Optimally Vague Contracts and the Law*

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Abstract

Many real-world contracts contain vague clauses despite the enforcement risk they entail. To study the causes and consequences of this phenomenon, we build a principal-agent model in which contracts can include vague clauses whose enforcement may be distorted by opportunistic litigants and biased judges. We find three results. First, the optimal contract is vague, even if courts are very imperfect. Second, the use of vague clauses is a public good: it promotes the evolution of precedents, so future contracts become more complete, incentives higher powered, and surplus larger. Third, as precedents evolve, vague contracts spread from sophisticated to unsophisticated parties, expanding market size. Our model sheds light on the evolution and diffusion of business-format franchising and equity finance.

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1 Introduction

Contracts in all areas of business routinely include vague provisions. Parties often choose to define performance on the basis of terms such as "best efforts," "good faith," or "reasonable cause." The use of such vague terms is systematic. Most distribution, franchising and licensing contracts impose a duty of best efforts and allow termination for reasonable cause. In other cases, vague terms are included by default in a contract, unless the parties explicitly opt out. For instance, the managers of common-stock corporations owe fiduciary duties to shareholders.[1] The widespread use of vague provisions places courts center stage. First, vagueness enables judges to exercise some discretion, making contract enforcement uncertain (Posner 2005; Scott and Triantis 2006; Burton 2008). Second, as we show in Section 2, vagueness influences the law and future contracts. Repeated litigation of a vague term systematically used in a certain type of contract establishes legal precedents that narrow down uncertainty in enforcing contracts of the same kind.

These phenomena cannot be explained using standard models of incomplete contracts. Such models posit a stark, exogenous and time-invariant dichotomy between contingencies that are perfectly verifiable and thus contractible, and contingencies that are unverifiable and thus non-contractible. As a result, the standard approach cannot answer two key questions. First, why would parties choose to contract upon vague contingencies that rely on limited court enforcement? Second, how does contracting evolve over time as courts verify vague contingencies?

This paper develops a model that answers both questions. We build upon a standard principal-agent problem with one-sided moral hazard, risk neutrality and limited liability. There is a multitude of informative proxies of the agent's performance. Contracts are incomplete because parties cannot fully describe ex ante how courts should use specific performance proxies ex post. However, parties can write a vague contract that leaves courts discretion to verify performance on the basis of litigation.

We model two key features of litigation. First, courts must respect precedents. In our setup, courts reliably recognize and verify performance proxies used in past decisions. Second, the possibility of raising novel arguments creates litigation risk. In our setup, courts may also consider new performance proxies, but these are subject to enforcement frictions. To begin with, litigants have limited ability to collect a new proxy and can choose strategically to withhold it. In addition, judges may be biased and can choose strategically

[1] In a minority of cases, parties cannot contract out entirely of implicit contract terms. This is most famously the case of the duty of good faith implicit in any contract. Even then, parties retain some leeway to specify behavior that might seem questionable but that they deem consistent with the unwaivable duty.

to discard new proxies so as to favor either litigant.

In this setup, we solve for the optimal contract from a mechanism-design perspective. This means that the schedule of payments from the principal to the agent is written with the expectation of litigation and enforcement frictions. It satisfies incentive-compatibility constraints so that partisan litigants and biased judges report new proxies and interpret the contract ex post the way the contracting parties want them to ex ante.

We find that the optimal contract is at the same time systematic and vague. It is systematic because payment is contingent on proxies embodied in precedents established by litigating contracts of the same type. It is vague because payment is also contingent on new proxies that litigants must present and judges verify. Thus, parties choose to leave judges some discretion when enforcing the contract.

With the optimal contract, the benefit of vagueness is that judges can exploit new proxies to verify bad performance. As a result, bad performance is less likely to be mistakenly rewarded. The cost of vagueness is that judges or litigants may selectively discard new proxies pointing to good performance. Then good performance is also less likely to be correctly rewarded. Crucially, vague terms can be specified so as to mitigate the latter cost while retaining the former benefit. First, the stipulated payment can be raised to compensate the agent for the risk that good performance is not rewarded. Second, if judicial bias is extreme, the contract can place the burden of proof entirely on the principal to show evidence of bad performance. By adjusting these two dimensions, parties secure the informational benefit of allowing courts to use new performance proxies ex post.

We find that the use of systematic but vague optimal contracts provides a public good. Litigation of such contracts steadily uncovers new performance proxies. Judges verify them, creating more informative precedents. As a result, performance endogenously becomes more verifiable. This positive development has three key consequences. First, the agent's incentives and the surplus generated under the optimal contract increase. Second, parties choose to employ higher-powered incentives because the risk of rewarding bad performance falls. Third, judges' biases and litigants' legal capabilities become less important because novel proxies become less decisive over time. As a consequence, contracting spreads to less sophisticated parties, who have lower evidence-collection skills, and who found it too costly to contract when enforcement uncertainty was very high. Legal evolution fosters market development though the creation and diffusion of reliable contracts.

Our analysis underscores a positive feedback loop between private contracting and the development of contract law. Unlike in standard contract theory, contracts are not written only for the contracting parties, taking enforcement as an exogenous constraint. The contract influences the behavior of enforcers and the evolution of the law. As a result, contracts become public goods that affect the contract space of future parties. To illustrate this phenomenon, Section 6 discusses two real-world examples. The first concerns the development of franchising (Blair and Lafontaine 2005). The franchise contract systematically relies on the judicial interpretation of vague provisions to regulate a complex agency relationship (Hadfield 1990). Accordingly, Lafontaine, Perrigot and Wilson (2017) document lower usage of franchising contracts in countries with worse legal institutions. Crucially, precedents set by American courts in enforcing vague clauses have enabled and shaped the growth of business-format franchising by helping define when and how the franchisor has a right to enforce its quality standards. Our second example concerns the development of corporate law, and in particular the definition of shareholder rights. This process occurred gradually, in court, through the accumulation of judicial rulings when litigating fiduciary duties, a vague implicit provision in equity contracts (Easterbrook and Fischel 1989; Jacobs 2015). In both examples, the use and litigation of initially vague terms allowed contracts to become less vague but also better adapted over time, and thus attractive to many parties.

By accounting for these real-world patterns, our model complements and advances the literature on incomplete contracts. Closest to our approach, Battigalli and Maggi (2002) model how contractual incompleteness can take the form of either rigidity or discretion.[2] In contrast to our analysis, however, they view discretion as enabling the agent to choose ex post between different actions. They do not model courts as a third actor that exercises discretion in enforcing the contract. Likewise, Hart and Moore (2008) present a model in which discretion requires renegotiation, but such renegotiation creates ex post aggrievement costs, not enforcement costs.

Legal scholars have studied the optimal rules of contract interpretation, but have mostly taken contracting as given (Goetz and Scott 1981; Ayres and Gertner 1989; Schwartz 1992; Schwartz and Scott 2003, 2010; Eric Posner 1998; Richard Posner 2005; Burton 2008). Few papers consider, like us, the interplay between ex-post contract interpretation by fallible courts and ex-ante contract drafting by parties that anticipate the risk of judicial mistakes (Hadfield 1994; Shavell 2006; Choi and Triantis 2008, 2010).[3] Gennaioli (2013) studies

[2] Battigalli and Maggi (2002) build on prior models of contract incompleteness arising from drafting costs (Dye 1985; Anderlini and Felli 1994; Segal 1999).

[3] Hadfield (1994) shows that the risk of small judicial mistakes can make vague standards preferable to clear rules, which entail large changes in outcomes as a result of small changes in fact-finding. Shavell (2006) shows how welfare-maximizing courts can reduce contracting costs by committing to the optimal method of contract interpretation. Choi and Triantis (2008, 2010) study how contract clauses that raise litigation costs can improve screening and incentives.

optimal contracting in the shadow of a court that may be imperfectly informed and biased. We are the first to provide a model that explains vague contracts as an optimal mechanism that induces the ex-ante surplus-maximizing contract interpretation subject to ex-post incentive-compatibility constraints of judges and litigants. We are also the first to study formally the dynamic implications of systematically vague contracts.

Our dynamic analysis contributes to the literature on legal evolution. Conventional wisdom holds that the development of common law promotes economic efficiency thanks to the efficiency-oriented nature of its judges (Posner [1973] 2014) and the adaptability of precedents (Hayek 1960). Relative to early papers (Priest 1977; Rubin 1977), recent work has focused on judges' behavior (Gennaioli and Shleifer 2007; Ponzetto and Fernandez 2008; Fernandez and Ponzetto 2012; Niblett 2013; Anderlini, Felli, and Riboni 2014). Our view of legal evolution as a way to enrich the empirical content of the law is closest to Gennaioli and Shleifer's (2007) model of distinguishing. Departing from the earlier focus on tort law, we study the role of optimal private contracts and the feedback between contracting and the development of contract law.[4]

The remainder of the paper is organized as follows. In Section 2 we briefly present evidence on the two key building blocks of our analysis: judicial discretion in enforcing vague contracts and the role of legal precedents in narrowing down uncertainty in future litigation. In Section 3 we present the economic environment of our agency problem. Section 4 describes imperfect performance proxies and litigation. Section 5 studies the optimal mechanism for a given state of legal evolution. Section 6 presents the joint evolution of contracts and precedents. Section 7 concludes.

2 Judicial Discretion and Precedent in Contract Interpretation

Legal scholars emphasize that real-world contracts are systematically vague and ambiguous, and that business litigation is predominantly about contract interpretation (Posner 2005; Scott and Triantis 2006; Burton 2008; Schwartz and Scott 2010). We could distinguish three broad categories of contract ambiguity. First, as we stressed in the introduction, contracts use standard but intrinsically vague terms like "best efforts," "good faith," or "reasonable cause." Second, contracts include ambiguous provisions that admit multiple

[4] Anderlini, Felli and Riboni (2014) study contracting when precedents provide a partial solution to the problem of time inconsistency. They do not consider contract design as a mechanism to induce the optimal judicial interpretation.

reasonable meanings.[5] Third, contracts fail to specify in full the parties' obligations. Our model applies most directly to the first case. Yet, it is also consistent with the second and the third whenever they result not from inadvertent drafting errors, but rather from a deliberate decision to economize on drafting costs or to retain some ex-post flexibility.

In this section, we discuss some textbook examples of contract interpretation that illustrate two key assumptions of our model. First, judges have some discretion in assessing performance in light of the litigants' conflicting claims and evidence. Second, landmark decisions create legal precedents that reduce uncertainty in the enforcement of analogous vague clauses in subsequent contracts.

Consider, to begin with, an exclusive distributor's promise to exert its "best efforts" to promote sales. The leading precedent for interpreting this vague term is *Bloor v. Falstaff Brewing Corp.*,[6] which set a standard that both textbooks and subsequent cases often cite. The plaintiff represented a regional brewer that had sold its assets to the defendant, a larger national brewer. Falstaff contracted to use its best efforts to maintain sales of the brands it acquired. After a severe decline in sales, Bloor sued for breach of this best effort clause. Falstaff argued its distribution efforts were sufficient in light of its poor financial situation.

The court found in plaintiff's favor. It held that best efforts cannot require losses that threaten the distributor's solvency, but conversely they are inconsistent with mere profit-maximization on his part. In practice, the court chose as a performance benchmark the sales of comparable products from other companies facing similar market conditions. *Bloor*'s adoption of a standard that requires assessing both subjective and objective factors (the promisor's capabilities and the performance of comparable third parties) has shaped the subsequent interpretation of best efforts clauses (Miller 2006). While such clauses remain ambiguous, the existence of an influential precedent has narrowed the scope of their ambiguity. *Bloor*'s comparison to other companies in the same industry was applied again in subsequent cases.[7] Moreover, contracting parties themselves have followed the strategy of the Bloor court: "Many commercial contracts include explicit benchmarks similar to the ones the court adopted in *Bloor*" (Scott and Triantis 2006, p. 839).

Consider next the case of construction contracts. The vast majority of contracts be-

[5] For instance, the construction contract in the textbook case of *Jacob & Youngs, Inc. v. Kent*, 230 N.Y. 239 (1921), included a clause specifying a brand of pipes to be installed. Ambiguously, mention of the brand could have been included to define an easily understood quality standard, or to mandate the use of a specific brand permitting no substitutions irrespective of quality.

[6] 454 F. Supp. 258 (S.D.N.Y. 1978), *aff'd,* 601 F.2d 609 (2d Cir. 1979).

[7] For instance, *Olympia Hotels Corp. v. Johnson Wax Development Corp.*, 908 F.2d 1363 (7th Cir. 1990), *First Union National Bank v. Steele Softward Systems Corp.*, 838 A.2d 404 (Md. Ct. Spec. App. 2003), and *Carlson Distributing Co. v. Salt Lake Brewing Co.*, 95 P.3d 1171 (Utah Ct. App. 2004).

tween private parties, and all those involving the Federal government, include a "differing site conditions" clause. These clauses enable a contractor to claim additional time and/or money to complete construction if the physical conditions of the building site are different from those originally anticipated. The provision is invariably accompanied by a notice requirement, whose purpose is to allow the owner to inspect the changed site conditions and consider redesigning the project accordingly.

Judges have enforced these notice requirements with an understanding that they admit multiple reasonable interpretations. It is hard for the parties to specify in complete detail when notice satisfies its purpose. This difficulty creates ambiguity over how narrowly the notice requirement should be construed. The lack of substantive compliance with the notice requirement may result in a denial of the entire claim. However, Federal courts have developed a liberal approach, excusing lack of strict compliance. Without any landmark decisions, a series of consistent rulings has gradually reduced uncertainty in contract interpretation. It is by now unambiguous that notice need not follow a specific format, and notably that oral notice is sufficient even if the statutory language in Federal contracts calls for written notice. This consistent judicial interpretation is well known to practitioners and underpins industry practice (Sgarlata and Brasco 2004). The opinion in *Brinderson Corp. v. Hampton Roads Sanitation Dist.*[8] aptly summarizes the evolution of precedents:

> "When a standardized provision widely used in construction contracts receives consistent judicial and administrative interpretation, it acquires a gloss that lends color to the words. When the same words are incorporated in later contracts, it may be presumed that the intention of the parties conforms to the earlier, consistent judicial and administrative interpretation. If, at the beginning, the words were ambiguous in the sense that they were open either to a strict, technical construction or to a more liberal construction in furtherance of their purpose, consistent judicial and administrative rulings that they are to be interpreted with liberality dissolve the ambiguity until, finally, there is no room for a contention that they should be construed strictly."

Our last example is the textbook case of *Wood v. Duff-Gordon.*[9] It concerns a distribution contract that granted plaintiff an exclusive right to market defendant's designs in exchange for a share of the profits. The contract, however, was ambiguous because the distributor had neither paid an advance nor explicitly guaranteed he would market the designs. As a consequence, defendant breached the exclusivity clause but argued it was

[8] 825 F.2d 41, 44 (4th Cir. 1987).
[9] 222 N.Y. 88 (1917).

unenforceable for lack of consideration. This contention was plausible at the time, and Goldberg (2006) argues it ought to have carried the day.

Instead, Judge Cardozo found in plaintiff's favor. Noting that the distributor had in fact actively marketed the design and generated revenues, he enforced exclusivity, ruling that consideration was provided by an implied promise of best efforts. Vagueness here can be seen as deriving from the failure of the contract to fully specify the parties' obligations, an incompleteness that allowed Cardozo to deliver an innovative ruling. This decision created an important precedent that established two legal rules concerning exclusivity provisions. First, an exclusivity provision is unambiguously enforceable without either an advance payment or an express guarantee of minimum revenues. Second, such a contract unambiguously implies a best efforts obligation.[10]

To summarize, these examples illustrate two cornerstones of our analysis. First, vague contract terms allow judges to exert discretion in enforcement. Second, the exercise of judicial discretion creates precedents that narrow down future judicial interpretation. These principles hold true whether vagueness was a deliberate choice of contracting parties (as in the usage of best effort clauses) or an unintended consequence of drafting errors. We now introduce a model to study what these principles imply for the drafting of optimal contracts and their co-evolution with the law.

3 Economic Environment

3.1 Agency Problem

We build our analysis on the repetition of a transaction that has three features that are conventional in models of incomplete contracts: non-contractible effort, risk neutrality and limited liability (Bolton and Dewatripont 2004). Time is discrete, with an infinite horizon. At the beginning of each period $t = 0, 1, ...$ a penniless agent and a wealthy principal meet and choose whether to form a partnership that involves the supply of a relationship-specific service, such as the management of a public company, or a distributorship, or a franchise, or any other business service. Parties live for one period. We keep time subscripts implicit until Section 4.

During each period t, production occurs in two stages. First, the agent exerts effort $a \in [0, 1]$ at a non-pecuniary cost $C(a)$. Second, with probability a the service is realized

[10] Once again, however, ambiguity was not entirely eliminated. Subsequent courts have found no implied best efforts provision in contracts with a substantial advance payment. It remains ambiguous whether an implied duty of best efforts exists in contracts with only a small advance or minimum (Coplan 2001; Goldberg 2006).

to be "good," taking value $v > 0$; with probability $1 - a$, the service is "bad," taking value zero. The principal learns the value of the service only after it has been rendered.

We impose the following restrictions on the agent's cost function:

$$C(a) > 0,\ C'(a) > 0,\ C''(a) > 0, \text{ and } C'''(a) \geq 0 \text{ for all } a \in (0,1), \tag{1}$$

with limit conditions $C(0) = 0$, $\lim_{a \to 0} C'(a) = 0$, and $\lim_{a \to 1} C'(a) > v$.

If in period t the partnership is formed, the agent's first-best effort level is:

$$a_{FB} = \arg\max_a \{av - C(a)\} = C'^{-1}(v) \in (0,1), \tag{2}$$

which corresponds to joint surplus:

$$\Pi_{FB} = \max_a \{av - C(a)\} = vC'^{-1}(v) - C\left(C'^{-1}(v)\right) > 0. \tag{3}$$

If the partnership is not formed, the agent obtains 0, while the principal obtains utility $\underline{u}_P \geq 0$. We interpret $\underline{u}_P$ as the surplus obtained by the principal if she acquires a general-purpose service in the market. In the case of managing a corporation, $\underline{u}_P$ can be viewed as the payoff from short-term management, devoid of any firm-specific investment. Forming the partnership is first-best efficient if and only if $\Pi_{FB} \geq \underline{u}_P$.

3.2 The Complete Contract

At the start of period t, the agent and the principal meet. If they form a partnership, the agent makes a take-it-or-leave-it contract offer to the principal (so the agent has full bargaining power). Next, the agent exerts effort, which determines the likelihood of providing a valuable service. The service is produced, the principal consumes it, and the contract is enforced.

Under full observability, the first best would be implemented by requiring the principal to pay the agent a price $p = a_{FB}v - \underline{u}_P$ if he exerted effort a_{FB}, and zero otherwise. Unfortunately, effort is unobservable (and non-contractible), so this solution does not work.

