The Volatility of Inflation Expectations and Interest Rates*

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Abstract

Using the consensus forecasts of major banks during 1989-2022, we link inflation expectations to interest rates in 18 advanced economies. We detect horizon-increasing overreaction: high expected inflation today predicts future inflation overestimation, especially at long horizons, and higher real returns on nominal bonds, especially at long maturities. As a result, high expected inflation today predicts a future redistribution of wealth from borrowers to lenders. To understand the drivers of such redistribution, we offer a learning model where investors overweight states that are salient in memory due to their past frequency or similarity to current inflation. The model endogenizes belief under- and overreaction based on features of the inflation DGP, helping account for observed cross-country variation in belief biases and return predictability.

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1 Introduction

Inflation expectations are central to macro-financial dynamics. By the Fisher equation, the nominal interest rate i is equal to the ex-ante real rate r plus expected inflation π^e:

$$i = r + \pi^e. \tag{1}$$

Classic studies of interest rates and inflation, theoretical (Woodford, 2003b; Galí, 2015) and empirical (Fama, 1975; Mishkin, 1992; Gürkaynak et al., 2010), assume rational expectations. In this approach, expected inflation π^e affects resource allocation via the ex-ante real rate r. With flexible prices, higher π^e leaves r unchanged: it only affects the nominal rate i. With nominal rigidities, by contrast, higher π^e affects the real rate r – in part through monetary policy – shaping consumption, investment and output.

Contrary to this approach, growing work detects biases in professional inflation forecasts, taking the form of predictable forecast errors (Coibion and Gorodnichenko, 2015; Bordalo et al., 2020). Accounting for these biases is important. First, they could distort the ex-ante real rate r and output. Second, by causing systematic forecast errors, they could drive a wedge between r and the average ex post real rate $i - \pi$. By redistributing wealth between borrowers and lenders with different propensities to spend, this wedge could affect output even under flexible prices. Fisher (1933) stressed the cost of debt deflation; Doepke and Schneider (2006) document the redistributive effects of inflation. Critically, while in a rational world such redistributions are random, they systematically occur and affect output when beliefs are biased. What are, then, the drivers of biases in inflation expectations? What are their consequences for interest rates?

We address these questions by linking the inflation forecasts of major banks from Consensus Economics to interest rates in 18 advanced economies over 1989-2022. We then present a psychologically founded learning model that sheds light on biases and their economic effects. Sections 2 and 3 document our key motivating facts. First, consensus inflation forecasts sharply overreact at medium and long horizons, while exhibiting a small or even reverse bias (underreaction) at short horizons. Horizon-increasing overreaction is a broad feature of beliefs: D'Arienzo (2020) detects it in individual interest rate forecasts; Afrouzi et al. (2023) in an abstract lab experiment and Halperin and Mazlish (2025) in consensus forecasts for various macro variables. Second, and crucially, overreaction ties to interest rates: high expected inflation today predicts high future real yields to maturity and excess returns on nominal bonds, especially at long maturities (which are also less influenced by monetary policy). Consistent with a direct role of beliefs, return predictability is robust to controlling for proxies of time-varying ex-ante real rates and to correcting our estimates for Stambaugh bias, following Boudoukh et al. (2022).

These facts point to a link between belief overreaction and excess nominal rate volatility: high current inflation prompts overestimation of future inflation, especially at long horizons, causing investors to demand too high a nominal rate, especially for long-term bonds. Because future inflation is on average less than anticipated, high returns follow. The entailed wealth redistribution from borrowers to lenders is sizable: a 2% higher expected inflation today predicts a higher real repayment cost of about 2% of the value of a 2-year loan, and 12% of that of a 10-year one. In heterogeneous agent models, this redistribution can cause output to drop. If expected inflation is high in booms, the effect is countercyclical, amplifying boom-bust cycles.

To study the direction and strength of this mechanism we must understand the source of biases, which arguably vary across environments as in the Lucas critique (Lucas, 1972). The data is at odds with leading models of expectations. Rational inattention predicts consensus underreaction, inconsistent with our evidence of long term overreaction. Diagnostic expectations (DE) (Bordalo et al., 2018; Bianchi et al., 2024b)) capture overreaction, but do not explain why its presence and strength depend on the forecast horizon. Adaptive expectations (Cagan, 1956) are backward-looking, so they make no predictions across horizons.

Section 4 offers a model of learning under selective recall. Memory limits have been used to justify investor overweighting of recent states in constant-gain learning (Evans and Honkapohja, 2009; Marcet and Nicolini, 2003) or of representative states in DE (Bordalo et al., 2020).[1] Even if investors use statistical models, they can overweight salient states when making qualitative adjustments to model outputs, consistent with the role of past experiences for professionals (Malmendier et al., 2021). Our key innovation is to model memory limits explicitly, based on robust regularities in cued recall (Kahana, 2012): frequency and similarity. When forecasting inflation, the agent overweights frequent past episodes, say inflation at 2%. At the same time, current inflation at 3% cues the agent to overweight past states similar to 3%, or that followed 3% episodes. We build on the model in Bordalo et al. (2023) but allow for mnemonic learning: the database evolves with realized inflation and interacts with time-varying retrieval cues. Unlike in models exhibiting recency effects, belief updating is state-dependent. Compared to DE, belief distortions are founded in frequency and similarity.

We find that long-run beliefs converge to a stable distortion of the true DGP, with memory forces pulling in different directions. Frequency anchors beliefs to the long-run mean, promoting underreaction. Similarity anchors them to current inflation, promoting forward-looking overreaction. Combined together, these forces produce horizon-increasing overreaction, whose strength depends on the features of the DGP. If, for instance, inflation follows an AR(1) process, higher persistence promotes underreaction, while higher volatility promotes overreaction. More broadly, biases evolve with changing histories and cues, leading to new predictions.

To assess these predictions, Section 5 uses the inflation database and the current inflation cue – the model's observables – to estimate two similarity parameters – its only degrees of

[1] Bianchi et al. (2022) also assumes limited memory of inflation regimes and Farmer et al. (2024) assume slow learning under an incorrect prior, which may stem from memory of past history.

freedom. We do so by fitting model-implied beliefs to CE forecasts in the pooled sample. The model produces realistic horizon-increasing overreaction and return predictability, even under constant ex-ante real rates. Crucially, it captures observed cross-country and cross-horizon variation in the strength of belief extrapolation and in the predictability of forecast errors and returns. We show that recency-based adaptive learning cannot explain this variation.

Growing work studies non-rational investor beliefs. Sims (2003), Woodford (2003a), Mankiw and Reis (2002) and Gabaix (2014) stress rigidity due to costly attention, detected in US short-term inflation forecasts (Coibion and Gorodnichenko, 2015). Constant-gain learning (Evans and Honkapohja, 2009; Marcet and Nicolini, 2003) and DE models (Bordalo et al., 2018, 2020; Bianchi et al., 2024b,a) produce belief volatility tied to memory limits. We rely on memory forces, frequency and similarity, that account for beliefs about novel risks (Bordalo et al., 2024a), stock returns forecasts (Jiang et al., 2024), and household inflation expectations (Gennaioli et al., 2024). The latter paper develops survey tests of memory effects given the observable households' database. Our learning setting unveils the role of the DGP, endogenizing rigidity and overreaction in forecasts, interest rates, and their variation across countries. The conclusions argue that memory opens new avenues in macrofinance, also for monetary policy.[2]

A second strand of work studies excess volatility of interest rates Shiller (1979); Mankiw and Summers (1984) and predictability of bond returns based on yield curve "factors" (Fama and Bliss, 1987; Cochrane and Piazzesi, 2005). These patterns violate the expectations hypothesis and are explained by adding time-varying risk premia to rational models (Wachter, 2006; Bansal and Shaliastovich, 2013; Vayanos and Vila, 2021). Giglio and Kelly (2018) show that excess volatility of long rates relative to short ones is at odds with these models. We directly link excess interest rate volatility to overreacting measured inflation forecasts. Cieslak (2018) offers complementary evidence of bond return predictability based on short-term interest rates forecasts. Bianchi et al. (2022) and Nagel (2024) study how rigid household inflation expectations drive low-frequency changes in nominal and real interest rates by shaping monetary policy, with a focus on the inflation of the 1970s and disinflation of the 1980s. We instead show high frequency overreaction starting from the 1990s, a period of more stable inflation, and account for cross-country variation in biases and return predictability. We contribute to work showing that investor beliefs are key to macro-financial volatility. (Bordalo et al., 2024b,c) link stock price volatility to overreaction in analysts' long-term earnings forecasts. Bouchaud et al. (2019) link stock price rigidity to underreacting short-term earnings forecasts. By producing horizon-increasing overreaction, selective memory can help reconcile these patterns.

[2]Alternative approaches to modeling beliefs in macroeconomics include natural expectations (Fuster et al. (2012)), incorrect models (Gabaix (2019), Angeletos et al. (2021), Molavi et al. (2024)), and level-k thinking (García-Schmidt and Woodford (2019), Farhi and Werning (2019), Iovino and Sergeyev (2023)).

2 Overreaction in Inflation Expectations

2.1 Data Sources and Sample

Expectations are drawn from Consensus Economics (CE), which surveys macroeconomic forecasts from major financial institutions worldwide. Our main variable is the consensus forecast for year-on-year CPI inflation h calendar years ahead, with $h = 0, 1, 2, 3, 4, 5,$ 6-10. The $h = 0$ forecast pertains to current year inflation; the $h =$ 6-10 forecast pertains to average inflation between 6 to 10 years ahead. We also use CE consensus forecasts of next year's 3-month and 10-year interest rates to control for monetary policy and other factors. Forecasts are reported quarterly after 2014, semiannually before.

Realized year-on-year CPI inflation is obtained from the IMF Data Mapper and the World Development Indicators (WB); when both sources are available, we take the average.[3] To compute interest rates and returns, we use bond prices for maturities from 1 to 10 years from Refinitiv. The 4-year maturity is excluded due to missing data. We focus on IMF-defined "advanced economies" with consistent data coverage. For each horizon h, countries are included if they have data on bond yields and forecasts since at least 1999Q4. The sample comprises 18 countries and ranges from 1989Q4 to 2022Q1. Table 7 describes the data; Table 8 reports the countries at each maturity.[4]

In our sample, which starts after the deflation of the 1980s, average yearly inflation (across countries and over time) is 4.21% with a standard deviation of 4.29%. CE forecasts do not exhibit systematic upward or downward bias: average forecast error at the 1-year horizon is -0.0046 with $p = 0.9507$. Arguably, this is a feature of relatively stable inflation in our sample period. A systematic bias could arise during or shortly after unstable inflation, also due to a "Peso problem". Figure 5 in Appendix A.1 plots in-sample realized inflation for the 18 countries.

2.2 Rationality Tests

To understand whether interest rates properly compensate for inflation risk, a first-order question is whether investors systematically over- or underestimate future inflation during high inflation states. To assess this issue, we predict the consensus forecast error for country c at horizons $t + h$ using two proxies for the inflation state at t. The first one is the time t inflation forecast for h calendar years ahead, $\mathbb{F}_t(\pi_{c,t+hY})$. A high forecast signals that the country is in a high inflation state at t. We thus pool countries and time periods, and estimate:

$$\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right) = \alpha_h + \delta_h \cdot \mathbb{F}_t\left(\pi_{c,t+hY}\right) + \varepsilon_{c,t+hY}. \tag{2}$$

[3] IMF and World Bank data cover up to 2023; the sources for 2024Q1 data are reported in Table 7.

[4] The UK is excluded from our main sample because its CE coverage starts only in 2004. Results are robust to having all countries, including the UK, at all maturities, see Tables 24 to 28.

A negative coefficient $\delta_h < 0$ implies that forecasters overestimate future inflation in a high inflation state, and vice-versa in a low state. The opposite occurs if $\delta_h > 0$. Our second proxy is based on recent inflation, which we observe at yearly frequency. The state in year t is average inflation in the two prior years $\overline{\pi}_{c,t} = (\pi_{c,(t-1)Y} + \pi_{c,(t-2)Y})/2$. The forecast in year t is then computed by averaging forecasts within the year. Again, a negative (positive) coefficient detects over- (under-) estimation of future inflation in a high inflation state.

Table 1 reports the estimates, with Panel A using the first proxy and Panel B the second. We denote the forecast error at horizon h by $FE_{c,h} \equiv \pi_{c,t+hY} - \mathbb{F}_t(\pi_{c,t+hY})$. All regressions in the paper include country dummies and use Driscoll-Kraay standard errors to control for heteroskedasticity and spatio-temporal correlation.

Table 1: Predicting the consensus forecast error using the current inflation state

	Panel A					
	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{c,0}$	$FE_{c,1}$	$FE_{c,2}$	$FE_{c,3}$	$FE_{c,5}$	$FE_{c,6\text{-}10}$
$\mathbb{F}_t(\pi_{c,t+hY})$	0.037	-0.356**	-1.004***	-1.114***	-1.655***	-1.207***
	(0.045)	(0.140)	(0.300)	(0.274)	(0.273)	(0.108)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	1029	1029	1171	1033	1093	656
Adj. R^2	0.020	0.062	0.157	0.176	0.252	0.553
	Panel B					
	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{c,0}$	$FE_{c,1}$	$FE_{c,2}$	$FE_{c,3}$	$FE_{c,5}$	$FE_{c,6\text{-}10}$
$\overline{\pi}_{c,t}$	-0.041	-0.218*	-0.295**	-0.339**	-0.626***	-0.490***
	(0.039)	(0.119)	(0.141)	(0.127)	(0.189)	(0.074)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	420	420	462	416	462	331
Adj. R^2	-0.002	0.048	0.057	0.088	0.229	0.426

Note. Each column reports the country pooled regression at horizon $h = 0, ..., 6\text{-}10$: the dependent variable is the consensus forecast error $\pi_{c,t+hY} - \mathbb{F}_t(\pi_{c,t+hY})$ at horizon h and country c. The independent variables are: (i) the consensus forecast for country c at horizon h, $\mathbb{F}_t(\pi_{c,t+hY})$ (Panel A), and (ii) the average realized inflation of country c over the past two years, $\overline{\pi}_{c,t} = (\pi_{c,(t-1)Y} + \pi_{c,(t-2)Y})/2$ (Panel B). We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

In Panel A, δ_h is positive but not significant at the shortest horizon. For $h \geq 1$, the coefficient turns negative: high current expected inflation predicts overestimation of future inflation. Critically, the effect is stronger at longer horizons h. These results are robust when estimating equation (2) at the annual frequency (see Appendix Table 18). Panel B confirms the result using our second proxy: high recent inflation predicts overestimation of future inflation, more strongly so at longer horizons. Magnitudes are meaningful. In Panel A, a one standard deviation increase in the current forecast reduces the future forecast error by 0.217 standard deviations at the 1-year horizon, and by 0.826 standard deviations at the 6-10-year horizon. In Panel B, a one standard deviation increase in recent inflation reduces the error by 0.205 standard deviations at the 1-year and by 0.667 at the 6-10-year horizons.

De Silva and Thesmar (2024) show that noise or measurement error in expectations could create a spurious negative coefficient when predicting the forecast error with the current forecast (or its revision). This mechanism cannot explain our results for two reasons. First, noise or measurement error in individual analyst forecasts is averaged out in the consensus. Second, we see negative predictability also in Panel B, using past inflation as a predictor.

A high inflation state can prompt future inflation overestimation for two reasons. First, forecasters may overreact to recent "high inflation" news, proxied by an upward consensus revision. Second, holding news constant, forecasters may be too pessimistic following periods during which their forecasts were high, indicating a persistent overreaction to past inflation. Assessing these mechanisms is key for evaluating theories of belief formation. Coibion and Gorodnichenko (2015) stress underreaction of professional consensus forecasts to *recent* news, detected through a positive correlation between the one-quarter revision and future forecast errors for US short-term (three quarters ahead) inflation. They view the evidence as supporting rational inattention (Sims, 2003; Woodford, 2003a; Mankiw and Reis, 2002). By contrast, Bordalo et al. (2020) detect overreaction of individual forecasts, pointing to non-rational updating.

To test for these mechanisms, we follow Bordalo et al. (2024b) and predict consensus forecast errors using the four-quarter revision – $\mathbb{F}_t\left(\pi_{c,t+hY}\right) - \mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$ – our proxy for news, and the consensus forecast from four quarters earlier – $\mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$ – our proxy for past pessimism. Besides being informative about mechanisms, this test is methodologically more appropriate for assessing under- or overreaction to news than the correlation between forecast errors and revisions. The latter test is well specified if forecasters are rational, the case considered by Coibion and Gorodnichenko (2015), not otherwise. The reason is that with non-rational updating the forecast revision is correlated with the lagged forecast – a pattern that we observe in the data (see Appendix Table 9) – which creates an omitted variable problem if the lagged forecast is not itself used as a predictor. Using analyst forecasts of US firms' long-term earnings growth, Bordalo et al. (2024b) find that the consensus overreacts both to recent news and to its lagged value. In the context of inflation, we analogously estimate:

$$\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right) = \alpha_h + \delta_{1,h} \cdot \left[\mathbb{F}_t\left(\pi_{c,t+hY}\right) - \mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)\right] + \delta_{2,h} \cdot \mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right) + \varepsilon_{c,t+hY}. \quad (3)$$

A negative $\delta_{1,h}$ indicates overreaction to recent news; a negative $\delta_{2,h}$ indicates persistent overreaction to past high inflation. Positive coefficients instead indicate underreaction.

Table 2 shows that, at horizons up to one year, $\delta_{1,h}$ is positive, although not statistically significant, suggesting mild underreaction to news. At longer horizons, $\delta_{1,h}$ is significantly negative, indicating sharp overreaction to news. The lagged forecast coefficient $\delta_{2,h}$ is negative and significant at all horizons, showing persistent overreaction to past inflation. The rigidity of consensus inflation expectations documented by Coibion and Gorodnichenko (2015) is thus a feature of short-term (within-year) forecasts. At long horizons, consensus beliefs overreact, consistent with sharply non-rational updating.[5] Interestingly, for $h \geq 2$, overreaction to recent news and to past states are comparable: the magnitude of the two coefficients converges ($\delta_{1,h} = \delta_{2,h}$ cannot be rejected at conventional levels). This means that the simpler predictability regression in equation (2), which imposes $\delta_{1,h} = \delta_{2,h}$, captures the key pattern in the data: a horizon-increasing overreaction of consensus inflation forecasts. As we will see, this is also the specification emerging from our memory model.

Table 2: Predicting consensus error from recent news and past inflation states

	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{c,0}$	$FE_{c,1}$	$FE_{c,2}$	$FE_{c,3}$	$FE_{c,5}$	$FE_{c,6\text{-}10}$
$\mathbb{F}_t(\pi_{c,t+hY}) - \mathbb{F}_{t-4}(\pi_{c,t+hY})$	0.160	0.215	-0.847*	-1.092***	-1.297***	-1.123***
	(0.098)	(0.240)	(0.448)	(0.411)	(0.257)	(0.110)
$\mathbb{F}_{t-4}(\pi_{c,t+hY})$	-0.105**	-0.625**	-1.232***	-1.318***	-1.765***	-0.988***
	(0.045)	(0.246)	(0.433)	(0.399)	(0.380)	(0.107)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	973	970	1104	969	1008	618
Adj. R^2	0.088	0.103	0.137	0.151	0.183	0.398

Note. Each column reports the country pooled regression at horizon $h = 0, ..., 6\text{-}10$: the dependent variable is the inflation consensus forecast error $\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The dependent variables are the consensus revision $\mathbb{F}_t\left(\pi_{c,t+hY}\right) - \mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$ and the lagged inflation forecast $\mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$. We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Overall, consensus inflation forecasts exhibit horizon-increasing overreaction. As we discuss in Section 5, this pattern cannot be accommodated by leading models of belief formation, motivating our mnemonic approach. Before presenting the model, the logic of the Fisher equation raises a key question: does overreaction link to excess interest rate volatility?

[5]If we replicate the Coibion and Gorodnichenko (2015) test, regressing the forecast error on the four quarters forecast revision, we see positive predictability at short horizons and negative but not statistically significant predictability at longer horizons, see Appendix Table 10. If we estimate equation (3) using US short-term CPI forecasts in the SPF as in Coibion and Gorodnichenko (2015) and US quarterly realized inflation from the Bureau of Labor Statistics, we obtain no predictability from the revision and negative predictability from the lagged forecast – consistent with our findings in columns (1) and (2) of Table 2; see Appendix Table 11.

3 Belief Overreaction and Interest Rate Volatility

Under the expectations hypothesis (Campbell and Shiller, 1991), ex-ante real rates are constant and nominal rates are shaped by inflation expectations. Let $r_c^{(h)}$ be the ex-ante real rate on a zero-coupon bond of maturity h in country c. The nominal rate $i_{c,t}^{(h)}$ satisfies:

$$i_{c,t}^{(h)} = r_c^{(h)} + \mathbb{E}_t\left(\overline{\pi_{c,t+1,h}}\right), \tag{4}$$

where $\overline{\pi_{c,t+1,h}} = (\pi_{t+1Y} + ... + \pi_{t+hY})/h$ is average inflation between t and $t + hY$, and $\mathbb{E}_t(\cdot)$ denotes market expectations. Under the expectations hypothesis, a unit slope in a regression of interest rates on CE forecasts would point to excess volatility: nominal rates overreact in tandem with beliefs. This conclusion, however, neglects that the real rate $r_c^{(h)}$ may itself vary over time, due to monetary policy or risk premia. For instance, the central bank may raise short-term nominal and real rates in reaction to expected inflation, as in Bianchi et al. (2022) and Nagel (2024). In addition, real rates can respond to shocks that simultaneously affect expectations. To isolate the role of beliefs, we need a proxy for the real rate.[6]

The most widely used proxies rely on "factors" built from time variation in nominal rates. These factors cannot be used as controls in equation (4) because they mirror the dependent variable. We thus develop excess volatility tests based on bond return predictability. Under the expectations hypothesis, which assumes both constant $r_c^{(h)}$ and market efficiency, returns should be unpredictable. If the required return varies over time, predictability from beliefs *conditional* on a proxy for $r_c^{(h)}$ would point to a violation of market efficiency.

Real yield to maturity. The real yield to maturity of a bond is defined as the difference between the bond's nominal yield at issuance $i_{c,t}^{(h)}$ and the average realized inflation over its life. From equation (4), the ex-post real yield on a bond issued at t with maturity h is then

$$\widetilde{r}_{c,t,t+h}^{(h)} = r_c^{(h)} - \left[\overline{\pi_{c,t+1,h}} - \mathbb{E}_t\left(\overline{\pi_{c,t+1,h}}\right)\right]. \tag{5}$$

If inflation is lower than expected, $[\overline{\pi_{c,t+1,h}} - \mathbb{E}(\overline{\pi_{c,t+1,h}})] < 0$, the real yield exceeds the ex-ante real rate $\widetilde{r}_{c,t,t+h}^{(h)} > r_c^{(h)}$; the opposite holds when inflation is higher than expected. Under rational expectations, inflation surprises – and thus returns – are unpredictable. This is no longer true if investors overreact, which motivates our first test (all proofs are in the Appendix).

Prediction 1. *If investors overreact, more strongly so at longer maturities, the regression:*

$$\widetilde{r}_{c,t,t+h}^{(h)} = \theta_{0,h}^r + \theta_h^r \cdot \mathbb{F}_t(\overline{\pi_{c,t+1,h}}) + \varepsilon_{c,t,t+h}^{(h)} \tag{6}$$

[6] In the typical NK model, the coefficient on expected inflation would capture both the reactivity of the Taylor rule to expectations and the co-movement of beliefs with supply and demand shocks.

should yield a positive coefficient θ^r_h, with its magnitude increasing in h.

In high inflation states, expected inflation and hence nominal rates are excessively high, predicting higher realized real rates, especially at long maturities. The reverse occurs in low inflation states. In equation (6), the ex-ante real rate $r_c^{(h)}$ is embedded in the intercept $\theta^r_{0,h}$. To account for its potential time variation, we control for a direct measure of $r_c^{(h)}$ at t: the nominal rate minus the inflation forecast at t (details are provided below). Under the rational null, forecasts should not have predictive power conditional on such proxy. Finding $\theta^r_h > 0$, by contrast, ties predictability to belief overreaction.

