



Università Commerciale  
Luigi Bocconi

# Unspanned Macroeconomic Factors in the Yield Curve

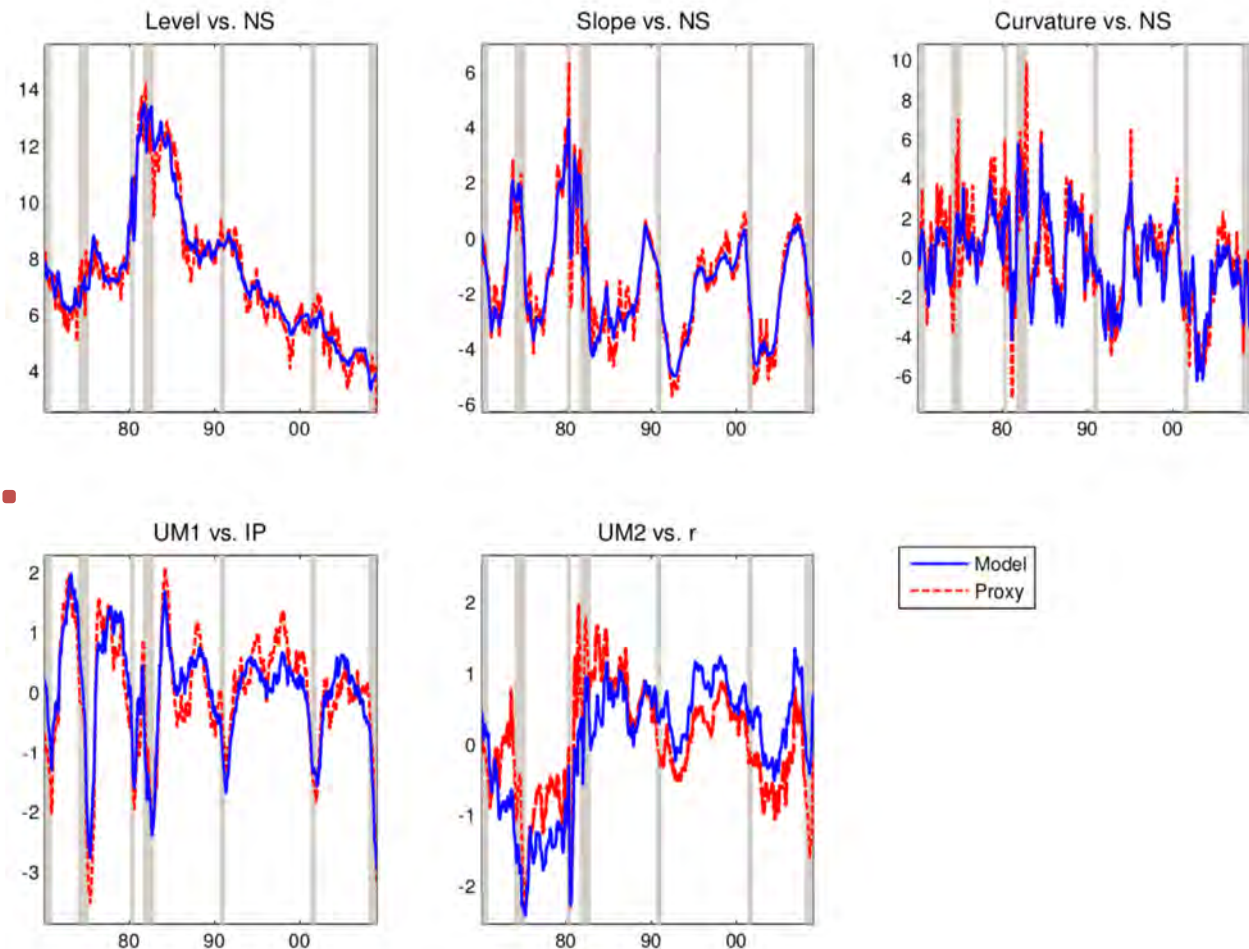
Laura Coroneo, Domenico Giannone,  
Michele Modugno (2015)

