SIGNALING COVERTLY ACQUIRED INFORMATION[1]

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We study the interplay between information acquisition and signaling. A sender decides whether to learn his type at a cost prior to taking a signaling action. A receiver observes the signaling action and responds. We characterize equilibria and apply a version of NWBR refinement in the environment where the information acquisition is observable as well as in the environment where it is covert. Observability of acquisition has important consequences on predicted behavior. In one of the commonly used environments, information is never acquired if acquisition is observable, yet is acquired if information is cheap and the acquisition covert.

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1. Introduction

With signaling, Spence (1973) introduced an elegant solution to the problem of asymmetric information and the inefficiencies it can lead to. The underlying idea is simple but powerful: the informed party can transmit information through the choice of their action. In separating equilibria, which have been the focus of much applied work, each type undertakes a distinct observable action thereby fully revealing the sender's information. This leads to a natural question: if the sender's information is revealed through his action, what incentives does he have to acquire it?[1] In some signaling environments it is reasonable to assume that the sender is informed for reasons outside of the model, in others such exogenous explanations are less convincing:

- Prior to attempting to take over a target firm, the raider can acquire information about the synergies generated from the merger. The take-over offer, however, might reveal some of the raider's private information.
- Before posturing aggressively to signal its resolve, a belligerent country might investigate how likely it is to win a potential conflict.
- During a currency crisis, a central bank may study how ready it is to stave off a potential currency attack. Its costly policy interventions might signal both the likelihood it attaches to a devaluation as well as how informed it is about the prospects.[2]

Motivated by the question and the examples, we study the interplay between information acquisition and signaling. A sender first decides whether to learn his type at a cost c, then takes a signaling action. The majority of the paper focuses on the case of two types. We analyse both the case where the sender's information acquisition is observable as well as where it is covert. In either case, the receiver observes the sender's signaling action and replies with an action of his own. The sender would like the receiver to take as high an action as possible. The receiver, on the other hand, would like to match the state. Our study of information acquisition in signaling is part of a wider recent effort to understand the sources of information in models of private information. See Rüdiger and Vigier (2019) for a study of information acquisition in dealer markets and Roesler and Szentes (2017) and Condorelli and Szentes (2020) for explorations of optimal buyer's information in a monopoly.

[1] A similar idea, that agents in a market might not inform themselves if their information gets revealed through the price was developed in Grossman and Stiglitz (1980).

[2] For signaling in take-overs see Burkart and Lee (2015), Ekmekci and Kos (2014). Signaling in conflicts was explored in Fearon (1997) and more recently in Wolton (2019). Signaling in currency attacks is studied for example in Angeletos et al. (2006). This body of work, however, takes the sender's information as given.

We focus on Nash equilibria in which the receiver's strategy is not weakly dominated, equilibria for short.[3] As is often the case in the games with signaling, our games admit a wide array of equilibria. To remedy this nuisance, we invoke a type of a *never weak best response* (NWBR) refinement. The refinement operates by constructing the set of all equilibria with a given outcome and iteratively deleting strategies that are never weak best response to any strategy in the set of equilibria of the game. Precise definition of the procedure is provided in the main text. We use the NWBR refinement since the most commonly used refinements (the intuitive criterion and D1 of Cho and Kreps (1987), D2 and divinity of Banks and Sobel (1987)) are defined and operate on signaling games—games where a privately informed sender takes an action that is followed by the receiver's action—which the games analyzed in this paper are not. Furthermore, these refinements do not readily extend to the games studied here.[4] Strategic stability of Kohlberg and Mertens (1986)—the refinement on which the above-mentioned belief based refinement are built upon—, on the other hand, is only defined for finite games and even there impractical to use, to say the least.

First, we examine the model where the information acquisition decision is observable. While the set of equilibrium outcomes is in general not unique, the refinement does pin down a unique equilibrium outcome. The sender either acquires information and plays the least cost separating equilibrium or does not acquire information and takes the least cost action depending on which of the two outcomes leads to a higher payoff. A stronger result obtains in an additively separable environment where the sender's benefit from the receiver's action is concave and independent of the state of the world; an environment commonly analyzed in applications, e.g. see Spence (1973) and Feltovich et al. (2002). There, the sender never acquires costly information in the unique equilibrium outcome surviving the refinement.

The set of equilibria in the game with unobservable information acquisition is overwhelming; among other things, one can support as equilibrium no information acquisition followed by a strictly costly action. When information is free, these equilibria coexist with information acquisition followed by the well-known equilibria arising in Spence model: pooling, separating, and semi-separating. However, the set of equilibria shrinks significantly when information becomes costly. In any equilibrium where information is acquired with positive probability the two types must separate themselves strongly, that is, each type must strictly prefer their own equilibrium action(s). As a consequence, when in-

[3] Among other things, this eliminates implausible equilibria where receiver responds off path with actions that are not a best response to any of his beliefs.

[4] The above mentioned refinements are based on comparisons of sets of beliefs for which each type could profitably deviate to a given action. Such type-by-type comparisons do not suffice here. For example, in the equilibrium outcome of the game with covert information acquisition in which the sender does not acquire information and undertakes the least costly action the high type does not exist.

formation is costly the Riley outcome—the most efficient separating equilibrium in the Spence model—can not be supported as an equilibrium after information acquisition. Information acquisition, however, can be supported (as long as it is not too costly) by requiring the high type of the sender to burn more surplus than in the Riley outcome. In the most efficient such equilibrium, the sender is indifferent between his equilibrium play and deviating to not acquiring information and mimicking the high type. The amount of surplus the high type burns in such equilibria increases with the cost of information. The multiplicity of equilibrium outcomes and the associated low predictive power of the model, all but necessitate a refinement.

We show that the usual single-crossing condition is not enough to guarantee uniqueness of the outcome surviving the NWBR refinement and provide sufficient conditions that do guarantee uniqueness. Single-crossing, however, is sufficient for uniqueness when information is cheap: only the most efficient outcome with information acquisition survives the refinement. Stronger results are provided for more specialized environments. For example, if the sender's cost of signaling does not depend on the state (termed *type-dependent benefits*) there is always a unique equilibrium outcome that survives the refinement.[5] The comparison between observable and covert information acquisition is most striking in the case where the sender's benefits from receiver's action are concave and independent of the state of the world. In such an environment information is never acquired when information acquisition is observable, but is acquired when information acquisition is covert and cheap.[6]

Finally, we establish the robustness of information acquisition when it is cheap and covert to environments with more than two states.

Related Literature. Our paper builds on the seminal work of Spence (1973) and the subsequent literature on refinements, see Kohlberg and Mertens (1986), Cho and Kreps (1987), Banks and Sobel (1987), Cho and Sobel (1990); most of which was discussed above. The majority of work on signaling is focused on signaling games, which games analyzed in this paper are not. A comprehensive review of the literature on signaling games is beyond the scope of this paper; the interested reader is recommended to consult Riley (2001) and Sobel (2009).

[5]This environment is commonly employed in applications ranging from advertising (e.g. Kihlstrom and Riordan (1984)) to political science (e.g. Fearon (1997)).

[6]Our results complement similar conclusions obtained in the disclosure literature. There, information is not acquired if the information acquisition decision is observable and acquired when the information acquisition decision is covert; see Matthews and Postlewaite (1985) and Kartik et al. (2017). When information is costly, in the covert case, information is acquired with positive but (unlike here) interior probability.

Information acquisition and signaling appears in the models of Grassi and Ma (2016) and Rüdiger and Vigier (2019). The first paper studies referrals where two experts compete for clients. An expert may acquire information about whether a client is a good fit or not, and may refer the client to the other expert for a fee. Rüdiger and Vigier (2019) study a financial market in which the market makers and the participants can acquire information. The focus of the paper is on the case where all players move simultaneously, i.e., signaling is absent. It is shown that if the market makers moved first, signaling opportunities arise, and an equilibrium that is qualitatively similar to the equilibrium of the simultaneous move game exists. Both of these papers focus on a particular equilibrium of the game rather than comprehensively studying the interplay of signaling and information acquisition.

The question of information acquisition on the side of the receiver, which likewise leads outside of the scope of signaling games, has received much more attention. Under various degrees of generality (and in different applications) it has been studied in Banks (1992), Bester and Ritzberger (2001), Stahl and Strausz (2017), Bester et al. (2019). The trade-off between the sender's incentive to signal and the receiver's incentives to acquire information arises as the central theme.[7]

In and Wright (2017) provide a comprehensive analysis of games where the sender (who has no private information to start with) takes two actions in a row and only the second one is observed by the receiver; the second action serves as a signal of the first.[8] The authors introduce a refinement that is easily applied and delivers a unique equilibrium in many environments. Their results rely on the assumption that there are no nature's moves between the two actions of the sender, thus they cannot be applied to the environment studied in this paper. For a particularly nice application to advertising see Cho et al. (2019). Brighi and D'Amato (2020) analyze a job market signaling environment where the low types can invest in early education to reduce their cost of signaling.

2. Setting

We study a game between two players: a sender and a receiver. The sender decides whether to acquire information, then takes an action. We will analyze both the model where the sender's information acquisition is observable as well as where it is covert,

[7]Similar ideas appear in the body of work where the receiver observes a signal of given precision about the sender for free; see Feltovich et al. (2002), Alós-Ferrer and Prat (2012), Daley and Green (2014), Kurlat and Scheuer (2017).

[8]The literature on endogenous signaling games is too large to survey here properly, a prominent strand being the study of how firms use price as a signal of their quality; see Klein and Leffler (1981) and Wolinsky (1983).

though, the emphasis will be on the latter. The environment is a canonical signaling game preceded by an information acquisition stage.

There are two states of the world $\Theta = \{\theta_L, \theta_H\}$, a low and a high state, respectively, with $\theta_H > \theta_L \geq 0$.[9] The prior probability of state θ_H is $\lambda \in (0,1)$. In the first stage the sender decides whether to learn the state θ at some cost $c \geq 0$, in the second stage he chooses an action $s \in S = [0, \infty)$. In the final stage, the receiver chooses an action $r \in R = [0, \infty)$ after having observed s. We refer to the set $T := \{\theta_u\} \cup \Theta$ as the set of types of the sender, where t denotes a generic element of the set. Type θ_u is the sender's type when he does not acquire information.

The sender's payoff in state $\theta \in \Theta$ is $u(s, r, \theta)$, where u satisfies the following properties: it is twice continuously differentiable in all of its arguments, $u_s < 0$, $u_r > 0$ and $-\frac{u_s}{u_r}$ decreasing in θ for $s, r > 0$. Moreover, $\lim_{s \to \infty} u(s, r, \theta) = -\infty$ for each $\theta \in \Theta$. The sender wants the receiver to take as high an action as possible. Signaling is, however, costly, and the sender's utility function satisfies the single-crossing condition. The receiver, on the other hand, maximizes $-(\theta - r)^2$. The receiver, thus, takes an action that is equal to the expected value of θ given his posterior.[10]

Two special cases of the environment under consideration are of particular interest:

- In the first,

$$u(s, r, \theta) = \tilde{u}(r) - g(s, \theta),$$

  for some strictly increasing $\tilde{u}$, and g that satisfies $g_s > 0$, $g_\theta < 0$ and $g_{s\theta} < 0$. We refer to this environment as *type-dependent signaling cost*. Its most prominent application is job market signaling; see Spence (1973), Feltovich et al. (2002) and Kurlat and Scheuer (2017).
- In the second,

$$u(s, r, \theta) = \hat{u}(r, \theta) - h(s),$$

  where $\hat{u}$ satisfies $\hat{u}_r > 0$ and $\hat{u}_{r\theta} > 0$ and $h_s > 0$. This case is referred to as *type-dependent benefits*. It is employed in applications to advertising (e.g. Kihlstrom and Riordan (1984)), currency attacks (e.g. Angeletos et al. (2006)) and assortative matching

[9] For consistency, most of the results are presented in the framework with two types. We comment as we go along which results extend to more general settings. An analysis of a case with more than two types is provided in Section 6.

[10] The exact form of the receiver's payoff function does not matter for the rest of our analysis as long as the receiver's optimal action is an affine function of her beliefs.

(e.g. Hoppe et al. (2009)), to name just a few.

We will first study the benchmark model where the receiver can observe whether the sender acquired information. In this scenario, a strategy for the sender, σ_S, is an element of $\{I, U\} \times \mathbb{R}^3_+$ that represents the sender's information acquisition choice (I stands for informed and U for uninformed) and his signaling action for each of his possible types, $\theta_L, \theta_u, \theta_H$. A strategy for the receiver is a mapping $\sigma_R : \{I, U\} \times \mathbb{R}_+ \to \mathbb{R}_+$. We then study the scenario where the receiver cannot observe whether the sender acquired information, i.e., covert information acquisition. In this case, a sender's strategy is an element of $\{I, U\} \times \mathbb{R}^3_+$ while a receiver's strategy is a mapping $\sigma_R : \mathbb{R}_+ \to \mathbb{R}_+$.

