Consumption Insurance Under Default: Endogenizing Liquidity Constraints

Lecture I

Nicola Pavoni

September 22, 2016

Two Benchmarks

There are two standard class of models

1. Models with (complete markets and) no aggregate uncertainty $\Rightarrow$ Arrow-Debreu & Radner sequential equilibrum
2. Models with (complete markets and) aggregate uncertainty $\Rightarrow$ the reppresentative agent RBC and the Lucas' tree model.

- Complete market models seems to be inconsistent with the data. Essentially, they predict that consumption should not depend on individual income, only on aggregate shocks. In contrast, individual consumption growth is positively correlated with individual income (Attanasio and Davis, 1995).
- The equity premium puzzle is sometimes interpreted as a failure of the complete market model.

Idiosyncratic Shocks and Market Arrangements

1. Complete Markets (CM) $\rightarrow$ Full risk sharing (Benchmark I)
2. Self-Insurance (LC) $\rightarrow$ Only risk free asset
3. Models with limited commitment (DC)
4. Models with asymmetric information (MH and AS)

T. Kehoe and D. Levine, Econometrica 2001

What do they do?

- Consider two simple (standard) models of incomplete markets
  a. The model with exogenous incomplete markets (liquidity) (LC)
  b. A model with endogenous incomplete markets (default) (DC)
- They compare the steady state allocations in two ‘similar’ environments (deterministic flipping and stochastic endowment)
- They show that in (b) the equilibrium allocation is PO, it is easier to show existence, and easier to compute than in (a)
- They also show that DC can be seen as a special case of LC for a given level of the liquidity constraint (micro-foundation)

Model (Physical Environment)

- Infinite discrete time periods $t = 0, 1, ...$
- Two type of consumers $i = 1, 2$ (a continuum of them)
- A single consumption good each period c_t^i hence the lifetime consumption vector of type i consumer is $\mathbf{c}^i = \left\{ c_t^i \right\}_{t=0}^{\infty} \in l_{\infty}^{++}$
- Preferences are the same across consumers:

$$(1-\delta) \sum_t \delta^t u(c_t^i),$$

with $\delta \in (0, 1)$; $u \in C^2$ and $u' > 0$ and $u'' < 0$; (increasing, concave) with Inada: $\lim_{c \to 0} u'(c) = \infty$.

Endowments

Two type of endowments.

- Human capital (or wages, or idiosyncratic income) $w_t^i \in \{\omega^b, \omega^g\}$ with $w_t^i \neq w_t^{-i}$ and $w_t^i = w_{t+1}^{-i}$. That is, the human capital endowments change deterministically over time switching from ω^g to $\omega^b < \omega^g$ every period, with $w_t^1 + w_t^2$ constant for all $t \geq 0$.
- One unit of physical capital (or durable goods, e.g. trees) with per period divident $d > 0$ in consumption goods. We denote by θ_t^i the consumer of type i holding of physical capital in period t (shares).

Obviously: $\theta_t^1 + \theta_t^2 \leq 1$. The other social feasibility constraint is

$$c_t^1 + c_t^2 \leq \omega^g + \omega^b + d \equiv \omega.$$

Market Arrangements:

1. Complete Markets
2. Liquidity Constrained Economy
3. Debt Constrained Economy

1. Complete Markets

Definition A Competitive AD-equilibrium is an allocation $\mathbf{c} = \{c_t^1, c_t^2\}_{t=0}^{\infty}$ and a set of prices $\mathbf{p} = \{p_t\}_{t=0}^{\infty}$ such that given $\mathbf{p}$, for $i = 1, 2$, $\{c_t^i\}_{t=0}^{\infty}$ solves the following agent i problem

$$\max_{\mathbf{c}^i} \; (1-\delta)\sum_t \delta^t u(c_t^i) \quad \text{s.t.}$$

and

$$\sum_t p_t c_t^i \;\; \leq \;\; \sum_t p_t \left[w_t^i + d\theta_0^i\right], \text{ with } \theta_0^i \text{ given.}$$

and markets clear, that is, $c_t^1 + c_t^2 = \omega$ for any t.

