Online Appendix

Moderating Political Extremism:

Single Round vs Runoff Elections under Plurality Rule*

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August 2014

Abstract

This Appendix provides additional materials that are also discussed in the paper. In particular, the Online Appendix I contains the proofs of the main propositions in the text. The Online Appendix II develops a set of extensions of the baseline model. The Online Appendix III provides further empirical evidence and validity tests.

*We thank Pierpaolo Battigalli, Carles Boix, Daniel Diermeier, Massimo Morelli, Giovanna Iannantuoni, Francesco de Sinopoli, Ferdinando Colombo, Piero Tedeschi, Per Petterson-Lindbom, the coeditor Debraj Ray, four anonymous referees, and seminar participants at Berkeley, CIFAR, Pompeu Fabra, Brescia, Cattolica, Munich, Warwick, Cesifo Workshop, IGIER workshop in Political Economics, IIPF annual conference, NYU conference in Florence for several helpful comments. We also thank Massimiliano Onorato for excellent research assistance, and Veruska Oppedisano, Paola Quadrio, and Andrea Di Miceli for assistance in collecting the data. Financial support is gratefully acknowledged from the Italian Ministry for Research and Catholic University of Milan for Bordignon, from ERC (grant No. 230088) and Bocconi University for Nannicini, and from the Italian Ministry for Research, CIFAR, ERC (grant No. 230088), and Bocconi University for Tabellini.

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Online Appendix I

Main proofs

Proof of Proposition 1

i) Suppose that (3,4) have merged and have agreed to the policy platform $q^{34} \in [t^3, t^4]$. If 1 and 2 run alone, they lose the election with certainty and get the utility:

$$\bar{u}^P = -C(|q^{34} - t^P|), \qquad P = 1,2 \tag{1}$$

Let $U^P(q, r^P)$ denote candidate P utility (for $P = 1,2$) if 1 and 2 merge into a single party and agree to the policy q and rent allocation r^P. By (1) and eq. (1) in the paper, we have:

$$U^P(q, r^P) - \bar{u}^P = \frac{1}{2}[V(r^P) - C(|q - t^P|) + C(|q^{34} - t^P|)] \tag{2}$$

Note first of all that, for any $q \in [t^1, t^2]$ and any $r^P \geq \bar{r}$, and since $\lambda > 1/4$, the RHS of (2) is strictly positive. Hence both 1 and 2 are always strictly better off by forming the party $\{1, 2\}$ than by running alone against $\{3, 4\}$. This implies that, if (3,4) have merged (or are expected to merge) into a single party, candidates 1 and 2 will also merge, irrespective of the sequence of proposals.

To determine q and r^P we solve for the Nash bargaining equilibrium. Thus, we solve $Max_{q,r^1,r^2}(U^1(q, r^1) - \bar{u}^1)(U^2(q, r^2) - \bar{u}^2)$ subject to the constraints that $R \geq r^1 + r^2$, $r^P \geq \bar{r}$ for $P = 1,2$ and $q \in [t^1, t^2]$.

At an interior optimum the first order conditions imply:

$$\frac{U^1(q, r^1) - \bar{u}^1}{U^2(q, r^2) - \bar{u}^2} = \frac{V_r(r^1)}{V_r(r^2)} = \frac{C_q(|q - t^1|)}{C_q(|t^2 - q|)} \tag{3}$$

Given strict convexity of C and since $q^{34} > t^2 > t^1$, it is easy to verify that $U^1(q, r^1) - \bar{u}^1 > U^2(q, r^2) - \bar{u}^2$ at the the symmetric outcome, $r^P = R/2$ and $q = (t^1 + t^2)/2$ - intuitively, the extremist has more to lose from disagreement since his bliss point is further away from q^{34}. Furthermore, at the symmetric outcome, both ratios on the RHS of (3) are equal to 1. Hence this cannot be an equilibrium. The Nash bargaining equilibrium must entail $r^2 > r^1$ and $q > (t^1 + t^2)/2$ so that all three ratios in (3) exceed unity (note that the left-most side of (3) is decreasing in q, decreasing in r^2 and increasing in r^1). Finally, with enough concavity in $V(.)$ and enough convexity in $C(.)$, the solution to the Nash bargaining equilibrium must be an interior optimum. By symmetry, a similar conclusion applies to Nash bargaining between

3 and 4 (with the appropriate changes), given that 1 and 2 have merged.

ii) Next suppose that 3 and 4 have not merged (or are expected not to merge). In this case, if 1 and 2 also run alone, the two moderate candidates win with probability 1/2 each on a policy plartfrom corresponding to their respective bliss point. In this case, the expected utility of 1 and 2 respectively is:

$$\bar{u}^1 = -\frac{1}{2}C(|t^2 - t^1|) - \frac{1}{2}C(|t^3 - t^1|) \tag{4}$$

$$\bar{u}^2 = \frac{1}{2}V(R) - \frac{1}{2}C(|t^3 - t^2|) \tag{5}$$

Again, let $U^P(q, r^P)$ denote candidate P utility (for $P = 1, 2$) if 1 and 2 merge into a single party and agree to the policy q and rent allocation r^P. Now party $\{1, 2\}$ wins with certainty on any feasible policy platform q, given that 3 and 4 are running alone. We thus have:

$$U^1(q, r^1) - \bar{u}^1 = V(r^1) - C(|q - t^1|) + \frac{1}{2}[C(|t^2 - t^1|) + C(|t^3 - t^1|)] \tag{6}$$

$$U^2(q, r^2) - \bar{u}^2 = V(r^2) - C(|t^2 - q|) - \frac{1}{2}V(R) + \frac{1}{2}C(|t^3 - t^2|)] \tag{7}$$

At an interior optimum, the Nash bargaining outcome between 1 and 2 must still satisfy (3) above. Repeating the same logic as above, evaluate the left-most expression of (3) at the symmetric outcome, $r^P = R/2$ and $q = (t^1 + t^2)/2$. Here too, at the symmetric outcome we have $U^1(q, r^1) - \bar{u}^1 > U^2(q, r^2) - \bar{u}^2$. Hence, by the same argument as above, the Nash bargaining equilibrium must again favor the moderate candidate, so that $r^2 > r^1$ and $q > (t^1 + t^2)/2$ even if 3 and 4 have not merged.

Finally, consider the first stage of party formation. Suppose that 3 and 4 have not merged (or are expected not to merge). We want to show that both 1 and 2 are better off merging into a single party, given that once they have done so the policy and rent allocation will be set according to the Nash bargaining outcome just described. Note that the RHS of (6) is positive since $|q - t^1| < |t^2 - t^1|$ and $C(.)$ is convex. Hence not surprisingly player 1 is better off by merging than running alone. Consider the RHS of (7) evaluated at the Nash bargaining outcome. Since in the Nash bargaining outcome $r^2 > R/2$ (and the function $V(.)$ is concave), we have $V(r^2) - \frac{1}{2}V(R) > 0$. Moreover, we also have $\frac{1}{2}C(|t^3 - t^2|) = \frac{1}{2}C(2\lambda) > C(\lambda) > C(|t^2 - q|) = C(\frac{1}{2} - \lambda - q)$ where the first inequality follows from convexity of $C(.)$ and the second inequality follows from $\lambda > \frac{t^2}{2} = \frac{1}{4} - \frac{\lambda}{2} > t^2 - q$ (since $\lambda > 1/4$ and $q > (t^1 + t^2)/2 = \frac{t^2}{2}$). Hence, the RHS of (7) is also strictly positive at the Nash bargaining outcome, and the moderate candidate too is better off merging (given the anticipated Nash bargaining outcome) rather than running alone, if 3 and 4 have not (or will not merge).

Combining (i) and (ii), we conclude that forming a coalition of the moderate and extremist candidate is a dominant strategy in the first stage, irrespective of the behavior of the opponents. Hence in equilibrium both $\{1,2\}$ and $\{3,4\}$ will form, and the Nash bargaining outcome is as described in part (i) of the proof. QED

Proof of Proposition 3

Suppose that 3 and 4 are running alone (or are expected to do so). If 1 and 2 also run alone, then 2 and 3 win with probability 1/2 each, on a policy platform corresponding to their respective bliss points. Hence, the expected utility of 2 in this case is given by (5) above. If 1 and 2 merge, then their probability of victory remains 1/2 irrespective of the policy q, since, given $\lambda > 1/4$ and sincere voting, in the second round the moderate candidate 3 is able to capture all extremist voters in group 4. Hence, the expected utility of candidate 2, given that he has merged with 1 and that 3 and 4 are running alone, is:

$$U^2(r^2, q) = \frac{1}{2}V(r^2) - \frac{1}{2}C(|t^2 - q|) - \frac{1}{2}C(|t^3 - t^2|) \tag{8}$$

Note that $r^2 \leq R - \bar{r}$ (where $\bar{r} > 0$ denotes the minimal rents that must be given to candidate 1 when party $\{1,2\}$ is formed). Comparing (8) and (5), we see that candidate 2 is strictly better off running alone than under the merger, for any q (even for $q = t^2$). Hence, even if candidate 1would be better off under a merger, there is nothing that he can offer to the candidate 2 to convince him to merge.

Next, suppose that 3 and 4 have merged (or are expected to merge) and run on a policy platform q^{34}. The probability of final victory for candidate 2 in the final ballot is 1/2, irrespective of whether he has merged with 1 or not, since in any case he can collect the votes of extremist voters close to him. Hence, the expected utility of 2 if he runs alone is given by:

$$\bar{u}^2 = \frac{1}{2}V(R) - \frac{1}{2}C(|q^{34} - t^2|)$$

and his expected utility if he merges with 1 on a policy platform q is:

$$U^2(r^2, q) = \frac{1}{2}V(r^2) - \frac{1}{2}C(|t^2 - q|) - \frac{1}{2}C(|q^{34} - t^2|)$$

Comparing these two expressions and repeating the same argument as above, we see that candidate 2 is always better off running alone than merging with 1, for any policy q.

Given the model's symmetry, the only equilibrium of the runoff thus has both moderate candidates running alone on a policy that coincides with their respective bliss points. QED

Online Appendix II

Extensions

Strategic voters

Suppose that a fraction $0 \leq s \leq 1$ of voters in each group J behaves strategically, while the remaining ones vote sincerely.[1] Strategic voters take into account the probability of victory of each candidate, and may thus vote for a less preferred candidate who is more likely to win or pass the post. This depends on the beliefs about the voting behavior of all other voters. We study a Nash equilibrium where each strategic voter maximizes expected utility, given correct beliefs about the equilibrium behavior of all the others.[2] Strategic voting may affect our previous results because candidates, by correctly anticipating the voting equilibrium, might be induced to change their choices concerning merger with other candidates and/or proposed policy platforms. We continue to assume $\lambda > 1/4$.

Strategic voting in single round elections. Here there are several equilibria, some of which replicate our previous results with sincere voting, while others produce different results. In particular, it is possible to prove that, even if all voters are strategic ($s = 1$), there is a two party equilibrium in which extremist candidates exert even more influence on policy than under sincere voting.

Specifically, suppose that the voting stage is reached with four parties: $\{1\}, \{2\}, \{3\}, \{4\}$. With strategic voting and symmetry, equilibrium implies that only two parties (one on each side of 1/2) have a positive probability of victory, and that for both, this probability is 1/2. But which parties (whether extremists or moderates) depends on voters beliefs. Suppose that voters coordinate on the following sunspot equilibrium with symmetric beliefs: with equal probabilities, either all votes converge on the extremist parties on each side ($\{1\}$ and $\{4\}$), or they converge on the moderate parties on each side ($\{2\}$, and $\{3\}$). In the first case, it

[1] Degan and Merlo (2006) estimate that only 3% of individual voting profiles are inconsistent with sincere voting in US elections, a figure below measurement error. Sinclair (2005) estimates a bigger fraction of strategic voters in the UK, but still of limited empirical relevance. Kawai and Watanabe (2012) use heterogeneity of Japanese municipalities in electoral districts to estimate both the share of strategic voters (voters who would potentially be willing to vote for a candidate other than the one they most prefer) and the share of *misaligned voting* (voters that effectively cast a vote for a candidate different from the most preferred). While the former are up to 85% of all voters, the latter is only 1,4% - 4,2% of all votes. Spenkuch (2013), exploiting the simultaneous presence of both a list vote and a candidate vote in German national elections, reaches similar conclusions. Potential strategic voters are up to 30% of all voters, but misaligned votes are only 5,8%. The reason for these differences is that many strategic voters find it optimal to vote sincerely. In Section 6 we show that widespread strategic voting is not supported by our data.

[2] This is the standard definition of a voting equilibrium with strategic voters (Myerson and Weber, 1993). For an alternative approach, see Myatt (2007). See also Cox (1997) and Bouton (2013) for a runoff model with strategic voters.

is optimal for all voters in groups 1 and 2 to vote for candidate 1, in the second to vote for candidate 2, and symmetrically for voters in groups 3 and 4. Then, in a four party system, each candidate wins with probability 1/4.

Suppose instead that the voting stage is reached with three parties, say $\{1\}, \{2\}, \{3,4\}$. In line with the previous assumption, suppose that here too voters in groups 1 and 2 coordinate on a sunspot equilibrium with the same symmetric beliefs as above, namely with equal probabilities either all votes converge on party $\{1\}$ or they converge on party $\{2\}$. Again, voters in groups 1 and 2 find it optimal to validate these beliefs, so that, if this three parties equilibrium is reached, party $\{3,4\}$ wins with probability 1/2, while $\{1\}$ and $\{2\}$ win each with probability 1/4. The same outcome occurs (in reverse) in the party system $\{1,2\}, \{3\}, \{4\}$.