Let Γ be the normal form representation of either version of the sender-receiver game. Let A_1 be the set of pure strategies for the sender, and A_2 be the set of pure strategies for the receiver. A mixed strategy for Player i is a probability distribution over Player i's pure strategies, i.e., $\sigma_i \in \Delta A_i$.

A strategy profile $\sigma = (\sigma_1, \sigma_2) \in \Sigma := \Delta A_1 \times \Delta A_2$ is a Nash equilibrium if $u_i(\sigma_1, \sigma_2) = \max_{\sigma'_i} u_i(\sigma'_i, \sigma_{-i})$ for each $i \in \{1, 2\}$. A Nash equilibrium is admissible for the receiver if σ_2 is a weakly undominated strategy for the receiver. For brevity, we call a Nash equilibrium in which the receivers strategy is admissible an *equilibrium*.

3. Refinements

Games with signaling are notorious for the wide array of equilibria they admit and for the corresponding low predictive power. Our model(s) is no different, a variety of behavior can potentially be sustained as an equilibrium: the sender can acquire information, not acquire it, even undertake a strictly costly action after remaining uninformed. In an attempt to narrow down the players' behavior an assortment of refinements has been developed. Perhaps the most commonly used refinement in signaling games—the intuitive criterion of Cho and Kreps (1987)—reduces the set of equilibria in the model with two types to a single outcome. With more than two types stronger refinements, for example D1 (Cho and Sobel (1990)), are required for uniqueness. The two mentioned refinements are defined on signaling games, games where a privately informed sender takes an action that is followed by the receiver's action. The games analyzed in this paper are not signaling games and therefore the before-mentioned refinements do not apply directly, nor can they be easily extended to accommodate our environment.[11]

[11] The two mentioned refinements belong to a class of stability-based refinements, thus named due to their connection to strategic stability of Kohlberg and Mertens (1986). An alternative approach would be to explore the refinement proposed in Mailath et al. (1993), which too is defined for signaling games and would therefore have to be generalized for our game. This is beyond the scope of our paper.

Consider the model with covert information acquisition and an equilibrium in which the sender does not acquire information and undertakes the least costly action; this outcome can indeed be supported as an equilibrium by, say, passive beliefs. The above-mentioned refinements would start by characterizing the set of beliefs for which a type, say the high type, can profitably deviate to each action. However, the high type in the considered equilibrium does not exist, and neither does, for that matter, the low type. To contemplate the high type's deviations, the sender would first need to deviate to acquiring information. But to discern whether the sender has an incentive to do so one would need to determine how both the low and the high type would behave if the sender was informed. Looking at each type's deviations in isolation will not do. The above-mentioned refinements rely on the set of types being fixed, whereas it is endogenous in our game.

A refinement that does apply to our setting is the strategic stability of Kohlberg and Mertens (1986). Strictly speaking, strategic stability is defined for finite games, thus, it only applies to a discretized version of our game. The point is, however, moot as strategic stability is notoriously difficult to verify—it requires checking every sequence of particular trembles. We, instead, resort to a simplified refinement called never weak best response (NWBR); for a good primer see chapter 11 in Fudenberg and Tirole (1991) and Cho and Kreps (1987).[12] The building block of NWBR is a property of strategically stable sets: if one erases from a game some strategies that are never weak best response to any strategy in a stable set, the newly obtained game has a stable set that is contained in the stable set of the original game; Kohlberg and Mertens (1986). In addition, generically there exists a stable set such that the distribution over outcomes is unique. One can, therefore repeatedly apply NWBR to refine away equilibria. Starting with a set of equilibria with a unique outcome one erases (possibly iteratively) never weak best response strategies. If one arrives at a game where the starting outcome cannot be supported as an equilibrium, then what one started with can not be a stable set. NWBR has commonly been used, in various forms, to validate equilibrium refinements, the standard result being that the refinement does not eliminate anything NWBR would not eliminate itself. In fact, Cho and Sobel (1990) show that D1 is equivalent (in terms of outcomes) to a version of NWBR in monotonic signaling games.

The main reason for adopting the NWBR refinement (defined below) is the following. The model with information acquisition when information is free is the natural counterpart of the standard signaling game. To facilitate the comparison of our findings with

[12]The development of equilibrium refinements took a rather interesting path. The introduction of strategic stability (Kohlberg and Mertens (1986)) was followed by the development of simpler refinements that can be more readily applied to signaling games while guaranteeing that the outcome(s) they deliver is stable.

the existing results in signaling games we wanted a refinement that is general enough to apply to the game studied here while at same time replicating the results obtained under the stability-based refinements in signaling games.

The Refinement. Recall that Γ is the normal form representation of either version of the sender-receiver game. Each strategy profile leads to an outcome o which is a probability distribution over the terminal nodes of the game.

Fix another sender-receiver game in the normal form Γ', where the set of pure strategies for player i is $A_i' \subseteq A_i$, and fix an outcome o of an equilibrium. Let $\Sigma'(o, \Gamma')$ be the set of all equilibria of Γ' that lead to the outcome o.

Definition 1 *Γ'' is a pruning of (Γ', o) where o is an equilibrium outcome of Γ' if:*

1. *$A_1'' \subset A_1'$ and $A_2'' = A_2'$.*
2. *If $a_1' \in A_1'$, and if $a_1' \notin A_1''$, there exists no $\sigma' \in \Sigma'(o, \Gamma')$ such that a_1' is a weak best reply to σ_2'.*

Pruning of a game with respect to an outcome o erases a strategy of the sender (Player 1) only if that strategy is never a weak best response to any of receiver's (Player 2's) strategies in the set of all equilibrium strategy profiles. Pruning, as we define it, does not erase any strategy of the receiver. We chose this specification because the only strategies of the receiver that are never weak best responses in a set of equilibria that lead to a unique outcome are those that do not lead to that outcome. Hence, erasing such strategies would not lead to any change in the power of the NWBR test we define below. Despite not pruning any of receiver's strategies, pruning operates with respect to the set of all equilibria.

Definition 2 *An equilibrium outcome o of the game Γ fails the NWBR test if there exists a sequence $\{\Gamma^n\}_{n=1,2..,k}$ of games which satisfy:*

1. *Γ^1 is a pruning of (Γ, o).*
2. *Γ^n is a pruning of (Γ^{n-1}, o) for every $n = 2, 3, ..., k$.*
3. *o is not an equilibrium outcome in Γ^k.*

4. Observable Information Acquisition

We start with the benchmark model where the receiver observes whether the sender acquired information. For the receiver, any weakly undominated strategy σ_R satisfies

$\sigma_R(x, y) \in [\theta_L, \theta_H]$, and $\sigma_R(U, y) = \theta_u$ for $x \in \{U, I\}$ and $y \in \mathbb{R}_+$.[13]

The restriction that the receiver's strategy not be weakly dominated implies that outcomes where the sender does not acquire information and takes an action $s > 0$ cannot be supported as equilibria. The implications are even stronger in the environment with type-dependent signaling cost. One can conjecture information acquisition to be followed by various kinds of separation or semi-separation of the two types. However, all the continuations have a feature in common: the beliefs are a martingale on the equilibrium path and, therefore, the receiver's expected action is $E[\theta]$. By deviating to not acquiring information and choosing $s = 0$, the sender could induce the receiver to play $E[\theta]$, yet forgo the cost of information and possible costs of signaling. In consequence, if the receiver's payoff is weakly concave in the receiver's action and $c > 0$, Jensen's inequality implies that the sender is better off not acquiring information. By implication, the only equilibrium outcome that can be supported in equilibrium is the one where the sender does not acquire information and refrains from taking a strictly costly action ($s > 0$). A second equilibrium outcome arises when $c = 0$, the sender acquires information and both types pool on $s^* = 0$.[14] We summarize these findings in the proposition below, and skip a formal proof since it follows straightforwardly from the discussion above.

PROPOSITION 1 *In the environment with type-dependent cost of signaling and $\tilde{u}''(r) \leq 0$ for all r, the only equilibrium outcome that can be supported in equilibrium is the one where no information is acquired, the sender chooses $s = 0$ and the receiver $r = E[\theta]$. For $c = 0$, a second equilibrium outcome can be supported where information is acquired, but both types pool on $s^* = 0$.*

The above result delivers a unique equilibrium outcome in a game with signaling and sets a somewhat daunting tone.[15] In the environment with type-dependent signaling cost where the sender's benefit of the receiver's action is weakly concave—a commonly analyzed environment (e.g., see Spence (1973), Feltovich et al. (2002), Kurlat and Scheuer (2017)) —, if information acquisition is observable and the information costly, it will not be acquired. Even if the information is costless, it will not be conveyed. In such environments, the sender's private information can, therefore, not be justified through observable information acquisition. For consistency we maintain the environment with

[13] Nash equilibria allow the receiver to take actions outside of the set $[\theta_L, \theta_H]$ off path.

[14] These equilibria can be refined away with the refinement discussed in the preceding section. The continuation game after information acquisition corresponds to the standard signaling model where the refinements like the intuitive criterion and D1 yields the separating outcome as the only reasonable.

[15] The same result holds under more commonly used solution concept of Perfect Bayesian equilibrium, as defined in Fudenberg and Tirole (1991), where no signaling of what you do not know is interpreted to imply that the receiver's posterior is equal to the prior when the sender does not acquire information; on or off the equilibrium path.

two types through the most of the paper. The above result, however, relies only on the martingale property and the Jensen's inequality and therefore holds for any number of types.

In a more general single-crossing framework there can be a multiplicity of equilibria. To get sharper predictions, we apply the refinement introduced above. Let V_I be the sender's payoff in any equilibrium where the sender acquires information and chooses the least cost separating equilibrium:

$$V_I = \lambda u(s_H, \theta_H, \theta_H) + (1-\lambda)u(0, \theta_L, \theta_L),$$

where s_H is defined through the low type's indifference:

$$u(s_H, \theta_H, \theta_L) = u(0, \theta_L, \theta_L). \tag{1}$$

Let V_U be the sender's payoff from not acquiring information and choosing the least costly action and the receiver responding with $r = E[\theta]$: $V_U = E[u(0, E[\theta], \theta)]$.

PROPOSITION 2 *Generically a unique equilibrium outcome survives the refinement. If $V_I > V_U$, it is the outcome where the sender acquires information and the least cost separating equilibrium is played. If $V_U > V_I$, in the unique outcome surviving the refinement no information is acquired and the least costly signaling action taken.*

PROOF: The proof of this and all the subsequent results are in the Appendix. □

The refinement only ever selects the equilibrium outcome with information acquisition followed by the most efficient separating outcome or no information acquisition followed by the least costly action, depending on which of the two outcomes results in a larger payoff for the sender. More can be said about when the condition $V_I > V_U$ holds, and thus information is acquired, in the two commonly studied special cases. For the type-dependent signaling envrionment, we argued already that if the sender's benefit is concave, he will never acquire information in an equilibrium. The refinement delivers a stronger result. Note

$$V_U - V_I = (\tilde{u}(E[\theta]) - E[\tilde{u}(\theta)]) + (E[g(s(\theta), \theta)] - E[g(0, \theta)]) + c,$$

where $s(\theta_L) = 0$ and $s(\theta_H) = s_H$, and thus the term $E[g(s(\theta), \theta)] - E[g(0, \theta)]$ is positive. If, in addition, $\tilde{u}$ is weakly concave, the term in the first brackets is also non-negative and therefore $V_U > V_I$. On the other hand, for $V_I > V_U$ one needs that $\tilde{u}$ is "convex enough" so that the sender's love of risk compensates him both for the cost of information as well as the costs of signaling.[16]

Consider, instead, the environment with type-dependent benefits and assume that the benefits are multiplicatively separable: there exist functions v_1, v_2 and h such that $u(s, r, \theta) = v_1(r)v_2(\theta) - h(s)$ where v_1 and g are strictly increasing. Then

$$V_U - V_I = (E[v_1(E[\theta])v_2(\theta)] - E[v_1(\theta)v_2(\theta)]) + (E[h(s(\theta)] - h(0)) + c,$$

where the difference in the cost of signaling, $E[h(s(\theta)] - h(0)$, is non-negative. If v_2 is decreasing and v_1 weakly concave $V_U > V_I$.[17] On the other hand, if v_2 is increasing (the case more commonly studied in applications), $E[v_1(\theta)v_2(\theta)] > E[v_1(\theta)]E[v_2(\theta)]$. In this case, it is possible that $V_I > V_U$.

5. Covert Information Acquisition

Equilibria. The opportunity to covertly acquire information opens venues for behavior not present in signaling games. The sender has an opportunity to save on the cost of information and merely pretend that he is informed—to bluff, so to say. The question thus becomes when one can sustain information acquisition as an equilibrium.