Advanced Financial Econometrics III

Giulia Scaglioni - March 4th, 2020

# In-sample results

## Model fit (cont'd)



- Two bottom plots = **unspanned macro factors**.
- Bottom left = **UM1 vs industrial production index. Correlation is high (0.90).**
- Bottom right = **UM2 vs real interest rate. The correlation here is lower (0.74) because nominal variables are partly explained also by the yield curve factors.**

This figure displays the estimated factors of the macro-yields model. The dashed red lines in the three top graphs refer to the NS yield curve factors estimated by ordinary least squares as in Diebold and Li (2006). The red dashed line in the bottom left plot refers to the industrial production index (IP), while the red dashed line in the bottom plot refers to the real interest rate (FFR-CPI). The grey-shaded areas indicate the recessions as defined by the NBER.

# In-sample results

## Predicting bond risk premia

- The relationship between yield risk premium and return risk premium:
$$yrp_t^{(\tau)} = \frac{1}{\tau} E_t[rrp_t^{(\tau)} + rrp_{t+12}^{(\tau-12)} + \dots + rrp_{t+\tau-24}^{(24)}]$$
- **Expectation hypothesis** of term structure of interest rates: yield risk premium is constant. This implies that expected excess returns are time-invariant and, thus, excess bond returns should not be predictable with variables in the info set at time  $t$ .
- However, this theory has been empirically rejected: excess returns are in fact partially predictable, with some models that we will compare:

1. **Macro-yields model**
2. **CP model** = A linear combination of forward rates
3. **LN model** = Macroeconomic factors, constructed as linear/non linear combination of principal components, extracted from a large dataset of macroeconomic variables
4. **CP and LN factors combined**
5. **Only-yields model**

