Diagnostic Expectations and Credit Cycles

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Abstract

We present a model of credit cycles arising from diagnostic expectations – a belief formation mechanism based on Kahneman and Tversky's (1972) representativeness heuristic. Diagnostic expectations overweight future outcomes that become more likely in light of incoming data. The expectations formation rule is forward looking and depends on the underlying stochastic process, thus being immune to the Lucas critique. Diagnostic expectations reconcile extrapolation and neglect of risk in a unified framework. In our model, credit spreads are excessively volatile, over-react to news, and are subject to predictable reversals. These dynamics can account for several features of credit cycles and macroeconomic volatility.

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1.Introduction

The financial crisis of 2008-2009 revived economists' and policymakers' interest in the relationship between credit expansion and subsequent financial and economic busts. According to an old argument (e.g., Minsky 1977), investor optimism brings about the expansion of credit and investment, and leads to a crisis when such optimism abates. Stein (2014) echoes this view by arguing that policy-makers should be mindful of credit market frothiness and consider countering it through policy. In this paper, we develop a behavioral model of credit cycles with micro-founded expectations, which yields the Minsky narrative but is also consistent with a great deal of evidence.

Recent empirical research has developed a number of credit cycle facts. Schularick and Taylor (2012) demonstrate, using a sample of 14 developed countries between 1870 and 2008, that rapid credit expansions forecast declines in real activity. Jorda, Schularick, and Taylor (2013) further find that more credit-intensive expansions are followed by deeper recessions. Mian, Sufi, and Verner (2016) show that the growth of household debt predicts economic slowdowns. Baron and Xiong (2017) establish in a sample of 20 developed countries that bank credit expansion predicts increased crash risk in both bank stocks and equity markets more broadly. And Fahlenbrach, Prilmeier, and Stulz (2016) find, in a cross-section of U.S. banks, that fast loan growth predicts poor loan performance and low bank returns in the future.

Parallel findings emerge from the examination of credit conditions. Greenwood and Hanson (2013) show that credit quality of corporate debt issuers deteriorates during credit booms, and that high share of risky loans in the total forecasts low, and even negative, corporate bond returns. Gilchrist and Zakrajsek (2012) and Krishnamurthy and Muir (2015) relatedly establish that eventual credit tightening correctly anticipates the coming recession. Lopez-Salido, Stein, and Zakrajsek (hereafter

LSZ 2017) find that low credit spreads predict both a rise in credit spreads and low economic growth afterwards. They stress predictable mean reversion in credit market conditions.[2] In Section 2 we offer preliminary evidence that survey forecasts of future credit spreads are excessively optimistic when these spreads are low, and that both errors and revisions in forecasts are predictable. Overall, the existing evidence is hard to square with rational expectations, indicating a need for a behavioral approach to modeling credit cycles.

We propose a behavioral model that accounts for this evidence, and describes in a dynamic setup how credit markets overheat. We offer a psychologically-founded model of beliefs and their evolution in light of new data.[3] The model implies that in a boom investors are too optimistic and systematically become more pessimistic in the future, leading to a crisis even without deteriorating fundamentals. The model unifies the phenomena of extrapolation (Cagan 1956, Cutler et al. 1990, DeLong et al. 1990, Barberis and Shleifer 2003, Greenwood and Shleifer 2014, Barberis et al. 2015, 2016, Gennaioli, Ma, and Shleifer 2015) and the neglect of risk (Gennaioli, Shleifer, and Vishny 2012, Coval, Pan, and Stafford 2014, Arnold, Schuette, and Wagner 2015). Critically, households in our model are forward looking. As a result, different stochastic processes of fundamentals entail different expectation formation rules, making the model immune to the Lucas critique.

Our model was developed not to match credit cycles, but rather to account for well-documented judgment biases in the lab, such as the conjunction and disjunction fallacies and base rate neglect. Here it is adapted to macroeconomic problems. The

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[2] See also Bernanke (1990), Friedman and Kuttner (1992), and Stock and Watson (2003), among others.

[3] Many models of beliefs in finance are motivated by psychological evidence, but often use specifications specialized to financial markets (e.g., Muth 1961, Barberis, Shleifer, and Vishny 1998, Rabin and Vayanos 2010, Fuster, Laibson, and Mendel 2010, Hirshleifer et al 2015, Greenwood and Hanson 2015, Barberis et al. 2015). Fuster et al (2010) review lab and field evidence on deviations from rational expectations.

model is thus portable in the sense of Rabin (2013). The model is based on Gennaioli and Shleifer's (2010) formalization of Kahneman and Tversky's (KT 1972, TK 1983) representativeness heuristic describing how people judge probabilities. According to KT, a certain attribute is judged to be excessively common in a population when that attribute is diagnostic for the population, meaning that it occurs more frequently in this population than in a relevant reference population. For example, after seeing a patient test positive on a medical test, doctors overestimate the likelihood that he has the disease because sick people are more numerous in the population of positive tests relative to the population of negative tests, even when they are few in absolute terms (Casscells et al. 1978).

This idea can be naturally applied to modeling expectations. Analogously to the medical test example, agents overweight in their beliefs the future states whose likelihood increases the most in light of current news *relative to what they know already*. Just as doctors overestimate the probability of sickness after a positive test result, agents overestimate the probability of a good future state when the current news is good. Following TK (1983)'s description of the representativeness heuristic as overweighting diagnostic information, we refer to such beliefs as diagnostic expectations.

This approach has significant implications. For example, a path of improving news leads to excess optimism, and a path of deteriorating news to excess pessimism, even when these paths lead to the same fundamentals. There is a kernel of truth in assessments: revisions respond to news, but excessively. When change slows down, the agent no longer extrapolates. This leads to a reversal, even absent bad shocks. Excessively volatile expectations drive cyclical fluctuations in both financial and economic activity.

We construct a neoclassical macroeconomic model in which the only non-standard feature is expectations. To begin, we do not include financial or any other frictions. The model accounts for many empirical findings, some of which also obtain under rational expectations, but some do not. In our model:

1) In response to good news about the economy, credit spreads decline, credit expands, the share of high risk debt rises, and investment and output grow.
2) Following this period of narrow credit spreads, these spreads predictably rise on average, credit and the share of high risk debt decline, while investment and output decline as well. Larger spikes in spreads predict lower GDP growth.
3) Credit spreads are too volatile relative to fundamentals and their changes are predictable in a way that parallels the cycles described in points 1) and 2).
4) Investors commit predictable forecast errors and forecast revisions. Bond returns are also predictable in a way that parallels points 1) and 2).

Prediction 1) can obtain under rational expectations, and the same is true about prediction 2) provided fundamentals are mean reverting. Predictions 3) and 4), in contrast, critically depend on our model of diagnostic expectations. After presenting the basic model, we briefly explore the interaction between diagnostic expectations and leverage by proposing an extension with a preferred habitat for safe debt, a version of financial frictions. In this setting, excess volatility in expectations causes strong market reactions by inducing a re-classification of debt.

Our paper is related to three strands of research. First, the prevailing approach to understanding the link between financial markets and the real economy is financial frictions, which focus on the transmission of an adverse shock through a leveraged economy (Bernanke and Gertler 1989, Kiyotaki and Moore 1997)[4]. The adverse shock in such models is either a drop in fundamentals, or a "financial shock" consisting of the tightening of collateral constraints or an increase in required returns. These models do not usually explain the sources of "financial shocks". As importantly, because they assume rational expectations, these models do not explain predictable negative or low abnormal returns on debt in over-heated markets or systematic errors in expectations. Our model accounts for both market crises and abnormal returns.

Second, our paper is related to recent work on limited attention (e.g., Sims 2003, Gabaix 2014). In general, these models predict sluggish expectations and under-reaction to information, consistent with empirical evidence for inflation (Coibion and Gorodnichenko 2012, 2015). Also related is research on momentum and slow reaction to information in financial markets (Jegadeesh and Titman 1993, Hong and Stein 1999, Bouchard et al. 2016). Our model most naturally delivers over-reaction to information, although we discuss briefly how the two approaches can be unified.

Third, our paper continues research on behavioral credit cycles, initiated by Minsky (1977). Gennaioli, Shleifer, and Vishny (2012) present a model of neglected risk. Gennaioli, Shleifer, and Vishny (2015) sketch a model of credit cycles exhibiting both under-reaction and over-reaction based on the Bordalo, Coffman, Gennaioli, Shleifer (BCGS 2016) model of stereotypes. Jin (2015) models extrapolation in credit markets. Greenwood, Hanson, and Jin (2016) present a model of extrapolation of default rates which also delivers many of the credit cycle facts. We unify several theories, as well as a good deal of evidence in a micro-founded model of beliefs.

[4] Some papers add to financial frictions Keynesian elementd, such as the zero lower bound on interest rates or aggregate demand effects (e.g., Eggertson and Krugman 2012, Rognlie et al. 2015).

In section 2, we document the predictability of both forecast errors and forecast revisions. Section 3 introduces diagnostic expectations, describes how they evolve, and relates our formulation to extrapolation and neglect of risk. Section 4 presents our model of credit cycles. Section 5 develops the predictions of the model for the behavior of credit spreads, expectations about credit spreads, and the link between spreads and economic activity. Section 6 incorporates safe debt. Section 7 concludes. An Appendix discusses some alternative specifications of the diagnostic expectations model.

2. Some Evidence on Expectations and Credit Spreads

We start by offering some motivating evidence on analysts' expectations of the Baa bond – Treasury credit spread. Both here and in our theory we focus on the credit spread, a commonly used indicator of credit market conditions (Greenwood and Hanson 2013). We do not have enough data to perform a stringent test of rational expectations, but can illustrate how the analysis of credit cycles can be deepened with expectations data, and to establish some facts that a model of expectations formation should account for[5].

We use data from Blue Chip Financial Forecasts, a monthly survey of around 40 panelists' forecasts of various interest rates for the current quarter and for each of 6 quarters ahead. We average quarterly forecasts to obtain 12-month forecasts; we then construct consensus forecasts by averaging expectations across analysts. We use data from the March, June, September, and December publications. Forecasts of the Baa spread are obtained as the difference between the forecasts of the Baa corporate bond

[5] One may worry that Blue Chip professional forecasts are distorted for signaling or entertainment reasons since participants are not anonymous. However, these forecasts tend to be very similar to the anonymous forecasts collected by the Philadelphia Fed Survey of Private Forecasters. Moreover, unlike in the case of stock analysts, there is no unconditional bias in the Blue Chip forecasts we study here.

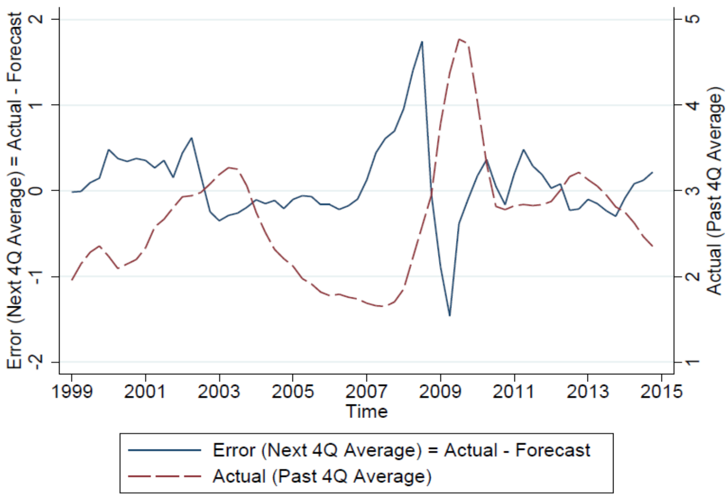
yield and of the 10Y Treasury yield. Our data covers the period from 1999Q1 to 2014Q4.

2.1 Predictability in Forecast Errors

Under the assumption of rational expectations (and knowledge of the data generating process), analysts' forecast errors should be unpredictable from past data. Figure 1 plots, over time, the current spread against future forecast errors. The data suggest predictability: when the current spread is low (high), the expected spread is too low (high). Likewise, when the current spread is high, the expected spread is too high. The 1999-2000 and 2005-2008 periods witness low spreads and excessive optimism, the early 2000s and the recent crisis witness high spreads and excess pessimism.

Table 1 reports an econometric test of predictability. Column 1 estimates an AR(1) process for the Baa-10Y spread, column 2 regresses analysts' forecast on the current spread, column 3 regresses the future forecast error on the current spread.