We start by examining equilibria in which the sender refrains from acquiring information. Let $\bar{s}_u$ be such that the uninformed sender is indifferent between $(0, \theta_L)$ and $(\bar{s}_u, E[\theta])$:

(2) $$E[u(0, \theta_L, \theta)] = E[u(\bar{s}_u, E[\theta], \theta)],$$

and $\bar{s}^*$ such that the low type is indifferent between $(0, \theta_L)$ and $(\bar{s}^*, E[\theta])$:

(3) $$u(0, \theta_L, \theta_L) = u(\bar{s}^*, E[\theta], \theta_L).$$

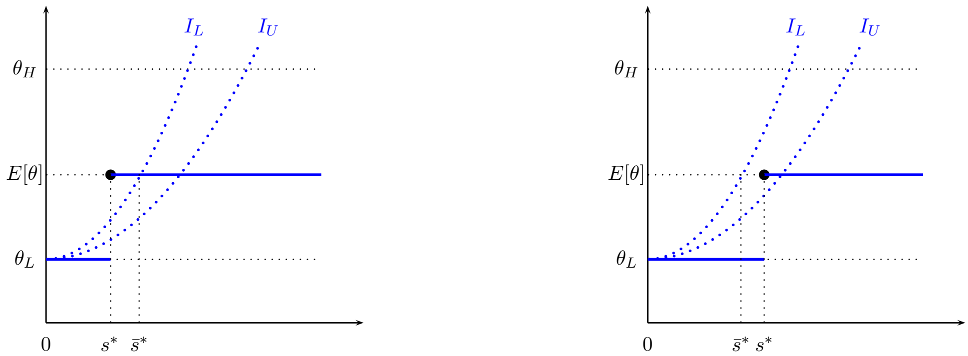
Assumptions imposed on u imply $\bar{s}^* < \bar{s}_u$.

Proposition 3 *The following statements are true:*

- *For every $s^* \leq \bar{s}^*$, not acquiring information followed by s^* can be supported in an equilibrium for every $c \geq 0$.*

[16] In particular, one needs $E[g(s(\theta), \theta)] - E[g(0, \theta)]) + c < \tilde{u}(\theta_H) - \tilde{u}(\theta_L)$.

[17] Since $v_1(x)$ and $v_2(x)$ are negatively correlated, $E[v_1(\theta)v_2(\theta)] \leq E[v_1(\theta)]E[v_2(\theta)] \leq v_1(E[\theta])E[v_2(\theta)]$.



(a) After deviating to information acquisition both types choose s^*.

(b) After a deviation the low type would choose $s = 0$.

Figure 1: Equilibria with signaling but no information. The solid lines are the receiver's equilibrium response, and the dotted lines are the indifference curves of the sender's types.

- *For every $s^* \in (\bar{s}^*, \bar{s}_u]$, there exists a $c_{s^*} > 0$ such that not acquiring information followed by s^* can be supported in an equilibrium if and only if $c \geq c_{s^*}$.*
- *Not acquiring information followed by an action $s^* > \bar{s}_u$ cannot be supported in equilibrium.*

The most straightforward way to support equilibria is to have the receiver respond to non-equilibrium actions s with $r = \theta_L$. The range of equilibrium signaling actions, s^*, can be split into three regions. For low s^*, equilibria where the sender does not acquire information followed by s^* can be supported for every level of cost c; see figure 1a. In this case, s^* would be optimal for both types after the deviation to acquiring information. The cost of information acquisition would, therefore, be wasted. For the intermediate values of s^*, after the deviation to acquiring information the high type sender would choose s^*, but the low type would strictly prefer $s = 0$; see Figure 1b. For the sender not to deviate the cost of acquiring information must be large enough. Finally, large s^* can never be supported as an equilibrium with no information acquisition. The sender would rather deviate to $s = 0$ without the need to acquire information.

Equilibria with information acquisition introduce several new challenges. When the cost of information acquisition is nil one can sustain equilibria with information acquisition followed by a large set of outcomes familiar from signaling games: pooling, separating and semi-separating. However, when the cost of acquiring information rises above 0 the threat of deviations to not acquiring information alters the set of equilibria.

LEMMA 1 *For $c > 0$, in any equilibrium where the sender acquires information with positive probability, each type $\theta \in \Theta$ of the sender must strictly prefer every equilibrium* *signaling action s they play with positive probability to every equilibrium action the other type plays with positive probability.*

If after acquiring information one of the two types, say θ_L, was indifferent between his own equilibrium action and some equilibrium action of the high type, the sender could deviate to not acquiring information and take the high type's action. Due to the low type's indifference, the sender would replicate the payoff in the low state. The sender would without acquiring information achieve the same utility state by state as if he acquired it but forgo the cost of information. The above lemma has the following implications. For $c > 0$:

- there are no equilibria where the sender acquires information with positive probability and the low and the high type pool with positive probability;
- information acquisition followed by the Riley outcome (the separating outcome where the low type chooses s=0, and the high type chooses $s = s_H$ such that the low type is indifferent) cannot be supported as an equilibrium.

The implications are particularly strong for equilibria where the sender acquires information with certainty. Since the two types separate, the low type can be identified and thus has no incentive to signal his type, $s_L = 0$. The high type's equilibrium behavior, on the other hand, must be curtailed in order to prevent the sender from deviating to not acquiring information and pretending to be one of the two types. The two constraints can be written as

$$u(s_H, \theta_H, \theta_H) - \frac{c}{\lambda} \geq u(0, \theta_L, \theta_H); \tag{4}$$

and

$$u(0, \theta_L, \theta_L) - \frac{c}{1-\lambda} \geq u(s_H, \theta_H, \theta_L). \tag{5}$$

If the sender were to deviate to not acquiring information and pretend to be, say, the high type, his behavior would differ from the prescribed only in the low state. In addition, he would save on the cost of information acquisition. Not to benefit from the said deviation, the low type must prefer his equilibrium action to the high type's sufficiently enough to outweigh the savings on the cost of information. The deviation constraints preventing the sender from deviating to not acquiring information are, therefore, stronger than the deviation constraints preventing the agent from simply misrepresenting the type after having acquired information.

For low costs of information, a wide array of actions after information acquisition can be

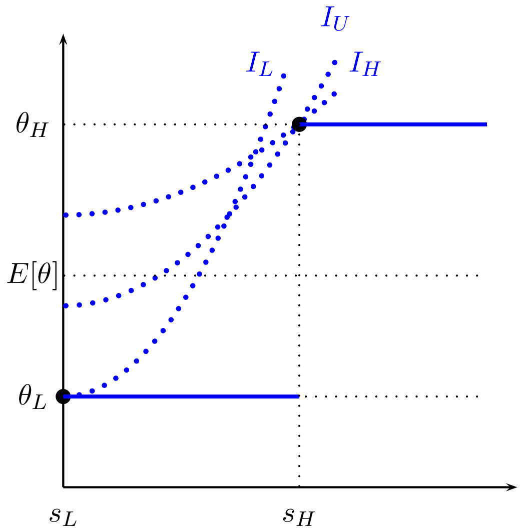


Figure 2: An equilibrium with information acquisition for $c > 0$. The high type burns enough surplus for the low type to strictly prefer his own option.

supported in equilibrium for the high type. Of particular interest is the most efficient such equilibrium—the one where the high type burns the least surplus. The lower bound on the high type's signaling action is imposed by the constraint preventing the sender from deviating to not acquiring information and choosing the high type's action; inequality (5). In the most efficient equilibrium with information acquisition, where the high type's action is denoted s_H^* the said incentive constraint is binding:

$$(6) \qquad u(0, \theta_L, \theta_L) - \frac{c}{1-\lambda} = u(s_H^*, \theta_H, \theta_L).$$

The equality uniquely pins down s_H^* for every c; and vice-versa. It is immediate to see that s_H^* is increasing in c. At $c = 0$ the Riley outcome obtains, but as c increases, so does s_H^*; one obtains a *generalized Riley outcome*, so to say. Reducing the high type's payoff reduces the deviation payoff more than the equilibrium payoff; the uninformed sender always chooses s_H, the informed only when he is the high type.

There is an upper bound on the cost of information, $\bar{c}$, for which equilibria with information acquisition can be sustained. The incentive constraints (4) and (5) represent the upper and the lower bound on the high type's action s_H, respectively.[18] The upper bound is decreasing and the lower increasing in the cost of information. The upper bound on the cost of the information under which information acquisition with certainty can be sustained as equilibrium, $\bar{c}$, is, therefore, reached when the two constraints bind simultaneously. The only value of s_H that can be sustained under $\bar{c}$ is denoted by $\bar{s}_H^*$. $\bar{s}_H^*$ is such

[18]For the related idea of using incentive constraints as bounds on transfers in mechanism design see Carbajal and Ely (2013), Kos and Messner (2013a) and Kos and Messner (2013b).

that the uninformed sender is indifferent between $(0, \theta_L)$ and $(\bar{s}_H^*, \theta_H)$; see Figure 3. The above result is summarized in the next proposition.

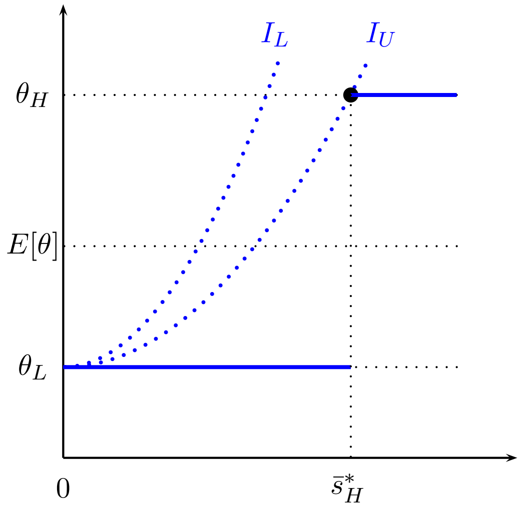


Figure 3: An equilibrium with information acquisition and the highest amount of signaling for the high type.

PROPOSITION 4 *Equilibria in which information is acquired with probability one exist for $c \leq \bar{c}$. In any such equilibrium $s_L = 0$. In the most efficient equilibrium with information acquisition the high type's action, s_H^*, is given by equality (6), and s_H^* is increasing in c.*

Finally, there is a myriad of equilibria in which the sender randomizes over information acquisition decisions. We omit the characterization of those, as they will not play a prominent role in the subsequent analysis.

Refining Equilibria. We split equilibrium outcomes into three groups—with information acquisition, without it, and with randomization over information acquisition—and study when the outcomes in each group survive the refinement. First, we take a closer look at outcomes without information acquisition. These are divided further into the ones followed by signaling ($s^* > 0$) and the ones without signaling ($s^* = 0$).

DEFINITION 3 *For a fixed set of equilibria, $\mathbb{T}$, we say that an action s is never weak best response for type i, $i \in \{\theta_L, \theta_H\}$, if there is no strategy where the sender acquires information and the type i chooses s that is a weak best response to some receiver's strategy in the set $\mathbb{T}$.*

Recall that $\bar{s}^*$ is such that the low type is indifferent between $(0, \theta_L)$ and $(\bar{s}^*, E[\theta])$;

see equation (3). Strategies where the low type chooses an action $s > \bar{s}^*$ and the receiver responds with $E[\theta]$ are never a weak best response.

LEMMA 2 *Outcomes with no information acquisition and $s^* > \bar{s}^*$ can be refined away for every cost of information, while each equilibrium outcome with no information acquisition and $s^* \in (0, \bar{s}^*]$ can be refined away for every cost of information bar one; denoted by $b(s^*)$. In particular, at $c = b(s^*)$, the outcome with no information acquisition and $s^* \leq \bar{s}^*$ survives the refinement.*

Consider an outcome in which the sender does not acquire information and chooses s^*. In any equilibrium with the outcome, the receiver responds to s^* with $E[\theta]$. Proposition 3 established limits to the amount of signaling s^* that follows no information acquisitions—if too much signaling was required, the uninformed sender is better off being considered to be the low type. The refinement restricts the signaling further. If the sender were to choose an $s^* > \bar{s}^*$ after not acquiring information, the actions between $\bar{s}^*$ and s^* would be never weak best response for the low type. In the game obtained after pruning all the strategies where the low type chooses an action in $(\bar{s}^*, s^*)$, the uninformed sender could profitably deviate to an action in the same interval.

On the other hand, equilibria where the sender does not acquire information and signals moderately, $s^* \leq \bar{s}^*$, cannot be refined away invariably. For each $s^* \leq \bar{s}^*$ such an equilibrium survives the refinement for precisely one value of the cost of information. Fix an equilibrium outcome with no information acquisition and $s^* \leq \bar{s}^*$ and let s_u be such that the uninformed sender is indifferent between $(s^*, E[\theta])$ and (s_u, θ_H):

$$(7) \qquad E[u(s^*, E[\theta], \theta)] = E[u(s_u, \theta_H, \theta)];$$

see Figure 4. Actions above s_u are NWBR neither for the low type nor the uninformed sender. One can, therefore, prune away all the strategies where the two play such actions. In the newly obtained game, termed *the refined game*,[19] only the high type can choose actions $s > s_u$. The receiver, therefore, responds to any such action with θ_H; the solid line in the figure. Single-crossing implies that the best option for the high type is s_u (to which he expects the reply θ_H). Furthermore, the sender must be indifferent between his equilibrium play and the deviation to acquiring information followed by s^* as the low type and s_u as the high type. If the deviation yielded a smaller payoff, any strategy with information acquisition would too and the actions slightly below s^* would never be a best response for the low type. Upon erasing the strategies where the low type plays the

[19]The refined game plays a prominent role in our analysis, thus warranting the special name. It should also be noted that it is a set of games, one for each s^*.

mentioned signaling actions, one would arrive at a game where the uninformed sender would find it profitable to reduce his signaling action. The indifference condition is:

$$E[u(s^*, E[\theta], \theta)] = \lambda u(s_u, \theta_H, \theta_H) + (1 - \lambda)u(s^*, E[\theta], \theta_L) - b(s^*), \tag{8}$$

where the left-hand side is the equilibrium payoff and the right-hand side the payoff from the deviation to (I, s^*, s_u). Since s_u is pinned down by s^* through equation (7), the indifference can obtain only for one cost of information.

