Forecasting Inflation

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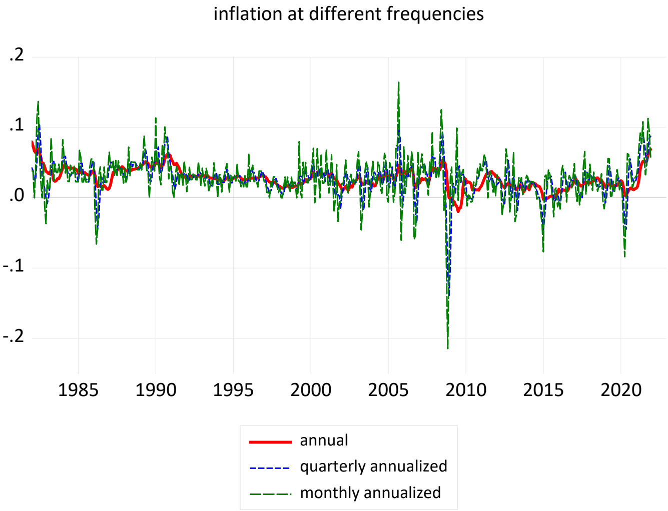
March 2022

Inflation

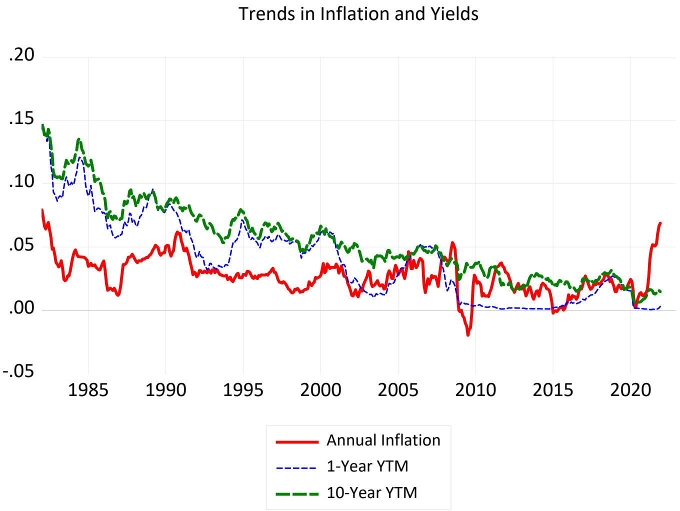
The object of our interest is forecasting inflation.

- As an illustrative case consider the case of US Consumer Price Index for All Urban Consumers: All Items, seasonally adjusted observed at monthly frequency over the sample 1982-2022.
- Graphical analysis of the data shows clearly that inflation at different frequencies (monthly, quarterly and annual) has different time-series properties in terms of persistence and volatility.
- Asset Prices have (common) trends while inflation is much less persistent

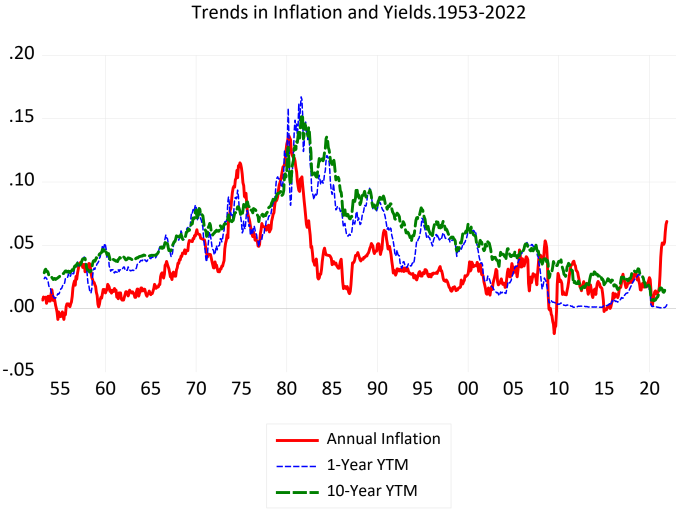
Inflation at different frequencies



Inflation and Bond Yields:1980-2022



Inflation and Bond Yields:1950-2022



Alternative Forecasting Methods

- Univariate time-series methods (ARMA and ARIMA)
- Multivariate TS methods (VAR, and model based forecasts)
- Model Based Forecast (for example the Phillips Curve)
- Survey based
- Using big data (collecting online prices by webscraping, using NPL techniques) http://www.thebillionpricesproject.com/
- Asset price based forecasts