Figure 1: Predictable Errors in Forecasts of Credit Spread



	Actual	Forecast	Error
Avg. spread past year	0.3927 [1.67]	0.6519 [4.62]	-0.2592 [-2.20]

Constant	1.6280 [2.56]	0.8596 [2.25]	0.7684 [2.40]
Observations	64	64	64
R^2	0.158	0.472	0.161

Table 1: Actual, Forecast, and Error of Next Year Average Baa-10Y Credit Spread
(Actual Forecast Error=Actual-Forecast, Newey West t-statistics in brackets)

Table 1 confirms the message of Figure 1. In column 3, the higher the current spread, the higher is the forecast relative to the realization. This may occur because analysts see excessive persistence in current conditions: in Column 1 the estimated persistence of the actual Baa-10Y spread is about 0.4, but in column 2 forecasts follow the current spread with a coefficient of about 0.6.

2.2 Tests of Expectations' Revisions

We next examine forecast revisions, which should also be unpredictable under rational expectations. Figure 2 plots the current spread against the future forecast revision, defined as the difference between the forecast for the spread in quarter t + 4 made in quarter t + 3 and the current (quarter t) forecast of the same spread. The evidence again suggests predictability: when the current spread is low, forecasts are revised upwards, when the current spread is high, forecasts are revised downward.

Figure 2: Predictable Revisions in Forecasts of Credit Spread

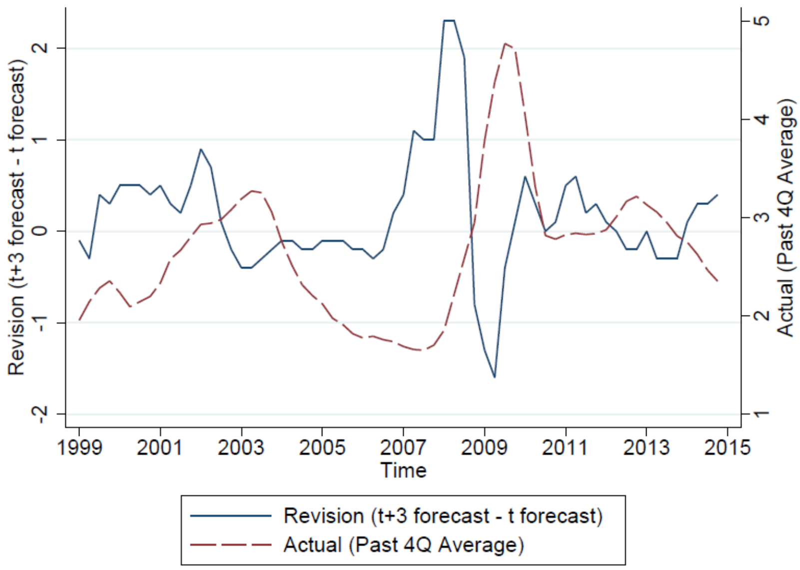


Table 2 shows that this predictability is statistically robust.

	Revision of Baa-10Y spread
Avg. spread past year	-0.3636 [-2.13]
Constant	1.1334 [2.44]
Observations	64
R^2	0.152

Table 2: Forecast Revisions in Credit Spreads
(Revision=Forecast(t+3)-Forecast(t), Newey West t-statistics in brackets)

This evidence is difficult to reconcile with rational expectations, but suggests that analysts' forecasts follow a boom bust pattern. During booming bond markets (low spreads), expectations are too optimistic and systematically revert in the future, planting the seeds of a cooling of bond markets. The extrapolative dynamics of forecasts that we document here are in line with other studies, such as Greenwood and Hanson's (2013) evidence of systematic reversal in bond spreads, and the extrapolative nature of CFO's expectations about their company's earnings growth (Gennaioli et al. 2015).

3. Diagnostic Expectations

3.1 A Formal Model of Representativeness

We build our model of expectations from first principles, starting with Kahneman and Tversky's representativeness heuristic, which they define as follows: "an attribute is representative of a class if it is very diagnostic; that is, the relative frequency of this attribute is much higher in that class than in the relevant reference class (TK 1983)." KT argue that individuals often assess likelihood by representativeness, thus estimating types or attributes as being likely when they are instead representative, and present a great deal of experimental evidence to support this claim. Gennaioli and Shleifer (2010) construct a model in which judgment biases arise because decision makers overweight events that are representative precisely in the sense of KT's definition. To motivate diagnostic expectations, we briefly describe this model and a related application to stereotype formation by Bordalo et al. (BCGS 2016).

A decision maker judges the distribution of a trait T in a group G. The true distribution of the trait is $h(T = t|G)$. GS (2010) define the representativeness of the trait $T = t$ for group G to be:

$$\frac{h(T = t|G)}{h(T = t| - G)},$$

where $-G$ is a relevant comparison group. As in KT's quote, a trait is more representative if it is *relatively* more frequent in G than in $-G$. GS (2010) assume that representative types are easier to recall. Due to limited working memory, the agent overweighs these types in his assessment. Note that in this model beliefs about a group G are context dependent because they depend on features of the comparison group $-G$.

To illustrate, consider an individual assessing the distribution of hair color among the Irish. The trait T is hair color, the conditioning group G is the Irish. The comparison group $-G$ is the world at large. The true relevant distributions are:[6]

	$T = red$	$T = blond\,/$ $light\ brown$	$T = dark$
$G \equiv Irish$	10%	40%	50%
$-G \equiv World$	1%	14%	85%

The most representative hair color for the Irish is red because it is associated with the highest likelihood ratio among hair colors:

$$\frac{Pr(red\ hair|Irish)}{Pr(red\ hair|World)} = \frac{10\%}{1\%} = 10.$$

Our model thus predicts that assessments exaggerate the frequency of red haired Irish. After hearing the news "Irish", the representative red-haired type comes to mind and its likelihood is inflated. The agent discounts the probability of blond and dark hair types because they are less available when thinking about the Irish.

In sum, representativeness causes the agent to inflate the likelihood of types whose *objective* probability rises the most in G relative to the reference *context* $-G$.[7] GS (2010) and BCGS (2016) show that this model unifies widely documented errors in probabilistic judgment, such as base rate neglect (the medical test example of the introduction) or the conjunction fallacy, and sheds light on key features of social stereotypes, and of context dependent beliefs.

3.2 Diagnostic Expectations

The same logic can be applied to belief formation about aggregate economic conditions. Time is discrete $t = 0,1,\ldots$ The state of the economy at t is a random

[6] Shares of red hair are accurate, shares of and dark and brown are approximate. See http://www.eupedia.com/genetics/origins_of_red_hair.shtml and http://www.eupedia.com/europe/genetic_maps_of_europe.shtml.

[7] This example also illustrates context dependence of beliefs. It is the paucity of red haired people in the "rest of the world" that renders red hair so distinctive for the Irish. The judgment bias would be smaller if the share of red haired people in the rest of the world were to rise, or equivalently if the agent was primed to think about the Irish in the context of a more similar group (e.g., $-\ G = Scots$).

variable ω_t that follows the AR(1) process $\omega_t = b\omega_{t-1} + \epsilon_t$, with $\epsilon_t \sim N(0, \sigma^2)$, $b \in [0,1]$. The model is easily generalized to richer AR(N)-normal processes.

When forming a forecast the agent assesses the distribution of a certain future state, say $\widehat{\omega}_{t+1}$, entailed by the realized current conditions $\omega_t = \widehat{\omega}_t$. This is similar to the medical test example, where the doctor assesses the health of the patient conditional on a positive test outcome. Pursuing the analogy, the agent must predict the distribution of future prospects ω_{t+1} in a group $G \equiv \{\omega_t = \widehat{\omega}_t\}$ that summarizes current conditions.

The rational agent solves this problem by using the true conditional distribution of ω_{t+1} given $\omega_t = \widehat{\omega}_t$, denoted $h(\omega_{t+1}|\omega_t = \widehat{\omega}_{t+1})$. The agent whose judgments are shaped by representativeness has this true distribution in the back of his mind, but selectively retrieves and thus overweighs realizations of ω_{t+1} that are representative or diagnostic of $G \equiv \{\omega_t = \widehat{\omega}_t\}$ relative to the background context $-G$. But what is $-G$ here?

In the Irish example, representativeness assesses $G = Irish$ against $-G =$ *Rest of the world*. In the medical test example, $G = Positive\ test$ is assessed against $-G = Not\ taking\ the\ test$, so that context captures absence of new information. We adopt this dynamic perspective by taking context at time t to reflect only information held at $t-1$. Formally, context is the state prevailing if there is no news, which is $-G \equiv \{\omega_t = b\widehat{\omega}_{t-1}\}$ under the assumed AR(1). A certain future state $\widehat{\omega}_{t+1}$ is thus more representative at t if it is more likely to occur under the realized state $G \equiv \{\omega_t = \widehat{\omega}_t\}$ than on the basis of past information $-G \equiv \{\omega_t = b\widehat{\omega}_{t-1}\}$. Representativeness of $\widehat{\omega}_{t+1}$ is then given by:

$$\frac{h(\widehat{\omega}_{t+1}|\omega_t = \widehat{\omega}_t)}{h(\widehat{\omega}_{t+1}|\omega_t = b\widehat{\omega}_{t-1})}. \quad (3)$$

The most representative state is the one exhibiting the largest increase in its likelihood based on recent news. The comparison group $-G$ could alternatively be slow moving, including more remote recollections. In the Appendix, we discuss these cases.[8]

The psychology of diagnostic expectations works as follows. Decision makers have in the back of their mind the conditional distribution $h(\hat{\omega}_{t+1}|\omega_t = \hat{\omega}_t)$. After seeing current news $\Omega_t = \omega_t$, those states whose probability increased the most come immediately to mind. Memory limits then imply that the agent over samples these states from his memory database $h(\hat{\omega}_{t+1}|\omega_t = \hat{\omega}_t)$. Beliefs inflate the probability of more representative states and deflate the probability of less representative states.

We formalize overweighting of representative states "as if" the agent uses the distorted density:

$$h_t^\theta(\hat{\omega}_{t+1}) = h(\hat{\omega}_{t+1}|\omega_t = \hat{\omega}_t) \cdot \left[\frac{h(\hat{\omega}_{t+1}|\omega_t = \hat{\omega}_t)}{h(\hat{\omega}_{t+1}|\omega_t = b\hat{\omega}_{t-1})}\right]^\theta \frac{1}{Z},$$

where the normalizing constant Z ensures that $h_t^\theta(\hat{\omega}_{t+1})$ integrates to one, and $\theta \in [0, +\infty)$ measures the severity of judging by representativeness. When $\theta = 0$, the agent has no memory limits and he appropriately uses all information, forming rational expectations. When $\theta > 0$, memory is limited. The distribution $h_t^\theta(\hat{\omega}_{t+1})$ then inflates the likelihood of representative states that quickly come to mind, while it deflates the likelihood of non-representative ones. Because they overweight the most diagnostic future outcomes, we call the expectations formed in light of $h_t^\theta(\hat{\omega}_{t+1})$ diagnostic.

In our analysis, we take θ to be fixed. In principle, however, recall can depend on the agent's deliberate effort, in which case θ may vary across situations. In section 5 we calibrate θ given a model of fundamentals and the facts in Section 2.

[8] The analysis is less tractable if context is defined to be the past state $-G \equiv \{\omega_{t-1} = \hat{\omega}_{t-1}\}$. In fact, the distributions $h(\hat{\omega}_{t+1}|\omega_t = \hat{\omega}_t)$ and $h(\hat{\omega}_{t+1}|\omega_{t-1} = \hat{\omega}_{t-1})$ have different variances, which distorts not only the mean, but also the variance of the target distribution. The representation obtained in (4) below – and the tractability it entails – extends to other distributions, including lognormal and exponential.

The distorted distribution $h_t^\theta(\hat{\omega}_{t+1})$ shows that the effect of news is not just to alter the objective likelihood of certain states. They also change the extent to which the agent focuses on them. An event that increases the likelihood of a future state $\hat{\omega}_{t+1}$ also makes it more representative, so $h_t^\theta(\hat{\omega}_{t+1})$ overshoots. The reverse occurs when the likelihood of $\hat{\omega}_{t+1}$ decreases. If the likelihood ratio in (2') is monotone increasing, "rationally" good news cause overweighting of high future states, and underweighting of low future states (the converse is true if news are bad). In this sense, good news cause neglect of downside risk. Diagnostic beliefs have a convenient representation.