Finally, suppose that the voting stage is reached with parties $\{1\}, \{2,3\}, \{4\}$. Here, given $\lambda > 1/4$, a plausible set of beliefs is that voters on both sides of 1/2 coordinate on the extremist candidates, so that parties $\{1\}$ and $\{4\}$ each win with probability 1/2.

Repeating the steps in the proof of Proposition 1 on the bargaining game between candidates, it can then be verified that a similar equilibrium still holds. Namely, under these beliefs, the equilibrium is a two-party system, where rents are split in half inside each coalition, and the policy platforms are set at the mid point between the bliss points of moderates and extremists on each side of 1/2 (i.e., $q = (t^e + t^m)/2$, where e and m denote the extremist and moderate candidate respectively).[3] Note that, with these voters' beliefs, the extremist candidates have more bargaining power and hence more influence than in the equilibrium with sincere voting described in Proposition 1. The reason is that here the extremist candidates have a chance of winning the election on their own (in fact they have the same chance as the moderate candidates). Both candidates continue to have an incentive to merge (since the sunspot creates uncertainty about who has a chance of victory if running alone); but the symmetry in the sunspot realizations enhances the bargaining power of the extremist relative to sincere voting.[4]

This is not the only possibility, however. Suppose that a fraction $s > 1 - \frac{2e}{\alpha}$ of voters is strategic. Then there is also another equilibrium where, irrespective of the number of parties, a strategic extremist voter would vote for the moderate candidate because she expects all

[3]The proof follows the same steps as that of Proposition 1. The first order condition that pins down the equilibrium policy platform and rent allocations is the same as (3) in Online Appendix I. But here equation (2) is replaced by:

$$u^P(q, r^P) - \bar{u}^P = \frac{1}{2}[V(r^P) - C(|q - t^P|)] - \frac{1}{4}[V(R) - C(t^2)]$$

for $P = 1, 2$.

[4]Of course, with different and non symmetric sunspot uncertainty, either the moderate or the extremist candidate could have more bargaining power.

other strategic voters to do the same. Realizing this, each moderate candidate prefers to run alone or to merge with the extremist on a policy platform more moderate than in the equilibrium with sincere voting, depending on the size of s. Indeed, given these beliefs, the equilibrium under single round elections is perfectly analogous to the runoff equilibrium with attached voters described in Online Appendix I, except that we need to replace δ (the fraction of attached voters) with $1-s$ (the fraction of sincere voters) in the definition of h in Lemma 1. Intuitively, here the extremist strategic voters in single round elections behave like the non-attached voters under runoff elections with sincere voting. The moderate candidates thus know that they can capture some of the votes of the extremists even if running alone, and this reduces the extremists' bargaining power (or induces the moderates to run alone if s is large enough).

Strategic voting in runoff elections. Here strategic voting only bites in the first round, since in the second round with only two candidates strategic voters always find it optimal to vote sincerely. This immediately implies that the equilibrium with sincere voting in Proposition 3 remains an equilibrium even under strategic voting. To see this, note that, even if all voters are strategic, there is always a voting equilibrium in the first round where the two moderates pass the post with probability 1. Given this outcome and the absence of strategic voting in the second round, the proof of Proposition 3 immediately follows. In particular, the beliefs described above under single round elections are not compatible with equilibrium under runoff elections, if voters within each group can coordinate amongst themselves and act as a bloc (i.e. if they are bloc-strategic voters). Specifically, consider a four party system and suppose that (at the first ballot) all extremist voters vote for their own candidate. Then it cannot be optimal for the moderate voters as a bloc to also vote for the extremists, since by voting for their own moderate candidate, the two moderates pass the first round even without the support of the extremists. Hence, the sunspot beliefs described above are not consistent with any equilibrium under runoff elections.

Here too, however, other equilibria are possible, for some special configuration of parameters and if the fraction of strategic voters is not the same in all groups. Specifically, consider the model with attached voters, and suppose that the first round voting stage is reached with three candidates, say $\{1\}, \{2\}, \{3,4\}$. Here, provided that the attached voters are many, the strategic voters of groups (3,4) may find it optimal to converge part of their votes on candidate 1, so that this candidate rather than 2 reaches the final ballot with certainty. The reason is that, with many attached voters and more attached voters in group 2 than in group 1, party {3,4} wins for sure against candidate 1 in the second round.[5] For this

[5] A sufficient condition for this to happen is that $\delta\overline{\alpha} > 2e$. This behavior by voters in groups 3 and 4 is known as "push over" in the relevant literature; see Bouton and Gratton (2013).

first round outcome to be incentive compatible, however, strategic voters in group 1 must accept it without shifting their vote towards candidate 2; this may happen if the fraction of strategic voters in group 1 is sufficiently smaller than in groups 3 and 4.[6] Anticipating this result at the first round, candidate 2 is then induced to seek an agreement with 1 even at the price of an extremist policy platform. This example is rather special, of course, but it reverts the previous results, that runoff elections weaken the bargaining power of extremists and induce policy moderation.

Summing up, strategic voting adds considerable ambiguity to the predictions of our model. If strategic voters are few, nothing changes with respect to previous results. And even if strategic voters are many and act as a bloc, there are equilibria in which the contrast between single round vs runoff elections described above under sincere voting continues to hold or is even stronger. Nevertheless, other equilibria are possible if many voters are strategic and if they are unevenly distributed across groups. In some of these, strategic voting blurs the sharp distinction between the two electoral rules, inducing policy moderation under single round, or vice versa enhancing the bargaining power of extremists under runoff.[7]

Runoff system with attached voters

Proof of Lemma 1

Suppose that candidates 3 and 4 have merged, while candidate 2 runs alone. Consider the second round of voting. Given the behavior of the attached extremists in group 1, candidate 2 wins if:

$$(1-\delta)\underline{\alpha} + \overline{\alpha} + \eta > \underline{\alpha} + \overline{\alpha} - \eta \tag{9}$$

or more succinctly if:

$$\eta > \delta\underline{\alpha}/2$$

Since η is distributed over the interval $[-e, e]$, this event has probability :

$$1 - \Pr(\eta \leq \delta\underline{\alpha}/2) = 1/2 - h$$

and $1/2 > h > 0$, where the first inequality follows from $\delta\underline{\alpha}/2 > 0$ and the second inequality is implied by (A2). QED

[6] This can only happen if, given that all voters in groups 3 and 4 are strategic, the share of strategic voters in group 1 does not exceed $\frac{1}{6} + \frac{4}{3}e$.

[7] Not all these equilibria would survive suitable refinements of the equilibrium notion. For instance, Bouton and Gratton (2013) are able to rule out "push over" behavior in runoff elections by imposing strict perfection on equilibria.

We now describe the equilibrium.

Proposition 1 *Suppose that* $(A1)$, $(A2)$ *hold and that* $\lambda > 1/4$. *Define*

$$\bar{h} = \frac{V(R) - V(R - \bar{r})}{2[V(R - \bar{r}) + C(|t^3 - t^2|)}$$

(i) If $h < \bar{h}$, *then the unique equilibrium under runoff elections is a four-party system where all candidates run alone, and each moderate candidate wins with probability 1/2 on a policy platform that coincides with his bliss point and grabs all the rents if he wins.*

(ii) If $h > \bar{h}$, *then the unique equilibrium under runoff elections is a two party system where moderates and extremists merge on both sides and each party wins with probability 1/2. In this case, the equilibrium policies under runoff are always closer to the moderate candidates' bliss points, and moderate candidates get a larger share of rents if elected, than in the equilibrium under single round elections. Moreover, the smaller is* h, *the closer are the equilibrium policies under runoff election to the moderate candidates' bliss points, and the larger are the rents that go to the moderates if elected.*

(iii) If $h = \bar{h}$, *the equilibrium might either be a four party system or a two party system. In both cases, the equilibrium policies will coincide with the bliss points of the moderates.*

Proof

We repeat the steps in the proof of Proposition 3, but now taking into account the attached voters. Throughout we assume $\lambda > 1/4$ and that $(A1)$, $(A2)$ hold.

Suppose that 3 and 4 have not merged (or are expected not to merge). In this case, if 1 and 2 also run alone, the two moderate candidates win with probability 1/2 each on a policy platform corresponding to their respective bliss point, and their expected utility are still given by (4) and (5) respectively.

If 1 and 2 merge into a single party, they win with probability $1/2 + h$,and their expected utility can then be written as:

$$U^P(q, r^P; h) = (\frac{1}{2} + h)[V(r^p) - C(|q - t^P|)] - (\frac{1}{2} - h)C(|t^3 - t^P|), \quad P = 1, 2 \tag{10}$$

where $U^P(.)$ is now expressed also as a function of h. Consider candidate 2, and evaluate (10) at his most favorable policy and rent allocation, namely $q = t^2$ and $r^2 = R - \bar{r}$. He is indifferent between merging with 1 on these terms or running alone if:

$$U^2(t^2, R - \bar{r}; h) - \bar{u}^2 = 0 \tag{11}$$

Now solve (11) for h, and denote the solution by $\bar{h}$. Using (5) and (10) we get:

$$\bar{h} = \frac{V(R) - V(R - \bar{r})}{2[V(R - \bar{r}) + C(|t^3 - t^2|)}$$

For $h < \bar{h}$, candidate 2 prefers to run alone, and given the indivisibility of rents below $\bar{r}$, there is nothing that candidate 1 can do to induce him to merge. For $h > \bar{h}$, instead, the electoral advantage of merging is sufficiently large that candidate 2 is willing to merge with 1 for at least some feasible policy q and rent allocation, given that 3 and 4 run alone. Repeating the same procedure for candidate 1, it is easy to verify that 1 is always willing to merge with 2 on the terms most favorable for the latter (intuitively, he stands to gain the minimal rents and a higher probability of a policy closer to his bliss point). By symmetry, the same results holds for candidate 3, given that 1 and 2 run alone.

Now suppose that 3 and 4 have merged (or are expected to do so) on a policy platform of q^{34}. If 1 and 2 also merge, they win with probability 1/2, and their expected utility is given by

$$U^P(q, r^P) = \frac{1}{2}[V(r^P) - C(|q - t^P|) - C(|q^{34} - t^P|)] \tag{12}$$

If instead they run alone, then candidate 2 wins with probability $(1/2 - h)$ while candidate 1 has no chances. Hence their expected utilities are respectively:

$$\bar{u}^1(h) = -(\frac{1}{2} - h)C(|t^2 - t^1|) - (\frac{1}{2} + h)C(|q^{34} - t^1|) \tag{13}$$

$$\bar{u}^2(h) = (\frac{1}{2} - h)V(R) - (\frac{1}{2} + h)C(|q^{34} - t^2|) \tag{14}$$

where $\bar{u}^P(h)$ has been expressed as a function of h. Combining these expressions, we get:

$$U^1(q, r^1) - \bar{u}^1(h) = \frac{1}{2}[V(r^1) - C(|q - t^1|)] + (\frac{1}{2} - h)C(|t^2 - t^1|) + hC(|q^{34} - t^1|) \tag{15}$$

$$U^2(q, r^2) - \bar{u}^2(h) = \frac{1}{2}[V(r^2) - C(|q - t^2|)] - (\frac{1}{2} - h)V(R) + hC(|q^{34} - t^2|) \tag{16}$$

Again evaluate (16) at the policy and rent allocation most favorable for candidate 2, namely $q = t^2$ and $r^2 = R - \bar{r}$, and then solve $U^2(t^2, R - \bar{r}) - \bar{u}^2(h) = 0$ for h. Denoting the solution by $\underline{h}$, we get:

$$\underline{h} = \frac{V(R) - V(R - \bar{r})}{2[V(R) + C(|q^{34} - t^2|)}$$

Again, for $h > \underline{h}$, candidate 2 prefers to merge for at least some feasible policy and rent allocation, while he cannot be induced to merge if $h < \underline{h}$. Repeating the same procedure for candidate 1, again it can be verified that 1 is always willing to merge with 2 even on the terms most favorable to 2. By symmetry, similar results hold for 3 and 4, given that 1 and 2 have merged.

Note that $1/2 > \bar{h} > \underline{\mathrm{h}} > 0$, where the inequality $\bar{h} > \underline{h}$ follows from $q^{34} \geq t^3$ and $\bar{r} > 0$. Hence, combining these two results, we conclude that if $h < \underline{h}$ then the equilibrium is unique and consists of a four party system, where each moderate candidate wins with probability 1/2 on a policy platform that coincides with his bliss point. The reason is that, in stage 1 of the game when deciding on party formation, if $h < \underline{h}$ then it is a dominant strategy for the moderate candidate to say no to any merger proposal made by the extremists.

Conversely, if $h > \bar{h}$ then the equilibrium is unique and consists of a two party system where moderate and extremist candidates have merged on a policy platform and rent allocation that coincides with the Nash bargaining outcome (to be derived below), and both parties win with probability 1/2. The reason is that, in stage 1 of the game and if $h > \bar{h}$, it is a dominant strategy for both the moderate and the extremist to merge, irrespective of what the opponents do.

