Università Commerciale Luigi Bocconi

Unspanned Macroeconomic Factors in the Yield Curve

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Overview

- Building the Macro-Yields Model
- Estimation and preliminary results
- In–sample results
- Out-of-sample forecasts
- Conclusions

Building the Macro-Yields Model

Empirical evidence shows that government bond yields and macroeconomic variables comove and have a strong interaction.

- Aim of the paper is therefore to build a dynamic factor model for the **joint behavior of government bond yields and macroeconomic indicators.**
- Let us look at bond yields and macroeconomic factors individually.

Closer look at: bond yields

- Bond yields at different maturities are driven by 3 traditional factors: level, slope and curvature.
- The cross-section of bond yields is modeled using the Dynamic Nelson-Siegel (NS) framework of Diebold and Li (2006):

$$y_t = a_y + \Gamma_{yy} F_t^y + v_t^y$$

In fact equal to 0

Factor loadings

3 yield-curve factors

Idiosyncratic components

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Closer look at: bond yields (cont'd)

- The 3 yield curve factors in F_t^y are identified by constraining the factor loadings to follow the smooth pattern proposed by NS (1987):

$$\Gamma_{yy}^{(\tau)} = \left[1 \quad \frac{1-e^{-\lambda\tau}}{\lambda\tau} \quad \frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right] \equiv \Gamma_{NS}^{(\tau)}$$

Whereby λ is the decay parameter.

- Observations on the factor loadings:

1. The first factor represents the level of the yield curve and is proxied by the long-term yield
2. The second factor represents the slope of the yield curve and is proxied by the spread between the long and short maturity yield
3. The third factor represents the curvature of the yield curve and is proxied by the sum of spreads between and medium and long-term yield, and between a medium and short-term yield

Building the Macro-Yields Model

Closer look at: macroeconomic variables

- Macroeconomic variables are assumed to be potentially driven by 2 sources of co-movement: the 3 yield curve factors F_t^y and macro specific factors, as per below:

$$x_t = a_x + \Gamma_{xy} F_t^y + \Gamma_{xx} F_t^x + v_t^x$$

Factor loadings on the yield curve factors

Yield-curve factors

Factor loadings on the macro factors

Macro latent factors

Idiosyncratic components

Building the Macro-Yields Model

Joint dynamics of yield curve and macroeconomic factors

- Need to estimate the two models simultaneously:

$$(1) \qquad \begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} 0 \\ a_x \end{pmatrix} + \begin{bmatrix} \Gamma_{yy} & \Gamma_{yx} \\ \Gamma_{xy} & \Gamma_{xx} \end{bmatrix} \begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix} + \begin{pmatrix} v_t^y \\ v_t^x \end{pmatrix}$$

- Whereby $\boldsymbol{\Gamma_{yy} \equiv \Gamma_{NS}}$ **and** $\boldsymbol{\Gamma_{yx} = 0}$**, but** we allow $\boldsymbol{\Gamma_{xy} \neq 0}$ → macro factors capture only those sources of co-movement in the macro variables that are **not** already spanned by the yield curve.
- The idiosyncratic components v_t follow independent AR processes:

$$(2) \qquad v_t = B v_{t-1} + \xi_t$$

- The factors themselves, then, follow a VAR(1):

$$\begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix} = \begin{pmatrix} \mu_y \\ \mu_x \end{pmatrix} + \begin{bmatrix} \mathrm{A}_{yy} & \mathrm{A}_{yx} \\ \mathrm{A}_{xy} & \mathrm{A}_{xx} \end{bmatrix} \begin{pmatrix} F_{t-1}^y \\ F_{t-1}^x \end{pmatrix} + \begin{pmatrix} \mu_t^y \\ \mu_t^x \end{pmatrix}$$

Estimation and preliminary results

Data

- The paper uses monthly US Treasury zero-coupon **yield curve** data
  Period: Jan 1970 – Dec 2008 (CRSP)
- **Macroeconomic** dataset consists instead of 14 macroeconomic variables (5 inflation measures + 7 real variables + federal funds rate + a money indicator)

Estimation

- Rewrite eq 1 and 2 in a compact form:

$$(3)\qquad z_t = a + \Gamma F_t + v_t$$
$$(4)\qquad F_t = \mu + \mathrm{A} F_{t-1} + u_t$$
$$v_t = B v_{t-1} + \xi_t$$

