Time for Memorable Consumption*

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A consumption event is memorable if the memory of the event affects well-being at times after the material consumption, as originally introduced by Gilboa, Postlewaite, and Samuelson (2016) and Hai, Krueger, and Postlewaite (2020). Our main contribution is to develop an axiomatic foundation of memorable consumption in a dynamic setting. Preferences are represented by the present value of the sum of utilities derived at each date from the current consumption and from recollecting the past. Our model accommodates well-known phenomena in psychology, such as the peak-end rule, duration neglect, and adaptation trends. We also provide foundations for a prominent special case of the representation with the Markovian property. The model is illustrated with applications in two different contexts: risk-taking behavior in a principal-agent problem and life-cycle consumption-savings decisions.

Keywords: memorable consumption, history dependence, Peak-end rule, Markovian preferences, subjective well-being, life-cycle dynamics.

"We actually don't choose between experiences, we choose between memories of experiences. Even when we think about the future, we don't think of our future normally as experiences. We think of our future as anticipated memories." — Daniel Kahneman (2010)

1 Introduction

Psychology and behavioral science have widely recognized that one's subjective well-being at any point in time is not determined simply by the consumption at that moment — the recollection of past experiences plays a crucial role. This idea is at the core of a well-known

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literature initiated by Kahneman and is supported by sizable experimental evidence.[1] Evoking early ideas of Bentham (1789) and Edgeworth (1881), Kahneman describes hedonic experiences as consisting of sequences of moments that give rise to two distinct measurements, so-called 'moment utility' and 'remembered utility.' The former expresses the instant degree of pain or pleasure associated with moments, while the latter refers to the judgement arising from the ex post recollection of the overall experience.[2] This encompassing notion of subjective well-being has reached out even to public policy; searching for more adequate indicators of well-being beyond GDP per capita, policy makers often resort to large-scale surveys, such as the World Happiness Report and the European Social Survey, that inquire about citizens' happiness and life satisfaction.[3]

From a behavioral perspective, a critical aspect lies in the elicitation of these more inclusive measures of well-being from observables. Experienced utilities reflect hedonic states, such as the perceived intensity of pain or pleasure, and their measurements are traditionally based on self-reports of these feelings. The controversies are well-known and have to do with inherent biases associated with self-reports, the difficulties to attribute ordinal meanings, and to make interpersonal comparisons.[4] The literature report puzzling findings in different domains: controversial examples are the limited correlation between economic growth and well-being in the well-known Easterlin's Paradox (1974, 1995), as well as the significant differences in life satisfaction among comparable countries (Kahneman et al., 2004). The psychological phenomenon of hedonic adaptation may tempt one to conclude that even life-changing events may not have any impact on long-run happiness (Brickman, Coates, and Janoff-Bulman, 1978).

The main goal of this paper is to bring into formal analysis the psychological ideas of moment and remembered utilities and develop the notion of *memorable consumption* by

[1]Among many, see Elster and Loewenstein (1992), Diener, Suh, Lucas, and Smith (1999), Arieli and Carmon (2000), Frey and Stutzer (2002), Kahneman, Diener, and Schwarz (1999), Kahneman (2000a, 2000b), and Kahneman and Thaler (2006). The idea that past memories may influence well-being goes back, at least, to Smith (1759).

[2]Kahneman, Wakker, and Sarin (1997) provide a formal analysis of this distinction and propose a measurement method based on the notion of 'temporally extended outcomes.'

[3]At the national level, see, e.g., the Integrated Household Survey in UK. Diener (2000), Kahneman, Krueger, Schkade, Schwarz, and Stone (2004), and Benjamin, Heffetz, Kimball, and Szembrot (2014) inquire on the development of national indices of subjective well-being.

[4]Bond and Lang (2019) provide an econometric analysis that reports the difficulties encountered in comparing two groups of individuals by their average happiness using scale-based surveys.

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modeling the dynamic effects of memories of past consumption on well-being. Natural examples of memorable consumption include extraordinary events, such as celebrations of various occasions, life achievements, lottery wins, exotic dining and entertainment, vacations, as well as bad customer experiences, job termination, or more traumatic life-changing events. Apart from extraordinary events, enduring effects on well-being may grow from daily experiences that are related, for instance, to the person's income, place and conditions of living, or social status.

Our contribution is threefold. Our first and main contribution is an axiomatic theory of memorable consumption in a dynamic setting. Our model accommodates well-known phenomena in psychology, such as the peak-end rule, duration neglect, and adaptation trends. The axiomatic theory shows that the model has strong uniqueness properties and allows identification of the memorable effects of consumption in later periods separately from its material effects at present. The axioms also help to distinguish memorability from other history-dependent behavioral traits, such as habits and anticipation effects, as well as other effects and interactions among them. Importantly, we do not rely on an exogenous identification of what goods or experiences are memorable — the quality of being memorable is endogenously derived and subjective, and, thus, is allowed to vary across individuals. Our second contribution is an axiomatic Markovian model that represents a special case of the general model. In this model, the agent makes decisions on the basis of the "stock" of accumulated memories, and that stock of memories serves as the sole state variable in the dynamic process. Markovian evolution of memory makes the model particularly tractable and suitable for solving it with standard dynamic programming methods. The Markovian specification also makes it clear that the effect of memories may vary along two dimensions: they may generate greater utility values ("stronger effect") or last longer ("longer effect"); in turn, we can make interpersonal comparisons along these dimensions. Third, we illustrate the applied relevance and the analytical tractability of the model by introducing memorable effects in two distinct contexts. We study risk-taking behavior in a principal-agent setting and show that memorability may alter the agent's willingness to take risks; and, we study consumption-savings decisions in the presence of memorability, where we obtain a closed-form solution and illustrate the way memorability interacts with the permanent income hypothesis, excess sensitivity puzzle, and life-cycle decision patterns.

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1.1 The model's essential components

We study memorable consumption in a dynamic framework of preferences over consumption streams. As in Koopmans (1960, 1972), preferences are defined from an ex ante perspective, before time starts unfolding. A typical consumption stream of length t is denoted by $f = (f_0, f_1, \ldots, f_{t-1})$, where $f_\tau \in \mathcal{C} \subseteq \mathbb{R}^N$ for $N \geq 1$ is the consumption bundle at time $\tau = 0, \ldots, t-1$. In its simplest and most general form, our agent evaluates a stream f by computing its value (utility) according to the following formula:

$$V(f) = \sum_{\tau=0}^{t-1} \beta^\tau \big[u(f_\tau) + M(f_{\tau-1}, \ldots, f_0, 0, 0, \ldots)\big]. \tag{1}$$

As in the standard theory of exponential discounting, the parameter $\beta \in (0,1)$ is a discount factor, and the value of $u(f_\tau)$ represents the direct utility of consuming bundle f_τ at time τ. The novel component is $M(f_{\tau-1}, \ldots, f_0, 0, 0, \ldots)$, which represents the utility derived from the memory of the consumption history $(f_{\tau-1}, \ldots, f_0)$, while the expression $u(f_\tau) + M(f_{\tau-1}, \ldots, f_0, 0, 0, \ldots)$ captures the agent's total subjective well-being that can be attributed to time τ. The value of $M(f_{\tau-1}, \ldots, f_0, 0, 0, \ldots)$ is positive for pleasant memories and negative for unpleasant memories that the agent would prefer not to carry over into the future, if possible. We emphasize that our agent is aware that different consumption profiles may generate different memories. Thus, among other things, Representation (1) formalizes Kahneman's idea that people choose among memories of experiences, not simply among experiences.[5]

History-dependent phenomena As mentioned earlier, memorable effects are not the only potential reason for the past to affect the current utility. One striking example of history dependence is the well-known Mom's Treat (Machina, 1989, p. 1643). Suppose that a mom has a single indivisible treat that she can give either to her daughter or to her son. In principle, she is indifferent between giving the treat to either child. However, if her son got a treat just yesterday, she will strictly prefer to give the treat to her daughter today. Naturally, such a preference does not rely on whether a treat to a child is a memorable experience — it is guided by concerns about fairness. There are many other reasons for

[5] Anticipation emerges as a manifestation of agents' rationality and it is distinct from the notion of anticipation understood as savoring. See, e.g., Kahneman and Riis (2005).

history dependence, including an intrinsic preference for variety,[6] habit formation, and anticipatory feelings. Therefore, our model is not intended to be a universal theory of history-dependent utility; rather, we are interested in the phenomenon of memorability, its effect on individual choices, and its relevance for economic analysis. Our focus manifests noticeably in the proposed axiomatization, our results on identification and uniqueness, and applications.

1.2 Special cases and applied relevance

Representation (1) provides a general structure for analyzing different processes by which the memorable effect of past consumption may accrue over time. We look closely at three special cases of that functional form.

Peak-end rule Our first example provides a time-dependent specification of the memory function M that accommodates the so-called *peak-end rule* and *duration neglect* (Fredrickson and Kahneman, 1993). In these experimentally observed phenomena, the recollection of a prolonged experience is driven by only two salient points — the peak of the intensity and the most recent moment — while neglecting the duration of the experience.

Adaptation trends Our second example proposes a time-dependent specification of the memory function M that captures adaptation to repeated similar experiences. In particular, an experience becomes memorable and generates utility at later dates depending on its contrast with previous experiences. The proposed specification can be used in a wide range of contexts, from capturing the role of breaks in repeated consumption experiences to thinking about prevention of adaptation in the design of compensation schemes and promotion of job satisfaction.

Markovian memory We study in more detail a special case of representation (1) in which memory evolves according to a time-invariant Markov law. A consumption stream $f = (f_0, \dots, f_{t-1})$ is evaluated according to

$$V(f) = \sum_{\tau=0}^{t-1} \beta^\tau \big[u(f_\tau) + m_{\tau-1}\big], \tag{2}$$

[6]Indeed, on the next occasion, a person would prefer to have different meals and read different fiction books even if those meals or books are not worth recollecting and not memorable at all.

where m_τ is computed as $m_\tau = \psi(m_{\tau-1}, f_\tau)$ for $\tau = 0, \ldots, t-2$ and $m_{-1} = 0$. The key feature of this specification is that the utility of memorable consumption can be thought of as a "stock" variable that is determined at each point in time only by its value in the previous period and the current consumption. This recursive specification favors simplicity in keeping track of memorable effects and has the advantage of being highly tractable, as shown in Section 5.2.

Memorable goods and durable goods Goods that exhibit memorable effects have certain similarities with durable goods since they both generate flows of utilities for a significant period of time after the instant of their purchase. Nevertheless, they cannot be assimilated to one another. Hai et al. (2020) develop an extensive empirical analysis identifying memorable goods as a distinct category from durable goods. Gilboa et al. (2016) provide further support for this distinction. In this paper, we also argue that there are many types of spendings that have never been regarded as durable goods (such as wedding ceremonies, vacations, social events, and even restaurant meals); yet, they may generate additional flows of utilities after the time of their direct "consumption." Moreover, we argue that those additional flows are taken into account by economic agents and affect their choice behavior. Our theory is revealed-preference based, and agents' attitudes towards different types of consumption are entirely subjective and identifiable by the analyst due to the strong uniqueness properties of the model.

We observe that a typical durable-goods model would not be sufficient to accommodate the variety of patterns exhibited by memorable effects. Following an inventory-type dynamic, durable goods are subject to a decay rate, and, typically, they are replaced after complete deterioration. Memorable effects have a different nature. For instance, memories can be good or bad, and their "sign" may lead to noticeable asymmetries of the corresponding decay rates.[7] Moreover, there might be no decay at all: a person may derive pleasure from the narrative value of recollecting past experiences, and storytelling itself may reinforce the value of memory. Furthermore, as mentioned earlier, the utility from memories may be subject to effects such as the peak-end rule or adaptive trends that are extraneous to the usual turnover of durable goods. Our model can accommodate these empirical

[7]Both introspection and empirical evidence indicate that bad experiences persist into one's mind longer and more strongly than good experiences. See, e.g., Baumeister, Bratslavsky, Finkenauer, and Vohs (2001).

observations, as discussed in Section 2.4.

The rest of the paper is organized as follows. We next discuss the related literature. Section 2 presents the general model and illustrates its richness with two examples of time-dependent (non-Markovian) laws of motion for memory that are well-known in psychology — the peak-end rule and the adaptation-level theory. Section 3 proposes a special case of our model that features time-invariant Markovian dynamics. Section 4 provides an extensive discussion of Hai et al. (2020) and of our contributions relative to theirs. Section 5 develops two applications that illustrate our model in Micro and Macro settings. Section 6 concludes with a brief summary. All proofs are contained in the appendix.

1.3 Related literature

The key feature of our theory is that it breaks the separability of preferences across time periods. The agent's well-being is determined by two components — the joy of instantaneous consumption and the joy of memories. Preferences typically violate time separability with respect to the memory component — and this is a natural property of memorable effects. Indeed, memories generated by a fine dining experience may depend on the reference point set by past experiences of that sort. From this perspective, our paper belongs to a broad list of decision-theoretic papers in which the prize obtained by the decision maker in one time period has effects beyond that period, and, hence, the decision maker's preferences have a non-time-separable utility representation.[8] What distinguishes us from this literature is our specific interest in memorability.

In the study of memorability, two key papers are the empirical work of Hai, Krueger, and Postlewaite (2020), who introduced the notion of memorable good, and the theoretical work of Gilboa, Postlewaite, and Samuelson (2016). Both works are particularly interested in functional forms of the utility generated by memorable goods that exhibit adaptation traits (see Section 2.4). Among their key insights, these papers show that optimal consumption profiles of memorable goods should exhibit spikes that are clearly distinct from those generated by the consumption of durable goods. Furthermore, those spikes provide a net distinction of memorable consumption from the predictions of habit formation's models.

[8]See, among others, Rozen (2010) and Tserenjigmid (2020) who study habit formation; He, Dyer, and Butler (2013) who study habit formation and satiation; and Dillenberger and Rozen (2015) who study history-dependent risk attitudes.

After presenting our model, we devote Section 4 to discuss Hai et al.'s (2020) seminal work and its relationship with our paper.

The key assumption of Gilboa et al.'s (2016) model is that consumption of the memorable good generates additional flows of utility only if it exceeds a certain threshold level determined by previous memorable experiences. They study the optimal consumption of ordinary and memorable goods for different levels of "acclimatization," and show how the presence of memorable consumption generates non-smooth patterns. In the domain of axiomatic analysis, they characterize a preference relation that captures memorable effects across two time periods. Their utility representation takes the form of $u(x, y) + v(y, z)$, where the term $u(x, y)$ represents the current utility of consuming ordinary goods in quantity x and memorable goods in quantity y, and the term $v(y, z)$ represents the memorable utility from consuming memorable goods in quantities y in the current period and z in the past (Gilboa et al., 2016, Section 4).

In comparison with Gilboa et al. (2016), our paper has a different scope. Our main contribution is to propose an axiomatic foundation for a general multiperiod model. Our goal is to identify the minimal departures from the classic Discounted Expected Utility model that are necessary to accommodate memorable effects. The general functional form proposed in this paper encompasses several alternative specifications that include threshold-type specifications, as well as other psychology-driven specifications, such as the peak-end rule, and tractable specifications such as the Markovian one.[9] Another important contribution of our work is that, in terms of primitives of the model, we do not make assumptions about whether any particular good is ordinary or memorable. The property of being memorable is endogenous, and, by taking a subjective perspective, we let the agent reveal through her choices what is memorable for her. As a more technical remark, there is also a difference between our models in terms of timing: in Gilboa et al. (2016), consumption gives rise to memories and generates additional utility starting from the period in which the memorable good is consumed, while, in our model, memory starts to generate additional utility starting from the next period.