What happens if $\bar{h} > h > \underline{h}$? Note that both moderate candidates are better off in the four party equilibrium than in the two party equilibrium, since they have larger expected rents and (weakly) more favorable policies, and the probability of victory is 1/2 in both cases. But then, given that party formation occurs in sequence, the four party system is the unique equilibrium even in this range parameters. Specifically, the moderate candidate who speaks first will say no to any merger proposal received by the extremist, since he knows that, for $\bar{h} > h$, this will also induce the other moderate to reject any subsequent merger proposal by the other extremist. Hence here too the unique equilibrium is a four party system. Only in the knife-edge case $h = \bar{h}$, where the moderates are indifferent between a two-party and a four party system, there can be multiple equilibria, depending on the moderates' beliefs about what the moderate opponent will do.

Finally, we want to compare the Nash bargaining outcome under runoff elections with that under single round elections in the two party system. This can be achieved comparing (15) and (16) with (2). Specifically, holding fixed all equilibrium variables (q, r^P, q^{34}), define

$$G(h) \equiv \frac{U^1(q, r^1) - \bar{u}^1(h)}{U^2(q, r^2) - \bar{u}^2(h)} \tag{17}$$

where $U^P(q, r^P) - \bar{u}^P(h)$ are given by (15) and (16) respectively, corresponding to the expressions under runoff elections. The function $G(h)$ has the following properties. First, for $h = 1/2$, it reduces to the same expression under single round elections. This can be verified comparing (15) and (16) with (2). This in turn implies that, for $h = 1/2$, the Nash bargaining outcome is identical under the two electoral rules. Second, and holding (q, r^P, q^{34}), fixed, the function $G(h)$ is strictly decreasing in h. This can be verified from (15), (16) and the definition of $G(h)$.[8] This in turn implies that, for any $h < 1/2$, the moderate candidate 2 has more bargaining power under runoff elections than under single round elections, and hence the Nash equilibrium outcome characterized by (3) gives a policy closer to his bliss point and a rent allocation more favorable to him. Moreover, and by the same argument, the smaller is h, the closer is the policy to candidate 2's bliss point, and the larger is the share of rents that goes to this candidate. QED

Victory at the first round

Consider a three-party system consisting of say $\{1, 2\}$, $\{3\}$, and $\{4\}$. Let both ε_1 and ε_2 be distributed with density $f(.)$ and cumulative distribution $F(.)$ over the interval $[-e/2, e/2]$. As stated in the text, $f(.)$ is symmetric around 0 and ε_1 and ε_2 are independently distributed. The probability that $\{1, 2\}$ wins is: $\Pr(\varepsilon_1 > 0) + Pr(\varepsilon_1 \leq 0, \varepsilon_1 + \varepsilon_2 > 0) = 1/2 + \int_{-\frac{e}{2}}^{0} [1 - F(-\varepsilon_1)] f(\varepsilon_1) d\varepsilon_1$, where we have used the fact that $Pr(\varepsilon_1 + \varepsilon_2 > 0) = 1 - F(-\varepsilon_1)$. The handicap of running alone for candidate 3 is thus

$$h = \int_{-\frac{e}{2}}^{0} [1 - F(-\varepsilon_1)] f(\varepsilon_1) d\varepsilon_1 = 1/2 - \int_{-\frac{e}{2}}^{0} F(-\varepsilon_1) f(\varepsilon_1) d\varepsilon_1.$$

Note that:

(i) $\int_{-\frac{e}{2}}^{e/2} F(-\varepsilon_1) f(\varepsilon_1) d\varepsilon_1 = \int_{-\frac{e}{2}}^{0} F(-\varepsilon_1) f(\varepsilon_1) d\varepsilon_1 + \int_{0}^{e/2} F(-\varepsilon_1) f(\varepsilon_1) d\varepsilon_1 = 1/2$, where the last equality follows from the assumption that ε_1 and ε_2 are independently and symmetrically distributed around 0.

(ii) $\int_{-\frac{e}{2}}^{0} F(-\varepsilon_1) f(\varepsilon_1) d\varepsilon_1 > \int_{0}^{e/2} F(-\varepsilon_1) f(\varepsilon_1) d\varepsilon_1 > 0$, since $F(.)$ is increasing and $f(.)$ is symmetric around zero.

Combining (i) and (ii), we have that $1/2 > \int_{-\frac{e}{2}}^{0} F(-\varepsilon_1) f(\varepsilon_1) d\varepsilon_1 > 1/4$, implying that $1/4 > h > 0$.

In the special case in which ε_1 and ε_2 are both uniformly distributed over $[-e/2, e/2]$ with density $1/e$, we have:

$$h = \int_{-\frac{e}{2}}^{0} [1 - F(-\varepsilon_1)] f(\varepsilon_1) d\varepsilon_1 = \frac{1}{e} \int_{-\frac{e}{2}}^{0} (\frac{1}{2} + \frac{\varepsilon_1}{e}) d\varepsilon_1 = 1/8$$

[8] Specifically, after some algebra, the sign of $G_h(h)$ is the same as that of the following expression:

$$-[C(q^{34}) - C(t^2)] \left[V(R) - V(r^2) + C(t^2 - q)\right] - [V(r^1) + C(t^2) - C(q)][V(R) + C(q^{34} - t^2)]$$

It is easy to verify that the sign of all square brackets is positive, as $q^{34} > t^3 > t^2 > q$, and $V(R) > V(r^2)$.

since $\frac{1}{e}\int_{-\frac{e}{2}}^{0}(\frac{1}{2}+\frac{\varepsilon_1}{e})d\varepsilon_1 = \frac{1}{4}+\frac{1}{2e^2}((0)^2-(\frac{e^2}{4})) = \frac{1}{4}-\frac{1}{8} = \frac{1}{8}$.

Equilibrium with endorsements

In this section we discuss what happens when extremists are allowed to endorse the moderates after the first round of voting (if the latter accept). Recall the assumption that $\eta = \varepsilon_1 + \varepsilon_2$, where ε_1 and ε_2 are independently and identically distributed, with a uniform distribution over the interval $[-e/2,\ e/2]$. Exploiting the properties of uniform distributions, we obtain that η is distributed over the interval $[-e, e]$, it has zero mean, and a symmetric cumulative distribution given by

$$\begin{aligned} G(z) &= \frac{1}{2}+\frac{z}{e}-\frac{z^2}{2e^2} \text{ for } e \geq z \geq 0 \\ G(z) &= \frac{1}{2}+\frac{z}{e}+\frac{z^2}{2e^2} \text{ for } -e \leq z \leq 0 \end{aligned}$$

We start with the case in which rents are not contractable at the endorsement stage, and in case of victory the endorsing extremist gets rents $\bar{r}$ while the endorsed moderates retains rents $R-\bar{r}$. Suppose that both moderate candidates have passed the first round and that no coalition has formed before the first round. Define

$$\breve{\varepsilon} \equiv \frac{\delta\underline{\alpha}\left[V(R)+C(|t^2-t^1|)\right]}{2\left[V(R)-V(R-\bar{r})\right]}-\frac{e}{2} \gtrless 0$$

We have:

Lemma 2 *Irrespective of what candidate 3 does, candidate 2 prefers to be endorsed by candidate 1 if $\varepsilon_1 < \breve{\varepsilon}-\frac{\delta\underline{\alpha}}{2}$, and he prefers no endorsement if $\varepsilon_1 > \breve{\varepsilon}$. In between, if $\breve{\varepsilon}-\frac{\delta\underline{\alpha}}{2} \leq \varepsilon_1 \leq \breve{\varepsilon}$, then 2 prefers to seek the endorsement of the extremist if 3 has also been endorsed, while 2 prefers no endorsement if 3 has not been endorsed. Candidate 3 behaves symmetrically (in the opposite direction), depending on whether $-\varepsilon_1$ is below or above these same thresholds.*

Proof

Suppose that both 2 and 3 have been endorsed by their extremist neighbors. By our previous assumptions, candidate 2 wins if $\varepsilon_1+\varepsilon_2>0$. When decisions over endorsements are made, the realization of ε_1 is known, but ε_2 is not. Hence the probability that candidate 2 wins is

$$\Pr(\varepsilon_2 > -\varepsilon_1) = \frac{1}{2}+\frac{\varepsilon_1}{e} \tag{18}$$

where the RHS follows from the assumptions on the distribution of the two electoral shocks.

Candidate 2's expected utility is:

$$(\frac{1}{2}+\frac{\varepsilon_1}{e})V(R-\bar{r})-(\frac{1}{2}-\frac{\varepsilon_1}{e})C(|t^3-t^2|) \tag{19}$$

Suppose instead that 2 refuses the endorsement of 1, while 3 is endorsed by 4. Now 2 loses the support of $\delta\underline{\alpha}$ voters, the attached extremists in group 1, while 3 carries all voters in group 4. Hence, repeating the analysis in (9), the probability that 2 wins is:

$$\Pr(\varepsilon_2 > \frac{\delta\underline{\alpha}}{2}-\varepsilon_1)=\frac{1}{2}+\frac{\varepsilon_1}{e}-\frac{\delta\underline{\alpha}}{2e} \tag{20}$$

if $\varepsilon_1 \geq \frac{\delta\underline{\alpha}}{2}-\frac{e}{2}$, and it is 0 if $\varepsilon_1 < \frac{\delta\underline{\alpha}}{2}-\frac{e}{2}$. Candidate 2's expected utility is then:

$$(\frac{1}{2}+\frac{\varepsilon_1}{e}-\frac{\delta\underline{\alpha}}{2e})V(R)-(\frac{1}{2}-\frac{\varepsilon_1}{e}+\frac{\delta\underline{\alpha}}{2e})C(|t^3-t^2|)$$

provided that the first expression in brackets is strictly positive and the second expression in brackets is strictly less than 1, which again occurs if $\varepsilon_1 \geq \frac{\delta\underline{\alpha}}{2}-\frac{e}{2}$. If instead $\varepsilon_1 < -\frac{e}{2}+\frac{\delta\underline{\alpha}}{2}$, then the probability that 2 wins is 0 and his expected utility is $-C(|t^3-t^2|)$.[9]

Equalizing the two expected utilities, candidate 2 is indifferent between these two alternatives if:

$$\varepsilon_1 = \breve{\varepsilon} \equiv \frac{\delta\underline{\alpha}\,[V(R)+C(|t^3-t^2|)]}{2\,[V(R)-V(R-\bar{r})]}-\frac{e}{2} \tag{21}$$

Note that $\frac{e}{2} > \breve{\varepsilon} > -\frac{e}{2}$, where the first inequality follows from (A2) and the second by inspection of the equation above. If $\varepsilon_1 > \breve{\varepsilon}$ then candidate 2 strictly prefers no endorsement, given that 3 has been endorsed. While if $\varepsilon_1 < \breve{\varepsilon}$ then candidate 2 strictly prefers to be endorsed, given that 3 has been endorsed.

Next, suppose that no moderate candidate has been endorsed by the extremist. By symmetry, the probability that 2 wins if he is not endorsed is still described by (18), but, as 2 does not have to share rents with 1 if elected, his expected utility is now

$$(\frac{1}{2}+\frac{\varepsilon_1}{e})V(R)-(\frac{1}{2}-\frac{\varepsilon_1}{e})C(|t^3-t^2|) \tag{22}$$

If instead candidate 2 accepts to be endorsed and 3 refuses, the probability that 2 wins is:

$$\Pr(\varepsilon_2 > -\frac{\delta\underline{\alpha}}{2}-\varepsilon_1)=\frac{1}{2}+\frac{\varepsilon_1}{e}+\frac{\delta\underline{\alpha}}{2e} \tag{23}$$

[9] By (A2), the first expression in brackets is always strictly less than 1 and the second expression in brackets is always positive.

if $\varepsilon_1 \leq \frac{e}{2} - \frac{\delta\alpha}{2}$ and it is 1 if $\varepsilon_1 > \frac{e}{2} - \frac{\delta\alpha}{2}$.[10] In this case, candidate 2's expected utility is:

$$(\frac{1}{2} + \frac{\varepsilon_1}{e} + \frac{\delta\alpha}{2e})V(R - \bar{r}) - (\frac{1}{2} - \frac{\varepsilon_1}{e} - \frac{\delta\alpha}{2e})C(|t^3 - t^2|)$$

provided that the first expression in brackets is strictly less than 1 and the second expression in brackets is strictly positive, which occurs if $\varepsilon_1 \leq \frac{e}{2} - \frac{\delta\alpha}{2}$. If instead $\varepsilon_1 > \frac{e}{2} - \frac{\delta\alpha}{2}$, then the probability that 2 wins is 1 and his expected utility cannot exceed $V(R - \bar{r})$.[11]

Candidate 2 is then indifferent between these two options if $\varepsilon_1 = \breve{\varepsilon} - \frac{\delta\alpha}{2}$. If $\varepsilon_1 > \breve{\varepsilon} - \frac{\delta\alpha}{2}$ then candidate 2 strictly prefers no endorsement, given that 3 has not been endorsed. While if $\varepsilon_1 < \breve{\varepsilon} - \frac{\delta\alpha}{2}$ then candidate 2 strictly prefers to be endorsed, given that 3 has not been endorsed.

