Identifying the Real Effects of Zombie Lending*

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Abstract

The policy response to Covid-19 includes the provision of credit guarantees to firms, which may generate zombie lending. This might slow down the recovery because, according to a recent literature, the relative performance of healthy firms deteriorates as the fraction of zombies increases. We argue that this literature faces a serious identification problem because firm performance is often used to define zombies (sometimes implicitly). We show that, under general conditions on the distribution of firm performance, the correlation between healthy firms performance and zombies is a mechanical consequence of an increase in the fraction of zombies with no causal meaning.

Keywords: Zombie lending, capital misallocation

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1 Introduction

Many countries adopted a strict lockdown policy to tackle the diffusion of the Covid-19 pandemic, forcing firms in nonessential sectors to completely shut down operations. Even as these restrictions are lifted, economic activity is bound to remain below potential for a prolonged period of time in several sectors. Loss of revenues and lower productivity could push solvent but illiquid firms into bankruptcy. The response of many governments has been to set up schemes of credit guarantees for bank loans, particularly to small and medium enterprises (SME). According to an OECD report, as of April 20$^{\text{th}}$, 2020, 52 out of the 54 countries considered had set up some form of government-provided financial support for SMEs (see OECD 2020, Table 3). Clearly, avoiding the bankruptcy of solvent firms due to liquidity shortages is the right thing to do in the acute phase of the crisis. However, the need to act fast, the reduction in incentives to screen borrowers created by the government guarantees, and the large number of applications received by banks may lead to a substantial amount of funds flowing to weak firms – the "zombies".

In a seminal paper on the 1990s Japanese stagnation, Caballero, Hoshi & Kashyap (2008) argue that zombie lending reduces restructuring and delays recovery because it impedes reallocation of assets from low productivity uses (the zombies) to high productivity uses (the non-zombies), increasing misallocation (Hsieh & Klenow 2009). Zombie lending can hurt healthy firms in two ways: first, it reduces the flow of bank credit that is available to healthy firms if credit supply is limited; second, lending to non-viable firms is equivalent to a subsidy that hurts their healthy competitors in product and input markets. While lending extensively and indiscriminately might be a sensible response in the acute phase of the crisis, the zombie lending arising as a consequence of the credit guarantees might slow down the recovery once it starts.

It is therefore important to assess if zombie lending will be an issue during the Covid-19 crisis. The unprecedented nature of the crisis makes it difficult to draw lessons from the past. Yet, the experience of the recent financial crisis in Europe offers some guidance to test the effects of zombie lending on healthy firms. A recent literature has documented that when banks have a weak capital structure, they tend to engage in zombie lending to avoid provisioning and raise capital. Consequently, zombie lending is the subject of a growing number of papers studying the European financial and sovereign debt crisis (see, for example, Acharya, Eisert, Eufinger & Hirsch 2019, McGowan, Andrews & Millot 2018, Schivardi, Sette & Tabellini 2019, Storz, Koetter, Setzer & Westphal 2017, Blattner, Farinha & Rebelo 2018, Anderson, Riley & Young 2019). In line with the results of Caballero et al. (2008), most (but not all) of these papers conclude that a larger presence of zombies hurts healthy firms, slowing the recovery down.

Compared to the 2008-2009 financial crisis and the 2011-2012 sovereign debt crisis, the banking sector was in much better conditions at the outset of the Covid-19 crisis in most countries. One could therefore argue that nowadays, zombie lending would not arise. However, there are other mechanisms that might create zombie lending. Rather than deriving from the distorted incentives of weak banks to subsidize zombies, in the current situation, zombie lending might be a result of Government programs designed to boost bank corporate lending, particularly to SMEs. If the default risk is absorbed by the Government, banks' incentives to allocate credit according to underlying firm risk are weakened so that credit might accrue disproportionately to non-viable firms more likely to demand it. The concerns put forward by the literature on zombie lending during financial crises might therefore apply to the Covid-19 crisis too.

In this paper, we argue that despite the results of the literature mentioned above, there is no solid support for the assertion that Government policies to sustain corporate lending will have negative consequences due to zombie lending. First, we offer some evidence using Italian data that the bulk of liquidity needs during the crisis will come from firms that were financially sound before the crisis. This is a reflection of the fact that the negative effects of the pandemic on firm performance, including the policies enacted to contrast its spread, are unrelated to a firm's financial health.

Second, and more importantly, we argue that the empirical framework commonly applied in the literature to estimate the effects of zombie lending on healthy firms suffers from a serious identification problem that can bias the results towards finding a negative spillover, even when this is not actually the case. In a nutshell, the framework correlates the performance of non-zombies relative to that of zombies with the sectoral share of zombies, interpreting a negative coefficient as measuring the negative spillovers: when more zombies are active, healthy firms perform relatively worse. To account for common shocks, the regression is saturated with sector-year dummies. We show that this correlation can arise naturally from standard shocks which, by shifting the distribution of firms performance to the left, mechanically increase the share of zombies and reduce the *relative* performance of healthy firms, absent any spillover. We provide analytical conditions on the performance distribution under which this is actually the case and show that such conditions are likely to be satisfied in the settings typically considered in the literature. This effect is not accounted for by sector-year dummies, which only control for shocks that hit firms equally. The danger of defining zombie firms solely on the basis of firm performance is recognized by Caballero et al. (2008), who propose that a zombie is defined based only on whether it receives subsidized financing. In Section 4, we argue that this definition is in fact not immune to our critique.

The remedy to these problems is the usual one: finding exogenous variation in the share of zombies with respect to aggregate shocks. However, the literature has not generally followed this approach given its reliance on sector-year dummies. In the final section, we review the main contributions to the literature on zombie lending and argue that none of them are completely immune from the possibility that the share of zombies is correlated with aggregate shocks. As a consequence, we conclude that the correlation on which the literature bases the claim that zombies hurt healthy firms is likely be spurious and cannot offer a solid scientific basis for policy prescriptions.

Finally, there are other reasons specific to the Covid-19 crisis to question the usefulness of the approach of the literature in the current situation. First, given the policy responses to the Covid-19 shock, identifying zombies through subsidized credit may create additional identification problems in the current scenario. Governments provide credit subsidies, including caps on interest rates, to facilitate firms' access to credit, and central banks enact aggressive policy measures to bring interest rates to unprecedented low levels. In this setting, distinguishing between subsidized credit to zombies and cheap credit to healthy firms will be problematic. More generally, as argued by Barrero, Bloom & Davis (2020), the Covid-19 shock will require substantial reallocation of production across sectors. Therefore, firms that looked healthy on the basis of operating performance in January 2020 may suddenly become weak performers for the long term and vice versa. The traditional definitions used by the literature to identify zombies will therefore be of limited use during this crisis.

Our paper contributes to an important academic and policy debate. In fact, while keeping zombies alive increases misallocation in the long run, there might be beneficial effects in downturns. First, avoiding bankruptcies prevents layoffs. This in turn can mitigate the adverse aggregate demand externalities that are important during a recession (Mian, Sufi & Trebbi 2015). Firms closures can also disrupt input-output relationships that, at least in the short run, can be difficult to substitute for (Barrot & Sauvagnat 2016, Bernstein, Colonnelli, Giroud & Iverson 2019).[1] Given that the effects are ex-ante ambiguous, policy prescriptions should be based on uncontroversial empirical evidence. These considerations are particularly important during the crisis generated by the Covid-19 pandemic, whose speed of diffusion and severity is unprecedented.

The rest of the paper is organized as follows. Section 2 sets up a scheme to predict firms' liquidity shortages during the crisis and applies it to Italy, distinguishing between the needs of zombies and those of non-zombies. Section 3 illustrates the approach of the literature and the identification problem it faces. Section 4 assesses to what extent the various definitions of zombies used in the literature are vulnerable to the it. Section 5 concludes.