We conclude by remarking that memory is traditionally studied as a cognitive constraint.

[9]In passing, we note that the models of Gilboa et al. (2016) and Hai et al. (2020) are generally not Markovian in the sense of our Section 3.

Notable references in economic theory, among others, are Gilboa and Schmeidler (1995), where choice behavior is driven by similarity with past cases; Mullainathan (2002) and Bordalo, Gennaioli, and Shleifer (2020) who develop models of selective memory; Fudenberg, Lanzani, and Strack (2022), who study the effect of imperfect memory on long-run behavior; Battigalli and Generoso (2021), who develop a general framework and a formal language to describe flows of information and players' ability to recall information for a large class of sequential games. Finally, we refer to Kahana (2012) for a classic reference on existing theories and experiments on memory in psychology. Departing from the aforementioned literature, we view memory not as a constraint for the agent in achieving the best possible outcome, but rather as a channel for the past to make additional contributions to the current well-being. We therefore focus on the way this channel works and relate to the psychological evidence on mechanisms (such as the peak-end rule) leading to memorable effects.

2 The general model

This section presents formally our general model of memorable consumption. Our goal is to study memorability in its purest form; we propose a minimal deviation from the standard exponential discounting paradigm that, nevertheless, allows capturing the essential features of memorable consumption and distinguishing it from other forms of history-dependent preferences. As we will illustrate in Section 2.4 and, to a greater extent, in our applications, our minimal notion of memorability can still accommodate a wide range of psychological features and generate nontrivial effects in different economic contexts.

2.1 Setup

Let $\mathcal{C} \subseteq \mathbb{R}^N$ for some $N \in \mathbb{N}$ be the space of consumption bundles, which we assume to be nondegenerate and connected. Its typical element is denoted by $c = (c^1, \dots, c^N)$. The set $\mathcal{F}_t = \mathcal{C}^t$ for $t \in \mathbb{N}$ represents the collection of consumption streams of finite length t, with the typical element given by $f = (f_0, f_1, \dots, f_{t-1})$. Also, let $\oslash$ denote the stream of length zero and let $\mathcal{F}_0 = \{\oslash\}$. We denote by $\mathcal{F} = \bigcup_{t=0}^{\infty} \mathcal{F}_t$ the collection of all consumption streams of finite length. The sets $\mathcal{F}_t$ for $t \in \mathbb{N}$ are endowed with the sup-norm topology. For an element $f \in \mathcal{F}$, let $\ell(f) \coloneqq t$ if $f \in \mathcal{F}_t$.

For $t \in \mathbb{N}$, let $\mathcal{L}_t = \Delta(\mathcal{F}_t)$ be the space of lotteries (probability distributions with finite support) over streams of length t, and let $\mathcal{L} = \Delta(\mathcal{F})$ be the space of lotteries over all consumption streams of finite length. The agent's behavior is described by a preference relation (complete preorder) $\succsim$ on $\mathcal{L}$.

As usual, for every $P, Q \in \mathcal{L}$ and $\alpha \in [0,1]$, the lottery $\alpha P + (1-\alpha)Q \in \mathcal{L}$ is defined by $\alpha P(f) + (1-\alpha)Q(f)$ for every $f \in \mathcal{F}$. The spaces $\mathcal{L}_t$ for $t \in \mathbb{N}$ are endowed with the weak* topology: A net $\{P_\alpha\}_\alpha$ in $\mathcal{L}_t$ converges to $P \in \mathcal{L}_t$ iff, for any continuous and bounded function $U : \mathcal{F}_t \to \mathbb{R}$, we have $\int U\, dP_\alpha \to \int U\, dP$.

Our object of interest is a complete and transitive binary relation $\succsim$ on $\mathcal{L}$. From the viewpoint of interpretation, if a lottery P delivers the stream $f \in \mathcal{F}_t$ for some $t \in \mathbb{N}$ with probability p, then with probability p the agent receives the sequence of consumption bundles f_0 at date 0, f_1 at date 1, and so on up to date $t-1$, which represents the last date of the agent's lifespan.[10] Naturally, the space of lotteries makes the agent's lifespan uncertain.

Note that our framework of preferences on lotteries over consumption streams is akin to the classic setup of Koopmans (1960, 1972), where the modeler takes the perspective of an agent who makes a single choice among consumption streams at the ex ante stage, i.e., before time starts and payoffs in the streams begin to realize.[11]

We emphasize that, following the route of departing minimally from standard textbook assumptions, this framework implies that our agent does not have constraints on her capacity to reason, is capable of thinking in advance about her future well-being, and perfectly able to assess the value of alternative consumption streams in terms of material utility and generation of memorable effects.[12]

Throughout the paper, we use the following notation.

[10]The domain of preferences can be extended to also include infinite consumption streams (as in our applications in Section 5). Finite length streams are, however, both sufficient and necessary for developing our theory.

[11]In the seminal works of Koopmans (1960, 1972), preferences are defined on deterministic consumption streams of infinite length. Several axiomatic works adopt similar settings: see, e.g., Epstein (1983), Bleichrodt, Rohde, and Wakker (2008), Rozen (2010), Olea and Strzalecki (2014), and Dillenberger, Gottlieb, and Ortoleva (2020).

[12]Note that Gilboa et al. (2016) and Hai et al. (2020) focus on cognitively capable and sophisticated agents, as well. Habit formation models, notably Rozen (2010), have also traditionally maintained the assumption of unconstrained cognitive abilities.

Notation. For any $f = (f_0, f_1, \ldots, f_k)$ and $h = (h_0, h_1, \ldots, h_m)$ in $\mathcal{F}$ and $P \in \mathcal{L}$,

- let $h|f \in \mathcal{F}$ denote the concatenated stream $(h_0, h_1, \ldots, h_m, f_0, f_1, \ldots, f_k)$.
- let $h|P$ denote the lottery Q obtained from P by prepending h to the streams in the support of P: Formally, Q is defined as $Q(f) = P(f')$ if $f = h|f'$ for some $f' \in \mathcal{F}$, and $Q(f) = 0$ otherwise.

As usual, we identify a degenerate lottery that gives some stream $f \in \mathcal{F}$ with probability one with the stream itself.

2.2 Axioms

We next introduce the behavioral properties that characterize memorable consumption. They are organized into three groups: properties pertaining to the framework of lotteries over consumption streams; key axioms capturing memorability effects; and few technical assumptions.

Framework assumptions The first three assumptions are standard for models that deal with both time and uncertainty.

The first axiom states that advancing or postponing the timing of a consumption stream by one period does not affect preferences. It guarantees stability of tastes over time and is closely related to the standard formulation of Koopmans (1960):

Axiom A1 (Stationarity)**.** *There exists a neutral consumption bundle that we identify with* 0 *such that, for any* $P, Q \in \mathcal{L}$*, we have*

$$P \succsim Q \quad \Leftrightarrow \quad (0)|P \succsim (0)|Q.$$

The second axiom imposes the basic economic principle that positive experiences diminish in value if postponed:

Axiom A2 (Impatience)**.** *For any* $P \in \mathcal{L}$ *such that* $P \succ (0)$*, we have*

$$P \succ (0)|P \quad \textit{and} \quad (0)|P \succ (0).$$

Recall that $(0)|P$ is the lottery obtained from P by prepending the consumption stream that has the length of one and offers the neutral consumption element to all the streams in the support of P. Thus, Impatience states that, for any lottery P that is deemed better than "nothing," the agent prefers to consume P immediately rather than postpone it by one period. At the same time, she prefers to consume lottery P delayed by one period, rather than not consuming it at all.

Note that Stationarity and Impatience guarantee that our agents make dynamically consistent choices, regardless of whether or not consumption has memorable effects.

Finally, Independence is the classic property that delivers the underlying expected utility form:

Axiom A3 (Independence)**.** *For any* $P, Q, R \in \mathcal{L}$ *and* $\alpha \in (0,1]$*, we have*

$$\alpha P + (1-\alpha)R \gtrsim \alpha Q + (1-\alpha)R \quad \Leftrightarrow \quad P \gtrsim Q.$$

Overall, the above three axioms adapt to our notation the classical assumptions of rational behavior in the presence of time and risk.

Axioms pertinent to memory The next two key axioms delineate the behavioral features of memorable consumption. As frequently occurs in axiomatic theory, they do not give an explicit definition of the phenomenon of memorability by claiming that an agent is sensitive to memorable effects if and only if a certain axiom holds. Instead, they have more of a suggestive nature — they describe patterns that definitely hold for a standard textbook decision maker and that should also be *preserved* under memorability. More importantly, they clarify the notion of memorability by *ruling out* behavior that is not necessarily associated with memory and by distinguishing memorability from other forms of history dependence.

Axiom A4 (Risk Preference Consistency)**.** *For any* $f, g \in \mathcal{F}$*,* $p, q \in \mathcal{L}_1$*,* $P \in \mathcal{L}$*, and* $\alpha \in (0,1]$*, we have*

$$\alpha(f|p) + (1-\alpha)P \gtrsim \alpha(f|q) + (1-\alpha)P \quad \Rightarrow \quad \alpha(g|p) + (1-\alpha)P \gtrsim \alpha(g|q) + (1-\alpha)P.$$

Risk Preference Consistency guarantees that the agent's *tastes* remain unchanged after varying histories. Despite being a relatively simple property, this axiom is rather powerful. It rules out various types of backward-looking or reference-dependent evaluations of the current consumption. First, it immediately rules out the well-known habit formation phenomenon. Indeed, the latter rests on the idea that people feel an additional discomfort if their consumption falls below the level or departs from the pattern that was established in the past. Second, it rules out intrinsic preference for variety. The latter captures, for example, the reason why a person would prefer to go to The Nutcracker ballet on the second night if she watched Swan Lake on the previous one, even if she likes Swan Lake more than The Nutcracker.

Furthermore, since the streams f and g in the statement of the axiom can have different lengths, this axiom ensures that tastes (as in Swan Lake vs. The Nutcracker) remain unchanged with the passage of time. Moreover, p and q in the statement of the axiom may be nondegenerate lotteries. Hence, the axiom postulates also that the attitude towards risk depends neither on the calendar time nor on the realization of outcomes in preceding periods, thus ruling out direct psychological effects that the realization of extreme outcomes may have on risk-taking behavior.

The Risk Preference Consistency axiom clearly holds in the standard Discounted Expected Utility model. In that model, however, the consistent ranking between p and q is expected to be observed when those one-period lotteries are offered at any position in otherwise identical streams. Our axiom demands much less than that, and imposes the consistent ranking only in the last period of the agent's planning horizon for the reason that that period surely does not generate any memories that are relevant for the agent's problem. By weakening the replacement property in this way, histories of past consumption are given the freedom to affect memories but not tastes.

As a final note, we highlight that the above axiom imposes consistency on the way the agent treats one stream — $f|p$ vs. $g|p$ or $f|q$ vs. $g|q$ — in various mixtures that have the general form of $\alpha(f|p) + (1 - \alpha)P$, where the lottery P may potentially have hundreds of other streams in its support. Hence, in its statement, the axiom does not insist that the agent knows the future with certainty. Rather, it asks a question of the form: "If you have an opportunity to choose your consumption along *one possible* path of future events, would you prefer the left option or the right one?" We also note that our memory-related axioms are formulated having in mind also a normative perspective, so that one can deliberate if a rational person may agree with the statement of the axiom and adopt it as a rule for her decision making.

The second memory-related axiom is concerned with tradeoffs between the direct value of consumption and its contribution to the well-being in subsequent periods. As Risk Preference Consistency, it involves comparisons of mixtures consisting of a common lottery component: this allows to introduce an uncertain time horizon in the formulation of the axioms by relying on the linear structure of preferences.

Axiom A5 (Memory-Consumption Tradeoff Consistency)**.** *For any* $t \in \mathbb{N}, f, g \in \mathcal{F}_t, p, q \in \mathcal{L}_1, P \in \mathcal{L}$, *and* $\alpha \in (0,1]$, *we have*

$$\alpha(f|p)+(1-\alpha)P \gtrsim \alpha(g|(\tfrac{1}{2}p+\tfrac{1}{2}q))+(1-\alpha)P \quad \Leftrightarrow \quad \alpha(f|(\tfrac{1}{2}p+\tfrac{1}{2}q))+(1-\alpha)P \gtrsim \alpha(g|q)+(1-\alpha)P.$$

To interpret this axiom, consider the following tradeoff between consumption in a fixed (the last) period and pleasant memories that are generated by the consumption stream from period zero to the second-to-last. Suppose that changes in the initial part of the consumption stream can be counterbalanced by replacing a consumption lottery p in the last period with a lottery that is a midpoint between p and some q. The axiom postulates that, in this case, a similar replacement in the last period of the midpoint between p and q with q — which is a replacement that has the same distance and direction in the space of last-period consumption lotteries — should have the same counterbalancing effect. Thus, it calibrates the relative effects of memory and consumption in quantitative terms. Note that, together with the other assumptions, this axiom implies the following simple property:[13]

- For all $f, g \in \mathcal{F}_t$, $p, q \in \mathcal{L}_1$, $P \in \mathcal{L}$, and $\alpha \in (0,1]$, we have that

$$\alpha(f|p)+(1-\alpha)P \gtrsim \alpha(g|p)+(1-\alpha)P \quad \Leftrightarrow \quad \alpha(f|q)+(1-\alpha)P \gtrsim \alpha(g|q)+(1-\alpha)P.$$

This property ensures that the agent's preferences are driven by the elements for which the two mixtures differ, i.e., f and g. Formally, the desirability of the stream f versus the stream g is independent of the consumption element appended to them, as well as it is independent of the common element in the mixtures, P, and the probability α with which the mixtures are played. The axiom thus rules out additional effects on the subjective well-being that the agent may obtain in early periods from the mere anticipation of her consumption in the last period (say, positive anticipation of high consumption p versus negative anticipation of low consumption q).[14] The full-fledged Memory-Consumption Tradeoff Consistency rules out additional forms of forward-looking psychological effects.

[13]We refer to Lemma 7 in the Appendix for a proof of this statement.

[14]Note that the simplest forms of anticipation effect — for instance, the case of a consumption p in a stream $f|p$, where $f \in \mathcal{F}_t$, giving a constant f-independent utility boost in each of the first t periods — is ruled out by the Risk Preference Consistency axiom. Indeed, that axiom asserts that p has the same value after the initial stream f of arbitrary length t as after the stream $g = \oslash$ of length zero.

The aforementioned property is complementary to Risk Preference Consistency — both axioms constitute separability assumptions (that always hold in the standard expected discounted utility model). However, our assumptions are weaker than standard ones. Indeed, in Koopmans' formalization of the standard theory, separability is required with respect to each period, while we impose it only with respect to two "coordinates" — the periods from 0 to second-to-last and the last period. This way, our axioms allow for reference dependence in *memory* (as will be illustrated in subsequent examples) but rule out reference dependence in the *direct value* of consumption, thereby precluding behavioral phenomena such as habit formation (ruled out by Risk Preference Consistency) and anticipation (ruled out by Memory-Consumption Tradeoff Consistency).