By symmetry, 3 has similar preferences, but in the opposite direction and with respect to the symmetric thresholds $-\breve{\varepsilon} + \frac{\delta\alpha}{2}$ and $-\breve{\varepsilon}$ (eg. 3 prefers no endorsement, given that 2 has not been endorsed, if $\varepsilon_1 < -\breve{\varepsilon} + \frac{\delta\alpha}{2}$, and 3 prefers no endorsement, given that 2 has been endorsed, if $\varepsilon_1 < -\breve{\varepsilon}$). QED

Invoking Lemma 2, we now describe the equilibrium continuation if the two moderate candidates have passed the first round and compete over the second round. Equilibrium endorsements depend on whether the thresholds in Lemma 2 are positive or negative. These thresholds are positive for high values of δ (the fraction of attached voters) and low values of $\bar{r}$ (the minimal rents that have to be left to the extremists). This in turn increases the willingness of the moderates to accept endorsements. This provides the intuition for the proposition to follow. Specifically, under (A1-A2), we have:

Proposition 2 *(i) Suppose $\breve{\varepsilon} - \frac{\delta\alpha}{2} > 0$. Then, the equilibrium is unique and at least one moderate candidate accepts the endorsement of the ideologically closer extremist. Specifically, if $\varepsilon_1 > \breve{\varepsilon}$, 3 accepts the endorsement while 2 does not. Symmetrically, if $\varepsilon_1 < -\breve{\varepsilon}$, 2 accepts the endorsement while 3 does not. For all other realizations of ε_1, both 2 and 3 accept the endorsements.*

(ii) Suppose that $\breve{\varepsilon} < 0$. Then, the equilibrium is unique and at most one of the two moderate candidates accepts the endorsement of his extremist neighbor. Specifically, if $\varepsilon_1 < \breve{\varepsilon} - \frac{\delta\alpha}{2}$, 2 accepts the endorsement while 3 does not. Symmetrically, if $\varepsilon_1 > -\breve{\varepsilon} + \frac{\delta\alpha}{2}$, 3 accepts the endorsement while 2 does not. For all other realizations of ε_1, neither 2 nor 3 accept the endorsements.

(iii)Suppose $\breve{\varepsilon} \geq 0 \geq \breve{\varepsilon} - \frac{\delta\alpha}{2}$. Here, there are two cases to consider. If $\frac{\delta\alpha}{2} \geq 2\breve{\varepsilon}$, then the equilibrium is identical to the one described under point (ii). If $\frac{\delta\alpha}{2} < 2\breve{\varepsilon}$, then the

[10]By (A2), $\Pr(\varepsilon_2 > \frac{\delta\alpha}{2} - \varepsilon_1) < 1$ and $\Pr(\varepsilon_2 > -\frac{\delta\alpha}{2} - \varepsilon_1) > 0$ for any $\varepsilon_1 \in [-e/2, e/2]$.

[11]Assumption (A2) implies that the first expression in brackets is always positive and the second one is always less than 1.

equilibrium is unique and depending on the realization of ε_1, both moderates are endorsed by the extremists, none are, or one moderate only is endorsed by the closer extremist.

Proof of Proposition 6

Suppose first that $\breve{\varepsilon} - \frac{\delta\alpha}{2} > 0$. This then implies $\breve{\varepsilon} > 0 > -\breve{\varepsilon} + \frac{\delta\alpha}{2}$.The equilibrium is illustrated in Figure A1. If $\varepsilon_1 > \breve{\varepsilon}$ by Lemma 2, 2 does not accept the endorsement of 1 whatever 3 does; and as $\varepsilon_1 > \breve{\varepsilon}$ implies $\varepsilon_1 > 0 > -\breve{\varepsilon} + \frac{\delta\alpha}{2}$, 3 accepts the endorsement of 4 even if 2 is not endorsed. By symmetry, if $\varepsilon_1 < -\breve{\varepsilon}$, 3 does not accept to be endorsed, while 2 is endorsed. If $\varepsilon_1 \in [-\breve{\varepsilon} + \frac{\delta\alpha}{2}, \breve{\varepsilon} - \frac{\delta\alpha}{2}]$, then both moderates find it optimal to seek the endorsement of the extremists, no matter what their opponent does. If $\varepsilon_1 \in (\breve{\varepsilon} - \frac{\delta\alpha}{2}, \breve{\varepsilon}]$, then candidate 3 still finds it optimal to seek the endorsement of 4 no matter what 2 does; and given 3's behavior, 2 also finds it optimal to seek the endorsement of 1. The same conclusion holds, but with the roles of 2 and 3 reversed, if $\varepsilon_1 \in [-\breve{\varepsilon} + \frac{\delta\alpha}{2}, -\breve{\varepsilon})$.

Next suppose that $\breve{\varepsilon} < 0$. This then implies $\breve{\varepsilon} - \frac{\delta\alpha}{2} < \breve{\varepsilon} < 0$ and $-\breve{\varepsilon} + \frac{\delta\alpha}{2} > -\breve{\varepsilon} > 0$. This equilibrium is illustrated in Figure A2. If $\varepsilon_1 \in [\breve{\varepsilon}, -\breve{\varepsilon}]$, then both moderates find it optimal to seek no endorsement, no matter what their opponent does. If $\varepsilon_1 \in [\breve{\varepsilon} - \frac{\delta\alpha}{2}, \breve{\varepsilon})$, 3 does not seek for an endorsement as $\varepsilon_1 < \breve{\varepsilon} < -\breve{\varepsilon}$, and given 3's behavior then candidate 2 also seeks no endorsement. If $\varepsilon_1 < \breve{\varepsilon} - \frac{\delta\alpha}{2}$ 2 seeks an endorsement no matter what 3 does, and 3 does not seek an endorsement for the same reason spelled above. The same conclusion holds, but with the roles of 2 and 3 reversed, if $\varepsilon_1 \in (-\breve{\varepsilon}, -\breve{\varepsilon} + \frac{\delta\alpha}{2}]$. Finally, if $\varepsilon_1 > -\breve{\varepsilon} + \frac{\delta\alpha}{2}$ then candidate 2 still finds it optimal to seek no endorsement no matter what 3 does, while 3 finds it optimal to seek the endorsement of 4 no matter what 2 does.

Finally, suppose that $\breve{\varepsilon} > 0 > \breve{\varepsilon} - \frac{\delta\alpha}{2}$. Suppose also that $\frac{\delta\alpha}{2} \geq 2\breve{\varepsilon}$, so that $-\breve{\varepsilon} + \frac{\delta\alpha}{2} \geq \breve{\varepsilon}$ and $\breve{\varepsilon} - \frac{\delta\alpha}{2} \leq -\breve{\varepsilon}$. This equilibrium is illustrated in Figure A3. As shown in the Figure, if $\varepsilon_1 \in [-\breve{\varepsilon}, \breve{\varepsilon}]$ each moderate candidate would accept to be endorsed only if the other moderate is also endorsed. However, by the assumed sequentiality of the endorsement proposals, the first moderate receiving an offer of endorsement by the closer extremist would always be better off by refusing this offer, knowing that this will induce the other moderate to refuse the offer by the other extremist as well. Hence, for $\varepsilon_1 \in [-\breve{\varepsilon}, \breve{\varepsilon}]$ no endorsement occurs. No endorsement also occur if $\varepsilon_1 \in [\breve{\varepsilon} - \frac{\delta\alpha}{2}, -\breve{\varepsilon})$ (or symmetrically, if $\varepsilon_1 \in [\breve{\varepsilon}, -\breve{\varepsilon} + \frac{\delta\alpha}{2}))$ as at least one moderate always prefers to run alone and the other accepts to be endorsed only if the other moderate is endorsed. Hence, in this case at most one moderate is endorsed, 2 if $\varepsilon_1 < \breve{\varepsilon} - \frac{\delta\alpha}{2}$ and 3 if $\varepsilon_1 > -\breve{\varepsilon} + \frac{\delta\alpha}{2}$. Suppose next that $\frac{\delta\alpha}{2} < 2\breve{\varepsilon}$ so that $-\breve{\varepsilon} + \frac{\delta\alpha}{2} < \breve{\varepsilon}$ and $\breve{\varepsilon} - \frac{\delta\alpha}{2} > -\breve{\varepsilon}$. This equilibrium is illustrated in Figure A4. Here, if $\varepsilon_1 \in [\breve{\varepsilon} - \frac{\delta\alpha}{2}, -\breve{\varepsilon} + \frac{\delta\alpha}{2}]$ for the previous argument, no candidate accepts to be endorsed. For $\varepsilon_1 \in (-\breve{\varepsilon} + \frac{\delta\alpha}{2}, \breve{\varepsilon}]$ 3 always accepts to be endorsed and 2 too accepts to be endorsed if he expect 3 to be endorsed.

Hence, both candidates are endorsed. Symmetrically, for $\varepsilon_1 \in [-\breve{\varepsilon}, \breve{\varepsilon} - \frac{\delta\alpha}{2})$, 2 always accepts to be endorsed and 3 too accepts to be endorsed if he expect 2 to be endorsed. Hence, both candidates are endorsed. Finally, for $\varepsilon_1 > \breve{\varepsilon}$ or $\varepsilon_1 < -\breve{\varepsilon}$ only one candidate is endorsed, 2 in the former case and 3 in the latter. QED

A centrist party

Single round elections. Consider first the party formation stage. As stated in the text, if $\underline{\alpha} \geq \alpha^c$ and $\lambda > 1/4$, it is immediate to show that the extremist party always merges with the moderate. Whether the centrist party is also included or not in one of the coalitions, depends on its size. If α^c is small, then including it is not worth the cost of rents and policy accommodation that this would require in the subsequent bargaining stage. In this case, the equilibrium under single round elections is identical to that described in Section 3. If α^c is sufficiently large, then the increase in the probability of victory compensates for the cost of including it. In this case, whether c merges with $\{1,2\}$ or $\{3,4\}$ depends on the order of moves at the party formation stage.

Suppose that α^c is large enough and party $\{1,2,c\}$ has formed. How are policy and rents determined by this party? The first step is to compute the disagreement points of all candidates. Since disagreement implies unilateral party breakup, we have:

$$\bar{u}^P = -C(q^{34} - t^P), \qquad P = 1, 2, c$$

where as before q^{34} denotes the equilibrium policy set by party $\{3,4\}$. We thus have:

$$\begin{aligned} U^P(q, r^P) &= p[V(r^P) - C(|q - t^P|) - (1-p)C(q^{34} - t^P)], \qquad P = 1, 2, c \\ U^P(q, r^P) - \bar{u}^P &= p[V(r^P) - C(|q - t^P|) + C(q^{34} - t^P)], \qquad P = 1, 2, c \end{aligned} \tag{24}$$

where $p = \Pr[\eta \geq -\frac{\alpha^c}{2}]$ denotes the probability that party $\{1,2,c\}$ wins. The Nash bargaining outcome is the solution to the problem of maximizing $\{[U^1(q,r^1) - \bar{u}^1][U^2(q,r^2) - \bar{u}^2]$ $[U^c(q,r^c) - \bar{u}^c]\}$ by choice of q, r^1, r^2, r^c. After some transformations, at an interior optimum the first order conditions of this problem imply:

$$\begin{aligned}\sum\nolimits_{1,2,c} \frac{U_q^P(q,r^P)}{U_r^P(q,r^P)} &= 0 \\ \frac{U_r^2(q,r^2)}{U_r^c(q,r^c)} &= \frac{U^2(q,r^2)-\bar{u}^2}{U^c(q,r^c)-\bar{u}^c} \\ \frac{U_r^2(q,r^2)}{U_r^1(q,r^1)} &= \frac{U^2(q,r^2)-\bar{u}^2}{U^1(q,r^1)-\bar{u}^1}\end{aligned} \tag{25}$$

Manipulation of these conditions can be shown to imply $q > t^2$, confirming the intuition stated in the text that the centrist party has more bargaining power than the extremist, despite its possibly smaller size.

Runoff elections. Repeating the same logic as in Section 3, it is easy to show that moderate candidates never want to merge with the extremists, since they can capture the extremists vote at the second round. What about a merger with the centrist? If say candidate 2 merges with the centrist, its expected utility cannot exceed $pV(R-\bar{r})-(1-p)C(|t^3-t^2|)$ (recall that in the equilibrium under runoff elections, candidate 3 does not merge with 4), where as above $p = \Pr[\eta \geq -\frac{\alpha^c}{2}]$. If instead no such merger takes place and both 2 and 3 run alone, then candidate 2's expected utility is $\frac{1}{2}[V(R)-C(|t^3-t^2|)]$. Combining these two expressions, we obtain that the moderates prefers to merge with the centrist candidate rather than to run alone, given that the other moderate is running alone, if

$$\Pr(\eta > -\frac{\alpha^c}{2}) > \frac{V(R)+C(t^3-t^2)}{2[V(R-\bar{r})+C(t^3-t^2)]} \tag{26}$$

If p exceeds the threshold on the RHS of (26), then there is a feasible combination of q and r^2 that leaves candidates 2 and c better off with a merger than without it - of course candidate c has nothing to loose from such merger.[12]

If condition (26) is satisfied and party $\{2,c\}$ is formed, then policy and rents inside this party are set according to the Nash bargaining outcome. Repeating the procedure in the previous proofs, the Nash bargaining solution implies:

$$\begin{aligned}\sum\nolimits_{2,c} \frac{U_q^P(q,r^P)}{U_r^P(q,r^P)} &= 0 \\ \frac{U_r^2(q,r^2)}{U_r^c(q,r^c)} &= \frac{U^2(q,r^2)-\bar{u}^2}{U^c(q,r^c)-\bar{u}^c}\end{aligned} \tag{27}$$

[12]Note that the LHS of (26) increases in α^c, and the RHS equals 1/2 if $\bar{r}=0$ while it rises above 1/2 as $\bar{r}$ increases above 0. Hence (26) is certainly consistent with $\alpha^c \leq \underline{\alpha}$ for sufficiently small $\bar{r}$ or large $\underline{\alpha}$.

where now $U^P(q, r^P) - \bar{u}^P = p[V(r^P) - C(|q - t^P|) + C(t^3 - t^P)]$, which is the same expression as in (24), except that q^{34} has been replaced by t^3. Manipulation of these first order conditions can be shown to imply that $q > \frac{\lambda}{2}$, which is the mid point between t^2 and $1/2$.