# In-sample results

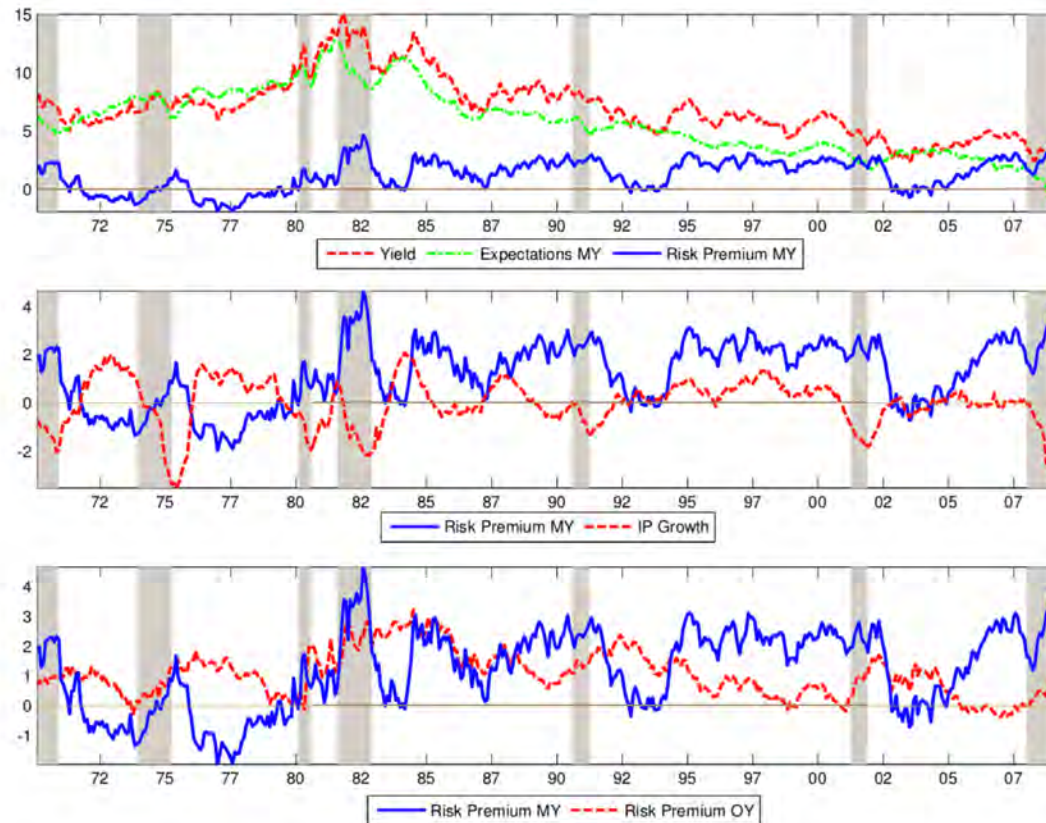
## Predicting bond risk premia

- The relationship between yield risk premium and return risk premium:
$$yrp_t^{(\tau)} = \frac{1}{\tau} E_t[rrp_t^{(\tau)} + rrp_{t+12}^{(\tau-12)} + \dots + rrp_{t+\tau-24}^{(24)}]$$
- **Expectation hypothesis** of term structure of interest rates: yield risk premium is constant. This implies that expected excess returns are time-invariant and, thus, excess bond returns should not be predictable with variables in the info set at time t.
- However, this theory has been empirically rejected: excess returns are in fact partially predictable, with some models that we will compare:
  1. **Macro-yields model**
  2. **CP model** = A linear combination of forward rates
  3. **LN model** = Macroeconomic factors, constructed as linear/non linear combination of principal components, extracted from a large dataset of macroeconomic variables
  4. **CP and LN factors combined**
  5. **Only-yields model**

# In-sample results

## Predicting bond risk premia (cont'd)

Figure 2: Yield risk premium, 5-year bond



This figure displays the yield risk premium using the 5 years to maturity bond. The top panel shows the 5 years to maturity yield (red dashed line) along with the corresponding expectation (green dot-dashed line) and the yield risk premium (blue line) components, computed as in Equation (9) using the macro-yields model. The middle panel reports the yield risk premium according to the macro-yields model (blue line) and the standardized industrial production growth (red dashed line). The bottom plot shows the yield risk premium obtained from the macro-yields model (blue line) and the only-yields model (red dashed line). The grey-shaded areas indicate the recessions as defined by the NBER.

- Top panel: yield risk premium MY vs actual yield and expected yield. We can **reject the expectation hypothesis** (see substantial variation of the risk premium).
- Mid panel: yield risk premium MY vs industrial production index. Note countercyclicity (correlation = - 0.33) → **MY model consistent with real life.**
- Bottom panel: yield risk premium MY vs OY. OY acyclical w.r.t. industrial production index (correlation = - 0.07) → **OY model not consistent with real life.**



# In-sample results

## Predicting bond risk premia (cont'd)

Table 3: In-sample fit of excess bond returns

Maturity	MY	OY	CP	LN	LN+CP
2y	0.55	0.12	0.22	0.33	0.41
3y	0.53	0.12	0.24	0.33	0.43
4y	0.50	0.14	0.27	0.32	0.43
5y	0.46	0.15	0.24	0.30	0.40

This table reports the  $R^2$  for one-year ahead one year holding period excess bond returns from different models. The columns MY and OY refer to the model-implied expected excess bond returns from the macro-yields model (MY) and the only-yields model (OY) respectively. The columns CP, LN and CP+LN refer to the predictive regression using the Cochrane and Piazzesi (2005) factor (CP), the Ludvigson and Ng (2009) factor (LN), and both the Cochrane and Piazzesi (2005) and the Ludvigson and Ng (2009) factors jointly.