2 Predicting zombie lending during the Covid-19 crisis

The crisis induced by the Covid-19 pandemic has hit the economy with unprecedented speed and strength in several countries, forcing many businesses to completely shut down operations for months. Sales and cash flows dropped dramatically and will remain low for an extended period of time. To prevent the ensuing liquidity crisis from pushing many solvent but illiquid firms out of business, almost all countries set up some form of public guarantee scheme for loans to non-financial firms (OECD 2020). This is a necessary measure to avoid a massive wave of bankruptcies. At the same time, it might have unintended consequences in the medium run. The public guarantee and the pressure on the banking system to act quickly imply that lending standards will be weakened and that loans will also be extended to firms that were already suffering before the crisis: that is, the crisis might be characterized by massive zombie lending. There is evidence, to which we come back in the next sections, that zombie lending may slow down the recovery.

[1] Historically, one of the causes of the long lasting effects of the Great Depression is often connected to Andrew Mellon, who famously defended the cleansing effect of the Depression by urging President Hoover to " Liquidate labor, liquidate stocks, liquidate the farmers, liquidate real estate... purge the rottenness out of the system."

In this section, we use data from Italy to supply some evidence on the potential dimension of this phenomenon. The data come from the Firm Register compiled by the data provider Cerved and contain detailed balance sheets information for *all* incorporated businesses. We consider non-financial firms excluding agriculture. The data refer to almost 650,000 firms, accounting for approximately 70% of private sector value added. Most of the firms are SMEs, the main target of the public guarantee schemes. Cerved produces an Altman Z-score that measures firms' creditworthiness which is extensively used by banks when evaluating credit applications. The score takes values from 1 to 10, with higher values signaling higher probability of default. We define " zombies" as firms with scores of 8, 9 or 10, the categories used to flag troubled firms.[2]

We use the most recent available data, referring to 2018. This means that flows, such as sales and costs, are for the full year, and stocks, such as liquidity, are measured at the end of 2018.[3] Table 1 reports descriptive statistics for firms, separately for zombies and non-zombies. Firms in the 3 riskiest score classes – the zombies – are 11% of the total. They are on average smaller in terms of both sales (average 1.3 million against 4.3) and employment (9 vs. 16). Zombie firms are clearly financially fragile: the median firm only has 2,000 euros of equity and the 25^{th} percentile has negative equity. Their average holdings of liquid assets (liquidity in what follows) is 122,000 euros, a little more than one fourth of that of non-zombies.

Next, we forecast the liquidity needs of firms during the Covid-19 crisis which we take as a measure of the credit that these firms will demand. We construct an accounting framework that allows us to predict the month in which each firm will become illiquid and the amount of the liquidity shortages afterwards.[4] The general logic is simple and is based on three ingredients: the initial stock of liquidity, an estimate of the evolution of cash flow month by month and the budget equation governing the evolution of liquidity. Specifically, given sales S and costs C, the evolution of liquidity L for firm i in month m

[2]The score takes accounting measures of profitability and financial structure to assess a firm's probability of default. As we show in detail below, these are the typical measures used by the literature to identify zombies.

[3]Ideally, one would use data for 2019. However, using 2018 data is not a major limitation. In fact, while the exercise predicts liquidity needs firm by firm, we are interested in the aggregate values. As long as the distribution of firms conditions is invariant between 2018 and 2019, the aggregate results will be unaffected by idiosyncratic firm movements that leave the distribution unchanged.

[4]See Schivardi & Romano (2020) for further details.

Table 1: Zombies and Non-Zombies: Descriptive statistics

	N. obs.	Mean	p25	Median	p75	Std. Dev.
			Panel A: Zombies			
Sales	70,993	1,365	72	234	692	22,730
Employment	70,993	9	0	1	5	80
Equity	70,993	204	-20	2	55	15,380
Debt	70,993	968	0	49	220	47,292
Liquid assets	70,993	122	3	11	38	3,548
			Panel B: Non-Zombies			
Sales	578,237	4,317	135	432	1413	102,871
Employment	578,237	16	0	2	8	293
Equity	578,237	1728	25	87	350	79,086
Debt	578,237	1397	0	47	234	112,485
Liquid assets	578,237	459	8	32	129	24,903

Note: The table shows descriptive statistics of firms according to zombie status. A firm is classified as zombie if its score is 8, 9 or 10. Data are from the Italian Firm Register (Cerved) and refer to 2018. Sales, Equity, Debt and Liquid assets are in thousands euros.

of 2020 is:

$$L_{im} = L_{im-1} + S_{im} - C_{im} \tag{1}$$

where we consider the months from March to December 2020. Given an initial value of the stock of liquidity and an estimate of the evolution of cash flow (sales minus costs), the equation allows to detect for each firm the month in which liquidity turns negative and the amount of liquidity shortages afterward.

We compute the values of the variables above at the firm level based on the balance sheets of 2018 (the most recent available). For liquidity, we assume that the initial stock is that which is reported in the balance sheets. For sales, we consider the sales of 2018 and assume that absent the Covid-19 crisis, monthly sales would have been equal to 1/12 of the total sales of 2018. We then apply forecasts of sales growth for approximately 500 sectors produced by Cerved sector experts. The forecasts are based on a scenario in which the lockdown applies at the sectoral level according to the Government decrees. There is then a period of partial opening that also varies by sector, and after that, the activity gradually recovers. The sectoral estimates take into account sectoral exposure to Covid-19 specific effects including the possibility to work remotely, the effects of social distancing, and the reduction in mobility.

Firms' outlays are for capital, labor, intermediates and taxes. As far as capital is concerned, the Decree *Cura Italia* (Cure Italy) of March 17 freezes all interest and mortgage payments, as well as the granted credit on credit lines. It also suspends (and a later decree partially cancels) corporate taxes. Moreover, we also assume that firms freeze their investment expenditures. The only outlays that are left are cost of labor and intermediate good and services. Given an elasticity of each of them to sales, $\varepsilon_{WS}, \varepsilon_{MS}$, and sectoral estimates of the drop in sales for each month compared to the pre Covid-19 value d_{im}, Equation 1 becomes:

$$L_{im} = L_{im-1} + (1 - d_{im})S_i - (1 - \varepsilon_{WS} d_{im}) * W_i - (1 - \varepsilon_{MS} d_{im}) * M_i \tag{2}$$

where $L_i = L_{i2018}/12$ is monthly sales according to 2018 sales. Similarly, monthly labor costs W and intermediates M are according to their 2018 values.

To implement this scheme, we need to fix the two elasticities of labor and intermediates to sales. We use the balance sheet of Italian non financial incorporated companies between 2005 and 2015 with a total of 3.9 million firm-year observations. We regress the percentage annual change (the log difference) in intermediate expenditure and in the wage bill on the percentage change in sales, controlling for year and firm fixed effects. We obtain $\varepsilon_{MS} = 0.70$ and $\varepsilon_{WS} = 0.46$. However, these estimates are based on all changes in sales, including both small and positive ones. To check how firms respond to large sales drops, we repeated the regressions using only observations for which the change in sales was below -0.1. The number of observations drop to 1.05 million and the elasticities to $\varepsilon_{MS} = 0.62$ and $\varepsilon_{WS} = 0.40$, indicating that large shocks are more difficult to accommodate. We therefore assume a conservative value for the elasticity of intermediates of 0.5. For labor, the Government provides a job retention scheme that allows all firms to reduce paid work by any amount, and let the Government pay workers for the income loss (the *Cassa integrazione guadagni*, that is, the Fund to integrate income). This greatly increases the elasticity of the wage bill to production. The scheme has been extensively used by Italian firms: in April, the number of hours paid was almost 900 millions, equal to the *total* amount paid in the 2009, the worst year of the financial crisis, when GDP contracted by more than 5%. To account for this, we set ε_{WS} to 0.75. Changing these elasticities obviously affects the absolute values, but not the relative effects on zombies and non-zombies.