Excess holding period return. The excess holding period return is defined as the one-period return on a bond with maturity $h > 1$ in excess of the short rate. Formally, an investor buying a bond (with unit face value) at price $\ln P_t^{(h)} = -h \cdot i_t^{(h)}$ and selling it at $t+1$ obtains a return $y_{t,t+1}^{(h)} = P_t^{(h)}/P_{t+1}^{(h-1)} - 1$, which can be approximated as $y_{t,t+1}^{(h)} \approx i_t^{(h)} + (h-1)(i_t^{(h)} - i_{t+1}^{(h-1)})$. Relative to the short rate $i_t^{(1)}$ the excess return is then:

$$rx_{t,t+1}^{(h)} = rx^{(h)} - (h-1)\left[\mathbb{E}_{t+1}\left(\overline{\pi_{t+2,h}}\right) - \mathbb{E}_t\left(\overline{\pi_{t+2,h}}\right)\right], \tag{7}$$

where $rx^{(h)} \equiv -(h-1)r^{(h-1)} + hr^{(h)} - r^{(1)}$ is constant. If investors revise beliefs upward at $t+1$, $\mathbb{E}_{t+1}(\overline{\pi_{t+2,h}}) - \mathbb{E}_t(\overline{\pi_{t+2,h}}) > 0$, the nominal rate increases, depressing the bond's resale price and lowering the realized return. Conversely, a downward revision raises the realized return. Under rationality, future revisions are unpredictable and so are returns. If, instead, investors overreact, revisions tend to be low following periods of high expected inflation and high after periods of low expected inflation – a pattern observed in the data (see Appendix Table 9) – which motivates our second test.

Prediction 2. *If investors overreact, more strongly so at longer maturities, the regression:*

$$rx_{c,t,t+1}^{(h)} = \theta^{ex}_{0,h} + \theta^{ex}_h \cdot \mathbb{F}_t(\overline{\pi_{c,t+1,h}}) + \varepsilon_{c,t,t+1}^{(h)} \tag{8}$$

should yield a positive coefficients θ^{ex}_h, with its magnitude increasing in h.

Due to horizon-increasing overreaction, nominal rates are excessively high in high-inflation states, depressing bond prices. As a result, bonds appreciate over time, especially at longer maturities. Predictability here originates from belief revisions, not errors. To control for possible time variation in required returns (embedded in the intercept $\theta^{ex}_{0,h}$), we also control for their leading proxies in the excess return predictability literature Duffee (2013): the level and slope of the yield curve at t.[7] Conditional on these proxies, evidence of predictability from

[7]Fama and Bliss (1987) offer early evidence of predictability based on the spread between the h-year forward and spot rates. Cochrane and Piazzesi (2005) build a related predictor based on forward rates.

expectations, $\theta_h^{ex} > 0$, would again point to market inefficiency driven by belief overreaction.

Our tests, which estimate equations (6) and (8) controlling for yield-curve proxies of real rates, are demanding: changes in the yield curve today reflect not only shifts in ex-ante real rates, but also in investor beliefs, for which CE forecasts are a noisy proxy. Conditional predictability from measured beliefs would then indicate that CE consensus contains information for market beliefs, linking predictability to market inefficiency. By comparison, current tests of rational models are weak: they do not directly measure time-varying risk aversion or other determinants of ex ante real rates, and thus cannot rule out that the yield curve may predict returns through the biased beliefs it embeds.

One concern in our tests is that maturity increasing return predictability may be contaminated by a bias due to a persistent predictor Stambaugh (1999). For this reason, we perform robustness tests using the adjustment for such bias developed by Boudoukh et al. (2022).

3.1 Expectations and Interest Rates

We begin by testing whether nominal rates correlate with expectations – a necessary condition for beliefs to cause excess interest rate volatility. Using CE data, we construct expected average inflation over the next h calendar years, $\mathbb{F}_t(\overline{\pi_{c,t+1,h}})$, and estimate:

$$i_{c,t}^{(h)} = \alpha_h + \lambda_h \cdot \mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right) + \varepsilon_{c,t}^{(h)}. \tag{9}$$

We obtain $i_{c,t}^{(h)}$ from the zero-coupon price $P_{c,t}^{(h)}$ using the identity $\ln P_{c,t}^{(h)} = -h \cdot i_{c,t}^{(h)}$. We compute quarterly rates by averaging monthly ones. We do not expect $\lambda_h = 1$: real rates may vary over time and CE forecasts may imperfectly proxy for market beliefs, the drivers of nominal rates.

Table 3, Panel A, reports our estimates, with maturity ranging from 1 year (Column 1) to 6-10 years (Column 5). Panel B controls for CE forecasts of nominal rates. the latter forecasts reflect – conditional on inflation forecasts – beliefs about real rates, including expectations about central bank policy (Bauer et al., 2024). To match maturities, we use the expected 3-month rate for $h = 1, 2, 3$ and the 10-year rate for $h = 5$, 6-10. We cannot include both measures due to collinearity.[8]

In Panel A, higher expected inflation over the next h years is associated with higher nominal rates at the same maturity. The R^2 is sizable, ranging from 31% to 54%, with the lowest fit observed at the 6-10-year maturity, likely reflecting missing data. The estimated coefficient is statistically indistinguishable from 1 at $h = 1$, and approaches 2 at longer maturities. A coefficient above one means that higher expected inflation is associated with higher ex-ante real rates. Interestingly, this occurs for long-term bonds, where monetary policy likely plays

[8] Variance Inflation Factors (VIFs) for one or both measures exceed the standard threshold of 10 for horizons 1 to 5. VIFs for inflation forecasts remain well below 10.

Table 3: Nominal yields and consensus inflation forecast

	Panel A				
	(1)	(2)	(3)	(4)	(5)
	$i_{c,t}^{(1)}$	$i_{c,t}^{(2)}$	$i_{c,t}^{(3)}$	$i_{c,t}^{(5)}$	$i_{c,t}^{(6\text{-}10)}$
$\mathbb{F}_{\rm t}\left(\overline{\pi_{\rm c,t+1,h}}\right)$	1.090***	2.144***	2.324***	2.658***	2.545***
	(0.173)	(0.303)	(0.269)	(0.282)	(0.543)
Country fixed effects	✓	✓	✓	✓	✓
N	972	1163	1079	1296	550
Adj. R^2	0.339	0.488	0.540	0.527	0.314
	Panel B				
	(1)	(2)	(3)	(4)	(5)
	$i_{c,t}^{(1)}$	$i_{c,t}^{(2)}$	$i_{c,t}^{(3)}$	$i_{c,t}^{(5)}$	$i_{c,t}^{(6\text{-}10)}$
$\mathbb{F}_{\rm t}\left(\overline{\pi_{\rm c,t+1,h}}\right)$	0.039	0.067**	0.243***	0.369***	0.213**
	(0.026)	(0.034)	(0.074)	(0.089)	(0.103)
$\mathbb{F}_{\rm t}(i_{\rm c,t+1}^{(k)})$	0.982***	1.029***	0.949***	1.011***	0.938***
	(0.018)	(0.016)	(0.023)	(0.033)	(0.029)
Country fixed effects	✓	✓	✓	✓	✓
N	972	1146	1060	1124	550
Adj. R^2	0.962	0.973	0.938	0.965	0.948

Note. Each column of Panel A reports a pooled regression at horizon $h = 1, ..., 6\text{-}10$ whose dependent variable is the nominal yield at t to maturity h in country c and the independent variable is the consensus forecast for average inflation over h years in the same country and time, computed as $\mathbb{F}_{\rm t}\left(\overline{\pi_{\rm c,t+1,h}}\right) = \frac{1}{h}\sum_{h'=1}^{h}\mathbb{F}_{\rm t}\left(\pi_{c,t+h'Y}\right)$. Panel B controls for the expected 3 months rate for $h = 1, 2, 3$, Columns (1)-(3), and for the expected 10 year rate for $h = 5, 6\text{-}10$, Columns (4) and (5). We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

a smaller direct role. Arguably, then, either higher expected inflation correlates with higher longer term real rates, or CE forecasts are more sluggish than market beliefs at long horizons.

In Panel B, we control for expected rates, which are tightly aligned with current rates, with estimated coefficients close to one.[9] The correlation between current rates and expected inflation remains positive and significant. Overall, the sharp comovement of nominal rates and overreacting inflation expectations suggests that the latter may create excess volatility in interest rates. We now assess this hypothesis using our return predictability tests.

[9]Controlling for expected rates sharply increases the R^2, reflecting their strong correlation with current rates. Expected inflation has a much lower correlation (between 0.606 and 0.730) with expected rates.

3.2 Predictable Real Yields to Maturity

Following equation (6), we regress the future real yield to maturity $\widetilde{r}^{(h)}_{c,t,t+h}$ on the current forecast $\mathbb{F}_t(\overline{\pi_{c,t+1,h}})$. We proxy for time varying ex-ante rates using the expected real rate, i.e., the nominal rate at t minus expected inflation, $i^{(h)}_{c,t} - \mathbb{F}_t(\overline{\pi_{c,t+1,h}})$. For validation, we compare this proxy to real rates inferred from inflation-adjusted bonds (King and Low, 2014), which are actively traded and liquid in the US and UK. The correlation between the two measures in these countries ranges from 70% to 90%, supporting our proxy (see Appendix Table 12).

Table 4 reports the baseline estimates in Columns (1)-(5); columns (6)-(10) control for real rates. Consistent with Prediction 1, higher expected inflation predicts higher ex-post yields. The result is robust to controlling for ex-ante rates and becomes stronger as h increases – consistent with horizon-increasing overreaction.[10] Predictability of real yields does not simply reflect changing expected real rates: it also reflects a distorted inflation compensation.

Belief-driven excess volatility is sizable. In the baseline specification, a one standard deviation increase in the inflation forecast lowers the real yield by 0.163 standard deviations at $h = 1$ and by 0.688 at $h = 5$. When we control for ex-ante real rates, the effect remains sizable, amounting to a 0.127 standard deviation increase in the ex-post real return at the 1-year horizon and to a 0.238 standard deviation increase at the 5-year horizon.

3.3 Predictable Excess Returns

To test Prediction 2, we compute for each quarter the 1-year excess bond return $rx^{(h)}_{c,t,t+1} = i^{(h)}_{c,t} + (h-1)\big(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\big) - i^{(1)}_{c,t}$, where quarterly yields average monthly yields. Results are robust to using annual data. To reduce the influence of short-term price volatility, which is notoriously large, we average excess returns over two years:

$$\overline{rx}^{(h)}_{c,t,t+1} = \frac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}. \tag{10}$$

Table 5, columns (1)-(4), reports estimates of equation (8). Columns (5)-(8) control for "level" and "slope" at t: "level" is the 1-year nominal rate, "slope" is the difference between the 10- and 1-year rates – these are standard proxies for time-varying premia (Duffee, 2013).

In line with Prediction 2, higher inflation forecasts predict higher excess returns, more strongly so at longer maturities – consistent with horizon-increasing overreaction. Predictability survives when we control for the level and slope of the yield curve, pointing to market inefficiency. The effect is robust to controlling for expected future interest rates and to smoothing returns over three years (see Appendix Tables 22 and 23). The magnitude is meaningful.

[10] Results are robust at the annual frequency (see Table 20). We have only few data points at the 6-10 maturity because we need ten years of realized inflation to construct the real yield.

Table 4: Predicting ex-post real rates using consensus inflation forecast

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$\tilde{r}^{(1)}_{c,t,t+1}$	$\tilde{r}^{(2)}_{c,t,t+2}$	$\tilde{r}^{(3)}_{c,t,t+3}$	$\tilde{r}^{(5)}_{c,t,t+5}$	$\tilde{r}^{(6\text{-}10)}_{c,t+6,t+10}$	$\tilde{r}^{(1)}_{c,t,t+1}$	$\tilde{r}^{(2)}_{c,t,t+2}$	$\tilde{r}^{(3)}_{c,t,t+3}$	$\tilde{r}^{(5)}_{c,t,t+5}$	$\tilde{r}^{(6\text{-}10)}_{c,t,t+6,t+10}$
$\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$	0.483	1.798***	2.003***	2.302***	-0.894	0.375***	0.470***	0.579***	0.796***	0.613***
	(0.299)	(0.392)	(0.329)	(0.264)	(0.563)	(0.131)	(0.143)	(0.145)	(0.125)	(0.208)
$i^{(h)}_{\mathrm{c,t}} - \mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$						1.202***	1.161***	1.098***	1.031***	1.054***
						(0.125)	(0.105)	(0.076)	(0.059)	(0.090)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
N	972	1163	1019	1080	230	972	1163	1019	1080	230
Adj. R^2	0.058	0.235	0.326	0.384	0.885	0.682	0.736	0.802	0.880	0.885

Note. We predict the real yield to maturity $\tilde{r}^{(h)}_{c,t,t+h}$ using the consensus forecast for average inflation over h years (Columns (1) to (5)), measured as $\mathbb{F}_{\mathrm{t}}(\overline{\pi_{\mathrm{c,t+1,h}}}) = \frac{1}{h}\sum_{h'=1}^{h} \mathbb{F}_{\mathrm{t}}\left(\pi_{c,t+h'Y}\right)$. Columns (6) to (10) add the ex-ante real rate proxy $i^{(h)}_{\mathrm{c,t}} - \mathbb{F}_{\mathrm{t}}(\overline{\pi_{\mathrm{c,t+1,h}}})$. We pool countries and control for country-fixed effects. The sample ranges from 1989Q4 to 2022Q1, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 5: Predicting realized excess returns using consensus inflation forecasts

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6\text{-}10)}_{c,t,t+1}$	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6\text{-}10)}_{c,t,t+1}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.307***	0.429**	3.059***	5.402**	0.097	0.409***	0.926**	3.291***
	(0.103)	(0.165)	(0.916)	(2.311)	(0.072)	(0.149)	(0.407)	(1.158)
$i^{(1)}_{c,t}$					0.270***	0.137	0.867***	0.750**
					(0.036)	(0.089)	(0.137)	(0.326)
$i^{(10)}_{c,t} - i^{(1)}_{c,t}$					0.381***	0.349**	1.458***	2.439***
					(0.081)	(0.148)	(0.234)	(0.520)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓
N	944	873	712	466	941	871	712	466
Adj. R^2	0.138	0.267	0.211	0.121	0.417	0.300	0.516	0.409

Note. Columns (1)-(4) predict the smoothed excess return $\overline{rx}^{(h)}_{c,t,t+1} = \frac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}$ using the consensus forecast for average inflation over h years $\mathbb{F}_t(\overline{\pi_{c,t+1,h}}) = \frac{1}{h}\sum_{h'=1}^{h} \mathbb{F}_t\left(\pi_{c,t+h'Y}\right)$. Columns (5)-(8) additionally control for the level $i^{(1)}_{c,t}$ and slope $i^{(10)}_{c,t} - i^{(1)}_{c,t}$ of the yield curve. The raw one-year excess return is $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)\left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. We pool countries and control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

In columns (5)-(8), a one standard deviation increase in expected inflation predicts a 0.134 standard deviation rise in excess returns at $h = 3$, and a 0.405 rise at $h = 6$–10.

One concern is that our results may be biased due the use of persistent predictors, along the lines of Stambaugh (1999). Indeed, Boudoukh et al. (2022) show that such bias becomes stronger with the forecast horizon, potentially contaminating our results. In Appendix A.6, we apply the bias adjustment method developed by Boudoukh et al. (2022) to our univariate and multivariate predictive regressions, for both the real yield to maturity and the excess return. Due to our panel structure (Boudoukh et al. (2022) develop their tests for time series regressions), we perform the analysis country by country and report the median OLS and adjusted coefficients. Figures (7) and (8) in the Appendix show that maturity increasing predictability is robust to the adjustment, confirming the role of horizon increasing overreaction in affecting interest rates.

3.4 Economic Implications

Our empirical findings suggest that non-rational inflation expectations can have significant economic effects. Return predictability in fact implies that high expected inflation today predicts a future redistribution of wealth from borrowers to lenders, and vice-versa for low expected inflation. In heterogeneous-agent models such effects can affect output, because borrowers typically have a higher propensity to consume and invest than lenders (Auclert, 2019; Sterk and Tenreyro, 2018). Fisher (1933) famously stressed the costs of debt deflation and Doepke and Schneider (2006) offer more recent evidence on inflation-induced redistribution.

To understand the drivers of this effect, we must understand belief formation, which motivates our mnemonic learning approach. Before doing so, however, we show with a simple example that inflation expectations can lead to sizable wealth redistribution.

Consider a borrower who takes a simple loan with maturity h^* years. The payment is constant in real terms (i.e., in $t = 0$ dollars) and equal to $P^{(h^*)}$. Expected inflation is π^e for all future periods. Thus, the contractually stipulated nominal payment is $P^{(h^*)} \cdot p_s^e$, where p_s^e is the expected price level at time s. We denote the ex-ante real rate at time s by $r_s(\pi^e)$, which we allow to increase with expected inflation, consistent with our data. The ex-ante real rate is indeed another channel through which biased inflation expectations can affect output, which has been studied theoretically by Bianchi et al. (2024b) and L'Huillier et al. (2024) and also empirically by Bianchi et al. (2022) and Nagel (2024). Given these parameters, the real value of the borrower's future payments is on average equal to

$$\hat{L}^{(h^*)} \equiv \sum_{h=1}^{h^*} \prod_{s=1}^{h} \frac{P^{(h^*)}}{1 + r_s(\pi^e)} \cdot \frac{p_s^e}{p_s} \approx \sum_{h=1}^{h^*} \prod_{s=1}^{h} \frac{P^{(h^*)}}{(1 + r_s(\pi^e))(1 + \pi_s - \pi^e)}, \tag{11}$$

where p_s is the price level realized on average at s, and π_s is average inflation in the same period

Table 6: A simple quantification of the redistribution from borrowers to savers

	Loan Maturity				
	1	2	3	5	10
$-\Delta \log L^{(h^*)}$	0.09	1.19	2.10	4.18	8.21
$\Delta \log \hat{L}^{(h^*)}$	0.36	0.86	1.39	2.77	6.04
$L^{(h^*)}$	\$100,000	\$100,000	\$100,000	\$100,000	\$100,000
$P^{(h^*)}$	\$100,090	\$50,069	\$33,540	\$20,365	\$10,263
$-\Delta L^{(h^*)}$	\$90	\$1,187	\$2,100	\$4,181	\$8,206
$\Delta \hat{L}^{(h^*)}$	\$356	\$857	\$1,387	\$2,772	\$6,038

*Note.*The top rows report the semi-elasticity of the ex-ante loan value and the ex-post additional cost to a parallel shift in inflation expectations. The lower rows quantify the effects based on an loan of \$100,000 across various maturities, reporting also the repayment amount as well as the absolute ex-ante and ex-post changes resulting from a 1 p.p. parallel increase in inflation expectations. Computations are based on the estimates from Tables 1 and 3.

(we work under a certainty equivalent approximation). An increase in expected inflation π^e matters in two ways. First, it leads to a higher ex-ante rate $r_s(\pi^e)$, as shown by the above-one slope coefficients in Panel A of Table 3, reducing the amount the agent can borrow. This effect is not necessarily causal: it may reflect fundamental shocks to which π^e responds. Second, higher expected inflation causes higher ex-post real payments because average future inflation is below expectations, $\pi_s - \pi^e < 0$, as shown in Panel A of Table 1. This effect, due to belief overreaction, reduces the borrower's future resources and hence spending.

We use equation (11) and our estimated coefficients to evaluate the effect of an increase in π^e, starting from a scenario in which expected and realized inflation are equal to zero. We distinguish between a pure ex-ante effect – in which we allow the ex-ante real rate to change but set future forecast errors to zero – and a pure ex-post effect – where we allow for predictable forecast errors but keep the ex-ante loan amount constant. The ex-post effect is novel and directly ties to beliefs, the ex-ante effect is a useful benchmark. The top of Table (6) reports a measure of the two costs: the ex-ante percent loan reduction after a 1 percentage point increase in expected inflation (first row), and the ex-post increase in real payments caused by the same event, again as a percentage of the initial loan amount (second row).

The two costs are sizable and comparable. For a 2-year loan, higher expected inflation is associated with lower current and future resources of about 1% of the loan amount each. Consistent with horizon-increasing overreaction, for a 10-year loan the costs increase, respectively, to 8% and 6%, adding to a hefty 14%. If the increase in expected inflation is 2%, the effects reach 16% and 12% at the 10-year maturity.[11]

To see the ex-post redistribution in dollars, consider a $100.000 loan. An extra 1% expected inflation raises the future real repayment by approximately $\$6,038$, while an extra 2% expected inflation entails a $\$12,076$ cost in real (i.e., $t = 0$ dollars) terms, which is significant. Ex-ante, the same events are associated with reductions in the value of the loan of $\$8,206$ and $\$16,412$, respectively. These costs, which obviously scale with the loan amount, show the sizable real costs associated with higher expected inflation.

Following this example, overreaction can hinder aggregate demand in several ways. In one channel, households taking a long-term fixed rate mortgage during times of high expected inflation will have to cut consumption as inflation drops in the future.[12] Another channel entails indebted firms, especially if facing tight borrowing constraints. As shown by Gomes et al. (2016), a systematic higher repayment due to lower than expected inflation can sharply reduce investment. Real appreciation of borrowers' assets may soften these costs, but this force is limited because households and firms hold many illiquid assets, consistent with the costs of debt deflation. In a similar vein, the drop in inflation can raise the debt burden for banks that finance illiquid real investments using nominal liabilities (Brunnermeier and Sannikov, 2016), and hurt governments issuing long-term nominal debt during high inflation states. If expected inflation is demand-driven, so that investors expect higher inflation during economic expansions, these adverse forces materialize after booms, amplifying the business cycle.

To understand the nature and severity of these effects we must understand where belief overreaction comes from and how it varies across environments. We address this next.

4 Selective Memory and Inflation Expectations

Horizon-increasing overreaction challenges leading theories of beliefs. Models of information frictions or inattention (Sims, 2003; Woodford, 2003a), motivated by underreaction of US short-term forecasts (Coibion and Gorodnichenko, 2015), predict consensus underreaction *only* to recent news. In equation (3) this means $\delta_{1,h} > 0$ and $\delta_{2,h} = 0$, which is counterfactual: we observe consensus overreaction to news and to the lagged forecast, $\delta_{1,h} < 0$ and $\delta_{2,h} < 0$.

Diagnostic expectations (DE), (Bordalo et al., 2018, 2020; Bianchi et al., 2024a), motivated

[11] This variation in inflation expectations is realistic. A one-percent increase in expected inflation corresponds, in our zero-inflation benchmark, to a forecast error of 1%. By our estimates, this error is fairly common: at the 10-year horizon, it is attained from an increase in expected inflation equal to 1%. This follows from dividing a forecast error of 1% by taking an average coefficient of -1 across maturities from Panel A of Table 1.

[12] In the Survey of Consumer Finances (SCF, 2022 wave) a 40-year-old household has a nominal net position of roughly $-\$100,000$. Nominal assets comprise transaction accounts, certificates of deposit, investment funds (except stock mutual funds), savings bonds, directly held bonds, and cash value of whole life insurance; nominal liabilities comprise debt secured by primary residence, debt secured by other residential property, other lines of credit, credit card balances, installment loans (e.g., education and vehicle loans), other types of debt.

by individual-forecaster overreaction (Bordalo et al., 2020), predict $\delta_{1,h} < 0$. In formulations with a long look-back period (Bianchi et al., 2024b; Bordalo et al., 2024b), they also predict $\delta_{2,h} < 0$. However, DE cannot account for no overreaction (or even underreaction) in the short term, failing to produce the observed horizon-increasing pattern. Merged with information frictions, DE can produce consensus underreaction to news $\delta_{1,h} > 0$, as in Bordalo et al. (2020), and possibly even overreaction to the lagged forecast $\delta_{2,h} < 0$. However, they do not yield individual underreaction, which is documented for several analysts (Bordalo et al., 2020).

Adaptive expectations, also originally motivated by rigid inflation expectations (Nerlove, 1958; Cagan, 1956), are a distributed lag of past inflation. This model may produce overreaction to news, $\delta_{1,h} < 0$, if the weight on recent inflation is high. In this case, though, the weight on the past forecast is low. Thus, it is difficult for this model to produce sizable overreaction to the lagged forecast, $\delta_{2,h} < 0$, which we observe.[13] Furthermore, this model is not forward-looking (Lucas, 1972), so it makes no predictions on the term structure of forecasts.

To account for the data, we offer a model in which, due to selective memory, beliefs about the future overweight events that are similar to the current inflation state. Memory limitations are often invoked to justify sophisticated departures from rationality. In constant-gain learning models (Evans and Lewis, 1995; Marcet and Nicolini, 2003), which offer a forward-looking version of adaptive expectations, forecasters overweight recent conditions compared to more remote ones. Experience based learning models take a similar route (Malmendier and Nagel, 2016). Also diagnostic overreaction relies on mnemonic overweighting, of representative states in this case. Compared to these approaches, we explicitly formalize memory limitations based on well established regularities in cued recall, following Bordalo et al. (2023). Combined with a learning setting this foundation proves crucial: rather than the time-decaying adaptive weights produced by recency or the invariant overreaction bias of DE, our model produces state- and horizon-dependent distortions. The latter allow to reconcile diagnostic overreaction with short-term rigidity, yielding new implications for the variation of belief biases across environments.