Where $z_t = \begin{pmatrix} y_t \\ x_t \end{pmatrix}$, $F_t = \begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix}$, $a = \begin{pmatrix} 0 \\ a_x \end{pmatrix}$, $\Gamma = \begin{bmatrix} \Gamma_{yy} & \Gamma_{yx} \\ \Gamma_{xy} & \Gamma_{xx} \end{bmatrix}$, $\mathrm{A} = \begin{bmatrix} \mathrm{A}_{yy} & \mathrm{A}_{yx} \\ \mathrm{A}_{xy} & \mathrm{A}_{xx} \end{bmatrix}$,

$$\mu = \begin{pmatrix} \mu_y \\ \mu_x \end{pmatrix}.$$

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Where $z_t = \binom{y_t}{x_t}$, $F_t = \begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix}$, $a = \begin{pmatrix} 0 \\ a_x \end{pmatrix}$, $\Gamma = \begin{bmatrix} \Gamma_{yy} & \Gamma_{yx} \\ \Gamma_{xy} & \Gamma_{xx} \end{bmatrix}$, $\mathrm{A} = \begin{bmatrix} \mathrm{A}_{yy} & \mathrm{A}_{yx} \\ \mathrm{A}_{xy} & \mathrm{A}_{xx} \end{bmatrix}$,

$$\mu = \binom{\mu_y}{\mu_x}.$$

Estimation and preliminary results

Estimation (cont'd)

- We can put the macro-yields model of eq. 4 and 5 into a **state-space form**, augmenting F_t with the idiosyncratic components v_t and a constant c_t (which will be restricted to 1 at every t):

$$z_t = \Gamma^* F_t^* + v_t^* \qquad F_t^* = \begin{bmatrix} F_t \\ c_t \\ v_t \end{bmatrix}$$
$$F_t^* = \mathrm{A}^* F_{t-1}^* + u_t^*$$

We restrict $\boldsymbol{\Gamma}^*$ and $\boldsymbol{A}^*$: $H_1 vec(\Gamma^*) = q_1, \qquad H_2 vec(\mathrm{A}^*) = q_2$

- Now we are ready to write the **joint likelihood** of z_t and F_t :

$$(z, F^*; \theta) = -\sum_{t=1}^{T} \left(\frac{1}{2}[z_t - \Gamma^* F_t^*]' (\mathrm{R}^*)^{-1} [z_t - \Gamma^* F_t^*]\right) - \frac{T}{2}\log|\mathrm{R}^*|$$

$$-\sum_{t=2}^{T} \left(\frac{1}{2}[F_t^* - \mathrm{A}^* F_{t-1}^*]' (\mathrm{Q}^*)^{-1} [F_t^* - \mathrm{A}^* F_{t-1}^*]\right) - \frac{T-1}{2}\log|\mathrm{Q}^*| + \frac{1}{2}[F_1^* - \pi_1]' V_1^{-1} [F_1^* - \pi_1]$$

$$-\frac{1}{2}\log|V_1| - \frac{T(p+k)}{2} log 2\pi + \lambda_1'(H_1 vec(\Gamma^*) - q_1) + \lambda_2' H_2 vec(\mathrm{A}^*) - q_2)$$

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Estimation and preliminary results

Estimation (cont'd)

- The computation of Maximum Likelihood estimates is performed using the Kalman smoother with the Expectation Restricted Maximization (ERM) algorithm, because of the restrictions on the factor loadings of the yields on NS factors.

- At each iteration of this procedure:

1. Expectation-step: compute the expected log-likelihood conditional on the data and the estimates from previous iteration:

$$L(\theta) = E[L(z, F^*; \theta^{(j-1)}|z]$$

Which depends on 3 expectations (computed through Kalman filter):

$$\bar{F}_t^* \equiv E[F_t^*; \theta^{(j-1)}|z]$$
$$P_t \equiv E[F_t^*(F_t^*)'; \theta^{(j-1)}|z]$$
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2. Restricted Maximization-step: update parameters maximizing the expected log-likelihood w.r.t. θ:

$$\theta^{(j)} = \underset{\theta}{argmax}\, L(\theta)$$

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Estimation and preliminary results

Estimation (cont'd)

- As such, we initialize the yield curve factors → NS factors estimated through the 2-step OLS procedure introduced by Diebold and Li (2006) – NS restrictions apply.
- Then we project the macroeconomic variables on the NS factors and use the principal components of the **residuals** of this regression to initialize the unspanned macroeconomic factors.
- For comparison, you can also estimate only-yields model, just imposing $\Gamma_{xy} = A_{yx} = 0$.