Models ranked from best to worst in explaining the variation of excess bonds returns:

1. **Macro-yields model** (explains 46-55%)
2. LN and CP combined (explain 40-43%)
3. LN model (explains about a third). Issue: LN factors are constructed aggregating principal components extracted from a set of macroeconomic and financial variables, without imposing that they are unspanned by the cross section of yields → this means that those factors duplicate information that is already spanned by the yields factors.
4. CP (explains 22-27%)
5. Only-yields model (explains 12-15%)

# In-sample results

Unspanning conditions: empirical test.

- Results so far have shown that **unspanned macro factors play an important role in explaining the term premium, despite being constrained to not affect current yields.**
- This can happen in 2 ways:
  1. **Automatically**, if unspanned macro factors have offsetting effects on average expected future short rates and term premia.
  2. **By imposing** our restriction on the factor loadings of the yields on the macro factors, i.e.  $\Gamma_{yx} = 0$ .
- To test point 1., we compute the risk premium of an **unrestricted** macro-yields model, which does **not** impose the restriction of 2.
- Outcome of the test: **the results of this unrestricted model are practically indistinguishable from those of the restricted model** (correlation between the 2 different estimates of bond premium = 0.99).

# In-sample results

Unspanning conditions: empirical test.

- Results so far have shown that **unspanned macro factors play an important role in explaining the term premium, despite being constrained to not affect current yields.**
- This can happen in 2 ways:
  1. **Automatically**, if unspanned macro factors have offsetting effects on average expected future short rates and term premia.
  2. **By imposing** our restriction on the factor loadings of the yields on the macro factors, i.e.  $\Gamma_{yx} = 0$ .
- To test point 1., we compute the risk premium of an **unrestricted** macro-yields model, which does **not** impose the restriction of 2.
- Outcome of the test: **the results of this unrestricted model are practically indistinguishable from those of the restricted model** (correlation between the 2 different estimates of bond premium = 0.99).



# In-sample results

Unspanning conditions: empirical test (cont'd)

- What does this mean? That, **even without imposing restrictions on factor loadings, the macro factors are truly and naturally unspanned by the yield curve.**
- First practical implication: point 1 is valid. Indeed:
  - a. In periods of **recessions**: unspanned macro factors increase risk premium and decrease expected future short rates by the same amount → no steepening of the yield curve.
  - b. In periods of **expansion**: unspanned macro factors decrease risk premium and increase expected future short rates by the same amount → no flattening of the yield curve.
- Second practical implication: changes in the current shape of the yield curve are only determined by changes in the yield curve factors.

# In-sample results

## Unspanning conditions: empirical test (cont'd)

- What does this mean? That, even without imposing restrictions on factor loadings, the macro factors are truly and naturally unspanned by the yield curve.
- First practical implication: point 1 is valid. Indeed:
  - a. In periods of **recessions**: unspanned macro factors increase risk premium and decrease expected future short rates by the same amount → no steepening of the yield curve.
  - b. In periods of **expansion**: unspanned macro factors decrease risk premium and increase expected future short rates by the same amount → no flattening of the yield curve.
- Second practical implication: changes in the current shape of the yield curve are only determined by changes in the yield curve factors.

# In-sample results

## Unspanning conditions: empirical test (cont'd)

- What does this mean? That, even without imposing restrictions on factor loadings, the macro factors are truly and naturally unspanned by the yield curve.
- First practical implication: point 1 is valid. Indeed:
  - a. In periods of recessions: unspanned macro factors increase risk premium and decrease expected future short rates by the same amount → no steepening of the yield curve.
  - b. In periods of expansion: unspanned macro factors decrease risk premium and increase expected future short rates by the same amount → no flattening of the yield curve.
- Second practical implication: changes in the current shape of the yield curve are only determined by changes in the yield curve factors.

# In-sample results

Unspanning conditions: formal test.