Proposition 1 *When the process for ω_t is AR(1) with normal $(0, \sigma^2)$ shocks, the diagnostic distribution $h_t^\theta(\hat{\omega}_{t+1})$ is also normal, with variance σ^2 and mean:*

$$\mathbb{E}_t^\theta(\omega_{t+1}) = \mathbb{E}_t(\omega_{t+1}) + \theta[\mathbb{E}_t(\omega_{t+1}) - \mathbb{E}_{t-1}(\omega_{t+1})]. \tag{4}$$

Proposition 1 is a representation result for diagnostic expectations in terms of the distributions held in the agent's memory. Diagnostic beliefs can be represented as a linear combination of the rational expectations of ω_{t+1} held t and at $t-1$. If the agent had in mind a subjective distribution, based for instance on his personal knowledge, diagnostic expectations would entail a similar transformation of that distribution. The general point of Equation (4) is the "kernel of truth" logic: diagnostic expectations over-react to the information received at t by the term $\theta[\mathbb{E}_t(\omega_{t+1}) - \mathbb{E}_{t-1}(\omega_{t+1})]$. Crucially, it is *not* that decision-makers compute and combine rational expectations. Rather, oversampling representative future states of a specific random variable, as defined in (3), implies that the "news" term in the right hand side of (4) excessively influences the agent's subjective beliefs. This is consistent with Kahneman's (2011) view that "our mind has a useful capability to focus spontaneously on whatever is odd, different, or unusual."

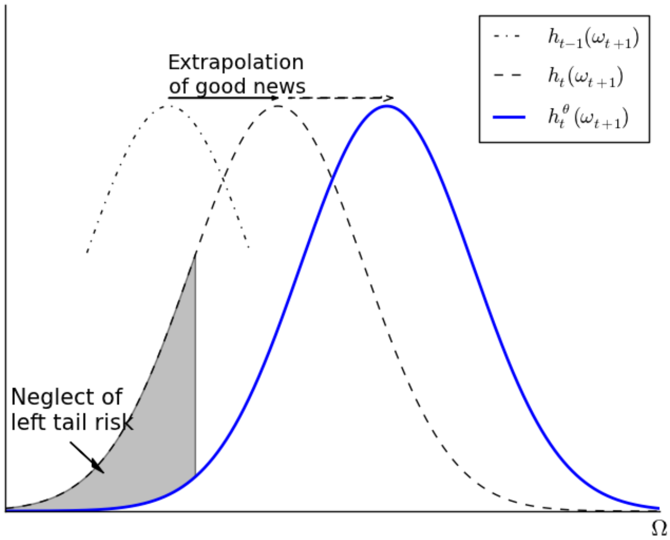


Figure 3: Neglect of risk and Extrapolation

Figure 3 illustrates the entailed neglect of risk. After good news, the diagnostic distribution of ω_{t+1} is a right shift of the objective distribution, which under-estimates probabilities in the left tail (the shaded area). There are two main differences with the model of neglect of risk in Gennaioli, Shleifer and Vishny (2012), which assumes that investors neglect unlikely events. First, under diagnostic expectations neglect of tail events is selective: one tail is neglected, the other tail is exaggerated. This is due to the emphasis on probability changes, not levels. After good news, investors neglect the left tail but overweight the right tail, even if the latter remains unlikely in absolute terms.

Second, neglect of risk depends on the volatility of economic conditions. Indeed, representativeness varies as follows across future states of the world ω_{t+1}:

$$\frac{\partial}{\partial \widehat{\omega}_{t+1}} ln\left[\frac{h(\widehat{\omega}_{t+1}|\omega_t = \widehat{\omega}_t)}{h(\widehat{\omega}_{t+1}|\omega_t = b\widehat{\omega}_{t-1})}\right] = \frac{\mathbb{E}_t(\omega_{t+1}) - \mathbb{E}_{t-1}(\omega_{t+1})}{\sigma^2}.$$

After good news, when $\mathbb{E}_t(\omega_{t+1}) > \mathbb{E}_{t-1}(\omega_{t+1})$, the above expression is positive and the least representative states are the lowest ones, particularly so if volatility σ^2 is low. After bad news, bad states are the most representative ones, particularly so when volatility σ^2 is low. As a result, a given diagnostic distortion of mean beliefs $\mathbb{E}_t^\theta(\omega_{t+1})$ causes a larger distortions in the perception of tail risk under stable economic conditions.

This mechanism differentiates distortions of mean beliefs and distortions of tail events. In a stable environment, namely when σ^2 is low, mild positive news cause a mild exaggeration of mean optimism $\mathbb{E}_t^\theta(\omega_{t+1})$ but a drastic neglect of downside risk. This aspect plays an important role for credit spreads, which reflect default risk and thus depend (even in the rational benchmark) on the volatility of economic conditions.[9]

One important consequence of selective neglect of tail events is that it connects neglect of tail risk with extrapolation by the same psychological mechanism. For the AR(1) process $\omega_t = b\omega_{t-1} + \epsilon_t$, with persistence parameter b, Equation (4) becomes:

$$\mathbb{E}_t^\theta(\omega_{t+1}) - \omega_t = [\mathbb{E}_t(\omega_{t+1}) - \omega_t] + b \cdot \theta \cdot [\omega_t - \mathbb{E}_{t-1}(\omega_t)],$$

namely the current shock $\omega_t - \mathbb{E}_{t-1}(\omega_t)$ is extrapolated into the future, but only if the data are serially correlated, $b > 0$. The key difference with mechanical extrapolative formulas is that under diagnostic expectations the extent of extrapolation depends on the true persistence b of the process that the agent tries to forecast. This is a testable prediction that also makes our model invulnerable to the Lucas critique.[10]

It is straightforward to extend diagnostic expectations to longer term forecasts.

Corollary 1 *When the process for ω_t is AR(1) with normal $(0, \sigma^2)$ shocks, the diagnostic expectations for ω_{t+T} is given by:*

[9] As we show in the Appendix, Proposition 1 extends to GARCH processes in which not only the mean but also the variance varies over time. In this more general case, both the perceived mean and variance are distorted by news, and the distortion of mean beliefs $\mathbb{E}_t^\theta(\omega_{t+1})$ depends on the variance innovation.

[10] The Lucas critique holds that mechanical models of expectations cannot be used for policy evaluation because expectation formation in such models does not respond to changes in policy. Indeed, empirical estimates of mechanically extrapolative processes revealed parameter instability to policy change. Muth (1961) generalizes rational expectations to allow for systematic errors while preserving forward looking behavior. His formula takes the linear form of Equation (4): relative to rationality, expectations distort the effect of recent news. Muth's formulation naturally follows from the psychology of representativeness.

$$\mathbb{E}_t^\theta(\omega_{t+T}) = \mathbb{E}_t(\omega_{t+T}) + \theta[\mathbb{E}_t(\omega_{t+T}) - \mathbb{E}_{t-1}(\omega_{t+T})], \tag{5}$$

Furthermore, we have that $\mathbb{E}_t^\theta(\omega_{t+T}) = \mathbb{E}_t^\theta\left(\mathbb{E}_{t'}^\theta(\omega_{t+T})\right)$ *for any* $t < t' < t+T$.

Longer term forecasts can also be represented as a linear combination of past and present rational expectations. Furthermore, diagnostic expectations obey the law of iterated expectations with respect to the distorted expectations $\mathbb{E}_t^\theta$, so that forecast revisions are unpredictable from the vantage point of the decision maker.

However, forecast revisions are predictable using the true probability measure, because errors in expectations correct on average in the future. Using (4) we find:

$$\mathbb{E}_{t-1}\left[\mathbb{E}_t^\theta(\omega_{t+T})\right] = \mathbb{E}_{t-1}(\omega_{t+T}).$$

The average diagnostic forecast is rational. On average, diagnostic expectations revert to rational expectations because the diagnostic distortion is a linear function of news, and the average news is zero by definition. Even if expectations are inflated at $t-1$, they return to rationality on average at t. As we show in Section 5, this behavior allows us to account for the empirical findings in Section 2.

4. A Model of Credit Cycles

We next introduce diagnostic expectations into a simple macroeconomic model and show that the psychology of representativeness generates excess volatility in expectations about credit spreads, over-heating and over-cooling of credit markets, as well as predictable reversals in credit spreads and economic activity that are consistent with the evidence of Section 2, as well as with many other features of credit cycles.

4.1. Production

A measure 1 of atomistic firms uses capital to produce output. Productivity at t depends on the state ω_t, but to a different extent for different firms. A firm is identified by its risk $\rho \in \mathbb{R}$. Firms with higher ρ are less likely to be productive in any state ω_t. If a firm ρ enters period t with invested capital k, its current output is given by:

$$y(k|\omega_t, \rho) = \begin{cases} k^{\alpha} & if \quad \omega_t \geq \rho \\ 0 & if \quad \omega_t < \rho \end{cases}, \tag{6}$$

where $\alpha \in (0,1)$. The firm produces only if it is sufficiently safe, $\rho < \omega_t$. The safest firms, for which $\rho = -\infty$, produce k^{α} in every state of the world. The higher is ρ, the better the state ω_t needs to be for the firm to pay off. At the same capital k, two firms produce the same output if they are both active, namely if $\omega_t \geq \rho$ for both firms.

A firm's riskiness is common knowledge and it is distributed across firms with density $f(\rho)$. Capital for production at $t+1$ must be installed at t, before ω_{t+1} is known. Capital fully depreciates after usage. At time t each firm ρ demands funds from a competitive financial market to finance its investment. The firm issues risky debt that promises a contractual interest rate $r_{t+1}(\rho)$. Debt is repaid only if the firm is productive: if at t the firm borrows $k_{t+1}(\rho)$ at the interest rate $r_{t+1}(\rho)$, next period it produces and repays $r_{t+1}(\rho)k_{t+1}(\rho)$ provided $\omega_{t+1} \geq \rho$, and defaults otherwise.

Because there are no agency problems and each firm's output has a binary outcome, the model does not distinguish between debt and equity issued by the firm. Both contracts are contingent on the same outcome and promise the same rate of return. For concreteness, we refer to the totality of capital invested as debt.

4.2 Households

A risk neutral, infinitely lived, representative household discounts the future by a factor $\beta < 1$. At each t, the household allocates its current income between non-negative consumption and investment by maximizing its expectation of the utility function:

$$\sum_{s=t}^{+\infty} \beta^{s-t} c_s.$$

The household purchases the claims issued by firms, which then pay out or default in the next period. His income consists of the payout last period's debt, the profits of firms (which he owns), and a fixed endowment w that we assume to be large enough:[11]

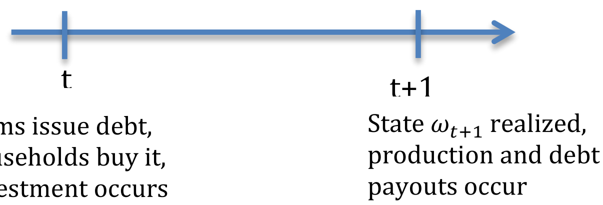
A.1 $w \geq (\alpha\beta)^{\frac{1}{1-\alpha}}$.

At each time s and state ω_s, then, the household's budget constraint is:

$$c_s + \int_{-\infty}^{+\infty} k_{s+1}(\rho) f(\rho) d\rho = w + \int_{-\infty}^{+\infty} I(\rho, \omega_s)[r_s(\rho) k_s(\rho) + \pi_s(\rho)] f(\rho) d\rho,$$

where $c_s \geq 0$ is consumption, $k_{t+1}(\rho)$ is capital supplied to firm ρ, $I(\rho, \omega_s)$ is an indicator function equal to one when firm ρ repays, namely when $\omega_s \geq \rho$, and $\pi_s(\rho)$ is the profit of firm ρ when active. The worse is the state of the economy (the lower is ω_s), the higher is the fraction of defaulting firms and thus the lower is the household's income.

The timeline of an investment cycle in the model is illustrated below.



[11] As we show later, this condition ensures that the equilibrium expected return is equal to β^{-1}.

Investment by households and firms depend on the *perceived* probability with which each firm ρ repays its debt in the next period. At time t the perceived probability with which firm ρ produces output and repays at time $t+1$ is given by:

$$\mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right) = \int_\rho^{+\infty} h_t^\theta(x)\, dx = \frac{1}{\sigma\sqrt{2\pi}} \int_\rho^{+\infty} e^{-\frac{\left(x - \mathbb{E}_t^\theta(\omega_{t+1})\right)^2}{2\sigma^2}} dx. \tag{7}$$

The perceived probability of default is then $1 - \mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)$. A perfectly safe firm $\rho \to -\infty$ never defaults, since $\lim_{\rho \to -\infty} \mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right) = 1$.

When $\theta = 0$, expectations are rational and the probability of default is computed according to the true conditional distribution $h(\Omega_{t+1} = \omega | \Omega_t = \omega_t)$. When $\theta > 0$, the distortions of diagnostic expectations affect the perceived safety of different firms. In what follows, we refer to $\mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)$ as the "perceived creditworthiness" of firm ρ.