Comparing single round vs runoff elections. Clearly single round elections have a smaller equilibrium number of parties than runoff elections, since in the latter the extremists are always on their own. What about policy moderation?

Obviously party $\{3\}$ under runoff has a more moderate policy than party $\{3, 4\}$ under single round elections. The comparison between party $\{2, c\}$ under runoff and party $\{1, 2, c\}$ under single round is more subtle, however. On the one hand, the extremist candidate is only included under single round elections, and this pushes party $\{1, 2, c\}$ towards a more extreme policy than party $\{2, c\}$. This can be seen formally by noting that the summation of the marginal rates of substitutions between rents and policies includes the extremist candidate in (25) but not in (27). On the other hand, the bargaining power of the centrist candidate is stronger under single round elections than under runoff. The reason is that the opponent runs on a more extreme policy under single round (q^{34}) than under runoff (t^3), and the threat of electoral defeat is less fearsome for the centrist candidate than for the other party members, the more so the more extreme is the policy platform of the opponent. This can be seen by comparing the remaining expressions in (25) vs (27). Hence, a priori and without additional restrictions on functional form we cannot rule out the possibility that, despite the inclusion of the extremist candidate, party $\{1, 2, c\}$ under single round elections enacts a more moderate policy than party $\{2, c\}$ under runoff.

Relaxing the restrictions on party formation

Here we allow the formation of parties consisting of up to *three* adjacent candidates, and show that Propositions 1-3 still identically hold provided that candidates care sufficiently about policy relative to rents and that polarization (i.e., λ) is sufficiently high. Though we assume that there are no attached voters, although with suitable changes to the proofs and conditions the results with attached voters would also go through. Below we discuss the possible formation of a three candidate party consisting of $\{1, 2, 3\}$; given symmetry, the proposition below holds identically for a party consisting of $\{2, 3, 4\}$.

Before going through a formal proof, here is the intuition. If $\lambda > 1/4$ and if party $\{1, 2, 3\}$ was formed, it would have to run on a policy sufficiently close to the bliss point of candidate 3, t^3; otherwise all moderate voters in group 3 would be lost to extremist candidate 4. Specifically, the policy set by $\{1, 2, 3\}$ would have to satisfy $q \geq 2\lambda$, where $q = 2\lambda$ is such that $t^4 - t^3 = t^3 - q$, so that group 3 voters are indifferent between q and t^4.

If this constraint is satisfied, then party $\{1,2,3\}$ wins the election with certainty, otherwise it wins with probability 1/2. But it only makes sense to form party $\{1,2,3\}$ if it wins with certainty, because otherwise the extremist and at least one moderate candidate would be strictly better off with the symmetric two party system $\{1,2\}$ and $\{3,4\}$. Of course, the constraint $q \geq 2\lambda$ benefits candidate 3, but hurts candidates 1 and 2. If candidates care sufficiently about policy relative to rents and if λ is sufficiently high, then either candidate 1 or candidate 2 cannot be compensated enough for this unpleasant policy choice through a more favorable rent allocation, and party $\{1,2,3\}$ is not formed in equilibrium.

When discussing the possible formation of party $\{1,2,3\}$, we need to be explicit about what is the disagreement point under which Nash bargaining is conducted inside this party. It is natural to assume that disagreement implies that the party breaks up and no further renegotiation about party formation is possible.

Consider first single round elections. We start with the following (r^{*P} and q^* denote the equilibrium outcomes described in Proposition 1).

Lemma 3 *If the following condition is satisfied*

$$Max\left\{[C(2\lambda - t^2) + \frac{1}{2}V(R) - \frac{1}{2}C(t^3 - t^2)], [C(2\lambda) + \frac{1}{2}V(r^{*1}) - \frac{1}{2}C(q^*) - \frac{1}{2}C(1-q^*)]\right\}$$
$$> \ V(R - 2\bar{r}) \tag{28}$$

then under single round elections there is no feasible outcome under party $\{1,2,3\}$ *that leaves both candidates 1 and 2 better off than in the equilibrium outcome of some other feasible party system.*

Proof

Consider candidate 1. His most favorable outcome under party $\{1,2,3\}$ is that he gets all the feasible rents, $r^1 = R - 2\bar{r}$, and the policy is as low as possible subject to the constraint of winning with certainty, namely, $q = 2\lambda$. In this case the utility of candidate 1 is $V(R - 2\bar{r}) - C(2\lambda)$. His best alternative to party $\{1,2,3\}$ is a symmetric two party system. By Proposition 1, in the equilibrium outcome of a two party system, candidate 1 gets rents r^{*1} (if $\{1,2\}$ win the election) and the policy is q^* if $\{1,2\}$ win and $1 - q^*$ otherwise. Hence in the symmetric two party equilibrium the expected utility of candidate 1 is: $\frac{1}{2}V(r^{*1}) - \frac{1}{2}C(q^*) - \frac{1}{2}C(1-q^*)$, and candidate 1 prefers this symmetric equilibrium outcome to any feasible outcome under party $\{1,2,3\}$ if:

$$C(2\lambda) + \frac{1}{2}V(r^{*1}) - \frac{1}{2}C(q^*) - \frac{1}{2}C(1-q^*) > V(R - 2\bar{r}) \tag{29}$$

Next consider candidate 2. His most favorable outcome under party $\{1, 2, 3\}$ is that he gets all the feasible rents, $r^2 = R - 2\bar{r}$, and the policy is again as low as possible, namely $q = 2\lambda$. In this case the utility of candidate 2 is $V(R - 2\bar{r}) - C(2\lambda - t^2)$. From his perspective, the best alternative to party $\{1, 2, 3\}$ is a four party system in which all candidates run alone. Candidate 2 expected utility in this case is: $\frac{1}{2}V(R) - \frac{1}{2}C(t^3 - t^2)$. Hence, candidate 2 prefers the four party system to any feasible outcome under party $\{1, 2, 3\}$ if:

$$C(2\lambda - t^2) + \frac{1}{2}V(R) - \frac{1}{2}C(t^3 - t^2) > V(R - 2\bar{r}) \tag{30}$$

Combining these two inequalities we get (A3). QED

We are now ready to state our first result.

Proposition 3 *Consider single round elections. If (A3) is satisfied, a party resulting from the merger of three candidates cannot formed in equilibrium, and Propositions 1 and 2 hold.*

Proof

Start with single round elections and suppose that $\lambda > 1/4$. Let e and m (or e', m') denote an extremist and moderate candidate respectively, and index by s the substages of the party formation stage. Since there are four candidates, and at most each one of them has a proposal right, $s = 1, 2, 3, 4$ (or less if a partition is reached before everyone has made a proposal). We first prove the following:

Lemma 4 *If party $\{e, m, m'\}$ is formed, this can only happen for $s = 1, 2$.*

Proof of Lemma 4

Consider the last substage, $s = 4$. If it is reached with a four party system, then the proposer will propose party $\{e, m\}$ (or $\{e', m'\}$) and this proposal will be accepted. This being the last substage of the game, such a party will win the election for sure, leaving both e and m better off than in the four party system.

Now consider $s = 3$, and suppose again that it is reached with a four party system. Anticipating the outcome $s = 4$, a party consisting of $\{e, m\}$ (or $\{e', m'\}$) will again be formed at $s = 3$. The reason is that leaving a four party system to whoever will make a proposal at $s = 4$ is suboptimal (strictly or weakly depending on the identity of the proposers). And proposing party $\{e, m, m'\}$ is also suboptimal, because either m or m' will reject this proposal, anticipating that in $s = 4$ they will be able to merge with the nearby extremist, and thus win the election for sure. This completes the proof of Lemma 3.

Now consider $s = 1$ or 2. We now prove the following:

Lemma 5 *If (A3) holds, the extremist candidates (say e) will never propose party $\{e, m, m'\}$ in $s = 1, 2$, and will always say no to any proposal to form such a party.*

Proof of Lemma 5

Condition A3 says that either: (i) candidate e prefers the symmetric two party equilibrium to the outcome under party $\{e, m, m'\}$; or (ii) candidate m or m' prefers the four party system to the outcome under party $\{e, m, m'\}$. Consider case (i). By Lemma 4 and the proof therein, if party $\{e, m, m'\}$ is not formed in $s = 1, 2$, it will also not be formed in later substages. By the reasoning in Proposition 1, we will then have a symmetric two party system, which by case (i) of (A3) is better than the outcome under $\{e, m, m'\}$ for extremist candidates. Hence the extremist will never allow party $\{e, m, m'\}$ to be formed. Next, consider case (ii). Here, if party $\{e, m, m'\}$ was formed, it will not survive once the bargaining stage is reached, since by case (ii) of (A3) candidate m is strictly better off by breaking the party and moving to the four party system (recall the restriction that a party must consist of adjacent candidates, and if a party breaks up then no renegotiation can take place amongst candidates). But this disagreement outcome is worse than the two party equilibrium from the perspective of extremist candidates, who will thus veto the formation of party $\{e, m, m'\}$ in case (ii) as well. This completes the proof of Lemma 5.

If the party $\{e, m, m'\}$ is not formed in equilibrium and $\lambda > 1/4$, then Proposition 1 holds. Finally suppose that $\lambda \leq 1/4$. Then the centrist party $\{m, m'\}$ is viable and dominates any other party from the perspective of both moderates. Hence, Proposition 2 always holds. QED

Next, turn to runoff elections. Here the two party system cannot be reached in equilibrium, so we need the following result instead:

Lemma 6 *If the following condition is satisfied*

$$Max\left\{[C(2\lambda - t^2) + \frac{1}{2}V(R) - \frac{1}{2}C(t^3 - t^2)], [C(2\lambda) - \frac{1}{2}C(t^2) - \frac{1}{2}C(t^3)]\right\} > V(R - 2\bar{r}) \tag{A4}$$

then there is no feasible outcome under party $\{1, 2, 3\}$ *that leaves both candidates 1 and 2 better off than under a four party system.*

Proof of Lemma 6

Consider candidate 1. Under the four party system, candidate 1 gets an expected utility of $-\frac{1}{2}C(t^2) - \frac{1}{2}C(t^3)$. Hence candidate 1 prefers the four party system to any feasible outcome under party $\{1, 2, 3\}$ if:

$$C(2\lambda) - \frac{1}{2}C(t^2) - \frac{1}{2}C(t^3) > V(R - 2\bar{r}) \tag{31}$$

Combining (31) and(30), we get (A4). QED

We can then prove:

Proposition 4 *Consider runoff elections. If (A4) is satisfied, a party resulting from the merger of three candidates cannot be formed in equilibrium, and Proposition 3 holds.*

Here the proof is simpler, since if party $\{e, m, m'\}$ is not formed, then by Proposition 3 we end up with a four party system. But by condition (A4), either the moderates or the extremists prefer the four party system to the most favorable outcome under party $\{e, m, m'\}$. Hence under (A4) there is always a candidate who will veto the formation of $\{e, m, m'\}$, and thus Proposition 3 holds. QED

Note that (29) is less restrictive than (31), since candidate 1 prefers the symmetric two party equilibrium to the outcome under four parties, so that condition (A3) is less restrictive than (A4). Intuitively, runoff elections reduce the bargaining power of extremist candidates, and so extremists are more likely to favor party $\{1, 2, 3\}$ than under single round elections. That is, to rule out the formation of party $\{1, 2, 3\}$ we need to impose a more restrictive condition.

Finally, to better assess the implications of conditions (A3-A4), suppose that the function $C(x)$ takes the form $C(x) = \sigma x^2$. Then after some algebra conditions (A3) and (A4) can be rewritten respectively as:

$$\begin{aligned} & Max\left\{[\frac{1}{2\sigma}V(R) + 7\lambda^2 - 3\lambda + \frac{1}{4}], [\frac{1}{2\sigma}V(r^{*1}) + 4\lambda^2 + q^*(1-q^*) - \frac{1}{2}]\right\} \\ > \;& V(R - 2\bar{r})/\sigma \end{aligned} \tag{32}$$

$$\begin{aligned} & Max\left\{[\frac{1}{2\sigma}V(R) + 7\lambda^2 - 3\lambda + \frac{1}{4}], [3\lambda^2 - \frac{1}{4}]\right\} \\ > \;& V(R - 2\bar{r})/\sigma \end{aligned} \tag{33}$$

Both conditions are more likely to be satisfied for values of λ above 1/4 (i.e., a more polarized political system), and for high values of σ (i.e., if the value of policy relative to rents is high).

