Solution to Exercise 1

1 Question 1

Consider a two-period economy in which the government keeps a balanced budget with a level of public investment equal to g. At time $t = 0$, the government makes an additional investment in infrastructure ε, financed with one-period bonds paying an interest of R, and taxes households at $t = 1$ to fully repay the debt.

Prices are fully rigid, so output at $t = 0$ is fully demand-determined.

Consumer demand, income, and disposable income in period 0 are given as follows:

$$c = \beta_0 + 0.9 \cdot y_{disp}$$

$$y_{disp} = (1 - 0.3)y = 0.7y$$

$$y = c + (g + \varepsilon)$$

where y_{disp} is disposable income. Total demand consists solely of private consumption and government investment. There is no private investment and no government consumption. The government collects proportional taxes at a flat rate of 30 percent.

1.1 Evaluating the Normal Level of Public Investment g

At equilibrium before the shock ε:

$$c = \beta_0 + 0.9(0.7y) = \beta_0 + 0.63y$$

Using the output equation:

$$y = c + g$$

Substituting c:

$$y = \beta_0 + 0.63y + g$$

Rearranging:

$$0.37y = \beta_0 + g$$

Solving for y:

$$y = \frac{\beta_0 + g}{0.37}$$

Substituting this back into the equation for g:

$$g = \frac{0.37\beta_0}{1 - 0.37} = \frac{0.37\beta_0}{0.63}$$

$$g \approx 0.587\beta_0$$

Thus, the normal level of public investment g is directly expressed as a function of β_0.

1.2 Evaluating the Fiscal Multiplier

With the additional investment ε:

$$y' = c' + (g + \varepsilon)$$

Using the updated consumption function:

$$c' = \beta_0 + 0.9(0.7y')$$

$$y' = \beta_0 + 0.63y' + g + \varepsilon$$

Rearranging:

$$0.37y' = \beta_0 + g + \varepsilon$$

Solving for y':

$$y' = \frac{\beta_0 + g + \varepsilon}{0.37}$$

The impact on output due to ε is:

$$\Delta y = \frac{\varepsilon}{0.37}$$

So, the fiscal multiplier is:

$$\frac{1}{0.37} \approx 2.7$$

1.3 Extra Tax Revenue Needed at t = 1

The government must repay $(1 + R)\varepsilon$, but it already collected additional tax revenue at $t = 0$:

$$\Delta T_0 = 0.3 \cdot \Delta y = 0.3 \cdot \frac{\varepsilon}{0.37} = \frac{0.3}{0.37}\varepsilon \approx 0.81\varepsilon$$

The remaining amount to be financed at $t = 1$ is:

$$T_1 = (1 + R)\varepsilon - \Delta T_0$$

$$T_1 = (1 + R)\varepsilon - 0.81\varepsilon$$

$$T_1 = (1 + R)\varepsilon \cdot (1 - 0.81)$$

$$T_1 = (1 + R) \cdot 0.19\varepsilon$$

Thus, the correct answer is:

$$T_1 = (1 + R) \cdot 0.19\varepsilon$$

This confirms that the government needs to raise less than $(1 + R)\varepsilon$ in taxes at $t = 1$ due to the automatic increase in tax revenue at $t = 0$.

2 Question 2

The following code in R allows to answer to the first part of the question and, with small adaptations, to all other parts.

```r
#clear the environment
rm(list=ls())
#setwd(path)
setwd(dirname(rstudioapi::getActiveDocumentContext()$path))
## ------------------------------------------------------------------------
# Define parameters
r_W <- 0.02  # Interest rate on wealth
n <- 5  # Number of periods
net_income <- c(0,100, 200, 300, 200, 150)
discount_factors <- c(0, 1 / (1 + r_W)^(0:(n-1)))
print(discount_factors)
sum_discount_factors <- sum(discount_factors)

# Case 1: W_1 - W_6 / (1+r^W)^5 = 0
wealth <- numeric(n+1)
wealth[1] <- 200
wealth[6]<- (1+r_W)^n*wealth[1]
C <- (sum(net_income * discount_factors) + ((1+r_W)*wealth[1] - wealth[6] / (1 + r_W)^4)) / sum_discount_factors

for (t in 2:n) {
  wealth[t] <- (1 + r_W) * wealth[t-1] + net_income[t] - C
}

case1_table <- data.frame(
  Year = 1:(n+1),
```

```r
  Net_Income = net_income,
  Consumption = c(0,rep(C, n)),
  Wealth = wealth
)
print(case1_table)

# Case 2: W_t+5 = 0
# Define parameters
r_W <- 0.02  # Interest rate on wealth
n <- 5  # Number of periods
net_income <- c(0,100, 200, 300, 200, 150)
discount_factors <- c(0, 1 / (1 + r_W)^(0:(n-1)))
print(discount_factors)
sum_discount_factors <- sum(discount_factors)

#
wealth2 <- numeric(n+1)
wealth2[1] <- 200
wealth2[6]<- 0
C2 <- (sum(net_income * discount_factors) + ((1+r_W)*wealth2[1] - wealth2[6] / (1
    + r_W)^4)) / sum_discount_factors

for (t in 2:n) {
  wealth2[t] <- (1 + r_W) * wealth2[t - 1] + net_income[t ] - C2
}

case2_table <- data.frame(
  Year = 1:(n+1),
  Net_Income = net_income,
  Consumption = c(0,rep(C2, n)),
  Wealth = wealth2
)
print(case2_table)
```