4.3 Capital Market Equilibrium and Credit Spreads

At time t firm ρ demands capital $k_{t+1}(\rho)$ at the market contractual interest rate $r_{t+1}(\rho)$ so as to maximize its expected profit at $t+1$:

$$\max_{k_{t+1}(\rho)} (k_{t+1}(\rho)^\alpha - k_{t+1}(\rho) \cdot r_{t+1}(\rho)) \cdot \mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right). \tag{8}$$

The first order condition for the profit maximization problem is given by:

$$k_{t+1}(\rho) = \left[\frac{\alpha}{r_{t+1}(\rho)}\right]^{\frac{1}{1-\alpha}}, \tag{9}$$

which is the usual downward sloping demand for capital.

Households are willing to supply any amount of capital to firm ρ provided the interest rate $r_{t+1}(\rho)$ makes the household indifferent between consuming and saving:

$$r_{t+1}(\rho) \cdot \mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right) = \beta^{-1} \quad \Leftrightarrow \quad r_{t+1}(\rho) = \frac{1}{\beta\mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)}. \tag{10}$$

In equilibrium, this condition must hold for all firms ρ. On the one hand, no arbitrage requires all firms to yield the same expected return. On the other hand, such expected return cannot be below β^{-1}. If this were the case, the household would not invest and the marginal product of capital would be infinite, leading to a contradiction. But the expected return of debt cannot be above β^{-1} either. If this were the case, the household would invest the totality of its income. Under A.1, however, this implies that the marginal product of capital would fall below β^{-1}, again leading to a contradiction.

From Equation (10), we can compute the spread obtained on the debt of risky firm ρ at time t as the difference between the equilibrium $r_{t+1}(\rho)$ and the safe rate β^{-1}:

$$S\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right) = \left(\frac{1}{\mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)} - 1\right)\beta^{-1}. \tag{11}$$

Risky firms must compensate investors for bearing their default risk. The spread at t depends on the firm's riskiness ρ and on expectations of the aggregate economy. Greater optimism $\mathbb{E}_t^\theta(\omega_{t+1})$ improves perceived creditworthiness $\mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)$, lowering spreads. Greater riskiness ρ enhances spreads by reducing perceived creditworthiness.

By combining Equations (11) and (9) we obtain:

$$k_{t+1}(\rho) = \left[\frac{\alpha\beta}{1 + \beta S\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)}\right]^{\frac{1}{1-\alpha}}, \tag{12}$$

which links expectations to credit spreads, investment and output. In good times households are optimistic, $\mathbb{E}_t^\theta(\omega_{t+1})$ is high. Spreads are compressed, firms issue debt and expand investment. When times turn sour, households become pessimistic; Spreads rise and firms cut debt issuance and investment. Equation (12) can be aggregated across different values of ρ to obtain aggregate investment at time t and output at $t + 1$.

We can now generate some testable implications of our model. Using Equation (11), define the average spread at time t as:

$$S_t = \int_{-\infty}^{+\infty} S\left(\rho, \mathbb{E}_t^{\theta}(\omega_{t+1})\right) f(\rho) d\rho. \tag{13}$$

S_t is an inverse measure of optimism, which is strictly monotonically decreasing in expectations. When $\mathbb{E}_t^{\theta}(\omega_{t+1})$ is higher, average perceived creditworthiness is higher, and hence the average spread S_t charged on risky debt is lower.

We can substitute S_t for expectations $\mathbb{E}_t^{\theta}(\omega_{t+1})$ in Equations (11) and (12). We then obtain the following result for the cross section of firms:

Proposition 2. *Lower optimism $\mathbb{E}_t^{\theta}(\omega_{t+1})$ and thus higher spread S_t at time t causes:*

i) a disproportionate rise in the spread of riskier firms:

$$\frac{\partial^2 S\left(\rho, \mathbb{E}_t^{\theta}(\omega_{t+1})\right)}{\partial S_t \partial \rho} > 0.$$

ii) a disproportionate decline in debt issuance and investment by riskier firms:

$$\frac{\partial}{\partial S_t} \frac{k_{t+1}(\rho_1)}{k_{t+1}(\rho_2)} < 0 \quad \text{for all } \rho_1 > \rho_2.$$

Because it is more sensitive to aggregate conditions, investment by riskier firms fluctuates more with expectations and displays more co-movement with credit markets.

These predictions of the model are consistent with the evidence of Greenwood and Hanson (2013). They document that when the Baa-credit spread falls, bond issuance increases and the effect is particularly strong for firms characterized by higher expected default rates. As a consequence, the share of non-investment grade debt over total debt (the "junk share") increases, as has also been documented by LSZ (2017).[12]

[12] Giroud and Mueller (2016) find that "high leverage firms" were worst hit during the 2008 financial crisis. Consistent with the above, such firms have most increased their debt to leverage ratio, but are also less

This behavior of the junk share follows directly from property ii) above, which implies that the share of debt issued by firms riskier than an arbitrary threshold $\hat{\rho}$:

$$\frac{\int_{\hat{\rho}}^{+\infty} k_{t+1}(\rho) f(\rho) d\rho}{\int_{-\infty}^{+\infty} k_{t+1}(\rho) f(\rho) d\rho},$$

unambiguously increases as S_t drops and spreads become compressed (for any $\hat{\rho}$).

The qualitative effects described in Proposition 2 do not rely on diagnostic expectations and obtain even if households are fully rational. Diagnostic expectations have distinctive implications for the behavior over time of equilibrium credit spreads as well as of their expectations by market participants. We now turn to this analysis.

5. Diagnostic Expectations and Equilibrium Credit Spreads

To study the equilibrium spread and expectations, we consider a linearized version of Equation (11). A first order expansion of Equation (11) with respect to investors' expectations $\mathbb{E}_t^{\theta}(\omega_{t+1})$ around the long run mean of zero yields:

$$S\left(\rho, \mathbb{E}_t^{\theta}(\omega_{t+1})\right) \approx \frac{1}{\beta} \cdot \left[\frac{1}{\mu(\rho, 0)} - 1\right] - \frac{\mu'(\rho, 0)}{\beta \mu(\rho, 0)^2} \cdot \mathbb{E}_t^{\theta}(\omega_{t+1})$$

The spread drops as expectations improve (since $\mu'(\rho, 0) > 0$), but more so for riskier firms (the slope coefficient increases in ρ). Aggregating this equation across all firms ρ and denoting by $\sigma_0, \sigma_1 > 0$ the average intercept and slope, we find that the average spread at time t approximately satisfies:

$$S_t = \sigma_0 - \sigma_1 \mathbb{E}_t^{\theta}(\omega_{t+1}). \tag{14}$$

Inserting into (14) the expression for $\mathbb{E}_t^{\theta}(\omega_{t+1})$ of Equation (4), under the maintained assumption of AR(1) fundamentals $\omega_t = b\omega_{t-1} + \epsilon_t$, we establish:

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productive and more financially constrained than firms "low leverage firms". However, our stylized setting does not capture the fact that such firms also have higher leverage on average.

Proposition 3. *The average credit spread S_t follows an ARMA(1,1) process given by:*

$$S_t = (1-b)\sigma_0 + b \cdot S_{t-1} - (1+\theta)b\sigma_1\epsilon_t + \theta b^2\sigma_1\epsilon_{t-1}. \tag{15}$$

This is a key result. Under rational expectations ($\theta = 0$) the equilibrium spread, like fundamentals, follows an AR(1) process characterized by persistence parameter b. Starting from the long run spread σ_0, a positive fundamental shock $\epsilon_t > 0$ causes expectations to improve and the spread to decline. Subsequently, the spread gradually reverts to σ_0. The reverse occurs after a negative piece of news $\epsilon_t < 0$: spreads go up on impact and then monotonically revert to σ_0.

Under diagnostic expectations, $\theta > 0$, credit spreads continue to have an autoregressive parameter b but now also contain a moving average component. The spread at time t now depends also on the shock experienced at $t-1$. If the news received in the previous period were good, $\epsilon_{t-1} > 0$, so that S_{t-1} was low, there is a discrete hike in the spread at time t. If the news received in the previous period were bad, $\epsilon_{t-1} < 0$, so that S_{t-1} was high, there is a discrete drop in the spread at time t.

These delayed corrections occur on average (controlling for mean reversion in fundamentals) and correspond to the systematic correction of errors in diagnostic forecasts described in Section 3. In fact, the over-reaction $\theta\epsilon_{t-1}$ at $t-1$ reverses on average at t. Such reversal of expectations about fundamentals contaminates the spread, which exhibits the predictable non-fundamental reversal of Equation (15). When at $t-1$ news are good, $\epsilon_{t-1} > 0$, optimism is excessive and the spread S_{t-1} drops too far. Next period this excess optimism wanes on average, so that S_t is corrected upwards. The reverse occurs if at $t-1$ news are bad. In this sense, the making and un-making of expectational errors cause boom bust cycles and mean reversion in spreads.

We next show that the equilibrium behavior of credit spreads in (15) accounts for the findings on expectational errors of Section 2. In section 5.2, we show that the model also accounts for the evidence on the link between credit spreads and economic activity that is hard to explain under rational expectations.

5.1 Credit Spread Forecasts

In Section 2 we showed that forecasts of credit spreads exhibit predictable errors due to an extrapolative nature of expectations, and that these forecasts exhibit systematic reversals. To connect the model to this evidence, we now describe how agents with diagnostic expectations form forecasts of credit spreads.

Investors forecast future credit spreads using Equation (14). As a consequence, the forecast made at t for the spread at $t+T$ is given by:

$$\mathbb{E}_t^\theta(S_{t+T}) = \sigma_0 - \sigma_1 \mathbb{E}_t^\theta\left[\mathbb{E}_{t+T}^\theta(\omega_{t+T+1})\right].$$

By exploiting Proposition1 and Corollary 1 we obtain:

Lemma 1 *The T periods ahead diagnostic forecast of the spread is given by:*

$$\mathbb{E}_t^\theta(S_{t+T}) = \sigma_0(1 - b^T) + b^T S_t. \tag{16}$$

Diagnostic expectations project the current spread into the future via the persistence parameter b. The more persistent is the process for fundamentals, the greater is the influence of the current spread S_t on forecasts of future spreads.

Critically, unlike the equilibrium process for the spread in (15), the forecast process in (16) does not exhibit reversals. The intuition is simple: diagnostic forecasters fail to anticipate the systematic reversal in the equilibrium spread realized when their extrapolation of current news turns out to be incorrect.

This idea can help account for the evidence of Section 2.

Proposition 4. *If the equilibrium spread follows (15) and expectations follow (16):*

i) the forecast error at $t+1$ *is predictable in light of information available at* t*:*

$$\mathbb{E}_t\left[S_{t+1} - \mathbb{E}_t^\theta(S_{t+1})\right] = \theta b^2 \sigma_1 \epsilon_t. \tag{17}$$

ii) the revision of expectations about S_{t+T} *occurring between* t *and* $t+s$ *is predictable in light of information available at time* t*:*

$$\mathbb{E}_t\left[\mathbb{E}_{t+s}^\theta(S_{t+T}) - \mathbb{E}_t^\theta(S_{t+T})\right] = \theta b^{T+1} \sigma_1 \epsilon_t. \tag{18}$$

Forecast errors and forecast revisions are predictable because agents neglect the reversal of their expectations errors. Thus, good news about fundamentals, $\epsilon_t > 0$, predict both that the realized spread next period is on average above the forecast (Equation 17) and that longer term forecasts of spreads will be revised upward in the future (Equation 18). The reverse pattern of predictability occurs after bad news $\epsilon_t < 0$. Equations (17) and (18) can therefore account for the evidence of Section 2, whereby the spread S_t is negatively correlated with the forecast error $\mathbb{E}_t\left[S_{t+1} - \mathbb{E}_t^\theta(S_{t+1})\right]$ and with the future forecast revision $\mathbb{E}_t\left[\mathbb{E}_{t+s}^\theta(S_{t+T}) - \mathbb{E}_t^\theta(S_{t+T})\right]$.

We can draw a comparison between our model and "Natural Expectations" (Fuster et al 2010). In both models forecast errors are predictable because people underestimate the possibility of reversals. The underlying mechanism is however very different. Under natural expectations, long-term reversals are assumed and errors in expectations arise because agents fit a simpler AR(1) model to the data. In our model, in contrast, both the process for spreads and forecast errors are endogenous to diagnostic expectations. Agents extrapolate current news too far into the future, which in turn endogenously generates unanticipated reversals in the equilibrium process for spreads.