Computation of the Equilibrium I

- The first order conditions imply

$$p_t \lambda^i = \delta^t u'(c_t^i) \quad \forall i, t,$$

  since there is no aggregate uncertainty, from market clearing we can show that for each agent $i = 1, 2$ we have $c_t^i = \bar{c}^i \ \forall t$. Using this and normalizing $p_0 = 1$, (hence $\lambda^i = u'(c_0^i)$) the first order conditions imply $p_t = \delta^t$.

- The budget constraint for agent i becomes

$$(1 - \delta) \left(\sum_{t=0}^{\infty} \delta^t (w_t^i + d\theta_0^i) \right) = \bar{c}^i.$$

Computation of the Equilibrium II

- Hence, assume agent 1 starts with ω^g

$$\bar{c}^1 = (1-\delta)\left[\left(1+\delta^2+\delta^4+\ldots\right)\omega^g + \delta\left(1+\delta^2+\delta^4+\ldots\right)\omega^b\right] + d\theta_0^1$$

$$= \frac{1-\delta}{1-\delta^2}\omega^g + \delta\frac{1-\delta}{1-\delta^2}\omega^b + d\theta_0^1$$

$$= \frac{1}{1+\delta}\omega^g + \delta\frac{1}{1+\delta}\omega^b + d\theta_0^1 = \frac{\omega^g+\delta\omega^b}{1+\delta} + d\theta_0^1,$$

and, similarly for agent 2 we have

$$\bar{c}^2 = \frac{\delta\omega^g+\omega^b}{1+\delta} + d\theta_0^2.$$

(Q) Suppose that one of the consumers markets a derivative asset that promises to pay x units of consumption each period. What would the price of that asset be?

(A) Since we are in complete markets, the derivative security is redundant and can be priced using the AD prices determined above. The time zero price of the derivative is ...

Radner Sequential Equilibrium in Complete Markets

- The consumer problem can be written as

$$\max_{\{c_t^i,\theta_t^i\}} \ (1-\delta)\sum_t \delta^t u(c_t^i) \quad \text{s.t.} \qquad (LC)$$

$$c_t^i + \nu_t \theta_{t+1}^i \leq w_t^i + (\nu_t + d)\,\theta_t^i,\ \theta_t^i \geq -\Theta,\ \theta_0^i \text{ given.}$$

  where ν_t is the period t price of of capital and Θ is chosen high enough so that to rule out Ponzi Games but to not make the constraint $\theta_t^i \geq -\Theta$ otherwise binding.

- Definition. An equilibrium is an infinite sequence of consumption levels $\{c_t^1, c_t^2\}_{t=0}^{\infty}$, capital holdings $\{\theta_t^1, \theta_t^2\}_{t=0}^{\infty}$, and capital prices $\{\nu_t\}_{t=0}^{\infty}$ such that consumers solve the consumer maximization problem and such that the social feasibility conditions are satisfied.

2. Liquidity Constrained Economy

In a Radner Sequential Equilibrium, the consumer's i problem is

$$\max_{\{c_t^i, \theta_t^i\}} \ (1-\delta)\sum_t \delta^t u(c_t^i) \quad \text{s.t.} \qquad (LC)$$

$$c_t^i + \nu_t \theta_{t+1}^i \quad \leq \quad w_t^i + (\nu_t + d)\,\theta_t^i,\ \theta_t^i \geq -B,\ \theta_0^i \text{ given.}$$

Two key imperfections characterize the LC model

1. $B \geq 0$ can be very small, for example, $B = 0$. In this case agents can only carry out intertemporal trade to smooth consumption by exchanging physical capital which can only be kept in positive amounts. This constraint can be enforced in a fully decentralized anonymous way.
2. The other key feature of the Liquidity constrained model is that there are no securities or other assets that can be traded besides physical capital. Obviously, this will be especially important when we will consider a stochastic environment.