- By a formal definition, **an unspanned factor by the yield curve:**
  - Does not affect the current cross-section of yields (i.e.  $\Gamma_{yx} = 0$ )**
  - Has predictive ability for the yield curve factors ( $A_{yx} \neq 0$ ).**

- To test for these 2 conditions, we perform a likelihood test ratio:

$$LR = 2(L_u - L_r)$$

Whereby  $L_u$  denotes the loglikelihood of an unrestricted model and  $L_r$  is our restricted macro-yield model. LR has a chi-squared probability distribution with d.o.f. equal to the number of restrictions imposed.

- We thus conclude: conditions a. and b. satisfied.

$H_0$	Test statistic	p-value
$\Gamma_{yx} = 0$	12.85	0.38
$A_{yx} = 0$	79.03	0.00

- Moreover, we can say that specifically, the first unspanned factor (proxied by economic growth) Granger-causes the slope and curvature, while the second, (proxied by real interest rates) Granger-causes the level.

# In-sample results

Unspanning conditions: formal test.

- By a formal definition, **an unspanned factor by the yield curve**:
  - Does not affect the current cross-section of yields (i.e.  $\Gamma_{yx} = 0$ )**
  - Has predictive ability for the yield curve factors ( $A_{yx} \neq 0$ ).**
- To test for these 2 conditions, we perform a likelihood test ratio:

$$LR = 2(L_u - L_r)$$

Whereby  $L_u$  denotes the loglikelihood of an unrestricted model and  $L_r$  is our restricted macro-yield model. LR has a chi-squared probability distribution with d.o.f. equal to the number of restrictions imposed.

- We thus conclude: conditions a. and b. satisfied.

$H_0$	Test statistic	p-value
$\Gamma_{yx} = 0$	12.85	0.38
$A_{yx} = 0$	79.03	0.00

- Moreover, we can say that specifically, the first unspanned factor (proxied by economic growth) Granger-causes the slope and curvature, while the second, (proxied by real interest rates) Granger-causes the level.

# In-sample results

Unspanning conditions: formal test.

- By a formal definition, **an unspanned factor by the yield curve:**
  - a. Does not affect the current cross-section of yields (i.e.  $\Gamma_{yx} = 0$ )**
  - b. Has predictive ability for the yield curve factors ( $A_{yx} \neq 0$ ).**
- To test for these 2 conditions, we perform a likelihood test ratio:

$$LR = 2(L_u - L_r)$$

Whereby  $L_u$  denotes the loglikelihood of an unrestricted model and  $L_r$  is our restricted macro-yield model. LR has a chi-squared probability distribution with d.o.f. equal to the number of restrictions imposed.

- We thus conclude: conditions a. and b. satisfied.

$H_0$	Test statistic	p-value
$\Gamma_{yx} = 0$	12.85	0.38
$A_{yx} = 0$	79.03	0.00

- Moreover, we can say that specifically, **the first unspanned factor (proxied by economic growth) Granger-causes the slope and curvature, while the second, (proxied by real interest rates) Granger-causes the level.**



# Out-of-sample forecasts

- To evaluate the predictive ability of the macro-yields model, we generate **OOS iterative forecasts of the factors**, as follows:

$$E_t(F_{t+h}^*) \equiv \hat{F}_{t+h|t}^* = (\hat{A}_{|t}^*)^h \hat{F}_{t|t}^*$$

Whereby  $h$  denotes the forecast horizon and  $\hat{A}_{|t}^*$  is estimated using the info available until time  $t$ .

## OOS forecasts of yields

- We adapt the formula above to compute OOS yields:

$$E_t(z_{t+h}) \equiv \hat{z}_{t+h|t}^* = \hat{\Gamma}_{|t}^* \hat{F}_{t+h|t}^*$$

- Used data for recursive forecasts from Jan 1970, while used data for evaluating the forecast performances on Jan 1990 – Dec 2008.
- To evaluate prediction accuracy: relative MSFE against the benchmark (i.e. random walk):

$$rMSFE_{t_0}^{t_1}(\tau, h, M) = \frac{MSFE_{t_0}^{t_1}(\tau, h, M)}{MSFE_{t_0}^{t_1}(\tau, h, RW)}$$

# Out-of-sample forecasts

- To evaluate the predictive ability of the macro-yields model, we generate **OOS iterative forecasts of the factors**, as follows:

$$E_t(F_{t+h}^*) \equiv \hat{F}_{t+h|t}^* = (\bar{A}_{|t}^*)^h \hat{F}_{t|t}^*$$

Whereby  $h$  denotes the forecast horizon and  $\bar{A}_{|t}^*$  is estimated using the info available until time  $t$ .