4.1 The Model

We consider an agent who has lived through or studied a country's inflation path, which forms her database. When forecasting at horizon h, current inflation cues her to retrieve similar past states and overweight them in the future. Overweighting may act on a model output. Due to concerns about misspecification, upon observing an inflation surprise the agent may adjust an initial model forecast using qualitative judgments, causing recall to bias her final belief. We

[13] The adaptive one-step ahead inflation forecast is $\pi^e_{t+1} = p[\pi_t - \pi^e_{t-1}] + (1-p)\pi^e_{t-1}$. If inflation is AR(1) with persistence ρ, the coefficient on the inflation news is $(\rho - \alpha)$, and the one on the lagged forecast is $(\rho + \alpha - 1)$. Higher α reduces the former coefficient and increases the second; the two coefficients are equal when $\alpha = 0.5$, which entails underreaction for realistic values of ρ.

do not explicitly formalize this process, but this is without much loss of generality. As we will show, model-free mnemonic learning implies that beliefs take the form of a distorted version of the true DGP, "as if" the agent distorts an underlying statistical model.

Longstanding research on human recall documents three regularities (Kahana, 2012): frequency, similarity to cues, and interference.[14] Frequency implies that frequent past states, say inflation at 2%, are more likely to be recalled. Similarity implies that the current inflation state cues recall of past states having similar numerical features. One such feature is *contextual*: if current inflation is 3%, numerically similar states are more likely to be recalled and overweighted for the future. A second numerical feature is *semantic*, based on "cause-effect" logic: the agent recalls what historically followed similar conditions. If current inflation is 3% and 3% typically led to 2%, the latter outcome is more accessible and hence overweighted. Finally, interference implies that retrieving certain episodes blocks recall of others. Thus, changing cues re-weight recall, causing state-dependence.

Frequency and similarity are not just central in memory research. They have been shown to shape beliefs across domains. In Bordalo et al. (2024a) they affect household beliefs about novel risks, in Jiang et al. (2024) they affect investors' stock return expectations, in Cenzon (2025) and Taubinsky et al. (2024) they account for the influence of idiosyncratic experiences on household macroeconomic expectations, in Gennaioli et al. (2024) they offer novel survey tests that account for US households' inflation expectations beyond conventional learning mechanisms. Here we embed these forces in a learning setup and theoretically show that they help account for biases in investors' inflation expectations and interest rate volatility. Relative to the model in Gennaioli et al. (2024), we do not include temporal context but characterize long run learning and convergence. Like them, on the other hand, we centrally rely on numerical similarity and its interaction with the database.[15]

Recall is stochastic, so agents with the same database may form different beliefs based on the same cue. Their average forecast will however reflect a systematic bias – our main object of interest. We characterize it building on Bonaglia and Gennaioli (2025), who generalize the model of Bordalo et al. (2023) and study its dynamic implications.

The Database. At each $t = 0, 1, ...$ the memory database $\Pi_t \equiv \{\pi_k\}_{0 \le k \le t-1}$ contains dated realizations of past inflation. In principle, memory traces could encode macroeconomic condi-

[14] In classic experiments, subjects recall words from a studied list. Frequency means that a word, say "apple", is more likely recalled if it is repeated in the list. Similarity has two dimensions: semantic (e.g., the word "wheel" cues "car"), and contextual (e.g., "wheel" may cue "eel" due to similar spelling and phonetics). Interference reflects the competition among memories: frequent or similar items crowd out others.

[15] Regarding temporal context, our current model cannot include primacy effects, which are shown to be relevant also for professionals (Malmendier et al., 2021), because we do not know the identity of forecasters (while recency effects are subsumed under similarity). Regarding numerical similarity, here we allow for a semantic component (i.e., "cause-effect" logic), which is arguably important for professional forecasters and investors

tions (e.g., the monetary policy regime, unemployment). In this case, recall may also depend on these features. For parsimony we focus on numerical similarity and assess its explanatory power. Bonaglia and Gennaioli (2025) formalize high dimensional structures.

Recall and Beliefs. When forecasting at time t, inflation at $t+h$, the agent draws a number of episodes π_k from her database with i.i.d. probability $r_{t,h}(\pi_k)$. The agent's expectation is then equal to the average retrieved episode. Aggregating across many agents who share the same database and cue, the consensus forecast is equal to:

$$\mathbb{E}_t^m(\pi_{t+h}) = \sum_{\pi_k \in \Pi_t} \pi_k r_{t,h}(\pi_k). \tag{12}$$

The database Π_t encodes the frequency of past episodes. The current state π_t and horizon h affect the recall probability function $r_{t,h}(\pi_k)$ through similarity.

Cues, Similarity, Retrieval. A cue is a salient signal q_t triggering associative recall, which is current inflation in our setting, $q_t = \pi_t$. Following multidimensional scaling (Nosofsky, 1992; Torgerson, 1958), numerical similarity between π_t and the past state π_k decays exponentially with distance along two features. The first is contextual, captured by the numerical discrepancy between the cue and the past state, namely $|\pi_t - \pi_k|$. Upon seeing 2% inflation, past 2% episodes are easier to retrieve. The second feature is semantic, captured by the numerical discrepancy between the cue and the h-step predecessor of π_k, namely $|\pi_t - \pi_{k-h}|$. When thinking about two-year-ahead inflation, current inflation at 3% fosters recall of what happened two years after 3% inflation in the past. These distances enter a discrepancy function $d_h(\pi_k, \pi_t)$. The probability of recalling π_k is then equal to

$$r_{t,h}(\pi_k) = \frac{\exp[-d_h(\pi_k, \pi_t)]}{\sum_{\pi_s \in \Pi_t} \exp[-d_h(\pi_s, \pi_t)]}. \tag{13}$$

By the numerator, it is easier to retrieve numerical values that are similar to the current state and that are more frequent. The denominator captures interference: a highly frequent or similar episode π_s hinders recall of π_k.

Expectations. Equations (12) and (13) characterize beliefs at every horizon h as a function of: i) the database Π_t, and ii) numerical and semantic distances with the cue π_t, namely $|\pi_t - \pi_k|$ and $|\pi_t - \pi_{k-h}|$. These objects are observable from a country's inflation history. Beliefs are then pinned down by the similarity "weights" of the two features, which we later estimate. Before doing so, we study the long-run properties of beliefs.

4.2 Long Run Beliefs

We characterize long-run beliefs following Bonaglia and Gennaioli (2025). To get tractability, we take a second-order approximation of the discrepancy function, so that:

$$d_h(\pi_k, \pi_t) = \alpha (\pi_{k-h} - \pi_t)^2 + \beta (\pi_k - \pi_t)^2, \tag{14}$$

where $\alpha \geq 0$ and $\beta \geq 0$ capture the weights of semantic and contextual features, respectively.

Given the DGP $(\pi_t)_{t\in\mathbb{N}}$, the expectation $\mathbb{E}_t^m(\pi_{t+h})$ in equation (12) is a stochastic process that depends on the cue π_t and the database Π_t. We say that this process has a long-run approximation $\mathbb{E}_t^\infty(\pi_{t+h})$ if, fixing $\epsilon > 0$ arbitrarily, the probability of the event $\{|\mathbb{E}_t^m(\pi_{t+h}) - \mathbb{E}_t^\infty(\pi_{t+h})| > \epsilon\}$ tends to zero under the DGP as $t \to \infty$. Under general assumptions described in the appendix, Bonaglia and Gennaioli (2025) show that $\mathbb{E}_t^m(\pi_{t+h})$ admits a long-run approximation, which here we characterize for a broad class of DGPs.

Proposition 1 *Let $(\pi_t)_{t\in\mathbb{N}}$ be zero-mean, stationary, ergodic and Gaussian. Then, under the discrepancy in equation (14), there is a coefficient $\phi(\alpha, \beta, h) \in \mathbb{R}$ such that the expectation $\mathbb{E}_t^m(\pi_{t+h})$ admits the long-run approximation:*

$$\mathbb{E}_t^\infty(\pi_{t+h}) := \phi(\alpha, \beta, h)\pi_t.$$

We call "mnemonic expectation" the long-run approximation above. Mnemonic learning causes beliefs to take the form of a possibly distorted model that relies on the attended to similarity features (with weights α and β) and the true parameters of the data generating process (through the database). With the discrepancy in equation (14) such model is akin to an AR(1): it loads on current inflation with a proportionality coefficient $\phi(.)$ that depends on the horizon h, similarity weights α and β, and DGP features (which are kept implicit for now).

We now characterize the role of similarity weights, of the database, and their interaction.

Proposition 2 *The mnemonic expectation obeys the following limit cases with respect to similarity weights α and β:*

$$\lim_{\alpha\to\infty,\ \beta\to 0} \phi(\alpha, \beta, h) = \frac{cov(\pi_{t+h}, \pi_t)}{var(\pi_t)}, \qquad \text{(Least Squares)}$$

$$\lim_{\alpha\to 0,\ \beta\to\infty} \phi(\alpha, \beta, h) = 1, \qquad \text{(Adaptive)}$$

$$\lim_{\alpha\to 0,\ \beta\to 0} \phi(\alpha, \beta, h) = 0. \qquad \text{(Frequentist)}$$

If $cov(\pi_{t+h}, \pi_t) > 0$ converges to zero as $h \to \infty$ and $\beta > 0$, there is a horizon $\tilde{h}$ such that, for *all horizons $h \geq \tilde{h}$, the mnemonic expectation overreacts:*

$$\phi(\alpha, \beta, h) > \frac{cov(\pi_{t+h}, \pi_t)}{var(\pi_t)}.$$

Semantic similarity fosters forward-looking beliefs. If this force dominates ($\alpha \to \infty$ and $\beta \to 0$), the forecast is a least square projection of future inflation on π_t. Expectations are rational if the DGP is AR(1). They select the best AR(1) fit otherwise, as in natural expectations (Fuster et al., 2012). Semantic similarity can thus capture the use of a statistical model.

Numerical similarity causes instead anchoring to the present. If this force dominates ($\alpha \to 0$ and $\beta \to \infty$), the agent expects π_t to persist at any future horizon, promoting overreaction. Critically, this is not just a recency effect. First, it depends on the state π_t and forecast horizon h. Second, it depends on the database: if $\beta < \infty$ the cue π_t is more potent if the frequency of similar episodes in Π_t is larger. An inflation surge will not change beliefs much if similar episodes were rare in the past. We later show this property formally.

Finally, frequency causes overweighting of the (zero) long-run mean around which inflation fluctuates. If this force dominates ($\alpha, \beta \to 0$), the agent expects immediate reversal to such mean. If the DGP exhibits any serial correlation, this force promotes underreaction to π_t.

Consider next the interaction between the database and retrieval cues. It is shaped by the parameters of the DGP, which determine the database. If the stationary DGP exhibits positive short-run auto-correlation, the mnemonic expectation moves in the right direction but – due to numerical similarity β – it exhibits long-run overreaction: by overweighting future scenarios that resemble the present, it causes neglect of mean reversion. To characterize more sharply this interaction, and in particular whether forecasts under- or overreact, we must pin down the long-run database, namely $\lim_{t\to\infty} \Pi_t$. To do so, we assume that inflation is AR(1):

$$\pi_k = \rho \cdot \pi_{k-1} + \sigma \cdot \epsilon_k, \tag{15}$$

with persistence $\rho \in (0, 1)$ and volatility $\sigma > 0$. We obtain the following result.

Proposition 3 *Let $\sigma_\pi^2 \equiv \sigma^2 / (1 - \rho^2)$. Expected inflation at t for $t + h$ is equal to*

$$\mathbb{E}_t^\infty (\pi_{t+h}) = \frac{\alpha\rho^h + \beta + 2\alpha\beta\sigma_\pi^2 \left(1 - \rho^{2h}\right)}{\alpha + \beta + 2\alpha\beta\sigma_\pi^2 \left(1 - \rho^{2h}\right) + \frac{1}{2\sigma_\pi^2}} \pi_t. \tag{16}$$

The mnemonic expectation extrapolates from current inflation by distorting the AR(1) persistence parameter. Critically, due to the interaction between retrieval cues and the database, the degree of extrapolation increases in true volatility σ and persistence ρ. When σ is large, the agent has experienced many high and low inflation states. By similarity, these states are selectively retrieved when π_t is high or low, so extrapolation is strong. If $\sigma \to \infty$ the mnemonic expectation fully projects the present into any future horizon, $\mathbb{E}_t^\infty(\pi_{t+h}) \to \pi_t$. Stable inflation (low σ) instead promotes belief rigidity through overweighting of the long-run mean. When $\sigma \to 0$ we have $\mathbb{E}_t^\infty(\pi_{t+h}) \to 0$.

Higher persistence ρ increases extrapolation in two ways: it increases volatility, but it also increases semantically-retrieved persistence from the past. If persistence is maximal $\rho \to 1$ the mnemonic expectation is rational for any similarity parameters, $\mathbb{E}_t^\infty(\pi_{t+h}) \to \pi_t$. If persistence is zero, the mnemonic expectation overreacts at any $\beta > 0$.

Whether belief under- or overreact depends on the balancing of these forces. By equation (16), the short-run forecast (i.e., $h = 1$) overreacts, $\phi(\cdot) > \rho$, if and only if

$$\rho(1+\rho) < 2\beta\sigma^2\left(1 + 2\alpha\sigma^2\right). \tag{17}$$

Overreaction occurs when cues are strong enough, α and β are large, volatility σ is high, and persistence ρ is low. At low persistence, even a small amount of numerical anchoring causes overreaction. At high persistence a sufficient weight on frequency (i.e., low enough similarity weights) causes excess anchoring to the long-run mean and thus underreaction.

Memory-driven overweighting reconciles models featuring exaggerated AR(1) persistence such as Gabaix (2019), Angeletos et al. (2021)) or DE with long look-back period ($J \to \infty$ in Bianchi et al. (2024b)), with models featuring inattention-driven rigidity (Sims, 2003; Woodford, 2003a). Critically, the nature and strength of biases depends on the inflation DGP and the forecast horizon h, rationalizing our findings and yielding new predictions which we later test.

4.3 Implications of Selective Memory

Consider the predictability of forecast errors from the forecast revision and lagged forecast – the counterpart of regression (3). the mnemonic expectation obeys the property below.

Proposition 4 *Under equation (16) the mnemonic revision at t and the mnemonic expectation at $t-1$ predict the forecast error at $t+h$ with coefficients:*

$$\delta_{1,h} = \delta_{2,h} = \delta_h \equiv \rho^h \frac{\alpha + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h}) + 1/2\sigma_\pi^2}{\alpha\rho^h + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h})} - 1. \tag{18}$$

If $\beta > 0$ the predictability coefficient δ_h monotonically decreases in the forecast horizon h and exhibits long-term overreaction: $\lim_{h\to\infty} \delta_h = -1$.

Consistent with Table 2, there is horizon-increasing overreaction to news and to the lagged forecast: $\delta_{1,h}$ and $\delta_{2,h}$ decrease as the horizon h increases. At short horizons, underreaction may prevail, depending on parameter values, as shown in equation (17). At longer horizons, overreaction eventually prevails if $\beta > 0$. Consistent with Table 2, under an AR(1) DGP,

$\delta_{1,h} = \delta_{2,h}$. In terms of DGP parameters, our discussion of extrapolation suggests that we should expect overreaction to be stronger (and short-term underreaction less likely to arise) when inflation is more volatile and less persistent, i.e., σ is higher or ρ is lower. The comparative static with respect to σ is easily proved analytically, the one on ρ is more complicated. Our simulations show that if beliefs overreact at a certain ρ, then overreaction generally weakens as ρ increases. For certain parameter values, δ_h may even shift from over to underreaction when ρ becomes large enough. If however β is large or the forecast horizon is long, the model produces overreaction, which decreases monotonically in ρ.[16]

Consider return predictability. It can be analytically studied under the expectations hypothesis if the DGP is AR(1), giving the mnemonic counterparts of equations (5) and (7).

Proposition 5 *If $\beta > 0$, the mnemonic expectation $\mathbb{E}_t^\infty(\overline{\pi_{t+1,h}})$ predicts the ex-post real yield to maturity h with coefficient*

$$\theta_h^r = -\sum_{j=1}^{h} \delta_j w_j, \tag{19}$$

where weights $(w_j)_{1\leq j\leq h}$ are positive and sum to one. If $\alpha = 0$, this simplifies to $\theta_h^r = 1 - \frac{\rho(1+2\beta-\rho^2)(1-\rho^h)}{2h\beta(1-\rho)}$, which is increasing in h and positive for sufficiently large h.

The same expectation also predicts the excess return on a maturity h bond with coefficient:

$$\theta_h^{rx} = h[\theta_h^r(1-\rho) + \delta_1 w_1 - \rho\delta_h w_h]. \tag{20}$$

When $\alpha = 0$, this simplifies to $\theta_h^{rx} = (h-1)(1-\rho)$, implying positive and maturity-increasing predictability of excess returns.

Numerical similarity ($\beta > 0$) causes nominal rates to increase excessively with higher π_t. As a result, future inflation is systematically below expectations and real yields to maturity are abnormally high. This effect drives the positive and maturity-increasing values of θ_h^r. In equation (19) predictability at maturity h is inversely related to the forecast error coefficient δ_h, averaged across the relevant horizons.

For excess returns, the link with error predictability is more complex: overreaction of long-term beliefs, captured in equation (20) by $\theta_h^r > 0$, can be offset by biased shifts in short-term beliefs, captured by δ_1. The balance between these forces depends on α: when the agent is very forward-looking (i.e., α is high), long-term rates respond less than short-term ones in absolute terms, potentially causing negative predictability; the opposite occurs when α is low. If $\alpha = 0$, we obtain positive maturity-increasing predictability of excess returns as in the data.

[16]The mapping between ρ and δ_h is non-monotonic if beliefs exhibit substantial underreaction at intermediate values of ρ, which can happen at short horizons. In this case, as ρ approaches one the predictability coefficient δ_h decreases toward zero.

Crucially, the direction and strength of forecast errors and returns predictability should depend on similarity weights and the DGP. We test these predictions by estimating the model.

5 Explanatory Power of Mnemonic Expectations

Section 5.1 estimates the model. Sections 5.2, 5.3 and 5.4 assess its ability to produce realistic variation in the degrees of extrapolation, as well as in forecast error and return predictability.

5.1 The Database and Similarity Weights

We implement mnemonic beliefs by applying equation (12) to the database of past annual inflation. For the generic time t and country c, the database is $\Pi_{c,t} = \{\pi_{c,t}\}_{k=t_{c,0},\ldots t-1}$, where $t_{c,0}$ is the first available inflation episode for c (ranging from 1960 to 1996, see Table 7). The probability $r_{t,h}(\pi_{c,k})$ is computed using the discrepancy function:

$$d_h(\pi_{c,k}, \pi_{c,t}) = \alpha(\pi_{c,k-h} - \pi_{c,t})^2 + \beta(\pi_{c,k} - \pi_{c,t})^2 \quad \text{for } \pi_{c,k} \in \Pi_{c,t}. \tag{21}$$

To apply this procedure, we do not need to rely on a specific DGP. For given weights (α, β), retrieval from the evolving database pins down the model implied forecast $\mathbb{E}_t^m(\pi_{c,t+h}|\alpha,\beta)$. We estimate (α, β) by matching this forecast with CE forecasts for all t, c, h. (α, β) is constant across time, space and horizon. As a result, the model's explanatory power for horizon and country variation stems *entirely* from temporal and cross-sectional differences in databases.

We build a grid of more than 10,000 points $(\alpha, \beta) \in [0, 1]^2$ with a step size of 0.01, each yielding a panel of model-implied forecasts. We then select (α, β) to minimize the total absolute deviation between CE and model-implied forecasts:

$$\min_{\alpha,\beta} \sum_{c,t,h} |\mathbb{F}_t(\pi_{c,t+h}) - \mathbb{E}_t^m(\pi_{c,t+h}|\alpha,\beta)|,$$

where $\mathbb{F}_t(\pi_{c,t+h})$ is the CE forecast (see Appendix B.3 for details). The point estimates and 99% confidence intervals are:[17]

$$\widehat{\alpha} = 0.07 \pm 0.006 \quad \text{and} \quad \widehat{\beta} = 0.18 \pm 0.015.$$

[17] We perform a case-resampling bootstrap on the panel of model-generated inflation expectations for each country over the in-sample period 1989–2022, drawing $N = 1000$ bootstrap samples by sampling with replacement a multiset of length 34 (the number of in-sample years). For each sample, we reconstruct the bootstrapped series for each (α, β) pair, adjust for country-specific start dates, and estimate (α, β) by minimizing the sum of absolute deviations between simulated and observed CPI from CE. See Appendix B.4 for details. Confidence intervals are 99% symmetric percentile bootstrap confidence intervals, which have the advantage of being robust under model misspecification (Kolesár, 2025).

These estimates reveal that contextual similarity matters, $\widehat{\beta} = 0.18$, producing overreaction. Semantic similarity receives a smaller weight, $\widehat{\alpha} = 0.07$, generating a forward-looking component. Finally, the fairly low magnitude of the weights implies that anchoring to frequent outcomes is also sizable. All mnemonic forces are thus active in the data. Hereafter, we refer to the predicted belief at the above point estimates as the "mnemonic forecast". Model fit is good: with only two parameters, mnemonic forecasts exhibit a correlation of 0.61 with CE forecasts, capturing a sizable share of variation across countries, time periods, and horizons.

With model estimates in hand, we now evaluate the model's explanatory power for three key patterns in the data: cross-country variation in i) strength of extrapolation, ii) forecast error predictability, and iii) return predictability.

5.2 Strength of Extrapolation

The strength of extrapolation refers to the sensitivity of forecasts to the inflation level recently observed by forecasters. We measure this strength, both in the data and in the model, by estimating the following regression separately for CE and mnemonic forecasts:

$$\mathbb{F}_t^i(\pi_{c,t+h}) = \alpha_{c,h}^i + \chi_{c,h}^i \cdot \overline{\pi}_{c,t} + \varepsilon_{c,t,t+h}^i,$$

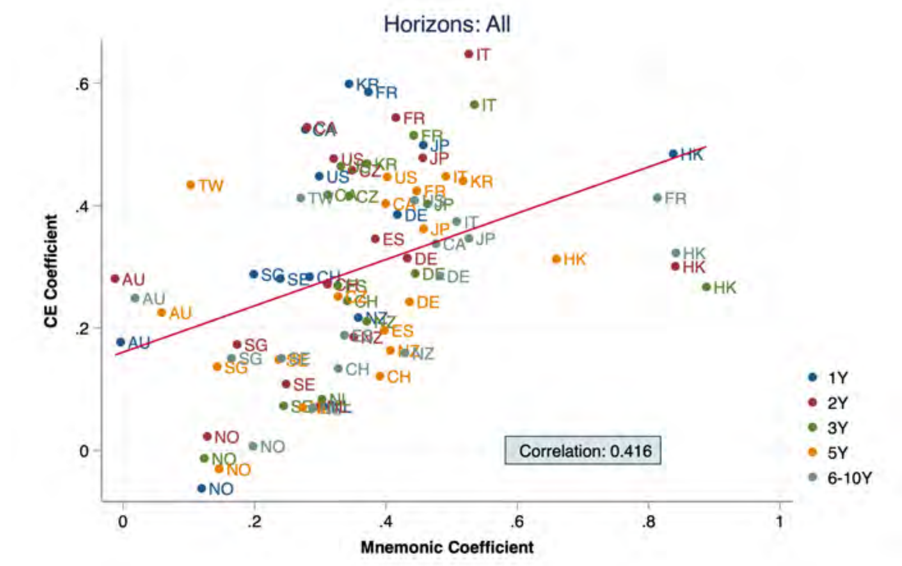
where $i = \text{CE}$ and $i = m$ denote CE and mnemonic forecasts, respectively. The coefficient $\chi_{c,h}^i$ captures the sensitivity of forecasts to the inflation state in country c at horizon h. Following Section (2), the inflation state is defined as $\overline{\pi}_{c,t} = (\pi_{c,t-1} + \pi_{c,t-2})/2$.