If performance is observable and perfectly verifiable after consumption, the parties can specify a quality-contingent price p_q for $q \in \{0, v\}$. In the optimal contract, the principal's participation constraint is binding. Otherwise, the agent could raise p_q for all q and still ensure participation without affecting effort provision. As a result, the agent chooses $p_v \geq 0$ and $p_0 \geq 0$ to maximize joint surplus $av - C(a) - \underline{u}_P$ subject to the principal's binding participation constraint $a(v - p_v) - (1 - a)p_0 = \underline{u}_P$, and to the agent's incentive-compatibility constraint $p_v - p_0 = C'(a)$. The problem can be rewritten as:

$$\max_{a\in[0,1]} \{av - C(a) - \underline{u}_P\} \tag{4}$$

subject to

$$av - \underline{u}_P \geq \min_{p_v,p_0\geq 0} \{ap_v + (1-a)p_0\} \text{ s.t. } p_v - p_0 = C'(a). \tag{5}$$

In (5), the optimal price p_q minimizes the cost of inducing any effort a. This minimum cost defines the set of effort levels that can be implemented given the principal's participation constraint. The agent chooses the surplus-maximizing effort a from this set.

Proposition 1 *When performance q is contractible, the optimal contract sets a positive price only when the service is good ($p_0 = 0$ and $p_v = C'(a_{SB}) > 0$).*

The first best is attained if and only if the principal's outside option is nil ($\underline{u}_P = 0$). The partnership is formed if and only if the principal's outside option is sufficiently low:

$$\underline{u}_P \leq \underline{U}_P = \max_{a\in[0,1]} \{a[v - C'(a)]\}. \tag{6}$$

When the partnership is formed, second-best effort and joint surplus decrease with the principal's outside option and increase with the value of a high-quality service ($\partial a_{SB}/\partial \underline{u}_P < 0 < \partial a_{SB}/\partial v$ and $\partial \Pi_{SB}/\partial \underline{u}_P < 0 < \partial \Pi_{SB}/\partial v$ for all $\underline{u}_P \in (0, \underline{U}_P)$).

The optimal contract specifies zero payment to the agent for a bad service ($p_0 = 0$). With this provision, wasteful payments are minimized, thereby reducing the cost of incentives. A similar property is at work when parties contract on vague terms.

When the principal's outside option is zero, the first best is attained by setting $p_v = v$, which makes the agent the full residual claimant. When the principal's outside option is positive, however, the payment must be reduced to $p_v < v$. As a result, the first best cannot be achieved because of the agent's wealth constraint. Ideally, the agent would like to pay $\underline{u}_P$ to the principal and "purchase the firm" from her, which would elicit first-best effort. This arrangement is infeasible because the agent is penniless. Hence, when $\underline{u}_P > 0$ second-best effort and joint surplus are below the first best and decrease with the principal's outside option. We assume that Condition (6) always holds, so the partnership is feasible when quality is fully contractible.

4 Evidence and Litigation

We now introduce factual ambiguity and litigation. To do so, we lay out a legal environment that, in line with previous evidence, consists of: i) a large set of conflicting performance proxies that courts can use in adjudication, and ii) a litigation process that must respect precedent but may also be distorted by opportunistic litigants and biased judges. We refer to the partnership occurring at time t as "partnership t."

4.1 Performance Proxies

There is a continuum I of performance proxies that can be used in adjudication, each of which is uniquely identified by an index $i \in [0,1]$. The binary value $e_t(i) \in \{-1,1\}$ of proxy i offers a noisy signal of quality: $e_t(i) = -1$ is a perfect signal of low quality, while $e_t(i) = 1$ is an imperfect signal of high quality.

Formally, if quality at t is high ($q_t = v$) all proxies are positive ($e_t(i) = 1$ for all i). If instead quality is low ($q_t = 0$) each proxy i takes value

$$e_t(i) = \begin{cases} 1 & \text{for} \quad i < \xi_t \\ -1 & \text{for} \quad i \geq \xi_t \end{cases}, \tag{7}$$

where ξ_t is an i.i.d. random variable that captures the difficulty of measuring performance. It has a cumulative distribution function $F_\xi(x)$ and continuous density $f_\xi(x) > 0$ on $[0,1]$.

For any distribution of ξ_t, proxies that carry a higher index i are more informative of performance. In the limit, $e_t(1)$ almost surely takes values 1 if quality is high and -1 if quality is low. A proxy $e_t(i)$ is a sufficient statistic for q_t given a lower-indexed proxy $e_t(j)$ for all $j \leq i$. The index i thus measures the informativeness of a performance proxy.

When the density $f_\xi(x)$ of legal uncertainty is concentrated around $\xi_t = 0$ all signals are very informative: they take value 1 if and only if quality is high. When instead $f_\xi(x)$ is concentrated around $\xi_t = 1$ all signals are uninformative: they always take value 1. If a random signal is drawn from the unit interval ($i \sim U[0,1]$), the probability that it detects low quality is

$$\Pr\{i \geq \xi_t\} = \int_0^1 F_\xi(i)\, di. \tag{8}$$

The probability of diagnosing low quality declines if the distribution of ξ_t shifts up in the sense of first-order stochastic dominance. The distribution of ξ_t thus captures the complexity of the transaction: in more sophisticated or innovative sectors it is harder to find proxies that are clearly informative of performance.

4.2 Litigation

At time t the set of performance proxies I is partitioned into a countable subset $J_t \subset I$ of judicial precedents and an uncountable subset $I \setminus J_t$ of novel proxies. Precedents J_t consist of all proxies $i \in J_t$ that have been used in past cases (before t) and cited in the judicial opinions justifying their outcome. As in Gennaioli and Shleifer (2007), precedents specify the facts or dimensions material to a transaction. Judges and lawyers have been trained to recognize and use them. As a result, the signal realizations $\{e_t(i)\}_{i \in J_t}$ are always used in court and parties know their informativeness, which coincides with the index of the most informative precedent available at t. Formally, the realization $e_t(i_t^J)$ for $i_t^J \equiv \max i \in J_t$ is a sufficient statistic for the entire vector $\{e_t(i)\}_{i \in J_t}$. In this sense, i_t^J summarizes the state of precedent at time t.[11]

Indices in $I \setminus J_t$ identify performance proxies that courts have not yet used in their decisions. Because of the factual uncertainties surrounding the transaction, such novel evidence is hard to collect and assess. During litigation, each party searches in an imperfect and undirected way for one random piece of novel evidence. With probability $\iota \in (0,1)$ search is successful: the litigant $L \in \{P, A\}$ randomly draws one piece of evidence in $I \setminus J_t$. The informativeness of the evidence collected by L, which we denote by $i_t^L \sim U[0,1]$, is not immediately observed in court: only the realization $e_t(i_t^L)$ of the signal is.[12] With probability $(1-\iota)$ search is unsuccessful and the litigant comes up empty-handed.

For both litigants and judges, novel evidence is "hard." Litigants can hide it but cannot falsify it. If a litigant chooses to present novel evidence in court, the judge can likewise choose to hide it (e.g., by declaring it inadmissible or immaterial). Due to the complexity of facts, the verification of novel evidence can be distorted ex post by opportunistic parties, and judges have discretion when verifying it.

Judicial verification is thus characterized as follows. Judges report the performance proxies $\{e_t(i)\}_{i \in J_t}$ embodied in precedents, but they discretionally choose whether to report also the pieces of novel evidence presented by the self-interested litigants.

[11] Litigants and judges may know exactly which precedent has informativeness i_t^J and suffices to asses the information conveyed by all precedents. Conversely, they may ignore completely the informativeness of individual elements of J_t, but know only the aggregate informativeness of the whole set. Our results are unchanged because the entire vector $\{e_t(i)\}_{i \in J_t}$ is observed in court.

[12] A successful search returns a piece of evidence with unknown informativeness $i_t^L \sim U[0,1]$ because the set of novel evidence $I \setminus J_t$ has full measure, given that there is only a countable number of precedents in J_t. The assumption that parties recognize the informativeness of the set of precedents but not of novel evidence is not crucial for our findings, but it simplifies the analysis. It is also realistic. Since novel evidence has never been used before, it is hard for parties to assess its precise informativeness ex post, once performance is realized and so is noise ξ_t, which is unobservable. By contrast, the informativeness of precedents can be inferred before contracting by observing the realization of the corresponding signals in many partnerships and by talking to industry peers.

Judges exercise their discretion on the basis of their preferences. Given that litigation is purely distributional (it occurs after performance has taken place), it is natural for a judge's preferences across litigants to matter. There are three types of judges. A fraction $\pi \in [0,1)$ have a pro-principal bias, and wish to minimize payment to the agent. A fraction $\alpha \in [0,1)$ have a pro-agent bias, and wish to maximize payment to the agent. The remaining $\omega = 1-\pi-\alpha$ judges are unbiased: they wish to enforce the contract faithfully. This formulation nests both the case in which pro-agent and pro-principal judges balance out ($\pi = \alpha$), for instance because they stem from personal idiosyncrasies, and the case in which one type of bias is more prevalent, for instance because the transaction is international and judges favor the local party.

5 The Optimal Incomplete Contract

Contract incompleteness stems from the parties' inability to specify in their contract which specific performance proxies judges should report. This inability is due to the prohibitive cost of identifying unambiguously performance proxies out of the dense set I. In line with real world practice, then, judges have some discretion in adjudication. Here we take such discretion to the extreme, assuming that no performance proxy is contractible, but we could relax this assumption without altering our main results.

Contracts specify a payment schedule $p(...) \geq 0$ from the principal to the agent contingent on the performance proxies verified by the judge. The contract can restrict payments to depend only on evidence based on precedents, or it may allow $p(...)$ to depend also on any novel performance proxy verified by the judge. Solving for the optimal contract is thus equivalent to asking: Do parties wish courts to report any novel evidence—understanding that its quality is uncertain and judges may be biased—or not?

In this section, we answer this question by deriving the optimal incomplete contract. This contract represents formally the real-world situation in which parties contract directly only on the ultimate outcome of judicial verification, but courts interpret ambiguous terms the way parties would have wanted. This is precisely the approach to contract interpretation that jurisprudence teaches and legal practice follows: "American courts universally say that the primary goal of contract interpretation is to ascertain the parties' intention at the time they made their contract" (Burton 2008, p. 1; cf. also Posner 2005). As we're about to see, the optimal contract induces even biased judges to abide by the parties' preferred strategy because it satisfies their incentive constraints.[13]

[13] An alternative interpretation is that parties can contract explicitly on evidence based on precedents.

5.1 The Contracting Problem

Parties contract at the beginning of the partnership, before any information on the value of the service or the preferences of the judge is revealed. By the revelation principle, any contract $p(...) \geq 0$ can be represented by a direct revelation mechanism.

In our setting, direct revelation follows a two-stage process. First, litigants report their private information to the judge. Second, the judge chooses which evidence to verify, and his verification is the sole determinant of contract enforcement. The payment schedule $p(...) \geq 0$ must fulfill incentive constraints that ensure truthful revelation by both litigants and the judge. Payment cannot be made directly contingent on the information the parties have presented in court, bypassing the judge's decision to verify it. This assumption captures a key feature of trial courts: judicial fact discretion (Frank 1949; Gennaioli and Shleifer 2008). In finding the facts of a case, trial courts have substantial leeway to emphasize or disregard evidence presented by litigants.

We also rule out the possibility for the contract to specify punishments for the parties as a whole, such as non-pecuniary criminal penalties or incentive payments to judges. These punishments are illegal in the real world and would not be robust to renegotiation.[14]

Figure 1 summarizes the sequence of events within a generic period t and illustrates the information possessed by different agents.

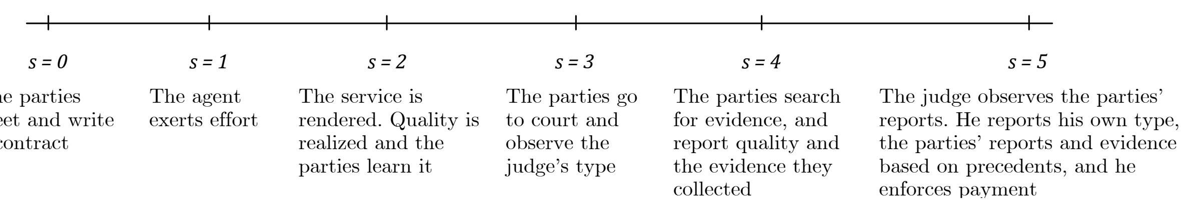

Figure 1: Timing

The contractual payment can be contingent on the realization of performance proxies based on precedents, $\{e_t(i)\}_{i \in J_t}$. A sufficient statistic for this entire vector is the signal $e_t(i_t^J)$ based on the most informative precedent-based proxy. Thus, for simplicity, we can restrict $p(...)$ to depend only on its realization, which we denot by e_J. This piece of information is perfectly verifiable. As a result, it is not subject to any truthful revelation constraints.

The non-vague contract, then, specifies that only this evidence should be used and how to enforce payment. The two interpretations are not mutually exclusive, and they are observationally equivalent because in equilibrium parties choose not to write contracts without vague terms.

[14]Common law prevents parties from stipulating by contract any kind of penalty, even pecuniary penal damages. "A term fixing unreasonably large liquidated damages is unenforceable on grounds of public policy as a penalty" under U.S. law (12 A.L.R. 4th 891, 899).

In addition, the mechanism needs to induce each litigant $L \in \{P, A\}$ to reveal truthfully two pieces of private information in the first enforcement stage ($s = 4$ in Figure 1). First, the parties must report quality $q_t \in \{0, v\}$, which they observe but the judge does not. We denote by q the realization of quality. Second, litigants must report the novel evidence they collected. Each litigant L privately observes a new performance proxy $e_t\left(i_t^L\right) \in \{-1, 1\} \cup \{0\}$, where i_t^L is its unknown informativeness, and $e_t\left(i_t^L\right) = 0$ denotes an unsuccessful search. We denote by e_L the realization of evidence collection by litigant L.

Finally, in the second enforcement stage ($s = 5$ in Figure 1), the mechanism needs to induce the judge to reveal the litigants' reports of quality and novel evidence as well as his own type $b_t \in \{b_P, u, b_A\}$, where b_L denotes a bias in favor of litigant L while u denotes an unbiased judge. The judge's type, which we denote by b, is observed during litigation by the litigants as well as the judge.

Because parties do not communicate directly with the mechanism designer, the payment schedule depends directly on precedent and on the judge's report; it reflects the litigants' reports to the judge only indirectly. Formally, the payment is a function $p\left(e_J, e'_P, e'_A; q'_P, q'_A; b'\right)$, where e'_P and e'_A denote the judge's report of the novel evidence presented by the parties, q'_P and q'_A stand for his report of the litigants' quality announcements, and b' is the judge's report of his own type.

The optimal payment schedule represents parties' preferences over the enforcement of incomplete contracts. If the specified payment depends at all on reports $(e'_P, e'_A; q'_P, q'_A; b')$, this means that parties wish to introduce vagueness in their contract and let courts fill in the gaps ex post. If payment depends (at most) on precedent-based proxies e_J, the parties want their contract to be perfectly unambiguous ex ante.

The optimal direct revelation mechanism is the contract that maximizes the agent's expected payoff:

$$\max_p \left\{a\mathbb{E}\left(p|q_t = v\right) + (1-a)\,\mathbb{E}\left(p|q_t = 0\right) - C\left(a\right)\right\}, \tag{9}$$

subject to the principal's participation constraint,

$$a\left[v - \mathbb{E}\left(p|q_t = v\right)\right] - (1-a)\,\mathbb{E}\left(p|q_t = 0\right) \geq \underline{u}_P, \tag{10}$$

the agent's incentive compatibility constraint,

$$C'(a) = \mathbb{E}\left(p|q_t = v\right) - \mathbb{E}\left(p|q_t = 0\right), \tag{11}$$

and the agent's wealth constraint,

$$p \geq 0, \tag{12}$$

for every possible realization of $(e_J, e'_P, e'_A; q'_B, q'_S; b')$.[15]

The mechanism must also satisfy truth-telling constraints that were absent from Proposition 1. First, parties cannot commit ex ante to report truthfully in court, but must be induced to do so ex post. Denote by $\Omega_t^P \equiv \{q_t = q, e_t\left(i_t^J\right) = e_J, e_t\left(i_t^P\right) = e_P, b_t = b\}$ the principal's information set when she reports $(\tilde{q}_P, \tilde{e}_P)$ to the judge, and by $\Omega_t^A \equiv \left\{q_t = q, e_t\left(i_t^J\right) = e_J, e_t\left(i_t^A\right) = e_A, b_t = b\right\}$ the corresponding set for the agent, who reports $(\tilde{q}_A, \tilde{e}_A)$. The principal's truth-telling constraints are

$$\mathbb{E}\left[p\left(e_J, e_P, e_A; q, q; b\right)|\Omega_t^P\right] \leq \mathbb{E}\left[p\left(e_J, \tilde{e}_P, e_A; \tilde{q}_P, q; b\right)|\Omega_t^P\right] \tag{13}$$

for any feasible report $\tilde{q}_P \in \{0, v\}$ and $\tilde{e}_P \in \{0, e_P\}$. That is, the principal cannot lower her expected payment by misreporting quality (which is cheap talk) nor by hiding her piece of evidence. The expectation here is computed with respect to the agent's search for private evidence, which the principal anticipates rationally when making her report.

Similarly, the agent's truth-telling constraints are

$$\mathbb{E}\left[p\left(e_J, e_P, e_A; q, q; b\right)|\Omega_t^A\right] \geq \mathbb{E}\left[p\left(e_J, e_P, \tilde{e}_A; q, \tilde{q}_A; b\right)|\Omega_t^A\right] \tag{14}$$

for any feasible report $\tilde{q}_A \in \{0, v\}$ and $\tilde{e}_A \in \{0, e_A\}$. The agent cannot raise his expected payment by making an untruthful report conditional on his information Ω_t^A.