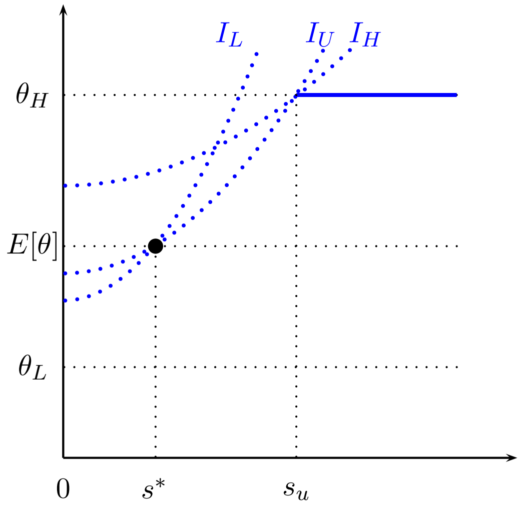


Figure 4: The equilibrium outcome where the sender does not acquire information and chooses s^*. Actions above s_u are NWBR for the low type or the uninformed sender.

Properties of $b(\cdot)$ will play a crucial role in the analysis to follow.

Lemma 3 *$\frac{db}{ds^*} > 0$ (< 0) if $\frac{u_s(s,r,\theta_H)}{u_s(s,r,\theta_L)}$ is decreasing (increasing) in s and r. The cost function b is constant in s^* when $\frac{u_s(s,r,\theta_H)}{u_s(s,r,\theta_L)}$ is constant in s and r.*

A more detailed characterization can be provided for the two additively separable case. In the case of type-dependent benefits, $\frac{u_s(s,r,\theta_H)}{u_s(s,r,\theta_L)} = 1$ and, therefore, $b(\cdot)$ is a constant function. On the other hand, in the environments with type-dependent signaling cost, b is increasing (decreasing) if $\frac{g_s(s,\theta_H)}{g_s(s,\theta_L)}$ is decreasing (increasing) in s. $b(\cdot)$ is constant if $\frac{g_s(s,\theta_H)}{g_s(s,\theta_L)}$ is constant in s. The latter, in turn, is the case if and only if g_s is multiplicatively separable.

Next, we turn attention to equilibria with information acquisition.

Lemma 4 *There exists a $c_I > 0$ such that no equilibrium outcome with information acquisition survives the refinement for $c > c_I$. For every $c \le c_I$ the only equilibrium*

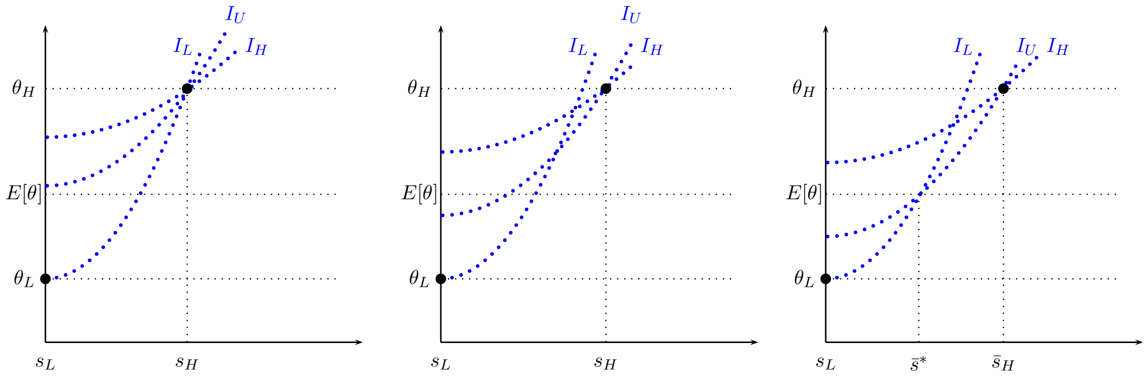


Figure 5: The most efficient separating equilibria with information acquisition.

outcome where information is acquired with probability one that survives the refinement is the most efficient equilibrium outcome with information acquisition (and separation of types).

Equilibria with information acquisition survive the refinement only when information is cheap. And when such an equilibrium survives, it is the most efficient one, that is, the one in which the sender is indifferent between acquiring and not acquiring information. When information is costless, the only information acquisition outcome that survives the refinement is the one in which the low type is indifferent between his own and the high type's action (the Riley outcome). As the cost of information increases, the equilibrium outcome with information acquisition that survives the refinement requires the high type to burn more and more surplus: s_H grows with c; see Figure 5. When information becomes costlier the receiver is more likely to question whether the sender truly acquired it. To validate his claim the sender must forgo more and more surplus. When cost becomes too large, the receiver ceases to believe that the sender acquired information all-together. The combined cost of acquiring information and signaling its acquisition are prohibitive.

The proof relies on the fact that pooling cannot occur in an equilibrium with information acquisition for $c > 0$, in fact, that in such equilibria each of the two types strictly prefers their own action; see Lemma 1. Furthermore, an equilibrium with information acquisition can be refined away unless the sender is indifferent between the equilibrium play and the deviation towards not acquiring information followed by pretending to be the high type. If the sender strictly preferred his equilibrium play, the actions just below the high type's action would be NWBR for the low type (by Lemma 1) as well as for the uninformed sender. After pruning away those NWBR strategies, the receiver should in any equilibrium respond to an action just below the high type's with $r = \theta_H$, providing a profitable deviation for the high type.

As c grows, the high type is forced to undertake more and more signaling, that is, to choose a higher s_H. Equilibria with too high an s_H, however, can be refined away. In particular, if one draws the uninformed's indifference curve through the high type's action s_H and the low type's indifference curve through $(0, \theta_L)$, the vertical intercept of the two indifference curves must be at least $E[\theta]$. Should it drop below, the actions just above the intercept are NWBR for the low type; see Figure 6. After pruning the strategies where the low type plays these actions, one obtains a game in which the original outcome cannot be supported as an equilibrium. The final part of the proof painstakingly verifies that the equilibria with information acquisition and indifference cannot be refined away for $c \leq c_I$.

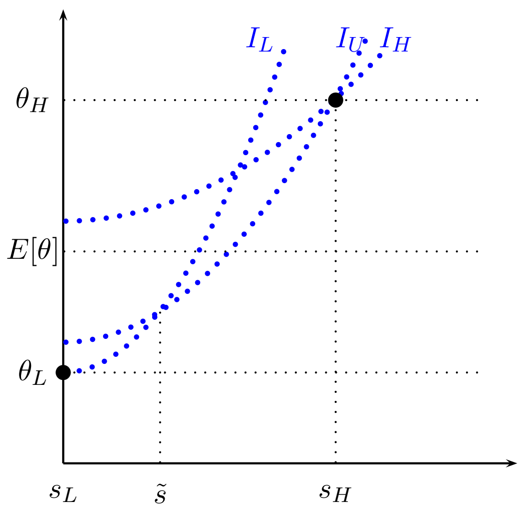


Figure 6: Too much signaling.

Remark 1 The following equality holds: $c_I = b(\bar{s}^*)$. The highest cost at which an equilibrium with information acquisition survives the refinement, c_I, coincides with the cost at which the equilibrium outcome without information acquisition and the largest amount of signaling, $b(\bar{s}^*)$, survives.

The only equilibria remaining to be studied are the equilibria where the sender randomizes over information acquisition decisions.

Lemma 5 *Equilibrium outcomes with randomization over information acquisition decisions survive the refinement at a single cost of information, denoted c_R. All such surviving equilibrium outcomes are separating and with the property that the low type is indifferent between his own and the uninformed sender's equilibrium action, and the uninformed* *sender between his own and the high type's equilibrium action.*

Equilibrium outcomes with randomization over information acquisition are much like a signaling game with three types with the added requirement that the sender is indifferent between acquiring and not acquiring information. In signaling games, stronger stability-based refinements (for example D1) are known to select the Riley outcome as the unique outcome. Similar result obtains here: the only outcomes with randomization that survive the refinement are the ones where the low type takes the least costly action and is indifferent between his own action and the uninformed agent's action while the uninformed agent is indifferent between his own and the high type's action. This pins down the signaling action and implies that the indifference between acquiring and not acquiring information can be achieved only at one cost of information. The probability with which information is acquired is, however, not pinned down.

Remark 2 The following equality holds: $c_R = c_I = b(\bar{s}^*)$.

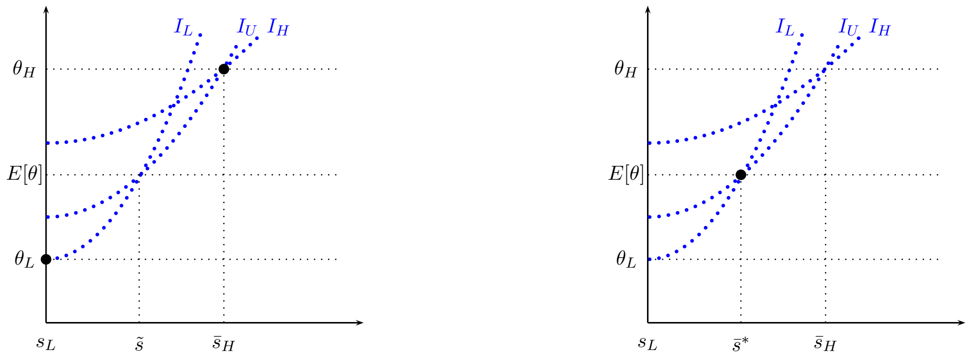
If $b(\cdot)$ is constant we write $b(s^*) = b$ for all $s^* \geq 0$ and consequently, there exists a c^* such that $c_R = c_I = b = c^*$.

Proposition 5 *If $b(\cdot)$ is constant, there exists $c^* > 0$ such that a unique equilibrium outcome survives the refinement for every $c \neq c^*$. For $c < c^*$ the only outcome surviving the refinement is the most efficient equilibrium outcome with information acquisition. For $c > c^*$, only the outcome where the sender does not acquire information nor engages in signaling survives.*

When $b(\cdot)$ is constant, a single equilibrium outcome survives the refinement at every cost of information bar one. When the information is cheap, the sender acquires information, when it is expensive, he does not.

That the lower bound on equilibria without information acquisition, c_N, coincides with the upper bound on equilibria with information acquisition, c_I, is most readily demonstrated graphically; see Figure 7. The cost c_I is determined by the equilibrium in which the low type's indifference curve and the uninformed sender's indifference curve intersect at a point with the vertical component $E[\theta]$.[20] Similarly, the cost of information c_N can be computed through indifference in equilibrium where the sender does not acquire information and chooses the amount of signaling $\bar{s}^*$ such that the low type would be

[20]The cost of information c_I is such that the sender is indifferent between strategies $(I, 0, \bar{s}_H)$ and (U, s_H).



(a) c_I (b) c_N

Figure 7: c_I and c_N coincide.

indifferent between $(0, \theta_L)$ and $(\bar{s}^*, E[\theta])$. In the computation of both costs, the high type chooses the same point. The choices of the uninformed sender and the low type are not the same, but they do lie on the same indifference curve. The case of c_R is analogous.

Corollary 1 *If $u(s, r, \theta)$ can be written as $\hat{u}(r, \theta) - g_1(s)g_2(\theta)$ for some strictly increasing g_1 and non-increasing g_2, there is a unique equilibrium outcome surviving the refinement for every cost of information except one.*

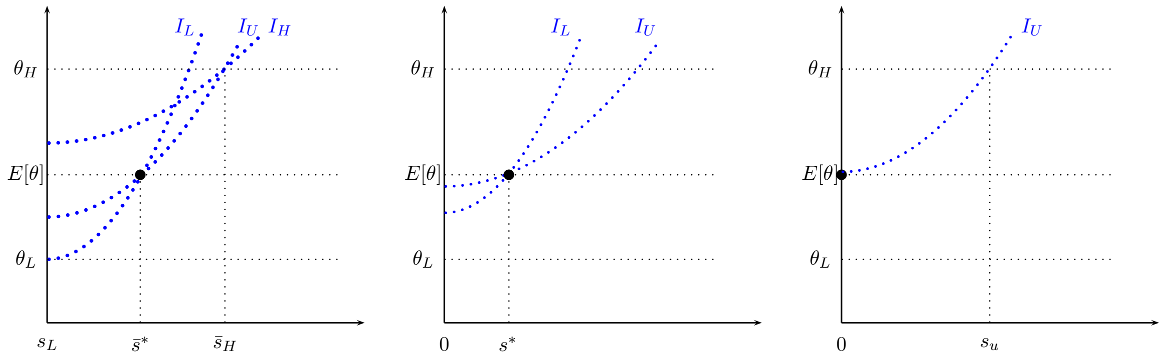
The corollary covers as special cases both the type-dependent benefits environment as well as the type-dependent signaling cost environment if the cost of signaling is mulitplicatively separable.