CE-based estimates reveal significant heterogeneity in the strength of extrapolation $\chi_{c,h}^{CE}$ across countries and horizons (see the boxplot in Appendix Figure 6, Panel A, for details). At the shortest horizon, the median coefficient $\chi_{c,1}^{CE}$ is about 0.48 with an inter-quartile range of $[0.32, 0.54]$. At the longest horizon, the median $\chi_{c,\text{6-10}}^{CE}$ is about 0.22 with an inter-quartile range of $[0.08, 0.27]$. Short-run forecasts extrapolate more than long-run ones, indicating that forecasters internalize mean reversion. Crucially, at each horizon we observe systematic cross-country variation in extrapolation: the equality of $\chi_{c,h}^{CE}$ across countries is rejected at standard confidence levels.[18] To assess whether our model can reproduce this variation, Figure 1 plots the coefficients $\chi_{c,h}^{CE}$ estimated using CE forecasts (y-axis) against those estimated using mnemonic forecasts $\chi_{c,h}^m$ (x-axis).

Our model captures a significant share of variation: the correlation between the two coefficients is 0.42. To isolate cross-country variation, let $\overline{\chi}_c^{CE}$ be the average strength of extrapolation in country c across horizons h. Figure 2 reports this coefficient estimated from CE data (y-axis) against its mnemonic counterpart (x-axis). The association is strong: the regression beta is

[18]Formally, we estimate a pooled regression in which the inflation state is interacted with country dummies. The p-values of the F-test are 0.0000 for all horizons $h = 1, 2, 3, 5, \text{6-10}$.

Figure 1: Performance for Forecast Predictability



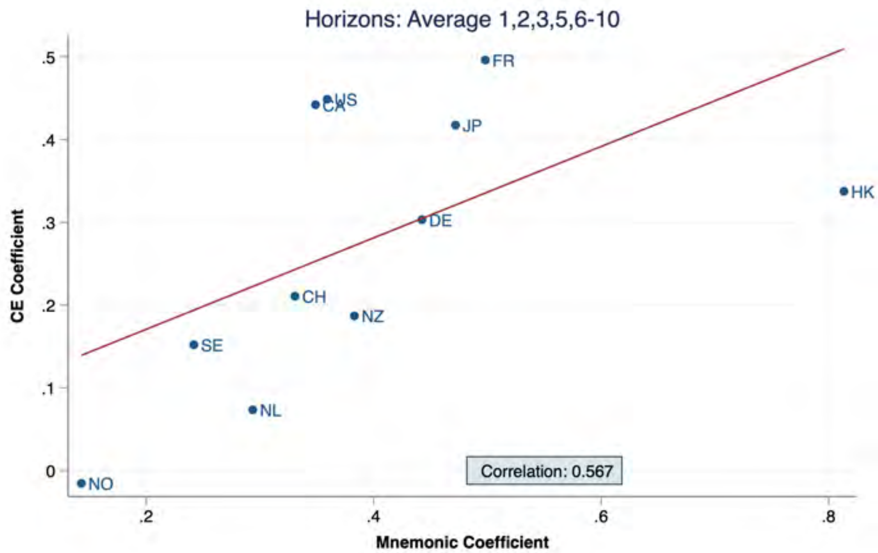
Note. The figure displays extrapolation coefficients estimated using CE forecasts (y-axis) against the those estimated using mnemonic ones (x-axis). The line shows the fit between the two, and their correlation is reported in the lower-right corner. Each data point is identified by the ISO 3166-2 country abbreviation.

0.552 ($p = 0.069$), the adjusted R^2 is 0.25 and the correlation is 0.57. Mnemonic forecasts successfully capture cross-country variation in extrapolation, confirming the model's ability to generate different belief functions – and, as we will see, distortions – in different environments.

To compare the explanatory power of mnemonic forecasts with standard approaches, we estimate a constant-gain learning model (Evans and Honkapohja, 2009), in which the agent infers the long-run mean and persistence of an AR(1) inflation process with gain parameter $\gamma > 0$. This model can be viewed as a forward-looking version of adaptive expectations. When matching CE forecasts, we find $\gamma \approx 0.2$, which is in line with conventional estimates (Nagel, 2024) (Appendix B.3 reports the details). When evaluating its explanatory power for extrapolation, we see that this model fails to account for cross-country variation: the coefficient $\overline{\chi}_c$ estimated under constant-gain learning does not meaningfully vary. The regression slope is insignificant ($p = 0.2$) and the explained variation is tiny ($R^2 = 0.07$). The constant-gain parameter introduces a form of "recency bias" in standard learning models. This force helps the model account for excess belief volatility, but it makes it more difficult for it to produce systematic country differences in extrapolation, because it weakens "memory" of the DGP. Numerical similarity is instead capable of producing significant cross-country variation in extrapolation because its mechanism for belief volatility relies on an interaction of current cues with the memory database: the volatility generated by recent high inflation depends on how frequent and persistent high inflation was in the past.

A final question is whether our model's explanatory power is linked to intuitive features

Figure 2: Performance for Forecast Predictability, Horizon Averages



Note. The figure displays the country specific, averaged across horizons, coefficients from the simulated (y-axis) against the CE (x-axis) datasets. The line indicates the fit between the two. Pooled (across countries and horizons) correlation is reported in the lower-right corner of the graph. The identifier for each data point corresponds to the ISO 3166-2 country abbreviation.

of inflation databases and DGPs. The AR(1) model suggests that such power may be tied to volatility and persistence. In volatile environments, seeing high inflation today recalls many high inflation episodes from the past, fostering extrapolation. Persistence matters because, upon observing a high π_t, the agent recalls a similar event π_{k-h} and overweights its successor π_k, so that the persistence of high values from π_{k-h} to π_k causes stronger extrapolation.

To capture these forces, we regress $\pi_{c,t}$ on $\pi_{c,t-h}$ for country c and horizon h. We take the slope $\rho_{c,h}$ as a measure of persistence and the variance of the error term $\sigma_{c,h}$ as a proxy for volatility. Mnemonic learning also relies on frequency, which prompts the agent at each time t to overweight the average episode experienced up to that point. This force plays a significant role in our model because the estimated weights α and β are fairly low. It follows that another relevant statistic is the extent to which higher average inflation lived up to t predicts higher realized inflation at $t+h$. The stronger the association, the more reactive beliefs will be. To quantify this force, which is a longer term measure of persistence, we regress, for each horizon h, inflation $\pi_{c,t}$ on the average inflation $\overline{\pi}_{c,t-h}$ experienced up to $t-h$. We denote the resulting regression coefficient with $\overline{\rho}_{c,h}$. We estimate $\rho_{c,h}$, $\sigma_{c,h}$ and $\overline{\rho}_{c,h}$ using data from the period in which CE forecasts are available.[19] We then regress the CE extrapolation coefficient $\chi^{CE}_{c,h}$ on

[19] Computing these statistics in the full sample would contaminate them with the high inflation period of the 1970s-80s. This would act as a confound because these dissimilar episodes have limited impact on belief updating during our period of stable and low inflation.

our three statistics:

$$\chi^{CE}_{c,h} = \underset{(0.035)}{0.229^{***}} + \underset{(0.021)}{0.051^{**}} \cdot \sigma_{c,h} + \underset{(0.058)}{0.352^{***}} \cdot \rho_{c,h} + \underset{(0.013)}{0.099^{***}} \cdot \overline{\rho}_{c,h} + \varepsilon_{c,h}. \quad (22)$$

Countries with higher inflation volatility and persistence indeed exhibit stronger extrapolation. The regression's adjusted R^2 is 0.567, which is sizable. The explanatory power of mnemonic forecasts is more directly illustrated in Figures 1 and 2, but this approximation based on intuitive statistics shows that the observed variation in the strength of extrapolation aligns with intuitive features of a country's inflation history that influence memory, such as changing cues and databases. We now investigate whether our model, by producing variation in extrapolation strength, can also account for variation in the predictability of forecast errors and returns.

5.3 Strength of Overreaction

We predict the forecast error using the current forecast as in equation (2), separately for each country and horizon. Using CE data, we detect sizable variation in error predictability coefficients $\delta^{CE}_{c,h}$ (see Figure 6, Panel B). At the shortest horizon, the median $\delta^{CE}_{c,1}$ is about 0.06 with an inter-quartile range of $[0.01, 0.06]$, indicating a tendency toward underreaction. At the longest horizon, the median $\delta^{CE}_{c,6\text{-}10}$ is about -1 with an inter-quartile range of $[-1.3, -0.7]$, pointing to sharp overreaction. Cross-country differences in overreaction are significant.[20]

Figure 3, plots the estimated $\delta_{c,h}$ from CE data (y-axis) against their mnemonic counterparts (x-axis).[21] Our model captures significant country and horizon variation in belief biases, achieving a 64% correlation with CE-based coefficient estimates. To assess the extent of cross-country variation, we regress the estimates of $\delta^{CE}_{c,h}$ on forecast-horizon dummies. The R^2 of this regression is 0.21. Adding the mnemonic coefficient $\delta^{m}_{c,h}$ to the regression yields an R^2 of 0.48 (see Table 13, columns (1) and (2)). The explanatory power more than doubles: our model reproduces significant cross-country variation in overreaction.[22]

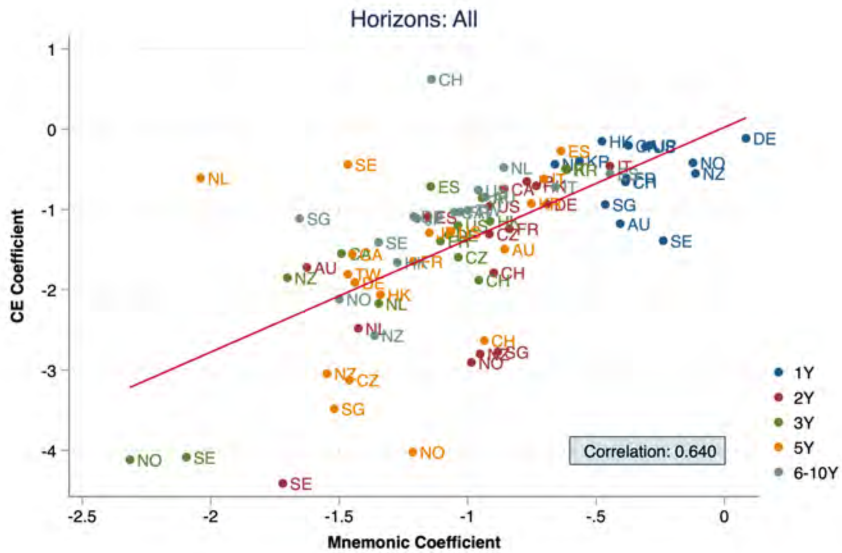
We can assess whether predictable overreaction is tied to inflation persistence and volatility. In the AR(1) model, higher persistence $\rho_{c,h}$ and $\overline{\rho}_{c,h}$ is associated with higher $\delta_{c,h}$ (less overreaction), while higher volatility $\sigma_{c,h}$ is associated with lower $\delta_{c,h}$ (more overreaction). This is

[20]Estimating a pooled error predictability regression in which expected inflation is interacted with country dummies, the F-test rejects coefficient equality at all horizons at standard confidence levels (with p-values of 0.0181 for $h = 1$, 0.0000 for $h = 2$, 0.0000 for $h = 3$, 0.0000 for $h = 5$, and 0.0000 for $h =$ 6-10).

[21]Since memory cues and databases vary annually, we benchmark the predictability coefficient $\delta_{c,h}$ under mnemonic forecasts to the annual CE-based estimates in Table 18. Looking at pooled estimates, the model successfully captures horizon-increasing overreaction. The coefficients δ_h estimated from mnemonic forecasts are statistically indistinguishable from their CE-based counterparts (see Table 15).

[22]We repeat this exercise for the error predictability specification in equation (3). Our model also matches the coefficients capturing overreaction to news $\delta_{1,h}$ and the lagged forecast $\delta_{2,h}$ (see Table 14). Notably, mnemonic forecasts can also reconcile the absence of significant overreaction to news at $h = 1$, $\delta_{1,h} = 0$, with significant overreaction to the lagged forecast at the same horizon, $\delta_{2,h} < 0$.

Figure 3: Performance for Forecast Errors Predictability



Note. The figure displays the country specific, averaged across horizons, coefficients from the simulated (y-axis) against the CE (x-axis) datasets. The line indicates the fit between the two. Pooled (across countries and horizons) correlation is reported in the lower-right corner of the graph. The identifier for each data point corresponds to the ISO 3166-2 country abbreviation.

indeed what we find in the data:

$$\delta^{CE}_{c,h} = \underset{(0.241)}{-0.803^{***}} - \underset{(0.142)}{0.237^{*}} \cdot \sigma_{c,h} + \underset{(0.402)}{1.770^{***}} \cdot \rho_{c,h} + \underset{(0.092)}{0.463^{***}} \cdot \overline{\rho}_{c,h} + \varepsilon_{c,h}. \tag{23}$$

Countries with lower persistence and higher inflation volatility indeed exhibit more overreaction. The regression's adjusted R^2 is 0.41, which remains unchanged when including forecast-horizon dummies. Consistent with mnemonic learning, belief biases are tied to intuitive features of inflation histories.

5.4 Explanatory Power for Bond Returns

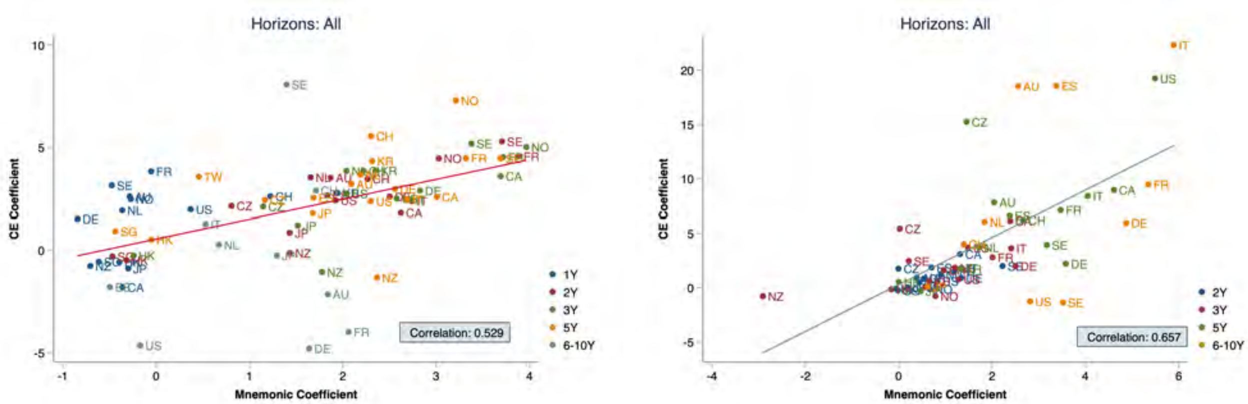
To study variation in market efficiency, following Section 3, we compute expected average inflation from t to $t+h$ using CE and mnemonic beliefs. We then use these measures to predict the real yield to maturity and excess bond returns observed in the data. We then obtain estimates for the return predictability coefficients $\theta^{r}_{c,h}$ and $\theta^{ex}_{c,h}$, for CE and mnemonic forecasts.

The CE data show substantial heterogeneity in return predictability. For the real yield (see Figure 6, Panel C), the median coefficient at one year is 0.6 with an inter-quartile range of $[0.007, 0.8]$. At 6-10 years, it rises to 0.9 with an inter-quartile range of $[0.3, 3.2]$. The same emerges for excess bond returns (see Figure 6, Panel D). At the $h = 2$ horizon, the median coefficient is 0.3 with interquartile range of $[-0.1, 0.6]$; at 6-10 years, it rises to 3.5 with interquartile range [2.4, 10]. The strength of positive, maturity-increasing return predictability varies systematically across countries, pointing to differences in market efficiency.[23][24]

To assess whether mnemonic forecasts capture variation in return predictability, Figure 4 reports, in Panel A, the real yield coefficients $\theta^{r}_{c,h}$ estimated from CE (y-axis) and mnemonic forecasts (x-axis). Panel B reports the excess return coefficients $\theta^{ex}_{c,h}$ estimated from CE (y-axis) and mnemonic forecasts (x-axis).

Figure 4: Performance for Real Yield to Maturity and Excess Returns

Panel A Panel B



Note. Panel A plots the return predictability coefficients $\theta^{r}_{c,h}$ from the CE dataset at annual frequency (y-axis) against the simulated dataset (x-axis). Panel B plots the excess return predictability coefficients $\theta^{ex}_{c,h}$ from the CE dataset at annual frequency (y-axis) against the simulated dataset (x-axis). Coefficients in both figures are country- and maturity-specific. The red lines indicate the fit between the two datasets' coefficients. Correlation is reported in the lower-right corner of the graphs. The identifier for each data point corresponds to the ISO 3166-2 country abbreviation.

The model captures sizable variation in market inefficiency. The correlation between CE and mnemonic coefficients is 53% for real yields and 66% for excess returns. A significant share of this explanatory power is tied to cross-country variation: for real yields, the R^2 for CE-based coefficients rises from 23% to 41% when adding the mnemonic estimates of $\theta^{r}_{c,h}$ to horizon dummies; for excess returns, it rises from 29% to 46% when adding the mnemonic estimates of $\theta^{ex}_{c,h}$ (see Table 13, columns (3), (4), (5) and (6)). In sum, selective recall applied to different databases accounts for significant variation in interest rate volatility and market inefficiency. Selective memory can have significant economic implications, as we discuss next.[25]

[23]Our F-test strongly rejects coefficient equality at all horizons. For real yields, the p-values are 0.0000 for all horizons $h = 1, 2, 3, 5,$ 6-10. For excess returns, they are 0.0000 for all horizons $h = 2, 3, 5,$ 6-10.

[24]The model reproduces pooled predictability estimates in CE. For the real yield, CE-based estimates are from Columns (1)-(5) of Table 20, mnemonic estimates are from Table 16. Both use annual data and give similar coefficients. For excess returns, mnemonic and CE estimates are in Table 17, which are again similar.

[25]We also evaluated the ability of mnemonic beliefs to produce a joint distribution of expectations and future returns that is similar to the one observed in the data. To do so, we imputed a constant ex-ante required return for each country c and maturity h, and then obtain "artificial" nominal rates by adding model-implied expectations to it, following the Fisher equation. Model-implied return predictability captures substantial variation in the data, with correlations of 42% and 43% with real yield and excess return predictability from

6 Conclusion

The consensus inflation forecasts of major banks in 18 advanced economies exhibit horizon-increasing overreaction. This is associated with maturity-increasing excess volatility of nominal rates, creating a systematic and sizable redistribution of wealth from borrowers to lenders after high expected inflation states. These patterns are at odds with leading models. We explain them using a model of mnemonic learning in which which forecasters overweight events that are salient due to their past frequency or similarity to current inflation. The model endogenizes belief biases based on features of the inflation DGP, making new predictions – supported in the data – on how these biases and return predictability should vary across countries.

We offer some concluding thoughts on the macro-financial implications of selective memory. Overreaction of long-run inflation expectations can affect macroeconomic volatility at business cycle frequencies. By creating a predictable redistribution from borrowing households, firms or governments to lenders, high expected inflation can cause a future reduction in aggregate demand. In general equilibrium, such systematic deflation "surprise" can generate a persistent recession, putting downward pressure on prices and feeding into further deflation. Work on news and sentiment (Lorenzoni, 2009; Angeletos and La'O, 2010, 2013; Angeletos et al., 2018) emphasizes how shifts in expectations about future real variables can generate demand-driven fluctuations without changes in current fundamentals. Mnemonic learning produces these shocks as a by-product of belief errors. First, a pure nominal shock such as an increase in inflation can cause, through excessively high expected inflation, a predictable future redistribution that has real effects. Second, real shocks can induce shifts in non-rational beliefs via general equilibrium channels, which will create seeming future belief "shocks" through the reversal of expectations, even absent contemporaneous disturbances.

Mnemonic beliefs may also throw new light on historical variation in the cost of disinflation. In the late 1970s and early 1980s, US inflation was high and volatile, and the Fed Chairman Volcker reduced it at a considerable employment cost (Ball, 1994). In contrast, the post pandemic inflation spike has been curbed relatively quickly and painlessly. In an NK model, such variation in the cost of disinflation can be due to shocks having different volatility or persistence or to changes in monetary policy conduct – such as a Taylor rule that responds more aggressively to inflation surges. Mnemonic beliefs can account for these episodes through the historical salience of different inflation scenarios. In the early 1980s, agents drew on a database dominated by high and volatile inflation, causing overweighting of high inflation states. This required a persistent and aggressive policy response. In the recent period, instead, the database was shaped by decades of low and stable inflation, which created, by similarity, a strong force toward rapid re-anchoring after the inflation spike subsided (Gennaioli et al. (2024) document

CE forecasts, respectively. Results are available upon request.

re-anchoring by US households). Similarity is key to allow for such state- and history-dependent speed of belief updating.

This logic can also shed light on the limited effectiveness of forward guidance. Following the financial crisis of 2008, nominal interest rates were constrained by the zero lower bound, prompting central banks to stimulate output by promising to generate higher inflation in the future (Eggertsson et al., 2003; Campbell et al., 2012; Del Negro et al., 2023). These announcements failed to raise expected inflation as intended, undermining their effectiveness, and challenging the predictions of standard NK models (Coibion et al., 2020; Del Negro et al., 2023). Several papers try to explain this phenomenon by departing from full-information rational-expectations (Angeletos and Lian, 2018; Farhi and Werning, 2019). Our model offers a different explanation: beliefs are shaped by similar inflation states. In the aftermath of the crisis, both the inflation database and the inflation cue rendered low inflation salient, making it difficult for agents to envision high-inflation scenarios – even if such scenarios were promised by central banks.

Finally, these memory forces offer a new perspective on optimal monetary policy. On the one hand, belief overreaction justifies gradualism, because an extreme cue may cause agents to overweight similarly extreme future events. On the other hand, memory implies that the level of beliefs is also shaped by historical experience. Ceteris paribus, this latter mechanism calls for aggressive policy: by responding swiftly to an inflation surge, the central bank prevents high-inflation states from accumulating in the database, thereby anchoring expected inflation at low levels, in the present and the future. The tradeoff between influencing memory cues and the database may offer new insights into the design of optimal policy.

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A Additional Material

A.1 The Data

Table 7: Description of Data Sources

Variable	Data Source	Series Identifier	Sample Period	Coverage Notes
Bond Yields	LSEG	RF *COUNTRY* GVT BMK BID YLD hY - RED. YIELD	1962m2–2022m10	$h = 1, 2, ..., 10$. Further information available upon request.
Realized CPI inflation	IMF Data Mapper	PCPIPCH	1980–2023	CZ from 1996. AU and NZ up to 2022. NL missing in 1980 and 1986.
Realized CPI inflation	World Bank's WDI	FP.CPI.TOTL.ZG	1960–2023	CZ from 1992, HK from 1982, SG from 1961. TW not covered.
Realized CPI inflation	Statistics Bureau	2020-Base Consumer Price Index	2024M1–2024M3	JP only.
Realized CPI inflation	OECD	Consumer price indices (CPIs, HICPs), COICOP 1999	2024Q1	HK, JP, SG, and TW not covered.
Realized CPI inflation	Census and Statistics Dept.	Table 510-60001	2024M1–2024M3	HK only.
Realized CPI inflation	Dept. of Stat., Directorate General of Budget, Accounting & Statistics	Price Indices	2024M1–2024M3	TW only.
Realized CPI inflation	Department of Statistics	Consumer Price Index	2024Q1	SG only.
Realized CPI inflation	Bureau of Labor Statistics	CUSR0000SA0	1947M1–2024M7	US only.

Note. the two-letter identifier in the coverage notes corresponds to the ISO 3166-2 country abbreviation.

Table 7: Description of Data Sources

Variable	Data Source	Series Identifier	Sample Period	Coverage Notes
Forecasted Bond Yields	Consensus Economics	3-month interest rate (12-month period)	1989m10–2022m1	NL, ES, SE, and CH from 1989m11. AU and NZ from 1990m1. SG from 1990m11. HK from 1990m12. KR and TW from 1994m12. CZ from 1998m5. NO from 1998m6. NZ missing from 1991m4 to 1991m12 and from 1993m1 to 1994m11.
Forecasted Bond Yields	Consensus Economics	Prime Lending Rate (12-month period)	1989m10–2022m1	AU from 1990m11. NZ, SG, and KR from 1994m12. NL, ES, and SE from 1995m1. NO and CH from 1998m6. CZ and TW from 2006m3. HK not covered.
Forecasted CPI Inflation	Consensus Economics	Consumer Prices	1989Q4–2022Q1	AU from 1991q2. HK, NL, NZ, SG, KR, ES, SE, and TW from 1995q2. NO and CH from 1998q4. Semiannual data, quarterly after 2014q2
Forecasted CPI Inflation	Survey of Professional Forecasters	CPIh, $h = 1, \dots 6$	1981Q3–2024Q2	US only. $h = 1$ denotes the "forecast" for the previous quarter, $h = 2$ denotes the forecast for the current quarter, $h = 3, ..6$ denote forecasts for the four quarters succeeding the current. Missing in 1969Q1-1969Q3 and 1970Q1 for $h = 6$.