Finally, the mechanism must also satisfy the truth-telling constraints of judges, since they have preferences over the outcome of a case and enjoy some degree of fact-finding discretion. In the final stage, the truth-telling constraints for pro-principal and pro-agent judges are respectively equal to

$$p\left(e_J, e_P, e_A; q_P, q_A; b_P\right) \leq p\left(e_J, e'_P, e'_A; q'_P, q'_A; b'\right) \tag{15}$$

and

$$p\left(e_J, e_P, e_A; q_P, q_A; b_A\right) \geq p\left(e_J, e_P, e_A; q'_P, q'_A; b'\right) \tag{16}$$

[15] In the agent's objective (9), as well as in the constraints, the expected payments to the agent $E\left(p|q_t = v\right)$ and $E\left(p|q_t = 0\right)$ are computed across realizations of the litigants' novel evidence collection (e_P, e_A), and the judge's type (b). When quality is high, $e_J = 1$ because no negative signals can be realized. When quality is low, expectations are computed also with respect to the realization of precedent. Formally, given truthful reporting the expectation of payment when quality is high can be written out in full as $E\left[p\left(1, e_P, e_A; v, v; b\right)|q_t = v\right]$ while the expectation of payment when quality is low can be written out in full as $E\left[p\left(e_J, e_P, e_A; 0, 0; b\right)|q_t = 0\right]$.

for any feasible ruling $e'_P \in \{0, e_P\}$, $e'_A \in \{0, e_A\}$, $q'_P, q'_A \in \{0, v\}$ and $b' \in \{b_P, u, b_A\}$. These constraints do not involve expectations because the judge moves last, after litigants reveal to him all the information he had not directly observed. The first constraint (Equation 15) means that pro-principal judges cannot lower payment by untruthfully verifying the litigants' reports nor by disguising their own preferences. The second constraint (Equation 16) means that, analogously, pro-agent judges cannot raise payment through such selective verification. These constraints ensure that judges abide by the verification strategy deemed optimal by the parties.

5.2 Optimal Vagueness

We can again solve the contracting problem in two steps. In the first step, the agent minimizes the cost of implementing effort subject to the incentive compatibility, non-negativity, and truth telling constraints (Equations 11 to 16). In the second step, the agent chooses optimal effort. In the Appendix we show that this is a linear programming problem whose solution minimizes the ratio of expected payments when quality is low relative to when it is high:

$$\frac{\mathbb{E}\left(p|q_t = 0\right)}{\mathbb{E}\left(p|q_t = v\right)}. \tag{17}$$

As in Proposition 1, the optimal contract minimizes wasteful payments when performance is bad and maximizes incentive payments when performance is good. Now, however, performance is not directly contractible. Parties, then, minimize the cost of effort provision by loading payment onto the judicial verification outcomes that are most indicative of high quality.

The optimal mechanism is implemented by a very simple contract.

Proposition 2 *The optimal incomplete contract requires the principal to pay the agent a price $p^* > 0$ if and only if the court verifies evidence of high quality both based on precedent and novel, while it verifies no evidence of low quality.*

The equivalent optimal direct revelation mechanism requires the principal to pay the agent a price $p^* > 0$ if and only if evidence based on precedent is positive, the agent presents novel positive evidence, and moreover either of the following two conditions holds.

1. *The judge is pro-agent ($p\left(1, e_P, 1; q_P, q_A; b_A\right) = p^*$ for all $e_P \in \{-1, 0, 1\}$ and all $q_P, q_A \in \{0, v\}$).*

2. *The judge is unbiased and the principal does not present novel negative evidence ($p(1, 0, 1; q_P, q_A; u) = p\left(1, 1, 1; q_P, q_A; u\right) = p^*$ for all $q_P, q_A \in \{0, v\}$).*

The optimal contract is systematic but optimally vague. It is systematic because it relies on performance proxies employed in similar transactions in the past. It is vague because it relies on judicial verification of novel evidence, despite enforcement frictions. Leaving the door open to vaguely defined new evidence allows biased judges to distort enforcement, but it also increases the information available to judges. The following tradeoff then ensues in our model. On the one hand, the use of novel evidence raises the standard for verifying good performance, reducing the probability of erroneously rewarding bad performance. This is good for incentives. On the other hand, the use of novel evidence also reduces the probability of correctly rewarding good performance, which is bad for incentives. This occurs because evidence of good performance may not be found or it may be strategically withheld by the principal or by judges. Parties alleviate the latter cost by raising the payment p^*. By so doing, they compensate the agent for enforcement risk but still benefit from the fact that vagueness allows the payment to track more closely the agent's performance.

The optimal incomplete contract in Proposition 2 is a close analogue of the full-verifiability contract of Proposition 1. In both cases, payment p^* is enforced only in the contingency that is most diagnostic of good performance. Now, however, payment cannot be perfectly conditional on high quality ($q = v$).[16] Instead, it is enforced when the judge verifies the most diagnostic evidence of high quality: precedent-based as well as novel signals of good performance, and no signals of bad performance.

Such evidence is verified less often than it is uncovered, because litigants and judges sometimes choose to discard it. Pro-principal judges are especially harmful: they always discard new proxies of good performance, causing the payment not to be enforced even when it should. If all judges are pro-principal (which we rule out in the main analysis by assuming $\pi < 1$), novel proxies of good quality are never verified. In this extreme case, parties change the optimal contract as follows.

Corollary 1 *Suppose all judges are pro-principal, $\pi = 1$. Then, the optimal contract requires the principal to pay the agent a price $p^* > 0$ if and only if the court verifies no evidence of low quality, either based on precedent or newly uncovered by the principal.*

To protect the agent against pervasive pro-principal bias, the optimal contract shifts the burden of proof from the agent to the principal. To obtain p^*, the agent no longer

[16] Direct revelation of performance is impossible because the litigants' reports (q_P, q_A) are cheap talk and their interests are perfectly opposed, given that outcomes in which both litigants are punished are impossible.

needs to present novel evidence of good quality. It is enough that the principal does not produce (and the judge verify) evidence of bad quality.

Although the case $\pi = 1$ is extreme, the contract of Corollary 1 is interesting for two reasons. First, it is optimal under less extreme conditions if the maximum payment is bounded ($p^* \leq p^{\max}$). With this constraint, parties cannot always increase the agent's protection by increasing p^*. As a result, when π is large enough, the only way to protect the agent is to place the burden of proof on the principal.

Second, and more generally, Corollary 1 shows that optimal contracts work around enforcement problems and include vague terms, even if judicial biases are extreme. If judges are very biased in favor of the principal, parties condition the payment on the only proxies that pro-principal judges will truthfully verify: negative ones. The burden of proof is allocated so as to maximize the accuracy of partisan information reporting.

Having characterized the vagueness of the optimal contract, we now analyze its features in detail, focusing again on the baseline case of $\pi < 1$. There are two endogenous outcomes of interest. First, the extent to which parties use high-powered incentives. Second, equilibrium effort and joint surplus from the partnership.

Corollary 2 *Incentives are higher-powered when precedents are more informative ($\partial p^*/\partial i_t^J > 0$), evidence is more informative (p^* increases if ξ_t shifts down in the sense of first-order stochastic dominance) and there are fewer pro-agent judges ($\partial p^*/\partial \alpha < 0$).*

These results complete and clarify our discussion above on the relationship between enforcement errors and the optimal level of performance pay p^*. When bad performance is less likely to be rewarded, incentive payments are more efficient and thus optimally higher. This explains why p^* rises with the information content both of precedent and of novel evidence. It also explains why it falls when there are more pro-agent judges. These judges discard the negative signals presented by the principal, a distortion that also wastes incentive payments.[17]

Instead, the effect on p^* of pro-principal bias (π) and evidence collection (ι) is ambiguous. When pro-principal bias rises or evidence collection falls, good performance becomes less likely to be rewarded. Then any given price elicits lower effort, so two countervailing effects ensue. On the one hand, p^* tends to rise to preserve incentives—an income effect. On the other, p^* tends to fall to economize on less efficient incentives—a substitution effect. The net outcome can be either a rise or a fall in p^*.

[17] Incentives are also more valuable and thus higher-powered when good performance is more valuable ($\partial p^*/\partial v > 0$) and when the principal's outside option is lower ($\partial p^*/\partial \underline{u}_P < 0$).

Turning from the structure of the optimal incomplete contract to its consequences, we establish that the efficiency of the legal environment is characterized by a single sufficient statistic.

Proposition 3 *The optimal incomplete contract has the following properties.*

1. *The likelihood ratio of low relative to high quality when payment is enforced,*

$$\Lambda = \frac{\Pr\left(p = p^* | q_t = 0\right)}{\Pr\left(p = p^* | q_t = v\right)}, \tag{18}$$

is a sufficient statistic for the efficiency consequences of all aspects of legal enforcement: the quality of precedents (i_t^J), judicial biases (π and α), evidence collection (ι) and the noisiness of evidence (the distribution of ξ_t).

2. *Equilibrium effort and joint surplus are monotone decreasing in Λ and reach their second-best levels when Λ is nil ($\partial a/\partial\Lambda < 0$, $\partial\Pi/\partial\Lambda < 0$ and $a = a_{SB} \Leftrightarrow \Pi = \Pi_{SB} \Leftrightarrow \Lambda = 0$).*

3. *The partnership is formed if and only if Λ is below a positive threshold $\hat{\Lambda}$ that rises with the value of good performance and falls with the principal's outside option ($\partial\hat{\Lambda}/\partial v > 0 > \partial\hat{\Lambda}/\partial \underline{u}_P$).*

The binary structure of the optimal incomplete contract implies that the minimized ratio of expected payments in Equation (17) equals the ratio of the respective probabilities that payment is enforced. This likelihood ratio provides an inverse measure of verifiability. When it is higher, the information that courts can verify is less diagnostic, in the sense that the court is more likely to enforce payment when quality is low. As a result, the optimal contract is more incomplete.

The extent of verifiability determines partnership formation and efficiency. When verifiability is too low ($\Lambda > \hat{\Lambda}$), the optimal contract cannot provide incentives for the agent while leaving the principal a sufficient payoff. As a result, the partnership cannot be formed.[18] The principal must purchase a service in the market, deriving utility $\underline{u}_P$, and any gains from relationship-specific trade are lost. This outcome is more likely if the outside option is higher and gains from trade lower. When instead verifiability is high enough

[18] The threshold for partnership formation is defined as $\hat{\Lambda}$ such that $\max_{a\in[0,1]}\{av - [\hat{\Lambda}/(1-\hat{\Lambda}) + a]\, C'(a)\} = \underline{u}_P$, reflecting that the minimum cost of inducing any effort a is increasing in Λ.

($\Lambda \leq \hat{\Lambda}$), the partnership is formed and some gains from relationship-specific trade are generated. These gains from trade increase monotonically as Λ falls. Accordingly, equilibrium effort and joint surplus increase ($\partial a/\partial\Lambda < 0$ and $\partial\Pi/\partial\Lambda < 0$).[19]

Partnership formation and efficiency depend on the legal environment entirely through its effect on verifiability, which we proceed to describe.

Proposition 4 *The optimal incomplete contract has the following properties.*

1. *Performance is more verifiable, and thus effort and welfare are higher, when there are fewer biased judges ($\partial\Lambda/\partial\pi \geq 0$ and $\partial\Lambda/\partial\alpha \geq 0$), precedents are more informative ($\partial\Lambda/\partial i_t^J \leq 0$), litigants are better at collecting novel evidence ($\partial\Lambda/\partial\iota \leq 0$) and evidence is less noisy (Λ decreases if ξ_t shifts down in the sense of first-order stochastic dominance). Full verifiability is achieved if and only if precedents are perfectly diagnostic of quality ($\Lambda = 0 \Leftrightarrow i_t^J = 1$).*

2. *When precedents are more informative, judicial biases are less detrimental ($\partial^2\Lambda/(\partial\pi\, \partial i_t^J) \leq 0$ and $\partial^2\Lambda/\left(\partial\alpha\partial i_t^J\right) \leq 0$) and litigants' ability to collect novel evidence is less important ($\partial^2\Lambda/\left(\partial\iota\partial i_t^J\right) \geq 0$).*

The first-order effect of the enforcement parameters is intuitive. Judicial bias reduces verifiability by destroying information embodied in novel evidence. This cost arises whether bias is idiosyncratic or systematic.[20] Precedents increase verifiability by generating more precise information (higher i_t^J) that biased judges cannot discard. Verifiability rises when performance proxies are more informative (ξ_t is systematically lower). Finaly, better collection of novel evidence naturally increases verifiability. Even though each party uses new proxies in a partisan way, evidence collection is beneficial because the contract optimizes the use of new evidence.

The second part of the proposition shows how the quality of precedents mitigates the impact of other enforcement frictions. When precedents are more informative, it is less important for litigants to generate additional information ($\partial^2\Lambda/\left(\partial\iota\partial i_t^J\right) \geq 0$). Moreover, judges are then constrained to use the more informative evidence based on precedents, so judicial biases have a smaller impact on verifiability ($\partial^2\Lambda/\left(\partial\pi\partial i_t^J\right) \leq 0$ and $\partial^2\Lambda/\left(\partial\alpha\partial i_t^J\right) \leq 0$).

[19] Just as in Proposition 1, effort and surplus are also increasing in the value of good performance and decreasing in the principal's outside option ($\partial a/\partial v > 0 > \partial a/\partial \underline{u}_P$).

[20] This does not mean that all biases are equally costly in our model. Pro-agent bias is more costly than pro-principal bias because pro-principal judges introduce white noise in the adjudication process (holding for the principal regardless of the true state), while pro-agent judges introduce a systematic distortion. They undesirably make payment more likely when quality is low, but they cannot also make it more likely when quality is high because they cannot fake novel informative evidence that the agent did not collect.

These results show that more informative precedents are associated with a more level contracting field. When precedents are more informative, novel proxies are less decisive. As a result, verifiability and efficiency depend less on the legal resources at the parties' disposal (ι) and on the biases (π, α) judges have in favor of or against certain parties. The implication is that improvements in the quality of precedents and reductions in legal uncertainty are followed by greater diffusion of contracts to less sophisticated or resourceful parties. As we discuss in the next section, this is consistent with real-world patterns.

6 The Dynamics of Verifiability

The vagueness of the optimal incomplete contract implies that courts are constantly required to verify novel evidence. As new performance proxies are incorporated into precedents, the informativeness of case law grows, improving the ability of parties to contract on performance.

In this section, we explore the dynamics of our model. We begin by studying the simplest setting in which all principal-agent pairs are identical, so legal evolution results only from contracting and litigation by the representative partnership. Then we let partnerships differ in their ability to collect novel evidence (ι). This captures differences in resourcefulness and thus sophistication. Partnerships involving richer or more skilled people are better able to unveil new evidence (i.e., they have higher ι), perhaps because they can hire better lawyers. In this richer setting, we can study the effect of evolving verifiability on more or less sophisticated parties. As in many models of legal evolution, we assume that litigation is costless, so parties always go to court to enforce their contract (e.g., Gennaioli and Shleifer 2007; Anderlini, Felli and Riboni 2014).[21]

To see how precedents are created in our model, note that under the optimal incomplete contract from Proposition 2, a judge deciding a case may write four different decisions.

1. The agent wins because evidence based on precedents is positive $\left(e_t\left(i_t^J\right)=1\right)$ and he presents novel positive evidence $\left(e_t\left(i_t^A\right)=1\right)$, while no negative evidence is verified. This decision establishes a new precedent $\left(J_{t+1}=J_t \cup \left\{i_t^A\right\}\right)$.

2. The principal wins because evidence based on precedents is negative $\left(e_t\left(i_t^J\right)=-1\right)$.

[21] This assumption is made only for simplicity. If litigation were costly, both litigants could prefer to settle out of court. The standard justification for why parties then go to court is that they hold different priors about the probability of winning the trial. We abstract from modeling this feature because none of our results would depend on the specific states leading or not leading to litigation. Reluctance to litigate would simply slow down legal evolution.

This decision is based on existing precedent and thus does not establish a new one ($J_{t+1} = J_t$).

3. The principal wins by presenting negative evidence ($e_t\left(i_t^P\right) = -1$). This decision establishes a new precedent ($J_{t+1} = J_t \cup \left\{i_t^P\right\}$).

4. The principal wins because the agent failed to present positive evidence. This decision is based on absence of evidence and does not establish a new precedent ($J_{t+1} = J_t$).

The stock of precedents is enriched with new evidence when judges write opinion 1 or 3.[22] Yet, changes in the set of precedents do not necessarily improve its informativeness. In case 1, informativeness improves if and only if the agent happens to draw a new piece of evidence that is more informative than precedents ($i_t^A > i_t^J$). In case 3, by contrast, informativeness necessarily improves. Under the structure of evidence we have assumed, a new signal with a negative realization is more informative than all precedents carrying a positive realization ($i_{t+1}^J = i_t^P > i_t^J$).

6.1 The Representative Partnership

As we prove in the Appendix, the probability that precedent improves if partnership t is formed under the optimal contract equals

$$\begin{aligned}&\Pr\left(i_{t+1}^J > i_t^J | i_t^J\right) = a_t \iota\left(1-\pi\right)\left(1-i_t^J\right) + \left(1-a_t\right)\iota \\ &\times \int_{i_t^J}^1 \left[\left(1-\alpha\right)\left(1-x\right) + \left(1-\pi\right)\left(x - i_t^J\right) - \iota\left(1-\pi-\alpha\right)\left(x-i_t^J\right)\left(1-x\right)\right] dF_\xi\left(x\right). \quad (19)\end{aligned}$$

If the partnership is formed, there is a strictly positive probability that any precedent is improved. The first term in Equation (19) captures the case of good performance, which occurs with probability a_t. Here, a new and more informative precedent can be created

[22] In some cases, a ruling in the principal's favor could be justified in several ways. Suppose that both precedent and the principal produced negative evidence ($e_t\left(i_t^J\right) = e_t\left(i_t^P\right) = -1$) while the agent failed to produce positive evidence ($e_t\left(i_t^A\right) \neq 1$): then each of the three decisions in the principal's favor is possible. We assume that judges choose which decision to render on the basis of two principles. First, in accordance with stare decisis, if evidence based on precedents suffices to settle the case, it is summarily decided without considering novel evidence. Second, due to the need to justify their decision, judges always prefer citing novel evidence than grounding their ruling on the insufficiency of available evidence. As a consequence, judges consider the four decisions in the order given above. They proceed down the line only if they cannot (or neither want nor have to) stop at a lower-numbered decision. This assumption does not qualitatively affect our results, but merely influences the speed of legal evolution. Precedents would evolve more rapidly if judges preferred decision 3 to decision 2, or more slowly if they preferred decision 4 to decision 3.

only through a ruling of type 1 in which the novel proxy i_t^A is more informative than the stock of precedents.[23] The second term captures the case of bad performance, which occurs with probability $1-a_t$. Here, precedent can improve through judicial opinions of both type 1 and type 3.[24]

Legal evolution can start from scratch only if it is profitable to form a partnership when there is no prior history of contract enforcement. This means that verifiability is high enough when there is no precedent: $\Lambda\left(i_0^J,\pi,\alpha,\iota\right)\leq\hat{\Lambda}\left(\underline{u}_P,v\right)$ for $i_0^J=0$. As Proposition 4 showed, this condition is met if and only if parties are sufficiently capable of generating novel evidence: $\iota\geq\underline{\iota}_0$.[25] In this case, the joint evolution of precedents and contracts admits the following characterization.