In what follows we explore the cases where b is decreasing and where it is increasing.

Proposition 6 *If $b(\cdot)$ is decreasing, generically a unique equilibrium survives the refinement:*

- *For $c < b(\bar{s}^*)$ it is the most efficient equilibrium outcome with information acquisition.*
- *For $c \in (b(\bar{s}^*), b(0))$, it is the equilibrium outcome in which the sender does not acquire information and chooses $b^{-1}(c)$.*
- *For $c > b(0)$, it is the equilibrium outcome in which no information is acquired and the sender chooses the least costly action.*

When b is decreasing generically a unique equilibrium outcome survives the refinement. When the information is cheap only the most efficient equilibrium outcomes with information acquisition survive. The familiar pattern arises: as the cost of information



(a) $c = b^{-1}(\bar{s}^*)$ (b) $c \in (b^{-1}(\bar{s}^*), b^{-1}(0))$ (c) $c = b^{-1}(0)$

Figure 8: The surviving equilibria for $c \in [b^{-1}(\bar{s}^*), b^{-1}(0)]$ when $b(\cdot)$ is decreasing.

increases, the high type burns more and more surplus to assure the receiver that he inquired information. This persists up to the cost c_I at which the uninformed's indifference curve through the high type's option and the low type's indifference curve through $(0, \theta_L)$ intersect at a point with the vertical component $E[\theta]$. At the same cost, an equilibrium without information acquisition and $s = \bar{s}^*$ survives too.

The most interesting behavior is exhibited when the cost of information exceeds c_I. The only equilibrium outcome that survives the refinement is the one in which the sender does not acquire information and chooses $s = b^{-1}(c)$. Strikingly, the uninformed sender undertakes costly signaling in order to convince the receiver that he is indeed uninformed rather than an informed sender with a low type. Since b is decreasing, the uninformed sender engages in less and less signaling as the cost of information increases. When the cost of information increases above $b(0)$, in the only equilibrium outcome that survives the refinement, the sender does not acquire information and chooses the least costly action.

Proposition 7 *If $b(\cdot)$ is increasing:*

- *A unique equilibrium outcome survives the refinement for $c < b(0)$: the most efficient equilibrium with information acquisition.*
- *A unique equilibrium outcome survives the refinement for $c > b(\bar{s}^*)$: the one in which no information is acquired and the sender chooses the least costly action, 0.*
- *For $c \in [b(0), b(\bar{s}^*)]$ multiple equilibrium outcomes survive NWBR. These outcomes include the most efficient equilibrium outcome with information acquisition, and the equilibrium outcome with no information acquisition and $s = b^{-1}(c)$.*

When b is increasing, there is a multiplicity of equilibrium outcomes that survive the refinement in the intermediate region of cost $c \in (b(\bar{s}^*), b(0))$. Equilibrium outcomes that survive are the most efficient equilibrium outcome with information acquisition, no information acquisition equilibrium outcome with $s^* = 0$ and no information acquisition equilibrium outcome with $b^{-1}(c)$. Despite the multiplicity in the intermediate region of cost, for low costs of information ($c < \min\{b(\bar{s}^*), b(0)\}$) a unique equilibrium outcome survives the refinement—the most efficient equilibrium outcome with information acquisition.

Above we only considered the cases where b is monotonic. More general statements can be made. First, if b is at any point increasing, there will be a multiplicity of equilibria that survive the refinement in the increasing region. Given that b is continuous, this implies that b non-increasing is also necessary for the generic uniqueness of equilibria; within the realm of single-crossing signaling cost functions g. Second, even if b is non-monotonic only the most efficient outcome with information acquisition survives the refinement for low enough c. Since b is continuous on the interval $[0, \bar{s}^*]$, it attains a minimum. Moreover, it is easy to verify $b(s^*) > 0$ for all $s^* \in [0, \bar{s}^*]$, thus the minimum on the interval is strictly above 0. By implication, unique equilibrium survives the refinement for $c < \min\{b(s^*) : s^* \in [0, \bar{s}^*]\}$.

Proposition 8 *For any $c < \min\{b(s^*) : s^* \in [0, \bar{s}^*]\}$, only the most efficient equilibrium outcome with information acquisition survives the refinement.*

The implications of the above proposition are the strongest in the environment with type-dependent signaling cost. When information acquisition is observable, in the type-dependent signaling cost environment with (weakly) concave benefits the sender never acquires information in an equilibrium that survives the refinement. In contrast, when information acquisition is covert, the sender does acquire information when it is cheap. The sender's private information in such environments can, therefore, be rationalized by covert information acquisition.

6. Multiple states

In this section we verify that our main results extend to the environment with multiple states. Suppose that there are n states of the world $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$, with $\theta_1 < \theta_2 < ... < \theta_n$ and the prior probability of state θ_i is $\lambda_i \in (0, 1)$. We establish two results. First, we show that the equilibrium outcome with no information acquisition and no signaling can be refined away when information is cheap. Second, we argue that an outcome with information acquisition cannot be refined away for sufficiently small costs of information.

Lemma 6 *There exists a $\gamma > 0$ such that the equilibrium outcome with no information acquisition and no signaling can be refined away for all $c < \gamma$.*

No information acquisition and no signaling outcome can be refined away for small costs of information. The proof establishes that the result holds for very low cost, rather than characterizing all such costs. The proof focuses on possible deviations of the highest type. This might lead one to believe that if one were to make types probabilisticaly smaller (moving to the continuum of types) γ in the above proposition would converge to 0, but this is not the case. The proof focuses only on the highest type to avoid unnecessary notation and analysis of cases. Consider an environment with a continuum of types $[\underline{\theta}, \overline{\theta}]$ and suppose $c = 0$.[21] Let s_u be such that the uninformed type is indifferent between $(0, E[\theta])$ and $(s_u, \overline{\theta})$ and let $\overline{r}$ be such that type $\overline{\theta}$ is indifferent between $(0, E[\theta])$ and $(s_u, \overline{r})$. By the single-crossing assumption $\overline{r} < \overline{\theta}$. Now, in any equilibrium where information is not acquired and $s = 0$ chosen, the receiver's response to s_u cannot be above $\overline{r}$. If it was the sender could profitably deviate to acquiring information and have all the types who prefer it choose s_u (there is an open set of such types since $\overline{\theta}$ would strictly prefer it). But then s_u is NWBR for any type $\theta < \overline{\theta}$. In the game obtained after pruning the strategies where types $\theta < \overline{\theta} - \epsilon$, for $\epsilon > 0$ small enough, play s_u, the sender has a profitable deviation to acquiring information and types in the ϵ neighborhood of $\overline{\theta}$ playing s_u. Continuity permits one to expand the argument to small but strictly positive costs of information.

Unlike in the two type case, when there are more than two types, information acquisition followed by the Riley outcome can be sustained as an equilibrium, provided the information is cheap.

Lemma 7 *Let $n > 2$. There exists a $\gamma_R > 0$ such that information acquisition followed by the Riley outcome can be sustained as an equilibrium for every $c < \gamma_R$.*

With two types, information acquisition followed by the Riley outcome can not be implemented. The sender can deviate to not acquiring information and choosing the high type's option. This enables him to replicate the same payoff as with information acquisition state by state without paying for information; the low type is indifferent between his own option and the high type's. When there are more than two types, in the Riley outcome each type is indifferent among at most two options. Therefore, after deviating to not acquiring information the sender can replicate the payoff of information

[21] We do not present the result for continuum of types formally, as it would require technically tedious definition of the refinement, without informing or adding substance.

acquisition in at most two states. In the remaining states, the sender is strictly better off acquiring information, which is the source of the value of information.

LEMMA 8 *Let $n > 2$. There exists a $\hat{c} > 0$ such that information acquisition followed by the Riley outcome cannot be refined away for $c \leq \hat{c}$.*

7. Concluding Remarks

We study information acquisition in signaling environments where the information acquisition is observable as well as where it is covert. We show that in the environment with two states a version of NWBR refinement always yields a unique equilibrium outcome when the information acquisition is observable. It is either information acquisition followed by the Riley outcome or no information acquisition followed by the least costly signaling action, depending on which of the two results in a higher payoff for the sender. When information acquisition is not observable, single-crossing assumption is not enough to yield a unique outcome surviving the refinement at every cost of information. Nevertheless, when information is cheap, the equilibrium outcome surviving the refinement is unique and information is acquired. The most striking comparison between the observable and covert information acquisition is in the environment with state-dependent signaling cost where the sender's benefit of the receiver's action is concave. There, the sender never acquires information if information acquisition is observable, but acquires it when the acquisition is covert and the information cheap; the latter result being reminiscent of Grossman and Stiglitz (1980). Of additional note is that in some cases the only outcome surviving the refinement is the one in which the sender does not acquire information yet undertakes a strictly costly action, signaling to the receiver that he is not the informed sender who learned that he is of the low type.

We study an environment where the sender and the receiver have opposing preferences over the receiver's action. One could alternatively consider circumstances in which the sender's preferred receiver's action depends on the state of the world and parametrize their disagreement. The conjecture is that as the two player's preferences become more aligned, the sender's incentive to acquire information grows.

While signaling games have been extensively studied, a comprehensive study of signaling in more general games is by and large an uncharted territory with few exceptions; for a recent take see In and Wright (2017). In separate work, we intend to study a game in which the sender undertakes an investment in his ability (productivity) with a stochastic outcome, and then undertakes a signaling action. The receiver observes the signaling action, but not the investment.

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A. Appendix

Notation: In the sender-receiver games that we study, a pure strategy of the sender specifies the sender's information acquisition action (I for acquiring information, U for not acquiring information), and a signaling action for each of the three types, θ_H, θ_L and θ_u. However, for purposes of applying the NWBR refinement, specifying the signaling action for a type that does not exist on the path generated by the information acquisition action of a strategy is redundant. Hence, we refer to a pure strategy in which the sender that acquires information, and chooses s if low type, and s' if high type as simply (I, s, s'), and a pure strategy of the sender that does not acquire information, and chooses s as simply (U, s).

Proof of Proposition 2: We split the proof into two cases. **Case 1:** $V_I > V_U$**.** Any equilibrium outcome other than the sender acquiring information followed by the Riley outcome does not survive the refinement.

To establish the statement we need to rule out three types of equilibrium outcomes: the other outcomes with information acquisition, the outcomes where no information is acquired and the outcomes where the sender randomizes over information acquisition. If the sender acquires information and the two types with positive probability choose the same $\tilde{s}$, then the receiver's equilibrium response $r(\tilde{s}) < \theta_H$. Define $\tilde{s}_H$ by $u(\tilde{s}_H, \theta_H, \theta_L) = u(\tilde{s}, r(\tilde{s}), \theta_L)$ to be such s that the low type is indifferent between $(\tilde{s}, r(\tilde{s}))$ and $(\tilde{s}_H, \theta_H)$. All the strategies where the low type chooses actions above $\tilde{s}_H$ are NWBR and can, therefore, be pruned. In the resulting game, type θ_H strictly prefers $\tilde{s}_H + \epsilon$ to $\tilde{s}$, for ϵ small enough, due to the single-crossing condition. But then, in the pruned game the original outcome can not be supported as an equilibrium. On the other hand, if one were to try to support an outcome where information is acquired and the two types do not choose the same action with positive probability, the low type would choose $s = 0$ with the receiver's response $r(0) = \theta_L$. Any outcome where the type θ_H would choose an action $s > s_H$, where s_H is given by (1) can be eliminated because strategies where the type θ_H chooses an action $s > s_H$ are NWBR.

The only equilibria with no information acquisition are the ones where the sender chooses $s = 0$, as the receiver responds to any s with no information acquisition by $r = E[\theta]$. Fix the outcome where the sender does not acquire information and chooses $s = 0$ and consider the set of all the equilibria with this outcome. The actions above s_H are still NWBR for θ_L and can thus be pruned. In the newly obtained game one can have the sender acquire information and the low type choose $s = 0$ and the high type $s_H + \epsilon$. Now for every $\delta > 0$ there exists an $\epsilon > 0$ so that the sender's payoff from prescribed strategy is at least $V_I - \delta$. Since we are in the case $V_I > V_U$ by choosing δ low enough we end in a game where the no information acquisition outcome cannot be supported as an equilibrium anymore. The outcomes where the sender randomizes over information acquisition are refined away in a similar fashion.

As for **Case 2:** $V_I < V_U$**.** Any equilibrium outcome other than the sender not acquiring information followed by $s = 0$ and the receiver responding with $r = E[\theta]$ does not survive the refinement.