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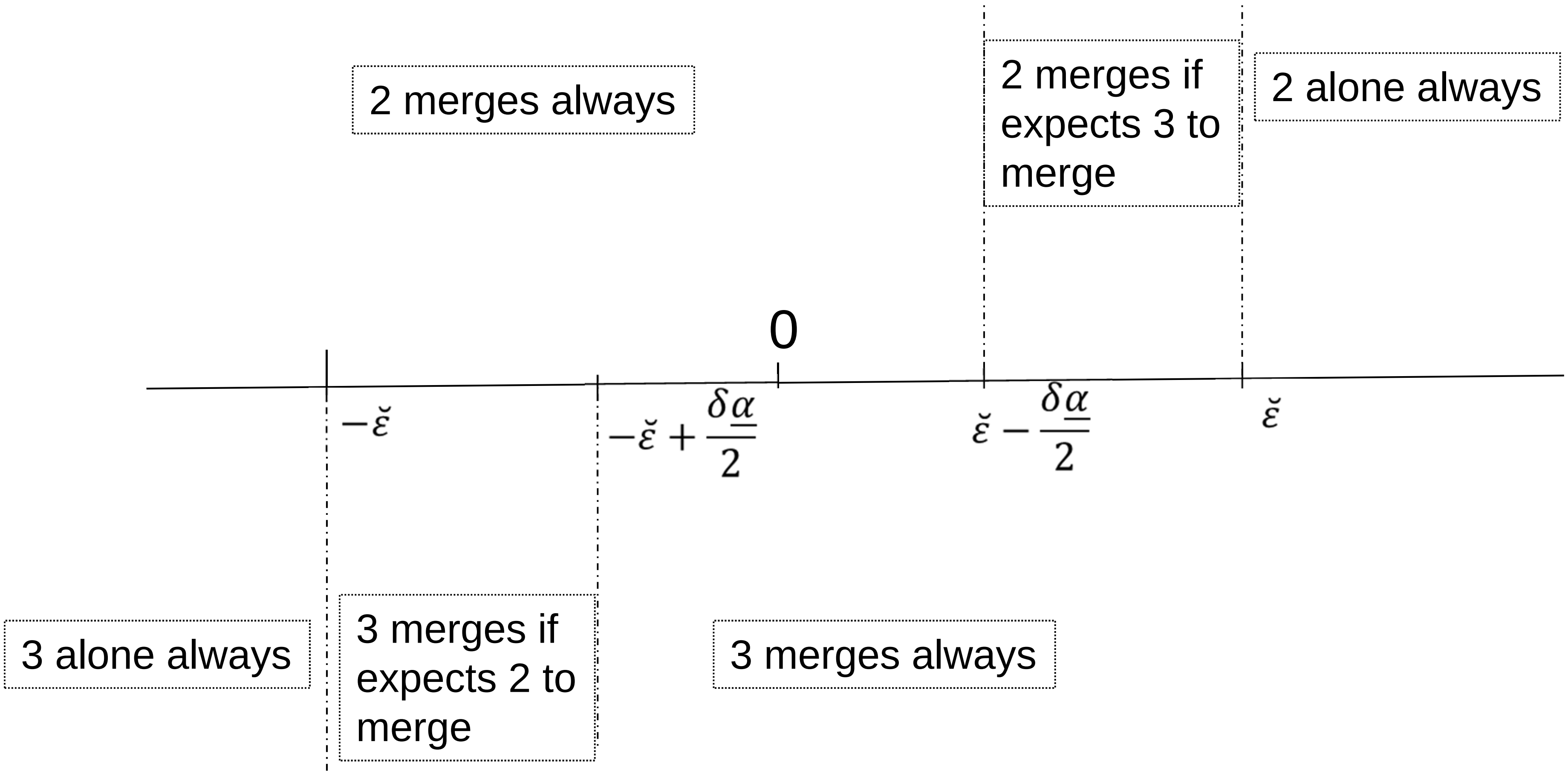
Figure A1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2 merges if expects 3 to merge
2 alone always
0
$-\breve{\varepsilon}$
$-\breve{\varepsilon}+\frac{\delta\underline{\alpha}}{2}$
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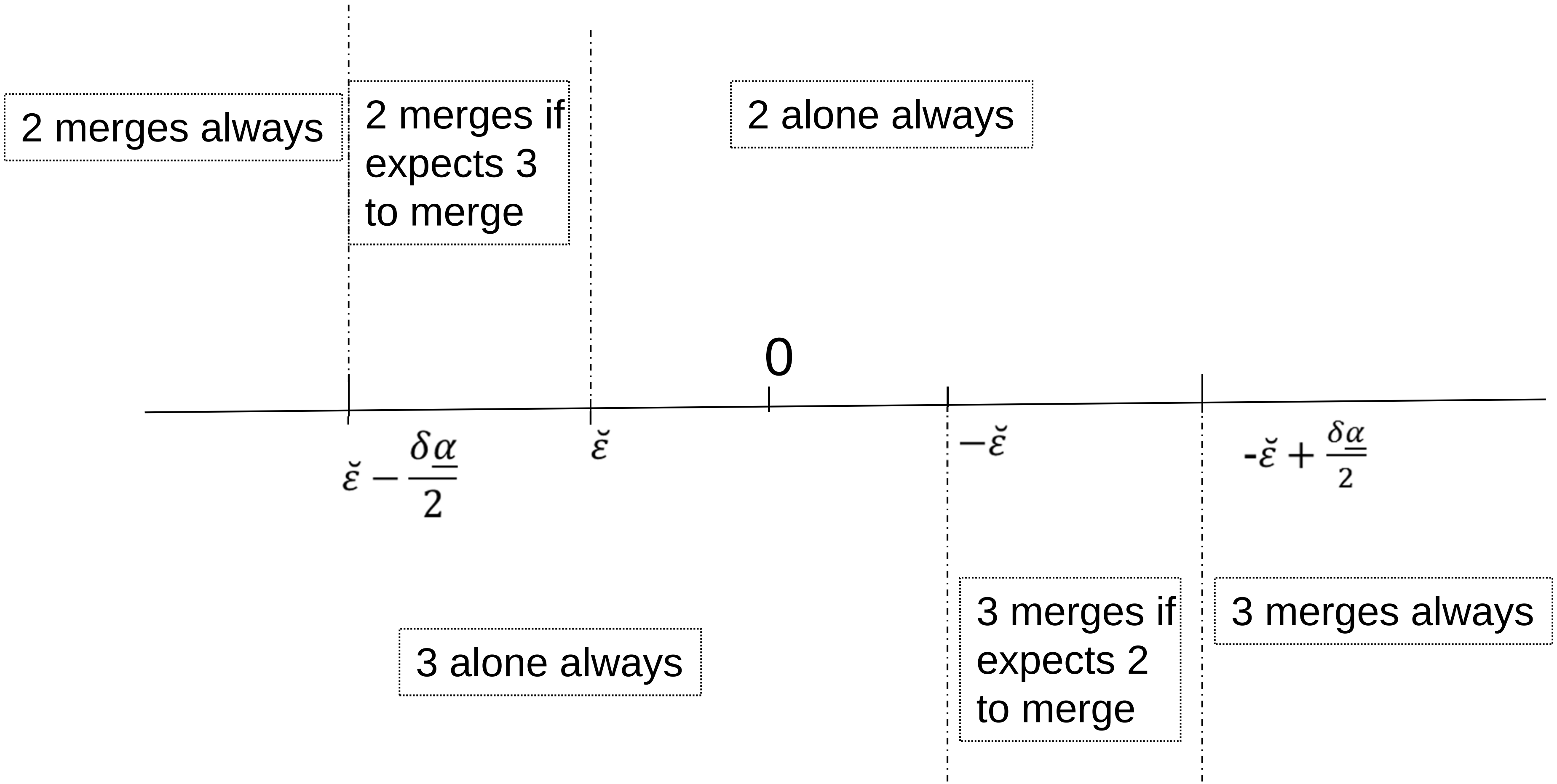
$\breve{\varepsilon}$
3 alone always
3 merges if expects 2 to merge
3 merges always

Figure A2


2 merges always
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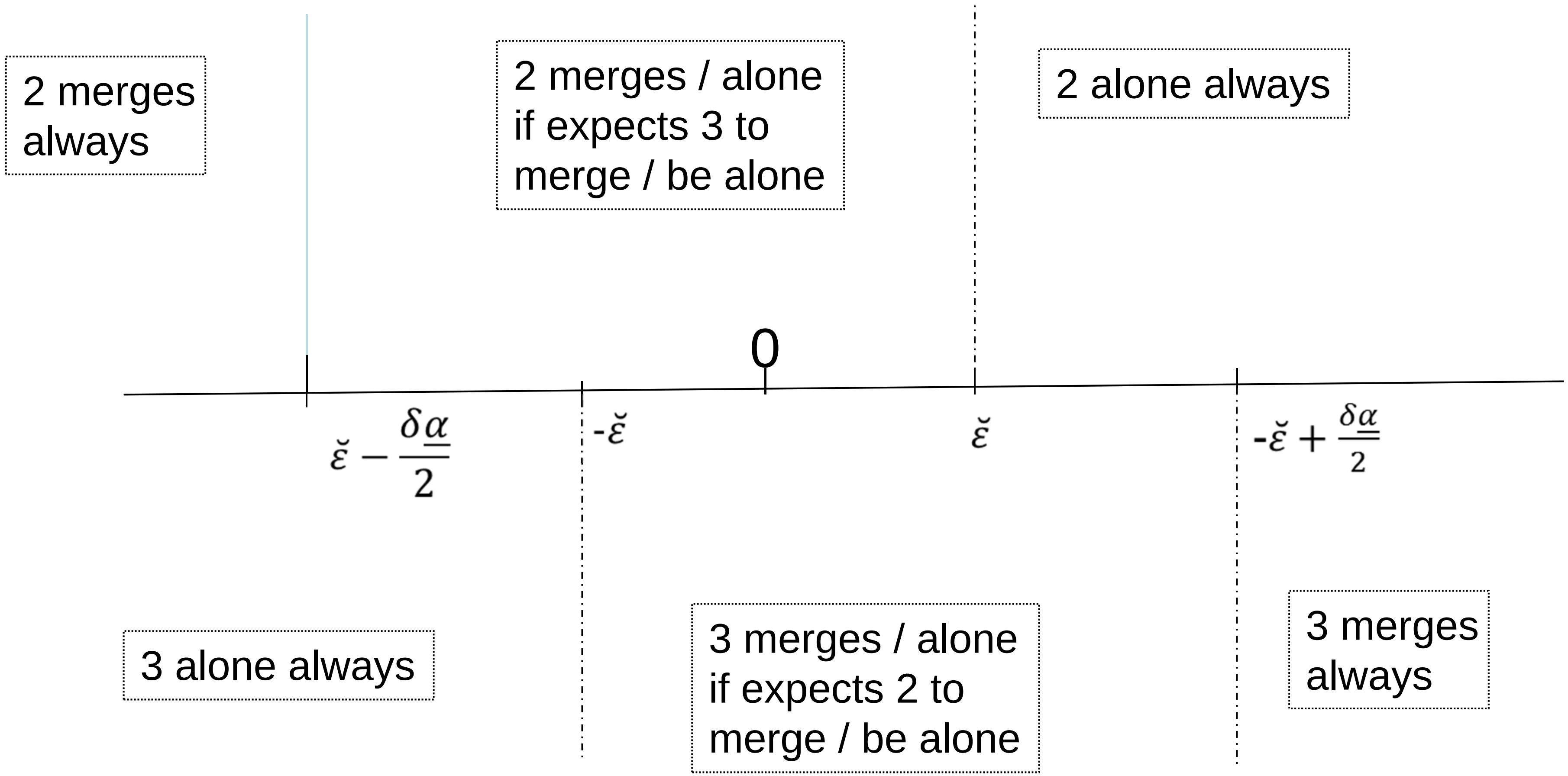
3 alone always
3 merges if expects 2 to merge
3 merges always