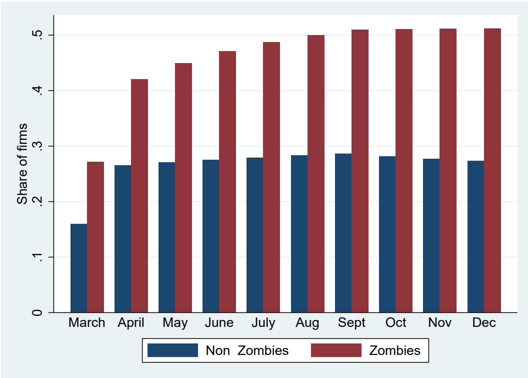
We apply this framework to identify the firms whose liquidity turns negative between March and December, separately for zombies and non-zombies. The results of this exercise are reported in Figure 1. Panel (a) shows the share of zombies and non-zombies that become illiquid. Already in March, more than 27% of zombies hit the negative liquidity constraint, consistent with the fact that the 25^{th} percentile had a liquidity of 3,000 euros (see Table 1). The share increases steadily to over 51% at the end of the year. The share of illiquid firms is less than half for non-zombies: not surprisingly, zombies are more likely to become illiquid. However, given that zombies are only 11% of the total, the absolute number of zombies turning illiquid is approximately one third of that of zombies (panel b). Zombies are on average smaller, so the value is lower in terms of workers: the number of workers in illiquid zombie firms is 10% of those in non-zombies. The difference is similar for total liquidity needs, defined as $\sum_{L_{mt}<0} |L_{mt}|$, arguably due to their lower sales. This can be seen as a proxy of loan demand. It starts below one billion in March and reaches 7 billion by the end of the year. For non-zombies, the values are 12 billion in March and 65 billion in December.

These numbers suggest that zombie lending will be an issue during the Covid-19 crisis but that most of the firms becoming illiquid will be firms that were solid before the start of the pandemic. This is expected as the shock has hit sectors differentially based on characteristics – such as producing nonessential goods and therefore having to shut down, the extent to which workers can work online, etc. – unrelated to financial health. However, some credit will inevitably be given to zombie firms. According to the received wisdom, this might slow down the recovery. We next show that this received wisdom might be based on problematic evidence.

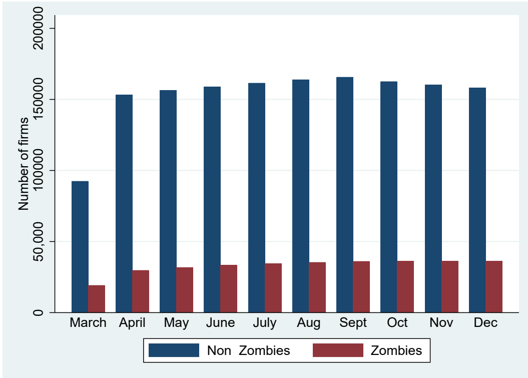
3 The Identification Problem

In this section, we illustrate the approach of the literature and the identification problem it faces.

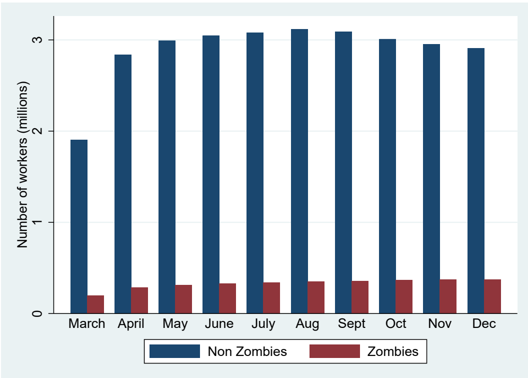
Figure 1: Zombies and non-zombies during the Covid Crisis



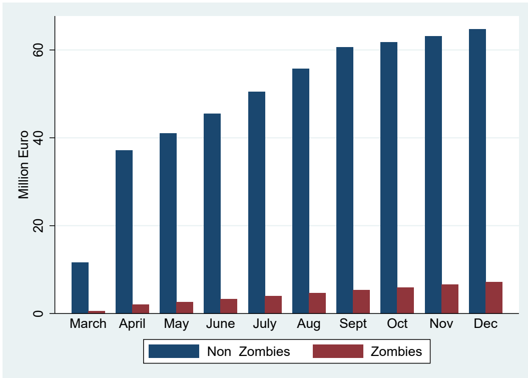
(a) Share of illiquid firms



(b) Number of illiquid firms



(c) Number of workers in illiquid firms



(d) Liquidity needs of illiquid firms

Note: The figure reports the estimation of the evolution of liquidity needs during the Covid-19 crisis for zombies and non-zombies. Panel (a) plots the share of zombies and non-zombies that become illiquid, Panel (b) the number of zombies and non-zombies that become illiquid, Panel (c) the number of workers in zombies and non-zombies, Panel (d) the liquidity needs of zombies and non-zombies.

3.1 The Approach in the Literature

The regression framework typically used to analyze the effects of zombies on heathy firms is the following:

$$X_{ijt} = \beta_0 + \beta_1 D_{ijt}^{NZ} + \beta_2 Z_{jt} + \beta_3 D_{ijt}^{NZ} * Z_{jt} + D_t + S_j + \varepsilon_{ijt} \quad (3)$$

where X is a measure of activity (say employment growth) of firm i in sector j and year t, D_{ijt}^{NZ} is a dummy equal to 1 for non-zombie firms, Z_{jt} measures the presence of zombies in a sector (in Caballero et al. (2008) it is the share of assets of zombie firms in total sectoral assets, in Acharya et al. (2019), McGowan et al. (2018), Gouveia & Osterhold (2018), Blattner et al. (2018) it is the share of zombie firms in a sector), D_t and D_j are year and sector dummies and ϵ_{ijt} an error term. The coefficient β_1 measures the correlation between the share of zombies in the sector and the zombie performance, and β_2 captures the differential effect for non-zombies. A negative estimate of β_2 is interpreted as evidence of negative spillovers from zombies to non-zombies: the higher the share of zombies, the worse the relative performance of healthy firms.

The key identification problem in estimating this regression is that the share of zombies is correlated with shocks affecting the performance of both zombies and non-zombies, such as demand shocks. An adverse demand shock in sector j is bound to increase the share of zombies while also negatively affecting the performance of healthy firms operating in the same sector. This problem is well understood by the literature, which addresses it by specifying the vector of dummy variables as a full set of (country-)sector-year[5] dummy variables D_{jt} and estimate the equation:

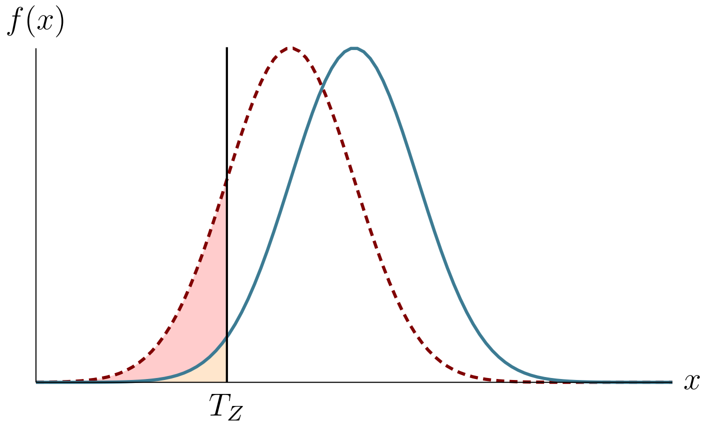
$$X_{ijt} = \beta_0 + \beta_1 D_{ijt}^{NZ} + \beta_3 D_{ijt}^{NZ} * Z_{jt} + D_{jt} + w_{ijt}. \quad (4)$$

In this equation, the absolute effect of the presence of zombies in a sector cannot be estimated anymore, as absorbed by the sector-year dummy; one can only estimate the relative effect on non-zombies, relative to zombies, β_3. Still, provided that the coefficient β_3 correctly identifies the negative spillovers, this is sufficient to state that, if $\beta_3 < 0$, zombies hurt healthy firms, at least in a relative sense. Given that the sector-year dummies take care of all unobserved heterogeneity at the sector-year level, this might seem like a robust empirical framework.