Technical requirements We conclude the list of axioms with few technical assumptions.

Axiom A6 (Continuity)**.** *(i) For all* $P \in \mathcal{L}$ *and all* $t \in \mathbb{N}$*, the sets* $\{Q \in \mathcal{L}_t : Q \succsim P\}$ *and* $\{Q \in \mathcal{L}_t : P \succsim Q\}$ *are closed. (ii) For all* $P, Q, R \in \mathcal{L}$*, the sets* $\{\alpha \in [0,1] : \alpha P + (1-\alpha)Q \succsim R\}$ *and* $\{\alpha \in [0,1] : R \succsim \alpha P + (1-\alpha)Q\}$ *are closed.*

The above continuity axiom combines two very well-known notions — the stronger Closed Continuity in Part (i) and the weaker Archimedean axiom in Part (ii). They are defined on different domains: the first one is applied to lotteries over streams of predetermined length, while the second one to arbitrary lotteries over consumption streams.

Finally, we assume nondegeneracy of preferences in the following form.

Axiom A7 (Nondegeneracy)**.** *There exist* $c^*, c_* \in \mathcal{C}$ *such that* $(c^*) \succ (0) \succ (c_*)$.

2.3 Basic representation

The following notation will be useful for stating the results throughout.

Notation. Let $\mathcal{C}_0^\infty$ denote the set of infinite sequences of elements of $\mathcal{C}$ for which only finitely many elements are distinct from 0, where 0 is the element of $\mathcal{C}$ given by Stationarity.

The space $\mathcal{C}_0^\infty$ is endowed with the following topology: a net $\{f^{(\alpha)}\}_\alpha$ converges to some f in $\mathcal{C}_0^\infty$ if and only if, for some $T \in \mathbb{N}$ such that $f_t = 0$ for all $t \geq T$, there exists an index α_0 such that $f_t^{(\alpha)} = 0$ for all $\alpha \geq \alpha_0$ and $t \geq T$, and $\sup_{0 \leq t \leq T} |f_t - f_t^{(\alpha)}|$ converges to zero. We also say that a function $\Phi : \mathcal{C}_0^\infty \to \mathbb{R}$ is *finite-horizon-bounded* if and only if, for any $T \in \mathbb{N}$, there exists $K > 0$ such that, for any $f \in \mathcal{C}_0^\infty$ such that $f_t = 0$ for all $t \geq T$, we have $|\Phi(f)| \leq K$.

We are ready to provide a behavioral characterization of preferences that exhibit memorable effects of consumption.

Theorem 1. *Let $\succsim$ be a complete preorder on $\mathcal{L}$. The following statements are equivalent:*

(i) $\succsim$ *satisfies Axioms (A1)–(A7);*

(ii) there exist a scalar $\beta \in (0,1)$, a continuous and bounded function $u : \mathcal{C} \to \mathbb{R}$ such that $u(0) = 0$ and its range contains positive and negative numbers, and a continuous and finite-horizon-bounded function $M : \mathcal{C}_0^\infty \to \mathbb{R}$ with $M(0,0,\ldots) = 0$, such that

$$V(P) = \sum_{f \in \operatorname{supp} P} P(f) \sum_{t=0}^{\ell(f)-1} \beta^t \big[u(f_t) + M(f_{t-1}, \ldots, f_0, 0, 0, \ldots) \big] \tag{3}$$

is a utility representation of $\succsim$ on $\mathcal{L}$.

Theorem 1 delivers a preference representation that enriches the standard exponential discounting formula to accommodate the memorable effects of consumption. The usual parameters of the evaluation formula are the scalar β, which represents the discount factor, and the function u, which measures the utility of a bundle of goods at the time of material consumption. Besides its direct value, consumption generates additional utilities in the future, and their flow is measured by a novel object — the function M. Given a stream f, the overall utility at time t is calculated as the sum $u(f_t) + M(f_{t-1}, \ldots, f_0, 0, 0, \ldots)$, in which the second term specifies the utility derived from the recollection of past memorable experiences. Thus, representation (3) can be interpreted as if the agent engages in two forms of consumption, the material one and the consumption of memories, giving rise to behaviorally distinct utilities. The notation for the arguments of the function M is backward-looking: first goes the most recent past consumption, then the second-to-most-recent, and so on. The sequence of arguments ends with an infinite sequence of zeroes since, at each point in time, the preceding history is assumed to be finite.[15] Note that, if no good is perceived as memorable, then $M(\cdot) = 0$ and the representation reduces to the standard exponential discounting model.

Uniqueness The parameters β, u, and M are identified uniquely, as shown next. In comparison to the standard uniqueness results in utility theory, the only minor difference is

[15] A single memory function operating on infinite (but vanishing) streams could be replaced by a collection of functions operating on finite streams — $M_1(f_0)$, $M_2(f_1, f_0)$, and so on. Specifying functions in this way would require imposing additional constraints — it must be that $M_2(\cdot, 0) \equiv M_1(\cdot)$, and so on.

that the functions u and M are unique only up to a positive multiplicative factor, whereas arbitrary additive constants are not allowed because we impose the convention of assigning the numeric value 0 to the neutral element identified by Stationarity. Importantly, our uniqueness result ensures that our model cannot be reinterpreted in terms of other history-dependent phenomena.

Proposition 2. *Suppose that (β, u, M) is a representation of a binary relation $\succsim$ on $\mathcal{L}$ as in Theorem 1. Then, $(\hat{\beta}, \hat{u}, \hat{M})$ is another such representation of the same binary relation if and only if $\beta = \hat{\beta}$, $\hat{u} = \lambda u$, and $\hat{M} = \lambda M$ for some $\lambda > 0$.*

2.4 Time- and history-dependent memory

One well-known heuristic about the way people recollect prolonged experiences is called the *peak-end rule*. Originally introduced by Fredrickson and Kahneman (1993), it builds upon the view that any hedonic experience can be thought of as consisting of a sequence of moments that can be identified, for instance, by the unfolding of time. The retrospective evaluation of a prolonged experience, whether positive or negative, is determined by the average of only two salient moments: the most intense moment — that is, the *peak* — and the latest moment experience — that is, the *end*. One notable implication is that the duration of an experience has no impact on its recollection. For instance, a short, but rather exotic, vacation may generate more intense memories than a longer, but more ordinary, vacation. This pattern, dubbed *duration neglect*, is observed in numerous experimental studies suggesting that prolonging an unpleasant experience by adding some extra moments of diminished discomfort may mitigate the subsequent assessment of the overall experience.[16] Our next example proposes a simple specification of the function M that accommodates the psychological evidence on the peak-end rule and duration neglect.

Example 1 (Peak-end rule)**.** *Assume that consumption bundles have one component that captures memorable experiences — their sign and intensity. For a bundle $c \in \mathcal{C} \subseteq \mathbb{R}^N$, let this component be N. Assume also that the utility function u is increasing in this component, so that positive values in this component contribute positively to the agent's utility. Let the*

[16] For experimental evidence, see Varey and Kahneman (1992), Kahneman, Fredrickson, Schreiber, and Redelmeier (1993), Fredrickson (2000), and Kahneman (2000a, 2000b). Arieli (1998) points out the relevance of additional factors, such as the rate of change and the intensity trend.

function M from representation (3) be defined as

$$\begin{aligned} M(f_{t-1},\dots,f_l,\dots,f_k,\dots,f_0,0,0,\dots) &= d(t-l-1)\left(\gamma \max_{i=k,\dots,l} v(f_i^N) + (1-\gamma)v(f_l^N)\right) \\ &\qquad \text{if } v(f_i^N) > 0 \text{ for all } i=k,\dots,l, \\ M(f_{t-1},\dots,f_l,\dots,f_k,\dots,f_0,0,0,\dots) &= d(t-l-1)\left(\gamma \min_{i=k,\dots,l} v(f_i^N) + (1-\gamma)v(f_l^N)\right) \\ &\qquad \text{if } v(f_i^N) < 0 \text{ for all } i=k,\dots,l. \end{aligned} \tag{4}$$

This specification covers both positive and negative experiences. The function $v : \mathbb{R} \to \mathbb{R}$ in the formula is the utility function for memory, and we assume that it is increasing and such that $v(0) = 0$: that is, the sign of memory is the same as the sign of the instantaneous utility. The variables k and l are the endpoints of the time interval $t = k,\dots,l$ that constitutes the most recent prolonged experience: formally, one prolonged experience is defined as a sequence of consecutive periods in which v-utilities of the memorable component of the consumption bundle are all positive or all negative, and such that it cannot be extended further in time while maintaining the sign. In both positive and negative cases, the additional utility generated by memories of an experience is determined by the intensity of the experience in the end period, f_l^N, and the intensity in the most extreme (i.e., the peak) period. Naturally, the peak of negative experiences is determined by the min *operator, hence formula (4) distinguishes between two cases. In the peak-end combination, the peak experience contributes to the utility with the weight $\gamma \in (0,1]$ and the latest experience contributes with the complementary weight. Since the formula takes into account only the utility values of the end and the peak of an experience, its duration $l-k+1$ is neglected. Finally, when time passes after the end of the experience, its memory is allowed to decay according to the function $d : \mathbb{Z}_+ \to [0,1]$ such as, for example, $d(\tau) = \beta^\tau$. We apply the peak-end rule for memory formation to a specific economic context in Section 5.1.*

Another important class of behavioral regularities related to past memories is studied in the well-known *adaptation-level theory* in psychology.[17] The most relevant economic prediction of the theory is that repeated exposure to the same good experience will gradually attenuate the initial feeling of pleasure; similarly, persistent exposure to the same bad experience will make the feeling of discomfort wane.

[17] See Helson (1947, 1948) for origins of the theory that started with the perceptual adaptation in vision. More recent works include Frederick and Loewenstein (1999) and Diener, Lucas, and Scollon (2006).

Our model is not intended to capture full-fledged adaptation-level theory. Indeed, our axioms imply that the current utility from consumption is not reference-dependent. However, the *memorability* of experiences and their value at the time of *recollection* may well depend on the past history of similar experiences and exhibit adaptation features, as shown next.

Example 2 (Adaptation)**.** *Let the function M from representation (3) be defined as*

$$M(f_t, \dots, f_0, 0, 0, \dots) = G(f_t, A(f_{t-1}, \dots, f_0, 0, 0, \dots)), \tag{5}$$

for all $f \in \mathcal{F}$ and $t \in \mathbb{Z}_+$, where $A : \mathcal{C}_0^\infty \to \mathbb{R}$ is defined as $A(f_t, \dots, f_0, 0, 0, \dots) = \alpha \sum_{\tau=0}^{\infty} (1 - \alpha)^\tau f_{t-\tau}$, $\alpha \in (0, 1)$, and $G : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a continuous function that is monotone in the first argument and such that $G(0, 0) = 0$.

The function $A(f_{t-1}, \dots, f_0, 0, 0, \dots)$ represents the adaptation level acquired from consumption up to time $t - 1$ and sets the reference point for new memories at time t. The formula for A can be equivalently written as $A(f_t, \dots, f_0, 0, 0, \dots) = \alpha f_t + (1 - \alpha) A(f_{t-1}, \dots, f_0, 0, 0, \dots)$, making it clear that the coefficient α is the weight attributed to the most recent experience in determining the new adaptation level. The function G measures the utility value of the memory from consuming bundle x after a history of consumption summarized by the reference level r. In Tversky and Griffin's (1991) terminology, $A(f_{t-1}, \dots, f_0, 0, 0, \dots)$ corresponds to the endowment level accumulated up to time t, whereas G quantifies the contrast effect. The simplest specification for G can be $G(x, r) = \max\{x - r, 0\}$, in which a positive flow of memory utility is generated only if the most recent consumption exceeds the reference level. A more general specification for G may accommodate a broader spectrum of adaptation trends in memories' recollection and, in particular, may not necessarily require new experiences to beat the prior record. Indeed, a person may have very high standards for fine dining and, at the same time, enjoy pleasant memories from having coffee and pastries in some regular bakery. Furthermore, reactions to new circumstances may result into only partial adaptation.[18] This is consistent with our subjective approach to memorability.

Adaptation-level theory gives rise to a number of well-known patterns. For instance, it suggests that introducing an interval of lower consumption in a lengthy stream of positive

[18] In an empirical study, Diener et al. (2006) suggest that individuals may have multiple adaptation points. They also report evidence on the heterogeneity of the adaptation process across individuals. Lucas, Clark, Georgellis, and Diener (2004) report evidence on limited adaptation to unemployment.

consumption may make the agent appreciate it more.[19] Within our setup, we can exemplify this idea by considering the following preference over mixtures of consumption streams:

$$\frac{1}{3}(c, c, 0, c, c, 0, \dots) + \frac{1}{3}(0, c, c, 0, c, c, \dots) + \frac{1}{3}(c, 0, c, c, 0, c, \dots) \succ \\ \frac{2}{3}(c, c, c, c, c, c, \dots) + \frac{1}{3}(0, 0, 0, 0, 0, 0, \dots).$$

According to the standard discounted expected utility, the agent should be indifferent between them because, at each date, she consumes c with two thirds probability and zero with one third probability on both the left-hand and right-hand sides. However, if memorability is taken into account, a constant stream of high consumption may generate less memory (and less utility from memory) than streams in which high consumption is interrupted. Our model can easily accommodate such a preference for intermittent consumption — if the utility from recollecting past experiences follows an adaptation process, then an intermittent profile will generate higher utility flows from memory than will an equivalent constant one. This observation has prescriptive implications, and it may offer new insights into, for instance, designing optimal wage schemes, promotions, or unemployment benefits.[20]

3 Markovian memory

This section studies a special case of our general representation that is particularly suitable for applications. Specifically, we present and provide a behavioral characterization of a version of the model in which the memory of past consumption follows a Markovian law of motion: the value of memories (i.e., the utility derived from them) at any time t is determined only by the corresponding value at time $t-1$ and the consumption at time t, and it does not depend directly on the patterns of consumption at earlier dates. Hence, the utility from memorable consumption can be thought of as a "stock" variable that is driven by the current consumption and evolves according to a time-invariant Markov process. Besides theoretical interest, the Markovian law of motion has a huge practical advantage over other specifications in terms of its simplicity and tractability. As shown in Section 5.2, the Markovian specification allows an analytical closed-form solution of

[19] This prediction is supported by evidence from psychology and marketing. See, e.g., Ratner, Kahn, and Kahneman (1999), Ariely and Zauberman (2000), and Nelson and Meyvis (2008).

[20] Kahneman and Thaler (1991) examine some implications of adaptation on job satisfaction.

the classical consumption-savings problem with the memorable component. The resulting formulas provide transparent insights and facilitate a deeper understanding of the effects of memorability on the response to uncertainty and the consumption-savings dynamics.

The Markovian property To present the Markovian model, we need to introduce a few concepts. We start with the notion of a tradeoff between memory and consumption.