## OOS forecasts of yields

- We adapt the formula above to compute OOS yields:

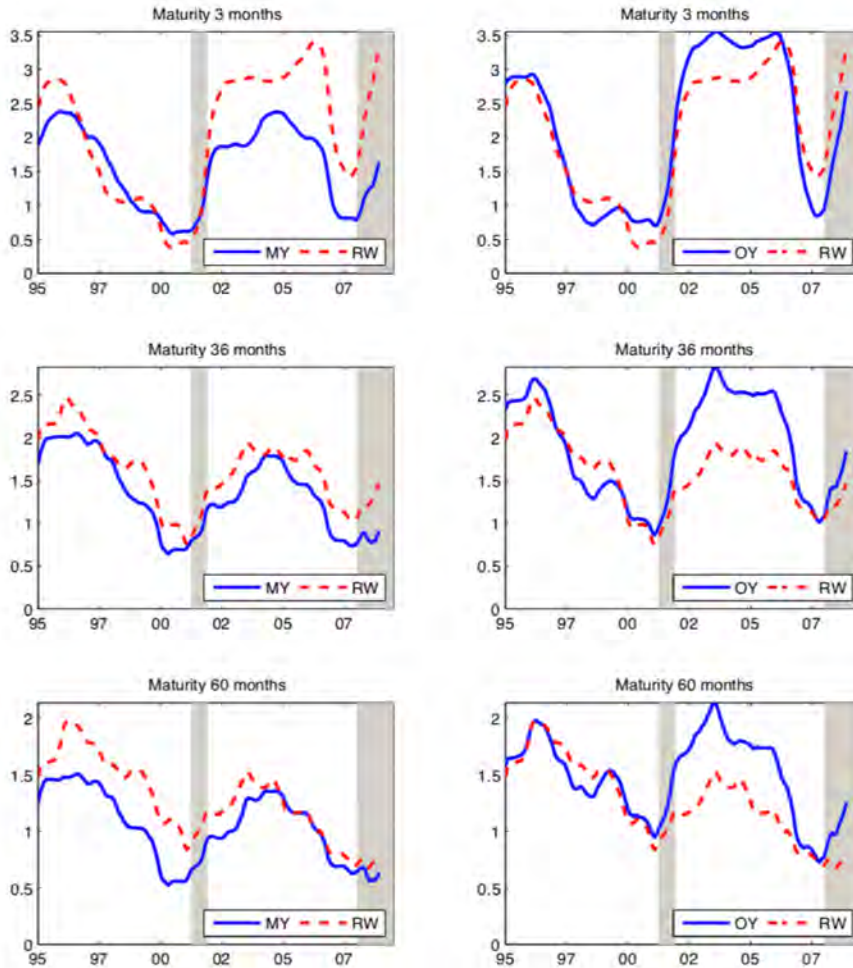
$$E_t(z_{t+h}) \equiv \hat{z}_{t+h|t}^* = \hat{\Gamma}_{|t}^* \hat{F}_{t+h|t}^*$$

- Used data for recursive forecasts from Jan 1970, while used data for evaluating the forecast performances on Jan 1990 – Dec 2008.
- To evaluate prediction accuracy: relative MSFE against the benchmark (i.e. random walk):

$$rMSFE_{t_0}^{t_1}(\tau, h, M) = \frac{MSFE_{t_0}^{t_1}(\tau, h, M)}{MSFE_{t_0}^{t_1}(\tau, h, RW)}$$