Unfortunately, this is not the case in a standard setting of firms' heterogeneity. This problem is illustrated in Figure 2, where the continuous curve depicts a hypothetical distribution of firms performance in a sector using a normal with mean five and unit standard deviation. The horizontal axis is a measure of firm "quality", such as growth prospects, which translates to firm performance, that is, actual growth. Zombie firms are those below a given threshold T_Z in the Figure. Healthy firms are those to the right of T_Z. We are interested in the difference between the average performance of healthy vs. zombie firms, namely $\mu^{NZ} - \mu^Z$, where $\mu^{NZ} \equiv E(X|X > T_Z)$ and $\mu^Z \equiv E(X|X \leq T_Z)$ denote the mean performance of healthy and zombie firms respectively, and X denotes firm performance. In particular, we want to know how exogenous changes in Z_{jt}, the

[5] Caballero et al. (2008) use data only for Japan, Blattner et al. (2018) for Portugal, while Acharya et al. (2019), McGowan et al. (2018) and Acharya, Crosignani, Eisert & Eufinger (2020) use data for multiple countries.

Figure 2: The effect of a common shock on zombies and non-zombies



The figure plots two normal distributions with unit variance and mean $\mu_L = 4$ and $\mu_H = 5$, respectively. T_Z is the threshold to be classified zombie.

share of zombies in sector j at t, affect $\mu^{NZ} - \mu^Z$ through possible spillover effects, such as distortions of competition or lower credit supply to healthy firms. According to the empirical framework of the literature, this can be assessed by the estimate of β_3 in equation (4): in fact, by OLS estimation, β_3 captures the conditional correlation between the share of zombies Z_{jt} and the relative performance of healthy firms $\mu^{NZ} - \mu^Z$.

The implicit identifying assumption behind this approach is that *in the absence of spillover effects*, shocks that change the share of zombies have the same effect on the average performance of zombies and healthy firms. That is, they do not affect $\mu^{NZ} - \mu^Z$. Under this assumption, observed variations in $\mu^{NZ} - \mu^Z$ associated with variations in the share of zombies can be entirely attributed to spillover effects. Unfortunately, this assumption is unlikely to hold in the data and, therefore, β_3 cannot identify the effects of zombies on non-zombies even if one includes sector-year dummy variables in equation (4). To see this, suppose that the sector is hit by a negative shock that shifts the whole distribution of firms to the left, to the dashed curve depicted in Figure 2. Three things happen. First, the share of zombie firms Z_j increases (the area to the left of T_Z rises, as illustrated by the shaded area in Figure 2). Second, both conditional means μ^{NZ} and μ^Z change and presumably drop.[6] This is the standard identification problem discussed

[6] Note that for some distributions, a leftward shift might actually increase μ^{NZ}, the conditional mean, above the threshold. However, the mean surely decreases for log-concave distributions (Barlow

above, addressed in the literature by the inclusion of area-sector-year dummy variables. Third, the difference between the conditional means $\mu^{NZ} - \mu^{Z}$ could also be affected in a manner that depends on the shape of the distribution of firms' performance. This third identification problem is neglected in the literature, but it can lead to totally spurious conclusions on the effects of zombies on healthy firms.

3.2 An Analytical Result

Consider first a symmetric distribution of firm performance, i.e., $X \sim F(.)$ where $F(.)$ is symmetric. A shift to the left of the distribution is equivalent to a shift to the right of the the threshold T_Z defining zombie firms. But for a large class of symmetric distributions, a shift to the right of the threshold T_Z automatically causes a reduction in the difference between the conditional means $\mu^{NZ} - \mu^{Z}$, provided that the threshold T_Z is below the unconditional mean of X, i.e., $T_Z < E(X)$. That is, a drop in the (average) performance of healthy firms relative to that of zombies cannot be interpreted as evidence of negative spillovers from zombies to healthy firms. It can be a mechanical consequence of a deterioration of the average performance of all firms, or equivalently of an increase in the fraction of zombie firms. Specifically, the appendix proves the following result:

Proposition 1. *Consider a random variable X with density $f(.)$ and cumulative distribution $F(.)$, and let $\mu^{NZ} \equiv E(X|X > T_Z)$ and $\mu^{Z} \equiv E(X|X \leq T_Z)$. Suppose that the following conditions hold:*

1. *X is integrable, i.e. $E[|X|] < +\infty$*
2. *f is symmetric around $E[X]$, that is $f(x + E[X]) = f(E[X] - x)$*
3. *f is differentiable almost everywhere and*

$$f'(x) \begin{cases} > 0 & \textit{if } x < E[X] \\ = 0 & \textit{if } x = E[X] \\ < 0 & \textit{if } x > E[X] \end{cases}$$

4. *$E[|X - E[X]|] > \frac{1}{4f(E[X])}$*

If $T_Z < E[X]$, then

$$\frac{\partial \left(\mu^{NZ} - \mu^{Z}\right)}{\partial T_Z} < 0.$$

& Proschan 1975), a family that includes many commonly used distributions, such as the normal, the Laplace and the logistic.

All four conditions in the Proposition are satisfied for any Normal distribution. The first three conditions are also satisfied for a large class of symmetric distributions, such as the Cauchy, the Logistic and the Student's t.[7]

We illustrate this result for a normal distribution with unit variance and mean equal to 5 (the choice of the mean and variance is inconsequential for the results since the conditions in Proposition 1 are satisfied for any normal distribution). Assume that firms below 3 are classified as zombies, i.e. $T_Z = 3$. We perform the following experiment. We generate negative shocks $s = 0.01, 0.02, ..., 3$ that progressively shift the distribution to the left, $\mu(s) = 5 - s$, and compute $\mu^{NZ}(s) - \mu^{Z}(s)$, that is, the difference in the average quality of non-zombies and zombies, for each value of s. Panel A of figure 3 plots $\mu^{NZ}(s) - \mu^{Z}(s)$ against the shock s and shows that it is decreasing for $s < 2$. as long as the zombie threshold T_Z is to the left of the mean of the distribution (for $s = 2, \mu(s) = 3 = T_Z$). Panel B plots $\mu^{NZ}(s) - \mu^{Z}(s)$ against the share of zombies, that is, the share of firms below 3, which obviously increases with s. Here too, we find a negative relationship, as long as the share of zombies is below 50%.