Definition 1. We say that the memory after a stream f k-dominates the consumption z, where $k > 0$, $f \in \mathcal{F}$, and $z \in \mathcal{C}$, if

$$f|0 \succsim \frac{1}{k+1} f + \frac{k}{k+1}(f|z). \tag{6}$$

We denote relationship (6) by $f \succsim^{\mathtt{m}:k} z$. A similar strict preference

$$f|0 \succ \frac{1}{k+1} f + \frac{k}{k+1}(f|z)$$

is denoted by $f \succ^{\mathtt{m}:k} z$.

To understand the gist of this definition, observe that the left-hand side of (6), in comparison to the right-hand side, offers the agent a greater chance of enjoying the memory of f in the subsequent time period — on the left-hand side, the probability of enjoying such a memory is one, while on the right-hand side it is only $\frac{k}{k+1}$, a difference of $\frac{1}{k+1}$. In exchange for that, the right-hand side offers the agent a potentially higher level of consumption in the last period, z instead of zero.[21] The additional consumption of z is available to the agent with probability $\frac{k}{k+1}$. Thus, the pattern in (6) describes a preference for enjoying the memory produced by f over the direct benefits of consuming z. Moreover, this preference is quantified: if the agent prefers the left-hand side, then, loosely speaking, the pleasure of the memory produced by f is at least k times greater than the pleasure of consuming z.[22]

The notion of consumption-memory tradeoff allows us to compare consumption streams in terms of their value for generating future memories. As formally stated next, a stream f memory-wise dominates another stream g if, for any consumption bundle z that the agent is willing to give up to enjoy the memory of g, she is willing to give it up to enjoy the memory of f a fortiori.

[21]The use of the neutral element 0 on the left-hand side of (6) is convenient but not mandatory. This and subsequent definitions can be modified to use a different reference point for measuring tradeoffs.

[22]As is usually the case, the usage of lotteries allows us to give cardinal meaning to relationships between utility levels.

Definition 2. For any $f, g \in \mathcal{F}$, we say that

- f generates a *higher value of memory* for the next period in comparison to g if $g \succsim^{\mathrm{m}:k} z \Rightarrow f \succsim^{\mathrm{m}:k} z$ for all $k > 0$ and $z \in \mathcal{C}$. We denote this relationship by $f\,\mathcal{R}^{\succsim}\,g$.
- f generates a *strictly higher value of memory* for the next period in comparison to g if $g \succsim^{\mathrm{m}:k} z \Rightarrow f \succ^{\mathrm{m}:k} z$ for all $k > 0$ and $z \in \mathcal{C}$. We denote this relationship by $f\,\mathcal{S}^{\succsim}\,g$.
- f generates *the same value of memory* for the next period as g if $g \succsim^{\mathrm{m}:k} z \Leftrightarrow f \succsim^{\mathrm{m}:k} z$ for all $k > 0$ and $z \in \mathcal{C}$. We denote this relationship by $f\,\mathcal{I}^{\succsim}\,g$.

The above definitions achieve two important goals. First, they provide a behavioral notion of what it means for one consumption stream $(f_0, \ldots, f_{t-1})$ to produce a higher-valued memory for the period t relative to another stream $(g_0, \ldots, g_{t-1})$, regardless of the utilities that these streams generate for periods $0, \ldots, t-1$. We will use this property shortly to set up the Markovian case. Second, these definitions enable the comparison of the value of memory for streams of different lengths. As a consequence, they can be used to verify that it is, indeed, behaviorally meaningful to attribute the memory utility $M(f_{t-1}, \ldots, f_0, 0, 0, \ldots)$ to date t in the general representation (3).

We use the above definition to formulate our key axiom for the Markovian representation.

Axiom A8 (Markovian Property)**.** *For any $f, g \in \mathcal{F}$,*

$$f\,\mathcal{I}^{\succsim}\,g \;\Rightarrow\; f|c\,\mathcal{I}^{\succsim}\,g|c \qquad \textit{for all } c \in \mathcal{C}.$$

The antecedent of this property considers the situation in which the memory effect of f is equivalent to that of g, that is, both streams generate the same value of memory, as measured in the time period that follows those respective streams. The axiom maintains that if these two streams are extended by an additional period of identical consumption, then the extended streams also produce memories of the same value — all differences between streams f and g are deemed irrelevant for the future.[23] The Markovian axiom (and, as will be seen later, the functional form for the memory evolution rule) is minimalistic in its nature: the history of past consumption may matter for the formation of memories, but only through one channel, the current stock of memories. From a psychological viewpoint, this may capture the behavior of a person who aims for simplicity and summarizes her

[23] Mathematically, this reflects the idea of Markov process: the value of memory generated by past consumption is a sufficient statistic of the past, so that this value and the current consumption uniquely determine the value of memory for the next period.

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experiences by a single number (such as star rating) to avoid overloading. We note that all other memory evolution rules that have been mentioned in the paper — the peak-end rule and the adaptation-level rule, as well as the rules specified in Gilboa et al. (2016) and Hai et al. (2020) — are generally not Markovian.

Characterization of the Markovian case Our next theorem shows that the above property, together with our basic axioms (A1)–(A7), delivers a convenient time-invariant Markovian representation.

For a function $\psi : I \times \mathcal{C} \to \mathbb{R}$, where $I \subseteq \mathbb{R}$ and $0 \in I$, we say that it is *normalized* if $\psi(0,0) = 0$, and it is *recursively bounded* if all sets I_t for $t \in \mathbb{N} \cup \{0\}$ defined recursively as $I_0 = \{0\}$ and $I_t = \psi(I_{t-1}, \mathcal{C})$ for $t \in \mathbb{N}$ are bounded.

Theorem 3. *Let $\succsim$ be a complete preorder on $\mathcal{L}$. The following statements are equivalent:*

(i) *$\succsim$ satisfies Axioms (A1)–(A7) and (A8);*

(ii) *there exist a scalar $\beta \in (0,1)$, a continuous and bounded function $u : \mathcal{C} \to \mathbb{R}$ with $u(0) = 0$, an interval I of $\mathbb{R}$ that contains 0, and a normalized, continuous, and recursively bounded function $\psi : I \times \mathcal{C} \to I$ with $\operatorname{range} \psi = I$ such that a utility representation of $\succsim$ on $\mathcal{L}$ is $V(P) = \sum_{f \in \operatorname{supp} P} P(f) V(f)$ for all $P \in \mathcal{L}$, where $V(f)$ for all $f \in \mathcal{F}$ is computed as*

$$V(f) = \sum_{t=0}^{\ell(f)-1} \beta^t \big[u(f_t) + m_{t-1} \big], \tag{7}$$

$$\text{where} \quad m_\tau = \psi(m_{\tau-1}, f_\tau) \text{ for } \tau = 0, \ldots, \ell(f) - 2,$$

$$m_{-1} = 0.$$

According to representation (7), the evaluation of a stream f at any time t is given by $u(f_t) + m_{t-1}$, where $u(f_t)$ is the material utility of f_t and m_{t-1} is the stock of memory accumulated up to time t. The function ψ describes the process of incorporating the memory effect of consuming f_t into m_{t-1}, giving rise to the next-period value, m_t. Similar to all specifications of the memory utility discussed earlier, memory may have long-lasting effects here, as well. However, the dependence of m_t on consumption in periods $t-1, \ldots, 1, 0$ is encapsulated in the previous stock of memory, m_{t-1}. This is the nature of our Markovian evolution of memory. Note that such a recursive process of computing the values of m_t is particularly tractable because the function ψ is independent of time.

Uniqueness Theorem 3 represents preferences in terms of quadruples of the form (β, u, I, ψ). These quadruples are unique up to rescaling, as shown next.

Proposition 4. *Suppose that (β, u, I, ψ) is a representation of a binary relation $\succsim$ on $\mathcal{L}$ as in Theorem 3. Then, $\left(\hat{\beta}, \hat{u}, \hat{I}, \hat{\psi}\right)$ is another such representation of the same binary relation if and only if $\hat{\beta} = \beta$ and there exists $\lambda > 0$ such that $\hat{u} = \lambda u$, $\hat{I} = \lambda I$, and $\hat{\psi}(m, c) = \lambda\psi(m/\lambda, c)$ for all $m \in \hat{I}$ and $c \in \mathcal{C}$.*

Comparative statics analysis The structure of the Markovian model makes salient two independent dimensions along which one can measure the influence of memorable consumption on choice behavior and, in turn, compare agents. One of them is longevity of memories in the agent's mind, that is negatively related to the rate at which past memories decay. The other one is the sensitivity of the agent's memory to new consumption and the ability of the latter to increase the stock of memories, which also captures the strength of memorable effects relative to the direct utility from consumption.

To illustrate, consider two agents with the same utility of consumption $u(c)$ and the same discount factor, and suppose that their Markovian evolution functions for memory are $\psi_i(m, c) = \alpha_i \max\{m, K_i u(c)\} + (1 - \alpha_i)K_i u(c)$ for $i = 1, 2$, where $\alpha_i \in [0, 1]$ and $K_i > 0$ are parameters. Then, we can say that α_i and K_i capture the effects of, respectively, the longevity of memory and the strength of memory for the agents. Moreover, these parameters are independent, in the sense that if $K_1 > K_2$ then Agent 1 exhibits stronger effects of memory, regardless of the values of α_1 and α_2; similarly, if $\alpha_1 > \alpha_2$, then Agent 1 exhibits longer effects of memory, regardless of the values of K_1 and K_2.

4 Comparison with Hai, Krueger, and Postlewaite (2020)

Hai, Krueger, and Postlewaite (2020) is the first paper to formalize the concept of memorable good and introduce it within a conventional consumption-saving model. They propose a model that splits total consumption into consumption of ordinary nondurable goods and memorable goods, and considers a household that solves the intertemporal consumption-savings problem of maximizing

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\xi \frac{C_{nt}^{1-\gamma}}{1-\gamma} + (1-\xi) \frac{(C_{mt} + \zeta M_t)^{1-\gamma}}{1-\gamma} \right],$$

where C_{nt} is the consumption of ordinary nondurable goods at date t, C_{mt} is the consumption of memorable goods at date t, M_t is the stock of memory at date t from past memorable consumption, ξ and ζ are weight parameters, γ is the risk aversion parameter, and β is the discount factor.

The key feature of the model is the assumption that extraordinary consumption of the memorable good generates new memories, that is, the stock of memory increases only if the current consumption C_{mt} exceeds a dynamic threshold N_t,

$$M_{t+1} = (1 - \delta_m)M_t + \max\{C_{mt} - N_t, 0\},$$

where δ_m is the decay rate for memory.[24] In turn, the threshold N_t evolves according to

$$N_t = (1 - \rho)N_{t-1} + \rho C_{m,t-1},$$

where ρ is a parameter such that $1 - \rho$ reflects the persistence of the threshold.

This parametric model is calibrated to the US economy in 1980–2003. Hai et al.'s (2020) most important result is an estimation of the impact of consumption fluctuations on welfare. They show that, despite their volatility, the presence of memorable goods significantly reduces welfare losses compared to an equivalent model without such goods. The reason lies in the fact that the memory stock acts as a substitute for the actual consumption of memorable goods in the utility function; thus, even when consumption exhibits spikes, the memory stock behaves as a mechanism to smooth overall utility. Second, Hai et al. (2020) show the role of memorable goods consumption in explaining the observed excess sensitivity of consumption to expected income changes.

Our paper has many complementarities with Hai et al.'s (2020). While Hai et al. (2020) are seminal in identifying memorable goods as a distinct category and in empirically demonstrating their relevance for economic analysis, our contribution is more conceptual — we establish an axiomatic foundation of the phenomenon of subjective memorability in a multiperiod setting. Our theory describes a minimal departure from the standard paradigm of expected utility that allows memorable effects, and provides a way to distinguish memorability from other psychologically-driven phenomena that lead to history-dependent felicity from consumption or experiences at the foundational level. Besides that, our uniqueness

[24] The threshold law of the evolution of memory is also used by Gilboa et al. (2016).

theorem provides foundation for model *identification*: for any parametric model that conforms to our general representation (3), our result can be used to show that the parameters of the model can be empirically identified — at least, when the data size is unbounded.

Regarding the form of the utility function that is assumed by Hai et al. (2020), one may ask how it relates with our representation (3). We observe, first, that Hai et al. (2020) measure the memory stock in consumption units and treat it on equal terms with the current consumption of memorable goods. In their expression $\frac{(C_{mt}+\zeta M_t)^{1-\gamma}}{1-\gamma}$, C_{mt} represents the direct contribution of consuming memorable goods and M_t is the contribution of past memories: the stock of memory and the current consumption of memorable goods are perfect substitutes. In our model, the memory stock is measured in utility units; similarly to Gilboa et al. (2016), the value of memory is combined additively with the *utility* derived directly from consumption. Therefore, our and Hai et al.'s (2020) models are generally non-nested.

Besides differences in measuring memorable effects, Hai et al. (2020) assume a law of memory evolution that incorporates adaptation motives in the form of thresholds. Pursuing our goal of developing an axiomatic foundation for the concept of memorability, we present a more general functional specification that can accommodate many different laws of motion, including the notable peak-end rule and the Markovian rule. Note that Hai et al.'s (2020) specification is not Markovian (in the sense of Section 3). Indeed, in their specification, the value of the memorable component of the total utility, $\frac{(C_{mt}+\zeta M_t)^{1-\gamma}}{1-\gamma}$, or its core term $C_{mt}+\zeta M_t$, are not sufficient to uniquely determine the future evolution of memory because the latter depends also on the second state variable, N_t. As we will illustrate in Section 5.2, Markovian specifications have the additional benefit of high tractability in comparison with non-Markovian ones. In our application, we complement the result of Hai et al.'s (2020) numerical simulation by deriving the excess sensitivity of consumption in the presence of memorability through a closed-form solution of the model.

5 Memorable consumption: two illustrations

This section presents two illustrations of our theory of dynamic memorable consumption in different applied contexts. Albeit simplified, our applications are suggestive of how our model can be used as a building block of larger economic models.

The first application considers a multiperiod principal-agent setting and builds up a simple model in which the agent intrinsically cares about the outcomes resulting from her actions; importantly, these outcomes may generate memorable effects for her. For the evolution of the agent's memories, we assume a version of the peak-end rule introduced in Section 2.4. As we illustrate, memorability does not simply increase the agent's intrinsic motivation, but creates nontrivial intertemporal tradeoffs for her. In turn, she may choose inefficient actions and behave either too conservatively or too recklessly since her choices depend on both the history of outcomes and the length of the relationship. Memorability of outcomes affects the principal's behavior, as well, and requires him to adjust the optimal incentive scheme. Overall, an agent for whom outcomes are memorable may become more costly for the principal in comparison with a similar agent with no memorable effects.

The second application is closer to the analysis of Hai et al. (2020) and introduces memorable consumption into a standard linear-quadratic consumption-savings problem. For the evolution of memories, we assume a version of our Markovian specification. In equilibrium, the solution exhibits two key features: (1) a higher sensitivity of consumption to income shocks in comparison with the standard models; and, (2) a negative dependence of the optimal consumption on the accumulated stock of memory, which highlights the potential relevance of memorable consumption in explaining some well-known puzzles about life-cycle dynamics of consumption and savings.