Proposition 5 *When $\iota\geq\underline{\iota}_0$, the evolution of precedent is described by a time-homogeneous Markov chain. Given any body of precedents i_t^J, any weakly higher informativeness $j\geq i_t^J$ is accessible, but any strictly lower informativeness $j<i_t^J$ is inaccessible. The Markov chain is absorbing: its unique absorbing state is perfectly informative precedent $j=1$, while all imperfectly informative states $j\in[0,1)$ are transient.*

If judicial biases are symmetric ($\pi=\alpha=(1-\omega)/2$) then precedent is more likely to improve when judicial bias is less prevalent ($\partial\Pr\left(i_{t+1}^J>i_t^J|i_t^J\right)/\partial\omega>0$) and parties are more capable of collecting novel evidence ($\partial\Pr\left(i_{t+1}^J>i_t^J|i_t^J\right)/\partial\iota>0$). When precedent is sufficiently developed, it is less likely to improve the better it is already ($\lim_{i_t^J\to1}\partial\Pr(i_{t+1}^J>i_t^J|i_t^J)/\partial i_t^J<0$).

The vagueness of the optimal incomplete contract induces a monotonic evolution of contract law towards greater verifiability. The quality of precedents is described by a monotone increasing and ratcheting process. If informativeness i_t^J has been attained at time t, then less informative states $j<i_t^J$ are unattainable in the future. Conversely, any higher level of informativeness can be reached from the initial state i_t^J. In fact it can be reached directly through a single ruling. The stationary distribution of the Markov chain is

[23] This term equals the probability that quality is high (a_t), the agent collects novel positive evidence (ι), the judge is willing to verify it because he doesn't have a pro-principal bias ($1-\pi$), and the new proxy is more informative than all those based on precedents ($1-i_t^J$).

[24] This term equals the probability that quality is low ($1-a_t$), precedent is nonetheless positive ($\xi_t>i_t^J$), and either (i) the principal collects (ι) and the judge is willing to verify ($1-\alpha$) a negative signal ($1-\xi_t$), which is certainly more informative than precedent; or (ii) the agent collects (ι) and the judge is willing to verify ($1-\pi$) a signal that is positive and yet more informative than evidence based on precedents ($\xi_t-i_t^J$). An unbiased judge bases his decision on such a positive signal only if the principal did not present negative evidence at the same time, as reflected in the last component.

[25] Meeting the condition is easier if biased judges are rarer ($\partial\underline{\iota}_0/\partial\pi\geq0$ and $\partial\underline{\iota}_0/\partial\alpha\geq0$), and it requires a sufficiently low outside option and sufficiently high value of high quality ($\partial\underline{\iota}_0/\partial\underline{u}_P\geq0\geq\partial\underline{\iota}_0/\partial v$).

fully concentrated on a unique absorbing state: perfect verifiability of performance, which attains the second-best outcome of Proposition 1 ($i_t^J = 1 \Leftrightarrow \Lambda = 0 \Leftrightarrow \Pi = \Pi_{SB}$).

The evolution of verifiability is monotonic because informative old precedents are not forgotten, and new precedents cannot reduce informativeness. The latter property follows from the assumption of hard evidence. Biased judges can decide cases incorrectly by discarding novel evidence, but not by mischaracterizing its sign. As a result, judicial mistakes cannot establish a precedent that interpets a true proxy of good performance as a legal proxy of bad performance, or vice-versa.[26]

The second part of Proposition 5 highlights the effect of enforcement frictions. Judges' biases and parties' inability to collect new evidence slow down legal evolution because they both reduce the probability that new evidence is verified in court. This slowdown is reflected directly in the partial derivatives of Equation (19) for a given level of effort (a_t). In addition, these enforcement frictions reduce equilibrium effort. With symmetric judicial biases ($\pi = \alpha$), reduced effort slows down legal evolution even further. The reason is that performance is less likely to be good when effort is lower, and thus novel evidence is less often used to enforce the contract.[27]

The effect of enforcement frictions on effort explains why the informativeness of precedents (i_t^J) and the complexity of the transaction (the distribution of ξ_t) have ambiguous effects. On the one hand, better precedents and simpler transactions reduce the need for new evidence. This reduction exerts a direct negative influence on legal change. On the other hand, better precedents also increase equilibrium effort, and thus the probability of good performance. Good performance raises the likelihood that judicial decisions hinge on and cite new evidence. Thus, increased effort exerts an indirect positive effect on legal change. The direct effect is eventually sure to prevail, so legal evolution slows down as it approaches the absorbing state ($i_t^J \to 1$).

One important message here is that different sources of legal uncertainty exert different effects in commercial disputes. When contract enforcement is unpredictable because a transaction is novel or complex, case law reliably fills in the gaps and attains greater legal certainty and economic efficiency, in the spirit of Posner ([1973] 2014). This beneficial

[26] In any case, such bad precedents could be neutered by subsequent contracting. Parties could specify in future contracts that the mistaken proxy is not to be used, or even that it has to be given the opposite interpretation. This is an important difference between the role of precedents in contract law and in tort law. In the former, there is scope for parties to remedy by contract the deficiencies of bad judicial decisions.

[27] When quality is good, the court is more likely to verify new positive evidence, but pro-principal bias reduces this effect. When quality is bad, the court is more likely to verify new negative evidence, but pro-agent bias reduces this effect. Finally, when quality is bad the judge may render a decision of type 2 that uses no new evidence. When judicial biases are symmetric ($\pi = \alpha$), the first two effects balance out, and the last one makes courts more likely to verify new evidence when performance is good.

evolution of precedent cannot instead be counted upon when litigation is distorted by judicial biases. Then material evidence is suppressed, so imperfect rules cannot be quickly weeded out by litigation, as in Priest (1977) and Rubin (1977).

6.2 Heterogeneity in the Collection of Novel Evidence

We now assume that each partnership t has an independently drawn realization ι_t of the probability of sampling novel evidence, whose cumulative distribution function $F_\iota(.)$ has full support on $[0, 1]$. Less sophisticated partnerships are those characterized by lower ι_t. Parties observe the realized probability ι_t right before deciding whether to contract or not. We then characterize optimal contracting and legal evolution as follows.

Proposition 6 *Partnership t is formed if and only if the parties' legal ability is high enough ($\iota_t \geq \underline{\iota}_t$). More partnerships are formed when there are fewer biased judges ($\partial \underline{\iota}_t / \partial \pi > 0$ and $\partial \underline{\iota}_t / \partial \alpha > 0$) and evidence is less noisy ($\underline{\iota}_t$ increases if ξ_t shifts up in the sense of first-order stochastic dominance).*

Some partnerships are formed irrespective of the informativeness of precedents provided evidence is not too noisy (if $\mathbb{E}\xi_t \leq \hat{\Lambda}$ then $\underline{\iota}_t \leq 1$ for all $i_t^J \geq 0$). Then legal evolution is described by a Markov chain with the same properties described in Proposition 5. As legal evolution proceeds, more partnerships are formed ($\partial \underline{\iota}_t / \partial i_t^J < 0$). When precedent is sufficiently informative, all partnerships are formed (there is a threshold $\underline{i}^0 < 1$ such that $\underline{\iota}_t = 0$ for all $i_t^J \geq \underline{i}^0$).

Initially, only parties with a sufficiently high ability to search for novel evidence ι_t choose to contract. The litigation of their optimal incomplete contract allows the law to evolve. As new performance proxies are added to the set of precedents, verification of performance becomes more precise and less subject to judicial biases.

This process improves welfare on both the intensive and the extensive margin. On the intensive margin, better precedents benefit parties that were contracting in the first place. On the extensive margin, better precedents allow less sophisticated parties to start contracting with each other. These parties refrained from trading in the presence of large legal uncertainty but they start to do so as enforcement becomes sufficiently reliable. This expansion in the volume of trade speeds up legal evolution even further.[28]

[28] Once again, in the limit performance becomes fully verifiable ($i_t^J = 1$ remains the unique absorbing state) and joint surplus reaches the second best. Even before legal evolution reaches the absorbing state, contracting becomes universal. A sufficiently high but still imperfect quality of precedent ($\underline{i}^0 < 1$) suffices to allow partnership formation even if the parties are completely unable to collect novel evidence ($\iota_t = 0$).

Contracting by sophisticated parties therefore exerts a positive externality on less sophisticated market participants. Writing and litigating systematically vague contracts is a public good. It fosters future improvements in verifiability that benefit society as a whole.

Our model thus predicts that flexible contract terms undergo a diffusion process. They are initially used by more sophisticated parties and then, if they manage to be incorporated into informative precedents, they gradually diffuse to less and less sophisticated parties. This result is consistent with the two-stage pattern of contract evolution documented by Choi, Gulati, and Posner (2013) in the sovereign bond market. In the first stage, major investment banks introduce new bond clauses to address litigation problems during sovereign default. In the second stage, successful contracts spread to other investors.[29] In the remainder of this section, we discuss two cases in which the legal system has played an important role in reducing the ambiguity of private contracts, favoring their diffusion: business-format franchising and equity financing.

6.3 Two Examples: Franchising and Equity Financing

Franchising provides an apt example of the beneficial feedback loop between the use of vague terms, legal evolution, and future contracting practices. Franchising is a law-intensive contract, because it relies on vague terms and judicial interpretation. Lafontaine, Perrigot and Wilson (2017) document that the same international hotel chain is less likely to use franchising contracts in countries with worse legal institutions. Instead, where contract enforcement is less reliable, the company is more likely to manage directly a property owned by local investors.

The most important development in the area of distribution contracts since World War II has been the rise of business-format franchising (Lafontaine and Blair 2008). Traditionally, franchising had been merely an arrangement for manufacturers to distribute their products through exclusive dealerships. Traditional product franchising nowadays consists mainly of car dealerships, gas stations and soft-drink bottlers, and it has been a stable or declining share of the U.S. economy for decades. Conversely, under the newer concept of business-format franchising, the franchisor provides the entire business model, but the goods and services sold are mostly produced by the franchisee. This type of franchising started booming with fast-food and hotel chains in the 1950s.

Crucially, the development of business-format franchising owes much to legal evolution. It is inextricably linked to McDonald's. Killion (2013, p. 11) notes that "business format

[29] Choi, Gulati, and Posner's (2013) analysis does not consider the role of the legal system in clarifying the interpretation of private contracts.

franchising as we know it today began on April 15, 1955. On that day, Ray Krock opened his first McDonald's restaurant in Des Plaines, Illinois." The company played a pioneering role not only in the market but also in the courts. The decision of its first major legal case, *McDonald's System, Inc. v. Sandy's Inc.*,[30] extended trade-secret protection to the novel setting of franchising. Most important, McDonald's subsequent successes in litigation against its franchisees made business-format franchising viable because courts recognized the franchisor's right to enforce its quality standards by disciplining franchisees or terminating their franchise. "Indeed, the McDonald's franchising cases established important legal precedents. ... The courts essentially used them to define franchising practices that were equitable, and taken together they created a body of case law that gave franchising a degree of legitimacy it had never had before" (Love 1995, p. 404).

The company's most significant victory was *Dayan v. McDonald's Corp.*[31] The ruling upheld franchisors' right to terminate a franchisee for failure to comply with contractual quality standards. It also clarified which evidence would be considered relevant in assessing such failure in court. "Combined with a number of other decisions regarding termination, non-renewal, and its policy of controlling the real estate of its stores, McDonald's set numerous precedents and convinced the courts that for franchising to function properly, franchisors must be allowed to control their franchisee's behavior and impose sanctions on those that do not abide by the rules set forth in the contract" (Blair and Lafontaine 2005, p. 127).

Another set of legal precedents that shaped the development of franchising lies at the intersection of contract law and antitrust law. Courts have gradually clarified to what extent the franchisor can dictate franchisee behavior without falling foul of antitrust restrictions, and companies have adjusted their franchise contracts in response. The landmark decision in *Siegel v. Chicken Delight, Inc.*[32] dramatically curtailed the use of input-purchase requirements. Such requirements are universal in traditional franchising: e.g., gas stations are uncontroversially required to buy fuel from their franchisor at a price that earns him a profit. Conversely, a tying requirement that earns a business-format franchisor a profit through captive sales to a franchisee is considered an illegal restraint of trade.

Reacting to this judicial interpretation, franchise contracts have come to rely instead on explicit fees and royalties to generate franchisor profits, and on lists of approved suppliers to ensure the franchisee's quality standards. However, explicit tying arrangements to guarantee quality can be legal, and once again McDonald's is responsible for the seminal

[30] 45 Ill. App. 2d 57 (1963).
[31] 125 Ill. App.3d 972 (1984).
[32] 448 F. 2d 43 (9th Cir. 1971).

precedent. The decision in *Principe v. McDonald's Corp.*[33] established that the franchisor can require franchisees to lease (at competitive rates) its own real estate, which is an integral component of the franchise business plan.

Building upon McDonald's success and the legal precedents it established, business-format franchising has spread widely to other companies and industries in the U.S. Its sales grew twice as fast as GDP from 1972 to 1986, the period for which Federal statistics are available (U.S. Department of Commerce 1988). In keeping with our model, the litigation episodes above—and others—seem to have helped jump start legal evolution. This process appears to have narrowed down legal uncertainty, enabling the use of business-format franchising by the broader business community. In line with this interpretation, business-format franchising has taken some time to spread from the United States to other legal systems, and its internationalization built upon the experience of American case law. The seminal *Dayan* decision decided McDonald's dispute in Illinois court against its franchisee in Paris, France.

Another illustration of the role of ambiguous contract terms in legal evolution comes from the development of corporate law and particularly of managers' fiduciary duties (Easterbrook and Fischel 1989; Jacobs 2015). Fiduciary duties are an implied term in corporate contracts that enables courts to protect shareholders against abuse by corporate executives. In principle, corporate charters could protect shareholders by specifying in detail which actions executives can or cannot take. This solution is obviously impractical and costly.

Fiduciary duties are a solution to this problem: they are broad principles that allow judges to scrutinize whether a given behavior is consistent with what parties would have agreed upon ex ante if contracting costs were zero (Easterbrook and Fischel 1989). The fiduciary principle is therefore akin to a vague contract term in our model. It is a residual concept that "can include situations that no one has foreseen or categorized ... and in fact has led to a continuous evolution in corporate law" (Clark 1986, p. 141).

One critical legal development in the way fiduciary duties are enforcd concerns the so called "business judgment rule". This doctrine has different formulations, but essentially it gives corporate directors the benefit of the doubt when reviewing transactions in which conflicts of interest are absent. The business judgment rule avoids excessive judicial intervention in corporate life, effectively balancing powers in the relationship between executives and sareholders.

As discussed in Arsht (1979), the creation and refinement of the business judgment rule

[33] 631 F.2d 303 (4th Cir. 1980)

has been the product of a series of landmark court decisions. The first landmark case in which the rule was outlined is the old and famous case of *Percy v. Millaudon*.[34] A few decades later it was expressed in similar form in *Godbold v. Branch Bank*.[35] In this case, a bank's board had appointed a fellow director as an agent of the bank, entitling him to receive an unlawful extra payment for $500 per year. The Alabama Supreme Court absolved the director from liability on the grounds that parties were unaware of the law. In its decision it held that:

> "The undertaking implies a competent knowledge of the duties of the agency assumed by them, as well as a pledge that they will diligently supervise, watch over, and protect the interests of the institution committed to their care. They do not in our judgment undertake that they possess such a perfect knowledge of the matters and subjects which may come under their cognizance, that they cannot err, or be mistaken, either in the wisdom or legality of the means employed by them."

Since these initial rulings, the interpretation of the business judgment rule has evolved through a series of decisions, sometimes broadening, other times restricting the power of courts.[36] This gradual development has improved predictability in the judicial interpretation of fiduciary duties and thus in the way courts enforce shareholder rights (Easterbrook and Fischel 1989). The adherence of the Delaware Court to the business judgment rule is often held as one key factor responsible for Delaware's role in corporate governance.

A large body of evidence documents that common law and its legal rules positively predict the development of financial markets, as well as the use of flexible and innovative contracts (Johnson et al. 2000; Lerner and Schoar 2005; Qian and Strahan 2007; La Porta, Lopez-de-Silanes and Shleifer 2008). Our model rationalizes the view that the legal system itself played an important role in this process. The significantly precedent-based corporate law of common-law countries like England and the United States ensured legal protection of outside investors (La Porta et al. 1998), enabling companies to be widely held by ordinary investors rather than majority owned by the most sophisticated or most connected market participants (La Porta, Lopez-de-Silanes and Shleifer 1999).

[34] 8 Mart. (n.s.) 68 (La. 1829).

[35] 11 Ala. 191 (1874).

[36] Grinstein and Rossi (2016) evaluate empirically the business judgment rule. They conduct an event study around a 1985 decision of the Delaware Supreme Court that held directors personally liable for breaching their fiduciary duties, signaling a weakening of the business judgment rule. The event study shows that high-growth Delaware firms under-performed matched non-Delaware firms by 1% in the three day event window. This suggests that weakening of the business judgment rule was perceived to be costly for these firms.

7 Conclusion

In standard theory, contracts specify parties' obligations as a function of fully verifiable states, so courts cannot interfere with the contract. In this paper, by contrast, we have presented a model in which optimal contracts deliberately contain vague terms that invite judicial interpretation, despite the presence of judicial biases. By structuring the contract to protect the party most vulnerable to legal error, contract drafters retain the benefit of leaving some ex-post flexibility to courts.

One key implication of our approach is that, thanks to the law, contracts become a public good that improves with repeated usage. The use of the same vague clause in contracts of the same type allows for the gradual buildup of precedents that progressively narrow down legal uncertainty. Contracts de facto become more complete over time, incentives become higher powered, and the volume of contracting and its efficiency expand.

This mechanism captures salient features of how contracts and law evolve in practice. Leading commercial contracts ranging from franchising to equity financing, loan commitments, underwriting, etc., all include vague terms whose judicial interpretation develops and becomes less uncertain with practice. Our analysis supports the view that in commercial domains, the law fundamentally enables the development and adaptation of efficient contracts. In the presence of dysfunctional courts, parties choose not to contract or, even if they contract, legal and contractual evolution are impaired by a distorted litigation process. Society is stuck with inefficient contracts for a long time. When courts are effective, by contrast, parties are willing to grant them at least some discretion. Vague contracts are used, their litigation is informative, and contractual arrangements get perfected over time. There is a beneficial feedback loop between the use of vague contract terms and legal evolution.

In future work this mechanism may help shed light on the diverging evolution of financial markets in different legal systems (Rajan and Zingales 2003), the evolution in the way debt contracts are enforced (Franks and Sussman 2005), the development of corporate law (Coffee 1999), or the adoption of flexible contracts across countries (Lerner and Schoar 2005). Our model can also be applied to the problem of legal reform. In a companion paper, for instance (Gennaioli, Ponzetto, and Perotti 2016) we show that reforms introducing standard contracts expand market size in the short run but may impede legal evolution. The general idea is that private contracts and the law cannot be studied separately, nor can we view this relationship as one-directional, with an exogenous law acting as a constraint on private contracts. The mutual influence between contracts and the law is critical to understanding how commercial practices evolve.