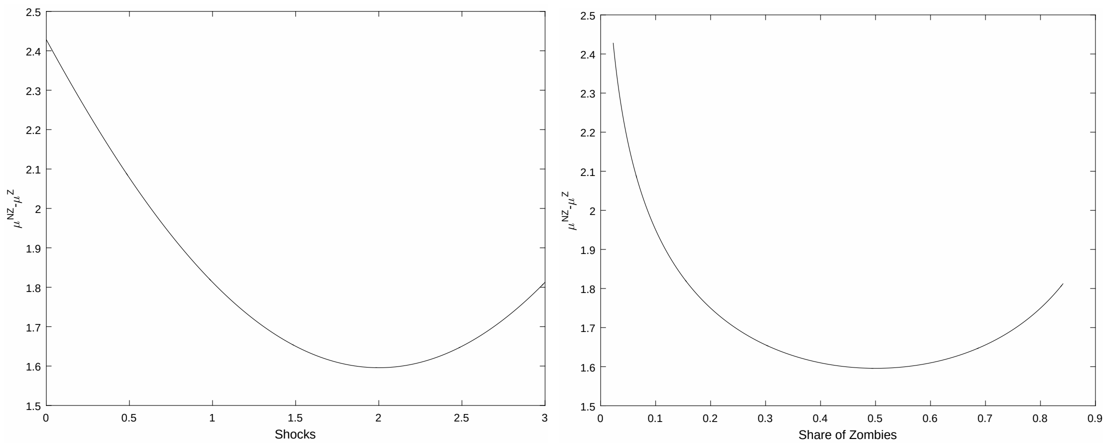
The condition that $T_Z < E(X)$ is generally met in the papers on zombie lending. For example, in Acharya et al. (2019) the share of zombies varies between 3% in Germany and 20% in Italy, while in Caballero et al. (2008) it varies between sector and over time, but it exceeds 20% only in a few years in Services and Real Estate (see their Figure 3).

Thus, in a very standard setting and without any negative spillovers occurring from zombies to non-zombies, the estimation of Equation 4 would deliver a negative coefficient β_3, corresponding to a negative relative performance of healthy firms as the share of zombies increases. But this simply reflects a property of the distribution of firms, and has nothing to do with the hypothesis that a larger share of zombies hurts healthy firms through spillovers in credit, product or input markets.

We have experimented numerically with some non-symmetric distributions typically used in the literature to model the distribution of firm performance. Specifically, we have computed the conditional expectations above and below a threshold T_Z as the distribution shifts leftward (and the share of zombies increases). In unreported results, we find that the correlation between the share of zombies and $\mu^{NZ} - \mu^{Z}$ can be both positive and negative,

[7] Condition 4 is a sufficient, but not necessary, condition. It says that the highest density must be sufficiently high, relative to the absolute deviation from the mean.

Figure 3: Difference in non-zombies vs. zombies average performance



(a) Relative performance and aggregate shocks (b) Relative performance and share of zombies

The graphs report the difference in the conditional mean of zombies and non-zombies, $\mu^{NZ} - \mu^{Z}$. In Panel (a) it is plotted against the aggregate shock $s = 0, 0.01..., 3$, which determines the leftward shift in the performance distribution, as illustrated in Figure 2. In Panel (b) it is plotted against the share of zombies implied by the leftward shift in the distribution shown in Panel (a).

depending on the distribution and the parameters that characterize it. For example, the relationship is continuously decreasing (that is, $\mu^{NZ} - \mu^{Z}$ decreases when the fraction of zombies increases) for the exponential distribution, while the opposite occurs for the Pareto distribution. For the Lognormal, it depends on the parametrization, and it can go from decreasing to increasing as for the normal case to continuous increasing as in the Pareto. Not surprisingly, the first case occurs for parameterizations that make the Lognormal similar to the normal (i.e., for a Lognormal with parameters μ, σ, with low σ) and the second to the Pareto (large σ). Only for the uniform distribution does $\mu^{NZ} - \mu^{Z}$ not vary with the share of zombies.

3.3 Direction of the Bias

We have shown that regressions of the relative performance of healthy firms on the share of zombies can give rise to estimates of β_3 different from zero even in the absence of any spillovers. The estimated coefficient can be positive or negative, depending on the shape of the performance distribution. While this is enough to show that such regressions are not able to identify spillovers, it is interesting to discuss the most likely direction of the bias, since the literature has generally estimated a negative coefficient using this methodology.

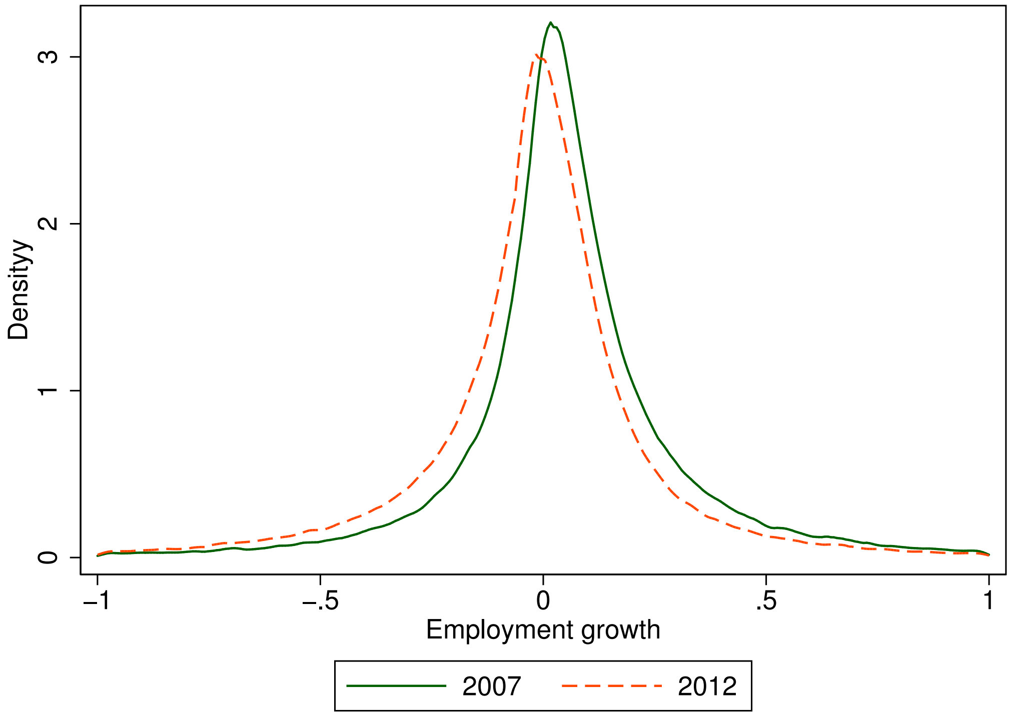
If the distribution is such that the difference in performance of non-zombies and zombies decreases with the share of zombies, then the regression results would imply negative spillovers even when there are none. When the difference in performance increases with the share of zombies, on the other hand, the regression coefficient is biased towards zero and it provide a lower bound for the true effect.

While it is not possible to give a general answer to the direction of the bias, we believe that in practice, the spurious negative estimate is the most likely result for two reasons. First, note that while distributions with a high σ and fat left tail, such as the Pareto and the Lognormal, are typically used to model the firm size distribution, in the zombie literature, the relevant measure of performance is a measure of firm *growth*, such as cost of labor or sales. Figure 4 plots the distribution of the first difference of the log of annual employment (defined in terms of wage bill) for a large sample of Italian incorporated businesses[8] for 2007, the last year before the financial crisis, and 2012, the trough of the European sovereign debt crisis. To make the graph readable, we exclude firms with values below -1 and above 1. We use the Epanechnikov kernel function with bandwith 0.0121 (the standard command in Stata). Two aspects are worth noticing: first, the densities are closer to the normal shape and do not resemble the Pareto at all. Second, during the crisis, there was a leftward shift in the distribution, similar to the one we constructed in Figure 2.

The second reason for which a spurious correlation is likely to emerge is that there is another factor that might induce a decrease in $\mu^{NZ} - \mu^{Z}$ as the distribution shifts to the left. Very low quality firms could exit the marketm which would limit the drop in performance of (surviving) zombies (and hence the drop in $\mu^{Z}(s)$) when shocks hit. This can be seen again in Figure 5, where we also added an exit threshold T_D. When we shift the distribution to the left, the drop in the average quality of zombies is reduced by the fact that extremely low quality zombies drop out of the market. At the same time, as long as the density is higher at the zombie threshold T_Z than at the exit threshold T_D, we still obtain that a leftward shift in the distribution increases the mass of zombies.