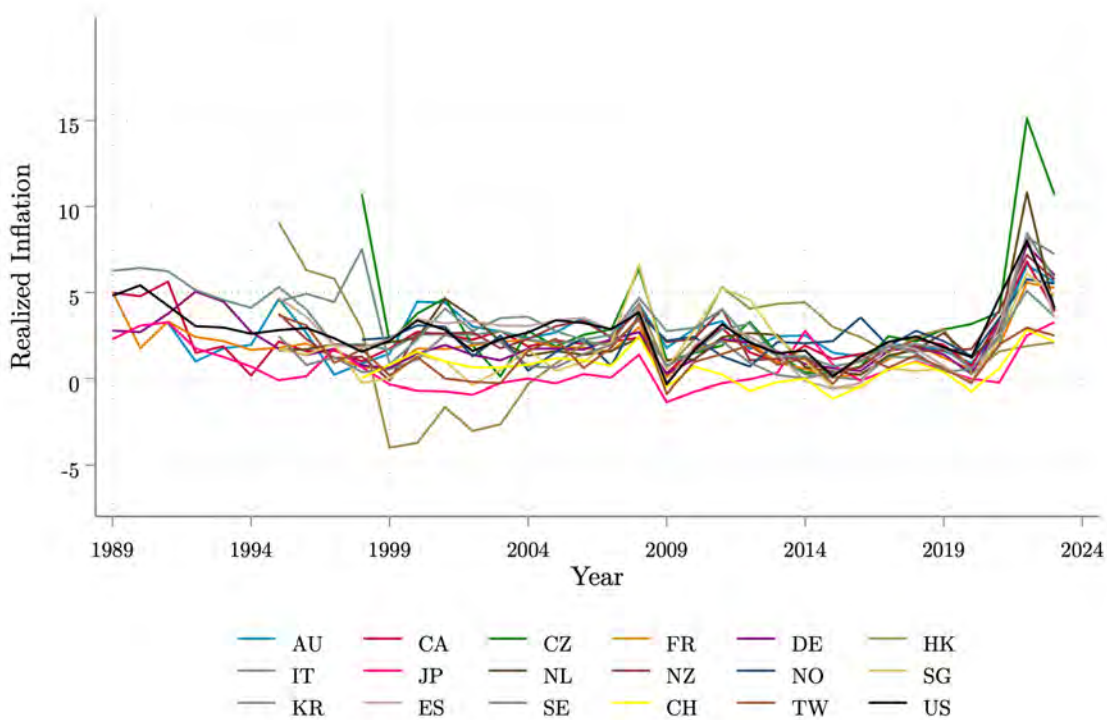
Note. the two-letter identifier in the coverage notes corresponds to the ISO 3166-2 country abbreviation.

Table 8: Composition of samples across different maturities

Country	Samples					
	Cumulative	By Maturity				
		H:0Y and H:1Y	H:2Y	H:3Y	H:5Y	H:(6-10)Y
Australia	✓	✓	✓		✓	✓
Canada	✓	✓	✓	✓	✓	✓
Czech Republic	✓		✓	✓	✓	
France	✓	✓	✓	✓	✓	✓
Germany	✓	✓	✓	✓	✓	✓
Hong Kong	✓	✓	✓	✓	✓	✓
Italy	✓		✓	✓	✓	✓
Japan	✓	✓	✓	✓	✓	✓
Netherlands	✓	✓	✓	✓	✓	✓
New Zealand	✓	✓	✓	✓	✓	✓
Norway	✓	✓	✓	✓	✓	✓
Singapore	✓	✓	✓		✓	✓
South Korea	✓	✓		✓	✓	
Spain	✓		✓	✓	✓	✓
Sweden	✓	✓	✓	✓	✓	✓
Switzerland	✓	✓	✓	✓	✓	✓
Taiwan	✓				✓	✓
United States	✓	✓	✓	✓	✓	✓

A.2 Additional Figures

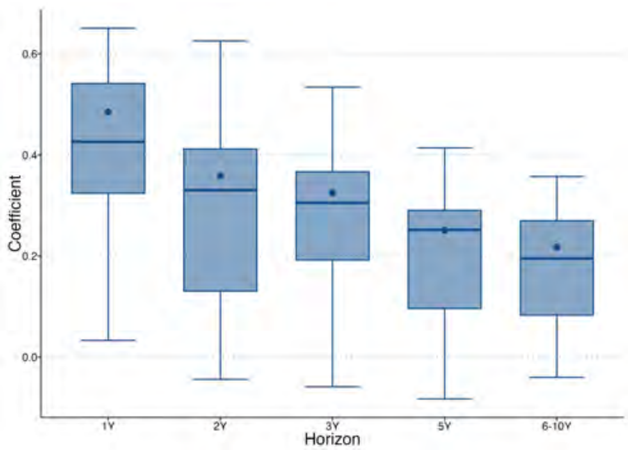
Figure 5: Realized Inflation over the sample



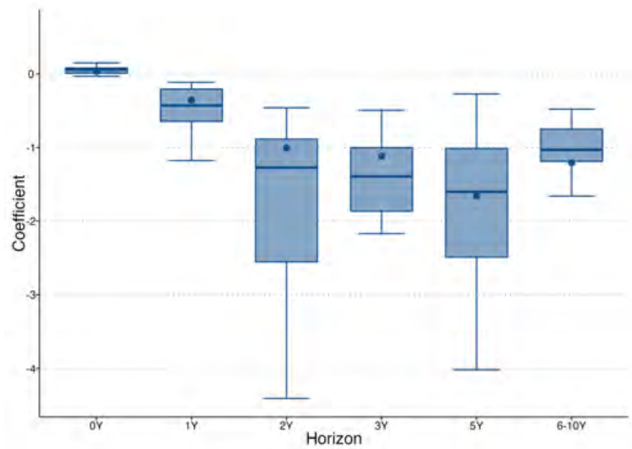
Note. Annual realized inflation for the 18 countries in our sample, from 1989 to 2023. Sample varies according to Table 7. The identifier for each series corresponds to the ISO 3166-2 country abbreviation.

Figure 6: Boxplots of country-specific coefficients.

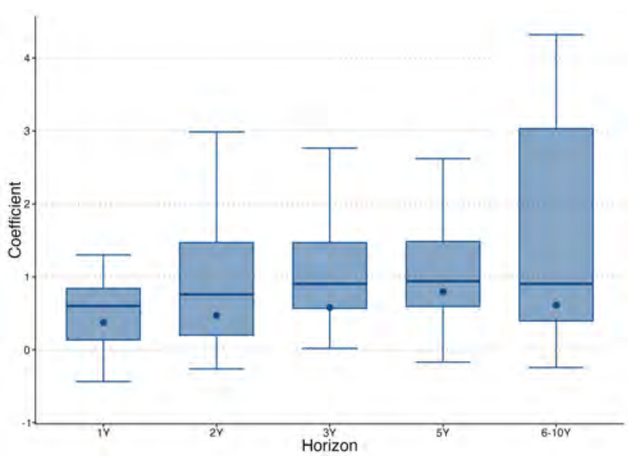
Panel A: Boxplot of Figure 1's results.



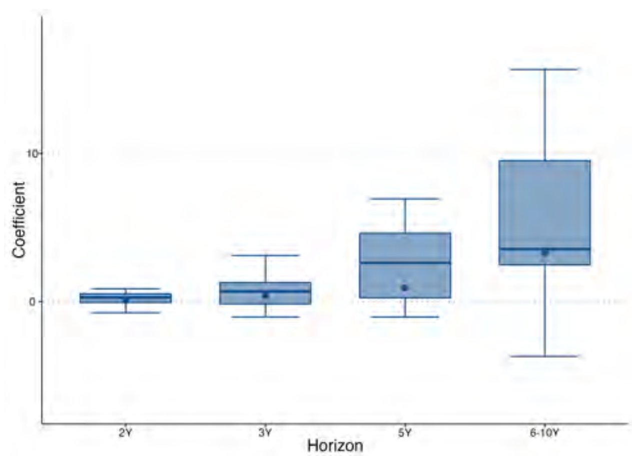
Panel B: Boxplot of Table 1 Panel A's results.



Panel C: Boxplots of Table 4 Panel B's results.



Panel D: Boxplots of Table 5 Panel B's results.



Note. The figure displays the boxplots and the pooled estimates from Figure 1 (Panel A), Table 1's Panel A (Panel B), Table 4's Panel B (Panel C), and Table 5's Panel B (Panel D). The y-axis measures the magnitude of the estimated coefficients, while the x-axis identifies the horizons $h = 0, 1, 2, 3, 5, 6\text{-}10$ for which estimates were computed. For each horizon, the sample varies according to Table 8. The boxplot illustrates the distribution of the country-specific coefficients. The box represents the interquartile (IQR) range, with the median indicated by the line inside the box. The whiskers extend to 1.5 times the IQR above the upper quartile and below the lower quartile. Points beyond the whiskers are considered outliers and are removed from the boxplots. The plot also displays the respective country-pooled estimates (dots) for each horizon and every specification.

A.3 Additional Tables and Figures

Table 9: Forecast revision predictability regressions by horizon

	(1)	(2)	(3)	(4)
	$FR_{c,2}$	$FR_{c,3}$	$FR_{c,5}$	$FR_{c,6-10}$
$\mathbb{F}_{\mathrm{t}}\left(\pi_{\mathrm{c,t+hY}}\right)$	-0.236**	-0.355***	-0.819***	-0.974***
	(0.090)	(0.097)	(0.131)	(0.150)
Country fixed effects	✓	✓	✓	✓
N	1107	973	1021	847
Adj. R^2	0.145	0.213	0.462	0.536

Note. We predict inflation consensus forecast revision from the current inflation state. Each column reports the country pooled regression at horizon $h = 2, 3, 5,$ 6-10: the dependent variable is the inflation consensus forecast revision $\mathbb{F}_{t+h-1}\left(\pi_{c,t+hY}\right) - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The current inflation state is the consensus forecast for country c and horizon h $\mathbb{F}_t\left(\pi_{c,t+hY}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2021, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 10: Forecast errors on inflation forecast revision

	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{c,0}$	$FE_{c,1}$	$FE_{c,2}$	$FE_{c,3}$	$FE_{c,5}$	$FE_{c,6-10}$
$\mathbb{F}_{\mathrm{t}}(\pi_{\mathrm{c,t+hY}}) - \mathbb{F}_{\mathrm{t-4}}(\pi_{\mathrm{c,t+hY}})$	0.173*	0.470*	-0.175	-0.256	0.242	-0.240
	(0.102)	(0.262)	(0.295)	(0.338)	(0.414)	(0.149)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	973	970	1104	969	1008	618
Adj. R^2	0.072	0.046	0.033	0.049	0.057	0.149

Note. We predict the inflation consensus forecast error from consensus revision (recent news). Each column reports the country pooled regression at horizon $h = 0, ...,$ 6-10: the dependent variable is the inflation consensus forecast error $\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The consensus revision is $\mathbb{F}_t\left(\pi_{c,t+hY}\right) - \mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2022, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 11: Inflation forecast error on inflation forecast revision from inflation forecast revision and lagged inflation forecast at the quarter horizon.

	(1)
	$\pi_{\mathrm{t+4}} - \mathbb{F}_{\mathrm{t}}^{SPF}(\pi_{\mathrm{t,t+4}})$
$\mathbb{F}_{\mathrm{t}}^{SPF}(\pi_{\mathrm{t+4}}) - \mathbb{F}_{\mathrm{t-1}}^{SPF}(\pi_{\mathrm{t+3}})$	0.017
	(0.407)
$\mathbb{F}_{\mathrm{t-1}}^{SPF}(\pi_{\mathrm{t+3}})$	-0.463***
	(0.103)
α	1.304***
	(0.322)
N	168
Adj. R^2	0.111

Notes. We predict the inflation consensus forecast error from consensus revision (recent news) and past inflation state. Column (1) reports the regression for the US at horizon $h = 4$, where h here denote quarters: the dependent variable is the CPI inflation SPF forecast error $\pi_{t+4} - \mathbb{F}_t^{SPF}(\pi_{t+4})$. The consensus revision is $\mathbb{F}_t^{SPF}(\pi_{t+4}) - \mathbb{F}_{t-1}^{SPF}(\pi_{t+3})$, while past inflation state is $\mathbb{F}_{t-1}^{SPF}(\pi_{t+3})$. The sample ranges from 1981Q4 to 2023Q3, coverage varies according to Tables 7. Standard errors in parenthesis. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 12: Correlation between CE ex-ante real yields and actual (FED or BoE) real yields.

	United Kingdom				
	$h = 2$	$h = 3$	$h = 4$	$h = 5$	$h = 6 - 10$
ρ	NA	.9183***	.9317***	.9365***	.9522***
N	NA	48	51	51	51
	United States				
	$h = 2$	$h = 3$	$h = 4$	$h = 5$	$h = 6 - 10$
ρ	.8723***	.9078***	.7080***	.9331***	.8899***
N	62	62	59	62	42

Note. For UK data, we use the "real spot rate" published by the Bank of England, i.e., the rate for a "index-linked zero coupon bond" having " its value linked to movements in a suitable price index to prevent inflation eroding its purchasing power" (see `https://www.bankofengland.co.uk/statistics/yield-curves/terminology-and-concepts`). We use the series at daily frequency, from 1999M1 to 2024M6, and aggregate it at quarterly frequency taking the simple average in order to make it comparable with the data we use in the rest of the analysis. To be consistent with CE data, we take the average of the rates for the horizons 6- to 10-year ahead. For US data, we use "TIPS yields" by the Federal Reserve Board following the methodology of Gürkaynak et al. (2010) (see `https://www.federalreserve.gov/data/tips-yield-curve-and-inflation-compensation.htm`). We used the series of zero-coupon inflation protected treasuries. We use the series at daily frequency, from 1999M1 to 2024M7, and then aggregate it at quarterly frequency taking the simple average in order to make it comparable with the data we use in the rest of the analysis. To be consistent with CE data, we take the average of the rates for the horizons 6- to 10-year ahead. Sample sizes and correlation coefficients vary with the horizons. ***, **, and * indicate significance at the 1, 5, and 10 percent levels respectively.

Table 13: Inflation forecast error predictability coefficients from Specifications (2), (6), and (8) on horizon dummies and simulated coefficients

	(1)	(2)	(3)	(4)	(5)	(6)
	$\delta_{\mathrm{c,h}}$	$\delta_{\mathrm{c,h}}$	$\theta^{r}_{\mathrm{c,h}}$	$\theta^{r}_{\mathrm{c,h}}$	$\theta^{ex}_{\mathrm{c,h}}$	$\theta^{ex}_{\mathrm{c,h}}$
$\Delta^{m}_{\mathrm{c,h}}$		1.549***		1.023***		1.733***
		(0.253)		(0.226)		(0.440)
α	-0.532**	-0.011	1.310**	1.393**	1.040	-0.254
	(0.251)	(0.222)	(0.596)	(0.526)	(1.188)	(1.099)
Horizon fixed effects	✓	✓	✓	✓	✓	✓
N	79	79	73	73	55	55
R^2	0.211	0.478	0.229	0.410	0.291	0.459

Notes. We study the relation between the actual and model-implied coefficients estimated for Specifications (2), (6), and (8). Columns (1), (3), and (5) report the regression of country- and horizon-specific coefficients on horizon dummies, while in Columns (2), (4), and (6) we control also for the model-implied coefficient, $\Delta^{m}_{c,h}$. $\Delta^{m}_{c,h} = \delta^{m}_{c,h}$ in column (2), $\Delta^{m}_{c,h} = \theta^{r,m}_{c,h}$ in column (4), and $\Delta^{m}_{c,h} = \theta^{ex,m}_{c,h}$ in column (6). Table 8 reports the countries available at each horizon $h = 1, \ldots, 6$-10. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 14: Inflation forecast error predictability coefficients from Specification (3) on horizon dummies and simulated coefficients

	(1)	(2)	(3)	(4)
	$\delta_{\mathrm{1,c,h}}$	$\delta_{\mathrm{1,c,h}}$	$\delta_{\mathrm{2,c,h}}$	$\delta_{\mathrm{2,c,h}}$
$\delta^{m}_{\mathrm{i,c,h}}$		1.044***		1.134***
		(0.328)		(0.190)
α	0.117	0.330	-1.280***	-0.528
	(0.369)	(0.354)	(0.384)	(0.341)
Horizon fixed effects	✓	✓	✓	✓
N	79	79	79	79
R^2	0.208	0.304	0.082	0.383

Notes. We study the relation between the actual and model-implied coefficients estimated for Specification (3). Columns (1) and (3) report the regression of country- and horizon-specific coefficients on horizon dummies, while in Columns (2) and (4) we control also for the model-implied coefficient, $\delta^{m}_{i,c,h}$. $\delta^{m}_{i,c,h} = \delta^{m}_{1,c,h}$ in column (2) and $\delta^{m}_{i,c,h} = \delta^{m}_{2,c,h}$ in column (4). Table 8 reports the countries available at each horizon $h = 1, \ldots, 6$-10. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 15: Predicting the mnemonic inflation forecast error using mnemonic inflation forecasts

	(1)	(2)	(3)	(4)	(5)
	$FE^m_{c,1}$	$FE^m_{c,2}$	$FE^m_{c,3}$	$FE^m_{c,5}$	$FE^m_{c,6-10}$
$\mathbb{F}^m_{\mathrm{t}}\left(\pi_{\mathrm{c,t+hY}}\right)$	-0.410***	-0.823***	-0.965***	-1.237***	-1.184***
	(0.082)	(0.183)	(0.165)	(0.174)	(0.059)
Country fixed effects	✓	✓	✓	✓	✓
N	420	462	416	462	331
Adj. R^2	0.095	0.203	0.292	0.365	0.779

*Note.*We predict mnemonic inflation forecast error from the mnemonic inflation forecast. Each column reports the country pooled regression at horizon $h = 1, ..., 6\text{-}10$: the dependent variable is the mnemonic inflation forecast error $\pi_{c,t+hY} - \mathbb{F}^m_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The consensus forecast for country c and horizon h is $\mathbb{F}_t\left(\pi_{c,t+hY}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2022, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels. Sample varies with the horizons according to Table 8.

Table 16: Ex-post real returns on mnemonic average forecast for inflation over the period $t + 1, t + h$

	$\tilde{r}^{(1)}_{\mathrm{c,t,t+1}}$	$\tilde{r}^{(2)}_{\mathrm{c,t,t+2}}$	$\tilde{r}^{(3)}_{\mathrm{c,t,t+3}}$	$\tilde{r}^{(5)}_{\mathrm{c,t,t+5}}$	$\tilde{r}^{(6\text{-}10)}_{\mathrm{c,t,t+6\text{-}t+10}}$
$\mathbb{F}^m_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$	-0.259	0.806*	1.015**	1.018**	0.988**
	(0.289)	(0.399)	(0.407)	(0.374)	(0.341)
Country fixed effects	✓	✓	✓	✓	✓
N	390	458	409	456	119
Adj. R^2	0.049	0.132	0.226	0.251	0.315

Notes. We study the association between ex-post real rates to maturity h and the mnemonic forecast for the average inflation over h years. Each column reports the country pooled regression at horizon $h = 1, ..., 6\text{-}10$. The mnemonic forecast for the average inflation over h years for country c, $\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$, is computed as: $\frac{1}{h}\sum_{h'=1}^{h}\mathbb{F}_{\mathrm{t}}\left(\pi_{c,t+h'Y}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2022, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 17: Bond excess returns (smoothed over two years) on mnemonic and consensus inflation forecasts

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$
$\mathbb{F}^i_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.142	0.082	1.898***	2.826**	0.244**	0.298*	2.941***	4.232*
	(0.092)	(0.138)	(0.507)	(1.070)	(0.102)	(0.157)	(0.836)	(2.195)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓
N	371	342	269	172	371	342	269	172
Adj. R^2	0.085	0.145	0.233	0.180	0.088	0.152	0.173	0.093

Note. We predict one-year holding period realized bond excess returns with maturity h from forecast of the average inflation over h years, both from mnemonic forecasts, Columns (1)-(4), and from consensus forecasts at annual frequency, Columns (5)-(8). Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6\text{-}10$: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^{(h)}_{c,t,t+1} = \frac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}$. The raw one-year holding period excess returns $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)\left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. The forecast for the average inflation over h years for country c, $\mathbb{F}^i_t\left(\overline{\pi_{c,t+1,h}}\right)$, with i indicating mnemonic or consensus forecasts, is computed as: $\frac{1}{h}\sum_{h'=1}^{h}\mathbb{F}^i_t\left(\pi_{c,t+h'Y}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2020, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

A.4 Robustness: Annual Frequency

Table 18: Predicting the consensus forecast error using consensus inflation forecast - Annual Frequency

	(1)	(2)	(3)	(4)	(5)
	$FE_{c,1}$	$FE_{c,2}$	$FE_{c,3}$	$FE_{c,5}$	$FE_{c,6\text{-}10}$
$\mathbb{F}_t\left(\pi_{c,t+hY}\right)$	-0.271*	-0.763**	-0.937***	-1.549***	-1.211***
	(0.151)	(0.286)	(0.275)	(0.297)	(0.123)
Country fixed effects	✓	✓	✓	✓	✓
N	420	462	416	462	331
Adj. R^2	0.036	0.117	0.154	0.268	0.545

Note. We predict inflation consensus forecast error from the consensus inflation forecast. Each column reports the country pooled regression at horizon $h = 1, ..., 6\text{-}10$: the dependent variable is the inflation consensus forecast error $\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The consensus forecast for country c and horizon h is $\mathbb{F}_t\left(\pi_{c,t+hY}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2022, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels. Sample varies with the horizons according to Table 8.

