $t_2$ .

The same results hold for the relationship between  $\frac{\partial \beta_1}{\partial \beta_2}$  and  $t_1$ .

### Appendix 4: Derivatives computation

It is possible to show that:

$$\frac{\partial \alpha^*_i}{\partial t_i} = \frac{\partial \alpha^*(t_i, \beta_i)}{\partial t_i} = \frac{-(1 - \beta_i \times \tau)}{g''} < 0$$

$$\frac{\partial \alpha^*_i}{\partial \beta_i} = \frac{\partial \alpha^*(t_i, \beta_i)}{\partial \beta_i} = \frac{t_i \tau}{g''} > 0$$

$$\frac{\partial \theta_i^*}{\partial t_i} = \alpha_i^* + (1 - \alpha_i^*) \times \beta_i \times \tau + \frac{\partial \alpha_i^*}{\partial t_i} \times (1 - \beta_i \times \tau) \times t_i \begin{matrix} < 0 \\ > 0 \end{matrix}$$

$$\frac{\partial \theta_i^*}{\partial \beta_i} = (1 - \alpha_i^*) \times t_i \times \tau + \frac{\partial \alpha_i^*}{\partial \beta_i} (1 - \beta_i \times \tau) \times t_i > 0$$

$$\frac{\partial g_i}{\partial t_i} = -\frac{\partial \alpha_i^*}{\partial t_i} (1 - \beta_i \times \tau) \times t_i > 0$$

$$\frac{\partial g_i}{\partial \beta_i} = -\frac{\partial \alpha_i^*}{\partial \beta_i} (1 - \beta_i \times \tau) \times t_i < 0$$

$$\left[ \frac{\partial \theta_i^*}{\partial t_i} + \frac{\partial g_i}{\partial t_i} \right] = \alpha_i^* + (1 - \alpha_i^*) \times \beta_i \times \tau > 0$$

$$\left[ \frac{\partial \theta_i^*}{\partial \beta_i} + \frac{\partial g_i}{\partial \beta_i} \right] = (1 - \alpha_i^*) \times t_i \times \tau > 0$$

$$\frac{\partial n_1}{\partial \beta_1} = -\frac{B \times \left[ \frac{\partial \theta_1}{\partial \beta_1} + \frac{\partial g_1}{\partial \beta_1} \right]}{2a} < 0$$

$$\frac{\partial n_1}{\partial \beta_2} = \frac{B \times \left[ \frac{\partial \theta_2}{\partial \beta_2} + \frac{\partial g_2}{\partial \beta_2} \right]}{2a} > 0$$

$$\frac{\partial n_1}{\partial t_1} = -\frac{B \times \left[ \frac{\partial \theta_1}{\partial t_1} + \frac{\partial g_1}{\partial t_1} \right]}{2a} < 0$$

$$\frac{\partial n_1}{\partial t_2} = \frac{B \times \left[ \frac{\partial \theta_2}{\partial t_2} + \frac{\partial g_2}{\partial t_2} \right]}{2a} > 0$$

$$\frac{\partial r_1}{\partial \beta_1} = \left[ B \times \frac{\partial \theta_1}{\partial \beta_1} - d'(\beta_1) \right] \begin{matrix} < 0 \\ > 0 \end{matrix}$$

$$\frac{\partial r_1}{\partial t_1} = \left[ B \times \frac{\partial \theta_1}{\partial t_1} \right] \begin{matrix} < 0 \\ > 0 \end{matrix}$$

$$\frac{\partial^2 \alpha^*(t_i, \beta_i)}{\partial \beta_i^2} = \frac{\frac{\partial \alpha^*}{\partial \beta_i} \times \tau \times t_i}{(g'')^2} \times g''' \leq 0 \quad \text{iff} \quad g''' \leq 0$$

$$\frac{\partial^2 \alpha^*(t_i, \beta_i)}{\partial t_i^2} = \frac{\frac{\partial \alpha^*}{\partial t_i} \times (1 - \beta_i \times \tau)}{(g'')^2} \times g''' \leq 0 \quad \text{iff} \quad g''' \geq 0$$

We assume  $g''' = 0$ .