5.1 Risk taking in the Principal-Agent setting

Consider a setting in which a principal hires an agent (a manager) who has to decide whether to implement innovative but risky projects. Suppose that the relationship lasts a predetermined finite number of periods $T > 1$, after which the employment terminates (and the agent presumably obtains a new job for a different principal). In each period t, the agent either runs business as usual ($a_t = 0$) or implements a project ($a_t = 1$) that can be one of two possible types, P_g or P_b. If a project is implemented, its duration is one period and it produces an outcome $x_t \in \{-2, -1, 1, 2\}$ at the end of the period. The outcome x_t generates revenue px_t for the principal, where $p > 0$ is a model parameter, while business as usual generates zero revenue. Conditional on the choice of projects, the outcomes x_t are independent across time periods t. The types of a project, P_g or P_b, can be thought of as probability distributions from which the outcome is drawn, and we assume that P_g is a good type with a positive expected value, and P_b is a bad type with a negative expected value.

The agent derives utility from her wage payment as well as from the outcome of the project that she chooses. The latter component of the utility captures her intrinsic motivation and can be attributed to psychological feelings of job satisfaction, sense of fulfillment, competence, and pride. Moreover, we assume that these feelings can be memorable, and generate not only instantaneous but also long-lasting utility. To formally incorporate memorable effects into an otherwise standard framework, we assume that the "consumption space" $\mathcal{C}$ consists of triples (w, a, x), where w is the wage that the agent receives in the current period, a is the decision to implement a project and x is the outcome of the project undertaken in that period. As formalized shortly, the wage w will be the ordinary (not memorable) part of the bundle. We assume that the realized outcome x of a project generates memories that can be either positive, if $x > 0$, or negative, if $x < 0$. For the evolution of memorable utility, we assume a version of the peak-end rule discussed in Section 2.4. The memory component $M(a_{t-1}, x_{t-1}; a_{t-2}, x_{t-2}; \ldots; a_0, x_0; 0, 0; \ldots)$ of the total utility from a stream evaluated at time t will be computed according to the rules described below. First, we postulate that the memory component is positive if $x_{t-i} > 0$ for the first index i such that $a_{t-i} = 1$;[25] it is negative if $x_{t-i} < 0$ for the first index i such that $a_{t-i} = 1$; and it is zero otherwise. The value of the memory component in the positive case is computed as

$$M = \kappa\Big((1-\gamma)a_{t-1}x_{t-1} + \gamma \max\{a_{t-1}x_{t-1}, a_{t-2}x_{t-2}, \ldots, a_{t-l}x_{t-l}\}\Big), \tag{8}$$

where $(a_{t-1}, x_{t-1}; a_{t-2}, x_{t-2}; \ldots; a_{t-l+1}, x_{t-l+1}; 1, x_{t-l})$ is the longest sequence that does not contain negative outcomes, that is, $l \in \{1, \ldots, t\}$ is the largest number such that $a_{t-l} = 1$, $x_{t-l} > 0$, and $x_{t-i} > 0$ for all $i = 1, \ldots, l-1$ such that $a_{t-i} = 1$. In this specification, the agent's utility from memories is determined by the last outcome (with weight $1-\gamma$, where $\gamma \in (0, 1]$ is a preference parameter) and the best result (with weight γ) in the continuous spell of non-negative outcomes. The periods in which she did not implement a project (and was doing business as usual) are not memorable, as they do not generate any positive psychological feelings. This way of computing utility also implies that negative outcomes

[25] That is, if $a_{t-i} = 1$, $x_{t-i} > 0$, and $a_{t-j} = 0$ for all $0 < j < i$.

generate a sense of failure, so that, after a negative outcome, the agent needs to accumulate positive memories over again. Furthermore, the preference parameter $\kappa \geq 0$ in the formula captures the strength of memorable effects. In the negative case, the memory component of the total utility is computed symmetrically,

$$M = \kappa\big((1-\gamma)a_{t-1}x_{t-1} + \gamma \min\{a_{t-1}x_{t-1}, a_{t-2}x_{t-2}, \dots, a_{t-l}x_{t-l}\}\big), \tag{9}$$

where $(a_{t-1}, x_{t-1}; a_{t-2}, x_{t-2}; \dots; a_{t-l+1}, x_{t-l+1}; 1, x_{t-l})$ is again the longest sequence that does not contain positive outcomes. Similarly to the specification discussed in Section 2.4, the formula contains the min operator because the agent's memory is driven by the most extreme outcome, which is negative in this case.

For the purpose of computing and interpreting the agent's optimal strategies, it is useful to rewrite specification (8)–(9) of the peak-end rule *recursively* using an auxiliary variable

$$\mu_t = \begin{cases} \max\{x_t, \mu_{t-1}\} & \text{if } a_t x_t > 0, \\ \min\{x_t, \mu_{t-1}\} & \text{if } a_t x_t < 0, \\ \mu_{t-1} & \text{if } a_t = 0 \end{cases} \tag{10}$$

and letting

$$M(a_{t-1}, x_{t-1}; a_{t-2}, x_{t-2}; \dots; a_0, x_0; 0, 0; \dots) = \kappa\big((1-\gamma)x_{t-1} + \gamma\mu_{t-1}\big).$$

To make this specification fully consistent with (11) and (8)–(9), the initial values for the recursion should be set as $\mu_{-1} = 0$ and $x_{-1} = 0$.

The overall utility function that the agent uses at time $t \in \{0, 1, \dots, T-1\}$ to evaluate the stream $(w_t, a_t, x_t; w_{t+1}, a_{t+1}, x_{t+1}; \dots; w_T, a_T, x_T)$ is

$$V_t = \mathbb{E}_{t-1}\left[\sum_{\tau=t}^{T-1} \beta^{\tau-t}\big(u(w_t) + d\, a_\tau x_\tau + M(a_{\tau-1}, x_{\tau-1}; \dots; a_0, x_0; 0, 0; \dots)\big)\right], \tag{11}$$

where $\mathbb{E}_{t-1}$ denotes the expectation conditional on variables known at period $t-1$ (inclusive); $\beta \in (0,1)$ is a parameter of preferences that captures the discount factor; u is a strictly increasing and strictly concave utility function for money; and $d > 0$ is another preference parameter that reflects the magnitude of the instantaneous psychological utility from the project's outcome. Note that this model uses an extension of the framework that has been assumed throughout the paper: previously, we have considered a decision maker who makes a single ex ante decision, while here the agent makes decisions in multiple periods, $t = 0, 1, \dots, T - 1$. However, it can be verified that the utility function (11) makes the agent *dynamically consistent*: her sequential decisions are the same as those she would make if she were asked ex ante to form a plan of actions for every period t and contingent on any possible realization of past stochastic outcomes $x_0, \dots, x_{t-1}$.

Finally, as usual for incomplete contracts, we assume that the principal cannot offer payments conditional on the agent's action and is restricted to contracts in which the payment w_t in each period is a linear function of obervables, i.e., $w_t = A a_t x_t + B$. The principal chooses the parameters of the contract $A \geq 0$ and $B \in \mathbb{R}$ to maximize his expected profit

$$\mathbb{E}\left[\sum_{t=0}^{T-1} \beta^t (p\, a_t x_t - w_t)\right],$$

where $p > 0$ is a model parameter and $\beta \in (0, 1]$ is the discount factor (that is the same as for the agent) subject to the agent's participation constraint $V_0 \geq \underline{U}$, where $\underline{U}$ is the agent's reservation utility. To make the problem interesting, we assume that the reservation utility is such that the principal can attract an agent who always implements P_g, and employing an agent who always runs business as usual generates negative profit.[26] As usual, to guarantee the existence of solutions, we also assume that if the agent is indifferent between actions, she chooses the one that is the most beneficial for the principal.

The solution of this model in the absence of memorability ($\kappa = 0$) is very simple.

Proposition 5. *If $\kappa = 0$ and $d > 0$, then the optimal contract for the principal is to set $A = 0$ and choose B so that the participation constraint binds. Under this contract, the agent always chooses the efficient-type project, P_g.*

The above result is intuitive: since the agent's utility contains the term dx_t, it follows that, even with a flat payment scheme, she has non-monetary incentives to choose the project with the highest expected value and, hence, the interests of the agent and the principal are perfectly aligned with respect to the choice of the project.

The situation changes drastically if the agent is affected by the memorability of the outcomes of the selected project. Memories from good outcomes affect the agent's utility over

[26] Formally, we assume that $\frac{1-\beta^T}{1-\beta} u(pe_g) > \underline{U} > \frac{1-\beta^T}{1-\beta} u(0)$, where e_g is the expected value of the outcome, $\mathbb{E}[x_t]$, when a P_g-type project is implemented.

multiple periods; thus, memorability increases the enjoyability of good outcomes and seemingly increases the power of incentives to implement good projects. However, the real effect of memorability on incentives is more complicated because the utility boosts have a non-trivial *temporal* structure. When the memorability of the outcomes is taken into account, the agent has additional concerns about the effects of her choices on the accumulated stock of memories and about the time horizon over which her current memories will continue to generate utility flows.

The following proposition illustrates these effects.

Proposition 6. *Let $T = 4$. There exist parameters of the model — preference parameters, payoffs ($p > 0$), and distributions of outcomes P_g and P_b — such that the optimal contract has $A > 0$ and the agent's strategy has the following features:*

1. *At $t = 3$, the agent always chooses an efficient P_g project.*
2. *At $t \in \{1, 2\}$, if $\mu_{t-1} = -1$ and $x_{t-1} = -1$ then the agent chooses an efficient P_g project.*
3. *At $t \in \{1, 2\}$, if $\mu_{t-1} = 2$ and $x_{t-1} = 2$ then the agent chooses $a_t = 0$.*
4. *At $t = 2$, if $\mu_{t-1} = -2$ and $x_{t-1} = -2$ then the agent chooses a risky inefficient P_b project.*
5. *At $t = 1$, if $\mu_{t-1} = -2$ and $x_{t-1} = -2$ then the agent chooses an efficient P_g project.*

The main point of the proposition is that, as a part of her optimal strategy, the agent may choose inefficient actions: her choices are affected by the realized history of outcomes as well as the calendar time. From the principal's perspective, the optimal contract for an agent with memorable effects may need to provide extrinsic motivation in the form of $A > 0$ on top of the intrinsic motivation to implement good projects that comes from the $d\,x_t$ term.

More specifically, the above proposition illustrates that there are two situations in which the agent may choose an inefficient project.

First, if she has very good memories generated by past outcomes, she may value those memories sufficiently high so that, in order to preserve them, she stops to take risks; she prefers to choose the safe option instead of a positive expected-value but risky project. Conventional wisdom and some empirical evidence suggest that success frequently leads to increased risk-taking behavior,[27] and, for that, there is a clear behavioral channel — disproportional increase in beliefs in one's abilities. In our model, memorability has a countervailing effect and makes the agent more conservative.

Second, if the agent is hit by a negative outcome, she may subsequently choose a negative expected-value project if it gives her higher chances of recovery.[28] Such a risk-seeking behavior following prior losses is not novel: the behavioral literature traditionally rationalizes it using Prospect Theory (see, e.g., Kahneman and Tversky, 1979, p. 287). In the finance literature, observably similar behavior is known as "gambling for resurrection." Here, we see that there exists another channel for risk-seeking, namely, memorability. We also note that our present model treats positive and negative memories symmetrically. In a recent experimental paper, Gödker, Jiao, and Smeets (2021) find that investors tend to distort memories of investment outcomes in the direction of excessive optimism. Asymmetries and biases in memories can further distort individuals' behavior in terms of the risks they take and the efficiency of their choices.

Finally, we observe that the agent's optimal choice of the project may also depend on the time period. Indeed, in the state $\mu_{t-1} = -2$ and $x_{t-1} = -2$, the agent may choose a P_g project at $t = 1$, but a P_b project at $t = 2$. The intuition for this behavior is that, in earlier time periods, there are more opportunities for the agent to gain positive memory by natural forces, without sacrificing the direct utility dx_t from projects' outcomes.

To sum up, we have shown that the memorability of outcomes creates nontrivial intertemporal tradeoffs and may lead to the agent choosing inefficient actions and behaving either too conservatively or taking excessive risks. In turn, these intertemporal tradeoffs affect the optimal incentive scheme that the principal should offer to the agent.

5.2 Consumption-savings decisions with memorable effects

Suppose that, in periods $t = 0, 1, 2 \ldots$, a consumer receives income y_t that is stochastic and i.i.d. across time. There are no borrowing constraints, hence she can reallocate income between periods by borrowing or saving at the gross interest rate $R > 0$. The time horizon

[27] For instance, Thaler and Johnson (1990) and Malmendier and Nagel (2011) support the idea that prior gains (resp., losses) make individuals more (resp., less) willing to take risks.

[28] For that, it is needed that $\Pr(x_t > 0)$ is greater under P_b than under P_g despite the lower expected value of x_t under P_b.

is infinite, and the future is discounted using discount factor $\beta \in (0,1)$. For simplicity, we assume that there is only one good ($N = 1$). Utility from physical consumption is given by $u(c) = c - \frac{1}{2}c^2$; utility from consuming memories conforms to a convenient special case of (7), where the memory stock follows an $AR(1)$-type law given by $\psi(m,c) = v\big(\alpha v^{-1}(m) + (1-\alpha)c\big)$, $m = v(\tilde{m})$, and $v(\tilde{m}) = b\tilde{m} - \frac{1}{2}a\tilde{m}^2$ with $a, b > 0$.[29]

Thus, the consumer faces the following maximization problem:

$$\begin{aligned}
&\underset{\{c_t\}_{t=0}^{\infty},\{s_t\}_{t=0}^{\infty},\{\tilde{m}_t\}_{t=0}^{\infty}\text{ adapted}}{\text{maximize}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(c_t - \frac{1}{2}c_t^2 + b\tilde{m}_{t-1} - \frac{1}{2}a\tilde{m}_{t-1}^2\right)\right] \qquad (12)\\
&\text{s.t. } c_t + s_t = y_t + Rs_{t-1} \qquad \text{for } t = 0,1,\ldots,\\
&\qquad \tilde{m}_t = \alpha\tilde{m}_{t-1} + (1-\alpha)c_t \quad \text{for } t = 0,1,\ldots,\\
&\qquad \tilde{m}_{-1} = 0,\\
&\qquad s_{-1} \text{ is given.}
\end{aligned}$$

Finally, assume that $R = \frac{1}{\beta}$, $s_{-1} \geq 0$, and $\mathbb{E}[y] > 0$.

Our goal here is to see how memorable effects change the standard Permanent-Income-type solution of the model.[30]

The Lagrangian of the problem is

$$\mathcal{L} = \mathbb{E}_0\Bigg[\sum_{t=0}^{\infty}\beta^t\Bigg(c_t - \frac{1}{2}c_t^2 + b\tilde{m}_{t-1} - \frac{1}{2}a\tilde{m}_{t-1}^2 - \lambda_t\left(c_t + s_t - y_t - \frac{1}{\beta}s_{t-1}\right) - \mu_t(\tilde{m}_t - \alpha\tilde{m}_{t-1} - (1-\alpha)c_t)\Bigg)\Bigg]$$

and the First-Order Conditions become

$$\begin{cases}
1 - c_t - \lambda_t + (1-\alpha)\mu_t = 0\\
-\lambda_t + \mathbb{E}_t[\lambda_{t+1}] = 0\\
\beta(b - a\tilde{m}_t) - \mu_t + \beta\alpha\mathbb{E}_t[\mu_{t+1}] = 0.
\end{cases}$$

By combining this system with the constraints, we eventually obtain the following solution:

$$c_t = \left((1-\beta)y_t + \beta\mathbb{E}[y] + \frac{1-\beta}{\beta}s_{t-1}\right)(1+\kappa) - \kappa\tilde{m}_{t-1}, \qquad (13)$$

[29] With this change of variable, the stock $\tilde{m}_t$ is measured in different "units" in comparison with (7), leading to a convenient specification for applications: $AR(1)$-type law for memory and non-linear utility. Note also that, in order to make a net comparison with the standard linear-quadratic consumption-saving model, we let u and ψ be unbounded in this application.