Definition. An equilibrium is an infinite sequence of consumption levels **c**, capital holdings $\left\{\theta_t^1, \theta_t^2\right\}_{t=0}^{\infty}$, and capital prices $\left\{\nu_t\right\}_{t=0}^{\infty}$ such that consumers maximize utility given their constraints, and such that the social feasibility conditions are satisfied.

- K & L focus on symmetric steady states, i.e. allocations that depend only on the today's endowment. The FOCs:

$$
\begin{aligned}
u'\left(c^g\right)\nu &= \delta\left(\nu+d\right)u'\left(c^b\right) \\
u'\left(c^b\right)\nu &\geq \delta\left(\nu+d\right)u'\left(c^g\right)
\end{aligned}
$$

  the budget constraints

$$
\begin{aligned}
c^g+\nu\theta^b &= \omega^g+\left(\nu+d\right)\theta^g \\
c^b+\nu\theta^g &= \omega^b+\left(\nu+d\right)\theta^b
\end{aligned}
$$

  and market clearing

$$
\begin{aligned}
\theta^b+\theta^g &= 1 \\
c^b+c^g &= \omega.
\end{aligned}
$$

Characterization

- In this deterministic environment the symmetric equilibrium always exists and in it can be of two types: Full insurance, and Partial insurance.
- If we have Full Insurance ($c^g = c^b$) then

$$u'(\omega - c^g)\nu = \delta\left(\nu + d\right)u'\left(c^g\right),$$

$\Rightarrow c^g = \frac{\omega}{2}$ and $\frac{(\nu+d)}{\nu} = (1+r) = \frac{1}{\delta}$.

- If we have Partial insurance, $c^g \in \left(\frac{\omega}{2}, \omega^g\right]$ and $\theta^b = -B$. In this case, it must be that $\omega^g > \frac{\omega}{2} = \frac{\omega^g + \omega^b + d}{2}$ since $\theta = 0$ is always feasible.

3. Debt Constrained Economy

- Here agents have access to a full set of securities (i.e. they solve an Arrow-Debreu like problem)
- However, in each period t they face an individual rationality constraint of the following type

$$(1-\delta)\sum_{n=0}^{\infty}\delta^n u(c^i_{t+n}) \geq (1-\delta)\sum_{n=0}^{\infty}\delta^n u(w^i_{t+n}) \tag{IC}$$

  this constraint can be either interpreted as a bankruptcy or opt out value. The agents can default on the debt. In this case they will lose the physical capital endowment and will be excluded from the market forever.
- The human capital is assumed to be inalienable.
- The model implicitly assumes the presence of a credit agency or government, who keep trace of who goes bankrupt.

Consumers' Problem

$$\max_{\mathbf{c}^i} \ (1-\delta)\sum_t \delta^t u(c_t^i) \quad \text{s.t.}$$

$$(\mathbf{IC}), \text{ and}$$

$$\sum_t p_t c_t^i \quad \leq \quad \sum_t p_t \left[w_t^i + d\theta_0^i\right], \text{ with } \theta_0^i \text{ given.}$$

- Notice that the central authority must also control how much each agent borrows, so that the trade are incentive compatible.
- The Radner sequential formulation of the constraint is similar to the liquidity constrained model

$$c_t^i + \nu_t \theta_{t+1}^i \leq w_t^i + (\nu_t + d)\,\theta_t^i, \ \theta_t^i \geq -\Theta$$

where again Θ is just to rule out Ponzi Games.

Equilibrium and Efficiency

Definition An equilibrium is an infinite sequence of consumption levels c, asset holding θ and prices for assets ν so that consumers maximize lifetime utility subject to he budget constraints, the restrictions for asset holdings, the (IC) constraints, and the allocation is socially feasible.