Time-series methods

- Univariate ARIMA methods and multivariate VAR based approaches are **backward-looking**.
- In addition, time-series methods have to make choices on the seasonal adjustment and on the level of aggregation at which the data are used in the forecasting model.
- Is it better to forecast directly the aggregate or to forecast inflation at higher frequencies and then aggregate them to obtain an annual forecast ?

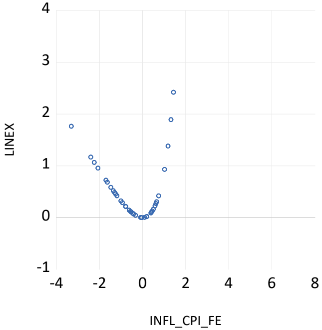
Survey based methods

- Survey based forecast reflect preferences of the forecasters.
- If the loss function is not symmetric, then inflation based forecast are different from expected inflation.
- Consider for example forecasters who have a LINEX loss function:

$$L = \frac{e^{\alpha x_t} - \alpha x_t - 1}{\alpha^2}$$

Survey based methods

If $\alpha > 0$, the Linex function is exponential to the right of the origin, and linear to the left. In this case, positive forecast errors for inflation are more costly than negative forecast errors.



Survey based methods

The optimal forecast under LINEX preferences is:

$$INFL^{f}_{t,t+h} = \mu_{t,t+h} + \frac{\alpha}{2}\sigma^{2}_{t,t+h}$$

Market Prices and Inflation Forecast

There are several market instruments from which inflation forecasts can be derived

- Inflation indexed bonds
- Inflation Swaps
- Inflation Options

Inflation indexed bonds

- The most common form of government bonds are nominal bonds that pay fixed coupons and principal.
- Inflation-indexed bonds, which in the U.S. are known as Treasury Inflation Protected Securities (TIPS), are bonds whose coupons and principal adjust automatically with the evolution of a consumer price index.
- They aim to pay investors a fixed inflation-adjusted coupon and principal, in other words they are real bonds and their yields are typically considered the best proxy for the term structure of real interest rates in the economy.

Inflation indexed bonds

- Investors holding either inflation-indexed or nominal government bonds are exposed to the risk of changing real interest rates.
- In addition to real interest rate risk, nominal government bonds expose investors to inflation risk while real bonds do not. When future inflation is uncertain, the coupons and principal of nominal bonds can suffer from the eroding effects of inflationary surprises.
- Finally, both the nominal and real bond are theoretically affected by a premium for liquidity risk. Liquidity risk, is, the risk of having to sell (or buy) a bond in a thin market and, thus, at an unfair price and with higher transaction costs.

Inflation indexed bonds

At time t the yields to maturity of nominal and real bonds maturing at T can be written as follows:

$$\begin{aligned} Y^n_{t,T} &= rr_{t,T} + E_t\pi_{t,T} + RP^{rr}_t + RP^{\pi}_t \\ Y^r_{t,T} &= rr_{t,T} + RP^{rr}_t + RP^{liq}_t \end{aligned}$$

the difference in the yield to maturity, usually referred to as the breakeven inflation rate $B_{t,T}$, can be written as:

$$B_{t,T} = E_t\pi_{t,T} + RP^{\pi}_t - RP^{liq}_t$$

Further technical details

Further details complicate the pricing of real bonds.