[8]The sample is that used by Schivardi et al. (2019) and comprises all the incorporated Italian firms with at least one banking relationship.

Figure 4: Distribution of employment growth in Italian firms, 2007 and 2012



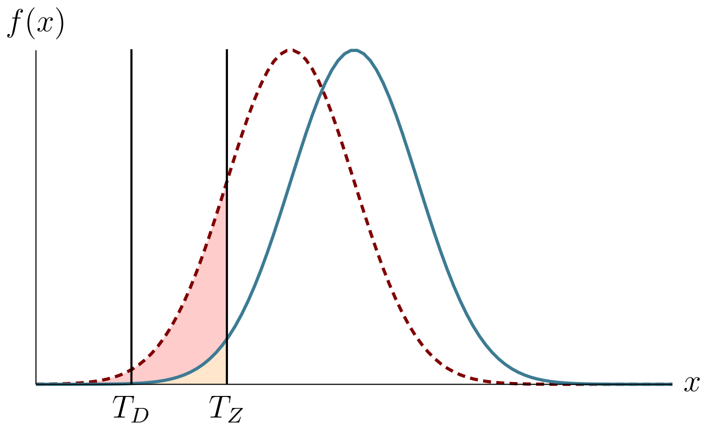
The graph reports the distribution of the growth rate of employment, defined as the first difference of the log of the annual wage bill, for 2007 and 2012 for the sample of incorporated businesses in Italy used in Schivardi et al. (2019).

4 Sources of variability in the share of zombies

The identification issue illustrated above can be restated in standard econometric language by saying the error term w_{ijt} in Equation 4 can be correlated with $D^{NZ}_{ijt} * Z_{jt}$, despite sector-year dummies. This formulation clarifies that, as usual, the causal interpretation of the β_3 coefficient requires some exogenous variation in the share of zombies with respect to aggregate performance shocks. The literature implicitly assumes that such variability comes from banks behaviour, as banks increase the share of zombies by engaging in zombie lending. However, possibly because this subtle identification problem is not fully understood, given that aggregate shocks are controlled for by time-sector effects, almost no paper takes the standard approach of finding an instrument.[9] Rather, bank behaviour is

[9]The only exceptions we are aware of is Blattner et al. (2018), which uses the industry exposure to a regulatory shock imposed by the European Banking Authority and Acharya et al. (2020), which uses the average capitalization, measured as Tier-1 to risk-weighted assets ratio, of the banks connected to the firms in a market, weighted by the number of firm relationships.

Figure 5: The effect of a common shock on zombies and non-zombies



The figure plots two normal distributions with unit variance and mean $\mu_L = 4$ and $\mu_H = 5$, respectively. T_Z is the threshold to be classified zombie and T_D is the threshold for exit.

incorporated in the definitions of zombies. It is therefore useful to discuss these definitions in some detail to assess if they are likely to satisfy the exogeneity requirement.

In the seminal work of Caballero et al. (2008), zombie firms are defined as firms that receive subsidized credit, that is, for which the interest payments over outstanding debt (a measure of average interest rate) are below the prime rate, which is measured by the average rate charged to high quality borrowers. If one believes that the number of zombies allowed to survive varies as a result of a change in lending policies by banks and that such a change is uncorrelated with shocks to the economy-wide distribution of firm performance, then our critique would not apply. Yet, such an assumption is fragile since banks' incentives to subsidize firms are stronger when banks themselves are more at risk of defaulting. Moreover, a negative aggregate shock to firm performance would increase the number of firms at risk and therefore increase the incentives for banks to extend subsidized credit. Therefore, even a definition based purely on whether firms obtain cheap financing may result in a larger number of firms being classified as zombies in sectors that are performing poorly.

Things get more problematic in subsequent literature, which moved towards definitions of zombies based on performance measures, in some cases integrated with indicators of banks lending policies. Storz et al. (2017) use the combination of negative ROA, negative net investment and low EBITDA over debt. Acharya et al. (2019) classify as zombies firms that, in addition to subsidized credit, have a rating of BB or lower while Acharya et al. (2020) use interest coverage below the median and leverage above it. McGowan et al. (2018) and Gouveia & Osterhold (2018) only use interest coverage ratio. Schivardi et al. (2019) define zombies as firms with a low ROA and a high leverage which, in their sample of mostly private, small firms, can be attributed to high bank debt. All these definitions contain a component of performance, possibly coupled with some indication of evergreening by banks (subsidized credit, high debt).[10] It seems unlikely that these definitions pass the exogeneity requirement, that is, that the share of zombies is orthogonal to shocks that shift the performance distribution.

To assess if the results of the literature are likely to be biased, in Schivardi et al. (2019), we have replicated the regressions run by the previous literature on a large sample of Italian firms for the period 2008-2013. The dependent variables are various measures of firm performance (the growth rates of the wage bill, capital spending and sales), and the specifications correspond to equations (3) and (4) above, with j denoting the province-sector and t the year. We define a zombie as a firm that is highly indebted and for which the returns on assets have been systematically below the cost of capital of the safest firms. We also find that as the share of zombies increases, all performance measures deteriorate more for healthy firms than they do for zombies under both specifications. Despite the substantial differences in the sample of firms and institutional setting, we obtain magnitudes similar to those in the literature: the estimates of β_3 for the the capital growth equation 4 are -0.08 in Caballero et al. (2008), -0.018 in Acharya et al. (2019) and -0.043 in our case, and are similar for employment growth. Unfortunately, this finding is likely to be a mechanical consequence of the leftward shift in the distribution and cannot in itself be interpreted as evidence of negative spillovers from zombies to non-zombies. In fact, when we replace the share of zombie firms on the right-hand-side of (3), Z_{jt}, with an arguably exogenous supply side indicator of bank lending to zombie firms, we find no evidence that zombie lending hurts healthy firms.[11]

[10] The definition of subsidized credit may also be affected by firms' performance when it is based on a firms' ratings. As these are proxies of firms' default probability, they also depend on firms' performance.

[11] The only other paper that we are aware of that tries to address this issue is Acharya et al. (2020), which shows that the negative estimate of β_3 disappears when defining zombies only on the basis of performance, that is, relaxing the subsidized credit condition. While this is a useful step toward addressing the identification issue we raise, we do not believe that this is sufficient to fully solve it for the reasons explained above. Within our framework, this result could be explained by the fact that the less stringent definition pushes the zombie threshold to the right in Figure 2, where the correlation between the share

5 Conclusions

The Covid-19 crisis is creating an unprecedented risk of massive bankruptcies due to firms' liquidity shortages following the drop in sales caused by the lockdowns that most countries enacted. Brunnermeier & Krishnamurthy (2020) argue that, contrary to "normal" recessions, the appropriate policy action to contrast the Covid-19 crisis is to incentive banks to engage in evergreen. In line with this prescription, many governments have set up credit guarantee schemes to foster corporate lending during the crisis (OECD 2020).

One potential side effect of such policy is the increase in zombie lending that, according to a recent literature, might slow down the recovery. We argue that this is not likely to be the case. First, using data for Italy, we show that the bulk of liquidity needs during the crisis comes from firms that were financially sound before crisis, as the shock hits firms with an intensity which is independent from their financial conditions.

Second, we show that the framework used in the literature to identify the effects of zombie lending on the real economy suffers from a serious identification problem, due to the fact that the share of zombies can be mechanically correlated with the relative performance of healthy firms. As a consequence, this correlation cannot be interpreted in a causal sense. We believe this is an important result, as it casts doubts on the policy prescription emerging from this literature, that zombie lending slows down the recovery. These prescriptions have played an important role in the policy debate following the financial and sovereign debt crisis, and they may also become relevant in addressing the COVID-19 crisis. It is therefore important that they are backed by flawless empirical evidence. Future research will need to put more emphasis on exogenous variation in the number of zombie firms to determine if these prescriptions are correct.