Table 19: Forecast errors on inflation forecast revision and lagged inflation forecast - Annual Frequency

	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{c,0}$	$FE_{c,1}$	$FE_{c,2}$	$FE_{c,3}$	$FE_{c,5}$	$FE_{c,6\text{-}10}$
$\mathbb{F}_t(\pi_{c,t+hY}) - \mathbb{F}_{t-4}(\pi_{c,t+hY})$	0.324	0.481**	-0.434	-0.635	-1.265***	-1.115***
	(0.199)	(0.232)	(0.356)	(0.378)	(0.313)	(0.137)
$\mathbb{F}_{t-4}(\pi_{c,t+hY})$	-0.091	-0.426*	-0.923**	-1.051**	-1.588***	-1.015***
	(0.055)	(0.235)	(0.421)	(0.405)	(0.359)	(0.120)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	406	405	445	400	437	314
Adj. R^2	0.137	0.092	0.091	0.114	0.165	0.389

Note. We predict the inflation consensus forecast error from consensus revision (recent news) and past inflation state. Each column reports the country pooled regression at horizon $h = 0, ..., 6\text{-}10$: the dependent variable is the inflation consensus forecast error $\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The consensus revision is $\mathbb{F}_t\left(\pi_{c,t+hY}\right) - \mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$, while past inflation state is $\mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2022, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 20: Ex-post real returns on current average forecast for inflation over the period $t+1, t+h$, controlling for required real rates - Annual frequency

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$\tilde{r}^{(1)}_{c,t,t+1}$	$\tilde{r}^{(2)}_{c,t,t+2}$	$\tilde{r}^{(3)}_{c,t,t+3}$	$\tilde{r}^{(5)}_{c,t,t+5}$	$\tilde{r}^{(6-10)}_{c,t,t+6,t+10}$	$\tilde{r}^{(1)}_{c,t,t+1}$	$\tilde{r}^{(2)}_{c,t,t+2}$	$\tilde{r}^{(3)}_{c,t,t+3}$	$\tilde{r}^{(5)}_{c,t,t+5}$	$\tilde{r}^{(6-10)}_{c,t,t+6,t+10}$
$\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$	0.376	1.562***	1.762***	2.115***	-0.816	0.262*	0.331**	0.446***	0.749***	0.556**
	(0.349)	(0.478)	(0.378)	(0.309)	(0.642)	(0.136)	(0.134)	(0.142)	(0.147)	(0.224)
$i^{(h)}_{\mathrm{c,t}} - \mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$						1.183***	1.146***	1.081***	1.010***	1.047***
						(0.135)	(0.119)	(0.081)	(0.066)	(0.113)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
N	390	458	409	456	119	390	458	409	456	119
Adj. R^2	0.046	0.220	0.293	0.384	0.865	0.706	0.773	0.831	0.883	0.865

Notes. We study the association between ex-post real rates to maturity h and the consensus forecast for the average inflation over h years (Columns (1) to (5)). Additionally, we control for ex-ante real rates (Columns (6) to (10)). Each column reports the country pooled regression at horizon $h = 1, ..., 6$-10. Real ex-post rates $\tilde{r}^{(h)}_{c,t,t+h}$ over horizon h and country c are defined as $i^{(h)}_{\mathrm{c,t}} - \overline{\pi_{\mathrm{c,t+1,h}}}$. The consensus forecast for the average inflation over h years for country c, $\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$, is computed as: $\frac{1}{h}\sum_{h'=1}^{h}\mathbb{F}_{\mathrm{t}}\left(\pi_{c,t+h'Y}\right)$. Ex-ante required real rates are defined as $i^{(h)}_{\mathrm{c,t}} - \mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$. We control for country fixed effects. The sample ranges from 1989 to 2022, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 21: Predicting realized excess returns using consensus inflation forecasts - Annual frequency

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$
$\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$	0.244**	0.298*	2.941***	4.232*	0.073	0.222	0.979**	2.723**
	(0.102)	(0.157)	(0.836)	(2.195)	(0.070)	(0.131)	(0.450)	(1.174)
$i^{(1)}_{c,t}$					0.257***	0.167*	0.789***	0.573*
					(0.028)	(0.085)	(0.115)	(0.306)
$i^{(10)}_{c,t} - i^{(1)}_{c,t}$					0.365***	0.357**	1.360***	2.242***
					(0.096)	(0.153)	(0.299)	(0.599)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓
N	371	342	269	172	370	341	269	172
Adj. R^2	0.088	0.152	0.173	0.093	0.364	0.191	0.461	0.346

Note. Columns (1)-(4) predict the smoothed excess return $\overline{rx}^{(h)}_{c,t,t+1} = \frac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}$ using the consensus forecast for average inflation over h years $\mathbb{F}_t(\overline{\pi_{c,t+1,h}}) = \frac{1}{h}\sum_{h'=1}^{h} \mathbb{F}_t\left(\pi_{c,t+h'Y}\right)$. Columns (5)-(8) additionally control for the level $i^{(1)}_{c,t}$ and slope $i^{(10)}_{c,t} - {}^{(1)}_{c,t}$ of the yield curve. The raw one-year excess return is $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)\left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. We pool countries and control for country fixed effects. The sample ranges from 1989 to 2022, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

A.5 Robustness: Smoothing excess returns over three years and controlling for interest rate forecasts

Table 22: Predicting realized excess returns (smoothed over three years) using consensus inflation forecasts

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$
$\mathbb{F}_{t}\left(\overline{\pi_{c,t+1,h}}\right)$	0.193**	0.268**	2.192***	3.401**	0.026	0.273**	0.557*	2.214**
	(0.077)	(0.119)	(0.604)	(1.399)	(0.079)	(0.129)	(0.313)	(0.851)
$i^{(1)}_{c,t}$					0.261***	0.133*	0.813***	0.785***
					(0.028)	(0.072)	(0.109)	(0.269)
$i^{(10)}_{c,t} - i^{(1)}_{c,t}$					0.304***	0.198	1.233***	2.089***
					(0.091)	(0.142)	(0.294)	(0.447)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓
N	880	813	656	426	877	811	656	426
Adj. R^2	0.154	0.314	0.247	0.161	0.446	0.338	0.581	0.511

Note. We predict one-year holding period realized bond excess returns with maturity h from consensus forecast of the average inflation over h years (Columns (1) to (4)). Additionally, we control for level and slope of nominal yields (Columns (5) to (8)). Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6$-10: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^{(h)}_{c,t,t+1} = \frac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2} + rx^{(h)}_{c,t+2,t+3}}{3}$. The raw one-year holding period excess returns $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)\left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. The level and slope for country c and maturity h are defined respectively as $i^{(1)}_{c,t}$ and $i^{(10)}_{c,t} - {}^{(1)}_{c,t}$. We control for country fixed effects. The sample ranges from 1989Q4 to 2019Q4, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 23: Predicting realized excess returns (smoothed over two years) using consensus inflation forecasts, controlling for interest rate forecasts

	(1)	(2)	(3)	(4)
	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$
$\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$	0.225***	0.999***	1.923***	3.271***
	(0.082)	(0.229)	(0.561)	(1.166)
$i^{(1)}_{\mathrm{c,t}}$	0.655***	0.541	0.975	0.598
	(0.163)	(0.403)	(0.692)	(1.203)
$i^{(10)}_{\mathrm{c,t}} - i^{(1)}_{\mathrm{c,t}}$	0.856***	0.849*	1.590**	2.266
	(0.163)	(0.468)	(0.746)	(1.440)
$\mathbb{F}_{\mathrm{t}}(i^{(10)}_{\mathrm{c,t+1}})$	-0.436**	-0.536	-0.132	0.163
	(0.177)	(0.427)	(0.799)	(1.330)
Country fixed effects	✓	✓	✓	✓
N	845	789	651	466
Adj. R^2	0.426	0.322	0.535	0.408

Note. We predict one-year holding period realized bond excess returns with maturity h from consensus forecast of the average inflation over h years. Additionally, we control for the level and slope of the yield curve and for nominal 10-year interest rate forecasts. We do not control for short-term interest rate forecasts because of multicollinearity. Each column reports the country pooled regression at horizon $h = 2, 3, 5,$ 6-10: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^{(h)}_{c,t,t+1} = \frac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}$. The raw one-year holding period excess returns $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)\left(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}\right) - i^{(1)}_{c,t}$. The level and slope for country c and maturity h are defined respectively as $i^{(1)}_{c,t}$ and $i^{(10)}_{c,t} - i^{(1)}_{c,t}$. The nominal 10-year interest rate forecast is $\mathbb{F}_{\mathrm{t}}(i^{(10)}_{\mathrm{c,t+1}})$. We control for country fixed effects. The sample ranges from 1989Q4 to 2020Q4, Table 8 reports the countries available at each maturity. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

A.6 Robustness: Stambaugh bias

We address the potential (Stambaugh (1999)) bias when predicting returns with persistent factors. In our analysis, this concern applies to the role of inflation expectations in Tables (4) and (5). Boudoukh et al. (2022) show that such bias becomes more severe at longer horizons, which is relevant for our analysis of maturity increasing overreaction, and develop a bias correction formula. We assess whether our predictability results are robust to such correction.

To see the idea, consider the univariate predictive regression of the 1-period ahead excess bond return onto a regressor X_t, namely:

$$rx_{t,t+1} = \alpha + \beta X_t + u_{t+1}. \tag{24}$$

Stambaugh (1999) showed that with a persistent predictor

$$X_{t+1} = \omega + \rho X_t + v_{t+1}, \tag{25}$$

the OLS estimate β^{OLS} exhibits a small–sample distortion, because the innovation v_t may be contemporaneously correlated with the return error u_t. Stambaugh (1999) further shows that, to a first order approximation in $1/T$:

$$\mathbb{E}[\hat{\beta}^{\mathrm{OLS}}] = \beta - \frac{1}{T}\,(1+\rho)\frac{\sigma_{uv}}{\sigma_v^2}, \tag{26}$$

where T is the sample length and σ_{uv} is the covariance between the error terms. Boudoukh et al. (2022) show that that in the case of multi-horizon predictions, for overlapping returns (and hence relevant to our long horizon regressions), the OLS estimate is biased as:

$$\mathbb{E}[\hat{\beta}^{\mathrm{OLS}}] = \beta - \frac{1}{T}\,K\frac{\sigma_{uv}}{\sigma_v^2}, \tag{27}$$

where

$$K = \; h\big(1+\rho\big) + 2\rho\frac{1-\rho^h}{1-\rho}. \tag{28}$$

We adapt this correction for our univariate tests, and also use the multivariate correction in Boudoukh et al. (2022) for the specification where we also control for time varying return proxies. In general, we assume that predictors $X_t = (X_{1t}, \ldots, X_{kt})^\top$ follow $AR(1)$ processes with possibly correlated innovations $v_t \sim (0, \Sigma_{vv})$. The vector of OLS slope coefficients then satisfies:

$$\mathbb{E}[\hat{\beta}^{\text{OLS}}] = \beta - \frac{1}{T}\Sigma_{uv}\Sigma_{vv}^{-1}K, \tag{29}$$

with $K = \text{diag}(K_1, \ldots, K_k)$ and K_j defined as

$$K_j = \ h\big(1+\rho_j\big) + 2\rho_j \frac{1-\rho_j^h}{1-\rho_j}. \tag{30}$$

Hence, our bias-adjusted estimator is

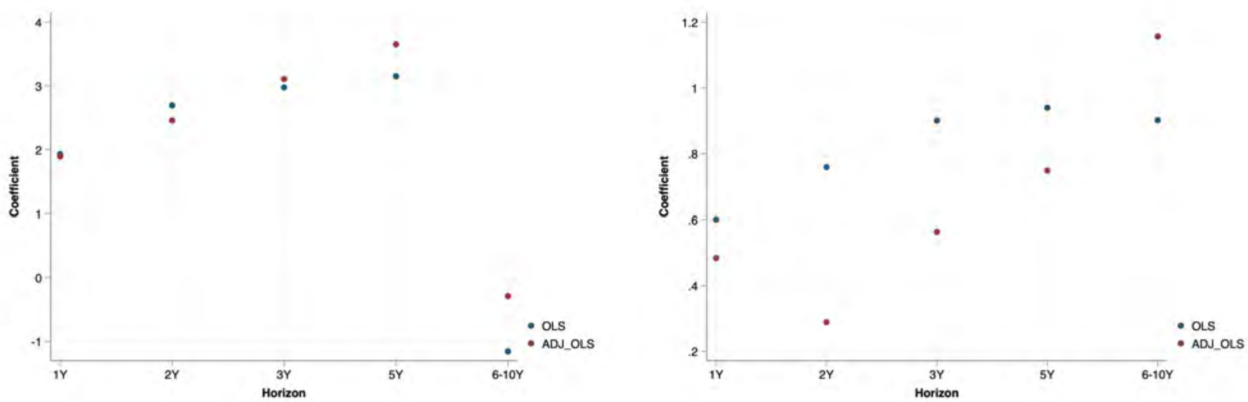
$$\hat{\beta}^{\text{BIR}} = \hat{\beta}^{\text{OLS}} + \frac{1}{T}\,\hat{\Sigma}_{uv}\,\hat{\Sigma}_{vv}^{-1}\,\hat{K},$$

which removes the leading $1/T$ bias simultaneously for all (correlated) regressors. We apply this method to the specifications in Tables (4) and (5). Given our panel structure, we run the predictive regressions by country and analyze the median coefficients, comparing the medians with and without adjustment.

Results for Table 4 are reported in Figure 7; results for Table 5 are reported in Figure 8. The reported coefficients are the median predictability coefficient for inflation expectations in the univariate and multivariate specifications, respectively.

Figure 7: Median country-level predictability coefficients from Table 4 - OLS vs bias-adjusted OLS.

Panel A: Columns (1)-(5)'s coefficients. Panel B: Columns (6)-(10)'s coefficients.

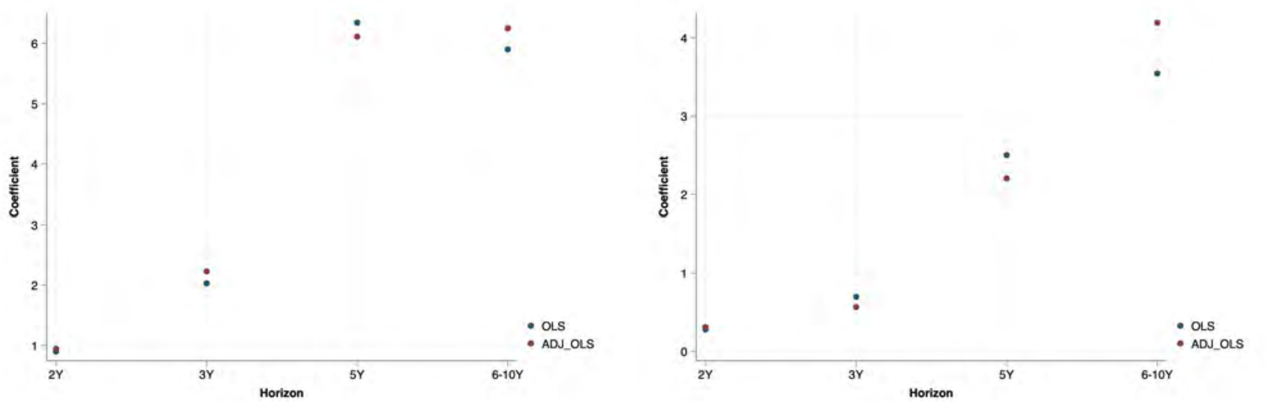


Note. Both panels plot the median return predictability coefficients $\theta^r_{c,h}$ from the CE dataset at quarterly frequency (y-axis) across horizons (x-axis) estimated using country-specific regressions. Panel A of this figure refers to Table 4's Columns (1)-(5); Panel B of this figure refers to Columns (6)-(10) of the same Table. The y-axis measures the magnitude of the estimated coefficients of $\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$ using OLS (OLS, blue points) and bias-adjusted OLS (ADJ_OLS, red points), while the x-axis identifies the horizons $h = 1, 2, 3, 5,$ 6-10 for which estimates were computed. For each horizon, the sample varies according to Table 8.

As evident from the two figures, our finding of maturity-increasing predictability is robust to the Boudoukh et al. (2022) adjustment. The OLS and the bias-adjusted coefficients are

Figure 8: Median country-level predictability coefficients from Table 5 - OLS vs bias-adjusted OLS.

Panel A: Columns (1)-(4)'s coefficients. Panel B: Columns (5)-(8)'s coefficients.



Note. Both panels plot the median return predictability coefficients $\theta^{ex}_{c,h}$ from the CE dataset at quarterly frequency (y-axis) across horizons (x-axis) estimated using country-specific regressions. Panel A of this figure refers to Table 5's Columns (1)-(4); Panel B of this figure refers to Columns (5)-(8) of the same Table. The y-axis measures the magnitude of the estimated coefficients of $\mathbb{F}_t\left(\overline{\pi_{c,t+1,h}}\right)$ using OLS (OLS, blue points(and bias-adjusted OLS (ADJ_OLS, red points), while the x-axis identifies the horizons $h = 2, 3, 5,$ 6-10 for which estimates were computed. For each horizon, the sample varies according to Table 8.

generally also close to each other, suggesting that the bias is small. In the figures the median country may differ across OLS and adjusted estimates, but the results are robust if we keep the country fixed (either by fixing the OLS median or the adjusted median).

A.7 Robustness: Including all countries and UK

Table 24: Inflation forecast errors on current inflation forecasts and past realized inflation. Full cumulative, including UK

	Panel A					
	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{\mathrm{c},0}$	$FE_{\mathrm{c},1}$	$FE_{\mathrm{c},2}$	$FE_{\mathrm{c},3}$	$FE_{\mathrm{c},5}$	$FE_{\mathrm{c},6\text{-}10}$
$\mathbb{F}_{\mathrm{t}}\left(\pi_{\mathrm{c,t+hY}}\right)$	0.039	-0.406**	-0.984***	-1.201***	-1.671***	-1.146***
	(0.054)	(0.155)	(0.286)	(0.279)	(0.277)	(0.110)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	1366	1366	1361	1285	1131	745
Adj. R^2	0.018	0.066	0.160	0.187	0.249	0.483
	Panel B					
	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{\mathrm{c},0}$	$FE_{\mathrm{c},1}$	$FE_{\mathrm{c},2}$	$FE_{\mathrm{c},3}$	$FE_{\mathrm{c},5}$	$FE_{\mathrm{c},6\text{-}10}$
$\overline{\pi}_{\mathrm{c,t}}$	-0.065	-0.265*	-0.298**	-0.340**	-0.638***	-0.428***
	(0.043)	(0.130)	(0.141)	(0.125)	(0.193)	(0.069)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	554	554	534	515	477	376
Adj. R^2	0.002	0.057	0.065	0.086	0.232	0.348

Note. We predict inflation consensus forecast error from two proxies for the current inflation state. Each column reports the country pooled regression at horizon $h = 0, ..., 6\text{-}10$: the dependent variable is the inflation consensus forecast error $\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The two proxies for the current inflation state are: (i) the consensus forecast for country c and horizon h $\mathbb{F}_t\left(\pi_{c,t+hY}\right)$ (Panel A), and (ii) the average realized inflation of country c over the past two years, $\overline{\pi}_{c,t} = (\pi_{c,(t-1)Y} + \pi_{c,(t-2)Y})/2$ (Panel B). We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1. For every horizon, we include all of the countries listed in Table 8, as well as United Kingdom. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 25: Inflation forecast error predictability from inflation forecast revision and lagged inflation forecast. Full cumulative, including UK

	(1)	(2)	(3)	(4)	(5)	(6)
	$FE_{\mathrm{c},0}$	$FE_{\mathrm{c},1}$	$FE_{\mathrm{c},2}$	$FE_{\mathrm{c},3}$	$FE_{\mathrm{c},5}$	$FE_{\mathrm{c},6\text{-}10}$
$\mathbb{F}_{\mathrm{t}}(\pi_{\mathrm{c,t+hY}}) - \mathbb{F}_{\mathrm{t}-4}(\pi_{\mathrm{c,t+hY}})$	0.193	0.299	-0.840*	-1.235***	-1.299***	-1.155***
	(0.122)	(0.317)	(0.425)	(0.438)	(0.272)	(0.125)
$\mathbb{F}_{\mathrm{t}-4}(\pi_{\mathrm{c,t+hY}})$	-0.108***	-0.612**	-1.181***	-1.427***	-1.805***	-0.952***
	(0.040)	(0.246)	(0.393)	(0.408)	(0.392)	(0.131)
Country fixed effects	✓	✓	✓	✓	✓	✓
N	1290	1286	1282	1205	1042	701
Adj. R^2	0.108	0.108	0.141	0.163	0.185	0.339

Note. We predict the inflation consensus forecast error from consensus revision (recent news) and past inflation state. Each column reports the country pooled regression at horizon $h = 0, ..., 6\text{-}10$: the dependent variable is the inflation consensus forecast error $\pi_{c,t+hY} - \mathbb{F}_t\left(\pi_{c,t+hY}\right)$ over horizon h and country c. The consensus revision is $\mathbb{F}_t\left(\pi_{c,t+hY}\right) - \mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$, while past inflation state is $\mathbb{F}_{t-4}\left(\pi_{c,t+hY}\right)$. We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1. For every horizon, we include all of the countries listed in Table 8, as well as United Kingdom. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 26: Nominal yields on current average forecast for average inflation over the period $t+1, t+h$ and forecast of nominal yields for $t+1$. Full cumulative, including UK

	Panel A				
	(1)	(2)	(3)	(4)	(5)
	$i_{c,t}^{(1)}$	$i_{c,t}^{(2)}$	$i_{c,t}^{(3)}$	$i_{c,t}^{(5)}$	$i_{c,t}^{(6\text{-}10)}$
$\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$	1.074***	2.109***	2.318***	2.637***	2.512***
	(0.161)	(0.295)	(0.268)	(0.284)	(0.518)
Country fixed effects	✓	✓	✓	✓	✓
N	1146	1326	1180	1347	652
Adj. R^2	0.357	0.488	0.526	0.519	0.291
	Panel B				
	(6)	(7)	(8)	(9)	(10)
	$i_{c,t}^{(1)}$	$i_{c,t}^{(2)}$	$i_{c,t}^{(3)}$	$i_{c,t}^{(5)}$	$i_{c,t}^{(6\text{-}10)}$
$\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$	0.049*	0.058*	0.234***	0.338***	0.134
	(0.026)	(0.031)	(0.071)	(0.094)	(0.104)
$\mathbb{F}_{\mathrm{t}}(i_{\mathrm{c,t+1}}^{(k)})$	0.986***	1.028***	0.949***	1.017***	0.953***
	(0.017)	(0.015)	(0.021)	(0.034)	(0.031)
Country fixed effects	✓	✓	✓	✓	✓
N	1146	1309	1161	1175	646
Adj. R^2	0.957	0.973	0.939	0.963	0.947

Note. We study the association between nominal yields to maturity h and the consensus forecast for the average inflation over h years (Panel A). Additionally, we control for the consensus forecast of interest rates (Panel B). Each column reports the country pooled regression at horizon $h = 1, ..., 6\text{-}10$. The dependent variable is the nominal yield to maturity h. The consensus forecast for the average inflation over h years for country c, $\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$, is computed as: $\frac{1}{h}\sum_{h'=1}^{h}\mathbb{F}_{\mathrm{t}}\left(\pi_{c,t+h'Y}\right)$. To roughly match short and long maturities, Panel B controls for the expected 3 months rate for $h = 1, 2, 3$, Columns (1)-(3), and for the expected 10 year rate for $h = 5, 6\text{-}10$, Columns (4) and (5). We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1. For every horizon, we include all of the countries listed in Table 8, as well as United Kingdom. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 27: Ex-post real returns on current average forecast for inflation over the period $t+1, t+h$, controlling for required real rates. Full cumulative, including UK

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$\tilde{r}^{(1)}_{c,t,t+1}$	$\tilde{r}^{(2)}_{c,t,t+2}$	$\tilde{r}^{(3)}_{c,t,t+3}$	$\tilde{r}^{(5)}_{c,t,t+5}$	$\tilde{r}^{(6-10)}_{c,t+6,t+10}$	$\tilde{r}^{(1)}_{c,t,t+1}$	$\tilde{r}^{(2)}_{c,t,t+2}$	$\tilde{r}^{(3)}_{c,t,t+3}$	$\tilde{r}^{(5)}_{c,t,t+5}$	$\tilde{r}^{(6-10)}_{c,t+6,t+10}$
$\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$	0.397	1.779***	2.025***	2.284***	-0.739*	0.308**	0.502***	0.621***	0.809***	1.037***
	(0.290)	(0.387)	(0.342)	(0.265)	(0.412)	(0.142)	(0.150)	(0.150)	(0.124)	(0.312)
$i^{(h)}_{\mathrm{c,t}} - \mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$						1.200***	1.151***	1.087***	1.026***	1.105***
						(0.136)	(0.108)	(0.079)	(0.060)	(0.103)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
N	1146	1325	1111	1118	268	1146	1325	1111	1118	268
Adj. R^2	0.098	0.238	0.328	0.383	0.878	0.615	0.726	0.799	0.880	0.878

Note. We study the association between ex-post real rates to maturity h and the consensus forecast for the average inflation over h years (Columns (1) to (5)). Additionally, we control for ex-ante real rates (Columns (6) to (10)). Each column reports the country pooled regression at horizon $h = 1, ..., 6\text{-}10$. Real ex-post rates $\tilde{r}^{(h)}_{c,t,t+h}$ over horizon h and country c are defined as $i^{(h)}_{\mathrm{c,t}} - \overline{\pi_{\mathrm{c,t+1,h}}}$. The consensus forecast for the average inflation over h years for country c, $\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$, is computed as: $\frac{1}{h}\sum_{h'=1}^{h}\mathbb{F}_{\mathrm{t}}\left(\pi_{c,t+h'Y}\right)$. Ex-ante required real rates are defined as $i^{(h)}_{\mathrm{c,t}} - \mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$. We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1. For every horizon, we include all of the countries listed in Table 8, as well as United Kingdom. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

Table 28: Bond excess returns on current forecasts for average expected inflation over the period $t, t+h$ and interest rate forecasts. Full cumulative, including UK

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$	$\overline{rx}^{(2)}_{c,t,t+1}$	$\overline{rx}^{(3)}_{c,t,t+1}$	$\overline{rx}^{(5)}_{c,t,t+1}$	$\overline{rx}^{(6-10)}_{c,t,t+1}$
$\mathbb{F}_{\mathrm{t}}\left(\overline{\pi_{\mathrm{c,t+1,h}}}\right)$	0.307***	0.452**	3.049***	6.089***	0.072	0.392**	0.934**	2.829***
	(0.104)	(0.181)	(0.929)	(2.113)	(0.067)	(0.154)	(0.426)	(1.026)
$i^{(1)}_{\mathrm{c,t}}$					0.270***	0.163*	0.867***	0.864**
					(0.040)	(0.095)	(0.158)	(0.355)
$i^{(10)}_{\mathrm{c,t}} - i^{(1)}_{\mathrm{c,t}}$					0.371***	0.364**	1.508***	2.667***
					(0.079)	(0.150)	(0.231)	(0.515)
Country fixed effects	✓	✓	✓	✓	✓	✓	✓	✓
N	1041	964	758	549	1038	962	758	549
Adj. R^2	0.136	0.260	0.201	0.117	0.411	0.300	0.511	0.415

Note. We predict one-year holding period realized bond excess returns with maturity h from consensus forecast of the average inflation over h years (Columns (1) to (4)). Additionally, we control for level and slope of nominal yields (Columns (5) to (8)). Each column reports the country pooled regression at horizon $h = 2, 3, 5, 6$-10: the dependent variable is the smoothed one-year holding period excess returns over horizon h and country c, defined as $\overline{rx}^{(h)}_{c,t,t+1} = \frac{rx^{(h)}_{c,t,t+1} + rx^{(h)}_{c,t+1,t+2}}{2}$. The raw one-year holding period excess returns $rx^{(h)}_{c,t,t+1}$ is defined as $i^{(h)}_{c,t} + (h-1)(i^{(h)}_{c,t} - i^{(h-1)}_{c,t+1}) - i^{(1)}_{c,t}$.