- Inflation figures are published with a lag and therefore the principal value of inflation indexed bonds is adjusted with a lag (three-month in the US and the UK).
- Moreover the mechanism of indexation could be different for coupons and principal and it might have different provisions for positive and negative inflation.
- Finally the tax treatment of the bond might also differ. In the UK principal adjustments of inflation-linked gilts are not taxed. In the US, on the other hand, inflation-adjustments of principal are considered ordinary income for tax purposes.

Inflation Swaps

- An inflation swap is a bilateral contractual agreement. It requires one party (the 'inflation payer') to make periodic floating-rate payments linked to inflation, in exchange for predetermined fixed-rate payments from a second party (the 'inflation receiver').
- The most common structure is the zero-coupon inflation swap. The inflation receiver pays on a contract maturing in T years pays the Fixed leg = (1 + fixed rate)T x Nominal value to receive from the the inflation payer the variable Inflation leg = (Final price index/Starting price index) x Nominal value.

$$SW_{t,T} = E_t \pi_{t,T} + RP_t^{\pi}$$

Further Technical Details

In practice, inflation swap contracts have indexation lags.

$$SW_{t,T} = E_t \pi_{t-m,T-m} + RP_t^{\pi}$$

As the object of interest is inflation from time t onward, the following decomposition becomes relevant:

$$SW_{t,T} = \frac{m}{T-t} \pi_{t-m,t} + \frac{T-m-t}{T-t} E_t \pi_{t,T-m} + RP_t^{\pi}$$

which, in absence of publication lags on inflation data allows to derive the relevant inflation compensation from the fixed swap rate.

The distribution of market-based inflation expectations

Inflation options are instruments with non-linear payoffs:

- inflation caps pay-out if inflation exceeds a certain threshold
- inflations floors pay-out if inflation falls short of a certain threshold

By comparing the price of options that insure against different outcomes it is possible to infer the probability that investors assign to those different outcomes.

The distribution of market-based inflation expectations

More formally:

$$p_t = \sum \frac{1}{1+r_f} P^Q(\pi) E_t\left(x_{t+1}(\pi)\right)$$

The price of an inflation linked-asset today depends on the risk neutral.probabilities $P^Q(\pi)$ and expected one-period ahead payoffs $E_t\left(x_{t+1}(\pi)\right)$ associated with different inflation events.
Given the knowledge of prices, payoffs and the risk-free rate it is then possible to recover risk-neutral probabilities of inflation.

Inflation Caps and Floors

- A (zero-coupon) inflation cap is a contract entered into at time t. The seller of the cap promises to pay a fraction $\max((1 + \pi(n))^n - (1 + k)^n; 0)$ of a notional underlying principal as a single payment in n years' time, where $\pi(n)$ denotes the average annual CPI inflation rate from t to t+n, and k denotes the strike of the cap. The notional underlying principal is normalized to 1. In exchange for this, the buyer makes an up-front payment of $p_t(k, n)$.
- A zero coupon inflation floor is identical except that the payment is A zero-coupon inflation floor is identical, except that the payment is $\max((1 + k)^n - (1 + \pi(n))^n; 0)$

Building the distribution of inflation expectations from caps and floor

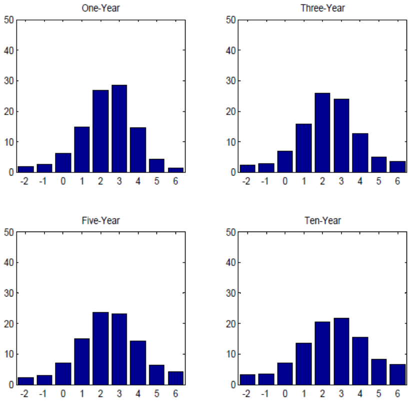
- Suppose you obtain daily quotes on zero-coupon inflation caps and floors at different strike prices (say -2,-1,0,1,2,3,4,5,6) and that inflation over the next n years can be approximated as having only integer support.
- A butterfly portfolio that involves buying one cap with strike of $(k-1)$ percent and one with strike $(k-1)$ percent while shorting two caps with a strike of k percent, is a pure Arrow-Debreu security with a payoff of 1 if inflation is k percent and zero otherwise, for any integer k. So the price of this security will be:

$$p_t = \frac{1}{1+r_f} P^Q(\pi = k)$$

- As for the tails, if an investor buys an inflation cap at 5 percent and shorts one at 6 percent, then this investor receives 1 if inflation is 6 percent or more and zero otherwise. This gives the probability of inflation being 6 percent or higher. The same works for the left tail