# Out-of-sample forecasts

## OOS forecasts of yields (cont'd)



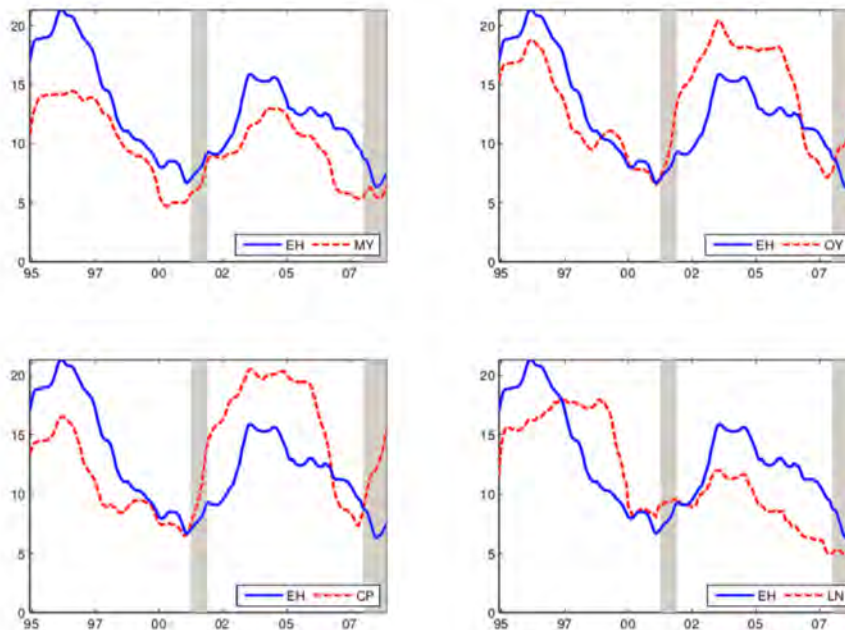
This figure displays the 5-years rolling 12-months ahead squared forecast error for the yields with 3, 36 and 60 months to maturity. The blue continuous line refers to the 5-years rolling squared forecast error of the macro-yields MY model (left plots) and of the only-yields OY model (right plots). The dashed red line refers to 5-years rolling squared forecast error of the random walk. The dates on the horizontal axis refer to the end of the rolling window period. The grey-shaded areas indicate the recessions as defined by the NBER.

- Left: MSFE for macro-yields model vs random walk. **MY outperforms RW.**
- Right: MSFE for **only-yields** model vs **random walk**. The two models show equivalent performances for short and medium maturities, but for longer maturities, RW outperforms the OY → unspanned macro factors do not explain contemporaneous variations in yields, but contain useful information to predict the evolution of the yield curve.

# Out-of-sample forecasts

## OOS forecasts of excess bond returns

- Even in this case, we will compare the MSFE of the benchmark (i.e. the expectation hypothesis  $\rightarrow$  constant excess return benchmark) with the MSFE of other models, among which macro-yields'.



This figure displays the 5-years rolling mean squared forecast error for one-year holding period excess bond returns from the expectation hypothesis EH (blue continuous line) and the corresponding values from different models (dashed red line). The dashed red line in the top plots refer to 5-years rolling mean squared forecast error of the macro-yields MY model (top right) and only-yields OY model (top left). The dashed red line in the bottom plots refer to the 5-years rolling mean squared forecast error from the predictive regressions using the CP factors (bottom left) and the LN factor (bottom right). The dates on the horizontal axis refer to the end of the rolling window period. The grey-shaded areas indicate the recessions as defined by the NBER.

- **OY and CP: similar performances** because they outperform the benchmark in short maturities, underperform over long term.
- **MY and LN: similar performances**, and they are better than the benchmark especially in the last part of the evaluation period.
- **Best performer on average: MY.**

# Conclusion

The good performance of the macro-yields model in predicting the future evolution of the yield curve suggests that **macroeconomic information have predictive power for the bond yields and excess returns in the US.**