To conclude, we use Proposition 3 and Lemma 1, together with the data on forecast errors analyzed in Section 2, to provide a "back of the envelope" calibration of θ. This calculation should be taken with caution. Data on expectations of credit spreads are quite limited, available only for about sixty quarters. Moreover, our predictions also rely on a number of assumptions, including that fundamentals follow an AR(1) process. Still, it is useful to illustrate how data can be used to calibrate θ and thus to quantitatively discipline the model.[13]

Lemma 2. *If fundamentals ω_t follow an AR(1) process with persistence b, the regression coefficient of spread forecast errors $S_{t+1} - \mathbb{E}_t^\theta(S_{t+1})$ on current spread S_t is given by:*

$$\beta = -\frac{(1+\theta)\theta b}{(1+\theta)^2 + \frac{b^2}{1-b^2}}$$

Regressing forecast errors on current levels of spreads yields an estimated coefficient that depends only on θ and the persistence b of fundamentals. In light of Lemma 1, persistence b can be estimated by regressing expectations $\mathbb{E}_t^\theta(S_{t+1})$ on current S_t. Table 1 of Section 2 shows the results of both regressions, and provides estimates of β (third column) and of b (second column). From these estimates we find $\theta = 0.91$.

This exercise suggests that forecast errors are sizable, in the sense that they are comparable to actual innovations in spreads – a point that is apparent upon visual inspection of Figure 1. In the model, a θ of the order of 1 implies that investors effectively treat shocks ϵ_t to the state of the world as an AR(1) process with the same persistence as the underlying process. As a result, reversals of credit spreads $\theta b^2\sigma_1\epsilon_t$ are comparable in magnitude to the actual innovations $(1+\theta)b\sigma_1\epsilon_{t+1}$. As we show

[13] A fuller quantification of θ based on field data requires to develop tests that do not rely on exact knowledge of the data generating process. This lies beyond the scope of this paper.

next, this has significant implications for the predictability and excess volatility of returns.

5.2 Predictability of Returns, Volatility of Spreads, and Economic Activity

Our model can account also for the evidence on abnormal bond returns and on excess volatility of credit spreads. To see this, define the "rational spread" S_t^r as one that would prevail at time t under rational expectations ($\theta = 0$). This is the compensation for default risk demanded by rational investors. Proposition 3 then implies:

Corollary 2. *Under diagnostic expectations, $\theta > 0$, the following properties hold:*

i) investors earn predictably low (resp. high) average returns after good (resp. bad) news:

$$S_t - S_t^r = -\theta b \sigma_1 \epsilon_t.$$

ii) credit spreads exhibit excess conditional volatility:

$$Var[S_t|\omega_{t-1}] = (1+\theta)^2 Var[S_t^r|\omega_{t-1}].$$

Predictability of returns comes from errors in expectations. After good news $\epsilon_t > 0$, investors are too optimistic and demand too little compensation for default risk, $S_t < S_t^r$. The average realized return on bonds is thus below the riskless rate β^{-1}. After bad news $\epsilon_t < 0$, investors are too pessimistic and demand excessive compensation for default risk, $S_t > S_t^r$. The average realized return is above β^{-1}.[14]

Expectational errors also underlie excess volatility of spreads. Equilibrium spreads vary too much relative to objective measures of default risk, which are captured by S_t^r, because spreads also reflect investor over-reaction to recent news. Over-reaction

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[14] Return predictability can also be gauged from Equation (10). The average return at t is below the risk free rate when investors are too optimistic $\mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right) > \mu(\rho, \mathbb{E}_t(\omega_{t+1}))$, and above it otherwise.

to good or bad news causes investors' risk perceptions to be too volatile, which in turn introduces excess volatility into market prices.[15]

Greenwood and Hanson (2013) document the pattern of return predictability in Corollary 2. They find that high levels of the junk share predict anomalously low, and even negative, excess returns, and that this occurs precisely after good news, measured by drops in expected default rates (point i). They consider conventional explanations for this finding, such as time varying risk aversion and financial frictions, but conclude that the evidence (particularly the observed frequency of negative returns) is more consistent with the hypothesis that the junk share is a proxy for investor sentiment and extrapolation. Diagnostic expectations offer a psychological foundation for this account.

Several papers document that credit spreads appear too volatile relative to what could be explained by the volatility in default rates or fundamentals (Collin-Dufresne et al. 2001, Gilchrist and Zakrasjek 2012). For instance, Collin-Dufresne et al. (2001) find that credit spreads display excess volatility relative to measures of fundamentals such as realized default rates, liquidity, or business conditions. They argue this excess volatility – up to 75% of total realized volatility – can be explained by a common factor that captures aggregate shocks in credit supply and demand. In our model, investors' temporary overreaction to news about fundamentals offers a source for such shocks.

We conclude by illustrating a general implication of our model. Suppose, as we do, that the economy is driven by a single aggregate factor ω_t. With diagnostic expectations, regressing the average spread S_t on this aggregate factor would detect excess volatility.

Lemma 3. *The R^2 of the regression of the average spread S_t on factor ω_t is given by:*

[15] Excess volatility is due to the fact that beliefs do not just depend on the *level* of the current fundamentals ω_t (as would be the case under rational expectations). They also depend on the magnitude ε_t of the recently observed news, which corresponds roughly speaking to the *change* in fundamentals.

$$R^2 = \frac{[1 + \theta(1 - b^2)]^2}{1 + 2\theta(1 - b^2) + \theta^2(1 - b^2)}.$$

Under rational expectations, $\theta = 0$, the spread is entirely explained by the level of fundamentals. Under diagnostic expectations, $\theta > 0$ (and provided there is persistence, $b > 0$), there is unexplained volatility in the spread, $R^2 < 1$. In fact, current fundamentals fail to capture reversals of past errors, which contaminate the spread dynamics. This feature does not arise under all belief distortions. For instance, if agents merely exaggerate the persistence of current fundamentals, changes in expectations are fully explained by movements in ω_t, yielding an R^2 of 1. In this sense, diagnostic expectations are a natural source of excess volatility.

The boom-bust cycles in credit spreads shape investment (see Equation 12) and cause in turn overbuilding, underbuilding, and excess volatility in the real economy. Gennaioli, Ma and Shleifer (2015) find that CFOs with more optimistic earnings expectations invest more. Greenwood and Hanson (2015) study empirically investment cycles in the ship industry. Consistent with our model, they find that returns to investing in dry bulk ships are predictable and tightly linked to boom-bust cycles in industry investment. High current ship earnings are associated with higher ship prices and higher industry investment, but predict low future returns on capital.

Our model also has implications for the link between credit markets and economic activity. Krishnamurthy and Muir (2015) and LSZ (2017) document that a tightening of credit spreads at t induces an output contraction in period $t + 1$. Our model yields this pattern as the result of a drop in confidence. A reduction in optimism $\mathbb{E}_t^\theta(\omega_{t+1})$ raises the current spread. Tighter financial conditions in turn cause current debt issuance and investment to decline, leading to a drop in aggregate output at $t + 1$.

There is also growing evidence of systematic reversion in credit conditions and of subsequent output drops. In particular, LSZ (2017) show that low credit spreads at $t-1$ systematically predict higher credit spreads at t and then a drop in output at $t+1$. LSZ (2017) suggest that a period of excessive investor optimism is followed by a period of cooling off, which they refer to as "unwinding of investor sentiment". This reversal contributes to a recession over and above the effect of changes in fundamentals. Diagnostic expectations can account for this "unwinding of investor sentiment", thereby reconciling predictable reversals in market conditions with abnormal returns and excess volatility of credit spreads.[16] Such reversals are once again due to the ARMA process followed by the equilibrium spread in Proposition 3.

Proposition 5. *Suppose that expectations are diagnostic,* $\theta > 0$*, and at* $t-1$ *credit spreads are too low due to recent good news, namely* $\epsilon_{t-1} > 0$*. Then:*

i) Controlling for fundamentals at $t-1$*, credit spreads predictably rise at* t*.*

ii) Controlling for fundamentals at $t-1$*, there is a predictable drop in aggregate investment at* t *and in aggregate production at* $t+1$*.*

Diagnostic expectations drive a cycle around fundamentals: over-reaction to good news causes credit markets and the economy to overshoot at $t-1$. The subsequent reversal of such over-reaction causes a drop in credit and economic activity that is more abrupt than could be accounted for by mean reversion in fundamentals. In fact, investor psychology can itself be a cause of volatility in credit, investment, and

[16] In related settings, Jorda, Schularick and Taylor (2012) document that strong growth of bank loans forecasts future financial crises and output drops. Baron and Xiong (2017) show that credit booms are followed by stock market declines. They document that in good times banks expand their loans, and this expansion predicts future negative returns on bank equity. The negative returns to equity might reflect the unwinding of initial investor optimism, or might be caused by abnormally low realized performance on the bank's credit decisions (as per Proposition 4). See also Fahlenbrach et al. (2016).

business cycles, even in the absence of mean reversion in fundamentals, for example if the process for aggregate productivity ω_t is a random walk ($b = 1$).

In sum, diagnostic expectations lead to short-term extrapolative behavior and systematic reversals. This is in line with a large set of recent empirical findings, including: i) excess volatility of spreads relative to measures of fundamentals, ii) excessive spread compression in good times and excessive spread widening in bad times (and a similar pattern in the junk share), iii) excessively volatile investment and output, iv) good times predicting abnormally low returns, and finally v) non-fundamental boom bust cycles in credit spreads, driven by transient overreaction to news.

6. Safe Debt

So far we focused on the issuance and pricing of debt of different levels of risk, without introducing ingredients that make safe debt special. However, recent work highlights special features of safe debt, such as its attractiveness for investors demanding "money-like" securities (Stein 2012). These features are important for thinking about financial crises. In these events, leverage takes center stage, giving rise to fire sales (when collateral is liquidated) or to a scarcity of safe assets (GSV 2012).

To illustrate the connection between safe debt, leverage and diagnostic expectations, we assume that investors have a preferred habitat for safe debt. Specifically, investors require significantly lower returns for claims issued by firms that default with probability lower than $\delta^* \in (0,1)$. GSV (2012) assumed an extreme form of preferred habitat, modelled through infinite risk aversion.[17]

Preferred habitat creates a friction whereby financing of investment becomes easier when more firms are perceived to be safe. After good news, leverage expands across the economy but disproportionately so for those firms that are reclassified as safe. Conversely, contractions in safe debt cause sudden tightenings of financial constraints.

Investors value safe and risky debt separately.[18] When valuing safe debt, the investor trades off the cost of lending one dollar today to a risky firm ρ and receiving repayment $r_{t+1}(\rho)$ tomorrow in case of no default using the usual discount factor β:

$$-1 + \beta r_{t+1}(\rho) \cdot \mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right). \qquad (20)$$

When valuing risky debt, namely a claim issued by a firm that defaults with believed probability $1 - \mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right) > \delta^*$ the investor instead applies the discount factor $\beta\psi$:

$$-1 + \beta\psi r_{t+1}(\rho) \cdot \mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right), \qquad (21)$$

where $\psi \leq 1$ captures the idea that the investor requires a higher return to hold risky debt (which can also be thought as equity). A preference for safe debt arises when $\psi < 1$ and $\delta^* < 1$. The model boils down to infinite risk aversion for $\psi = \delta^* = 0$.

We can now describe the equilibrium:

Proposition 6. *At time t define a threshold ρ_t^θ as the solution to:*

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[17] GSV (2012) also consider the case in which only some investors have a preference for safe debt. Preference heterogeneity can help understand phenomena such as trading of financial assets. For simplicity, we continue to assume that here is a representative investor, as in the previous sections.

[18] The fact that the investor values securities one by one, rather than valuing an overall portfolio, reflects the idea of preferred habitat: some investors specialize in holding a certain asset type. Without this assumption, even risk-averse investors with a preference for low default risk would be willing to hold some risky assets as long as the overall default probability of their portfolio is below δ^*.

$$\mu\left(\rho_t^\theta, \mathbb{E}_t^\theta(\omega_{t+1})\right) = 1 - \delta^*,$$

so that ρ_t^θ increases in optimism about the future $\mathbb{E}_t^\theta(\omega_{t+1})$. We then have:

i) Firms with $\rho \leq \rho_t^\theta$ issue safe debt, on which they promise the equilibrium interest rate

$$r_{t+1}(\rho) = \left[\beta\mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)\right]^{-1}$$

ii) Firms with $\rho > \rho_t^\theta$ issue risky debt, on which they promise the equilibrium interest rate

$$r_{t+1}(\rho) = \left[\beta\psi\mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)\right]^{-1}$$

Investors' preference for safe debt creates a discontinuity in required returns. Safe firms can borrow at an expected return of β^{-1}, which is lower than the expected return $(\beta\psi)^{-1}$ that risky firms must pay. The threshold ρ_t^θ that separates safe from risky firms is time varying. As more firms are categorized as safe in good times, the required returns for marginally risky firms declines discontinuously.