The distribution of market-based inflation expectations

Figure 2: Inflation PDFs February 28, 2011



YTM and Risk Premia

Apply the no arbitrage condition to a one-period bond (the safe asset) and a T-period bond:

$$
\begin{aligned}
E_t\left(r_{t,t+1}^T - r_{t,t+1}^1\right) &= E_t\left(r_{t,t+1}^T - y_{t,t+1}\right) = \phi_{t,t+1}^T \\
E_t\left(r_{t,t+1}^T\right) &= y_{t,t+1} + \phi_{t,t+1}^T
\end{aligned}
$$

Solving forward the difference equation $p_{t,T} = p_{t+1,T} - r_{t,t+1}^T$, we have :

$$
\begin{aligned}
y_{t,T} &= \frac{1}{(T-t)} \sum_{i=0}^{T-1} E_t\left(r_{t+i,t+i+1}^T\right) \\
&= \frac{1}{(T-t)} \sum_{i=0}^{T-1} E_t\left(y_{t+i,t+i+1} + \phi_{t+i,t+i+1}^T\right)
\end{aligned}
$$

$$
y_{t,T} - y_{t,t+1} = \sum_{i=1}^{T-t-1} \left(1 - \frac{i}{T-t}\right) E_t \Delta y_{t+i,t+i+1} + \frac{1}{T-t} \sum_{i=1}^{T-t-1} \phi_{t+i,t+i+1}^T
$$

HPR and risk-premia

The one-period uncertain holding-period return on a bond maturing at time T, $r_{t,t+1}^{T}$, is then defined as follows:

$$\begin{aligned} r_{t,t+1}^{T} &\equiv p_{t+1,T} - p_{t,T} = -\left(T-t-1\right) y_{t+1,T} + \left(T-t\right) y_{t,T} & (1) \\ &= y_{t,T} - \left(T-t-1\right)\left(y_{t+1,T} - y_{t,T}\right), \\ &= \left(T-t\right) y_{t,T} - \left(T-t-1\right) y_{t+1,T}, & (2) \end{aligned}$$

No arbitrage + RE implies:

$$\left(r_{t,t+1}^{T} - y_{t,t+1}^{1}\right) = \phi_{t,t+1}^{T} + u_{t,t+1}$$

HPR and risk-premia: a regression approach

Think of the following regression

$$(T-t)\,y_{t,T}-(T-t-1)\,y_{t+1,T}-y_{t,t+1}=\beta_0+\beta_1\left(y_{t,T}-y_{t,t+1}\right)+u_{t+1}$$

It is a predictive regression that will deliver predictability of excess returns under the two following hypothesis

- the one period risk premium is persistent
- the term spread it is not dominated by the expected monetary policy component

If the regression reveals predictability we have:

$$\begin{aligned}\hat{\phi}_{t,t+1}^{T} &= \hat{\beta}_0+\hat{\beta}_1\left(y_{t,T}-y_{t,t+1}\right)\\ \hat{\phi}_{t,t+T}^{T} &= \frac{1}{T}\sum_{i=1}^{T}E_t\hat{\phi}_{t+i-1,t+i}^{T}\end{aligned}$$

so the period term premium can be proxied by pairing the predictive regression with a model for the persistent one period risk premium