All in all, we conclude that the risk that government credit guaranteed schemes incentive zombie lending should not hold back governments and banks from providing firms with credit during the Covid-19 pandemic: it is essential to avoid that the liquidity crisis pushes many solvent firms out of the market.

---

of zombies and the relative performance of non-zombies becomes weaker. In particular, in Acharya et al. (2020) zombies are firms with interest coverage below the median and leverage above it. These two conditions are likely to be correlated, so that the resulting share of zombies could be close to 50%, where the effect becomes zero (see Figure 3, Panel B).

References

Acharya, V. V., Crosignani, M., Eisert, T. & Eufinger, C. (2020), Zombie credit and (dis-) inflation: Evidence from Europe. Mimeo, NYU Stern.

Acharya, V. V., Eisert, T., Eufinger, C. & Hirsch, C. W. (2019), 'Whatever it takes: The real effects of unconventional monetary policy', *The Review of Financial Studies* **32**, 3366–3411.

Anderson, G., Riley, R. & Young, G. (2019), Distressed banks, distorted decisions? Mimeo, University of Oxford.

Barlow, R. E. & Proschan, F. (1975), *Statistical Theory of Reliability and Life Testing*, Holt, Rinehart and Winston, New York.

Barrero, J. M., Bloom, N. & Davis, S. J. (2020), Covid-19 is also a reallocation shock. NBER Working Paper No. 27137.

Barrot, J.-N. & Sauvagnat, J. (2016), 'Input specificity and the propagation of idiosyncratic shocks in production networks', *The Quarterly Journal of Economics* **131**(3), 1543–1592.

Bernstein, S., Colonnelli, E., Giroud, X. & Iverson, B. (2019), 'Bankruptcy spillovers', *Journal of Financial Economics* **133**, 608–633.

Blattner, L., Farinha, L. & Rebelo, F. (2018), When losses turn into loans: The cost of undercapitalized banks. Mimeo. Harvard University.

Brunnermeier, M. & Krishnamurthy, A. (2020), COVID-19 SME evergreening proposal: Inverted economics. Mimeo, Princeton Economics. Availabel at https://scholar.princeton.edu/sites/default/files/markus/files/covid_sme_evergreening.pdf.

Caballero, R. J., Hoshi, T. & Kashyap, A. K. (2008), 'Zombie lending and depressed restructuring in Japan', *American Economic Review* **98**, 1943–77.

Gouveia, A. F. & Osterhold, C. (2018), Fear the walking dead: Zombie firms, spillovers and exit barriers. OECD Productivity working papers No. 13.

Hsieh, C.-T. & Klenow, P. J. (2009), 'Misallocation and manufacturing TFP in China and India', *The Quarterly Journal of Economics* **124**(4), 1403.

McGowan, M. A., Andrews, D. & Millot, V. (2018), 'The walking dead? Zombie firms and productivity performance in OECD countries', *Economic Policy* **33**.

Mian, A., Sufi, A. & Trebbi, F. (2015), 'Foreclosures, house prices, and the real economy', *The Journal of Finance* **70**(6), 2587–2634.

OECD (2020), Coronavirus (COVID-19): SME policy responses. Availabe at http://www.oecd.org/coronavirus/policy-responses/coronavirus-covid-19-sme-policy-responses-04440101/.

Schivardi, F. & Romano, G. (2020), A simple method to estimate firms liquidity needs during the Covid -19 crisis with an application to Italy . Mimeo. Luiss University and EIEF.

Schivardi, F., Sette, E. & Tabellini, G. (2019), Credit misallocation during the European financial crisis. CEPR Discussion Paper No. 11901 (revised version).

Storz, M., Koetter, M., Setzer, R. & Westphal, A. (2017), Do we want these two to tango? On zombie firms and stressed banks in Europe.

A Proof of Proposition 1

Proof. For ease of notation, we replace T_Z with c. Fix $c < E[X]$. Without loss of generality, assume $E[X] = 0$, otherwise we consider $Y = X - E[X]$ and $f_Y(y) = f(E[X] + y)$. By ease of notation call:

$$M(c) := E[X|X > c] \quad m(c) := E[X|X < c]$$

It is easy to show that:

$$M'(c) = \frac{f(c)}{1 - F(c)}(M(c) - c)$$

and:

$$m'(c) = \frac{f(c)}{F(c)}(c - m(c))$$

Where F is the cumulative distribution function of X.
Now notice that:

$$E[X] = F(c)m(c) + (1 - F(c))M(c)$$

By recognizing that $E[X] = 0$, we have:

$$m(c) = -\frac{1 - F(c)}{F(c)}M(c)$$

Then our proof becomes to prove that:

$$g(c) = M(c) - m(c) = M(c)\left[1 + \frac{1 - F(c)}{F(c)}\right] = \frac{M(c)}{F(c)}$$

is strictly decreasing as $c < 0$.

Our proof will rely on three different lemmas.

Lemma 1. $g'(0) = 0$.

Proof. Differentiating $g(c)$ we obtain:

$$
\begin{aligned}
g'(c) &= \frac{M'(c)F(c) - f(c)M(c)}{(F(c))^2} \\
&= \frac{f(c)}{(1-F(c))(F(c))}\left[M(c) - c\right] - \frac{f(c)M(c)}{(F(c))^2} \\
&= \frac{f(c)F(c)(M(c)-c) - (1-F(c))f(c)M(c)}{(1-F(c))(F(c))^2} \\
&= \frac{f(c)F(c)M(c) - f(c)F(c)c - (1-F(c))f(c)M(c)}{(1-F(c))(F(c))^2} \\
&= \frac{f(c)M(c)(F(c) - 1 + F(c)) - f(c)F(c)c}{(1-F(c))(F(c))^2} \\
&= \frac{f(c)M(c)(2F(c)-1) - f(c)F(c)c}{(1-F(c))(F(c))^2} \\
&= \frac{f(c)}{F(c)\,(1-F(c))}\left[\frac{2F(c)-1}{F(c)}M(c) - c\right]
\end{aligned}
$$

By symmetry, $F(0) = \frac{1}{2}$. Hence $g'(0) = 0$. QED.

Lemma 2. $\lim_{c\to 0^-} g'(c) < 0 < \lim_{c\to 0^+} g'(c)$

Proof. We have that:

$$
\begin{aligned}
g'(c) &= \frac{f(c)}{F(c)\,(1-F(c))}\left[\frac{2F(c)-1}{F(c)}M(c) - c\right] \\
&= \frac{f(c)}{(F(c))^2\,(1-F(c))}\left[(2F(c)-1)M(c) - cF(c)\right] \qquad (5) \\
&= \frac{f(c)}{(F(c))^2\,(1-F(c))}h(c)
\end{aligned}
$$

where $h(c) = (2F(c)-1)\,M(c) - cF(c)$. Thus, $g'(c) < 0$ if and only if $h(c) < 0$. Note that $h(0) = 0$ by symmetry.