References

[1] Anderlini, Luca, and Leonardo Felli. 1994. Incomplete Written Contracts: Undescribable States of Nature. *Quarterly Journal of Economics* 109(4): 1085–1124.

[2] Anderlini, Luca, Leonardo Felli, and Alessandro Riboni. 2014. Why Stare Decisis? *Review of Economic Dynamics* 17(4): 726–738.

[3] Arsht, S. Samuel. 1979. The Business Judgment Rule Revisited. *Hofstra Law Review* 8(1): 93–134.

[4] Ayres, Ian, and Robert Gertner. 1989. Filling Gaps in Incomplete Contracts: An Economic Theory of Default Rules. *Yale Law Journal* 99: 87–130.

[5] Battigalli, Pierpaolo, and Giovanni Maggi. 2002. Rigidity, Discretion, and the Costs of Writing Contracts. *American Economic Review* 92(4): 798–817.

[6] Blair, Roger D., and Francine Lafontaine. 2005. *The Economics of Franchising*. Cambridge: Cambridge University Press.

[7] Bolton, Patrick, and Mathias Dewatripont. 2004. *Contract Theory*. Cambridge, Mass.: MIT Press.

[8] Burton, Steven J. 2008. *Elements of Contract Interpretation*. Oxford: Oxford University Press.

[9] Choi, Stephen J., Mitu Gulati, and Eric A. Posner. 2013. The Dynamics of Contract Evolution. *New York University Law Review* 88(1): 1–50.

[10] Choi, Albert, and George Triantis. 2008. Completing Contracts in the Shadow of Costly Verification. *Journal of Legal Studies* 37(2): 503–534.

[11] ———. 2010. Strategic Vagueness in Contract Design: The Case of Corporate Acquisitions. *Yale Law Journal* 119: 848–924.

[12] Clark, Robert C. 1986. *Corporate Law*. Boston, Mass.: Little, Brown.

[13] Coffee, John C. 1999. Privatization and Corporate Governance: The Lessons from Securities Market Failure. *Journal of Corporation Law* 25: 1–39.

[14] Coplan, Daniel J. 2001. When Is "Best Efforts" Really "Best Efforts": An Analysis of the Obligation to Exploit in Entertainment Licensing Agreements and an Overview of How the Term "Best Efforts" Has Been Construed in Litigation. *Southwestern University Law Review* 31: 725–735.

[15] Dye, Ronald A. 1985. Costly Contract Contingencies. *International Economic Review* 26(1): 233–250.

[16] Easterbrook, Frank, and Daniel Fischel. 1989. *The Economic Structure of Corporate Law*. Cambridge, Mass.: Harvard University Press.

[17] Fernandez, Patricio A., and Giacomo A. M. Ponzetto. 2012. Stare Decisis: Rhetoric and Substance. *Journal of Law, Economics, and Organization* 28 (2): 313–336.

[18] Frank, Jerome. 1949. *Courts on Trial*. Princeton, N.J.: Princeton University Press.

[19] Franks, Julian, and Oren Sussman. 2005. Financial Innovations and Corporate Bankruptcy. *Journal of Financial Intermediation* 14(3): 283–317.

[20] Gennaioli, Nicola. 2013. Optimal Contracts with Enforcement Risk. *Journal of the European Economic Association* 11(1): 59–82.

[21] Gennaioli, Nicola, Giacomo A. M. Ponzetto and Enrico Perotti. 2016. Legal Evolution, Contract Evolution and Standardization. Working paper, Bocconi University.

[22] Gennaioli, Nicola, and Andrei Shleifer. 2007. The Evolution of Common Law. *Journal of Political Economy* 115: 43-68.

[23] ———. 2008. Judicial Fact Discretion. *Journal of Legal Studies* 37(1): 1–35.

[24] Goetz, Charles J., and Robert E. Scott. 1981. The Limits of Expanded Choice: An Analysis of the Interactions Between Express and Implied Contract Terms. *California Law Review* 73(2): 261–322.

[25] Goldberg, Victor. 2006. *Framing Contract Law: An Economic Perspective*. Cambridge, Mass.: Harvard University Press.

[26] Grinstein, Yaniv, and Stefano Rossi. 2016. Good Monitoring, Bad Monitoring, *Review of Finance* 20(5): 1719–1768.

[27] Hadfield, Gillian K. 1990. Problematic Relations: Franchising and the Law of Incomplete Contracts. *Stanford Law Review* 42: 927–992.

[28] ———. 1994. Judicial Competence and the Interpretation of Incomplete Contracts. *Journal of Legal Studies* 23: 159–184.

[29] Hart, Oliver, and John Moore. 2008. Contracts as Reference Points. *Quarterly Journal of Economics* 123(1): 1–48.

[30] Hayek, Friedrich A. 1960. *The Constitution of Liberty*. Chicago, Ill.: University of Chicago Press.

[31] Jacobs, Jack. 2015. Fifty Years of Corporate Law Evolution: A Delaware Judge's Retrospective. *Harvard Business Law Review* 5: 141–172.

[32] Johnson, Simon, Rafael La Porta, Florencio Lopez-de-Silanes, and Andrei Shleifer. 2000. Tunneling. *American Economic Review (Papers and Proceedings)* 90(2): 22–27.

[33] Killion, William L. 2013. The History of Franchising. In *Franchising: Cases, Materials, and Problems*, edited by Alexander M. Meiklejohn. Chicago, Ill.: American Bar Association.

[34] Lafontaine, Francine, and Roger D. Blair. 2008. The Evolution of Franchising and Franchise Contracts: Evidence from the United States. *Entrepreneurial Business Law Journal* 3(2): 381–434.

[35] Lafontaine, Francine, Rozenn Perrigot and Nathan E. Wilson. 2017. The Quality of Institutions and Organizational Form Decisions: Evidence from Within the Firm. *Journal of Economics and Management Strategy*, forthcoming.

[36] La Porta, Rafael, Florencio Lopez-de-Silanes, and Andrei Shleifer. 1999. Corporate Ownership Around the World. *Journal of Finance* 54(2): 471–517.

[37] ———. 2008. The Economic Consequences of Legal Origins. *Journal of Economic Literature* 46(2): 285–332.

[38] La Porta, Rafael, Florencio Lopez-de-Silanes, Andrei Shleifer, and Robert W. Vishny. 1998. Law and Finance. *Journal of Political Economy* 106(6): 1113–1155.

[39] Lerner, Josh, and Antoinette Schoar. 2005. Does Legal Enforcement Affect Financial Transactions? The Contractual Channel in Private Equity. *Quarterly Journal of Economics* 120(1): 223–246.

[40] Love, John F. 1995. *McDonald's: Behind the Arches*. Revised Ed. New York, N.Y.: Bantam.

[41] Miller, Zachary. 2006. Best Efforts? Differing Judicial Interpretations of a Familiar Term. *Arizona Law Review* 48: 615–638.

[42] Niblett, Anthony. 2013. Case-by-Case Adjudication and the Path of the Law. *Journal of Legal Studies* 42(2): 303–330.

[43] Ponzetto, Giacomo A. M., and Patricio A. Fernandez. 2008. Case law versus Statute Law: An Evolutionary Comparison. *Journal of Legal Studies* 37(2): 379–430.

[44] Posner, Eric A. 1998. The Parol Evidence Rule, the Plain Meaning Rule, and the Principles of Contractual Interpretation. University of Pennsylvania Law Review 146: 533–577.

[45] Posner, Richard A. [1973] 2014. *Economic Analysis of Law.* [1st edition, 1973] 9th edition. New York, N.Y.: Aspen Publishers.

[46] ———. 2005. Judicial Behavior and Performance: An Economic Approach. *Florida State University Law Review* 32: 1259–1279.

[47] Priest, George L. 1977. The Common Law Process and the Selection of Efficient Rules. *Journal of Legal Studies* 6(1): 65–82.

[48] Qian, Jun, and Philip E. Strahan. 2007. How Laws and Institutions Shape Financial Contracts: The Case of Bank Loans. *Journal of Finance* 62(6): 2803–2834.

[49] Rajan, Raghuram, and Luigi Zingales. 2003. The Great Reversals: The Politics of Financial Development in the Twentieth Century. *Journal of Financial Economics* 69(1): 5–50.

[50] Rubin, Paul H. 1977. Why is the Common Law Efficient? *Journal of Legal Studies* 6(1): 51–63.

[51] Schwartz, Alan. 1992. Relational Contracts in the Courts: An Analysis of Incomplete Agreements and Judicial Strategies. *Journal of Legal Studies* 21: 271–318.

[52] Schwartz, Alan, and Robert E. Scott. 2003. Contract Theory and the Limits of Contract Law. *Yale Law Journal* 113(3): 541–619.

[53] ———. 2010. Contract Interpretation Redux. *Yale Law Journal* 119(5): 926–964.

[54] Scott, Robert E., and George G. Triantis. 2006. Anticipating Litigation in Contract Design. *Yale Law Journal* 115(4): 814–879.

[55] Segal, Ilya. 1999. Complexity and Renegotiation: A Foundation for Incomplete Contracts. *Review of Economic Studies* 66(1): 57–82.

[56] Sgarlata, Mark A., and Christopher J. Brasco. 2004. Successful Claims Resolution Through an Understanding of the Law Governing Allocation of Risk for Delay and Disruption. *CM eJournal.*

[57] Shavell, Steven. 2006. On the Writing and the Interpretation of Contracts. *Journal of Law, Economics and Organization* 22(2): 289–314.

[58] U.S. Department of Commerce. 1998. *Franchising in the Economy 1986–1988.* Washington, D.C.: Government Printing Office.

A Mathematical Appendix

A.1. Proof of Proposition 1

The cost-minimizing way of inducing effort a given the non-negativity constraint is $p_0 = 0$ and $p_v = C'(a)$. Then second best effort solves the surplus-maximization problem:

$$\max_{a\in[0,1]} \{av - C(a)\} \tag{A1}$$

subject to the participation constraint

$$u_P(a) \equiv a[v - C'(a)] \geq \underline{u}_P. \tag{A2}$$

The principal's share of joint surplus $u_P(a)$ is a concave function:

$$u''_P(a) = -2C''(a) - aC'''(a) < 0 \tag{A3}$$

because $C''(a) > 0$ and $C'''(a) \geq 0$ for all $a \in (0,1)$. It has limits $u_P(0) = u_P(a_{FB}) = 0$ and thus a unique maximum,

$$U_P \equiv \max_{a\in[0,1]} u_P(a), \tag{A4}$$

that is reached at

$$a_P \equiv \arg\max_{a\in[0,1]} u_P(a) \in (0, a_{FB}). \tag{A5}$$

If $\underline{u}_P > U_P$ the partnership is infeasible. By the envelope theorem, $\partial \underline{U}_P/\partial v > 0$.

If $\underline{u}_P \leq U_P$, second-best effort is $a_{SB} \in [a_P, a_{FB})$ such that $u_P(a_{SB}) = \underline{u}_P$ and $u'_P(a_{SB}) < 0$ for all $\underline{u}_P \in (0, U_P)$. By the implicit-function theorem:

$$\frac{\partial a_{SB}}{\partial \underline{u}_P} = \frac{1}{u'_P(a_{SB})} < 0 \text{ and } \frac{\partial a_{SB}}{\partial v} = -\frac{a_{SB}}{u'_P(a_{SB})} > 0. \tag{A6}$$

Second-best surplus is $\Pi_{SB} = a_{SB}v - C(a_{SB})$, such that

$$\frac{\partial \Pi_{SB}}{\partial \underline{u}_P} = [v - C'(a_{SB})]\frac{\partial a_{SB}}{\partial \underline{u}_P} < 0 \text{ and } \frac{\partial \Pi_{SB}}{\partial v} = a_{SB} + [v - C'(a_{SB})]\frac{\partial a_{SB}}{\partial v} > 0 \tag{A7}$$

for all $a_{SB} < a_{FB} \Leftrightarrow \underline{u}_P > 0$.

A.2. Proof of Proposition 2

The first-step problem of minimizing the cost of eliciting effort a is

$$\min_p \mathbb{E}[p(e_J, e_P, e_A; 0, 0; b) | q_t = 0] \tag{A8}$$

subject to one equality constraints—the incentive-compatibility constraint in Equation (11)—and several inequality constraints—the non-negativity and truth-telling constraints in Equations (12) to (16).

Only some of the inequality constraints are binding. A first set of binding constraints reflects the impossibility of making payment contingent on direct revelation of high quality ($q_t = v$) without also paying for low quality ($q_t = 0$) to induce truthful revelation. When payment is contingent on novel evidence ($e_t\left(i_t^P\right)$ and $e_t\left(i_t^A\right)$), the ratio of expected payments in Equation (17) is minimized by lowering payment when more negative evidence is reported and increasing it when more positive evidence is reported. As a consequence, a second set of binding constraints reflects the litigants' and biased judges' ability to hide positive or negative evidence. Finally, the non-negativity constraint is binding.

Whenever $e_t\left(i_t^J\right) = -1$ precedent suffices to establish incontrovertible evidence of low quality and the optimal payment is nil. Since evidence based on precedent cannot be hidden nor distorted,

$$p\left(-1, e_P, e_A; 0, 0; b\right) = 0 \tag{A9}$$

for all $e_P, e_A \in \{0,1\}$ and all $b \in \{b_P, u, b_A\}$.

A.2.1. Pro-Principal Judges

Pro-principal judges use cheap talk to minimize payment, so for any report $(e_P, e_A; q_P, q_A)$ made by the litigants they enforce the same price regardless of cheap talk $q_P, q_A \in \{0, v\}$:

$$p\left(e_J, e_P, e_A; q_P, q_A; b_P\right) = p\left(e_J, e_P, e_A; b_P\right). \tag{A10}$$

Since the agent's payoff and a pro-principal judge's are antithetical, revelation of the agent's informative private signal ($e_t\left(i_t^A\right) \neq 0$) through a pro-principal judge requires a payment independent of $e_A \in \{-1, 0, 1\}$:

$$p\left(e_J, e_P, e_A; b_P\right) = p\left(e_J, e_P; b_P\right) \tag{A11}$$

for all $e_J \in \{-1, 1\}$ and $e_P, e_A \in \{-1, 0, 1\}$. When the principal reveals a positive signal $e_t\left(i_t^P\right) = 1$, pro-principal judges' ability to hide information implies the binding constraints:

$$p\left(e_J, 1; b_P\right) \leq p\left(e_J, 0; b_P\right) \text{ for } e_A \in \{0, 1\}. \tag{A12}$$

for all $e_J \in \{-1, 1\}$.

When the principal presents a negative signal $e_t\left(i_t^P\right) = -1$ it provides incontrovertible evidence of low quality. Thus the non-negativity constraints binds:

$$p\left(e_J, -1; b_P\right) = 0 \tag{A13}$$

for all $e_J \in \{-1, 1\}$. For non-negative realizations of the principal's private signals, the binding truth-telling constraints impose a single price:

$$p\left(e_J, 0; b_P\right) = p\left(e_J, 1; b_P\right) = p\left(e_J; b_P\right) \tag{A14}$$

for all $e_J \in \{-1, 1\}$.

If $e_J = -1$ the optimal price is nil, so the optimal price schedule for pro-principal judges consists of a single price:

$$p\left(1; b_P\right) \equiv p_P > 0, \tag{A15}$$

to be paid when neither precedent nor the principal reveal negative evidence. Intuitively, the best verification of novel evidence that can be obtained from pro-principal judges is to distinguish whether the principal can prove low quality ($e_t\left(i_t^P\right) = -1$). The contract cannot rely on evidence of low quality presented by the agent against his own interest ($e_t\left(i_t^A\right) = -1$), nor can it ask the pro-principal judge to raise payment when the parties have produced positive signals that he can hide ($e_t\left(i_t^L\right) = 1$).

Thus, pro-principal judges provide a reward for high quality

$$\mathbb{E}\left[p\left(1, e_P, e_A; v, v; b_P\right) | q_t = v\right] = p_P, \tag{A16}$$

and a wasteful payment for low quality

$$\begin{aligned} \mathbb{E}\left[p\left(e_J, e_P, e_A; 0, 0; b_P\right) | q_t = 0\right] &= p_P \Pr\left\{e_t\left(i_t^J\right) = 1, e_t\left(i_t^P\right) \neq -1 | q_t = 0\right\} \\ &= p_P \int_{i_t^J}^1 \left(1 - \iota + \iota x\right) dF_\xi\left(x\right). \end{aligned} \tag{A17}$$

If all judges have a pro-principal bias ($\pi = 1$) the minimand ratio of expected payments is

$$\Lambda = \int_{i_t^J}^1 \left(1 - \iota + \iota x\right) dF_\xi\left(x\right). \tag{A18}$$

If there are both pro-principal and unbiased judges, the truth-telling constraint (Equation 15) imposes

$$p_P \leq \min_{q \in \{0, v\}, e_P \in \{0,1\}, e_A \in \{-1,0,1\}} p\left(q, q; 1, e_P, e_A; u\right). \tag{A19}$$

A.2.2. Unbiased Judges

Unbiased judges impose no truth-telling constraints of their own because their preferences consist in faithfully applying the contract. On the other hand, unbiased judges introduce additional truth-telling constraints for the litigants, who must honestly report quality to a judge who is willing to make payment depend on their cheap talk if the contract so stipulates.

The principal must be induced to reveal truthfully $q_t = v$. Then $e_t\left(i_t^J\right) = 1$ with certainty, while $e_t\left(i_t^P\right) = 0$ with probability $1 - \iota$ and $e_t\left(i_t^P\right) = 1$ with probability ι independent of all other random variables. Hence, we can simplify his conditional expectation and write the constraints:

$$\mathbb{E}\left[p\left(1, e_P, e_A; v, v; u\right) - p\left(1, e_P, e_A; 0, v; u\right) | q_t = v\right] \leq 0 \text{ for } e_P \in \{0, 1\}. \tag{A20}$$

For ease of notation, define the conditional probability of individual evidence collection when quality is high,

$$F_e\left(e_A|v\right) \equiv \Pr\left\{e_t\left(i_t^A\right) = e_A|q_t = v\right\} \tag{A21}$$

for $e_A \in \{0,1\}$. Then the principal's truth-telling constraints are

$$\sum_{e_A\in\{0,1\}} F_e\left(e_A|v\right) p\left(1, e_P, e_A; v, v; u\right) \leq \sum_{e_A\in\{0,1\}} F_e\left(e_A|v\right) p\left(1, e_P, e_A; 0, v; u\right) \tag{A22}$$

for $e_P \in \{0,1\}$.