[30] We ignore the usual issues related to non-monotonicity of the utility from consumption and the exact structure of conditions at infinity.

where $\kappa \geq 0$ is a constant given by

$$\kappa = \frac{\sqrt{1-2\beta(\alpha^2 - a(1-\alpha)^2) + \beta^2(\alpha^2 + a(1-\alpha)^2)^2} - (1-\beta(\alpha^2 - a(1-\alpha)^2))}{2\alpha(1-\beta\alpha)}.$$ [31]

Expression (13) has a familiar intuition. If the effect of memorability is absent ($a = 0$ or $\alpha = 1$), then $\kappa = 0$ and we recover Hall's (1978) classic result that "consumption follows a random walk." In this case, the agent consumes the sum of the fraction $(1-\beta)$ of the income shock $y_t - \mathbb{E}[y]$, the average income $\mathbb{E}[y]$, and the interest from savings $\frac{1-\beta}{\beta}s_{t-1}$; the fraction β of her income shock and the body of the savings are kept as savings.

By contrast to the standard preferences, in the presence of memorability ($a > 0$ and $0 < \alpha < 1$), the agent exhibits a stronger reaction to income shocks and consumes more out of them ($\kappa > 0$). Albeit framed within a simplified setting, our finding suggests, similarly to Hai et al. (2020), that memorable consumption may help explain the well-known empirical evidence on excess sensitivity of consumption to income changes.[32] Note that Hai et al. (2020) make this observation on the basis of simulations in their calibrated model. We follow a purely theoretical path of including memorable effects into the textbook linear-quadratic consumption-savings model. Then, as can be seen from our closed-form solution, the excess sensitivity holds under a very wide range of parameters of the model. Furthermore, in comparison with Hai et al. (2020), we employ different specifications for the accumulation of memories — ours is Markovian whereas theirs does not satisfy the Markovian property but has adaptation features. Our result is, hence, complementary to theirs, and can be viewed as providing further support for the existence of a general channel through which memorability affects the sensitivity of consumption to income shocks.

Expression (13) also contributes to distinguishing the effects of memorability on consumption decisions from those produced by habit formation. Indeed, the optimal level of consumption is negatively correlated with the stock of memory — it increases as the stock of memory decreases, and vice versa. Such a negative relationship indicates that material consumption (of memorable goods) and consumption of memories behave as substitutes. This distinguishing feature contrasts typical patterns observed in habit formation models

[31] This consumption rule is supported by μ_t that depends on state variables also linearly, $\mu_t = \left((1-\beta)y_t + \beta\mathbb{E}[y] + \frac{1-\beta}{\beta}s_{t-1} - \tilde{m}_{t-1}\right)\kappa' + \frac{\beta}{1-\beta\alpha}(b - a\tilde{m}_{t-1})$, with a suitably chosen constant κ'.

[32] See, e.g., the surveys of Attanasio (1999) and Jappelli and Pistaferri (2010).

where the habit stock and the consumption level move in a complementary way, reinforcing each other.

To further study the properties of the solution, assume for a moment that there is no income uncertainty and $y_t = \mathbb{E}[y]$ for all t. Then, the consumption rule can be rewritten as $c_t = \bar{c}_t + \kappa(\bar{c}_t - \tilde{m}_{t-1})$, where $\bar{c}_t = \mathbb{E}[y] + \frac{1-\beta}{\beta} s_{t-1}$. In the standard linear-quadratic consumption-savings model, the expression for $\bar{c}_t$ corresponds to the permanent income. In our model, it becomes a reference that determines the level of consumption, taking into account the accumulation of memory. If $\tilde{m}_{t-1} = \bar{c}_t$, then the agent is in a steady state, and both her consumption and the stock of memory will stay constant; if $\tilde{m}_{t-1}$ exceeds $\bar{c}_t$, then she will consume less than $\bar{c}_t$ and opt for depleting part of her stock of memory; and, if $\tilde{m}_{t-1}$ has not reached $\bar{c}_t$, then she will consume more than $\bar{c}_t$ in order to build up her stock of memory. From the viewpoint of life-cycle profiles, these dynamics imply that agents tend to under-save and over-consume when they are young (as they start with $\tilde{m}_{-1} = 0 < \bar{c}_0$). As the stock of memory accumulates in subsequent periods, the gap will reduce and over-consumption will attenuate. If we compare consumption paths across agents, then those with higher κ over-consume more at young age, save less, and approach the steady state with lower savings. This behavior is rational, and can be interpreted as hidden savings in the form of investment in pleasant memories that substitutes for investment in financial assets. Furthermore, these dynamics may represent one key source of support for the empirical evidence according to which individuals consume too little at retirement age compared to the predictions of the canonical model.

The magnitude of the agent's excessive reaction to income shocks (relative to predictions of the standard model), as well as features of the life-cycle consumption pattern such as over-consumption when young, depend on the parameters of preferences through the value of κ. Holding everything else fixed, κ is an increasing function of the parameter a that, jointly with b, captures the strength of memorable effects of consumption. Hence, stronger memorable effects lead to greater over-consumption at young ages, as well as stronger reactions to income shocks. The longevity of memory is captured by the parameter α, and the strength of its effect has the inverse U-shaped form.

To sum, we have shown that the introduction of memory effects in the simplest consumption-savings model maintains the general random-walk structure of the solution; at the same time, it helps to explain the excess sensitivity puzzle and generates time profiles in which young agents under-save and over-consume, and old agents do the opposite. Moreover, memory effects are distinct (and, in a sense, opposite) to habit effects.

6 Conclusions

We present a model of memorable consumption in a dynamic setup. The model allows us to elicit from observables whether consumption is memorable or not for the agent. Due to the uniqueness properties, we can distinguish between the utility derived from material consumption and the utility derived from the "consumption of memories;" moreover, those utilities can be rightfully attributed to time periods. Our axioms consist of testable properties that distinguish memorable effects from other types of history-dependent behavior that violate time separability.

We show that our model can encompass various psychological evidence on intensity, duration, and interruption of experiences. In particular, we can accommodate the well-known peak-end rule. While the latter is a time-dependent, non-Markovian specification, we can also characterize the case of Markovian evolution of memory which should be particularly useful for macro and other dynamic applications. The Markovian setting brings to the fore two independent channels through which agents can be compared — according to the strength of memorable effects and to the longevity of their memory. Finally, we apply our model to a principal-agent problem and to a macroeconomic setting and show that memorable effects may have powerful implications on classic issues therein.

Appendix

A Proof of the basic representation

Lemma 7. *Suppose that $\succsim$ is a preference relation on $\mathcal{L}$ that satisfies the Memory-Consumption Tradeoff Consistency and Continuity axioms. Then, for any $t \in \mathbb{N}$, $f, g \in \mathcal{F}_t$, and $p, q \in \mathcal{L}_1$, we have*

$$f|p \succsim g|p \quad \Leftrightarrow \quad f|q \succsim g|q.$$

Proof. First, we claim that, for any $n \in \mathbb{N}$, and for any $t \in \mathbb{N}$, $f, g \in \mathcal{F}_t$, and $p, q \in \mathcal{L}_1$,

$$f|p \succsim g|(\tfrac{n-1}{n}p + \tfrac{1}{n}q) \quad \Leftrightarrow \quad f|(\tfrac{1}{n}p + \tfrac{n-1}{n}q) \succsim g|q. \tag{14}$$

Indeed, for $n = 1$, this statement is a triviality. Suppose that it holds for some $n \in \mathbb{N}$, and that $f|p \succsim g|(\frac{n}{n+1}p + \frac{1}{n+1}q)$ for some $t \in \mathbb{N}$, $f, g \in \mathcal{F}_t$, and $p, q \in \mathcal{L}_1$. Let $q' := \frac{1}{n+1}p + \frac{n}{n+1}q$. Note that $\frac{n-1}{n}p + \frac{1}{n}q' = \frac{n}{n+1}p + \frac{1}{n+1}q$ and, hence, it follows from assumptions that $f|(\frac{1}{n}p + \frac{n-1}{n}q') \succsim g|q'$. Now, observe that q' is the midpoint between $\frac{1}{n}p + \frac{n-1}{n}q'$ and q. Therefore, by Memory-Consumption Tradeoff Consistency, $f|q' \succsim f|q$, which completes the inductive step.

Now, the claim of the lemma follows from (14) by taking the limit $n \to \infty$. Indeed, fix arbitrary $t \in \mathbb{N}$, $f, g \in \mathcal{F}_t$, and $p, q \in \mathcal{L}_1$. If $f|p \succ g|p$ then, by continuity, for all sufficiently large n, we have $f|p \succ g|(\frac{n-1}{n}p + \frac{1}{n}q)$, which gives $f|(\frac{1}{n}p + \frac{n-1}{n}q) \succ g|q$ by the previous step and, in the limit as $n \to \infty$, $f|q \succsim g|q$. If $g|p \succ f|p$, then the claim similarly holds. By the symmetry of the claim with respect to renaming p and q, the only remaining case is $f|p \sim g|p$ and $f|q \sim g|q$, in which the claimed equivalence holds, as well. □

Lemma 8. *Let X be a connected separable topological space, Y a convex subset of a separable topological vector space, and $\succcurlyeq$ a continuous complete preorder on $X \times Y$ that has the following properties:*

(i) There exist $x, x', x_0 \in X$ and $y, y', y_0 \in Y$ such that $(x, y_0) \succ (x', y_0)$ and $(x_0, y) \succ (x_0, y')$.

(ii) For all $x, x' \in X$ and $y, y' \in Y$, $(x, y) \succcurlyeq (x', y) \Rightarrow (x, y') \succcurlyeq (x', y')$.

(iii) For all $x, x' \in X$ and $y, y' \in Y$, $(x, y) \succcurlyeq (x, y') \Rightarrow (x', y) \succcurlyeq (x', y')$.

(iv) For all $x, x' \in X$ and $y, y' \in Y$, $(x, y) \succcurlyeq (x', \frac{1}{2}y + \frac{1}{2}y') \Leftrightarrow (x, \frac{1}{2}y + \frac{1}{2}y') \succcurlyeq (x', y')$.

Then, there exist a continuous function $U_x : X \to \mathbb{R}$ and a continuous affine function $U_y : Y \to \mathbb{R}$ such that

$$(x, y) \succcurlyeq (x', y') \quad \Leftrightarrow \quad U_x(x) + U_y(y) \geq U_x(x') + U_y(y').$$

Proof. To verify the conditions of Wakker (1989, Th. III.4.1), observe that the assumptions of the lemma immediately guarantee the existence of two essential coordinates and that the coordinate independence property is satisfied.

It remains to show that the hexagon condition holds. Indeed, suppose that $a, b, c \in X$ and $u, v, w \in Y$ are such that $(b, u) \asymp (a, v)$[33] and $(c, u) \asymp (b, v) \asymp (a, w)$. Our goal is to show

[33]Here, we use $\asymp$ to denote the symmetric part of $\succcurlyeq$.

that $(c,v) \asymp (b,w)$. Let $r = \frac{1}{2}u + \frac{1}{2}w$. We claim that

$$(a,r) \asymp (a,v). \tag{15}$$

If $(b,u) \succ (a,r)$ then, on the one hand, $(a,v) \succ (a,r)$. On the other hand, $(b,u) \succ (a,r)$ implies by (iv) that $(b,r) \succ (a,w)$, which means that $(b,r) \succ (b,v)$. We obtained a contradiction with (iii). The situation $(a,r) \succ (b,u)$ similarly leads to a contradiction. We conclude that $(a,r) \asymp (b,u) \asymp (a,v)$. Then, observe that $(c,u) \asymp (b,v) \asymp (b,r)$, where the first part holds by assumption and the second follows from (15) and (iii). Then, $(c,r) \asymp (b,w)$ by (iv). As follows from (15) and (iii), we also have $(c,r) \asymp (c,v)$. The desired relationship follows by transitivity.

Now, we can apply Wakker (1989, Th. III.4.1) to obtain that there exist nonconstant continuous functions $U_x : X \to \mathbb{R}$ and $U_y : Y \to \mathbb{R}$ such that

$$(x,y) \succcurlyeq (x',y') \quad \Leftrightarrow \quad U_x(x) + U_y(y) \geq U_x(x') + U_y(y'). \tag{16}$$

It remains to show that U_y must be affine. Indeed, (16) and property (iv) imply that, for any $y, y' \in Y$,

$$U_y(y) - U_y(\tfrac{1}{2}y + \tfrac{1}{2}y') \geq U_x(x') - U_x(x) \Leftrightarrow U_y(\tfrac{1}{2}y + \tfrac{1}{2}y') - U_y(y') \geq U_x(x') - U_x(x) \quad \text{for all } x, x' \in X. \tag{17}$$

Fix an arbitrary $[a,b] \subseteq range\, U_x$, where $a < b$, and let $\varepsilon \in (0, b-a)$. Then, the arbitrariness of x and x' in (17) gives that, for any $y, y' \in Y$ such that $|U_y(y) - U_y(y')| \leq \varepsilon$,

$$U_y(y) - U_y(\tfrac{1}{2}y + \tfrac{1}{2}y') = U_y(\tfrac{1}{2}y + \tfrac{1}{2}y') - U_y(y').$$

Applying it repeatedly, this equation can be extended to all $y, y' \in Y$. Moreover, it can be rewritten as $U_y(\frac{1}{2}y + \frac{1}{2}y') = \frac{1}{2}U_y(y) + \frac{1}{2}U_y(y')$. By continuity, it implies that U_y is affine. ☐

Proof of Theorem 1. *Only if* part. Suppose that $\succsim$ is a complete preorder on $\mathcal{L}$ that satisfies Axioms (A1)–(A7). Throughout the proof, we will write z_t for $t \in \mathbb{N}$ to denote an element of $\mathcal{F}_t$ such that $z_t = (0, 0, \ldots, 0)$.

Step 1. On the subset $\mathcal{L}_1 \subset \mathcal{L}$, $\succsim$ admits an expected utility representation: there exists a continuous and bounded function $u : \mathcal{C} \to \mathbb{R}$ such that $p \succsim q \Leftrightarrow \mathbb{E}_p[u] \geq \mathbb{E}_q[u]$ for all

$p, q \in \mathcal{L}_1$. Note that u is continuous and bounded because $\mathcal{L}_1$ is endowed with the weak* topology and $\succsim$ on $\mathcal{L}_1$ satisfies the Closed Continuity axiom (Part (i) of Axiom A6).[34] Let u be normalized such that $u(0) = 0$. Moreover, Nondegeneracy directly implies that the range of u admits both positive and negative values.