The level and slope for country c and maturity h are defined respectively as $i^{(1)}_{c,t}$ and $i^{(10)}_{c,t} - i^{(1)}_{c,t}$.We control for country fixed effects. The sample ranges from 1989Q4 to 2022Q1. For every horizon, we include all of the countries listed in Table 8, as well as United Kingdom. Standard errors in parenthesis are Driscoll-Kraay. ***, **, and * indicate significance at the 1, 5, and 10 percent levels.

B Formal Analysis and Simulations

B.1 Derivation of Economic Implications

We are interested in how the ex-ante loan value, $L^{(h^*)}$, and ex-post loan value, $\hat{L}^{(h^*)}$, respond to changes in expected inflation π_j^e through changes in the ex-ante real rate and predictable forecast errors. Formally, $L^{(h^*)}$ is defined as

$$L^{(h^*)} := \sum_{h=1}^{h^*} \prod_{s=1}^{h} \frac{P^{(h^*)} p_s^e}{1 + r_s(\pi^e)}, \tag{31}$$

while the definition of $\hat{L}^{(h^*)}$ is given by equation (11).

Note that $\hat{L}^{(h^*)}$ is affected by both changes in the ex-ante real rate and by the predictable forecast errors, whereas $L^{(h^*)}$ responds only to variations in the ex-ante real rate. In addition, since we focus on an approximation around the point where $\pi_s = \pi_s^e$, the impact of a change in π_s^e on $L^{(h^*)}$ is a special case of its effect on $\hat{L}^{(h^*)}$. Therefore, in the derivations that follow, we focus on $\hat{L}^{(h^*)}$.

Formally, if we let $Q^{(h^*)} := \hat{L}^{(h^*)}/P^{(h^*)}$, then

$$\frac{\partial \hat{L}^{(h^*)}}{\partial \pi_j^e} = P^{(h^*)} \frac{\partial Q^{(h^*)}}{\partial \pi_j^e}$$

or, using the definition of $Q^{(h^*)}$,

$$\frac{1}{\hat{L}^{(h^*)}} \cdot \frac{\partial \hat{L}^{(h^*)}}{\partial \pi_j^e} = \frac{1}{Q^{(h^*)}} \cdot \frac{\partial Q^{(h^*)}}{\partial \pi_j^e}.$$

Note that the left-hand side captures the semi-elasticity of $\hat{L}^{(h^*)}$ to a change in expectations: $\partial \log \hat{L}^{(h^*)}/\partial \pi_j^e$.

More specifically, we will consider a parallel change of inflation expectations at all horizons, that is we assume $\pi_s^e = \tilde{\pi}_s^e + \pi^e$ and compute the derivative with respect to π^e. We have

$$\begin{aligned}
\frac{1}{Q^{(h^*)}} \cdot \frac{\partial Q^{(h^*)}}{\partial \pi^e} =& \frac{1}{Q^{(h^*)}} \cdot \frac{\partial}{\partial \pi^e} \sum_{h=1}^{h^*} \prod_{s=1}^{h} \frac{1}{(1+r_s)(1+\pi_s - \pi_s^e)} \\
=& -\frac{1}{Q^{(h^*)}} \left[\frac{1}{(1+r_1)(1+\pi_1-\pi_1^e)} \left(\frac{1}{1+r_1} \cdot \frac{\partial r_1}{\partial \pi^e} + \frac{1}{1+\pi_1-\pi_1^e} \cdot \frac{\partial(\pi_1 - \pi_1^e)}{\partial \pi^e} \right) + \ldots . \right. \\
&\left. + \prod_{s=1}^{h^*} \frac{1}{(1+r_s)(1+\pi_s-\pi_s^e)} \sum_{j=1}^{h^*} \frac{1}{1+r_j} \cdot \frac{\partial r_j}{\partial \pi^e} + \frac{1}{1+\pi_j-\pi_j^e} \cdot \frac{\partial(\pi_j - \pi_j^e)}{\partial \pi^e} \right].
\end{aligned}$$

We can further simplify the expression by approximating one-period inflation with average inflation over the loan duration. Similarly, the real rate at time s can be approximated with the average real rate over the loan duration:

$$r_s \approx r^{(h)} := \frac{1}{h}\sum_{j=1}^{h} r_s, \; \forall s.$$

Finally, we consider a change in expectations around the point where inflation and inflation expectations are both equal to zero: $\pi_s - \pi_s^e \approx 0, \forall s$. As a result,

$$\frac{1}{Q^{(h^*)}} \cdot \frac{\partial Q^{(h^*)}}{\partial \pi^e} = -\frac{1}{Q^{(h^*)}} \sum_{h=1}^{h^*} \frac{1}{(1+r^{(h)})^h} \left(\frac{h}{1+r^{(h)}} \cdot \frac{\partial r^{(h)}}{\partial \pi^e} + \sum_{j=1}^{h} \frac{\partial (\pi_j - \pi_j^e)}{\partial \pi^e} \right).$$

Therefore,

$$\frac{\partial \log \hat{L}^{(h^*)}}{\partial \pi^e} = \underbrace{-\frac{1}{Q^{(h^*)}} \sum_{h=1}^{h^*} \frac{h}{(1+r^{(h)})^{h+1}} \cdot \frac{\partial r^{(h)}}{\partial \pi^e}}_{\frac{\partial \log L^{(h^*)}}{\pi^e}} \underbrace{-\frac{1}{Q^{(h^*)}} \sum_{h=1}^{h^*} \frac{1}{(1+r^{(h)})^h} \sum_{j=1}^{h} \frac{\partial (\pi_j - \pi_j^e)}{\partial \pi^e}}_{\frac{\partial \log \hat{L}^{(h^*)}}{\pi^e} - \frac{\partial \log L^{(h^*)}}{\pi^e}}.$$

The first term in the round bracket is the change in ex-ante real rate over horizon h. Equation (9) estimates the sensitivity of the h-period nominal rate to a change in the average inflation expectation. Using the notation of this section, we can rewrite equation (9) as

$$i^{(h)} = \alpha_h + \lambda_h \left(\frac{1}{h} \sum_{j=1}^{h} \pi_j^e \right) + \varepsilon^{(h)}.$$

Then, using our assumption that $\pi_j^e = \tilde{\pi}_j^e + \pi^e$ for all $j = 1, ..., h$, it follows that $\frac{\partial}{\partial \pi^e} \sum_{j=1}^{h} (i_{j-1} - \pi_j^e) = \lambda_h - 1$ and, as a result,

$$\frac{\partial \log L^{(h^*)}}{\pi^e} = -\frac{1}{Q^{(h^*)}} \sum_{h=1}^{h^*} \frac{h(\lambda_h - 1)}{(1+r^{(h)})^{h+1}}.$$

Finally, the sum $\sum_{j=1}^{h} (\pi_j - \pi_j^e)$ is the cumulative forecast error over horizon h, thus, $\frac{\partial}{\partial \pi^e} \sum_{j=1}^{h} (\pi_j - \pi_j^e)$ captures the change in the cumulative forecast error following from a parallel change in expectations over the loan duration. Equation (2) estimates the sensitivity of the h-period-ahead forecast error to a change in the average inflation expectation. Using the notation of this section, we can rewrite equation (2) as

$$\pi_h - \pi_h^e = \alpha_h + \delta_h \pi_h^e + \varepsilon^{(h)}.$$

Then, using our assumption that $\pi_j^e = \tilde{\pi}_j^e + \pi^e$ for all h, it follows that

$$\frac{\partial \log \hat{L}^{(h^*)}}{\pi^e} - \frac{\partial \log L^{(h^*)}}{\pi^e} = -\frac{1}{Q^{(h^*)}} \sum_{h=1}^{h^*} \frac{\sum_{j=1}^{h} \delta_j}{(1+r^{(h)})^h}.$$

B.2 Proofs

Proof of Proposition 1 We apply the same line of reasoning used in Bonaglia and Gennaioli (2025) to prove more general convergence results. First we state some preliminary notation. We consider a generic horizon h. Under the assumptions of stationary, gaussian and zero mean DGP and with similarity specification described in (14), we sum up the model in three essential ingredients:

- A probability space $(\mathbb{R}^\infty, \mathcal{B}(\mathbb{R}^\infty), \mathcal{P})$ where $\mathcal{P}$ is the Kolmogorov extension law induced by the ergodic, stationary, gaussian, zero-mean, stochastic process $(X_t)_t$ whose time t realization is denoted by $\pi_t \in \mathbb{R}$ and $\mathcal{B}(\mathbb{R}^\infty)$ is the cylinder sigma algebra. We indicate by $\omega := (\pi_1, \pi_2, ...) \in \mathbb{R}^\infty$ any sequence of real numbers.

- A sequence of (random) mnemnonic expectations over the horizon h, $(\mathbb{E}_t^m)_{t\in\mathbb{N}}$, such that

$$\mathbb{E}_t^m : \omega \mapsto \frac{\sum_{k=h}^{t-1} \pi_k e^{-\alpha(\pi_t-\pi_{k-h})^2-\beta(\pi_t-\pi_k)^2}}{\sum_{k=h}^{t-1} e^{-\alpha(\pi_t-\pi_{k-h})^2-\beta(\pi_t-\pi_k)^2}}$$

  which is Borel measurable by standard arguments.

Proof. Fix $x \in \mathbb{R}$. For all $t \in \mathbb{N}$, consider the x-parameterized (random) mnemnonic expectation

$$\mathbb{E}_t^{m,x} : \omega \mapsto \frac{\sum_{k=h}^{t-1} \pi_k e^{-\alpha(x-\pi_{k-h})^2-\beta(x-\pi_k)^2}}{\sum_{k=h}^{t-1} e^{-\alpha(x-\pi_{k-h})^2-\beta(x-\pi_k)^2}}$$

We first show that for each $\mathbb{E}_t^{m,x}$ converges almost surely to some $\Lambda(x)$ and we algebraically characterize the limit $\Lambda(x)$. Then we we will prove that $\Lambda(X_t)$ is the p-lim of $(\mathbb{E}_t^m)_{t\in\mathbb{N}}$, namely

$$\lim_{t\to\infty} \mathcal{P}(\{|\mathbb{E}_t^m - \Lambda(X_t)| < \epsilon\}) = 1$$

for all $\epsilon > 0$.

P-lim characterization: Consider the numerator of $\mathbb{E}_t^{m,x}(\omega)$. We know that the map

$$S_x \circ T \circ ... \circ T : \omega \mapsto \pi_t e^{-\alpha(x-\pi_{t-h})^2-\beta(x-\pi_t)^2}$$

when the shift operator T is applied t times is bounded and $\mathbb{R}^2$-Borel measurable for each t (hence integrable with respect to any probability measure). Hence the Pointwise Bierkoff

Ergodic Theorem applies with respect to the shift operator which a well-known measure preserving and ergodic transformation of the probability measure $\mathcal{P} \in \Delta(\mathbb{R}^\infty)$ whose marginals for all $t \in \mathbb{N}$ coincides with the stationary distribution of $(X_t)_t$. We conclude that,

$$\lim_{t\to\infty} \frac{\sum_{k=h}^{t-1} \pi_k e^{-\alpha(x-\pi_{k-h})^2 - \beta(x-\pi_k)^2}}{t} = \int \tilde{\pi} e^{-\alpha(x-\tilde{\pi}_{-h})^2 - \beta(x-\tilde{\pi})^2} d\tau(\tilde{\pi}, \tilde{\pi}_{-h}) \tag{32}$$

$\mathcal{P}$-almost surely, where τ is the unique stationary probability measure for the process (X_{t-h}, X_t) with realizations (π_{t-h}, π_t). An analogous reasoning applies to the denominator. Hence we conclude that

$$\lim_{t\to\infty} \frac{\sum_{k=h}^{t-1} e^{-\alpha(x-\pi_{k-h})^2 - \beta(x-\pi_k)^2}}{t} = \int e^{-\alpha(x-\tilde{\pi}_{-h})^2 - \beta(x-\tilde{\pi})^2} d\tau(\tilde{\pi}, \tilde{\pi}_{-h}) \tag{33}$$

almost surely. By properties of the exponential function, we know that the denominator is strictly positive almost surely. Consequently we infer that

$$\lim_{t\to\infty} \mathbb{E}_t^{m,x} = \Lambda(x) := \frac{\int \tilde{\pi} e^{-\alpha(x-\tilde{\pi}_{-h})^2 - \beta(x-\tilde{\pi})^2} d\tau(\tilde{\pi}, \tilde{\pi}_{-h})}{\int e^{-\alpha(x-\tilde{\pi}_{-h})^2 - \beta(x-\tilde{\pi})^2} d\tau(\tilde{\pi}, \tilde{\pi}_{-h})} = \phi(\alpha, \beta, h)x \quad \mathcal{P} - a.e \tag{34}$$

where $\phi(\alpha, \beta, h)$ is the functional representation obtained by performing the algebra inside the integrals.[26] We said that τ is the stationary probability measure for the stochastic process (X_{t-h}, X_t). Denote by Y the bivariate normal having marginals X_{-h}, X distributed identically to the limit distribution of the one dimensional original stochastic process. Next, we prove that $\Lambda(X_t) = \mathbb{E}_t^\infty(\pi_{t+h})$, i.e that it characterizes the mnemonic long-run approximation of $\mathbb{E}_t^m(\pi_{t+h})$ of (12).

Convergence in Probability: We want to show that

$$\lim_{t\to\infty} \mathcal{P}(\{\ |\mathbb{E}_t^m - \Lambda(X_t)| \geq \epsilon\}) = 0$$

for all $\epsilon > 0$. Note that, for all $t \in \mathbb{N}$ and for all $B > 0$,

$$\mathcal{P}(\{\ |\mathbb{E}_t^m - \Lambda(X_t)| \geq \epsilon\}) \leq \mathcal{P}\left(\{X_t \in [-B, B]^c\}) + \mathcal{P}(\{\ |\mathbb{E}_t^m - \Lambda(X_t)| \cdot 1_{[X_t \in [-B,B]]} \geq \epsilon\}\right). \tag{35}$$

Fix $\delta > 0$ and pick $B > 0$ large enough so that, if $\mathbf{X}$ is distributed according to the stationary distribution, $\mathcal{P}(\{\mathbf{X} \in [-B, B]^c\}) < \delta$. Let us now focus on $\mathcal{P}(\{\ |\mathbb{E}_t^m - \Lambda(X_t)| \cdot 1_{[X_t \in [-B,B]]} \geq \epsilon\})$;

[26] See the proof of Proposition 2 for an explicit characterization.

in particular, let us prove that

$$\lim_{t\to\infty} |\mathbb{E}_t^m - \Lambda(X_t)| \cdot 1_{[X_t \in [-B,B]]} = 0 \quad \mathcal{P} - a.s.$$

Clearly, for all $\omega \in \{X_t \in [-B,B]^c, \text{ ultimately}\}$, the convergence holds. Hence let us focus on the event $\{X_t \in [-B,B], \text{ infinitely often}\}$. Pick $\omega \in \{X_t \in [-B,B], \text{ infinitely often}\}$. Clearly,

$$\limsup_{t\to\infty} |\mathbb{E}_t^m(\omega) - \Lambda(X_t(\omega))| \cdot 1_{[X_t\in[-B,B]]} \geq \lim_{t\to\infty} |\mathbb{E}_t^m(\omega) - \Lambda(X_t(\omega))| \cdot 1_{[X_t\in[-B,B]]},$$

hence let us prove that the LHS converges to zero. Let $(t_k)_k$ the subsequence where the limsup value is attained in the limit. Now, being $[-B,B]$ compact, $(X_{t_k}(\omega))$ has a convergent subsequence, say convergent to some $x \in [-B,B]$. Now, we claim that $S_{t_k} : (y_1,y_2) \mapsto \exp[-\alpha(X_{t_k}(\omega) - y_1)^2 - \beta(X_{t_k}(\omega) - y_1)^2]$ converges uniformly to $S_x : (y_1,y_2) \mapsto \exp[-\alpha(x - y_1)^2 - \beta(x - y_1)^2]$. This follows from the fact that the derivative of the negative exponential is always bounded by some arbitrarily large K, hence

$$|| \exp[-\alpha(X_{t_k}(\omega) - y_1)^2] - \exp[-\alpha(x - y_1)^2]||_\infty \leq K|X_{t_k}(\omega) - x| \to 0,$$

$$|| \exp[-\beta(X_{t_k}(\omega) - y_2)^2] - \exp[-\beta(x - y_2)^2]||_\infty \leq K|X_{t_k}(\omega) - x| \to 0,$$

whence

$$|| \exp[-\alpha(X_{t_k}(\omega) - y_1)^2 - \beta(X_{t_k}(\omega) - y_2)^2] - \exp[-\alpha(x - y_1)^2 - \beta(x - y_2)^2]||_\infty \to 0,$$

as claimed. Now, by construction,

$$\mathbb{E}_t^m(\omega) = \frac{\int y_2 S_{t_k}((y_1,y_2)) \; dm_e(y_1,y_2|\omega)}{\int S_{t_k}((y_1,y_2)) \; dm_e(y_1,y_2|\omega)},$$

where $m_e(\cdot|\omega) \in \Delta(\mathbb{R}^2)$ is the empirical measure generated by ω at time t_k. Since the process is ergodic and stationary, a standard implication of Bierkoff-Ergodic theorem in polish spaces (over which we are operating) is that m_e weakly converges to τ, $\mathcal{P}$-ae. WLOG we can assume that $\omega \in \{m_e \to \tau\}$, being $\{m_e \to \tau\}$ a full measure event. By combining the facts that $||S_{t_k}(\cdot) - S_x(\cdot)||_\infty \to 0$ and $m_e(\cdot|\omega) \to \tau$ weakly, we infer that

$$\lim_{k\to\infty} \mathbb{E}_{t_k}^m(\omega) = \lim_{k\to\infty} \frac{\int y_2 S_{t_k}((y_1,y_2)) \; dm_e(\cdot|\omega)}{\int S_{t_k}((y_1,y_2)) \; dm_e(\cdot|\omega)} = \frac{\int y_2 S_x((y_1,y_2)) d\tau}{\int S_x((y_1,y_2)) d\tau} = \lim_{k\to\infty} \Lambda(X_{t_k}(\omega)). \quad (36)$$

Because of (36) and (34), and by the definition of t_k, we deduce that

$$\limsup_{t\to\infty} |\mathbb{E}_t^m(\omega) - \Lambda(X_t(\omega)| \cdot 1_{[X_t(\omega)\in[-B,B]]} = 0,$$

but since ω is a generic element belonging to of a full measure event, our almost sure convergence claim holds. Plugging this result in (35) and recalling that almost sure convergence implies convergence in probability, we deduce also that

$$\lim_{t\to\infty} \mathcal{P}(\{\ |\mathbb{E}_t^m - \Lambda(X_t)| \geq \epsilon\}) \leq \lim_{t\to\infty} \mathcal{P}\left(\{X_t \in [-B,B]^c\}\right) + 0. \tag{37}$$

Being $(X_t)_t$ stationary, the distribution of X_t weakly converges to the distribution of $\mathbf{X}$. Since the boundary set of $[-B,B]$ has zero measure, Portmanteau Theorem yields that

$$\lim_{t\to\infty} \mathcal{P}(\{\ |\mathbb{E}_t^m - \Lambda(X_t)| \geq \epsilon\}) \leq \lim_{t\to\infty} \mathcal{P}\left(\{X_t \in [-B,B]^c\}\right) = \mathcal{P}\left(\{\mathbf{X} \in [-B,B]^c\}\right) < \delta.$$

Since $\delta > 0$ is arbitrary, the claim follows. ■

Proof of proposition 2 **Proof.** Reasoning as in the first step of proposition 1, we infer that $\mathbb{E}_t^\infty(\pi_{t+h}) = \dfrac{\int \tilde{\pi} e^{-\alpha(\pi_t - \tilde{\pi}_{-h})^2 - \beta(\pi_t - \tilde{\pi})^2} d\tau(\tilde{\pi}, \tilde{\pi}_{-h})}{\int e^{-\alpha(\pi_t - \tilde{\pi}_{-h})^2 - \beta(\pi_t - \tilde{\pi})^2} d\tau(\tilde{\pi}, \tilde{\pi}_{-h})}$, By applying the Radon-Nikodym Theorem and reformulating the RHS of (32) using quadratic forms we get

$$\mathbb{E}_t^\infty(\pi_{t+h}) = \frac{\int_{\mathbb{R}^2} \pi \exp\left(-0.5(\pi_{-h},\pi)^\top \Sigma_1^{-1}(\pi_{-h},\pi) - 0.5(\pi_{-h} - x, \pi - \pi_t)^\top \Sigma_2^{-1}(\pi_{-h} - \pi_t, \pi - \pi_t)\right) d(\pi_{-h},\pi)}{\int_{\mathbb{R}^2} \exp\left(-0.5(\pi_{-h},\pi)^\top \Sigma_1^{-1}(\pi_{-h},\pi) - 0.5(\pi_{-h} - \pi_t, \pi - \pi_t)^\top \Sigma_2^{-1}(\pi_{-h} - \pi_t, \pi - \pi_t)\right) d(\pi_{-h},\pi)}$$

Where one can note that the denominator is just the normalizing constant. By the characterization of the product of two bivariate Gaussian densities, we know it is normal $\mathcal{N}^*(\mathbf{m}, \mathbf{\Sigma_3})$ such that:

$$\begin{aligned}
\mathbf{\Sigma_3} &= \left(\begin{bmatrix} \mathrm{Var}(X_t) & \mathrm{Cov}(X_{t+h}, X_t) \\ \mathrm{Cov}(X_{t+h}, X_t) & \mathrm{Var}(X_t) \end{bmatrix}^{-1} + \begin{bmatrix} 2\alpha & 0 \\ 0 & 2\beta \end{bmatrix} \right)^{-1} \\
\mathbf{m} &= \mathbf{\Sigma_3} \begin{bmatrix} 2\alpha & 0 \\ 0 & 2\beta \end{bmatrix} \begin{pmatrix} \pi_t \\ \pi_t \end{pmatrix}.
\end{aligned}$$

Now, since the second component of $\mathbf{m}$ coincides with the definition of $\mathbb{E}_t^\infty(\pi_{t+h})$, by performing the algebra,

$$\mathbb{E}_t^{\infty}(\pi_{t+h}) = \frac{2\pi_t \operatorname{Cov}(X_t, X_{t+h})\, \alpha \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)}{\left(\operatorname{Var}(X_t) + 2\alpha \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)\right)\left(\operatorname{Var}(X_t) + 2\beta \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)\right) - \operatorname{Cov}(X_t, X_{t+h})^2}$$
$$+ \frac{2\pi_t\, \beta \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)\left(\operatorname{Var}(X_t) + 2\alpha \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)\right)}{\left(\operatorname{Var}(X_t) + 2\alpha \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)\right)\left(\operatorname{Var}(X_t) + 2\beta \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)\right) - \operatorname{Cov}(X_t, X_{t+h})^2}$$

Not, given this expression, standard algebraic manipulations yield that

$$\lim_{\alpha\to\infty,\ \beta\to 0} \mathbb{E}_t^{\infty}(\pi_{t+h}) = \frac{\operatorname{cov}(X_{t+h}, X_t)}{\operatorname{var}(X_t)}\pi_t, \qquad \text{(Least Square)}$$
$$\lim_{\alpha\to 0,\ \beta\to\infty} \mathbb{E}_t^{\infty}(\pi_{t+h}) = \pi_t, \qquad \text{(Adaptive)}$$
$$\lim_{\alpha\to 0,\ \beta\to 0} \mathbb{E}_t^{\infty}(\pi_{t+h}) = 0. \qquad \text{(Frequentist).}$$

Finally, In order to deduce overreaction for long enough horizons, let us focus on the coefficient that multiplies X_t. Letting $h \to \infty$, by relying on the vanishing auto-covariance hypothesis, we get that the first addendum divided by π_t goes to zero, while coefficient of the second one divided by π_t converges to

$$\frac{2\beta[\operatorname{Var}(X_t)]^2\,[2\alpha[\operatorname{Var}(X_t)]^2 + \operatorname{Var}(X_t)]}{[\operatorname{Var}(X_t) + 2\alpha\operatorname{Var}(X_t)^2][\operatorname{Var}(X_t) + 2\beta\operatorname{Var}(X_t)^2]} = \frac{2\beta\operatorname{Var}(X_t)^2}{\operatorname{Var}(X_t) + 2\beta\operatorname{Var}(X_t)^2} > 0 = \lim_{h\to\infty} \frac{\operatorname{Cov}(X_{t+h}, X_t)}{\operatorname{Var}(X_t)},$$

proving the desired claim.