In good times, the issuance of safe debt expands along both the intensive and the extensive margins. On the intensive margin, already safe firms issue too much safe debt, relative to the rational expectations benchmark. On the extensive margin, too many risky firms become safe, and start issuing safe debt (this occurs when ρ_t^θ is too high relative to the level $\rho_t^{\theta=0}$ that would prevail under rational expectations). In this sense, at the end of expansions borrowers (particularly marginally risky ones) are excessively burdened with liabilities perceived to be safe.

This implies that after periods of good news the debt structure is fragile, as in GSV (2012). Diagnostic expectations imply that crises occurs when good news stop coming, so that excess optimism reverts. To see this, consider a thought experiment outside the current model. Suppose that at an interim period between time t and time $t+1$, investors reassess their expectations of future economic conditions ω_{t+1}, which revert to rational expectations (e.g., because no news was received, in line with Proposition 1). We then have:

Proposition 7. *After good news, $\epsilon_t > 0$, the interim valuation of assets moves as follows:*

i) The valuation of safe and risky debt, i.e. of firms with $\rho \notin \left(\rho_t^{\theta=0}, \rho_t^{\theta}\right)$, drops by

$$\frac{\mu\left(\rho, \mathbb{E}_t^{\theta}(\omega_{t+1})\right)}{\mu\left(\rho, \mathbb{E}_t^{\theta=0}(\omega_{t+1})\right)} > 1$$

ii) The valuation of debt perceived to be safe, i.e. of firms with $\rho \in \left(\rho_t^{\theta=0}, \rho_t^{\theta}\right)$, drops by

$$\frac{\mu\left(\rho, \mathbb{E}_t^{\theta}(\omega_{t+1})\right)}{\psi\mu\left(\rho, \mathbb{E}_t^{\theta=0}(\omega_{t+1})\right)} \gg 1$$

Because in the interim period there is no new information, under rational expectations the valuation of different securities would not change. Instead, under diagnostic expectations the initial excess optimism wanes and the probability of repayment of all firms is reassessed downward, leading to negative realized returns.[19]

The key result in Proposition 7 is that realized returns are particularly low for the marginally risky firms $\rho \in \left(\rho_t^{\theta=0}, \rho_t^{\theta}\right)$. These firms were considered safe under the initial excess optimism, but not when investors appreciate their risk. Investors then demand a higher expected return to hold this debt, causing its price to exhibit a sharp decline.

As in GSV (2012), financial fragility arises from the combination of neglect of risk and preferred habitat. In the absence of preferred habitat (e.g, if $\psi = 1$), diagnostic expectations boost risk taking in good times as well as overvaluation and predictability

[19] The result carries through under the alternative assumption that information arrives also in the interim period. However, the analysis would be more complicated because i) ex post valuation would change also under rationality, because new information has arrived, and ii) ex ante valuation would take into account the probability that firms become safer or riskier than δ^* before repayment.

in bond returns, but in a continuous way. A preferred habitat for safe debt, in contrast, enhances volatility and concentrates it precisely on the safe debt segment. The over-issuance of safe debt causes neglected risks to be sharply underpriced, which exposes the economy to bad shocks when neglected risks resurface and safe debt is reclassified as risky. Even small downward revisions of expected fundamentals can trigger a large price decline when it causes a reassessment of safety. In line with the discussion in Section 3, unintended exposure to neglected risks is larger when economic volatility σ^2 is lower.

The price effects on debt described above can percolate through the economy in many ways. And while the price mechanism is symmetric with respect to good and bad news, its impact on the economy may be highly skewed. For example, if safe debt is an asset on the balance sheets of intermediaries, these intermediaries might become insolvent, or experience runs, when debt price falls (Baron and Xiong 2017, Fahlenbrach et al 2016). This can have large adverse effects on the economy through the bank lending channel (Chodorow-Reich 2014). Leveraged investors holding such “safe” debt might have to fire sell it, and this can further adversely affect balance sheets of other intermediaries or firms. Such compounding of adverse effects does not have a counterpart for positive news, creating a significant real asymmetry in response to movements in debt prices.

7. Conclusion

We have presented a new approach to modeling beliefs in economic models, diagnostic expectations, based on Kahneman and Tversky’s representativeness heuristic. Our model of expectations is portable in Rabin’s sense, meaning that the same framework accounts for many experimental findings, the phenomenon of stereotyping, but also critical features of beliefs in financial markets such as extrapolation, over-reaction, and neglect of risk. Diagnostic expectations are also forward-looking, which means that they are invulnerable to the Lucas critique of mechanical backward looking models of beliefs. We applied diagnostic expectations to a straightforward macroeconomic model of investment, and found that it can account for several empirical findings regarding credit cycles without resort to financial frictions.

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Proofs

Proposition 1. Let ω_t be an AR(1) process, $(\omega_t - \overline{\omega}) = b(\omega_{t-1} - \overline{\omega}) + \epsilon_t$, with i.i.d. normal $(0, \sigma^2)$ shocks ϵ_t. We now compute diagnostic expectations at a generic horizon $T > 1$. Writing ω_{t+T} as a function of ω_t plus subsequent shocks, we find

$$\omega_{t+T} = b^T \omega_t + (1-b)\overline{\omega} \sum_{s=0}^{T-1} b^s + \sum_{s=0}^{T-1} b^s \epsilon_{t+s+1}$$

so that the true distribution of ω_{t+T} given ω_t, namely $h(\omega_{t+T}|\omega_t)$, is a normal distribution $\mathcal{N}\left(\mathbb{E}_t(\omega_{t+T}), \sigma^2_{t,t+T}\right)$ with mean and variance given by:

$$\mathbb{E}_t(\omega_{t+T}) = b^T \omega_t + (1-b)\overline{\omega} \sum_{s=0}^{T-1} b^s, \quad \text{and} \qquad \sigma^2_{t,t+T} \equiv \sigma_T^2 = \sigma^2 \sum_{s=0}^{T-1} b^{2s} = \sigma^2 \frac{1-b^{2T}}{1-b^2}$$

The reference distribution is $h(\omega_{t+T}|b\omega_{t-1} + (1-b)\overline{\omega})$, characterized by:

$$\mathbb{E}_t(\omega_{t+T}) = b^{T+1} \omega_{t-1} + (1-b)\overline{\omega} \sum_{s=0}^{T} b^s, \quad \text{and} \qquad \sigma^2_{t,t+T} \equiv \sigma_T^2$$

The diagnostic distribution then reads (up to normalization constants):

$$h_t^\theta(\omega_{t+T}) \sim \exp - \frac{1}{2\sigma_T^2} \left[\left(\omega_{t+T} - \mathbb{E}_t(\omega_{t+T})\right)^2 (1+\theta) - \theta\left(\omega_{t+T} - \mathbb{E}_{t-1}(\omega_{t+T})\right)^2 \right]$$

The quadratic and linear terms in ω_{t+T} are as follows (the constant terms being absorbed by a normalization constant):

$$\exp - \frac{1}{2\sigma_T^2} \left[{\omega_{t+T}}^2 - 2\omega_{t+T}\left(\mathbb{E}_t(\omega_{t+T})(1+\theta) - \theta \mathbb{E}_{t-1}(\omega_{t+T})\right) \right]$$

It follows that the diagnostic distribution $h_t^\theta(\omega_{t+T})$ is also a normal distribution $\mathcal{N}\left(\mathbb{E}_t^\theta(\omega_{t+T}), \sigma_T^2\right)$ with mean:

$$\mathbb{E}_t^\theta(\omega_{t+T}) = \mathbb{E}_t(\omega_{t+T}) + \theta[\mathbb{E}_t(\omega_{t+T}) - \mathbb{E}_{t-1}(\omega_{t+T})]$$

In particular, for $T = 1$ we get

$$\mathbb{E}_t^\theta(\omega_{t+1}) = \mathbb{E}_t(\omega_{t+1}) + \theta[\mathbb{E}_t(\omega_{t+1}) - \mathbb{E}_{t-1}(\omega_{t+1})]$$

It is clear from the above that the proof carries through to a generic autoregressive process, provided the distributions $h(\omega_{t+T}|\omega_t)$ and $h(\omega_{t+T}|\mathbb{E}_{t-1}\omega_t)$ are normal and have the same variance. Here we present a generalization to GARCH processes. Suppose that the state of the world ω_{t+1} follows the GARCH(1,1) process:

$$\omega_{t+1} = b\omega_t + \sigma_{t+1}\epsilon_{t+1}, \quad \sigma_{t+1}^2 = \alpha_0 + \alpha_1(\sigma_t\epsilon_t)^2$$

where $\epsilon_t \sim \mathcal{N}(0,1)$ is white noise. Then the true conditional distribution $h(\omega_{t+1}|\omega_t)$ is given by $\mathcal{N}(b\omega_t, \alpha_0 + \alpha_1(\sigma_t\epsilon_t)^2)$, while the expected distribution $h\big(\omega_{t+1}|\mathbb{E}_{t-1}(\omega_t)\big)$ is given by $\mathcal{N}(b^2\omega_{t-1}, \alpha_0)$. This is because, in the absence of news at t, variance takes its baseline value $\sigma_{t+1}^2 = \alpha_0$.

For convenience, denote by $r_t = \frac{\alpha_0+\alpha_1(\sigma_t\epsilon_t)^2}{\alpha_0} = 1 + \frac{\alpha_1}{\alpha_0}(\sigma_t\epsilon_t)^2$ the increase in variance of ω_{t+1} given news ϵ_t relative to the comparison context where $\epsilon_t = 0$. Then, provided the variance does not increase excessively,[20] namely $r_t^2 < 1 + \frac{1}{\theta}$, diagnostic expectations at t are also described by a normal distribution, given by $\mathcal{N}\left(\mathbb{E}_t^\theta(\omega_{t+1}), var_t^\theta(\omega_{t+1})\right)$ where:

$$\mathbb{E}_t^\theta(\omega_{t+1}) = \mathbb{E}_t(\omega_{t+1}) + \tilde{\theta}_t[\mathbb{E}_t(\omega_{t+1}) - \mathbb{E}_{t-1}(\omega_{t+1})]$$

with distortions being modulated by the effective (time dependent) diagnostic parameter

$$\tilde{\theta}_t = \theta\frac{r_t^2}{1-\theta(r_t^2-1)} \geq \theta$$

Diagnostic variance is given by:

$$var_t^\theta(\omega_{t+1}) = \sigma_{t+1}^2\frac{1}{1-\theta(r_t^2-1)} \geq \sigma_{t+1}^2$$

[20] If the variance increases the tails become overweighted, and increasingly so as r_σ increases. If r_t is sufficiently large, the resulting density function becomes U-shaped and is not normalizable. The condition $r_t^2 < \frac{1+\theta}{\theta}$ ensures that diagnostic expectations are normalizable. This condition always holds in the limit of rational expectations, $\theta \to 0$.

For normal distributions, the kernel of truth property of diagnostic expectations applies to both the first and second moments. Suppose there is a shock at t, i.e. $\epsilon_t \neq 0$ so that $r_t^2 > 1$. Then the diagnostic mean $\mathbb{E}_t^\theta(\omega_{t+1})$ exaggerates the shock, as in Proposition 1, except that the effective diagnostic parameter is amplified, $\tilde{\theta}_t > \theta$. The intuition for this effect is clear: when there is a positive shock to the variance, the tails get thicker and the diagnostic tail gets more overweighted, thus compounding any average movement of the distribution.