Figure A3


2 merges always
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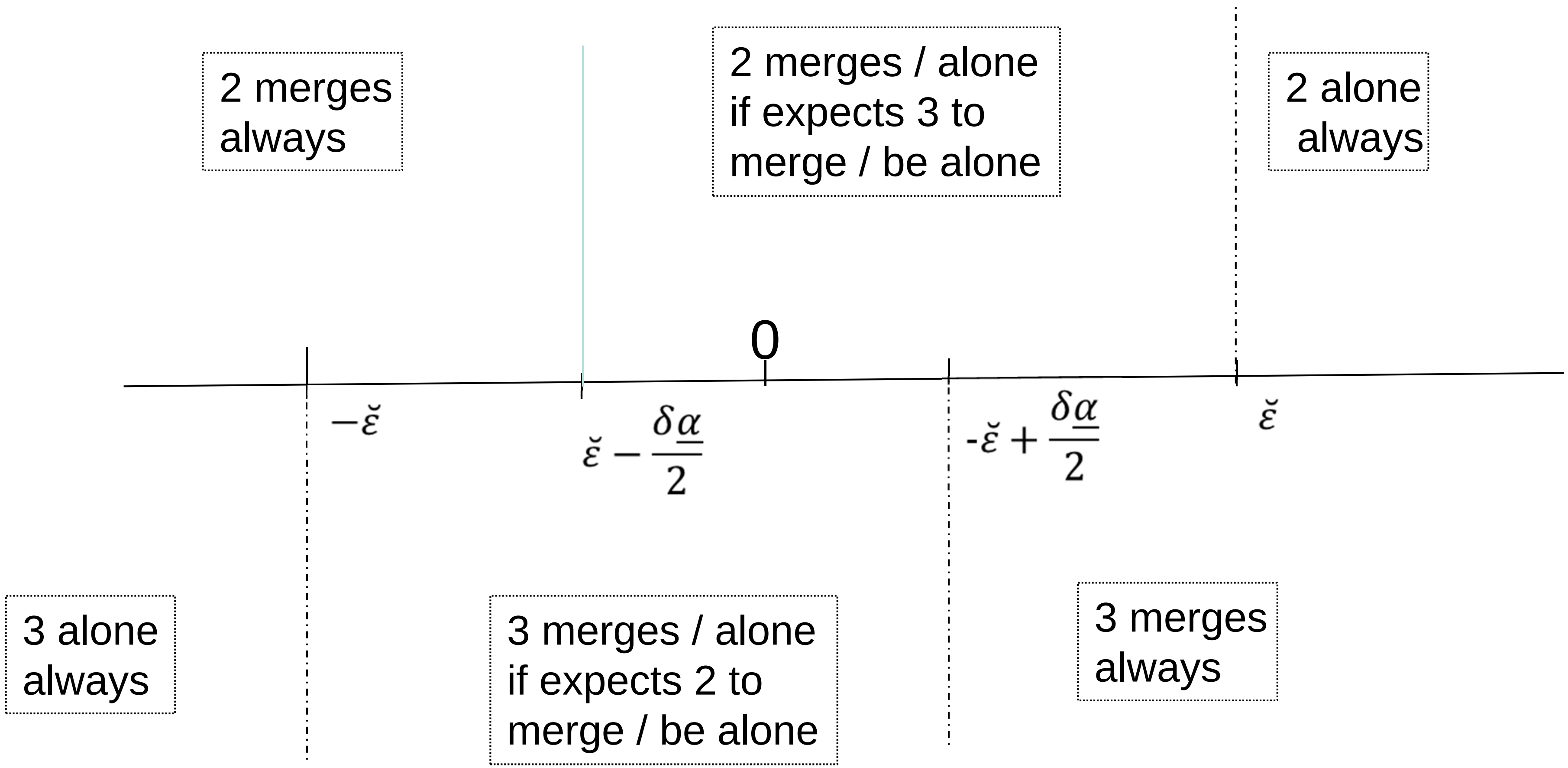
3 merges always

Figure A4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3 alone always
3 merges / alone if expects 2 to merge / be alone
3 merges always

Online Appendix III

Additional evidence and validity tests

Table A1: Impact of runoff on parties' vote shares in national elections, RDD estimates

	Spline 3^{rd}	Spline 2^{nd}	Spline 4^{th}	LLR (h)	LLR $(h/2)$	LLR $(2h)$
			A. Estimations without covariates			
Center-right	0.004	-0.026	-0.003	0.005	-0.013	-0.005
[Avg. 0.417]	(0.027)	(0.035)	(0.019)	(0.032)	(0.043)	(0.020)
Right	-0.023	-0.008	-0.003	-0.008	-0.011	-0.008
[Avg. 0.041]	(0.014)	(0.017)	(0.012)	(0.016)	(0.020)	(0.012)
Center-left	0.027	0.043	0.030	0.030	0.031	0.030
[Avg. 0.324]	(0.026)	(0.036)	(0.019)	(0.031)	(0.047)	(0.020)
Left	0.000	0.003	-0.005	0.001	-0.001	-0.001
[Avg. 0.088]	(0.008)	(0.010)	(0.006)	(0.010)	(0.011)	(0.007)
Centrist	-0.021	-0.034	-0.019	-0.035	-0.020	-0.021
[Avg. 0.061]	(0.019)	(0.025)	(0.013)	(0.023)	(0.023)	(0.014)
Obs.	2,027	2,027	2,027	364	175	761
			B. Estimations with covariates			
Center-right	0.005	-0.014	-0.002	0.007	0.001	-0.002
[Avg. 0.417]	(0.025)	(0.032)	(0.017)	(0.029)	(0.038)	(0.018)
Right	-0.006	-0.006	0.002	-0.004	0.001	0.001
[Avg. 0.041]	(0.009)	(0.011)	(0.007)	(0.010)	(0.012)	(0.007)
Center-left	0.014	0.027	0.021	0.020	0.016	0.016
[Avg. 0.324]	(0.022)	(0.030)	(0.016)	(0.025)	(0.033)	(0.017)
Left	-0.001	0.002	-0.005	0.001	-0.003	-0.002
[Avg. 0.088]	(0.008)	(0.010)	(0.006)	(0.009)	(0.011)	(0.006)
Centrist	-0.023	-0.031	-0.017	-0.034*	-0.027	-0.017
[Avg. 0.061]	(0.016)	(0.020)	(0.011)	(0.019)	(0.016)	(0.012)
Obs.	2,027	2,027	2,027	364	175	761

Notes. 2001 national election (results from the proportional tier of the mixed-member system for the House of Representatives); municipalities between 10,000 and 20,000. Dependent variables: vote shares of the main political parties/blocks. Specifically, the variable *Center-right* includes all parties that will merge into *Popolo della Libertà*; the variable *Center-left* includes all parties that will merge into *Partito Democratico*; the variable *Right* includes the (extremist) party *Lega Nord*; the variable *Left* includes the (extremist) party *Rifondazione Comunista*, *Lista Di Pietro*, and other minor communist lists; and the variable *Centrist* includes *CCD* and *Democrazia Europea*. Estimation methods: spline polynomial approximation as in equation (2), with 3^{rd}, 2^{nd}, and 4^{th} polynomial, respectively; local linear regression as in equation (3), with bandwidth $h = 1,000$, $h/2$, and $2h$, respectively. Estimations in Panel B also include the following covariates: macro-region dummies, area size, altitude, transfers, income, participation rate, elderly index, family size. Robust standard errors clustered at the city level are in parentheses. Significance at the 10% level is represented by *, at the 5% level by **, and at the 1% level by ***.

Table A2: Impact of runoff on strategic voting, RDD estimates

	Spline 3^{rd}	Spline 2^{nd}	Spline 4^{th}	LLR (h)	LLR $(h/2)$	LLR $(2h)$
	A. Electoral races with three candidates					
Top two candidates	0.013	0.038	0.012	0.026	0.074	0.015
	(0.036)	(0.047)	(0.026)	(0.044)	(0.068)	(0.029)
First candidate	-0.013	0.038	0.006	0.001	0.026	-0.006
	(0.046)	(0.060)	(0.033)	(0.057)	(0.083)	(0.036)
Second candidate	0.027	-0.000	0.006	0.025	0.048	0.021
	(0.029)	(0.035)	(0.024)	(0.032)	(0.040)	(0.024)
Third candidate	-0.024	-0.047	-0.020	-0.039	-0.090	-0.022
	(0.037)	(0.047)	(0.027)	(0.044)	(0.068)	(0.029)
Obs.	488	488	488	67	37	158
	B. Electoral races with more than three candidates					
Top two candidates	0.033	0.023	0.019	0.034	0.043	0.017
	(0.029)	(0.036)	(0.023)	(0.032)	(0.045)	(0.024)
First candidate	0.033	0.060	0.022	0.047	0.083	0.026
	(0.035)	(0.045)	(0.026)	(0.040)	(0.058)	(0.027)
Second candidate	0.000	-0.038*	-0.004	-0.013	-0.040	-0.008
	(0.018)	(0.022)	(0.013)	(0.019)	(0.026)	(0.014)
Third candidate	-0.035*	-0.045*	-0.013	-0.057**	-0.047	-0.021
	(0.020)	(0.025)	(0.015)	(0.023)	(0.029)	(0.015)
Obs.	879	879	879	184	82	363

Notes. Election years between 1993 and 2007; municipalities between 10,000 and 20,000 (with non-missing values of the mayoral candidates' vote shares). Dependent variables: mayoral candidates' vote shares. Estimation methods: spline polynomial approximation as in equation (2), with 3^{rd}, 2^{nd}, and 4^{th} polynomial, respectively; local linear regression as in equation (3), with bandwidth $h = 1,000$, $h/2$, and $2h$, respectively. Robust standard errors clustered at the city level are in parentheses. Significance at the 10% level is represented by *, at the 5% level by **, and at the 1% level by ***.

Table A3: Balance tests of time-invariant city characteristics

	Spline 3^{rd}	Spline 2^{nd}	Spline 4^{th}	LLR (h)	LLR $(h/2)$	LLR $(2h)$
South	0.024	-0.087	-0.039	-0.076	0.021	-0.016
	(0.145)	(0.183)	(0.108)	(0.167)	(0.215)	(0.114)
Area size	-1.511	16.541	-0.725	1.866	25.816	-0.048
	(17.800)	(23.509)	(12.562)	(20.913)	(25.982)	(13.746)
Altitude	115.904	99.701	26.494	-45.288	110.872	56.231
	(136.538)	(173.056)	(103.918)	(152.221)	(207.771)	(103.291)
Obs.	2,027	2,027	2,027	364	175	761

Notes. Election years between 1993 and 2007; municipalities between 10,000 and 20,000. Dependent variables: *South* is a dummy equal to 1 for Abruzzo, Molise, Campania, Puglia, Basilicata, Calabria, Sicilia, and Sardegna, and 0 otherwise; the *Area size* of the city is measured in km^2; the *Altitude* of the city is measured in meters. Estimation methods: spline polynomial approximation as in equation (2), with 3^{rd}, 2^{nd}, and 4^{th} polynomial, respectively; local linear regression as in equation (3), with bandwidth $h = 1,000$, $h/2$, and $2h$, respectively. Robust standard errors clustered at the city level are in parentheses. Significance at the 10% level is represented by *, at the 5% level by **, and at the 1% level by ***.

Table A4: Balance tests of pre-treatment city characteristics (Census 1991)

	Spline 3^{rd}	Spline 2^{nd}	Spline 4^{th}	LLR (h)	LLR $(h/2)$	LLR $(2h)$
Aged less than 25	0.002	-0.011	-0.003	-0.007	0.003	-0.001
	(0.017)	(0.023)	(0.012)	(0.021)	(0.029)	(0.013)
Aged 25-44	-0.006	-0.008	-0.004	-0.009	-0.005	-0.004
	(0.006)	(0.007)	(0.005)	(0.006)	(0.007)	(0.005)
Aged 45-64	-0.002	0.004	0.000	0.003	-0.005	-0.001
	(0.009)	(0.012)	(0.007)	(0.011)	(0.015)	(0.007)
Aged 65 or more	0.006	0.015	0.007	0.012	0.007	0.006
	(0.010)	(0.012)	(0.008)	(0.011)	(0.016)	(0.008)
Elementary	-0.014	0.000	-0.003	-0.001	-0.016	-0.008
	(0.011)	(0.013)	(0.008)	(0.012)	(0.015)	(0.008)
High school	0.010	0.008	0.007	0.016	0.021	0.006
	(0.012)	(0.015)	(0.009)	(0.013)	(0.018)	(0.009)
College	0.005	0.004	0.002	0.006	0.007	0.003
	(0.004)	(0.005)	(0.003)	(0.004)	(0.005)	(0.003)
Employed	-0.012	0.005	0.009	-0.007	-0.002	0.004
	(0.025)	(0.032)	(0.018)	(0.029)	(0.039)	(0.019)
Unemployed	0.002	0.003	-0.001	0.002	0.007	0.001
	(0.006)	(0.008)	(0.004)	(0.006)	(0.009)	(0.005)
Agriculture	-0.011	-0.008	-0.006	-0.013	-0.002	-0.006
	(0.012)	(0.016)	(0.009)	(0.015)	(0.018)	(0.010)
Manufacturing	0.004	0.007	0.018	0.006	0.003	0.013
	(0.022)	(0.028)	(0.017)	(0.025)	(0.031)	(0.017)
Public sector	0.001	0.001	0.001	0.002	-0.002	0.002
	(0.003)	(0.004)	(0.003)	(0.004)	(0.004)	(0.003)
Services	-0.002	0.003	0.004	-0.000	0.002	-0.002
	(0.012)	(0.015)	(0.009)	(0.014)	(0.019)	(0.009)
Water	-0.022	-0.000	-0.017	0.000	0.015	-0.020
	(0.023)	(0.027)	(0.017)	(0.024)	(0.032)	(0.017)
Heating	0.027	0.047	0.022	0.032	0.011	0.036
	(0.058)	(0.074)	(0.042)	(0.068)	(0.096)	(0.043)
Sewer	-0.003	-0.008	0.001	-0.008	-0.006	-0.002
	(0.006)	(0.009)	(0.006)	(0.006)	(0.007)	(0.005)
Obs.	2,027	2,027	2,027	364	175	761