- The symmetric equilibrium exists in this economy as well: either Full or Partial insurance (in the latter case $\omega^g > \frac{\omega}{2}$).
- If Partial Insurance is because the incentive constraint of the rich agent is binding

$$u(c^g) + \delta u(\omega - c^g) = u(\omega^g) + \delta u(\omega^b)$$

- The interest rate in the steady state is (from the Euler with $=$)

$$r = \frac{u'(c^g)}{\delta u'(c^b)} - 1.$$

  If $c^g > c^b$ then $r < \frac{1}{\delta} - 1$, (=the subjective discount rate).
- Borrowers are constrained but the lenders do not. In order to satisfy market clearing we must reduce the interest rate, otherwise the lenders (high endowment) will lend more than the equilibrium amount as the borrowers cannot fully post their requests.
- Result: We can find a B^* such that the equilibrium allocation in the liquidity model is the same as that in the default model. In this sense, we can rationalize the liquidity constraints as result of commitment problems.

Construction of the liquidity parameter B, I

Recall the problem of the consumer (Radner Equilibrium)

$$\text{Consumer Problem:} \quad V^i(\theta_0^i) = \max_{\mathbf{c}^i} \; (1-\delta)\sum_{t=0}^{\infty}\delta^t u(c_t^i) \quad \text{s.t.}$$

$$(1-\delta)\sum_{n=0}^{\infty}\delta^n u(c_{t+n}^i) \geq (1-\delta)\sum_{n=0}^{\infty}\delta^n u(w_{t+n}^i) \quad \forall t$$

$$c_t^i + \nu_t\theta_{t+1}^i \leq w_t^i + (\nu_t + d)\,\theta_t^i, \qquad \forall t$$

$$\theta_t^i \geq -\Theta \quad \text{with} \quad \theta_0^i \text{ given.}$$

Note that the autarky values solve:

$$A^b = (1-\delta)\,u(\omega^b) + \delta A^g$$

$$A^g = (1-\delta)\,u(\omega^g) + \delta A^b$$

with, for example,

$$A^b = (1-\delta)\left[u(\omega^b) + \delta u(\omega^g) + \delta^2 u(\omega^b) + \dots\right]$$

Construction of the liquidity parameter B, II

In the steady state, the consumer may face two situations:

$$\underline{\text{Good state}}: \quad V^g(\theta; B) = \max_{c^g, \theta^g} \; (1-\delta)\, u(c^g) + \delta V^b(\theta^g; B),$$

$$(1-\delta)\, u(c^g) + \delta V^b(\theta^g, B) \geq (1-\delta)\, u(\omega^g) + \delta A^b$$

$$c^g + \nu\theta^g \leq \omega^g + (\nu + d)\, \theta, \quad \theta^g \geq -B.$$

and

$$\underline{\text{Bad State}}: \quad V^b(\theta; B) = \max_{c^b, \theta^b} \; (1-\delta)\, u(c^b) + \delta V^g(\theta^b; B);$$

$$(1-\delta)\, u(c^b) + \delta V^g(\theta^b; B) \geq (1-\delta)\, u(\omega^b) + \delta A^g$$

$$c^b + \nu\theta^b \leq \omega^b + (\nu + d)\, \theta, \quad \theta^b \geq -B.$$

$-B^*$ is the weakest restriction that guarantees (IC), it solves:

$$V^g(-B^*; B^*) = (1-\delta)\, u(\omega^g) + \delta A^b.$$

First Welfare Theorem

Result: The debt constrained equilibrium is (Constrained) P. Eff.

- Assume it not the case, that is, there is an allocation c' so that both agents are made better off with respect to the equilibrium allocations. In this case, the new allocation cannot be feasible since it must be that at the new allocation c' the agent that stays better does not satisfy the budget constraint and that the other agent has the budget either violated or just satisfied. In particular, from the two agents' budget constraints
$$\sum_t p_t \left(c_t^{1\prime} + c_t^{2\prime} \right) > \sum_t p_t \omega.$$
since $p_t \geq 0$ it must be that the new allocation is not feasible.
- This is the firm's profits maximization violation in CE models of complete markets with firms (e.g., the RBC model).
- The whole reasoning does not affect the IC constraint since it will be even more satisfied by a Pareto dominating allocation.