Moreover, diagnostic expectations also exaggerate the variance of ω_{t+1}, namely $var_t^\theta(\omega_{t+1}) > \sigma_{t+1}^2$. In a world with risk aversion, this leads to systematic overreaction to bad news, but a more dampened reaction to good news. ■

Corollary 1. Equation (5) follows from the general proof given for Proposition 1. We now compute iterated diagnostic expectations. From the perspective of period t, the expectation $\mathbb{E}_{t'}(\omega_{t+T}) = b^{t+T-t'}\omega_{t'} + (1-b)\bar{\omega}\sum_{s=0}^{t+T-t'-1} b^s$ is, for any $t < t' < t+T$, a normal variable with mean $b^T\omega_t + (1-b)\bar{\omega}\sum_{s=0}^{T-1} b^s$ and variance $\sigma_{t+T-t'}^2$. Moreover, again from the perspective of period t, this variable is independent of the expectation $\mathbb{E}_{t'-1}(\omega_{t+T})$ in the previous period. As a consequence, we have that

$$\mathbb{E}_{t'}^\theta(\omega_{t+T}) = \mathbb{E}_{t'}(\omega_{t+T}) + \theta[\mathbb{E}_{t'}(\omega_{t+T}) - \mathbb{E}_{t'-1}(\omega_{t+T})]$$

is itself a normally distributed normal variable. Thus, the representation of diagnostic expectations from Proposition 1 can be applied. We find:

$$\mathbb{E}_t^\theta\left[\mathbb{E}_{t'}^\theta(\omega_{t+T})\right] = \mathbb{E}_t^\theta\left[\mathbb{E}_{t'}(\omega_{t+T}) + \theta[\mathbb{E}_{t'}(\omega_{t+T}) - \mathbb{E}_{t'-1}(\omega_{t+T})]\right] =$$

$$\mathbb{E}_t\left[\mathbb{E}_{t'}(\omega_{t+T}) + \theta[\mathbb{E}_{t'}(\omega_{t+T}) - \mathbb{E}_{t'-1}(\omega_{t+T})]\right] + \theta\mathbb{E}_t\left[\mathbb{E}_{t'}(\omega_{t+T}) + \theta[\mathbb{E}_{t'}(\omega_{t+T}) - \mathbb{E}_{t'-1}(\omega_{t+T})]\right]$$

$$-\theta\mathbb{E}_{t-1}\left[\mathbb{E}_{t'}(\omega_{t+T}) + \theta[\mathbb{E}_{t'}(\omega_{t+T}) - \mathbb{E}_{t'-1}(\omega_{t+T})]\right]$$

where we applied Proposition 1 in the second step. We now use linearity and the law of iterated expectations for the $\mathbb{E}$ operator to find:

$$\left[\mathbb{E}_t(\omega_{t+T}) + \theta[\mathbb{E}_t(\omega_{t+T}) - \mathbb{E}_t(\omega_{t+T})]\right] + \theta\left[\mathbb{E}_t(\omega_{t+T}) + \theta[\mathbb{E}_t(\omega_{t+T}) - \mathbb{E}_t(\omega_{t+T})]\right]$$

$$- \theta\left[\mathbb{E}_{t-1}(\omega_{t+T}) + \theta[\mathbb{E}_{t-1}(\omega_{t+T}) - \mathbb{E}_{t-1}(\omega_{t+T})]\right]$$

$$= \mathbb{E}_t(\omega_{t+T}) + \theta[\mathbb{E}_t(\omega_{t+T}) - \mathbb{E}_{t-1}(\omega_{t+T})] = \mathbb{E}_t^{\theta}(\omega_{t+T})$$

Intuitively, future distortions are in the kernel of the diagnostic expectations operator, because on average there is no news. As a result, the term structure of diagnostic expectations is fully consistent.

It is important to stress that the linear representation (4) of diagnostic expectations can be applied to the linear combination of variables $\mathbb{E}_{t'}^{\theta}(\omega_{t+T})$ only because the latter is itself a normal variable. Being defined in terms of representativeness, diagnostic expectations do not satisfy linearity in the following sense: $\mathbb{E}_t^{\theta}(x_{t+T} + y_t) \neq \mathbb{E}_t^{\theta}(x_{t+T}) + y_t$. In fact, representativeness must be defined with respect to the distribution of $x_{t+T} + y_t$, which yields:

$$\mathbb{E}_t^{\theta}(x_{t+T} + y_t) = \mathbb{E}_t(x_{t+T} + y_t)(1+\theta) - \theta\mathbb{E}_{t-1}(x_{t+T} + y_t) \neq$$

$$\mathbb{E}_t^{\theta}(x_{t+T}) + \mathbb{E}_t^{\theta}(y_t) = \mathbb{E}_t^{\theta}(x_{t+T}) + y_t.$$

In the case above, linearity breaks down because y_t is determined at time t. As a result, when computing diagnostic expectations of y_t, we find its infinitely representative state is y_t itself (formally, we represent y_t with a delta distribution). Thus, the $t-1$ distribution of y_t does not enter the diagnostic expectation $\mathbb{E}_t^{\theta}(y_t)$. In general, however, linearity holds for combinations of non-degenerate normal random variables. Namely, $\mathbb{E}_t^{\theta}(x_{t+s} + y_{t+r}) = \mathbb{E}_t^{\theta}(x_{t+s}) + \mathbb{E}_t^{\theta}(y_{t+r})$ whenever x_{t+s} and y_{t+r} are non degenerate ■

Proposition 2. For point i), write

$$\frac{\partial^2 S\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)}{\partial S_t \partial \rho} = \frac{\partial^2 S\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)}{\partial \mathbb{E}_t^\theta(\omega_{t+1}) \partial \rho} \cdot \frac{\partial \mathbb{E}_t^\theta(\omega_{t+1})}{\partial S_t}$$

where the last term is negative. Using the shorthand $\mu = \mu\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)$, the first term reads

$$\frac{\partial^2 S\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)}{\partial \rho \partial \mathbb{E}_t^\theta(\omega_{t+1})} = \partial_\rho \left[\partial_{\mathbb{E}_t^\theta(\omega_{t+1})} \frac{1}{\beta\mu}\right] = -\frac{1}{\beta\sigma} \partial_\rho \left[\frac{1}{\mu^2} \phi\left(\frac{\rho - \mathbb{E}_t^\theta(\omega_{t+1})}{\sigma}\right)\right]$$

where $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ is the Gaussian density function. Expanding the ρ derivative and re-arranging, we find

$$-\frac{1}{\beta\sigma^2\mu^2} \phi\left(\frac{\rho - \mathbb{E}_t^\theta(\omega_{t+1})}{\sigma}\right) \left[\frac{2}{\mu} \phi\left(\frac{\rho - \mathbb{E}_t^\theta(\omega_{t+1})}{\sigma}\right) + \frac{\phi'\left(\frac{\rho - \mathbb{E}_t^\theta(\omega_{t+1})}{\sigma}\right)}{\phi\left(\frac{\rho - \mathbb{E}_t^\theta(\omega_{t+1})}{\sigma}\right)}\right]$$

The second term in the parenthesis is equal to $-\frac{\rho - \mathbb{E}_t^\theta(\omega_{t+1})}{\sigma}$. To compute the first term, we use the identity $\int f'^{(x)} \cdot e^{f(x)} dx = f(x)$ to write

$$\frac{1}{\mu} \phi\left(\frac{\rho - \mathbb{E}_t^\theta(\omega_{t+1})}{\sigma}\right) = \frac{1}{\mu} \frac{1}{\sqrt{2\pi}} \int_{\frac{\rho - \mathbb{E}_t^\theta(\omega_{t+1})}{\sigma}}^{+\infty} z \cdot e^{-\frac{1}{2}z^2} dz = \mathbb{E}\left[z \middle| z > \frac{\rho - \mathbb{E}_t^\theta(\omega_{t+1})}{\sigma}\right]$$

where $z \sim \mathcal{N}(0,1)$. We thus find

$$\frac{\partial^2 S\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)}{\partial \rho \partial \mathbb{E}_t^\theta(\omega_{t+1})}$$

$$= -\frac{1}{\beta\sigma^2\mu^2} \phi\left(\frac{\rho - \mathbb{E}_t^\theta(\omega_{t+1})}{\sigma}\right) \left[2\mathbb{E}\left[z \middle| z > \frac{\rho - \mathbb{E}_t^\theta(\omega_{t+1})}{\sigma}\right] - \frac{\rho - \mathbb{E}_t^\theta(\omega_{t+1})}{\sigma}\right]$$

which is negative, and hence $\frac{\partial^2 S\left(\rho, \mathbb{E}_t^\theta(\omega_{t+1})\right)}{\partial S_t \partial \rho} > 0$.

To see point ii), use Equations (9) and (10) to write:

$$\frac{\partial}{\partial S_t}\frac{k_{t+1}(\rho_1)}{k_{t+1}(\rho_2)} = \frac{\partial \mathbb{E}_t^{\theta}(\omega_{t+1})}{\partial S_t}\frac{\partial}{\partial \mathbb{E}_t^{\theta}(\omega_{t+1})}\left[\frac{\mu\left(\rho_1, \mathbb{E}_t^{\theta}(\omega_{t+1})\right)}{\mu\left(\rho_2, \mathbb{E}_t^{\theta}(\omega_{t+1})\right)}\right]^{\frac{1}{1-\alpha}}$$

The first term is negative. The second term is proportional to:

$$\frac{\phi\left(\frac{\rho_1 - \mathbb{E}_t^{\theta}(\omega_{t+1})}{\sigma}\right)}{\mu\left(\rho_1, \mathbb{E}_t^{\theta}(\omega_{t+1})\right)} - \frac{\phi\left(\frac{\rho_2 - \mathbb{E}_t^{\theta}(\omega_{t+1})}{\sigma}\right)}{\mu\left(\rho_2, \mathbb{E}_t^{\theta}(\omega_{t+1})\right)}$$

$$= \mathbb{E}\left[z|z > \frac{\rho_1 - \mathbb{E}_t^{\theta}(\omega_{t+1})}{\sigma}\right] - \mathbb{E}\left[z|z > \frac{\rho_2 - \mathbb{E}_t^{\theta}(\omega_{t+1})}{\sigma}\right]$$

which is positive for any $\rho_1 > \rho_2$. In the first line, we used $\frac{\partial \mu\left(\rho, \mathbb{E}_t^{\theta}(\omega_{t+1})\right)}{\partial \mathbb{E}_t^{\theta}(\omega_{t+1})} = \phi\left(\frac{\rho - \mathbb{E}_t^{\theta}(\omega_{t+1})}{\sigma}\right)$ and in the second line we used the identity derived above.

■

Proposition 3. From Equation (14) we have:

$$S_t = \sigma_0 - \sigma_1[b\omega_t + (1-b)\overline{\omega} + \theta_1 b\epsilon_t]$$

$$= \sigma_0 - \sigma_1[b(b\omega_{t-1} + (1-b)\overline{\omega} + \epsilon_t) + (1-b)\overline{\omega} + \theta_1 b\epsilon_t]$$

where we used the AR(1) condition $\omega_t = b\omega_{t-1} + (1-b)\overline{\omega} + \epsilon_t$. Note that, rearranging the first line (valid for all t) we find:

$$\sigma_1[b\omega_{t-1} + (1-b)\overline{\omega}] = \sigma_0 - S_{t-1} - \sigma_1\theta_1 b\epsilon_{t-1}$$

Inserting above, we then get:

$$S_t = (1-b)(\sigma_0 - \sigma_1\overline{\omega}) + b \cdot S_{t-1} - (1+\theta_1)b\sigma_1\epsilon_t + \theta_1 b^2\sigma_1\epsilon_{t-1}$$

Spreads thus follow an ARMA(1,1) process.

■

Lemma 1 For notational convenience, rewrite the stochastic process driving credit spreads as $S_t = a + b \cdot S_{t-1} - c\epsilon_t + d\epsilon_{t-1}$, with $a = (1-b)(\sigma_0 - \sigma_1\overline{\omega})$, $c = (1+\theta)b\sigma_1$ and $d = \theta b^2\sigma_1$. The T periods ahead diagnostic forecast of the spread is given by:

$$\mathbb{E}_t^{\theta}(S_{t+T}) = \mathbb{E}_t^{\theta}(a + b \cdot S_{t+T-1} - c\epsilon_{t+T} + d\epsilon_{t+T-1})$$

Note that $\mathbb{E}_t^{\theta}(\epsilon_{t+s}) = 0$ for any $s > 0$, because rational expectations of future shocks are always zero. Thus, for $T > 1$, $\mathbb{E}_t^{\theta}(S_{t+T})$ becomes

$$\mathbb{E}_t^{\theta}(S_{t+T}) = a + \mathbb{E}_t^{\theta}(b \cdot S_{t+T-1}) = a\sum_{s=0}^{T-2} b^s + b^{T-1}\mathbb{E}_t^{\theta}(S_{t+1}) = a\sum_{s=0}^{T-1} b^s + b^T S_t$$

Inserting the coefficients we get :

$$\mathbb{E}_t^{\theta}(S_{t+T}) = (1-b^T)(\sigma_0 - \sigma_1\overline{\omega}) + b^T S_t$$

Consider now the case $T = 1$. Using (14) and the law of iterated expectations for diagnostic expectations, write $\mathbb{E}_t^{\theta}(S_{t+1}) = \sigma_0 - \sigma_1\mathbb{E}_t^{\theta}(\omega_{t+2})$. Inserting $\omega_{t+2} = b\omega_{t+1} + \epsilon_{t+2}$ and $\mathbb{E}_t^{\theta}(\epsilon_{t+2}) = 0$, we obtain the result.