■

Proof of Proposition 3 By relying on the general characterization for gaussian, ergodic, stationary processes, we use the stationary distribution of AR(1) to explicitly write the stationary distribution as a function of the parameter $\rho, \sigma \in \mathbb{R}$.

Proof. The claim follows by simple and immediate algebraic manipulations. Indeed by taking the general expression

$$\mathbb{E}_t^{\infty}(\pi_{t+h}) = \frac{2\pi_t \operatorname{Cov}(X_t, X_{t+h})\, \alpha \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)}{\left(\operatorname{Var}(X_t) + 2\alpha \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)\right)\left(\operatorname{Var}(X_t) + 2\beta \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)\right) - \operatorname{Cov}(X_t, X_{t+h})^2}$$
$$+ \frac{2\pi_t\, \beta \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)\left(\operatorname{Var}(X_t) + 2\alpha \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)\right)}{\left(\operatorname{Var}(X_t) + 2\alpha \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)\right)\left(\operatorname{Var}(X_t) + 2\beta \left(\operatorname{Var}(X_t)^2 - \operatorname{Cov}(X_t, X_{t+h})^2\right)\right) - \operatorname{Cov}(X_t, X_{t+h})^2}$$

and by plugging in the variance and covariance under the stationary distribution of the AR(1) process, we obtain that

$$\mathbb{E}_t^{\infty}(\pi_{t+h}) = \frac{\alpha\rho^h + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h})}{\alpha + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h}) + \frac{1}{2\sigma_\pi^2}}\pi_t.$$

■

Proof of Proposition 4. We first derive the expressions for the coefficients in the theoretical counterpart of the OLS regression (3), i.e.,

$$\pi_{t+h} - \mathbb{E}_t^m(\pi_{t+h}) = \alpha_h + \delta_{1,h}\left(\mathbb{E}_t^m(\pi_{t+h}) - \mathbb{E}_{t-1}^m(\pi_{t+h})\right) + \delta_{2,h}\mathbb{E}_{t-1}^m(\pi_{t+h}) + \varepsilon_{t+h}. \quad (38)$$

From (16), $e_h \equiv \mathbb{E}_t^m(\pi_{t+h})/\pi_t$ is independent of π_t. We can then define the forecast error $FE_{t,t+h} \equiv \pi_{t+h} - \mathbb{E}_t^m(\pi_{t+h}) = \pi_{t+h} - e_h\pi_t$ and the forecast revision $FR_{t,t+h} \equiv \mathbb{E}_t^m(\pi_{t+h}) - \mathbb{E}_{t-1}^m(\pi_{t+h}) = e_h\pi_t - e_{h+1}\pi_{t-1}$. To obtain the OLS coefficients, we use (38) and the assumption of orthogonality of ε_{t+h}, to obtain

$$Cov(FE_{t,t+h}, FR_{t,t+h}) = \delta_{1,h}Var(FR_{t,t+h}) + \delta_{2,h}Cov(FR_{t,t+h}, \mathbb{E}_{t-1}^m(\pi_{t+h})),$$
$$Cov(FE_{t,t+h}, \mathbb{E}_{t-1}^m(\pi_{t+h})) = \delta_{1,h}Cov(FR_{t,t+h}, \mathbb{E}_{t-1}^m(\pi_{t+h})) + \delta_{2,h}Var(\mathbb{E}_{t-1}^m(\pi_{t+h})).$$

Solving for $\delta_{1,h}$ and $\delta_{2,h}$ yields

$$\delta_{1,h} = \frac{Cov(FE_{t,t+h}, FR_{t,t+h})Var(\mathbb{E}_{t-1}^m(\pi_{t+h})) - Cov(FE_{t,t+h}, \mathbb{E}_{t-1}^m(\pi_{t+h}))Cov(\mathbb{E}_{t-1}^m(\pi_{t+h}), FR_{t,t+h})}{Var(FR_{t,t+h})Var(\mathbb{E}_{t-1}^m(\pi_{t+h})) - Cov(FR_{t,t+h}, \mathbb{E}_{t-1}^m(\pi_{t+h}))^2},$$

$$\delta_{2,h} = \frac{Cov(FE_{t,t+h}, \mathbb{E}_{t-1}^m(\pi_{t+h}))Var(FR_{t,t+h}) - Cov(FE_{t,t+h}, FR_{t,t+h})Cov(FR_{t,t+h}, \mathbb{E}_{t-1}^m(\pi_{t+h}))}{Var(FR_{t,t+h})Var(\mathbb{E}_{t-1}^m(\pi_{t+h})) - Cov(FR_{t,t+h}, \mathbb{E}_{t-1}^m(\pi_{t+h}))^2}.$$

We compute each of the theoretical moments of interests:

$$Var(FR_{t,t+h}) = (e_h^2 + e_{h+1}^2 - 2\rho e_h e_{h+1})Var(\pi_t),$$
$$Var(\mathbb{E}_{t-1}^m(\pi_{t+h})) = e_{h+1}^2 Var(\pi_{t-1}),$$
$$Cov(FR_{t,t+h}, \mathbb{E}_{t-1}^m(\pi_{t+h})) = (\rho e_h e_{h+1} - e_{h+1}^2)Var(\pi_{t-1}),$$
$$Cov(FE_{t,t+h}, \mathbb{E}_{t-1}^m(\pi_{t+h})) = (\rho^{h+1}e_{h+1} - \rho e_h e_{h+1})Var(\pi_{t-1}),$$
$$Cov(FE_{t,t+h}, FR_{t,t+h}) = (\rho^h e_h - \rho^{h+1}e_{h+1} - e_h^2 + \rho e_h e_{h+1})Var(\pi_t).$$

Substituting the above moments into the expression for $\delta_{1,h}$ yields

$$\begin{aligned}\delta_{1,h} &= \frac{(\rho^h e_h - \rho^{h+1}e_{h+1} - e_h^2 + \rho e_h e_{h+1})e_{h+1}^2 - (\rho^{h+1}e_{h+1} - \rho e_h e_{h+1})(\rho e_h e_{h+1} - e_{h+1}^2)}{(e_h^2 + e_{h+1}^2 - 2\rho e_h e_{h+1})e_{h+1}^2 - (\rho e_h e_{h+1} - e_{h+1}^2)^2} \\ &= \frac{\rho^h(1-\rho^2)e_h e_{h+1}^2 - (1-\rho^2)e_h^2 e_{h+1}^2}{(1-\rho^2)e_h^2 e_{h+1}^2} \\ &= \frac{\rho^h}{e_h} - 1.\end{aligned}$$

Using the definition of e_h, together with (16), gives $\delta_{1,h} = \delta_h$, where δ_h is given by (18).

Analogous steps prove that $\delta_{2,h} = \delta_h$.

Part (i). Differentiating (18) with repsect to h,

$$\begin{aligned}\frac{\partial \delta_h}{\partial h} =& \frac{\rho^h \log \rho}{2\sigma_\pi^2[\alpha\rho^h + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h})]^2} \times \\ &\big([2\sigma_\pi^2(\alpha+\beta) + 4\alpha\beta(\sigma_\pi^2)^2(1-3\rho^{2h}) + 1][\alpha\rho^h + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h})] \\ &-[2\sigma_\pi^2(\alpha+\beta) + 4\alpha\beta(\sigma_\pi^2)^2(1-\rho^{2h}) + 1][\alpha\rho^h - 4\alpha\beta\sigma_\pi^2\rho^{2h}]\big),\end{aligned}$$

which is negative if and only if

$$\begin{aligned}&\big([2\sigma_\pi^2(\alpha+\beta) + 4\alpha\beta(\sigma_\pi^2)^2(1-3\rho^{2h}) + 1][\alpha\rho^h + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^{2h})] \\ &-[2\sigma_\pi^2(\alpha+\beta) + 4\alpha\beta(\sigma_\pi^2)^2(1-\rho^{2h}) + 1][\alpha\rho^h - 4\alpha\beta\sigma_\pi^2\rho^{2h}]\big) > 0\end{aligned}$$

or, after straightforward algebra,

$$\beta + 2\beta^2\sigma_\pi^2 + 2\alpha\beta\sigma_\pi^2[2 + 4\beta\sigma_\pi^2(1-\rho^{2h}) + \rho^{2h}] + 4\alpha^2\beta(\sigma_\pi^2)^2[2\beta\sigma_\pi^2(1-\rho^{4h})^2 + 1 + \rho^{2h} - 2\rho^{3h}] > 0.$$

The first square bracket is clearly positive. The second square bracket is also positive since $\rho^{2h} > \rho^{3h}$ for $0 < \rho < 1$ and $h \geq 1$. As a result, δ_h is decreasing in h. Finally, from equation (18), it is immediate to verify that $\delta_h \to -1$.

Part (ii). From (18) with $h = 1$,

$$\begin{aligned}\delta_1 &= \frac{\alpha\rho + \beta\rho + 2\alpha\beta\rho\sigma_\pi^2(1-\rho^2) + \rho/2\sigma_\pi^2}{\alpha\rho + \beta + 2\alpha\beta\sigma_\pi^2(1-\rho^2)} - 1 \\ &= \frac{-2\beta(1-\rho) - 4\alpha\beta(1-\rho)\sigma^2 + (1-\rho^2)\rho/\sigma^2}{2\alpha\rho + 2\beta + 4\alpha\beta\sigma^2} \\ &= (1-\rho)\frac{\rho/\sigma^2 + \rho^2/\sigma^2 - 2\beta - 4\alpha\beta\sigma^2}{2\alpha\rho + 2\beta + 4\alpha\beta\sigma^2}.\end{aligned}$$

The latter is negative if and only if

$$\rho + \rho^2 - 2\beta\sigma^2(1 + 2\alpha\sigma^2) < 0,$$

which is condition (17). ■

Proof of Proposition 5. We derive an expression for the coefficient θ_h^r of the OLS regression

(6). From the definition of real rate (5), we have

$$\widetilde{r}_t^{(h)} = r^{(h)} - \frac{1}{h}\sum_{j=1}^{h}\pi_{t+j} + \frac{1}{h}\sum_{j=1}^{h}\mathbb{E}_t^m(\pi_{t+j})$$

or, using the process for inflation (15),

$$\widetilde{r}_t^{(h)} = r^{(h)} - \frac{1}{h}\sum_{j=1}^{h}(\rho^j\pi_t - \mathbb{E}_t^m(\pi_{t+j})) + \sigma\frac{1}{h}\sum_{j=1}^{h}\epsilon_{t+j}.$$

From (16), we let $e_h \equiv \mathbb{E}_t^m(\pi_{t+h})/\pi_t$, which is independent of π_t. As a result,

$$\widetilde{r}_t^{(h)} = r^{(h)} - \pi_t\frac{1}{h}\sum_{j=1}^{h}(\rho^j - e_j) + \sigma\frac{1}{h}\sum_{j=1}^{h}\epsilon_{t+j}.$$

Similarly,

$$\mathbb{E}_t^m\left(\overline{\pi_{t+1,h}}\right) = \frac{1}{h}\sum_{j=1}^{h}\mathbb{E}_t^m\pi_{t+j} = \pi_t\frac{1}{h}\sum_{j=1}^{h}e_j.$$

The OLS coefficient θ_h^r satisfies

$$\begin{aligned}
\theta_h^r &= \frac{Cov\left(\widetilde{r}_t^{(h)}, \mathbb{E}_t^m\left(\overline{\pi_{t+1,h}}\right)\right)}{Var\left(\mathbb{E}_t^m\left(\overline{\pi_{t+1,h}}\right)\right)} \\
&= -\frac{\left(\frac{1}{h}\sum_{j=1}^{h}(\rho^j - e_j)\right)\left(\frac{1}{h}\sum_{j=1}^{h}e_j\right)}{\left(\frac{1}{h}\sum_{j=1}^{h}e_j\right)^2} \\
&= -\frac{\sum_{j=1}^{h}e_j\delta_j}{\sum_{j=1}^{h}e_j},
\end{aligned}$$

where the last line uses the fact that $\delta_h = \rho^j/e_j - 1$ from Proposition 4. Equation (19) follows from letting

$$w_j \equiv \frac{e_j}{\sum_{j=1}^{h}e_j}. \tag{39}$$

Next, we derive an expression for the coefficient θ_h^{rx} of the OLS regression (8). Above we showed that $\mathbb{E}_t^m\left(\overline{\pi_{t+1,h}}\right) = \left(\pi_t\sum_{j=1}^{h}e_j\right)/h$. By the same logic,

$$\mathbb{E}_{t+1}^m\left(\overline{\pi_{t+2,h}}\right) = \pi_{t+1}\frac{1}{h-1}\sum_{j=1}^{h-1}e_j$$

and

$$\mathbb{E}_t^m\left(\overline{\pi_{t+2,h}}\right) = \pi_t \frac{1}{h-1} \sum_{j=2}^{h} e_j.$$

Substituting the latter into (7) gives

$$\begin{aligned} rx_{t,t+1}^{(h)} &= -(h-1)r^{(h-1)} + hr^{(h)} - r^{(1)} - \pi_{t+1} \sum_{j=1}^{h-1} e_j + \pi_t \sum_{j=2}^{h} e_j \\ &= -(h-1)r^{(h-1)} + hr^{(h)} - r^{(1)} - \pi_t \sum_{j=1}^{h-1} \rho e_j + \pi_t \sum_{j=2}^{h} e_j - \sigma \epsilon_{t+1} \sum_{j=1}^{h-1} e_j. \end{aligned}$$

Therefore,

$$\begin{aligned} \theta_h^{rx} &= \frac{Cov\left(rx_{t,t+1}^{(h)}, \mathbb{E}_t^m\left(\overline{\pi_{t+1,h}}\right)\right)}{Var\left(\mathbb{E}_t^m\left(\overline{\pi_{t+1,h}}\right)\right)} \\ &= -\frac{\left(\sum_{j=1}^{h-1} \rho e_j - \sum_{j=2}^{h} e_j\right)\left(\frac{1}{h}\sum_{j=1}^{h} e_j\right)}{\left(\frac{1}{h}\sum_{j=1}^{h} e_j\right)^2} \\ &= -\frac{\sum_{j=1}^{h-1} \rho e_j - \sum_{j=2}^{h} e_j}{\frac{1}{h}\sum_{j=1}^{h} e_j}. \end{aligned} \tag{40}$$

Using the fact that $\delta_j = \rho^j / e_j - 1$ yields

$$\begin{aligned} \sum_{j=1}^{h-1} \rho e_j - \sum_{j=2}^{h} e_j &= \sum_{j=1}^{h-1} (\rho^{j+1} - \rho e_j \delta_j) - \sum_{j=2}^{h} (\rho^j - e_j \delta_j) \\ &= \sum_{j=2}^{h} e_j \delta_j - \rho \sum_{j=2}^{h} e_{j-1} \delta_{j-1}. \end{aligned}$$

Substituting the latter into (40) and rearranging gives

$$\theta_h^{rx} = h\Big(-\frac{\sum_{j=2}^{h} e_j \delta_j}{\sum_{j=1}^{h} e_j} + \rho \frac{\sum_{j=2}^{h} e_{j-1} \delta_{j-1}}{\sum_{j=1}^{h} e_j} \Big).$$

From (19) and (39), the first term in the bracket equals $\theta_h^{rx} + \delta_1 w_1$, while the second term equals

$-\rho(\theta_h^{rx} + \delta_h w_h)$. Therefore,

$$\begin{aligned}\theta_h^{rx} &= h\Big(- \frac{\sum_{j=2}^{h} e_j \delta_j}{\sum_{j=1}^{h} e_j} + \rho \frac{\sum_{j=2}^{h} e_{j-1}\delta_{j-1}}{\sum_{j=1}^{h} e_j}\Big) \\ &= h\Big((1-\rho)\theta_h^{rx} + \delta_1 w_1 - \rho\delta_h w_h)\Big),\end{aligned}$$

which is (20).

Evaluating (16) at $\beta = 0$ yields

$$e_h = \frac{\alpha\rho^h}{\alpha + 1/2\sigma_\pi^2},$$

hence, $\rho e_h = e_{h+1}$, which implies $\theta_h^{rx} = 0$ from (40).

Evaluating (16) at $\alpha = 0$ yields

$$e_h = \frac{\beta}{\beta + 1/2\sigma_\pi^2},$$

hence, $e_h = e_{h+1}$, which implies $\theta_h^{rx} = (1-\rho)(h-1)$ from (40). ■

B.3 Simulation Algorithm: CE Estimation

1. **Parameters' Initialization:**

   - Define forecast horizons $h = 1, 2, \ldots, 10$ years.
   - Construct a grid of parameter values for $(\alpha, \beta) \in [0,1]^2$ with a step size of 0.01, yielding over 10,000 possible parameter pairs.

2. **Data Loading:**

   - For each country c, load annual realized inflation data, $\pi_{c,t}$, covering the period from t_0 to T as specified in Appendix Table 7, which details the starting and ending dates for each country based on the available data sources.

3. **Forecasts' Simulations:** For each parameter combination (α, β), for each country c, for each horizon h (from 1 to 10), and for each period t:

   - Calculate the model-implied expectation $\mathbb{E}_t^m(\pi_{c,t+h} \mid \alpha, \beta)$ as the weighted forecast:

$$\mathbb{E}_t^m(\pi_{c,t+h} \mid \alpha, \beta) = \frac{\sum_{k=t_0+h}^{t-1} \pi_{c,k} e^{-\alpha(\pi_{c,k-h}-\pi_{c,t})^2 - \beta(\pi_{c,k}-\pi_{c,t})^2}}{\sum_{k=t_0+h}^{t-1} e^{-\alpha(\pi_{c,k-h}-\pi_{c,t})^2 - \beta(\pi_{c,k}-\pi_{c,t})^2}}$$

- For the 6-10 year horizon, calculate the average model-implied expectation:

$$\mathbb{E}_t^m(\pi_{c,t+6,\dots,t+10} \mid \alpha, \beta) = \frac{1}{5} \sum_{h=6}^{10} \mathbb{E}_t^m(\pi_{c,t+h} \mid \alpha, \beta)$$

4. **Parameters' Estimation:**

   - For each (α, β), compute the average absolute distance between observed and model-implied forecasts, where $\mathbb{F}_t(\pi_{c,t+h})$ represents the observed forecast from the CE dataset averaged annually. Select the optimal parameter pair $(\hat{\alpha}, \hat{\beta})$ that minimizes this distance accordingly:

$$\min_{\alpha,\beta} \sum_{t,c,h} |\mathbb{F}_t(\pi_{c,t+h}) - \mathbb{E}_t^m(\pi_{c,t+h} \mid \alpha, \beta)|$$

B.4 Bootstrapping Procedure – Simulation Algorithm: CE Estimation

Let

$$X = \{\, t \,:\, t = 1989, 1990, \dots, 2022\}, \quad L = |X| = 34,$$

and denote by

$$\big\{\hat{p}_{c,t}^{(\alpha_i,\beta_j)} : (i,j) \in \{1, \dots, 101\}^2,\ t \in X\big\}$$

the $M = 10201$ series of model-produced inflation expectations for country c, one per (α_i, β_j) pair. Our bootstrap procedure consists of $N = 1000$ iterations. In each iteration n, we perform the following steps:

1. **Drawing the sample indices:**

   - Generate with replacement a multiset:

$$X^{(n)} = \{\, t_1^{(n)}, \dots, t_L^{(n)}\} \subseteq X$$

   of size L.

   - In what follows, the *position* $k = 1, \dots, L$ in $X^{(n)}$ matters; the actual calendar year $t_k^{(n)}$ is used only to pick the model value.

2. **Reconstructing bootstrapped model series:**

   - For each parameter pair (α_i, β_j), consider

$$\hat{p}_{c,k}^{(n,i,j)} = \hat{p}_{c,\,t_k^{(n)}}^{(\alpha_i,\beta_j)}, \quad k = 1, \dots, L.$$

3. **Adjusting for country-specific start dates:**
   If country c' has actual CE CPI data beginning at $t^0_{c'} > \min X$:

   (a) Let $L_{c'} = 2022 - t^0_{c'} + 1$.

   (b) Discard the first $L - L_{c'}$ positions in $X^{(n)}$, reindexing $k \leftarrow k - (L - L_{c'})$ for $k = L - L_{c'} + 1, \ldots, L$.

   (c) Whenever $t^{(n)}_k < t^0_{c'}$, replace $t^{(n)}_k \leftarrow t^0_{c'}$.

   This means that country-specific histories shorter than 1989–2022 are handled by truncation and year-replacement as explained above.

4. **Distance minimization problem:**

   - Compute distance to observed CPI from CE. Let $\{t^{\text{obs}}_k\}_{k=1}^{L_{c'}}$ be the chronological CPI (from CE) positions for country c'. Then for each (α_i, β_j) define

   $$D^{(n)}_{c'}(\alpha_i, \beta_j) = \sum_{k=1}^{L_{c'}} \left| p^{\text{obs}}_{c', t^{\text{obs}}_k} - \hat{p}^{(n,i,j)}_{c',k} \right|.$$

   - Select optimal parameters:

   $$(\alpha^*, \beta^*)^{(n)} = \arg\min_{i,j} \ D^{(n)}_{c'}(\alpha_i, \beta_j).$$

Once all N bootstrap replications have been performed, we exploit the empirical distribution of the N draws $\{(\alpha^*, \beta^*)^{(n)}\}_{n=1}^{N}$ to construct the confidence intervals through the relevant percentiles (Kolesár, 2025). For standard-error estimation, we conservatively set the sample size to $S = 535$, which equals the number of annual CPI-forecast observations available in the Consensus Economics dataset.

B.5 Simulation Algorithm: Constant-Gain Learning

1. **Parameters' Initialization:**

   - Define forecast horizons $h = 1, 2, \ldots, 10$ years.
   - Construct a grid of parameter values for $\gamma \in [0, 1)$ with a step size of 0.01.

2. **Data Loading:** For each country c, load annual realized inflation data, $\pi_{c,t}$, covering the period from t_0 to T as specified in Appendix Table 7, which details the starting and ending dates for each country based on the available data sources.

3. **Forecasts' Simulations:** Following Nagel (2024), we assume the true inflation process for country c follows the AR(1) $\pi_{c,t} = a_c + \rho_c \pi_{c,t-1} + \varepsilon_{c,t}$. Agents estimate the true parameters $\mathbf{b_c} := (a_c, \rho_c)'$ using:

$$\mathbf{b}_{c,t} = \mathbf{b}_{c,t-1} + \gamma \mathbf{R}_{c,t}^{-1} \mathbf{x}_{c,t-1}(\pi_{c,t} - \mathbf{b'}_{c,t-1}\mathbf{x}_{t-1,c})$$
$$\mathbf{R}_{c,t} = \mathbf{R}_{c,t-1} + \gamma(\mathbf{x}_{c,t-1}\mathbf{x'}_{c,t-1} - \mathbf{R}_{c,t-1}),$$

where $\mathbf{x}_{c,t} = (1, \pi_{c,t})'$ and with initial conditions $\mathbf{b}_{c,0} = (0,0)$ and $\mathbf{R}_{c,0} = diag(1,1)$.

We consider a global constant-gain learning parameter γ.

- We calculate the model-implied expectation $\mathbb{E}_t^m(\pi_{c,t+h} \mid \gamma)$ as the weighted forecast:

$$\mathbb{E}_t^m(\pi_{c,t+h} \mid \gamma) = a_{c,t} + \rho_{c,t}^h \pi_{c,t}$$

- For the 6-10 year horizon, calculate the average model-implied expectation:

$$\mathbb{E}_t^m(\pi_{c,t+6,\ldots,t+10} \mid \gamma) = \frac{1}{5}\sum_{h=6}^{10} \mathbb{E}_t^m(\pi_{c,t+h} \mid \gamma)$$

4. **Parameters' Estimation:**

- For each γ, compute the average absolute distance between observed and model-implied forecasts, where $\mathbb{F}_t(\pi_{c,t+h})$ represents the observed forecast from the CE dataset averaged annually. Select the optimal parameter ($\hat{\gamma}$) that minimizes this distance accordingly:

$$\min_{\gamma} \sum_{t,c,h} |\mathbb{F}_t(\pi_{c,t+h}) - \mathbb{E}_t^m(\pi_{c,t+h} \mid \gamma)|$$

- The estimated global constant-gain learning parameter is $\hat{\gamma} = 0.02$.