Any equilibrium outcome where the sender acquires information and the two types choose the same signaling action with positive probability can be refined away due to the single-crossing property the same way as in Case 1. Hence, the remaining possibility is that an outcome with information acquisition is followed by a separation of the two types. However, even the least costly separating equilibrium payoff to the sender is V_I, and $V_I < V_U$, so the sender has a profitable deviation to not acquiring information and choosing $s = 0$.

Verification. Consider Case 1. The sender's payoff from not acquiring information is $V_U < V_I$, hence not acquiring information is never a profitable deviation for the sender in any reduced game obtained through pruning. Actions $s > s^*$ will also never be a part of a profitable deviation. As for $s \in (0, s^*)$. In the first round, s is a best response for θ_L—for example, due to the equilibrium where the receiver's response coincides with the low type's indifference curve through $(0, \theta_L)$ and (s^*, θ_H)—but not for the high type (due to the single-crossing). But then, the action can be pruned for θ_L only after it was already pruned for θ_H and can therefore not be a part of a profitable deviation after any finite sequence of pruning.

In Case 2, when the sender does not acquire information, the receiver always responds with $r = E[\theta]$. Thus, after no sequence of pruning is there a profitable deviation for the sender where he does not acquire information. To rule out deviations to acquiring information after some rounds of pruning, let s^* be such that the sender's payoff from acquiring information and signaling s^* for each type and receiving θ_H is equal to V_U: $E[u(s^*, \theta_H, \theta)] - c = V_U$. Note that because $V_U > V_I$, $s^* < s_H$; at the end we consider the case where such an s^* does not exist. Observe that in any round of pruning, strategies where the sender acquires information and both types choose the same signal $s \leq s^*$ are never pruned. This is because, for every such s, there is an equilbrium where $\sigma_2(I, s') = \theta_L$ for every $s' \neq s$ and $\sigma_2(I, s)$ is such that the sender is indifferent between not acquiring information and acquiring information and choosing s. Hence, no action $s \in (0, s^*)$ can be pruned for all the strategies of some type and, therefore, there is no sequence of pruning where the sender has a profitable deviation to acquiring information in some stage of the pruning procedure. Finally, when $E[u(0, \theta_H, \theta)] - c < V_U$, s^*, as defined above, does not exist. However, in this case deviating to acquiring information is not profitable for the sender even if the receiver always responds with θ_H. □

Proof of Proposition 3: Action $\bar{s}_u$ as defined by (2) exists due to the assumption that $u(s, r, \theta)$ is continuous in s and goes to minus infinity with s. Not acquiring information followed by $s^* > \bar{s}_u$ cannot be supported as an equilibrium. The worst response the sender can expect after not acquiring information and $s = 0$ is $r = \theta_L$, which results in a strictly higher payoff.

As for $s^* \leq \bar{s}_u$, the easiest way to support not acquiring information followed by such an s^* is having the receiver respond to any non-equilibrium signaling action s by

$r = \theta_L$. The sender, then, does not have an incentive to deviate after not having acquired information. The question remains whether he can find it profitable to deviate to acquiring information.

If the sender were to deviate from not acquiring information and $s^* \leq \bar{s}_u$ (with the reply $r = \theta_L$ for out of equilibrium signaling actions s'), to acquiring information, the high type would choose s^* due to the single-crossing assumption. The low type, on the other hand, would choose s^* if $s^* \leq \bar{s}^*$ and 0 if $s^* \in (\bar{s}^*, \bar{s}_u]$. Consequently, for any $s^* \leq \bar{s}^*$ the sender would after deviating to information acquisition choose the same signaling action he is to choose when he does not acquire information regardless of his type, but incur a cost. He, therefore, has no incentive to deviate for any $c \geq 0$.

If one were to support no information acquisition followed by $s^* \in (\bar{s}^*, \bar{s}_u]$ as an equilibrium outcome, after a deviation to information acquisition the low type sender would strictly prefer $s = 0$—see figure 1b—thereby creating value for information acquisition. Let the threshold c_{s^*} be defined by the indifference condition:

$$Eu(s^*, E[\theta], \theta) = \lambda u(s^*, E[\theta], \theta_H) + (1 - \lambda)u(0, \theta_L, \theta_L) - c_{s^*},$$

where the left hand-side is the equilibrium payoff from not acquiring information and choosing s^* and the right hand-side from acquiring information, low type choosing $s = 0$ and the high type s^*. □

Proof of Lemma 1: On the way to a contradiction, assume that there is an equilibrium where the sender acquires information, there are $s_1, s_2 \geq 0$ such that type θ_L chooses s_1 and type θ_H chooses s_2 with positive probability, the receiver responds to s_1 with r_1, s_2 with r_2, and type θ_L is indifferent between s_1 and s_2:

$$u(s_1, r_1, \theta_L) = u(s_2, r_2, \theta_L) =: u_1.$$

Let $u_2 := u(s_2, r_2, \theta_H)$. Then, the sender's equilibrium payoff is $(1 - \lambda)u_1 + \lambda u_2 - c$. However, the strategy (U, s_2) gives the sender payoff $(1 - \lambda)u_1 + \lambda u_2$, leading to a contradiction. The other case in which type θ_H is indifferent between s_1 and s_2 leads to a contradiction analogously. □

Proof of Proposition 4: Recall the definition of s^*_H given by equality (6). If $c \leq \bar{c}$, the indifference curve of the uninformed type that passes through the point (s^*_H, θ_H) crosses the y-axis (weakly) above θ_L.

Consider the strategy profile in which the sender chooses $(I, 0, s^*_H)$, and the receiver chooses $r(s_H^*) = \theta_H$, $r(s) = \theta_L$ for every $s \neq s_H^*$. Consider the off-path beliefs for the receiver that attach probability 1 to the sender's type θ_L. Then, these strategy profiles form an equilibrium. The sender is indifferent between his equilibrium strategy and the strategy (U, s_H^*), because s_H^* satisfies equality (6). If the sender does not acquire information, then his most preferred action is s_H^* because the indifference curve of the uninformed type that passes through the point (s_H^*, θ_H) crosses the horizontal axis above θ_L. Hence, the sender does not have a profitable deviation that consists of not acquiring information. After acquiring information, the sender again does not have a profitable deviation, because the indifference curve of θ_H that passes through the point (s_H^*, θ_H), and the indifference curve of θ_L that passes through the point $(0, \theta_L)$ are both above the receiver's schedule.

In any equilibrium with information acquisition, $s_L = 0$. Indeed, by Lemma 1, any equilibrium with information acquisition is fully separating, implying that type θ_L's equilibrium action(s) is responded with θ_L. Due to the admissibility property of equilibria we consider, the receiver's response is never below θ_L. This implies $s_L = 0$, otherwise type θ_L could profitably deviate to $s = 0$ as he is guaranteed that the receiver responds with at least θ_L.

Finally, the most efficient separating equilibrium has s_H as small as possible while satisfying the two incentive constraints that type θ_L does not want to choose action s_H instead of action 0, and that the sender does not want to deviate to not acquiring information and choosing action s_H. The smallest s_H that satisfies both of these constraints is given by equation (6). Because $u(\cdot, \cdot, \theta)$ is decreasing in the first and increasing in the second argument, using (6) the implicit function theorem implies that s_H^* is increasing in the cost c. □

Proof of Lemma 2: Consider an equilibrium outcome where the sender does not acquire information and undertakes an $s^* > \bar{s}^*$. We claim that actions between $\bar{s}^*$ and s^* are never week best response for the low type. For the prescribed outcome to be an equilibrium outcome it has to be the case that the receiver's response $r(s)$ is never above the uninformed's indifference curve through $(s^*, E[\theta])$. On the interval $(\bar{s}^*, s^*)$ the low type's indifference curve through $(0, \theta_L)$ is strictly above the uninformed sender's indifference curve through $(s^*, E[\theta])$. This is due to $\bar{s}^*$ being defined so that the low type is indifferent between $(0, \theta_L)$ and $(\bar{s}^*, E[\theta])$. Therefore, any sender's strategy where he acquires information and the low type plays $s \in (\bar{s}^*, s^*)$ with positive probability is dominated by the same strategy where the low type chooses 0 instead of s. Having established that every $s \in (\bar{s}^*, s^*)$ is NWBR for the low type, one can prune away all the strategies in which the low type plays these actions. Since in the newly obtained game, actions $s \in (\bar{s}^*, s^*)$ can only be chosen by the uninformed agent or the high type, the receiver best responds with an $r \in [E[\theta], \theta_H]$. But then any action in $(\bar{s}^*, s^*)$ represents a profitable deviation for the uninformed sender.

Next we show that any outcome where the sender does not acquire information and chooses an $s^* \in (0, \bar{s}^*]$ can be refined away for all but one cost of information. Fix an equilibrium outcome with no information acquisition and $s^* \in (0, \bar{s}^*]$. Let s_u be the intersection of the uninformed agent's indifference curve through $(s^*, E[\theta])$ with the ray θ_H:

$$E[u(s^*, E[\theta], \theta)] = E[u(s_u, \theta_H, \theta)].$$

Actions $s' > s_u$ are never weak best response for the uninformed sender, as well as for the low type. Any strategy where the sender acquires information and the low type chooses $s \geq s_u$ is dominated by the same strategy altered so that the low type chooses s^*.

In the newly obtained game, the best option for the high type is (s_u, θ_H); formally, the high type can approach this payoff. This follows from the observation that $(s^*, E[\theta])$ and (s_u, θ_H) are connected by the uninformed's indifference curve and that the high type's indifference curve is flatter.

Next, we argue that if the equilibrium outcome is not to be refined away, the sender must be indifferent between his equilibrium play and deviation towards acquiring information followed by choosing s^* when the low type and s_u (to which the receiver responds with θ_H) when the high type. Suppose not, if the payoff with acquiring information was larger, then it would constitute a profitable deviation in the above-derived game. Strictly speaking $(I, s^*, s_u + \epsilon)$ would be a profitable deviation for an ϵ small enough. On the other hand, if the payoff with information acquisition is strictly lower, there exists an $\epsilon > 0$ such that action in $(s^* - \epsilon, s^*)$ are NWBR for the low type; the high type always optimally chooses s_u to which the receiver responds with θ_H. Indeed, since in every equilibrium with the outcome, the receiver's response cannot result in a pair above the uninformed's indifference curve through $(s^*, E[\theta])$, by moving slightly below s^* when the low type, the sender cannot increase his payoff discontinuously. One can, therefore, prune away all the strategies where the low type plays an action in $(s^* - \epsilon, s^*)$. In thus obtained game the receiver will respond to any $s \in (s^* - \epsilon, s^*)$ with an $r \in [E[\theta], \theta_H]$. But then not acquiring information and choosing an action in $s \in (s^* - \epsilon, s^*)$ is a profitable deviation for the sender.

We established that if an equilibrium outcome without information acquisition and $s^* \in (0, \bar{s}^*)$ is to survive the refinement the sender ought to be indifferent between not acquiring information followed by s^* and acquiring information followed by s^* if low type and s_u if high type:

$$E[u(s^*, E[\theta], \theta)] = \lambda u(s_u, \theta_H, \theta_H) + (1-\lambda)u(s^*, E[\theta], \theta_L) - c.$$

The cost that makes the sender indifferent for a given s^*, denoted $b(s^*)$, is

$$b(s^*) = \lambda u(s_u, \theta_H, \theta_H) + (1-\lambda)u(s^*, E[\theta], \theta_L) - E[u(s^*, E[\theta], \theta)].$$

Verification. The final step of the proof is to verify that any outcome where the sender does not acquire information and chooses $s^* \leq \bar{s}^*$, outcome o, survives the refinement when the cost of information is $b(s^*)$. On the way to a contradiction, suppose there is a sequence of pruning with respect to the outcome o such that in the stages $i = 1, ..., k-1$, o is an equilibrium outcome in Γ^i, but not in Γ^k.

We start with some observations: First, the set of equilibrium receiver strategies that lead to outcome o in the games $\{\Gamma^i\}_{i=1,...,k-1}$ are weakly below the indifference curve of the uninformed type that passes through $(s^*, E(\theta))$ (called IC_U), and weakly below the ray θ_H. These are necessary conditions for the optimality of the strategy (U, s^*) for the sender, and an implication of the equilibrium condition requiring that the receiver's strategy is not weakly dominated. Second, we claim that the strategy in which the sender acquires information, chooses s^* as the low and s_u as the high type, for short (I, s^*, s_u), cannot be pruned away in any step. The sender is indifferent between (U, s^*), and (I, s^*, s_u) when $r(s^*) = E(\theta)$, and $r(s_u) = \theta_H$ (by the definition of $b(s^*)$.). Therefore, the strategy in question could be pruned away only if the receiver could not assign the belief to θ_H after observing s_U, or equivalently, if all the strategies where the high type plays s_u have been pruned already, with the possible exception of (I, s_u, s_u). That is, for (I, s^*, s_u) to be NWBR it would have had to been pruned already.

Next, we argue that no s can be a part of a profitable deviation from the equilibrium outcome after any finite sequence of pruning. Two cases are to be considered: deviations in $[0, s^*)$ and deviations in (s^*, s_u). Having shown that (I, s^*, s_u) can never be erased, any deviation including some action $s' > s_u$ is dominated by a strategy where s' is replaced by s_u.