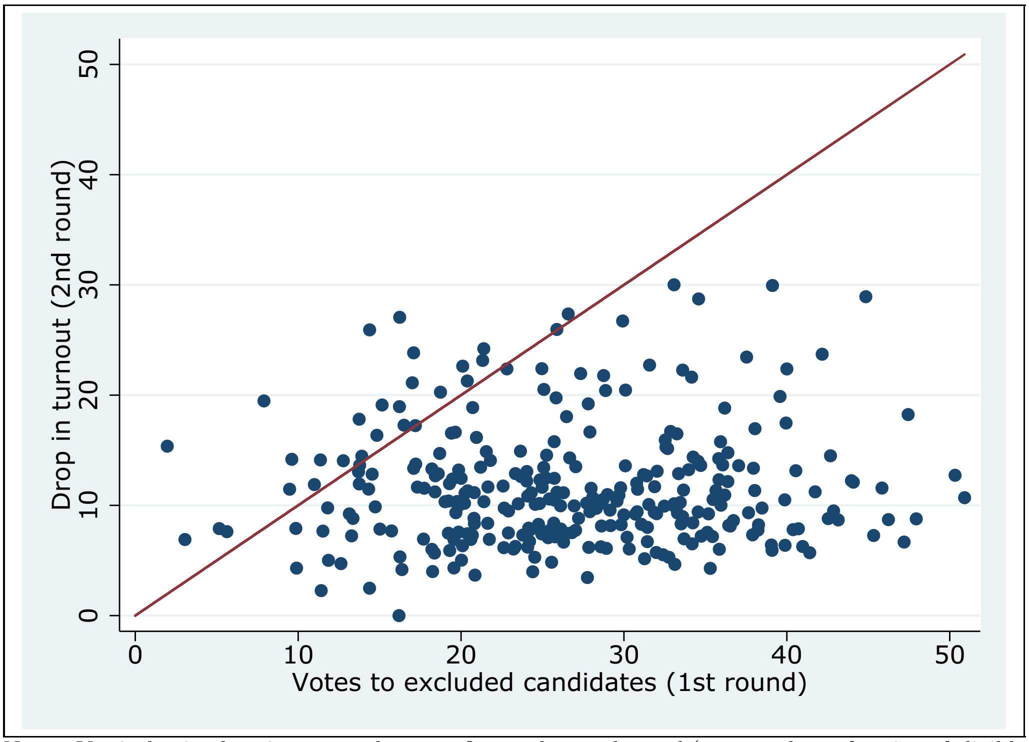
Notes. Election years between 1993 and 2007; municipalities between 10,000 and 20,000. Dependent variables: the age variables capture the share of individuals in the respective age bracket; *Elementary*, *High school*, and *College* capture the share of individuals with the respective educational attainment; *Employed* and *Unemployed* are the share of employed and unemployed individuals; *Agriculture*, *Manufacturing*, *Public sectors*, and *Services* capture the share of workers employed in the respective sector; *Water*, *Heating*, and *Sewer* capture the share of houses with access to the respective facility. All variables come from the 1991 Census. Estimation methods: spline polynomial approximation as in equation (2), with 3^{rd}, 2^{nd}, and 4^{th} polynomial, respectively; local linear regression as in equation (3), with bandwidth $h = 1,000$, $h/2$, and $2h$, respectively. Robust standard errors clustered at the city level are in parentheses. Significance at the 10% level is represented by *, at the 5% level by **, and at the 1% level by ***.

Table A5: Impact of runoff on political outcomes, decomposing diff-in-diff

	Municipalities moving above the threshold (UP_i)	Municipalities moving below the threshold ($DOWN_i$)
	A. Estimations without covariates	
No. of candidates	1.121**	-1.763**
	(0.448)	(0.887)
No. of lists	2.264***	-3.058***
	(0.516)	(1.021)
Lists/candidates	0.300	-0.438
	(0.214)	(0.423)
Opposition lists	1.383***	-2.968***
	(0.423)	(0.837)
Mayor's lists	0.363*	0.057
	(0.219)	(0.434)
Pre-treatment lists	-0.153	-0.186
	(0.239)	(0.473)
Obs.	518	518
	B. Estimations with covariates	
No. of candidates	1.063**	-1.833**
	(0.452)	(0.889)
No. of lists	2.411***	-3.387***
	(0.516)	(1.016)
Lists/candidates	0.408*	-0.568
	(0.214)	(0.421)
Opposition lists	1.374***	-3.105***
	(0.428)	(0.842)
Mayor's lists	0.426*	-0.000
	(0.223)	(0.438)
Pre-treatment lists	0.182	-0.410
	(0.225)	(0.444)
Obs.	518	518

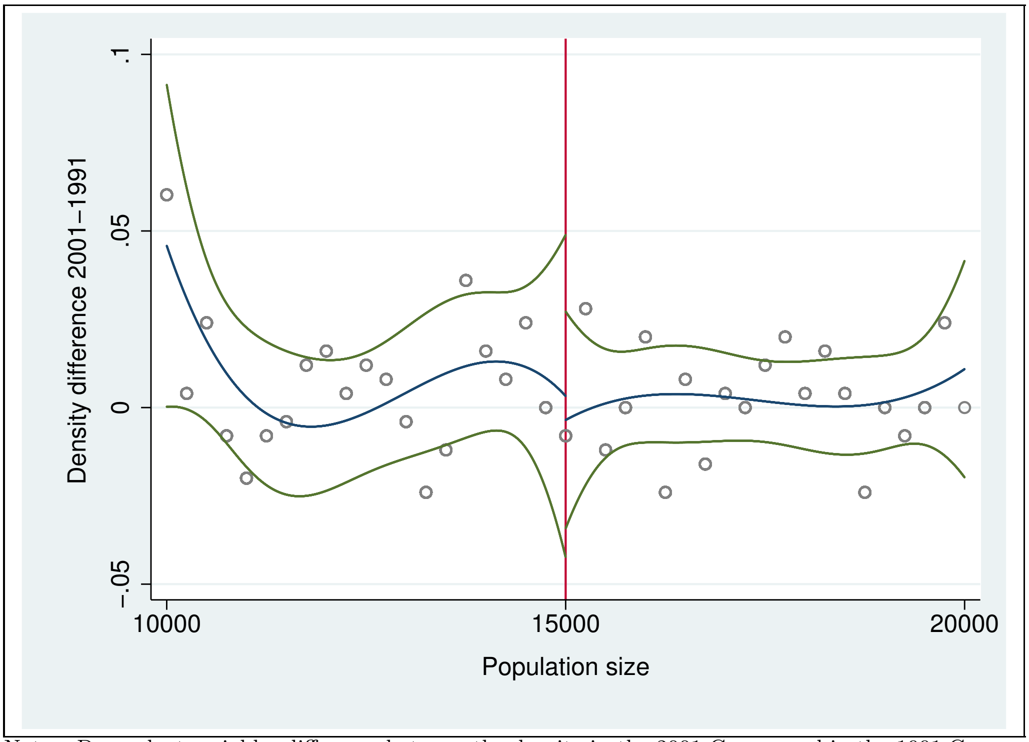
Notes. Municipalities between 10,000 and 20,000; 518 municipalities for which political outcomes are available both in the 1990s and in the 2000s. Dependent variables: *No. of candidates* running for mayor in the first round; *No. of lists* supporting mayoral candidates in the first round; *Lists/candidates* ratio; *Opposition lists* supporting the losing candidates; *Mayor's lists* supporting the winning candidate; *Pre-treatment lists* competing under proportional representation in the pre-treatment period (1985–1992). All dependent variables (excluding *Pre-treatment lists*) are expressed as the difference between the average value in the 2000s and the average value in the 1990s. Estimated equation: $\Delta Y_i = \alpha UP_i + \beta DOWN_i + x_i'\gamma + \epsilon_i$, where ΔY_i is the difference between the average outcome in the 2000s and in the 1990s, UP_i is a dummy equal to one if the municipality moved from below to above the threshold, $DOWN_i$ is a dummy equal to one if the municipality moved from above to below, and x_i is a vector of town-specific covariates. The reference group for the dummies UP_i and $DOWN_i$ is represented by municipalities that did not cross the threshold from 1991 to 2001 Census. Estimations in Panel B also include the following covariates: macro-region dummies, area size, altitude, transfers, income, participation rate, elderly index, family size. Robust standard errors are in parentheses. Significance at the 10% level is represented by *, at the 5% level by **, and at the 1% level by ***.

Figure A5: Drop in turnout between first and second round



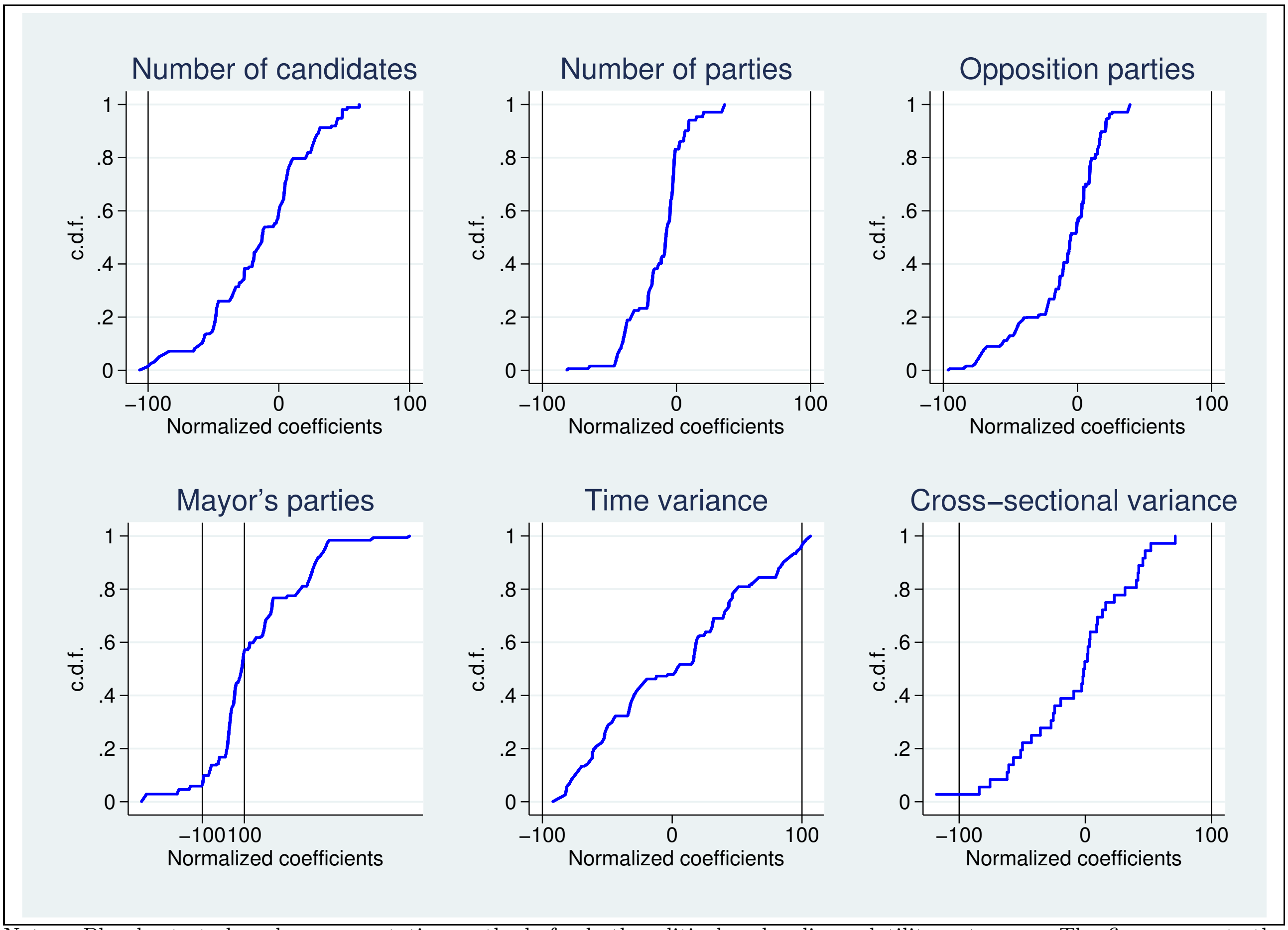
Notes. Vertical axis: drop in turnout between first and second round (expressed as a fraction of eligible voters). Horizontal axis: total votes for the excluded candidates in the first round (expressed as a fraction of eligible voters). Municipalities between 15,000 and 20,000 only.

Figure A6: Testing for sorting between 1991 and 2001 Census



Notes. Dependent variable: difference between the density in the 2001 Census and in the 1991 Census. The central line is a spline 3^{rd}-order polynomial in the normalized population size (i.e., population minus 15,000); the lateral lines are the 95% confidence interval of the polynomial. Scatter points are averaged over 250-inhabitant intervals. Municipalities between 10,000 and 20,000 only.

Figure A7: Placebo tests for political outcomes and policy volatility



Notes. Placebo tests based on permutation methods for both political and policy volatility outcomes. The figure reports the empirical c.d.f. of the normalized point estimates from a set of RDD estimations at 1,000 false thresholds: 500 below and 500 above the true 15,000 threshold (namely, any point from 13,501 to 14,000 and any point from 15,501 to 16,000). Only for the cross-sectional variance of the business property tax (where units of observations are 100-inhabitant bins), we consider 80 false thresholds: 40 below and 40 above the true 15,000 threshold (namely, any bin from 10,000 to 14,000 and any bin from 16,000 to 20,000). Each (false) estimate is normalized over the (true) baseline estimate from the paper; that is, a normalized coefficient equal to 100 indicates that the (false) estimate is exactly equal to the (true) baseline estimate. Dependent variables: *No. of candidates* running for mayor in the first round; *No. of lists* supporting mayoral candidates in the first round; *Opposition lists* supporting losing candidates; *Mayor's lists* supporting the winning candidate; *Time variance* (i.e., variance across terms averaged over the entire sample period) and *Cross-sectional variance* (i.e., variance across municipalities averaged over bins of 100 inhabitants) of the business property tax rate. Estimation method: spline polynomial approximation with 3^{rd}-order polynomial.