The agent must be induced to reveal truthfully $q_t = 0$ even if $e_t\left(i_t^J\right) = 1$:

$$\mathbb{E}\left[p\left(1, e_P, e_A; 0, 0; u\right) - p\left(1, e_P, 0; 0, v; u\right)|q_t = 0, e_t\left(i_t^J\right) = 1, e_t\left(i_t^A\right) = e_A\right] \geq 0. \tag{A23}$$

For ease of notation, define the conditional probability of individual evidence collection when quality is low:

$$F_e\left(e_P|0, e_J, e_A\right) \equiv \Pr\left\{e_t\left(i_t^P\right) = e_P|q_t = 0, e_t\left(i_t^J\right) = e_J, e_t\left(i_t^A\right) = e_A\right\} \tag{A24}$$

for $e_P \in \{-1,0,1\}$ given $e_J \in \{-1,1\}$ and $e_A \in \{-1,0,1\}$. Then the agent's truth-telling constraints are

$$\begin{aligned}\sum_{e_P\in\{-1,0,1\}} F_e\left(e_P|0,1,e_A\right) p\left(1, e_P, e_A; 0, 0; u\right) &\geq \sum_{e_P\in\{-1,0,1\}} F_e\left(e_P|0,1,e_A\right) p\left(1, e_P, e_A; 0, v; u\right) \\ &\geq \sum_{e_P\in\{0,1\}} F_e\left(e_P|0,1,e_A\right) p\left(1, e_P, e_A; 0, v; u\right) \text{ for } e_A \in \{-1,0,1\}. \end{aligned} \tag{A25}$$

The second inequality follows by the non-negativity constraint. It reflects the intuitive optimality of punishing the agent when he falsely reports $q_A = v$ and his lie is exposed by the principal's hard evidence $e_t\left(i_t^P\right) = -1$.

The principal's and the agent's constraints jointly imply that

$$\begin{aligned}&\sum_{e_P\in\{0,1\}} \frac{F_e\left(e_P|0,1,1\right)}{F_e\left(1|0,1,1\right)} \sum_{e_A\in\{0,1\}} F_e\left(e_A|v\right) p\left(1, e_P, e_A; v, v; u\right) \\ &\leq \sum_{e_P\in\{0,1\}} \frac{F_e\left(e_P|0,1,1\right)}{F_e\left(1|0,1,1\right)} \sum_{e_A\in\{0,1\}} F_e\left(e_A|v\right) p\left(1, e_P, e_A; 0, v; u\right) \\ &\leq \sum_{e_A\in\{0,1\}} \frac{F_e\left(e_A|v\right)}{F_e\left(1|0,1,e_A\right)} \sum_{e_P\in\{0,1\}} F_e\left(e_P|0,1,e_A\right) p\left(1, e_P, e_A; 0, v; u\right) \\ &\leq \sum_{e_A\in\{0,1\}} \frac{F_e\left(e_A|v\right)}{F_e\left(1|0,1,e_A\right)} \sum_{e_P\in\{0,1\}} F_e\left(e_P|0,1,e_A\right) p\left(1, e_P, e_A; 0, 0; u\right). \end{aligned} \tag{A26}$$

The first and last inequality are linear combinations of Equations (A22) and (A25), respectively. The inner inequality reduces to:

$$\frac{F_e(0|0,1,1)}{F_e(1|0,1,1)} p(1,0,0;0,v;u) \leq \frac{F_e(0|0,1,0)}{F_e(1|0,1,0)} p(1,0,0;0,v;u) \tag{A27}$$

which is true for all $p(1,0,0;0,v;u) \geq 0$ because the probability that the principal's search is unsuccessful is $F_e(0|0,1,1) = F_e(0|0,1,0) = 1-\iota$ independently of the agent's signal, while the probability that the principal uncovers a positive signal is increasing in the agent's signal:

$$F_e(1|0,1,1) = \iota \frac{\int_{i_t^J}^1 x^2 dF_\xi(x)}{\int_{i_t^J}^1 x dF_\xi(x)} > F_e(1|0,1,0) = \iota \frac{\int_{i_t^J}^1 x dF_\xi(x)}{1-F_\xi(x)}. \tag{A28}$$

Intuitively, a positive signal given low quality induces inference of high ξ_t and thus a higher likelihood that another signal is also positive.

We conjecture that the only binding constraint for the litigants' truthful reporting of quality q_t is

$$\sum_{e_P\in\{0,1\}} \sum_{e_A\in\{0,1\}} \frac{F_e(e_P|0,1,1)}{F_e(1|0,1,1)} F_e(e_A|v)\, p(1,e_P,e_A;v,v;u) \\ \leq \sum_{e_P\in\{0,1\}} \sum_{e_A\in\{0,1\}} \frac{F_e(e_P|0,1,e_A)}{F_e(1|0,1,e_A)} F_e(e_A|v)\, p(1,e_P,e_A;0,0;u). \tag{A29}$$

Then another binding constraint results from the need to induce the principal to reveal truthfully a positive signal ($e_t\left(i_t^P\right)=1$) when quality is high ($q_t = v \Rightarrow e_t\left(i_t^J\right)=1$):

$$\sum_{e_A\in\{0,1\}} F_e(e_A|v)\, p(1,1,e_A;v,v;u) \leq \sum_{e_A\in\{0,1\}} F_e(e_A|v)\, p(1,0,e_A;v,v;u). \tag{A30}$$

Any combination of the four prices $p(1,e_P,e_A;v,v;u) \geq p_P$ for $e_P, e_A \in \{0,1\}$ such that

$$\sum_{e_A\in\{0,1\}} F_e(e_A|v)\, p(1,e_P,e_A;v,v;u) = p_P + \iota p_U \text{ for } e_P \in \{0,1\} \tag{A31}$$

for some constant $p_U \geq 0$ is optimal given the truth-telling constraints we have considered so far. Only those with $p(1,0,1;v,v;u) \geq p(1,1,0;v,v;u)$ are feasible, because the litigants must be incentivized to disclose a positive private signal. Then unbiased judges provide a reward for high quality

$$\mathbb{E}\left[p(1,e_P,e_A;v,v;u)\,|q_t = v\right] = \\ \sum_{e_P\in\{0,1\}} \sum_{e_A\in\{0,1\}} F_e(e_P|v)\, F_e(e_A|v)\, p(1,e_P,e_A;v,v;u) = p_P + \iota p_U, \tag{A32}$$

recalling that the success of the two litigants' searches is independent.

The wasteful payment for low quality is minimized by minimizing payment whenever a negative signal is obtained. Thus, the non-negativity constraint is binding for $e_t\left(i_t^P\right)=-1$:

$$p\left(1,-1,e_A;0,0;u\right)=0 \text{ for all } e_A\in\{-1,0,1\}\,. \tag{A33}$$

The truth-telling constraint (Equation A19) is binding for $e_t\left(i_t^A\right)=-1$:

$$p\left(1,e_P,-1;0,0;u\right)=p_P \text{ for } e_P\in\{0,1\}\,. \tag{A34}$$

Intuitively, the agent should be punished when quality is revealed to be low. When the principal presents a negative signal ($e_t\left(i_t^P\right)=-1$) punishment is constrained because the agent is judgment proof. When the agent collects a negative signal ($e_t\left(i_t^A\right)=-1$) punishment is further limited by truth-telling constraints—as we are about to show, at the optimum $p\left(1,e_P,0;0,0;u\right)=p_P$ too.

For ease of notation, define the conditional probability of overall evidence generation,

$$F_{|q}\left(e_J,e_P,e_A|q\right)\equiv\Pr\left\{e_t\left(i_t^J\right)=e_J,e_t\left(i_t^P\right)=e_P,e_t\left(i_t^A\right)=e_A|q_t=q\right\}. \tag{A35}$$

Unbiased judges enforce a wasteful payment for low quality

$$\begin{aligned}&\mathbb{E}\left[p\left(e_J,e_P,e_A;0,0;u\right)|q_t=0\right]=\\&\quad\sum_{e_P\in\{0,1\}}F_{|q}\left(1,e_P,-1|0\right)p_P+\sum_{e_P\in\{0,1\}}\sum_{e_A\in\{0,1\}}F_{|q}\left(1,e_P,e_A|0\right)p\left(1,e_P,e_A;0,0;u\right).\end{aligned} \tag{A36}$$

The four prices $p\left(1,e_P,e_A;0,0;u\right)$ for $e_P,e_A\in\{0,1\}$ are optimally set to minimize it given the binding constraint for truthful reporting of q_t:

$$\begin{aligned}&\sum_{e_P\in\{0,1\}}\sum_{e_A\in\{0,1\}}\frac{F_e\left(e_P|0,1,e_A\right)}{F_e\left(1|0,1,e_A\right)}F_e\left(e_A|v\right)p\left(1,e_P,e_A;0,0;u\right)\\&\qquad\qquad=\left[1+\frac{F_e\left(0|0,1,1\right)}{F_e\left(1|0,1,1\right)}\right]\left(p_P+\iota p_U\right).\end{aligned} \tag{A37}$$

Thus, all prices should be minimized except those that minimize

$$L\left(e_P,e_A\right)\equiv F_{|q}\left(1,e_P,e_A|0\right)\frac{F_e\left(1|0,1,e_A\right)}{F_e\left(e_P|0,1,e_A\right)F_e\left(e_A|v\right)}, \tag{A38}$$

such that

$$L\left(0,0\right)=L\left(1,0\right)=\iota\int_{i_t^J}^1 xdF_\xi\left(x\right)>L\left(0,1\right)=L\left(1,1\right)=\iota\int_{i_t^J}^1 x^2dF_\xi\left(x\right) \tag{A39}$$

By the binding truth-telling constraint (Equation A19), the optimum is

$$p\left(1,e_P,0;0,0;u\right)=p_P \text{ for } e_P\in\{0,1\}\,, \tag{A40}$$

with any pair $p(1, e_P, 1; 0, 0; u) \geq p_P$ for $e_P \in \{0, 1\}$ such that

$$\sum_{e_P \in \{0,1\}} F_e(e_P|0,1,1)\, p(1, e_P, 1; 0, 0; u) = [F_e(0|0,1,1) + F_e(1|0,1,1)](p_P + p_U), \quad \text{(A41)}$$

recalling that $F_e(1|v) = \iota$. Any such pair is optimal given the truth-telling constraints we have considered so far. Only those with $p(1,1,1;0,0;u) \leq p(1,0,1;0,0;u)$ are feasible, because the principal must be induced to reveal truthfully a positive signal when quality is low.

Then unbiased judges enforce a wasteful payment for low quality

$$\begin{aligned}\mathbb{E}[p(e_J, e_P, e_A; 0, 0; u) | q_t = 0] \\ = p_P \int_{i_t^J}^1 (1 - \iota + \iota x)\, dF_\xi(x) + \iota p_U \int_{i_t^J}^1 (1 - \iota + \iota x)\, x dF_\xi(x). \quad \text{(A42)}\end{aligned}$$

Intuitively, pro-principal judges can be made to pay $p_P > 0$ when the principal fails to present evidence of low quality only if unbiased judges make the same payment in the same conditions. Moreover, unbiased judges can make an extra payment $p_U \geq 0$ when not only the principal fails to present evidence of low quality, but the agent also manages to present evidence of high quality.

If there are no pro-agent judges ($\alpha = 0$) the minimand ratio of expected payments is

$$\Lambda = \frac{p_P \int_{i_t^J}^1 (1 - \iota + \iota x)\, dF_\xi(x) + (1 - \pi)\, \iota p_U \int_{i_t^J}^1 (1 - \iota + \iota x)\, x dF_\xi(x)}{p_P + (1 - \pi)\, \iota p_U}, \quad \text{(A43)}$$

such that

$$\frac{\partial \Lambda}{\partial p_P} = \frac{(1 - \pi)\, \iota p_U}{[p_P + (1 - \pi)\, \iota p_U]^2} \int_{i_t^J}^1 (1 - \iota + \iota x)(1 - x)\, dF_\xi(x) \geq 0 \quad \text{(A44)}$$

and

$$\frac{\partial \Lambda}{\partial p_U} = -\frac{(1 - \pi)\, \iota p_P}{[p_P + (1 - \pi)\, \iota p_U]^2} \int_{i_t^J}^1 (1 - \iota + \iota x)(1 - x)\, dF_\xi(x) \leq 0. \quad \text{(A45)}$$

Thus, for $\alpha = 0$ and $\pi < 1$ the optimal contract has $p_P = 0 < p_U$ and the minimand ratio of expected payments is

$$\Lambda = \int_{i_t^J}^1 (1 - \iota + \iota x)\, x dF_\xi(x). \quad \text{(A46)}$$

A.2.3. Pro-Agent Judges

Pro-agent judges use cheap talk to maximize payment, so for any report $(e_P, e_A; q_P, q_A)$ made by the litigants they enforce the same price regardless of cheap talk $q_P, q_A \in \{0, v\}$:

$$p(e_J, e_P, e_A; q_P, q_A; b_A) = p(e_J, e_P, e_A; b_A) \quad \text{(A47)}$$

Since the principal's payoff and a pro-agent judge's are antithetical, revelation of the principal's informative private signal ($e_t\left(i_t^P\right) \neq 0$) through a pro-agent judge requires a payment independent of $e_P \in \{-1,0,1\}$:

$$p\left(e_J, e_P, e_A; b_A\right) = p\left(e_J, e_A; b_A\right) \tag{A48}$$

for all $e_J \in \{-1,1\}$ and $e_P, e_A \in \{-1,0,1\}$. When the agent presents a negative signal $e_t\left(i_t^A\right) = -1$, pro-agent judges' ability to hide information implies the binding constraints

$$p\left(e_J, -1; b_A\right) \geq p\left(e_J, 0; b_A\right) \tag{A49}$$

for all $e_J \in \{-1,1\}$ and $e_P \in \{-1,0,1\}$.

If $e_J = -1$ the optimal price is nil, so the optimal price schedule for pro-agent judges consists of at most two prices $\bar{p}_A \geq 0$ and $p_A \geq 0$ such that

$$\bar{p}_A \equiv p\left(1,-1; b_A\right) = p\left(1,0; b_A\right) \leq p\left(1,1; b_A\right) \equiv \bar{p}_A + p_A. \tag{A50}$$

Intuitively, the best verification that can be obtained from pro-agent judges is to distinguish whether the agent can present evidence of high quality ($e_t\left(i_t^A\right) = 1$). The mechanism cannot rely on evidence of high quality presented by the principal against his own interest ($e_t\left(i_t^P\right) = 1$), nor can it ask the pro-agent judge to lower payment when the parties have produced negative signals that he can hide ($e_t\left(i_t^L\right) = -1$).

Thus, pro-agent judges provide a reward for high quality

$$\mathbb{E}\left[p\left(1, e_P, e_A; v, v; b_A\right) | q_t = v\right] = \bar{p}_A + p_A \Pr\left\{e_t\left(i_t^A\right) = 1 | q_t = v\right\} = \bar{p}_A + \iota p_A \tag{A51}$$

and a wasteful payment for low quality

$$\begin{aligned}\mathbb{E}&\left[p\left(e_J, e_P, e_A; 0, 0; b_A\right) | q_t = 0\right] \\ &= \bar{p}_A \Pr\left\{e_t\left(i_t^J\right) = 1 | q_t = 0\right\} + p_A \Pr\left\{e_t\left(i_t^J\right) = 1, e_t\left(i_t^A\right) = 1 | q_t = 0\right\} \\ &= \bar{p}_A\left[1 - F_\xi\left(i_t^J\right)\right] + \iota p_A \int_{i_t^J}^1 x dF_\xi(x).\end{aligned} \tag{A52}$$

If all judges have a pro-agent bias ($\alpha = 1$) the minimand ratio of expected payments is

$$\Lambda = \frac{\bar{p}_A\left[1 - F_\xi\left(i_t^J\right)\right] + \iota p_A \int_{i_t^J}^1 x dF_\xi(x)}{\bar{p}_A + \iota p_A}, \tag{A53}$$

such that

$$\frac{\partial \Lambda}{\partial \bar{p}_A} = \frac{\iota p_A}{\left(\bar{p}_A + \iota p_A\right)^2} \int_{i_t^J}^1 (1-x)\, dF_\xi(x) \geq 0 \tag{A54}$$

and

$$\frac{\partial \Lambda}{\partial p_A} = -\frac{\iota \bar{p}_A}{\left(\bar{p}_A + \iota p_A\right)^2} \int_{i_t^J}^1 (1-x)\, dF_\xi(x) \leq 0. \tag{A55}$$

Thus, for $\alpha = 1$ the optimal contract has $\bar{p}_A = 0 < p_A$ and the minimand ratio of expected payments is

$$\Lambda = \int_{i_t^J}^{1} x dF_\xi\left(x\right). \tag{A56}$$

If there are both pro-agent and unbiased judges, the truth-telling constraint (Equation 16) imposes

$$\bar{p}_A \geq \max_{q\in\{0,v\},e_P\in\{-1,0,1\},e_A\in\{-1,0\}} p\left(q,q;1,e_P,e_A;u\right) \tag{A57}$$

and

$$\bar{p}_A + p_A \geq \max_{q\in\{0,v\},e_P\in\{-1,0,1\}} p\left(q,q;1,e_P,1;u\right). \tag{A58}$$

A.2.4. Optimal Contract

Since the optimal contract for pro-agent judges has $\bar{p}_A = 0 < p_A$, the binding truth-telling constraint (Equation 15) uniquely pins down the optimal combination of the four prices $p\left(1,e_P,e_A;v,v;u\right) \geq p_P$ for $e_P, e_A \in \{0,1\}$:

$$\begin{aligned} p\left(1,0,0;v,v;u\right) = p\left(1,1,0;v,v;u\right) = p_P \\ < p\left(1,0,1;v,v;u\right) = p\left(1,1,1;v,v;u\right) = p_P + p_U, \end{aligned} \tag{A59}$$

which enables the minimization of

$$\bar{p}_A = p_P. \tag{A60}$$

The optimal contracts for the extreme cases in which judges are respectively all pro-agent or all unbiased are ranked by

$$\Lambda_{\alpha=1} = \int_{i_t^J}^{1} x dF_\xi\left(x\right) > \Lambda_{\pi=\alpha=0} = \int_{i_t^J}^{1} \left(1-\iota+\iota x\right) x dF_\xi\left(x\right). \tag{A61}$$

Intuitively, unbiased judges provide the best verification, even if they cannot achieve perfect revelation of q_t for any $i_t^J < 1$. Thus, it is optimal to minimize p_A for any p_U, so the binding truth-telling constraint (Equation 15) also uniquely pins down the optimal pair $p\left(1,e_P,1;0,0;u\right) \geq p_P$ for $e_P \in \{0,1\}$:

$$p\left(1,e_P,1;0,0;u\right) = p\left(1,e_P,1;0,0;u\right) = p_P + p_U, \tag{A62}$$

which enables the minimization of

$$p_A = p_U. \tag{A63}$$

Then, for any $p_P = \bar{p}_A = p_0 \geq 0$ and $p_U = p_A = p_1 \geq 0$, the optimal contract provides a reward for high quality

$$\mathbb{E}\left(p|q_t = v\right) = p_0 + \left(1-\pi\right)\iota p_1 \tag{A64}$$

and a wasteful payment for low quality

$$\mathbb{E}\left(p|q_t=0\right)=\left[\int_{i_t^J}^1\left(1-\iota+\iota x\right)dF_\xi\left(x\right)+\alpha\iota\int_{i_t^J}^1\left(1-x\right)dF_\xi\left(x\right)\right]p_0$$
$$+\iota\left[\left(1-\pi\right)\int_{i_t^J}^1 xdF_\xi\left(x\right)-\left(1-\pi-\alpha\right)\iota\int_{i_t^J}^1 x\left(1-x\right)dF_\xi\left(x\right)\right]p_1. \quad \text{(A65)}$$

The minimand ratio of expected payments has derivatives

$$\frac{\partial\Lambda}{\partial p_0}=\frac{\left(1-\pi\right)\iota p_1}{\left[p_0+\left(1-\pi\right)\iota p_1\right]^2}\int_{i_t^J}^1\left(1-x\right)\left[1-\iota+\alpha\iota+\frac{1-\pi-\alpha}{1-\pi}\iota x\right]dF_\xi\left(x\right)\geq 0 \quad \text{(A66)}$$

and

$$\frac{\partial\Lambda}{\partial p_1}=-\frac{\left(1-\pi\right)\iota p_0}{\left[p_0+\left(1-\pi\right)\iota p_1\right]^2}\int_{i_t^J}^1\left(1-x\right)\left[1-\iota+\alpha\iota+\frac{1-\pi-\alpha}{1-\pi}\iota x\right]dF_\xi\left(x\right)\leq 0 \quad \text{(A67)}$$

Thus, for $\pi<1$ the optimal contract has $p_0=0<p_1=p^*$ and the minimand ratio of expected payments is

$$\Lambda=\int_{i_t^J}^1 xdF_\xi\left(x\right)-\frac{1-\pi-\alpha}{1-\pi}\iota\int_{i_t^J}^1 x\left(1-x\right)dF_\xi\left(x\right). \quad \text{(A68)}$$

The optimal mechanism stipulates that the price is nil ($p\left(...\right)=0$) except in the following two cases in which the principal must pay the agent a positive price $p^*>0$.