$$\frac{\partial^2 \theta_i^*}{\partial t_i^2} = \frac{\partial \alpha^*}{\partial t_i} \times 2 \times (1 - \beta_i \times \tau) + \frac{\partial^2 \alpha^*}{\partial t_i^2} (1 - \beta_i \times \tau) \times t_i < 0 \quad \text{iff} \quad g''' = 0$$

$$\frac{\partial^2 \theta_i^*}{\partial \beta_i^2} = -2 \times \frac{\partial \alpha^*}{\partial \beta_i} \times \tau \times t_i + \frac{\partial^2 \alpha^*}{\partial \beta_i^2} (1 - \beta_i \times \tau) \times t_i < 0 \quad \text{iff} \quad g''' = 0$$

$$\frac{\partial^2 r_1}{\partial \beta_1^2} = \left[ B \times \frac{\partial^2 \theta_1}{\partial \beta_1^2} - d''(\beta_1) \right] < 0 \quad \text{iff} \quad g''' = 0$$

$$\frac{\partial^2 r_1}{\partial t_1^2} = \left[ B \times \frac{\partial^2 \theta_1}{\partial t_1^2} \right] < 0 \quad \text{iff} \quad g''' = 0$$

$$\frac{\partial^2 g_i}{\partial t_i^2} = -\frac{\partial^2 \alpha^*}{\partial t_i^2} (1 - \beta_i \times \tau) \times t_i - \frac{\partial \alpha^*}{\partial t_i} (1 - \beta_i \times \tau) > 0$$

$$\frac{\partial^2 g_i}{\partial \beta_i^2} = -\frac{\partial^2 \alpha^*}{\partial \beta_i^2} (1 - \beta_i \times \tau) \times t_i + \frac{\partial \alpha^*}{\partial \beta_i} \times \tau \times t_i > 0$$

$$\left[ \frac{\partial^2 \theta_i^*}{\partial t_i^2} + \frac{\partial^2 g_i}{\partial t_i^2} \right] = \frac{\partial \alpha^*}{\partial t_i} \times (1 - \beta_i \times \tau) < 0$$

$$\left[ \frac{\partial^2 \theta_i^*}{\partial \beta_i^2} + \frac{\partial^2 g_i}{\partial \beta_i^2} \right] = -\frac{\partial \alpha^*}{\partial \beta_i} \times \tau \times t_i < 0$$

$$\frac{\partial^2 n_1}{\partial \beta_1^2} = -\frac{B \times \left[ \frac{\partial^2 \theta_2}{\partial \beta_1^2} + \frac{\partial^2 g_2}{\partial \beta_1^2} \right]}{2a} > 0$$

$$\frac{\partial^2 n_1}{\partial t_1^2} = -\frac{B \times \left[ \frac{\partial^2 \theta_2}{\partial t_1^2} + \frac{\partial^2 g_2}{\partial t_1^2} \right]}{2a} > 0$$

$$\frac{\partial^2 n_1}{\partial \beta_1 \partial t_1} = -\frac{B}{2a} \times \left[ \frac{\partial^2 \theta_i^*}{\partial \beta_i \partial t_i} + \frac{\partial^2 g_i}{\partial \beta_i \partial t_i} \right] = -\frac{B}{2a} \times \left( (1 - \alpha^*) \times \tau - \frac{\partial \alpha^*}{\partial t_i} \times \tau \times t_i \right) < 0$$

$$\frac{\partial^2 r_1}{\partial \beta_1 \partial t_1} = \left[ B \times \frac{\partial^2 \theta_1}{\partial \beta_1 \partial t_1} \right] > 0$$

$$\frac{\partial^2 n_1}{\partial t_1 \partial \beta_1} = -\frac{B}{2a} \times \left[ \frac{\partial^2 \theta_i^*}{\partial t_i \partial \beta_i} + \frac{\partial^2 g_i}{\partial t_i \partial \beta_i} \right] = -\frac{B}{2a} \times \left( \frac{\partial \alpha^*}{\partial \beta_i} \times (1 - \beta_i \times \tau) + (1 - \alpha^*) \times \tau \right) < 0$$

$$\frac{\partial^2 r_1}{\partial t_1 \partial \beta_1} = \left[ B \times \frac{\partial^2 \theta_1}{\partial t_1 \partial \beta_1} \right] > 0$$

$$\frac{\partial r_1}{\partial \beta_2} = \frac{\partial^2 r_1}{\partial \beta_1 \partial \beta_2} = \frac{\partial^2 n_1}{\partial \beta_1 \partial \beta_2} = \frac{\partial^2 n_1}{\partial \beta_2 \partial \beta_1} = \frac{\partial r_1}{\partial t_2} = \frac{\partial^2 r_1}{\partial t_1 \partial t_2} = \frac{\partial^2 n_1}{\partial t_1 \partial t_2} = \frac{\partial^2 n_1}{\partial t_2 \partial t_1} = \frac{\partial^2 n_1}{\partial t_1 \partial \beta_2} = 0$$