■

Proposition 4. The forecast error at $t+1$ is $\mathbb{E}_t\left[S_{t+1} - \mathbb{E}_t^{\theta}(S_{t+1})\right]$. The first result follows immediately from Lemma 1. Alternatively, write:

$$S_{t+1} - \mathbb{E}_t^{\theta}(S_{t+1}) = -\sigma_1\left[\mathbb{E}_{t+1}^{\theta}(\omega_{t+2}) - \mathbb{E}_t^{\theta}\left(\mathbb{E}_{t+1}^{\theta}(\omega_{t+2})\right)\right]$$

The first term is $\mathbb{E}_{t+1}^{\theta}(\omega_{t+2}) = \mathbb{E}_{t+1}(\omega_{t+2}) + \theta b\epsilon_{t+1}$, while the second term is $\mathbb{E}_t^{\theta}\left(\mathbb{E}_{t+1}^{\theta}(\omega_{t+2})\right) = \mathbb{E}_t(\omega_{t+2}) + \theta b^2\epsilon_t$. Taking expectations on the difference, we find $\mathbb{E}_t\left[S_{t+1} - \mathbb{E}_t^{\theta}(S_{t+1})\right] = \sigma_1\theta b^2\epsilon_t$. Thus, positive news today narrow the spread today and the predicted spread tomorrow, but the realized spread tomorrow is systematically larger than predicted.

Similarly, we can write

$$\mathbb{E}_t\left[\mathbb{E}_{t+s}^{\theta}(S_{t+T}) - \mathbb{E}_t^{\theta}(S_{t+T})\right] = -\sigma_1 \mathbb{E}_t\left[\mathbb{E}_{t+s}^{\theta}\left(\mathbb{E}_{t+T}^{\theta}(\omega_{t+T+1})\right) - \mathbb{E}_t^{\theta}\left(\mathbb{E}_{t+T}^{\theta}(\omega_{t+T+1})\right)\right]$$

Using the representation of Corollary 1, this becomes

$$\mathbb{E}_t\left[\mathbb{E}_{t+s}^{\theta}(S_{t+T}) - \mathbb{E}_t^{\theta}(S_{t+T})\right] = \sigma_1 \theta b^{T+1} \epsilon_t$$

Again, positive news today compress expected spreads in the future, and these expectations systematically widen going forward.

■

Lemma 2. Given normal and i.i.d. errors, the OLS regression coefficient of credit spread forecast error $S_{t+1} - \mathbb{E}_t^{\theta}(S_{t+1})$ on current spread levels S_t is given by the maximum likelihood estimator:

$$\beta \equiv \frac{cov\left(S_{t+1} - \mathbb{E}_t^{\theta}(S_{t+1}), S_t\right)}{var(S_t)}$$

From (15) and (16), the forecast error in the spread over the next 12 months is given by:

$$S_{t+1} - \mathbb{E}_t^{\theta}(S_{t+1}) = -(1+\theta) b \sigma_1 \epsilon_{t+1} + \theta b^2 \sigma_1 \epsilon_t$$

so that $cov\left(S_{t+1} - \mathbb{E}_t^{\theta}(S_{t+1}), S_t\right) = -(1+\theta)\theta b^3 \sigma_1^2 \sigma^2$.

To derive the sample variance $var(S_t)$, we write spreads iteratively in terms of the time series of shocks. Let $S_{t+1} = C + bS_t - A\varepsilon_{t+1} + B\varepsilon_t$, with $C = (1-b)\sigma_0$, $A = (1+\theta)b\sigma_1$ and $B = \theta b^2 \sigma_1$. For large t we have

$$S_t = \sum_{s \geq 0} b^s - A\varepsilon_t - b^2 \sigma_1 \sum_{s>0} b^{s-1} \varepsilon_{t-s}$$

where we used $B - bA = -b^2\sigma_1$. It then follows that $var(S_t) = b^2 \sigma_1^2 \left[(1+\theta)^2 + \frac{b^2}{1-b^2}\right] \sigma^2$. Replacing in the expression for β gives the result.

■

Corollary 2. Defining S_t^r as the credit spread that obtains under rational expectations, where $\theta = 0$, it follows immediately from Equation (14) that

$$S_t - S_t^r = -\theta b \sigma_1 \epsilon_t$$

Moreover,

$$Var[S_t|\omega_{t-1}] = Var[-(1+\theta)b\sigma_1\epsilon_t|\omega_{t-1}] = (1+\theta)^2 Var[S_t^r|\omega_{t-1}].$$

■

Proposition 5. Assume that at $t-1$ spreads are low due to recent good news, $\epsilon_{t-1} > 0$, It follows from the ARMA(1,1) structure for spreads derived in Proposition 3 that the expected future path of spreads is:

$$\mathbb{E}_{t-1}[S_t] = (1-b)(\sigma_0 - \sigma_1\overline{\omega}) + b \cdot S_{t-1} + \theta b^2 \sigma_1 \epsilon_{t-1}$$

from which the result follows.

Aggregate investment at t and aggregate production at $t+1$ are strictly decreasing functions of the average credit spread S_t. It follows from point i) that, under the assumptions of the Proposition and controlling for fundamentals at $t-1$, there is a predictable drop in these quantities from the perspective of $t-1$.

■

Lemma 3. We start from

$$S_t = \sigma_0 - \sigma_1 \mathbb{E}_t^\theta \omega_{t+1} = \sigma_0 - \sigma_1(b\omega_t + \theta b \epsilon_t)$$

Regressing S_t on ω_t yields coefficients:

$$\hat{\beta} = \frac{cov(S_t, \omega_t)}{var(\omega_t)} = -\sigma_1 b[1 + \theta(1-b^2)]$$

Explained variance is therefore equal to

$$Var(\hat{\alpha} + \hat{\beta}\omega_t) = (\sigma_1 b)^2 [1 + \theta(1-b^2)]^2 \frac{Var(\epsilon_t)}{1-b^2}$$

As was shown in Lemma 2, the actual variance is $Var(S_t) = (\sigma_1 b)^2[b^2 + (1+\theta)^2(1-b^2)]\frac{Var(\epsilon_t)}{1-b^2}$. The result follows from taking the ratio between explained and actual variance, or R^2. Note that $R^2 = 1$ for $\theta = 0$ or $b = 0$, but is smaller than 1 for $\theta > 0$ and $b > 0$.

■

Proposition 6. Let ρ_s^* be the riskiness threshold, at time s, below which firms can issue safe bonds whose default probability at $s+1$ is less than δ^*, and above which only risky debt can be issued. As in Section 4, we adopt assumption A.1, namely that wealth w is sufficiently large that debt of firm ρ is priced in such a way as to make investors indifferent between investing in it and consuming in the current period (i.e. the firm optimization is binding for the total investment).

The result now follows immediately from Equations (20) and (21). The rate of return that makes the household indifferent between consuming and investing in safe debt is given by Equation (10), namely $r_{t+1}(\rho) = \frac{1}{\beta\mu\left(\rho,\mathbb{E}_t^\theta(\omega_{t+1})\right)}$. For risky debt, it is $\psi r_{t+1}(\rho)\cdot\mu\left(\rho,\mathbb{E}_t^\theta(\omega_{t+1})\right) = \beta^{-1}$, that is $r_{t+1}(\rho) = \frac{1}{\psi\beta\mu\left(\rho,\mathbb{E}_t^\theta(\omega_{t+1})\right)}$.

■

Proposition 7. From Proposition 6, valuation of debt of firm ρ at time t is given by the inverse of the equilibrium interest rate $1/r_{t+1}(\rho)$. In the interim period, default probability is updated from $\mu\left(\rho,\mathbb{E}_t^\theta(\omega_{t+1})\right)$ to $\mu\left(\rho,\mathbb{E}_t^{\theta=0}(\omega_{t+1})\right)$. For firms that are perceived as safe both prior and during the interim period, $\rho < \rho_t^{\theta=0}$, valuation changes by the factor $\frac{\mu\left(\rho,\mathbb{E}_t^\theta(\omega_{t+1})\right)}{\mu\left(\rho,\mathbb{E}_t^{\theta=0}(\omega_{t+1})\right)}$. For firms that are perceived as risky both prior and during the interim period, $\rho > \rho_t^{\theta}$, valuation changes by the (same) factor $\frac{\psi\mu\left(\rho,\mathbb{E}_t^{\theta}(\omega_{t+1})\right)}{\psi\mu\left(\rho,\mathbb{E}_t^{\theta=0}(\omega_{t+1})\right)}$.

Finally, for marginally safe firms, $\rho \in \left(\rho_t^{\theta=0}, \rho_t^{\theta}\right)$, which are perceived as safe following good news but as risky in the interim period, valuation changes as $\frac{\mu\left(\rho,\mathbb{E}_t^{\theta}(\omega_{t+1})\right)}{\psi\mu\left(\rho,\mathbb{E}_t^{\theta=0}(\omega_{t+1})\right)}$.

■

Appendix A. Alternative specifications of $-G$

We briefly consider two alternative specifications of the reference group $-G$ used to define representativeness.

A.1. Lagged Diagnostic Expectations as Reference

We start by specifying $-G$ in terms of diagnostic expectations $\mathbb{E}^{\theta}_{t-1}(\omega_t)$. Assume that the agent compares the current distribution with the one implied by his past diagnostic expectation of Ω_t, namely $-G \equiv \{\Omega_t = \mathbb{E}^{\theta}_{t-1}(\omega_t)\}$. Diagnostic expectations at time t are then given by:

$$\mathbb{E}^{\theta}_{t}(\omega_{t+1}) = \mathbb{E}_{t}(\omega_{t+1}) + \theta\left[\mathbb{E}_{t}(\omega_{t+1}) - b\mathbb{E}^{\theta}_{t-1}(\omega_t)\right]. \tag{17}$$

The agent is overly optimistic when news point to an outcome that is sufficiently good as compared with his past expectations, $\mathbb{E}_{t}(\omega_{t+1}) > b\mathbb{E}^{\theta}_{t-1}(\omega_t)$, and overly pessimistic otherwise. By iterating Equation (17) backwards, for $\theta b < 1$ we obtain:

$$\mathbb{E}^{\theta}_{t}(\omega_{t+1}) = (1+\theta)\sum_{j\geq 0}(-\theta b)^{j}\mathbb{E}_{t-j}\left(\omega_{t-j+1}\right). \tag{18}$$

Diagnostic expectations are a weighted average of current and past one-period-ahead rational expectations, with weights that depend on θ. Again, when $\theta = 0$, expectations are rational. In Equation (18) the signs on rational expectations obtained in odd and even past periods alternate. This is an intuitive consequence of (17) and implies that news exert a non-monotonic effect in future expectations. Agents over-react on impact, but this over-reaction implies reference expectations are higher the next period, causing a reversal to pessimism (which in turn generates future optimism and so on). Specifying $-G$ in terms of diagnostic expectations thus preserves the two key properties of our basic model: expectations display over-reaction to news on impact but also reversal in the future.

A.2. Slow Moving – G

In our main specification, context – G is the immediate past. This assumption starkly illustrates our results and buys significant tractability. It is however possible that remote but remarkable memories influence the agent's background context. Our model can be easily enriched to capture this feature by defining representativeness in terms of a mixture of current and past likelihood ratios:.

Let representativeness be defined as:

$$\prod_{s\geq 1}\left(\frac{h(\Omega_{t+1}=\omega_{t+1}|\Omega_{t+1-s}=b^{s-1}\omega_{t+1-s})}{h(\Omega_{t+1}=\omega_{t+1}|\Omega_{t+1-s}=b^{s}\omega_{t})}\right)^{\alpha_s}$$

where $\alpha_s \geq 0$ capture the weights attached to present and past representativeness. In this case we have that:

$$\mathbb{E}_t^{\theta}(\omega_{t+1}) = \mathbb{E}_t(\omega_{t+1}) + \theta\sum_{s\geq 1}\alpha_s[\mathbb{E}_{t+1-s}(\omega_{t+1}) - \mathbb{E}_{t-s}(\omega_{t+1})]$$

The benchmark model in the paper has $\alpha_1 = 1$ and $\alpha_s = 0$ for $s > 1$. A constant rate of decay would have $\alpha_s = 1$ for all $s \geq 1$. Intermediate specifications, capturing recency effects but with some memory of past representativeness, would feature $\alpha_1 > \alpha_2 > \cdots$.

In models with a "slow moving" – G, namely where $\alpha_s > 0$ for some s>1, the agent can remain too optimistic even after minor bad news, $\mathbb{E}_t(\omega_{t+1}) - \mathbb{E}_{t-1}(\omega_{t+1}) < 0$, provided he experienced major good news in the past, e.g. $\mathbb{E}_{t-1}(\omega_{t+1}) - \mathbb{E}_{t-2}(\omega_{t+1}) \gg 0$. This feature can yield under-reaction to early warnings of crises (see Gennaioli et al. 2015 for a related formulation). At the same time, the main properties of over-reaction and reversal continue to hold in this specification with respect to repeated news in the same direction, which are plausible in the case of credit cycles.

In general, the robust predictions of the model remain over-reaction and reversals. Different specifications of $-G$ yield different ancillary predictions that may make it possible to uncover the structure of $-G$ in the data. This is an important avenue for future work.