1. Evidence based on precedent is positive, the principal does not present novel negative evidence, the agent presents novel positive evidence, and the judge is unbiased ($p(1,0,1;q_P,q_A;u)=p\left(1,1,1;q_P,q_A;u\right)=p^*$ for all $q_P,q_A\in\{0,v\}$).

2. Evidence based on precedent is positive, the agent presents novel positive evidence, and the judge is pro-agent ($p\left(1,e_P,1;q_P,q_A;b_A\right)=p$ for all $e_P\in\{-1,0,1\}$ and $q_P,q_A\in\{0,v\}$).

Under this optimal mechanism, all the truth-telling constraints we conjectured to be non-binding are slack. Pro-principal judges attain their bliss point because they never enforce payment. Thus, litigants are indifferent about their reports to pro-principal judges. Pro-agent judges have no avenue to increase payment further: they would need to disregard precedent or to fake positive evidence that the agent failed to present (because $p\left(e_J,e_P,-1;q_P,q_A;b_A\right)=p\left(e_J,e_P,0;q_P,q_A;b_A\right)=0$), both of which are impossible. The principal is indifferent about his reports to pro-agent judges, who will completely ignore them, while the agent is happy to report truthfully to a pro-agent judge because their goals coincide.

When the judge is unbiased litigants are incentivized to report truthfully quality q_t because the optimal mechanism ignores their cheap talk q_P,q_A. They are incentivized to report truthfully their private signals because they cannot improve their payoffs by hiding them. The principal may lower payment to zero by presenting $e_t\left(i_t^P\right)=-1$ but never raises it by presenting $e_t\left(i_t^P\right)=1$. The agent may increase it to $p>0$ by presenting $e_t\left(i_t^A\right)=1$ but never lowers it by presenting $e_t\left(i_t^A\right)=-1$.

The correspondence between the optimal direct revelation mechanism and the intuitive contract in Proposition 2 is straightforward. Under the latter, the principal hides positive evidence $e_t\left(i_t^P\right)=1$ to minimize payment and the agent hides negative evidence $e_t\left(i_t^A\right)=-1$ to maximize it. An unbiased judge reports truthfully all evidence presented in court. Thus, he enforces payment if and only if evidence based on precedent is positive ($e_t\left(i_t^J\right)=1$), the agent presented further positive evidence ($e_t\left(i_t^A\right)=1$), and the principal failed to present negative evidence ($e_t\left(i_t^P\right)\neq-1$). A pro-principal judge can and does hide any positive evidence presented by the agent. Thus, he never enforces payment. A pro-agent judge can and does hide any negative evidence presented by the principal. Thus, he enforces payment whenever the agent presents positive evidence ($e_t\left(i_t^A\right)=1$), unless evidence based on precedent is negative ($e_t\left(i_t^J\right)=-1$).

In the limit case $\pi=1$, the optimal mechanism stipulates instead that the price is nil ($p\left(...\right)=0$) except in a single case in which the principal must pay the agent a positive price $p_P=p^*>0$: evidence based on precedent is positive and the principal does not present novel negative evidence, the agent presents novel positive evidence, and the judge is unbiased ($p(1,0,e_A;q_P,q_A;b_P)=p\left(1,1,e_A;q_P,q_A;b_P\right)=p^*$ for all $e_A\in\{-1,0,1\}$ and $q_P,q_A\in\{0,v\}$).

Under this optimal mechanism, all the truth-telling constraints we conjectured to be non-binding are slack. Pro-principal judges have no avenue to reduce payment further: they would need to disregard precedent or to fake negative evidence that the principal failed to present, both of which are impossible. The agent is indifferent about his reports to pro-principal judges, who will completely ignore them, while the principal is happy to report truthfully to a pro-principal judge because their goals coincide.

The correspondence between the optimal direct revelation mechanism and the intuitive contract in Corollary 1 is straightforward. Under the latter, the agent hides negative evidence $e_t\left(i_t^A\right)=-1$ to maximize payment. A pro-principal judge is happy to verify any negative evidence he can, and the mechanism does not require him to verify positive evidence.

A.3. Proof of Corollary 2 and Propositions 3 and 4

Due to the binary nature of the optimal mechanism described by Proposition 2, we can define the probability that the incentive payment p^* is enforced given that $q_t=v$:

$$\eta_v\left(\iota,\pi\right)=\left(1-\pi\right)\iota; \tag{A69}$$

and the probability that it is enforced when $q_t=0$:

$$\eta_0\left(i_t^J,\iota,\pi,\alpha\right)=\iota\left[\left(1-\pi\right)\int_{i_t^J}^1 xdF_\xi\left(x\right)-\left(1-\pi-\alpha\right)\iota\int_{i_t^J}^1 x\left(1-x\right)dF_\xi\left(x\right)\right]. \tag{A70}$$

These probabilities characterize the minimized likelihood ratio

$$\Lambda\left(i_t^J,\iota,\pi,\alpha\right)\equiv\frac{\mathbb{E}\left(p|q_t=0\right)}{\mathbb{E}\left(p|q_t=v\right)}=\frac{\eta_0\left(i_t^J,\iota,\pi,\alpha\right)}{\eta_v\left(\iota,\pi\right)} \tag{A71}$$

and the solution of the first-stage cost-minimization problem.

A.3.1. Effort, Joint Surplus, and Partnership Formation

Then the agent's incentive-compatibility constraint implies that effort a is induced at minimum cost by an incentive payment

$$p^*\left(a;i_t^J,\iota,\pi,\alpha\right)=\frac{C'(a)}{\eta_v\left(\iota,\pi\right)-\eta_0\left(i_t^J,\iota,\pi,\alpha\right)}. \tag{A72}$$

Substituting this solution, the optimal contract induces effort

$$\hat{a}=\arg\max_{a\in[0,1]}\left\{av-C\left(a\right)\right\} \tag{A73}$$

subject to the principal's participation constraint

$$u_P\left(a;\Lambda,v\right)\equiv av-\left(a+\frac{\Lambda}{1-\Lambda}\right)C'(a)\geq\underline{u}_P. \tag{A74}$$

The principal's share of joint surplus $u_P\left(a;\Lambda,v\right)$ is a concave function of effort:

$$\frac{\partial^2 u_P}{\partial a^2}=-2C''(a)-\left(a+\frac{\Lambda}{1-\Lambda}\right)C'''(a)<0 \tag{A75}$$

because $C''\left(a\right)>0$ and $C'''\left(a\right)\geq 0$ for all $a\in(0,1)$. It has limit $u_P\left(0;\Lambda,v\right)=0$ and a unique maximum at

$$a_P\left(\Lambda,v\right)\equiv\arg\max_{a\in[0,1]}u_P\left(a;\Lambda,v\right). \tag{A76}$$

For sufficiently high values of $\Lambda\left(i_t^J,\iota,\pi,\alpha\right)$, contract enforcement is so poor that u_P is maximized at $a=0$:

$$a_P\left(\Lambda,v\right)=0\text{ for all }\Lambda\geq\frac{v}{v+C''(0)} \tag{A77}$$

because

$$\frac{\partial u_P}{\partial a}\left(0;\Lambda,v\right)=v-\frac{\Lambda}{1-\Lambda}C''(0). \tag{A78}$$

By the envelope theorem,

$$\frac{\partial u_P}{\partial\Lambda}\left(a_P\left(\Lambda\right);\Lambda,v\right)=-\frac{C'\left(a_P\left(\Lambda,v\right)\right)}{\left(1-\Lambda\right)^2}<0\text{ for all }\Lambda<\frac{v}{v+C''(0)}. \tag{A79}$$

In the limit as $\Lambda \to 0$, quality becomes perfectly contractible and

$$\lim_{\Lambda\to 0} u_P\left(a_P\left(\Lambda, v\right); \Lambda, v\right) = \max_{a\in[0,1]} \left\{a\left[v - C'\left(a\right)\right]\right\} \tag{A80}$$

as in Proposition 1. Condition (6) ensures that this is greater than $\underline{u}_P$. Therefore, there is a threshold

$$\hat{\Lambda}\left(\underline{u}_P, v\right) \in \left[0, \frac{v}{v + C''(0)}\right] \tag{A81}$$

such that partnership t is formed if and only if $\Lambda\left(i_t^J, \iota, \pi, \alpha\right) \leq \hat{\Lambda}\left(\underline{u}_P, v\right)$. By the implicit function theorem, $\hat{\Lambda}\left(\underline{u}_P, v\right)$ is decreasing in the principal's outside option $\underline{u}_P$ and increasing in the value of a high-quality service v.

If the partnership can be formed, optimal effort is $\hat{a}\left(\Lambda, \underline{u}_P, v\right)$ such that

$$u_P\left(\hat{a}; \Lambda, v\right) = \underline{u}_P, \tag{A82}$$

which implies

$$a_P\left(\Lambda, v\right) \leq \hat{a}\left(\Lambda, \underline{u}_P, v\right) < a_{FB} \tag{A83}$$

and

$$\frac{\partial u_P}{\partial a}\left(\hat{a}\left(\Lambda, \underline{u}_P, v\right); \Lambda, v\right) < 0 \text{ for all } \Lambda < \hat{\Lambda}\left(\underline{u}_P, v\right). \tag{A84}$$

By the implicit-function theorem,

$$\frac{\partial \hat{a}}{\partial \Lambda} = \frac{C'\left(\hat{a}\right)}{\left(1-\Lambda\right)^2}\left[v - C'\left(\hat{a}\right) - \left(\hat{a} + \frac{\Lambda}{1-\Lambda}\right)C''\left(\hat{a}\right)\right]^{-1} < 0. \tag{A85}$$

Joint surplus equal

$$\Pi = \hat{a}v - C\left(\hat{a}\right), \tag{A86}$$

which is monotone increasing in $\hat{a}$ for all $\hat{a} < a_{FB}$, namely whenever $\Lambda > 0$ or $\underline{u}_P > 0$. Quality is directly contractible if and only if $\Lambda\left(i_t^J, \iota, \pi, \alpha\right) = 0$. By Proposition 1, the first best is then attainable if and only if, furthermore, $\underline{u}_P = 0$.

A.3.2. Verifiability

Under the optimal mechanism described by Proposition 2,

$$\Lambda\left(i_t^J, \iota, \pi, \alpha\right) = \int_{i_t^J}^1 x dF_\xi\left(x\right) - \frac{1-\pi-\alpha}{1-\pi}\iota\int_{i_t^J}^1 x\left(1-x\right)dF_\xi\left(x\right), \tag{A87}$$

such that

$$\Lambda\left(i_t^J, \iota, \pi, \alpha, \alpha\right) = 0 \Leftrightarrow i_t^J = 1 \tag{A88}$$

and more generally

$$\frac{\partial \Lambda}{\partial i_t^J} = -\left[1 - \frac{1-\pi-\alpha}{1-\pi}\iota\left(1-i_t^J\right)\right] i_t^J f_\xi\left(i_t^J\right) \leq 0, \tag{A89}$$

$$\frac{\partial \Lambda}{\partial \iota} = -\frac{1-\pi-\alpha}{1-\pi} \int_{i_t^J}^{1} x\,(1-x)\, dF_\xi\,(x) \leq 0, \tag{A90}$$

$$\frac{\partial \Lambda}{\partial \pi} = \frac{\alpha \iota}{(1-\pi)^2} \int_{i_t^J}^{1} x\,(1-x)\, dF_\xi\,(x) \geq 0, \tag{A91}$$

and

$$\frac{\partial \Lambda}{\partial \alpha} = \frac{\iota}{1-\pi} \int_{i_t^J}^{1} x\,(1-x)\, dF_\xi\,(x) \geq 0. \tag{A92}$$

If we rewrite

$$\Lambda\left(i_t^J, \iota, \pi, \alpha\right) = \int_{i_t^J}^{1} \left[\left(1-\iota+\frac{\alpha\iota}{1-\pi}\right)x + \frac{1-\pi-\alpha}{1-\pi}\iota x^2\right] dF_\xi\,(x)\,, \tag{A93}$$

it is immediate that Λ increases if ξ shifts up in the sense of first-order stochastic dominance.

Furthermore,

$$\frac{\partial^2 \Lambda}{\partial \iota \partial i_t^J} = \frac{1-\pi-\alpha}{1-\pi} i_t^J \left(1-i_t^J\right) f_\xi\left(i_t^J\right) \geq 0, \tag{A94}$$

$$\frac{\partial^2 \Lambda}{\partial \pi \partial i_t^J} = -\frac{\alpha\iota}{(1-\pi)^2} \left(1-i_t^J\right) i_t^J f_\xi\left(i_t^J\right) \leq 0, \tag{A95}$$

and

$$\frac{\partial^2 \Lambda}{\partial \alpha \partial i_t^J} = -\frac{\iota}{1-\pi} \left(1-i_t^J\right) i_t^J f_\xi\left(i_t^J\right) \leq 0. \tag{A96}$$

A.3.3. Incentive Payment

The optimal price is

$$p^* = \frac{C'(\hat{a})}{\eta_v - \eta_0}, \tag{A97}$$

where $\hat{a}$ is defined as the right-most solution to

$$\hat{a}v - \left(\hat{a} + \frac{\eta_0}{\eta_v - \eta_0}\right) C'\left(\hat{a}\right) = \underline{u}_P, \tag{A98}$$

such that

$$0 < v - C'\left(\hat{a}\right) < \left(\hat{a} + \frac{\eta_0}{\eta_v - \eta_0}\right) C''(\hat{a}), \tag{A99}$$

where the first-inequality (inefficiency) comes from the definition itself,

$$v - C'\left(\hat{a}\right) = \frac{1}{\hat{a}}\left[\underline{u}_P + \frac{\eta_0}{\eta_v - \eta_0} C'\left(\hat{a}\right)\right], \tag{A100}$$

and the second (binding participation constraint) from selecting the right-most solution.

Comparative statics are then

$$\frac{\partial p^*}{\partial z} = \frac{C'(\hat{a})}{(\eta_v - \eta_0)^2}\left(\frac{\partial \eta_0}{\partial z} - \frac{\partial \eta_v}{\partial z}\right) + \frac{C''(\hat{a})}{\eta_v - \eta_0}\frac{\partial \hat{a}}{\partial z} \tag{A101}$$

for any parameter z. The implicit-function theorem implies

$$\frac{\partial \hat{a}}{\partial z} = \left[\left(\hat{a} + \frac{\eta_0}{\eta_v - \eta_0}\right) C''(\hat{a}) - v + C'(\hat{a})\right]^{-1} \times \left[\frac{\eta_0 C'(\hat{a})}{(\eta_v - \eta_0)^2}\frac{\partial \eta_v}{\partial z} - \frac{\eta_v C'(\hat{a})}{(\eta_v - \eta_0)^2}\frac{\partial \eta_0}{\partial z} + \hat{a}\frac{\partial v}{\partial z} - \frac{\partial \underline{u}_P}{\partial z}\right]. \tag{A102}$$

Therefore,

$$\frac{\partial p^*}{\partial i_t^J} = -\frac{C'(\hat{a})}{(\eta_v - \eta_0)^2}\frac{v - C'(\hat{a}) + (1-\hat{a})C''(\hat{a})}{\left(\hat{a} + \frac{\eta_0}{\eta_v - \eta_0}\right)C''(\hat{a}) - v + C'(\hat{a})}\frac{\partial \eta_0}{\partial i_t^J} > 0 \tag{A103}$$

because

$$\frac{\partial \eta_0}{\partial i_t^J} = -\iota\left[(1-\pi)\left(1 - \iota + \iota i_t^J\right) + \alpha\iota\left(1 - i_t^J\right)\right] i_t^J f_\xi\left(i_t^J\right) < 0; \tag{A104}$$

and

$$\frac{\partial p^*}{\partial \alpha} = -\frac{C'(\hat{a})}{(\eta_v - \eta_0)^2}\frac{v - C'(\hat{a}) + (1-\hat{a})C''(\hat{a})}{\left(\hat{a} + \frac{\eta_0}{\eta_v - \eta_0}\right)C''(\hat{a}) - v + C'(\hat{a})}\frac{\partial \eta_0}{\partial \alpha} < 0 \tag{A105}$$

because

$$\frac{\partial \eta_0}{\partial \alpha} = \iota^2 \int_{i_t^J}^1 x(1-x)\, dF_\xi(x) > 0. \tag{A106}$$

By the same token, p^* declines if ξ_t shifts up in the sense of first-order stochastic dominance.