Stochastic Case (quick sketch)

- Now, at each period the endowments to the agents are generated by a shock $z_t \in \{1, 2\}$ which denotes who gets the high endowment. The variable z_t is assumed to be Markov with transitions $\pi \in (0, 1)$, the probability of reversal. When $\pi = 1$ we are in the deterministic case above.
- We will denote by $z^t = (z_0, ...z_t)$ the history of shocks up to period t, where z_0 is the initial state of the economy. Finally, we denote by

$$\mu(z^t) = pr\left(z_t \mid z_{t-1}\right) pr\left(z_{t-1} \mid z_{t-2}\right) \ldots pr\left(z_1 \mid z_0\right)$$

the probability of history z^t (given z_0).

Agent Problems in Liquidity and Debt Constrained

In all cases, the objective function is the same:

$$\max_{\mathbf{c}^i,\theta^i} \ (1-\delta)\sum_t \sum_{z^t} \delta^t \mu(z^t) u\left(c_t^i\left(z^t\right)\right)$$

- The constraint in the (LC) model is

$$\begin{aligned} c_t^i\left(z^t\right) + \nu_t\left(z^t\right)\theta_{t+1}^i(z^t) &\leq w_t^i(z^t) + \left(\nu_t\left(z^t\right)+d\right)\theta_t^i\left(z^{t-1}\right), \\ \theta_{t+1}^i\left(z^t\right) &\geq -B,\ \theta_0^i \text{ fixed}, \end{aligned}$$

- The constraint faced by the consumer in the (DC) model is

$$\sum_n \sum_{z^{t+n}/z^t} \delta^n \frac{\mu(z^{t+n})}{\mu(z^t)} u(c_{t+n}^i(z^{t+n})) \geq \sum_n \sum_{z^{t+n}/z^t} \delta^n \frac{\mu(z^{t+n})}{\mu(z^t)} u(w_{t+n}^i(z^{t+n}))$$

$$\begin{aligned} c_t^i(z^t) + q_t(z^t,1)\theta_{t+1}^i(z^t,1) + q_t(z^t,2)\theta_{t+1}^i(z^t,2) &\leq w_t^i(z^t) + (\nu_t(z^t)+d)\theta_t^i(z^t), \\ \theta_{t+1}^i\left(z^t, z_{t+1}\right) &\geq -\Theta,\ \theta_0^i \text{ fixed}. \end{aligned}$$

- Notice that agents now decide state contingent plans, and $q_t\left(z^t, z_{t+1}\right)$ is the price of an Arrow-Debreu security traded in state z^t, period t that promises a unit of physical capital in state $\left(z^t, z_{t+1}\right)$ next period.
- The first order conditions in the liquidity constrained model are

$$u'(c_t) \geq \delta \mathbf{E}_t\left[\frac{\nu_{t+1}+d}{\nu_t} u'(c_{t+1})\right]$$

which is the first order condition for the situation where the agent is restricted to trades in which $\theta^i(z^t,1) = \theta^i(z^t,2)$. Indeed, standard arbitrage implies that

$$q_t\left(z^t,1\right) + q_t\left(z^t,2\right) = \nu_t\left(z^t\right).$$

- It turns out that the debt constrained economy has a unique symmetric steady state of the same form as before, whereas the liquidity constrained does not have it when $\pi \in (0, 1)$.
- In the debt model

$$\frac{dc^g}{d(1-\pi)} > 0$$

  more persistence in the shocks reduces trade, hence insurance.
- Alvarez and Jermann (Econometrica, 2000) describe how the equilibrium allocation in the default economy can be equivalently described by imposing state contingent (and possibly agent specific) liquidity constraints on the arrow securities:

$$\theta_{t+1}(z^t, z) \geq -B^i(z^t, z)$$

- Such constraints must be imposed so that agents' next period IC constraint is satisfied.