Step 2. Independence, Part (ii) of Continuity, and Nondegeneracy ensure that the conditions of the mixture space theorem (Herstein and Milnor, 1953) are satisfied and, therefore, there exists an affine function $V : \mathcal{L} \to \mathbb{R}$ that represents $\succsim$ on $\mathcal{L}$: $P \succsim Q \Leftrightarrow V(P) \geq V(Q)$ for all $P, Q \in \mathcal{L}$. By the uniqueness of the expected utility representation on $\mathcal{L}_1$, it must be that the restriction of V to $\mathcal{L}_1$ is a positive affine transformation of the mapping $p \mapsto \mathbb{E}_p[u]$ for $p \in \mathcal{L}_1$. Normalizing if necessary, assume that $V(p) = \mathbb{E}_p[u]$ for all $p \in \mathcal{L}_1$. Note that, by the continuity axiom, V must be continuous when restricted to convex sets $\mathcal{L}_t$ for all $t \in \mathbb{N}$.

Step 3. Risk Preference Consistency and Stationarity imply that, for all $f, g \in \mathcal{F}$ and $p, q \in \mathcal{L}_1$,

$$f|p \succsim f|q \quad \Leftrightarrow \quad g|p \succsim g|q \quad \Leftrightarrow \quad p \succsim q.$$

Hence, by the uniqueness of the expected utility representation, it must be that for all $t \in \mathbb{N}$, there exist $\alpha_t : \mathcal{F}_t \to \mathbb{R}$ and $\beta_t : \mathcal{F}_t \to \mathbb{R}_{++}$ such that

$$V(f|p) = \alpha_t(f) + \beta_t(f)\mathbb{E}_p[u] \qquad \text{for all } f \in \mathcal{F}_t \text{ and } p \in \mathcal{L}_1.$$

Step 4. This step establishes an alternative representation for $\succsim$ restricted to $\mathcal{F}_t \times \mathcal{L}_1$ for all $t \in \mathbb{N}$: we claim that there exist continuous functions $W_t : \mathcal{F}_t \to \mathbb{R}$ such that

$$f|p \succsim g|q \quad \Leftrightarrow \quad W_t(f) + \mathbb{E}_p[u] \geq W_t(g) + \mathbb{E}_q[u]$$

for all $f, g \in \mathcal{F}_t$ and $p, q \in \mathcal{L}_1$.

If, for some $t \in \mathbb{N}$, we have $f|p \sim z_t|p$ for all $f \in \mathcal{F}_t$ and $p \in \mathcal{L}_1$, then, as follows from Stationarity, we can let $W_t(f) = 0$ for all $f \in \mathcal{F}_t$.

Otherwise, we obtain the claim by Lemma 8: Assumption (iv) holds due to Memory-Consumption Tradeoff Consistency, (iii) by Risk Preference Consistency, (ii) by Lemma 7, and (i) with respect to the second coordinate holds by Nondegeneracy and Stationarity. Therefore, there exist continuous $W_t : \mathcal{F}_t \to \mathbb{R}$ and continuous affine $W'_t : \mathcal{L}_1 \to \mathbb{R}$ such

[34] This result can be found, e.g., is Kreps, 1988, Theorem 5.21.

that $f|p \succsim g|q \Leftrightarrow W_t(f) + W'_t(p) \geq W_t(g) + W'_t(q)$ for all $f, g \in \mathcal{F}_t$ and $p, q \in \mathcal{L}_1$. Then, as follows from Risk Preference Consistency, $p \succsim q \Leftrightarrow W'_t(p) \geq W'_t(q)$ for all $p, q \in \mathcal{L}_1$. Hence, by the uniqueness of the expected utility representation, it must be that, for all $t \in \mathbb{N}$, W'_t are positive affine transformations of our representation of $\succsim$ restricted to $\mathcal{L}_1$ obtained in Step 1. Then, normalizing if necessary, we can assume that, for all $t \in \mathbb{N}$, $W'_t(p) = \mathbb{E}_p[u]$ for all $p \in \mathcal{L}_1$.

Step 5. For all $t \in \mathbb{N}$, the range of the mapping $(f, p) \mapsto W_t(f) + \mathbb{E}_p[u]$ is convex, and, therefore, by the uniqueness of ordinal representations, there must exist continuous and strictly increasing functions $\zeta_t : \mathbb{R} \to \mathbb{R}$ such that $V(f|p) = \alpha_t(f) + \beta_t(f)\mathbb{E}_p[u] = \zeta_t(W_t(f) + \mathbb{E}_p[u])$ for all $f \in \mathcal{F}_t$ and $p \in \mathcal{L}_1$. Observe that, for any $t \in \mathbb{N}$ and any fixed $f \in \mathcal{F}_t$, the left-hand side of this equality is an affine function of $p \in \mathcal{L}_1$. Hence, $\zeta_t(\cdot)$ must be positive affine functions for all $t \in \mathbb{N}$: $\zeta_t(x) = A_t + B_t x$ for some $A_t \in \mathbb{R}$ and $B_t \in \mathbb{R}_{++}$. If we let $\tilde{W}_t(f) := A_t + B_t W_t(f)$ for all $t \in \mathbb{N}$ and $f \in \mathcal{F}_t$, we obtain:

$$V(f|p) = \tilde{W}_t(f) + B_t \mathbb{E}_p[u] \qquad \text{for all } t \in \mathbb{N},\ f \in \mathcal{F}_t, \text{ and } p \in \mathcal{L}_1. \tag{18}$$

Step 6. Since any $c \in \mathcal{C}$ can be expressed as a degenerate lottery $\delta_c \in \mathcal{L}_1$, the Impatience axiom directly implies that $(c) \succ (0, c) \succ (0)$ for all $c \in \mathcal{C}$ such that $(c) \succ (0)$. By taking the limit $c \to 0$ in the above and using continuity, we obtain $(0, 0) \sim (0)$. Using Stationarity and mathematical induction, it can be seen that $z_t \sim (0)$ and $V(z_t) = 0$; in turn, by (18), we also have $\tilde{W}_t(z_t) = 0$.

Let $\beta := B_1$ and note that Impatience implies that $\beta < 1$. For any $c \in \mathcal{C}$, let $p_c \in \mathcal{L}_1$ be defined as $p_c := \beta\delta_c + (1-\beta)\delta_0$, and observe that $V(0, c) = \beta u(c) = \mathbb{E}_{p_c}[u] = V(p_c)$, where the first equality holds by (18) and the last equality by construction of V in Step 2. For any $t \in \mathbb{N}$, Stationarity gives that $z_t|0|c \sim z_t|p_c$ and, hence, by (18), $B_{t+1}u(c) = B_t \mathbb{E}_{p_c}[u] = B_t \beta u(c)$. Since c was arbitrarily chosen, we have that $B_t = \beta^t$ for all $t \in \mathbb{N}$ and

$$V(f|p) = \tilde{W}_t(f) + \beta^t \mathbb{E}_p[u] \qquad \text{for all } t \in \mathbb{N},\ f \in \mathcal{F}_t, \text{ and } p \in \mathcal{L}_1. \tag{19}$$

This equation holds also for $t = 0$ by letting $\tilde{W}_0 := 0$.

Step 7. Let $M_0 : \mathcal{F}_0 \to \mathbb{R}$ be zero and $M_t : \mathcal{F}_t \to \mathbb{R}$ for $t \in \mathbb{N}$ be defined as

$$M_t(f_{t-1}, \ldots, f_0) := \beta^{-t}(\tilde{W}_t(f_0, \ldots, f_{t-1}) - V(f_0, \ldots, f_{t-1})). \tag{20}$$

Using this definition in (19), we obtain that, for all $t \in \mathbb{N}$ and $f \in \mathcal{F}_{t+1}$,

$$V(f_0, \ldots, f_{t-1}, f_t) = V(f_0, \ldots, f_{t-1}) + \beta^t M_t(f_{t-1}, \ldots, f_0) + \beta^t u(f_t).$$

Then, for all $t \in \mathbb{N} \cup \{0\}$,

$$V(f_0, \ldots, f_t) = \sum_{\tau=0}^{t} \beta^\tau \big[u(f_\tau) + M_\tau(f_{\tau-1}, \ldots, f_0)\big] \qquad \text{for all } f \in \mathcal{F}_{t+1}.$$

Step 8. We claim that, for all $t \in \mathbb{N}$ and $P \in \mathcal{L}$, $V(z_t|P) = \beta^t V(P)$. First, recall that it was shown in Step 6 that $V(z_t|0) = 0$ for all $t \in \mathbb{N}$. Now, fix $t \in \mathbb{N}$, and observe that, by Stationarity, both $P \mapsto V(P)$ and $P \mapsto V((0)|P)$ are representations of the restriction of $\succsim$ to $\mathcal{L}$. Hence, by uniqueness of affine representations, there exists $b > 0$ such that $V((0)|P) = bV(P)$ for all $P \in \mathcal{L}$. As follows from (19), it must be that $b = \beta$. The claim now follows by induction. Note that, by (19), we also have $\tilde{W}_{\ell(f)+t}(z_t|f) = \beta^t \tilde{W}_{\ell(f)}(f)$ for all $t \in \mathbb{N}$ and $f \in \mathcal{F}$.

Step 9. Now, we can define $M : \mathcal{C}_0^\infty \to \mathbb{R}$ for all $h \in C_0^\infty$ by letting $M(h) = M_l(h)$ for an arbitrary $l \in \mathbb{N}$ such that $h_\tau = 0$ for all $\tau \geq l$. (Note that, by the result of the previous step, this definition does not depend on the choice of l.) Then, for all $t \in \mathbb{N} \cup \{0\}$,

$$V(f_0, \ldots, f_t) = \sum_{\tau=0}^{t} \beta^\tau \big[u(f_\tau) + M(f_{\tau-1}, \ldots, f_0, 0, 0, \ldots)\big] \qquad \text{for all } f \in \mathcal{F}_{t+1}.$$

Step 10. Observe that M is continuous in the specified topology: for any $t \in \mathbb{N}$, $M(f)$ coincides with $M_t(f)$ for all f that are zero starting from time t. Functions M_t for $t \in \mathbb{N}$ are defined through V and W_t that are continuous (Steps 2 and 4).

Furthermore, M is finite-horizon-bounded: for that, it is sufficient to show that M_t for all $t \in \mathbb{N}$ are bounded. Indeed, note that we can rewrite (20) in Step 7 as $M_t(f_{t-1}, \ldots, f_0) = \beta^{-t}(V(f_0, \ldots, f_{t-1}, 0) - V(f_0, \ldots, f_{t-1}))$ for all $f \in \mathcal{F}_t$. Then, the function V restricted to $\mathcal{F}_t$ is bounded by the von Neumann-Morgenstern expected utility theorem because $\succsim$ restricted to $\mathcal{L}_t$ is a continuous preference relation. Representation (3) is now proven.

If part. Suppose that $\succsim$ admits a utility representation via a function V, as specified in (3). We next show that the axioms hold.

Stationarity. Let $0 \in \mathcal{C}$ denote an element that is mapped by u into $0 \in \mathbb{R}$. Then, equation (3) gives

$$V((0)|P) = \sum_{f \in \operatorname{supp} P} P(f) \left\{ u(0) + \sum_{t=1}^{\ell(f)} \beta^t \big[u(f_{t-1}) + M(f_{t-2}, \dots, f_0, 0, 0, \dots) \big] \right\} =$$

$$u(0) + \beta V(P) = \beta V(P)$$

for all $P \in \mathcal{L}$, which implies that $(0)|P \succsim (0)|Q \Leftrightarrow P \succsim Q$ for all $P, Q \in \mathcal{L}$.

Impatience. If $P \succ (0)$ for some $P \in \mathcal{L}$, then, by (3), $V(P) > 0$ and, hence, $V(P) > \beta V(P)$. As was proven above, $\beta V(P) = V((0)|P)$, so $V(P) > V((0)|P)$ and $V((0)|P) > 0$.

Independence. Follows immediately from the linear structure of the representation.

Risk Preference Consistency. For any $f, g \in \mathcal{F}$ and $p, q \in \mathcal{L}_1$, we have by (3) that $f|p \succsim f|q \Leftrightarrow \mathbb{E}_p[u] \geq \mathbb{E}_q[u]$ because the other terms in the inequality $V(f|p) \geq V(f|q)$ cancel out; in turn, $\mathbb{E}_p[u] \geq \mathbb{E}_q[u] \Leftrightarrow g|p \succsim g|q$. For $\alpha \neq 1$, the implication in the statement of the axiom follows from Independence.

Memory-Consumption Tradeoff Consistency. Define $S : \mathcal{F}_t \to \mathbb{R}$ as

$$S(f) := \sum_{\tau=0}^{t-1} \beta^\tau \big[u(f_\tau) + M(f_{\tau-1}, \dots, f_0, 0, 0, \dots) \big] + \beta^t M(f_{t-1}, \dots, f_0, 0, 0, \dots).$$

Then, for any $t \in \mathbb{N}$, $f, g \in \mathcal{F}_t$, and $p, q \in \mathcal{L}_1$, we have by (3) that

$$\begin{aligned} f|p \succsim g|(\tfrac{1}{2}p + \tfrac{1}{2}q) \quad &\Leftrightarrow \quad S(f) + \beta^t \mathbb{E}_p[u] \geq S(g) + \beta^t \left(\tfrac{1}{2}\mathbb{E}_p[u] + \tfrac{1}{2}\mathbb{E}_q[u] \right) \quad \Leftrightarrow \\ & S(f) - S(g) \geq \beta^t \left(\tfrac{1}{2}\mathbb{E}_q[u] - \tfrac{1}{2}\mathbb{E}_p[u] \right) \quad \Leftrightarrow \\ S(f) + \beta^t \left(\tfrac{1}{2}\mathbb{E}_p[u] + \tfrac{1}{2}\mathbb{E}_q[u] \right) &\geq S(g) + \beta^t \mathbb{E}_q[u] \quad \Leftrightarrow \quad f|(\tfrac{1}{2}p + \tfrac{1}{2}q) \succsim g|q. \end{aligned}$$

For $\alpha \neq 1$, the equivalence in the statement of the axiom follows from Independence.

Continuity. For each $t \in \mathbb{N}$, the mapping $\mathcal{F}_t \to \mathbb{R}$ defined as $f \mapsto \sum_{\tau=0}^{t-1} \beta^\tau \big[u(f_\tau) + M(f_{\tau-1}, \dots, f_0, 0, 0, \dots) \big]$ is continuous and bounded by the corresponding properties of u and M. Hence, when restricted to $\mathcal{L}_t$, V defined by (3) is continuous in the weak* topology, which establishes the first part of the axiom. The second part follows immediately from the expected utility structure of V.

Nondegeneracy. The property follows directly from the fact that the range of u contains both positive and negative values. □

Proof of Proposition 2. Let (β, u, M) and $(\hat{\beta}, \hat{u}, \hat{M})$ represent the same binary relation $\succsim$ on $\mathcal{L}$ as in Theorem 1. By Wakker (1989, Obs. III.6.6′), there exist $\lambda > 0$ and $d, d' \in \mathbb{R}$ such that $\hat{u} = \lambda u + d$, and $\hat{M} = \lambda M + d'$. As required by Theorem 1, it must be that $u(0) = 0 = \hat{u}(0)$. Thus, $d = 0 = d'$, implying that $\hat{u} = \lambda u$ and $\hat{M} = \lambda M$. Moreover, it clearly must be that $\beta = \hat{\beta}$ for the two triples to represent the same binary relation. The sufficiency of the conditions can be directly verified. $\square$

B Proofs of Theorem 3 and Related Results

We start with a preliminary lemma that will be useful to prove Theorem 3.