Case 1: $s \in (s^*, s_u)$. If for some $s \in (s^*, s_u)$ the strategy (U, s) is pruned away in some stage $l < k$, then it must be the case that in Γ_l the receiver does not assign positive probability to θ_H after observing s, i.e., the strategies (I, s', s) for every s', except for possibly $s' = s$, have been pruned at some earlier stage $l' < l$. To see this, suppose by a way of contradiction that there exists some $s' \neq s$ such that (I, s', s) is present in

Γ^l. Then, the receiver can attach a positive probability to s being played by the high type and the set of all equilibria that leads to outcome o contains an equilibrium with $r(s) = \tilde{r}$, where $(s, \tilde{r})$ is on IC_U. But this contradicts that (U, s) is never a best reply in any equilibrium that leads to outcome o in Γ^l.

The above implies that in Γ^k, either the strategies (U, s) and (I, s', s) exist for some $s' \neq s$, or all such strategies have been pruned away prior to stage k. In either case, some receiver strategy in which $r(s) \leq \tilde{r}$ is consistent with the admissibility of the receiver's strategy, and hence the sender does not have a profitable deviation to (U, s). He also does not have a profitable deviation to acquiring information and one of the two types playing s, since on the interval under study IC_U is strictly below the low type's indifference curve through $(s^*, E[\theta])$ and the high type's indifference curve through (s_u, θ_H).

Case 2: $s < s^*$. On this interval IC_U is below $E[\theta]$. Therefore, for a profitable deviation to occur one would need to prune all the strategies in which the low type plays s, with the possible exception of (I, s, s), while leaving at least one strategy where some other type plays it. That would force $r(s) \geq E[\theta]$ and the outcome $(s, r(s))$ above IC_U leaving the sender with a profitable deviation to not acquire information an choosing s. However, as we show in the following paragraph, if all the strategies (I, s, s'), $s' \neq s$, have been pruned at or before game Γ_l, $l < k$, then the strategy (U, s) and all strategies (I, s'', s) must have been pruned by some earlier stage $l' < l$. Thus, if all the strategies in which θ_L plays s have been pruned, no strategies in which any type plays s are left, and s cannot represent a profitable deviation.

We now prove the assertion that if all the strategies (I, s, s'), $s' \neq s$, have been pruned at or before game Γ_l, $l < k$, then the strategy (U, s) and all strategies (I, s'', s) must have been pruned by some earlier stage $l' < l$. Suppose on the way to a contradiction that at stage l, strategy (I, s, s'), for some $s' \neq s$, is available along with either (U, s) or (I, s'', s) for some s''. Then, in stage l, the set of equilibria that leads to outcome o is nonempty, and includes a receiver strategy in which $(s, r(s))$ is on IC_L, $r(s) \geq \theta_L$, and $r(s_u) = \theta_H$. The last property follows from our initial observation that (I, s^*, s_u) cannot be pruned, and the penultimate property from $s^* \leq \bar{s}^*$. Because the sender is indifferent between her payoff in o, and (I, s^*, s_u) when $r(s^*) = E(\theta)$ and $r(s_u) = \theta_H$, in stage l, the strategy (I, s, s_u) is not pruned, leading to a contradiction. □

Proof of Lemma 3: Differentiating the value of information $b(s^*)$, given by (8), with respect to s^* yields

$$\frac{db}{ds^*} = \lambda \left[u_s(s_u, \theta_H, \theta_H) \frac{ds_u}{ds^*} - u_s(s^*, E[\theta], \theta_H) \right].$$

On the other hand, differentiating the uninformed sender's indifference condition between $(s^*, E[\theta])$ and (s_u, θ_H) results in

$$\frac{ds_u}{ds^*} = \frac{E[u_s(s^*, E[\theta], \theta)]}{E[u_s(s_u, \theta_H, \theta)]}.$$

Combining the two equations yields

$$\begin{aligned}\frac{db}{ds^*} &= \lambda \left[u_s(s_u, \theta_H, \theta_H) \frac{E[u_s(s^*, E[\theta], \theta)]}{E[u_s(s_u, \theta_H, \theta)]} - u_s(s^*, E[\theta], \theta_H) \right] \\ &= \lambda(1-\lambda) \frac{u_s(s_u, \theta_H, \theta_L) u_s(s^*, E[\theta], \theta_L)}{E[u_s(s_u, \theta_H, \theta)]} \left(\frac{u_s(s_u, \theta_H, \theta_H)}{u_s(s_u, \theta_H, \theta_L)} - \frac{u_s(s^*, E[\theta], \theta_H)}{u_s(s^*, E[\theta], \theta_L)} \right)\end{aligned}$$

Since $u_s < 0$, $db/ds^* > 0$ (< 0) if $\frac{u_s(s,r,\theta_H)}{u_s(s,r,\theta_L)}$ is decreasing (increasing) in s and r. If $\frac{u_s(s,r,\theta_H)}{u_s(s,r,\theta_L)}$ is constant in s and r, b is constant in s^*. □

Proof of Lemma 4: The proof of the result is somewhat lengthy and, thus, broken down in steps: i) we argue that at $c = 0$ after information acquisition only separating equilibria can potentially survive the refinement (strictly speaking, we argue that all the other equilibria with information acquisition can be refined away); ii) only the equilibria with information acquisition in which the sender is indifferent between acquiring information (equilibrium play) and deviating to not acquiring information followed by pretending to be the high type can potentially survive the refinement; iii) we define a c_I and argue that no equilibrium with information acquisition survives the refinement for $c > c_I$; iv) we verify that the remaining equilibria—one for each $c \leq c_I$—survive the refinement.

Step 1: Let $c = 0$ and fix an equilibrium outcome with information acquisition in which the two types pool with positive probability. Denote such pooling action by $\tilde{s}$, and the receiver's response with $\tilde{r}$; notice that $\tilde{r} < \theta_H$. For $\tilde{s}$ to be a part of an equilibrium, the receiver's response to $s' > \tilde{s}$ cannot be above the high type's indifference curve through $(\tilde{s}, \tilde{r})$. Due to single-crossing nature of the preferences both the low type's as well as the uninformed sender's indifference curves through $(\tilde{s}, \tilde{r})$ are strictly above the high type's for $s' > \tilde{s}$. More precisely, for $s' > \tilde{s}$ every equilibrium with the given outcome is such that:

$$\begin{aligned}u(s', r(s'), \theta_L) &< E[u(s', r(s'), \theta)] \\ &< u(s', r(s'), \theta_H) \\ &\leq u(\tilde{s}, r(\tilde{s}), \theta_H),\end{aligned}$$

where the last inequality is guaranteeing that the high type does not have a profitable deviation from his equilibrium action $\tilde{s}$. Thus, actions above $\tilde{s}$ are NWBR for the low type or the uninformed sender. After removing all the strategies where the uninformed sender or the low type play actions above $\tilde{s}$, one obtains a game in which the receiver responds to an action $s' > \tilde{s}$ with θ_H. But then the high type would have a profitable deviation just above $\tilde{s}$.

Step 2: Fix a $c \geq 0$. We claim that if an equilibrium with information acquisition is to survive the equilibrium refinement it has to be the case that the sender is indifferent between following the equilibrium strategy and deviating to not acquiring information followed by the high type's equilibrium action; if an equilibrium outcome at c exists to start with. Lemma 1 established that for $c > 0$ in any equilibrium with information acquisition the two types separate themselves and, moreover, the low type chooses $s_L = 0$ and strictly prefers his action to the high type's. In the first step we showed that an analogous statement must hold for $c = 0$ if the equilibrium is to not be refined away.

Fix an equilibrium outcome with information acquisition and suppose, contrary to our claim, that the sender strictly prefers equilibrium play to not acquiring information followed by pretending to be the high type. Consider the set of all equilibria that lead to the outcome. Because the low type strictly prefers his equilibrium play, $s_L = 0$, to the high type's, s_H, and because the sender strictly prefers acquiring information, there exists an $\epsilon > 0$ such that $s \in (s_H - \epsilon, s_H)$ are NWBR for the low type nor for the uninformed sender (due to the assumption that the sender strictly prefers to acquire information); that is, any strategy with no information acquisition and $s \in (s_H - \epsilon, s_H)$ yields a strictly smaller payoff than the equilibrium strategy. Hence, these strategies can be pruned away. In the game obtained after the pruning, actions in $(s_H - \epsilon, s_H)$ can only be played by the high type, therefore in any equilibrium that leads to the outcome the receiver should respond to $s \in (s_H - \epsilon, s_H)$ with $r = \theta_H$. But then, the outcome is not an equilibrium of the reduced game, namely, type θ_H has a profitable deviation in $(s_H - \epsilon, s_H)$.

The above establishes that if an outcome with information acquisition is to survive the refinement, then it has to be the case that the sender is indifferent between acquiring and not acquiring information. We also know that any such outcome is separating, therefore we can conclude that this is the most efficient separating outcome—any separating outcome where the high type burns less surplus, at the fixed c, would have sender deviating to not acquiring information. We are yet to establish the range of c for which such equilibria survive the refinement.

Step 3: The most efficient (separating) equilibria with information acquisition have the following form. At $c = 0$ the outcome where the agent is indifferent between acquiring and not acquiring information is the Riley outcome. As c increases so does s_H, as can be seen from the sender's indifference between acquiring and not acquiring information (and pretending to be the high type):

$$\lambda u(s_H, \theta_H, \theta_H) + (1 - \lambda)u(0, \theta_L, \theta_L) - c = E[u(s_H, \theta_H, \theta)]$$

of after a simplification

$$(1 - \lambda)u(0, \theta_L, \theta_L) - c = (1 - \lambda)u(s_H, \theta_H, \theta_L).$$

In particular, to each c corresponds an s_H. Next we establish that such equilibria with s_H above some threshold can be refined away, or equivalently, such equilibria can be refined away for c above some threshold.

Fix a $c > 0$ and the equilibrium outcome with information acquisition such that the sender is indifferent between acquiring information and not acquiring information followed by mimicking the high type. Denote the intersection of the low type's indifference curve through $(0, \theta_L)$ and the uninformed sender's indifference curve through (s_H, θ_H) by (s_{LU}, r_{LU}). In particular, (s_{LU}, r_{LU}) solves the following pair of equations

$$\begin{aligned} u(0, \theta_L, \theta_L) &= u(s_{LU}, r_{LU}, \theta_L), \\ E[u(s_H, \theta_H, \theta)] &= E[u(s_{LU}, r_{LU}, \theta)]. \end{aligned}$$

We argue that an equilibrium can be refined away if $r_{LU} < E[\theta]$. This establishes an upper bound on s_H and thus on c.

If $r_{LU} < E[\theta]$, then there exist an $\epsilon > 0$, such that for actions in $(s_{LU}, s_{LU} + \epsilon)$ the receiver is responding with $r < E[\theta]$ in every equilibrium; in equilibrium his responses cannot be above the uninformed agent's indifference curve. However, the actions in question are strictly below the low type's indifference curve by the definition of (s_{LU}, r_{LU}) and the single-crossing property of our environment, and as such, NWBR for the low type. But then, in the game obtained after pruning the said NWBR strategies, in any equilibrium the receiver would have to reply to $(s_{LU}, s_{LU} + \epsilon)$ with at least $E[\theta]$, making a deviation to not acquiring information followed by one of those actions profitable. Therefore, if the equilibrium outcome is to survive the NWBR criterion, it must be the case that $r_{LU} \geq E[\theta]$. Since r_{LU} is decreasing in c (because s_H is increasing in c), the highest value of information acquisition cost where equilibria with indifference and information acquisition could possibly survive NWBR, c_I, is such that $r_{LU} = E[\theta]$.

Step 4: Verification. Fix a $c \leq c_I$ and the most efficient (separating) equilibrium outcome with information acquisition at c. We argue that there exists no finite sequence of pruning of the original game with respect to the outcome that leads to a game in which the outcome is not an equilibrium outcome.

Suppose on the way to a contradiction that there is a sequence of pruning such that in the game Γ^k o fails to be an equilibrium outcome; but not in any Γ^j, for $j < k$. Then in all the games Γ^l, for $l < k$, in all equilibria that give rise to outcome o the receiver's response function is weakly below the indifference curve of θ_u passing through (s_H, θ_H) (which we call IC_U), and the indifference curve of the low type passing through the point $(0, \theta_L)$ (which we call IC_L). The first condition must hold due to the nature of equilibrium outcome being that the sender is indifferent between acquiring information and not acquiring information followed by the high type's action. For o not to be an equilibrium outcome in Γ^k, there should exist some $s \neq \{0, s_H\}$ such that $r(s)$ is strictly above the minimum of IC_U and IC_L in every candidate for an equilibrium. It should also be noted, that it cannot be the case that up to Γ_k all the strategies in which the sender plays s are pruned, otherwise s could not be a part of a profitable deviation in Γ_k.