A.4. Proof of Proposition 5

The conditions under which each decision is written are the following:

1. Evidence based on precedent is positive ($e_t\left(i_t^J\right) = 1$), the agent presents positive evidence ($e_t\left(i_t^A\right) = 1$) and one of two additional contingencies is realized.
   (a) The judge is unbiased ($b_t = u$) and the principal does not present negative evidence ($e_t\left(i_t^P\right) \in \{0, 1\}$).
   (b) The judge has a pro-agent bias ($b_t = b_A$).
2. Evidence based on precedent is negative ($e_t\left(i_t^J\right) = -1$).
3. Evidence based on precedent is positive ($e_t\left(i_t^J\right) = 1$), the principal presents negative evidence ($e_t\left(i_t^P\right) = -1$) and the judge does not have a pro-agent bias ($b_t \in \{b_P, u\}$).

4. Evidence based on precedent is positive ($e_t\left(i_t^J\right)=1$) and one of three residual cases is realized.

   (a) The judge is pro-principal ($b_t = b_P$) and the principal does not present negative evidence ($e_t\left(i_t^P\right) \in \{0,1\}$).

   (b) The judge is unbiased ($b_t = u$), the agent does not present positive evidence ($e_t\left(i_t^A\right) \in \{-1,0\}$) and the principal does not present negative evidence ($e_t\left(i_t^P\right) \in \{0,1\}$).

   (c) The judge is pro-agent ($b_t = b_A$) and the agent does not present positive evidence ($e_t\left(i_t^A\right) \in \{-1,0\}$).

Suppose that given the current state of precedent i_t^J partnership t is formed with an innovative contract that induces optimal effort

$$a_t = \hat{a}\left(\Lambda\left(i_t^J, \iota, \pi, \alpha\right), \underline{u}_P, v\right) > 0. \tag{A107}$$

Then the probability that the informativeness of precedent remains unchanged is

$$\begin{aligned}
\Pr\left(i_{t+1}^J = i_t^J | i_t^J\right) = (1-\pi-\alpha)\,\iota i_t^J &\left[a_t + (1-a_t)\int_{i_t^J}^1 (1-\iota+\iota x)\, dF_\xi(x)\right] \\
&+ \alpha\iota\left\{a_t i_t^J + (1-a_t)\, i_t^J\left[1 - F_\xi\left(i_t^J\right)\right]\right\} \\
&\qquad + (1-a_t)\, F_\xi\left(i_t^J\right) \\
&+ \pi\left[a_t + (1-a_t)\int_{i_t^J}^1 (1-\iota+\iota x)\, dF_\xi(x)\right] \\
+ (1-\pi-\alpha)&\left\{a_t(1-\iota) + (1-a_t)\int_{i_t^J}^1 \left[1-\iota+\iota^2 x(1-x)\right] dF_\xi(x)\right\} \\
&\qquad + \alpha\left[a_t(1-\iota) + (1-a_t)\int_{i_t^J}^1 (1-\iota x)\, dF_\xi(x)\right],
\end{aligned} \tag{A108}$$

where the first two lines corresponds to each subcase of decision 1 with $i_t^A \leq i_t^J$, the third to decision 2, and the last three to each sub-case of decision 4. Simplifying,

$$\begin{aligned}
\Pr\left(i_{t+1}^J = i_t^J | i_t^J\right) &= 1 - a_t(1-\pi)\,\iota\left(1-i_t^J\right) - (1-a_t)\,\iota \\
&\times \int_{i_t^J}^1 \left\{(1-\alpha)(1-x) + \left[\alpha + (1-\pi-\alpha)(1-\iota+\iota x)\right]\left(x - i_t^J\right)\right\} dF_\xi(x).
\end{aligned} \tag{A109}$$

This rewriting highlights the cases in which the informativeness of precedent improves ($i_{t+1}^J > i_t^J$). If quality is high (with probability a_t), a valuable new precedent is created if the agent's search is successful (with probability ι), his evidence happens to be more informative than the best existing precedent (with probability $1 - i_t^J$), and the judge is willing to verify it because he doesn't have a pro-principal bias (with probability $1 - \pi$).

If quality is low (with probability $1 - a_t$), a valuable new precedent can be created only if evidence based on precedent is positive ($\xi_t > i_t^J$). Then, one possibility is that the principal finds negative evidence (with probability $\iota(1 - \xi_t)$), and the judge is willing to verify it because he doesn't have a pro-agent bias (with probability $1-\alpha$). The opposite possibility is that the agent finds evidence that is positive and yet more informative than precedents ($i_t^J < i_t^A < \xi_t$, with probability $\iota\left(\xi_t - i_t^J\right)$). A pro-agent judge always reports it to rule in the agent's favor (with probability α). An unbiased judge (who decides the case with probability $1 - \pi - \alpha$) does the same if and only if the principal does not simultaneously report negative evidence (with probability $1 - \iota + \iota\xi_t$).[37]

The informativeness of precedent improves when decision 3 is made, and also when decision 1 is made and the agent's novel evidence happens to be more informative than existing precedents ($i_{t+1}^J = i_t^A > i_t^J$). For every value $j \in \left[i_t^J, 1\right]$, the probability that the new precedent is more informative equals

$$\begin{aligned}
\Pr\left(i_{t+1}^J > j | i_t^J\right) &= (1 - \pi - \alpha)\,\iota\left[a_t(1-j) + (1-a_t)\int_j^1 (1 - \iota + \iota x)(x - j)\,dF_\xi(x)\right] \\
&\quad + \alpha\iota\left[a_t(1-j) + (1-a_t)\int_j^1 (x-j)\,dF_\xi(x)\right] \\
&\quad + (1-\alpha)\,\iota\,(1-a_t)\left[\int_{i_t^J}^j (1-j)\,dF_\xi(x) + \int_j^1 (1-x)\,dF_\xi(x)\right], \qquad \text{(A110)}
\end{aligned}$$

where the first two lines corresponds to each subcase of decision 1 with $i_t^A > j$, and the last one to decision 3. Simplifying,

$$\begin{aligned}
\Pr\left(i_{t+1}^J > j | i_t^J\right) &= a_t(1-\pi)\,\iota\,(1-j) \\
&\quad + (1-a_t)\,\iota\int_j^1 \left[\alpha + (1-\pi-\alpha)(1-\iota+\iota x)\right](x-j)\,dF_\xi(x) \\
&\quad + (1-a_t)\,\iota\,(1-\alpha)\left\{(1-j)\left[F_\xi(j) - F_\xi\left(i_t^J\right)\right] + \int_j^1 (1-x)\,dF_\xi(x)\right\}. \qquad \text{(A111)}
\end{aligned}$$

The first line describes the probability that precedent improves above informativeness j when quality is high (with probability a_t). The agent's search must be successful (with probability ι), his evidence must happen to be more informative than j (with probability $1-j$), and the judge must be willing to verify it because he doesn't have a pro-principal bias (with probability $1-\pi$). The second line represents the same decision in the agent's favor when quality is actually low (with probability $1 - a_t$). Then evidence based on precedent must be positive ($\xi_t > i_t^J$). The agent's search must be successful (with probability ι) and it must yield evidence that is positive and yet more informative than j ($j < i_t^A < \xi_t$, with probability $\xi_t - j$). Moreover, either the judge must have a pro-agent bias (with probability

[37] If an unbiased judge reports the principal's negative evidence, he may also report the agent's positive evidence, but the latter is not only irrelevant for the outcome of the case but also necessarily less informative: $e_t\left(i_t^P\right) = -1 < e_t\left(i_t^A\right) = 1 \Rightarrow i_t^A < \xi_t \leq i_t^P$.

α), or else he must be unbiased (with probability $1-\pi-\alpha$) and have observed no negative evidence produced by principal. The latter condition obtains when the principal's search fails or when it uncovers positive evidence (with probability $1-\iota+\iota\xi_t$).

If partnership t is formed, the informativeness of precedent i_t^J evolves as a time-homogeneous Markov chain with transition kernel

$$J(i,dj) = p(i,j)\,dj + r(i)\,1_i(dj), \tag{A112}$$

where 1_i denotes the indicator function $1_i(dj) = 1$ if $i \in dj$ and 0 otherwise;

$$\begin{aligned} r(i) = 1 - (1-\pi)\,\iota\,(1-i)\,\hat{a}\left(\Lambda(i,\iota,\pi,\alpha), \underline{u}_P, v\right) - \iota\left[1 - \hat{a}\left(\Lambda(i,\iota,\pi,\alpha,\alpha), \underline{u}_P, v\right)\right] \\ \times \int_i^1 \left\{(1-\alpha)(1-x) + \left[\alpha + (1-\pi-\alpha)(1-\iota+\iota x)\right](x-i)\right\} dF_\xi(x) \end{aligned} \tag{A113}$$

describes the discrete probability of a transition from $i_t^J = i$ to $i_{t+1}^J = i$; and finally

$$p(i,j) = 0 \text{ for all } j \in [0,i] \tag{A114}$$

and

$$\begin{aligned} p(i,j) = (1-\pi)\,\iota\hat{a}\left(\Lambda(i,\iota,\pi,\alpha), \underline{u}_P, v\right) + \iota\left[1 - \hat{a}\left(\Lambda(i,\iota,\pi,\alpha), \underline{u}_P, v\right)\right] \\ \times \left\{\int_j^1 \left[\alpha + (1-\pi-\alpha)(1-\iota+\iota x)\right] dF_\xi(x) + (1-\alpha)\left[F_\xi(j) - F_\xi(i)\right]\right\} \end{aligned} \tag{A115}$$

for all $j \in (i,1]$ jointly describe the continuous probability density of a transition from $i_t^J = i$ to $i_{t+1}^J = j$, which is positive if and only if $j > i$.

It follows that state j is accessible from state i if and only if $j \geq i$. The state $i = 1$ is absorbing because it is impossible to leave: $r(1) = 1$ and $p(1,j) = 0$ for all $j \in [0,1]$. The absorbing state is immediately accessible from any other state, so the Markov chain is absorbing.

Partnership t is formed for any $i_t^J \geq 0$ if it is formed for $i_0^J = 0$, namely if and only if

$$\hat{\Lambda}(\underline{u}_P, v) \geq \Lambda(0,\iota,\pi,\alpha) = \mathbb{E}\xi_t - \frac{1-\pi-\alpha}{1-\pi}\iota\mathbb{E}\left[\xi_t(1-\xi_t)\right]. \tag{A116}$$

This condition can be rearranged as:

$$\iota \geq \underline{\iota}_0 \equiv \max\left\{0, \frac{1-\pi}{1-\pi-\alpha}\frac{\mathbb{E}\xi_t - \hat{\Lambda}(\underline{u}_P, v)}{\mathbb{E}\left[\xi_t(1-\xi_t)\right]}\right\}, \tag{A117}$$

such that $\partial\underline{\iota}_0/\partial\pi \geq 0$, $\partial\underline{\iota}_0/\partial\alpha \geq 0$ and $\partial\underline{\iota}_0/\partial\underline{u}_P \geq 0 \geq \partial\underline{\iota}_0/\partial v$.

When judicial biases are symmetric ($\pi = \alpha = (1-\omega)/2$), the probability that precedent improves is

$$\Pr\left(i_{t+1}^J > i_t^J | i_t^J\right) = a_t \iota \frac{1+\omega}{2}\left(1 - i_t^J\right) + (1-a_t)\,\iota \int_{i_t^J}^1 \left[\frac{1+\omega}{2}\left(1-i_t^J\right) - \omega\iota\left(x - i_t^J\right)(1-x)\right] dF_\xi(x), \quad \text{(A118)}$$

such that

$$\frac{\partial}{\partial\omega}\Pr\left(i_{t+1}^J > i_t^J | i_t^J\right) = \left[\frac{1+\omega}{2}\iota\left(1-i_t^J\right)F_\xi\left(i_t^J\right) + \iota^2\int_{i_t^J}^1 \left(x-i_t^J\right)(1-x)\,dF_\xi(x)\right]\frac{\partial a_t}{\partial\omega} + \frac{1}{2}a_t\iota\left(1-i_t^J\right) + (1-a_t)\,\iota\int_{i_t^J}^1\left[\frac{1}{2}\left(1-i_t^J\right) - \iota\left(x-i_t^J\right)(1-x)\right]dF_\xi(x) > 0 \quad \text{(A119)}$$

because $\partial a_t/\partial\omega > 0$; and

$$\frac{\partial}{\partial\iota}\Pr\left(i_{t+1}^J > i_t^J | i_t^J\right) = \left[\frac{1+\omega}{2}\iota\left(1-i_t^J\right)F_\xi\left(i_t^J\right) + \iota^2\int_{i_t^J}^1 \left(x-i_t^J\right)(1-x)\,dF_\xi(x)\right]\frac{\partial a_t}{\partial\iota} + a_t\frac{1+\omega}{2}\left(1-i_t^J\right) + (1-a_t)\int_{i_t^J}^1\left[\frac{1+\omega}{2}\left(1-i_t^J\right) - 2\omega\iota\left(x-i_t^J\right)(1-x)\right]dF_\xi(x) > 0 \quad \text{(A120)}$$

because $\partial a_t/\partial\iota > 0$ while the integrand on the second line can be written

$$\frac{1+\omega}{2}\left(1-i_t^J\right) - 2\omega\iota\left(1-i_t^J\right)\left(x-i_t^J\right) + 2\omega\iota\left(x-i_t^J\right)^2, \quad \text{(A121)}$$

a quadratic that is always positive because its determinant is $-4\omega\iota\left[1+\omega-\omega\iota\left(1-i_t^J\right)\right]\left(1-i_t^J\right) < 0$.

Finally,

$$\frac{\partial}{\partial i_t^J}\Pr\left(i_{t+1}^J > i_t^J | i_t^J\right) = \left[\frac{1+\omega}{2}\iota\left(1-i_t^J\right)F_\xi\left(i_t^J\right) + \iota^2\int_{i_t^J}^1 \left(x-i_t^J\right)(1-x)\,dF_\xi(x)\right]\frac{\partial a_t}{\partial i_t^J} - a_t\frac{1+\omega}{2}\iota - (1-a_t)\,\iota\frac{1+\omega}{2}\left(1-i_t^J\right)f_\xi\left(i_t^J\right) - (1-a_t)\,\iota\int_{i_t^J}^1\left[\frac{1+\omega}{2} - \omega\iota(1-x)\right]dF_\xi(x) \quad \text{(A122)}$$

is generally ambiguous because $\partial a_t/\partial i_t^J > 0$ while the second line is negative. It is unambiguously negative in the limit:

$$\lim_{i_t^J \to 1} \frac{\partial}{\partial i_t^J} \Pr\left(i_{t+1}^J > i_t^J | i_t^J\right) = -a_t \frac{1+\upsilon}{2}\iota < 0. \tag{A123}$$

A.5. Proof of Proposition 6

Partnership t is formed if and only if $\Lambda\left(i_t^J, \iota_t, \pi, \alpha\right) \leq \hat{\Lambda}\left(\underline{u}_P, \upsilon\right)$, which can be rewritten as:

$$\iota_t \geq \underline{\iota}_t \equiv \max\left\{0, \frac{1-\pi}{1-\pi-\alpha} \frac{\int_{i_t^J}^1 x dF_\xi(x) - \hat{\Lambda}\left(\underline{u}_P, \upsilon\right)}{\int_{i_t^J}^1 x(1-x)\, dF_\xi(x)}\right\}, \tag{A124}$$

such that $\underline{\iota}_t = 0$ and all partnerships are formed if and only if

$$\int_{i_t^J}^1 x dF_\xi(x) \leq \hat{\Lambda}\left(\underline{u}_P, \upsilon\right), \tag{A125}$$

namely $i_t^J \geq \underline{i}^0$ for a threshold $\underline{i}^0 < 1$ given that $\hat{\Lambda}\left(\underline{u}_P, \upsilon\right) < 1$.

When $\underline{\iota}_t > 0$, it is implicitly defined by $\Lambda\left(i_t^J, \underline{\iota}_t, \pi, \alpha\right) = \hat{\Lambda}\left(\underline{u}_P, \upsilon\right)$. Since $\partial\Lambda/\partial\iota < 0$ for $\omega > 0$ while $\partial\Lambda/\partial i_t^J \leq 0$, $\partial\Lambda/\partial\pi \geq 0$, $\partial\Lambda/\partial\alpha \geq 0$ and Λ increases if ξ_t shifts up in the sense of first-order stochastic dominance, the implicit-function theorem implies that $\partial\underline{\iota}_t/\partial i_t^J \leq 0$, $\partial\underline{\iota}_t/\partial\pi \geq 0$, $\partial\underline{\iota}_t/\partial\alpha \geq 0$ and $\underline{\iota}_t$ increases if ξ_t shifts up in the sense of first-order stochastic dominance.

Some partnerships are formed if and only if $\Lambda\left(i_t^J, 1, \pi, \alpha\right) \leq \hat{\Lambda}\left(\underline{u}_P, \upsilon\right)$, and thus for $i_t^J = 0$ if and only if:

$$\hat{\Lambda}\left(\underline{u}_P, \upsilon\right) \geq \Lambda(0, 1, \pi, \alpha) = \mathbb{E}\xi_t - \frac{1-\pi-\alpha}{1-\pi}\mathbb{E}\left[\xi_t(1-\xi_t)\right]. \tag{A126}$$

Since $\xi_t \in [0,1]$, $\Lambda(0,1,\pi,\alpha) \leq \mathbb{E}\xi_t$ and thus a sufficient but not necessary condition is $\mathbb{E}\xi_t \leq \hat{\Lambda}\left(\underline{u}_P, \upsilon\right)$.

In period t, if the parties draw an ability to collect novel evidence $\iota_t < \underline{\iota}_t$ the partnership is not formed. If $\iota_t \geq \underline{\iota}_t$ the partnership is formed and the agent exerts effort

$$a_t = \hat{a}\left(\Lambda\left(i_t^J, \iota_t, \pi, \alpha\right), \underline{u}_P, \upsilon\right) > 0. \tag{A127}$$

Considering that ι_t is a random draw from the distribution $F_\iota(.)$, the evolution of precedent is described by:

$$\Pr\left(i_{t+1}^{J} > j | i_t^J\right) = (1-\pi)\,\iota\,(1-j)\int_{\underline{\iota}_t}^{1} \hat{a}\left(\Lambda\left(i_t^J, h, \pi, \alpha\right), \underline{u}_P, v\right) dF_\iota(h)$$
$$+\iota\left\{\int_j^1 \left[\alpha + (1-\pi-\alpha)(1-\iota+\iota x)\right](x-j)\,dF_\xi(x) + (1-\alpha)\int_{i_t^J}^1 (1-\max\{j,x\})\,dF_\xi(x)\right\}$$
$$\cdot\int_{\underline{\iota}_t}^{1}\left[1-\hat{a}\left(\Lambda\left(i_t^J, h, \pi, \alpha\right), \underline{u}_P, v\right)\right] dF_\iota(h) \text{ for all } j \in \left[i_t^J, 1\right]. \quad \text{(A128)}$$

Thus, it is represented by an absorbing Markov chain with the same qualitative properties described by Proposition 5.