Next, differentiate $h(c)$:

$$
\begin{aligned}
h'(c) &= 2f(c)M(c) + M'(c)(2F(c)-1) - F(c) - f(c)c \\
&= 2f(c)M(c) + \frac{f(c)}{1-F(c)}(M(c)-c)(2F(c)-1) - F(c) - f(c)c
\end{aligned}
$$

We have that:

$$\begin{aligned}
h'(c) > 0 \Leftrightarrow\ & 2f(c)M(c) + \frac{f(c)}{1-F(c)}(M(c)-c)(2F(c)-1) - F(c) - f(c)c > 0 \\
\Leftrightarrow\ & 2f(c)M(c) + \frac{f(c)}{1-F(c)}(2F(c)-1)M(c) - \frac{f(c)}{1-F(c)}(2F(c)-1)c - cf(c) > F(c) \\
\Leftrightarrow\ & f(c)M(c)\left[2 + \frac{2F(c)-1}{1-F(c)}\right] - f(c)\left[\frac{2F(c)-1}{1-F(c)} + 1\right] c > F(c) \\
\Leftrightarrow\ & f(c)M(c)\left[\frac{2-2F(c)+2F(c)-1}{1-F(c)}\right] - f(c)\left[\frac{2F(c)-1+1-F(c)}{1-F(c)}\right] c > F(c) \\
\Leftrightarrow\ & f(c)M(c)\left[\frac{1}{1-F(c)}\right] - f(c)\left[\frac{F(c)}{1-F(c)}\right] c > F(c) \\
\Leftrightarrow\ & M(c)\left[\frac{1}{1-F(c)}\right] - \left[\frac{F(c)}{1-F(c)}\right] c > \frac{F(c)}{f(c)}
\end{aligned}$$

So:

$$h'(c) > 0 \Leftrightarrow\ M(c)\left[\frac{1}{1-F(c)}\right] - c\left[\frac{F(c)}{1-F(c)}\right] > \frac{F(c)}{f(c)} \tag{6}$$

Now consider the point $c = 0$. By symmetry the above inequality reads:

$$M(0) > \frac{1}{4f(0)} \tag{7}$$

Moreover, also by symmetry, one can show that

$$M(0) = \frac{1}{F(0)} \int_c^{+\infty} xf(x)\,dx = 2\int_c^{+\infty} xf(x)\,dx$$

and:

$$\begin{aligned}
E[|X|] &= \int_{-\infty}^{0} -xf(x)\,dx + \int_0^{+\infty} xf(x)\,dx \\
&= \int_0^{+\infty} yf(-y)\,dy + \int_0^{+\infty} xf(x)\,dx \\
&= 2\int_0^{+\infty} xf(x)\,dx \\
&= M(0)
\end{aligned}$$

where the second equality follows from substituting $y = -x$ and the third is true thanks to the symmetry of $f(x)$. Hence, (7) and (6) can be written as:

$$h'(0) > 0 \Leftrightarrow E[|X|] > \frac{1}{4f(0)}$$

which holds by assumption 4.

We have shown that $h(0) = 0$ and $h'(0) > 0$. Since $h(c)$ is continuously differentiable, this means that $h(c) < 0$ in a left-neighbourhood of 0 and $h(c) > 0$ in a right-neighborhood of 0. As as a consequence, $g'(c) < 0$ and $g'(c) > 0$ in a left-neighbourhood and right-neighborhood of 0 respectively. QED

Lemma 3. *Suppose that there exists $c^* < 0$ such that $g'(c^*) = 0$. Then $g''(c^*) < 0$.*

Proof. Suppose that there exists $c^* < 0$ such that $g'(c^*) = 0$. By (5):

$$c^* = \frac{2F(c^*) - 1}{F(c^*)} M(c^*) \tag{8}$$

which can be rewritten as:

$$M(c^*) = \frac{c^* F(c^*)}{2F(c^*) - 1} \tag{9}$$

Then replacing $M(c^*)$ with the RHS of (9) into (6), we obtain:

$$\begin{aligned}
h'(c^*) > 0 \Leftrightarrow\ & M(c^*)[\frac{1}{1-F(c^*)}] - [\frac{F(c^*)}{1-F(c^*)}]c^* > \frac{F(c^*)}{f(c^*)} \\
\Leftrightarrow\ & \frac{c^* F(c^*)}{2F(c^*)-1}\left[\frac{1}{1-F(c^*)}\right] - \left[\frac{F(c^*)}{1-F(c^*)}\right] c^* > \frac{F(c^*)}{f(c^*)} \\
\Leftrightarrow\ & \frac{c^*}{1-F(c^*)}\left[\frac{1}{2F(c^*)-1} - 1\right] > \frac{1}{f(c^*)} \\
\Leftrightarrow\ & \frac{c^*}{1-F(c^*)}\left[\frac{2-2F(c^*)}{2F(c^*)-1}\right] > \frac{1}{f(c^*)} \\
\Leftrightarrow\ & \frac{2c^*}{2F(c^*)-1} > \frac{1}{f(c^*)} \\
\Leftrightarrow\ & 2c^* f(c^*) - 2F(c^*) + 1 < 0
\end{aligned}$$

where the sign of the inequality has been changed since:

$$2F(c^*) - 1 < 0$$

This follows from symmetry, given that:

$$c^* < 0 \Rightarrow F(c^*) < \frac{1}{2}$$

So, we have shown that $h'(c^*) > 0$ if and only if:

$$p(c^*) = 2c^* f(c^*) - 2F(c^*) + 1 < 0$$

Consider the function $p(c)$ with $c \leq 0$. Then: (i) $p(0) = 0$; (ii) $p'(c) = 2cf'(c) < 0$ if $c < 0$ by assumption 3. Since $c^* < 0$ and $p(c)$ is continuously differentiable, (i) and (ii) imply $p(c^*) > 0$.

Now we want to evaluate $g''(c)$. Note first of all that, by (5), $g'(c)$ can be written as:

$$g'(c) = \frac{f(c)h(c)}{(F(c))^2(1-F(c))} = \frac{N(c)}{D(c)}.$$

Differentiating w.r. to c :

$$g''(c) = \frac{N'(c)D(c) - D'(c)N(c)}{(D(c))^2}$$

where:

$$N'(c)D(c) = [f'(c)h(c) + h'(c)f(c)][F(c)]^2(1 - F(c)) \tag{10}$$

Now we can recognize that $h(c^*) = 0$, given that $g'(c^*) = 0$ by assumption. As a consequence $N(c^*) = 0$ and hence $D'(c^*)N(c^*) = 0$

Thus we have:

$$g''(c^*) = \frac{N'(c)}{D(c)} = \frac{f(c^*)}{(F(c^*))^2\,(1 - F(c^*))}h'(c^*)$$

where both equalities follow from from $h(c^*) = 0$. Since $h'(c^*) < 0$, we thus have $g''(c^*) < 0$. QED

Now we complete the proof of the Proposition by contraddiction. By Lemmas 1 and 2, $c = 0$ is a point of local minimum of $g(c)$. By Lemma 3, any point $c^* < 0$ such that $g'(c^*) = 0$ must be a local maximum of $g(c)$. Thus, Lemma 3 also implies that $c^* < 0$, if it exists, must be unique, since $g(c)$ is continuously differentiable and we cannot have two consecutive local maxima. Thus, existence of $c^* < 0$ implies that $g'(c) > 0$ for any $c < c^*$. But this is impossible because:

$$\lim_{c\to-\infty} g(c) = \lim_{c\to-\infty} \frac{M(c)}{F(c)} = \lim_{c\to-\infty} \frac{m(c)}{F(c) - 1} = +\infty$$

where the second equality follows from $m(c) = -\frac{1-F(c)}{F(c)}M(c)$. Thus, a point $c^* < 0$ such that $g'(c^*) = 0$ cannot exist. Therefore, by Lemmas 1 and 2, 0 is the only critical point and $g'(c) < 0$ for any $c < 0$.QED

As an application, we show that $X \sim N(\mu, \sigma^2)$ satisfies the assumptions. The first three ones are obvious, so we focus on the fourth. By the properties of the Gaussian distribution:

$$\begin{cases} E[|X - E[X]|] = \sigma\sqrt{\frac{2}{\pi}} \\ f(E[X]) = \frac{1}{\sqrt{2\pi\sigma^2}} \end{cases}$$

With easy computations:

$$E[|X - E[X]|] > \frac{1}{4f\,(E[X])} \Leftrightarrow \sqrt{\frac{2}{\pi}} > \frac{\sqrt{2\pi}}{4} \Leftrightarrow 4 > \pi$$

this is true.