Let the intersection of IC_U and IC_L be denoted (s_i, r_i). Because $c \leq c_I$, $r_i \geq E(\theta)$. Let also s_μ be such that IC_L crosses the ray $E(\theta)$ at s_μ:

$$u(s_\mu, E[\theta], \theta_L) = u(0, \theta_L, \theta_L).$$

There are three cases to consider: $s \leq s_\mu$, $s \in (s_\mu, s_i]$, $s > s_i$.

Case 1: $s \leq s_\mu$. Since in Γ_k the receiver's response $r(s)$ is strictly above IC_L and the latter is in this case below $E[\theta]$, it has to be that all the strategies in which θ_L plays s have been pruned, except for possibly the strategy (I, s, s). For s to be NWBR for the low type in some earlier game, the receiver's beliefs should have been restricted to θ_L, i.e., all the strategies where the other two types play s should have been pruned away even earlier. But then after pruning away also the strategies where the low type plays s, s is not available for any type and, therefore, cannot represent a profitable deviation.

More precisely, since Γ^k is the first game in which $r(s)$ is above IC_L, it has to be the case that in Γ^{k-1} there exists some strategy in which θ_L chooses s. If θ_L is the only type in Γ^{k-1} that can choose s, $r(s)$ cannot be above θ_L in any subsequent games obtained after pruning. On the other hand, if θ_L is not the only type in Γ_{k-1} who can choose s, any $r(s) \in [\theta_L, E(\theta)]$ that is weakly below IC_L can be sustained as an off equilibrium receiver action, hence (I, s, s_H) is a best reply to *some* receiver strategy that induces the outcome o, and is not pruned in Γ^k. Therefore, in Γ^k, there exists some strategy in which θ_L chooses s, so $r(s) = \theta_L$ is consistent with the requirement that the receiver's strategy is not weakly dominated; contradicting the idea that $r(s)$ must be above the minimum of the two indifference curves.

Case 2: $s \in (s_\mu, s_i]$. First, given the definitions of s_μ and s_i, on the interval under the consideration IC_L is below IC_U, and moreover, IC_L is above $E[\theta]$. Given that $r(s)$ is above IC_L in Γ^k, it has to be the case that $r(s) = \theta_H$. Moreover in Γ^{k-1} there exists some strategy in which type θ_L or θ_U chooses s. Let Γ^l, $l < k$, be the last game in which the strategies in which the low type plays s are removed. In Γ^l the receiver still considers that the low type can play s, but if he considers s possible for θ_U or θ_H, then one can construct an equilibrium where s is a best response for the low type. Thus, if s is to be NWBR for θ_L, it must be the case that the receiver believes only type θ_L can play s in Γ^l, i.e., all the strategies where θ_U or θ_H play s have been previously pruned away. If that is the case, s cannot be a part of a profitable deviation in Γ_k.

Case 3: $s > s_i$. For $s > s_i$ the indifference curve IC_U is below IC_L. Moreover, for $s \geq s_H$, $r(s) \leq \theta_H$ implies that the receiver's strategy is always strictly below IC_U. Therefore the only potential deviation actions are in (s_i, s_H). The idea is: to arrive at a game where only the high type can play s one would need to prune away all the strategies in which θ_U plays s. But for s to be NWBR for θ_U one would need to prune away the strategies where the high type plays s beforehand. One can not have it both ways.

More formally, suppose there is some $s \in (s_i, s_H)$ such that $r(s)$ is above IC_U in Γ^k. For this to be the case, in Γ^k, only type θ_H should have a strategy in which he chooses s. Let Γ^l, $l < k$, be the game where the last strategy in which θ_U plays s is pruned, that is, such that in no equilibrium with the outcome o is s a best response for θ_U. Since in Γ^l the high type has at least one strategy in which he plays s, that would mean that no other type can play it; otherwise the requirement that the receiver has to best respond to some belief would not restrict him in $[E[\theta], \theta_H]$ and s would be a best response for θ_U. But if only θ_H can play s in Γ_l, it must be the case that $r(s) = \theta_H$, contradicting the supposition that Γ_k is the first game in which o is not an equilibrium outcome. □

Proof of Lemma 5: First we argue that any equilibrium with randomization over information acquisition decisions must be separating—each type (low, high, uninformed) undertakes a different amount of signaling—if it is to survive the refinement.

We start by showing that the high type and the low type cannot pool in an equilibrium with information acquisition. The case $c > 0$ is covered by Lemma 1. We complete the argument by considering $c = 0$. Suppose on the way to a contradiction that the low and high types choose a signaling action s with positive probability and that the receiver responds with some r. Then, the sender's equilibrium payoff is equal to the payoff she would get by not acquiring information and choosing s. The high type's indifference curve through (s, r) is the flattest of the three, hence it crosses the ray θ_H at some s_H that is further above from those at which the other two types' indifference curves intersect the ray θ_H. Hence, for some $\epsilon > 0$, applying the pruning procedure we obtain that we can erase all strategies in which the uninformed or the low type chooses an action above $s_H - \epsilon$. But then, in the new game obtained after the pruning, the receiver's responses for these actions have to be θ_H, which makes the initial outcome not an equilibrium of the new game, a contradiction.

To show that the uninformed sender cannot pool with one of the two types, fix an equilibrium outcome with randomization over information acquisition in which the low type and the uninformed sender pool with positive probability on some action $\tilde{s}$, but not the high type. Then the receiver must respond with an $\tilde{r} < E[\theta]$. For $s' > \tilde{s}$ the uninformed sender's indifference curve through $(\tilde{s}, \tilde{r})$ is below the low type's indifference curve through the same point. For the prescribed outcome to be an equilibrium the receiver's response to actions above $\tilde{s}$ must, therefore, not be above the uninformed's indifference curve. Actions above $\tilde{s}$ are then NWBR for the low type. In the game obtained after pruning the strategies where the low type plays actions above $\tilde{s}$ the receiver should in every equilibrium respond to an $s' > \tilde{s}$ with an $r \geq E[\theta]$. But then the sender could profitably deviate to not acquiring information and choosing an s' just slightly above $\tilde{s}$. Equilibrium outcomes in which the uninformed sender and the high type pool are refined away similarly.

The above allows us to focus on equilibrium outcomes with randomization over information acquisition decisions and separation. Fix an outcome in which the uninformed agent strictly prefers his equilibrium action to the high type's. Due to single-crossing then so does the low type. The actions just below the high type's are then NWBR for the low type or the uninformed sender. After removing all the strategies where the uninformed sender or the low type play the mentioned actions one obtains a game where the actions could only be played by the high type, and therefore the receiver responds to them in any equilibrium with $r = \theta_H$. But then the sender has a profitable deviation. Likewise, if the low type were to strictly prefer his own action to the uninformed sender's the actions just below the uninformed's would be NWBR for the low type. In the game obtained after pruning the sender would have an incentive to deviate.

The only remaining equilibrium outcomes are the ones in which the low type chooses $s_L = 0$, the uninformed sender chooses an action that makes the low type indifferent and the high type an action that makes the uninformed sender indifferent. All the actions are, thus pinned down by the indifference conditions. Since the sender must be indifferent between acquiring and not acquiring information, there is only one cost of information at which such an equilibrium outcome can exist. It should be noted that there is a continuum of equilibrium outcomes, as the probability with which the information is acquired is not pinned down.

Finally, we argue that the outcome with randomization over information acquisition, outcome o, survives the refinement. On the way to a contradiction, suppose there is a sequence of pruning with respect to the outcome o such that in the stages $i = 1, ..., k-1$, o is an equilibrium outcome in Γ^i, and in Γ^k, o is not an equilibrium outcome. Observe that, in the proof of Lemma 2, we only used the property that $c = b(s^*)$ to argue that the outcome survives the refinement. In proving the lemma, we used the claims that it cannot be that at stage k, $r(s)$ is above IC_U for $s > s^*$ in all equilibria, neither can it be the case that $r(s)$ is above IC_L in all equilibria. These claims continue to hold in the equilibrium outcome under consideration here. But if the claims are true, then in Γ^k, o is an equilibrium, which is a contradiction.

□

Proof of Lemma 6: The proof proceeds in several steps. First we argue that it holds for $c = 0$, then that it extends to small cost of information.

Step 1: The slope of the indifference curve of the uninformed sender is flatter at any point of intersection than the indifference curve of the type θ_n. In particular, $-\frac{E[u_s(s,r,\theta)]}{E[u_r(s,r,\theta)]} > -\frac{u_s(s,r,\theta_n)}{u_r(s,r,\theta_n)}$. The result follows from:

$$\frac{-E[u_s(s,r,\theta)]}{E[u_r(s,r,\theta)]} - \frac{-u_s(s,r,\theta_n)}{u_r(s,r,\theta_n)} = \frac{E\left[u_r(s,r,\theta)\left(\frac{-u_s(s,r,\theta)}{u_r(s,r,\theta)} - \frac{-u_s(s,r,\theta_n)}{u_r(s,r,\theta_n)}\right)\right]}{E[u_r(s,r,\theta)]}$$
$$> 0,$$

where the inequality holds since $\frac{-u_s(s,r,\theta)}{u_r(s,r,\theta)}$ is assumed decreasing in θ and $u_r > 0$.

Step 2: Let s_u be such that $E[u(0, E[\theta], \theta)] = E[u(s_u, \theta_n, \theta)]$. That is, the uninformed sender is indifferent between the equilibrium option and (s_u, θ_n). From Step 1 it follows that type θ_n strictly prefers (s_u, θ_n) to $(0, E[\theta_n])$. Let r_n be defined by $u(0, E[\theta], \theta_n) = u(s_u, r_n, \theta_n)$. By above, $r_n < \theta_n$.

Step 3: When $c = 0$, in any equilibrium where the sender does not acquire information and chooses $s = 0$, the receiver's response to s_u cannot be above r_n. If it was, type θ_n would prefer s_u to 0 and the sender would have a profitable deviation. Since type θ_n is indifferent between $(0, E[\theta])$ and (s_u, r_n), all the lower types strictly prefer $(0, E[\theta])$.

Action s_u is, therefore, NWBR for any of the types $\theta < \theta_n$. Any strategy where such types play action s_u can thus be pruned.

In the newly obtained game, action s_u is available only to type θ_n. In consequence, the receiver responds to s_u with θ_n. But now the sender has a profitable deviation to acquiring information and having all types who prefer (s_u, θ_n) over $(0, E[\theta])$ choose s_u. The set is non-empty as it includes at least type θ_n.

Step 4: For $c > 0$, let $r_n(c)$ be defined by

$$E[u(0, E[\theta], \theta)] = \sum_{i=1}^{n-1} \lambda_i u(0, E[\theta], \theta_i) + \lambda_n u(s_u, r_n(c), \theta_n) - c.$$

That is, the sender is indifferent between not acquiring information paired with choosing $s = 0$ and acquiring information coupled with all the types except for θ_n choosing the option $(0, E[\theta])$ and type θ_n choosing $(s_u, r_n(c))$. Since $r_n(0) < \theta_n$, by continuity $r_n(c) < \theta_n$ for c small enough. This implies that for such low enough c, in any equilibrium where no information is acquired and $s = 0$ the receiver's response to s_u cannot be above $r_n(c)$. But then s_u is NWBR for types $\theta < \theta_n$ and the same logic as in the previous steps follows. □

Proof of Lemma 7: Fix the outcome where the sender acquires information and each type chooses the action corresponding to the Riley outcome (the most efficient separating equilibrium). In particular, type θ_1 chooses $s = 0$ and each type θ_i is indifferent between his own action and the action taken by θ_{i+1}. By the single-crossing property, each type θ_j, then strictly prefers action s_j to any action s_k with $k < j$ and also, each type θ_j strictly prefers s_j to any s_k with $k > j + 1$.

To support the Riley outcome, assume that the receiver replies to any action $s \in (s_i, s_{i+1})$ with $r = \theta_i$, with convention $s_{n+1} = \infty$. The sender would, therefore, after deviating to not acquiring information optimally choose one of the actions s_i, denote it s_l. Since every type θ_i, $i \notin \{l, l + 1\}$ strictly prefers action s_i to s_l, and there are at least three types, there is a type who prefers his own action to the one the uninformed sender would choose. Therefore, if c is small enough, the sender is better off acquiring information. □

Proof of Lemma 8: Fix the outcome with information acquisition and the Riley outcome. Each type is indifferent between his own action and the action of the upward-adjacent type. The indifference curves connecting all the type's options present an upper bound on the receiver's response in any equilibrium with the outcome.

To argue that the outcome cannot be refined away we can argue that no s that is not played in equilibrium can be a part of a profitable deviation after a finite sequence of deletion of NWBR strategies. Actions above s_n can clearly never be a a part of profitable deviation. Let's focus on some $s \in (s_i, s_{i+1})$. The receiver responds to s_i with θ_i and s_{i+1} with θ_{i+1}. For action s to become a profitable deviation it would have to be the case that after pruning it can be played only by types θ_{i+1} and above or the uninformed sender if $E[\theta] > \theta_i$.

Suppose that s indeed represented a profitable deviation after some number of rounds of deletions. Then s must have been NWBR for type θ_i at some earlier stage. However, to have been a NWBR for θ_i the action should have been eliminated for all the types above θ_i at an even earlier stage. But then s cannot be a profitable deviation. □