Estimation and preliminary results

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Estimation and preliminary results

Model selection

- The macro-yields model decomposes variations in yields and macroeconomic variables into:

1. Yield curve factors
2. Unspanned macroeconomic factors
3. Idiosyncratic noise

- The question is now the following: how many unspanned macroeconomic factors should we include?
- Idea: maximize the general fit of the model and minimize the penalty IC*

$$IC^*(s) = \log\left(V\left(s, \hat{F}_{(s)}\right)\right) + sg(N,T), \qquad where\ penalty\ factor\ g(N,T) = \frac{log C_{NT}^{*2}}{C_{NT}^{*2}}$$

In the above, $C_{NT}^{*2} = \min\left\{\sqrt{T}, \frac{N}{log N}\right\}$, i.e. rate of convergence of the estimate to the true value (Bai and Ng, 2002).

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Estimation and preliminary results

Model selection (cont'd)

- As per evidence in the table, the optimal number of factors to include is 5 :

i. 3 yield curve factors

ii. 2 unspanned macroeconomic factors, which will be **real economic activity** (proxied by industrial production index) and **real interest rate** (computed as the difference between federal funds rate and the consumer price index)

Number of factors	IC^*	V
3	0.02	0.44
4	-0.03	0.31
5	-0.11	0.22
6	0.01	0.18
7	0.23	0.17
8	0.43	0.16

This table reports the information criterion IC^*, as shown in (8) and (7), and the sum of the variance of the idiosyncratic components (divided by NT), V, when different numbers of factors are estimated.

In-sample results

Model fit

- The table below reports the share of variance of macroeconomic variables explained by macro-yields factors:

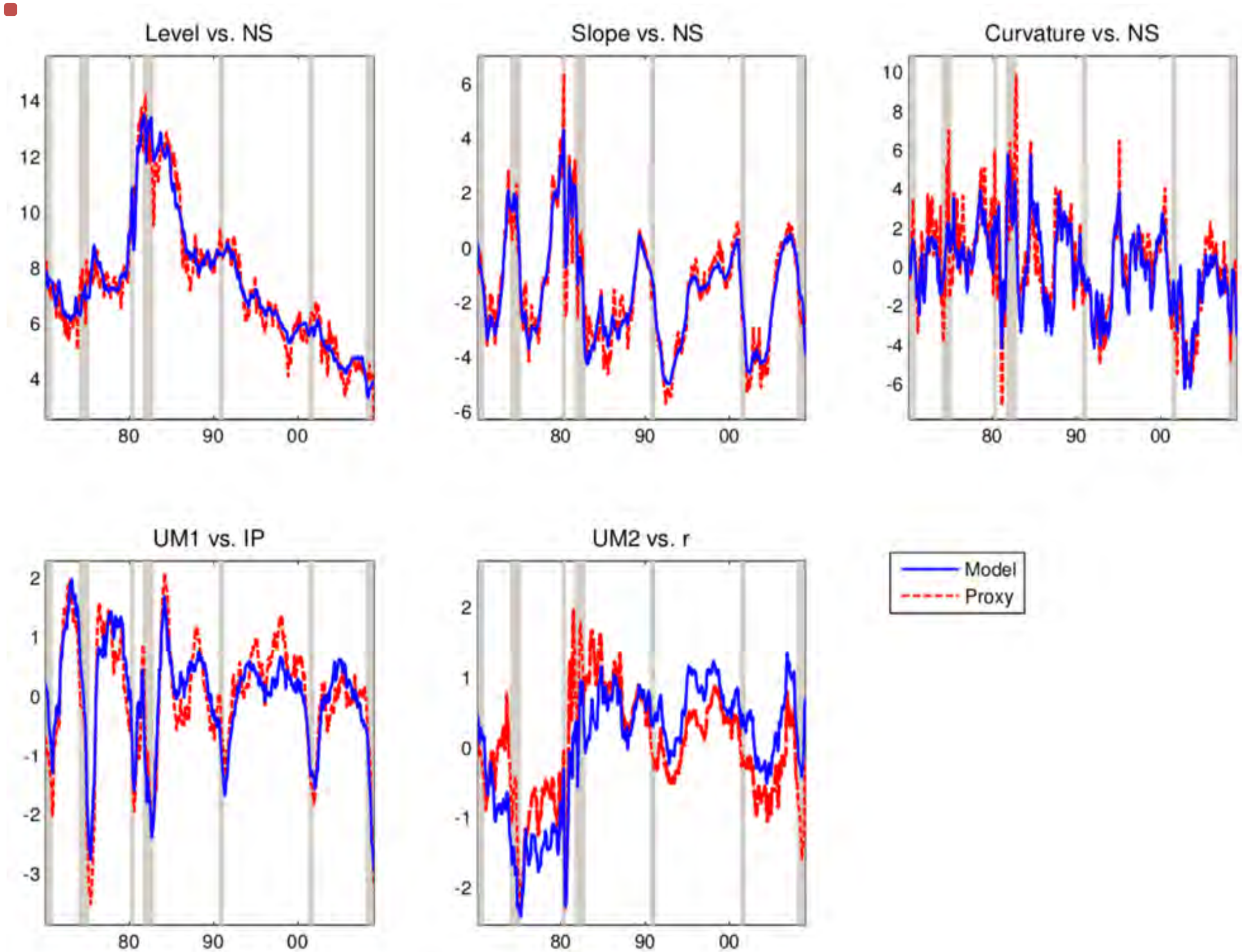
	Level	Slope	Curv	UM1	UM2
Government bond yield with maturity 3 months	0.59	0.94	1.00	1.00	1.00
Government bond yield with maturity 1 year	0.61	0.83	0.99	0.99	0.99
Government bond yield with maturity 2 years	0.65	0.78	0.99	0.99	0.99
Government bond yield with maturity 3 years	0.70	0.79	1.00	1.00	1.00
Government bond yield with maturity 4 years	0.74	0.80	0.99	0.99	0.99
Government bond yield with maturity 5 years	0.78	0.82	0.99	0.99	0.99
Average Hourly Earnings: Total Private	0.07	0.29	0.33	0.33	0.67
Consumer Price Index: All Items	0.19	0.48	0.48	0.50	0.85
Real Disposable Personal Income	0.00	0.02	0.03	0.34	0.36
Effective Federal Funds Rate	0.53	0.93	0.96	0.96	0.97
House Sales - New One Family Houses	0.00	0.19	0.19	0.23	0.23
Industrial Production Index	0.02	0.02	0.03	0.69	0.69
M1 Money Stock	0.17	0.25	0.25	0.25	0.31
ISM Manufacturing: PMI Composite Index (NAPM)	0.03	0.05	0.05	0.61	0.65
Payments All Employees: Total nonfarm	0.00	0.02	0.10	0.70	0.70
Personal Consumption Expenditures	0.16	0.23	0.33	0.46	0.78
Producer Price Index: Crude Materials	0.03	0.14	0.14	0.20	0.43
Producer Price Index: Finished Goods	0.03	0.32	0.32	0.33	0.80
Capacity Utilization: Total Industry	0.02	0.16	0.21	0.63	0.64
Civilian Unemployment Rate	0.44	0.54	0.55	0.65	0.68

This table reports the cumulative share of variance of yields and macro variables explained by the macro-yields factors. The first three columns refer to the yield curve factors (level, slope and curvature) and the last two to the unspanned macroeconomic factors ($UM1$ and $UM2$).

- Yield curve factors explain most of the variance in fed funds rate and in yields at different maturities
- But they also explain part of the variance in macroeconomic variables
- Same observation for the 2 unspanned macroeconomic factors

In-sample results

Model fit (cont'd)



This figure displays the estimated factors of the macro-yields model. The dashed red lines in the three top graphs refer to the NS yield curve factors estimated by ordinary least squares as in Diebold and Li (2006). The red dashed line in the bottom left plot refers to the industrial production index (IP), while the red dashed line in the bottom plot refers to the real interest rate (FFR-CPI). The grey-shaded areas indicate the recessions as defined by the NBER.

- Top three plots = yield curve factors as estimated within the macro-yields model vs unrestricted factors.
- As expected, **correlation is high but there are differences** due to the fact that, in macro-yields model, we extracted yield curve factors from both yields and macroeconomic variables, then imposing NS restrictions on the loadings of the yields to identify them as yield curve factors.