Breakeven Inflation and the Inflation Expectations Hypothesis

given that breakeven inflation is defined as follows:

$$b_{t,T} = y_{t,T} - y_{t,T}^{TIPS}$$

then the excess returns on breakeven inflation is given by

$$(T-t)\, b_{t,T} - (T-t-1)\, b_{t+1,T} - b_{t,t+1}$$

remebering that

$$b_{t,T} = E_t \pi_{t,T} + RP_t^{\pi} - RP_t^{liq}$$

When inflation and liquidity risk premia are constant the excess returns on break-even inflation are equal to a constant plus the expression

$$(T-t)\, \pi_{t,T}^{e} - (T-t-1)\, \pi_{t+1,T}^{e} - \pi_{t,t+1}$$

which fluctuates zero when expectations are rational

Breakeven Inflation and the Inflation Expectations Hypothesis

So the inflation (and liquidity) risk premia can be measured via the following regression strategy :

$$
\begin{aligned}
(T-t)\, b_{t,T} - (T-t-1)\, b_{t+1,T} - b_{t,t+1} &= \beta_0 + \beta_1 \left(b_{t,T} - b_{t,t+1} \right) + u_{t+1} \\
\hat{\phi}^T_{t,t+1} &= \hat{\beta}_0 + \hat{\beta}_1 \left(b_{t,T} - b_{t,t+1} \right) \\
\hat{\phi}^T_{t,t+T} &= \frac{1}{T} \sum_{i=1}^{T} E_t \hat{\phi}^T_{t+i-1,t+i}
\end{aligned}
$$

Implementation requires the following steps

- TIPS trade only at maturity of one year or longer so a short-term (one-quarter) real rate needs to be constructed
- run the predictive regression and extract the one period inflation risk premia
- given predictive model for the one period inflation risk premia construct the T period inflation risk premia
- subtract the T period inflation risk premia form the corrispondent measure of breakeven inflation to derive inflation expectations

The Construction of the short-term real rate

TIPS trade only at maturity of one year or longer so to construct a short term real rate assume zero inflation and risk premia over the quarter

$$y_{t,t+1}^{TIPS} = y_{t,t+1} - \pi_{t,t+1}^{e}$$

Next we assume that inflation expectations over the next quarter are rational and proxy for the ex-ante real short rate as the fitted value from the regression of this quarter's realized real rate onto last quarter's realized real rate, the nominal short rate,and annual inflation up to time t.

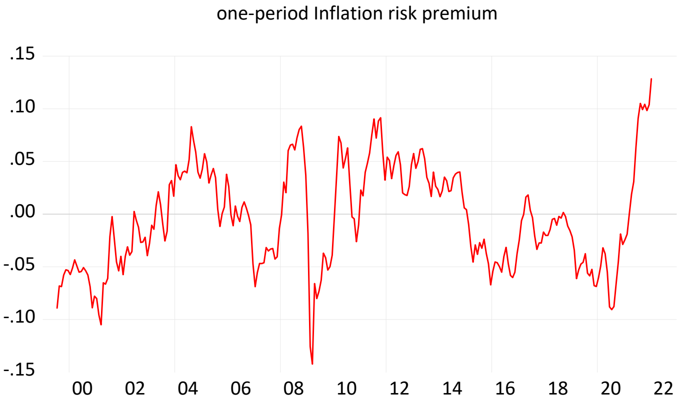
The Construction of the short-term real rate

Table 1
Forecasted Real Short Rate

$y^{\$}_{1,t} - \pi_{t+1}$	US	UK
$y^{\$}_{1,t}$	0.57**	0.46
	(0.22)	(0.29)
$y^{\$}_{1,t-1} - \pi_t$	0.08	−0.11
	(0.08)	(0.07)
$(\pi_{t-3} + \pi_{t-2} + \pi_{t-1} + \pi_t)/4$	0.08	0.03
	(0.09)	(0.09)
$p - value$	0.00	0.00
R^2	0.44	0.18

Overlapping quarterly real short rate returns onto the nominal short rate, last quarter's real short rate return and inflation over the past year.
Monthly data 1982.1-2009.12.
Newey-West standard errors with 4 lags in brackets.
* and ** denote significance at the 5% and 1% level respectively.
p-value of the F-test for no predictability.

One-period Inflation Risk premium



Break-even Inflation and Inflation Expectations