Lemma 9. *Suppose that a complete preorder $\succsim$ on $\mathcal{L}$ satisfies Axioms (A1)–(A7), and let (β, u, M) be its representation as in Theorem 1. Then, for all $z \in \mathcal{C}$ and $k > 0$,*

$$(f|0) \succsim \frac{1}{k+1} f + \frac{k}{k+1}(f|z) \quad \Leftrightarrow \quad M(f_{\ell(f)-1}, \ldots, f_0, 0, 0, \ldots) \geq k u(z).$$

Proof. Let $t := \ell(f)$. Using representation (3), we obtain

$$\begin{aligned}
&(f|0) \succsim \frac{1}{k+1} f + \frac{k}{k+1}(f|z) \quad \Leftrightarrow \\
&\quad V(f) + \beta^t M(f_{t-1}, \ldots, f_0, 0, 0, \ldots) \geq \\
&\qquad \frac{1}{k+1} V(f) + \frac{k}{k+1}\big[V(f) + \beta^t u(z) + \beta^t M(f_{t-1}, \ldots, f_0, 0, 0, \ldots)\big] \quad \Leftrightarrow \\
&\qquad M(f_{t-1}, \ldots, f_0, 0, 0, \ldots) \geq k u(z).
\end{aligned}$$

$\square$

Proof of Theorem 3. *Only if* part. Suppose that $\succsim$ is a complete preorder on $\mathcal{L}$ that satisfies the specified axioms.

Step 1. Let $V : \mathcal{L} \to \mathbb{R}$ be a utility representation of $\succsim$ as in (3), with β, u, and M as specified in Theorem 1. Let $I := \{M(f_{t-1}, \ldots, f_0, 0, 0, \ldots) \mid f \in \mathcal{F}_t, t \in \mathbb{N}\}$ and note that I contains 0 bacause $M(0, 0, \ldots) = 0$. Define $\psi : I \times \mathcal{C} \to I$ as follows: For $r \in \mathbb{R}$ and $c \in \mathcal{C}$, $\psi(r, c) := M(c, f_{t-1}, \ldots, f_0, 0, 0, \ldots)$, where $f \in \mathcal{F}_t$ for some $t \in \mathbb{N}$ is an arbitrary act such that $M(f_{t-1}, \ldots, f_0, 0, 0, \ldots) = r$.

Step 2. We claim that, in the above definition of ψ, the value of $\psi(r, c)$ is independent of the choice of f. Indeed, fix an arbitrary $c \in \mathcal{C}$, and let $f \in \mathcal{F}$ and $f' \in \mathcal{F}$ be such that $M(f_{t-1}, \ldots, f_0, 0, 0, \ldots) = M(f'_{t'-1}, \ldots, f'_0, 0, 0, \ldots)$, where $t = \ell(f)$ and $t' = \ell(f')$. Let $\hat{f} \in \mathcal{F}_{t-1}$ and $\hat{f}' \in \mathcal{F}_{t'-1}$ be the truncated streams: $f = \hat{f}|f_{t-1}$ and $f' = \hat{f}'|f'_{t-1}$. We have

$$M(f_{t-1}, \ldots, f_0, 0, 0, \ldots) \geq k u(z) \Leftrightarrow M(f'_{t'-1}, \ldots, f'_0, 0, 0, \ldots) \geq k u(z) \quad \text{for all } z \in \mathcal{C} \text{ and } k > 0,$$

and, therefore, by Lemma 9,

$$\hat{f}|f_{t-1} \succsim^{\mathtt{m}:k} z \quad \Leftrightarrow \quad \hat{f}'|f'_{t-1} \succsim^{\mathtt{m}:k} z \qquad \text{for all } z \in \mathcal{C} \text{ and } k > 0.$$

By Axiom A8, we have

$$f|c \succsim^{\mathtt{m}:k} z \quad \Leftrightarrow \quad f'|c \succsim^{\mathtt{m}:k} z \qquad \text{for all } z \in \mathcal{C} \text{ and } k > 0.$$

Applying Lemma 9 again, and since z and k are arbitrary, we conclude that $M(c, f_{t-1}, \ldots, f_0, 0, 0, \ldots) = M(c, f'_{t'-1}, \ldots, f'_0, 0, 0, \ldots)$.

Step 3. Now, we show that range $\psi = I$. Let $r \in I$ be chosen arbitrarily. By definition, $r = M(f_{t-1}, \ldots, f_0, 0, 0, \ldots)$ for some $t \in \mathbb{N}$ and $f \in \mathcal{F}_t$. Let $\tilde{r} = M(f_{t-2}, \ldots, f_0, 0, 0, \ldots)$, and observe that $\psi(\tilde{r}, f_{t-1}) = r$ by the result of the previous step. Hence, $r \in$ range ψ.

Step 4. We claim that ψ is recursively bounded. Indeed, the sets I_t, $t \in \mathbb{N} \cup \{0\}$ defined recursively as $I_0 = \{0\}$ and $I_t = \psi(I_{t-1}, \mathcal{C})$ for $t \in \mathbb{N}$ coincide with $\{M(f_{t-1}, \ldots, f_0, 0, \ldots) \mid f \in \mathcal{F}_t\}$ and, hence, are bounded for all $t \in \mathbb{N} \cup \{0\}$ because M is finite-horizon bounded.

Step 5. Finally, we prove that ψ is continuous. Suppose, by contradiction, that ψ is not continuous: there exist sequences $\{r_n\}_{n=1}^{\infty}$ in I and $\{c_n\}_{n=1}^{\infty}$ in $\mathcal{C}$ such that $r_n \to r \in I$, $c_n \to c \in \mathcal{C}$, $\psi(r_n, c_n) \to K \in \mathbb{R} \cup \{-\infty, +\infty\}$ as $n \to \infty$, but $K \neq \psi(r, c)$. Passing to a subsequence, we can assume that the sequence $\{r_n\}_{n=1}^{\infty}$ is either increasing or decreasing.

Note that $I = \bigcup_{t=1}^{\infty} I_t$, where $I_t = \{M(f_{t-1}, \ldots, f_0, 0, 0, \ldots) \mid f \in \mathcal{F}_t\}$. Recall that M is continuous; for each $t \in \mathbb{N}$, $\mathcal{F}_t$ is connected and, hence, I_t is an interval; moreover, $0 \in I_t$. Therefore, we can find some $t \in \mathbb{N}$ such that $r \in I_t$ and $r_n \in I_t$ for all $n \in \mathbb{N}$. Let $f^{(1)}$ and f in $\mathcal{F}_t$ be such that $M\left(f_{t-1}^{(1)}, \ldots, f_0^{(1)}, 0, 0, \ldots\right) = r_1$ and $M\left(f_{t-1}, \ldots, f_0, 0, 0, \ldots\right) = r$. For $n \in \mathbb{N}$, $n \geq 2$, let $f^{(n)} := (1 - \gamma_n) f^{(1)} + \gamma_n f$, where, for each $n \in \mathbb{N}$, $n \geq 2$, γ_n is chosen such that $M\left(f_{t-1}^{(n)}, \ldots, f_0^{(n)}, 0, 0, \ldots\right) = r_n$, which is possible by continuity. Passing to a subsequence, $\{\gamma_n\}_{n=1}^{\infty}$ converges, and, hence, $\{f^{(n)}\}_{n=1}^{\infty}$ converges to some $f^{(\infty)} \in \mathcal{F}_t$. Observe that $r = \lim_{n\to\infty} r_n = \lim_{n\to\infty} M\left(f_{t-1}^{(n)}, \ldots, f_0^{(n)}, 0, 0, \ldots\right) = M\left(f_{t-1}^{(\infty)}, \ldots, f_0^{(\infty)}, 0, 0, \ldots\right)$ by the continuity of M. By the result of Step 2, we have $\psi(r_n, c_n) = M\left(c_n, f_{t-1}^{(n)}, \ldots, f_0^{(n)}, 0, 0, \ldots\right)$ for all $n \in \mathbb{N}$ and $\psi(r, c) = M\left(c, f_{t-1}^{(\infty)}, \ldots, f_0^{(\infty)}, 0, 0, \ldots\right)$; by the continuity of M, we have $\lim_{n\to\infty} M\left(c_n, f_{t-1}^{(n)}, \ldots, f_0^{(n)}, 0, 0, \ldots\right) = M\left(c, f_{t-1}^{(\infty)}, \ldots, f_0^{(\infty)}, 0, 0, \ldots\right)$; and we obtain that $\lim_{n\to\infty} \psi(r_n, c_n) = \psi(r, c)$, a contradiction to our assumption.

If part. Assume that there exist a scalar $\beta \in (0, 1)$, a function $u : \mathcal{C} \to \mathbb{R}$, and a function

$\psi : I \times \mathcal{C} \to I$ for some interval $I \subseteq \mathbb{R}$ as described in the theorem, such that $V(P) = \sum_{f \in \operatorname{supp} P} P(f)V(f)$ for all $P \in \mathcal{L}$, where $V(f)$ is computed as in (7).

Let $M : \mathcal{C}_0^\infty \to \mathbb{R}$ be defined as follows. For $h = (h_0, h_1, \ldots, h_{t-1}, 0, 0, \ldots) \in \mathcal{C}_0^\infty$, where $t \in \mathbb{N} \cup \{0\}$, let $m_{-1} = 0$; for $\tau = 0, \ldots, t-1$, $m_\tau = \psi(m_{\tau-1}, h_\tau)$; and, finally, $M(h) = m_{t-1}$. Note that in this construction, the value of $M(h)$ does not depend on the choice of t as long as $h_\tau = 0$ for all $\tau \geq t$.

Clearly, M satisfies the normalization condition $M(0, 0, \ldots) = 0$.

Next, observe that it is also finite-horizon-bounded: for any $T \in \mathbb{N} \cup \{0\}$, the range of M when restricted to the set $\{f \in \mathcal{C}_0^\infty : f_t = 0 \text{ for all } t \geq T\}$ can be computed recursively as $I_0 = \{0\}$ and $I_t = \psi(I_{t-1}, \mathcal{C})$ for $t \in \mathbb{N}$ and is bounded since ψ is recursively bounded.

Finally, we establish the continuity of M. Suppose that a net $\{h^{(\alpha)}\}_\alpha$ converges to some h in $\mathcal{C}_0^\infty$. Hence, for some $T \in \mathbb{N}$ such that $h_t = 0$ for all $t \geq T$, there exists an index α_0 such that $h_t^{(\alpha)} = 0$ for all $\alpha \geq \alpha_0$ and $t \geq T$, and $\sup_{0 \leq t \leq T} |h_t - h_t^{(\alpha)}|$ converges to zero. Then, $M\left(h^{(\alpha)}\right) = \psi\left(\psi\left(\ldots \psi\left(0, h_{T-1}^{(\alpha)}\right), \ldots, h_1^{(\alpha)}\right), h_0^{(\alpha)}\right) \to M(h) = \psi(\psi(\ldots \psi(0, h_{T-1}), \ldots, h_1), h_0)$ because of the continuity of ψ.

Thus, we can apply the converse direction of Theorem 1 to conclude that Axioms (A1)–(A7) hold. It remains to show that Axiom (A8) holds, as well.

Suppose that $f, g \in \mathcal{F}$ and $x, y \in \mathcal{C}$ are such that

$$f|x \succsim^{\mathrm{m}:k} z \quad \Leftrightarrow \quad g|y \succsim^{\mathrm{m}:k} z \qquad \text{for all } z \in \mathcal{C} \text{ and } k > 0.$$

By Lemma 9, this gives

$$M(x, f_{\ell(f)-1}, \ldots, f_0, 0, 0, \ldots) \geq ku(z) \Leftrightarrow M(y, g_{\ell(g)-1}, \ldots, g_0, 0, 0, \ldots) \geq ku(z) \quad \forall z \in \mathcal{C}, k > 0.$$

Due to the arbitrariness of z and k and the fact that range u takes both positive and negative values, it must be that $M(x, f_{\ell(f)-1}, \ldots, f_0, 0, 0, \ldots) = M(y, g_{\ell(g)-1}, \ldots, g_0, 0, 0, \ldots)$. Fix an arbitrary $c \in \mathcal{C}$. Then, $M(c, x, f_{\ell(f)-1}, \ldots, f_0, 0, 0, \ldots) = \psi(M(x, f_{\ell(f)-1}, \ldots, f_0, 0, 0, \ldots), c) = \psi(M(y, g_{\ell(g)-1}, \ldots, g_0, 0, 0, \ldots), c) = M(c, y, g_{\ell(g)-1}, \ldots, g_0, 0, 0, \ldots)$. By Lemma 9, again,

$$f|x|c \succsim^{\mathrm{m}:k} z \quad \Leftrightarrow \quad g|y|c \succsim^{\mathrm{m}:k} z \qquad \text{for all } z \in \mathcal{C} \text{ and } k > 0.$$

□

Proof of Proposition 4. We will prove the necessity by using the fact that Theorem 3 is a special case of the general representation in Theorem 1. Let (β, u, I, ψ) and $\left(\hat{\beta}, \hat{u}, \hat{I}, \hat{\psi}\right)$ represent the same binary relation $\succsim$ on $\mathcal{L}$ as in Theorem 3.

Define $M : \mathcal{C}_0^\infty \to \mathbb{R}$ recursively via ψ in the same way as in the proof of the "*if* part" of Theorem 3, and, similarly, $\hat{M}$ via $\hat{\psi}$. As pointed out in that proof, such functions M and $\hat{M}$ satisfy the properties of Theorem 1. By the uniqueness result for the general representation (Proposition 2), $\beta = \hat{\beta}$, and there exists $\lambda > 0$ such that $\hat{u} = \lambda u$ and $\hat{M} = \lambda M$. Now, fix arbitrary $r \in \hat{I}$ and $c \in \mathcal{C}$. Let $t \in \mathbb{N}$ and $f \in \mathcal{F}_t$ be such that $r = \hat{M}(f_{t-1}, \ldots, f_0, 0, 0, \ldots)$, and note that $r = \lambda M(f_{t-1}, \ldots, f_0, 0, 0, \ldots)$. Then, by the construction of the functions M and $\hat{M}$, we have $\hat{M}(c, f_{t-1}, \ldots, f_0, 0, 0, \ldots) = \hat{\psi}(r, c)$ and $M(c, f_{t-1}, \ldots, f_0, 0, 0, \ldots) = \psi\left(\frac{1}{\lambda} r, c\right)$, which gives $\hat{\psi}(r, c) = \lambda \psi\left(\frac{1}{\lambda} r, c\right)$.

The sufficiency of the conditions can be verified directly. □

C Proofs of the results from Section 5.1

Proof of Proposition 5. If $\kappa = 0$, then the agent's objective function (11) can be simplified to $V_t = \mathbb{E}_{t-1}\left[\sum_{\tau=t}^{T-1} \beta^{\tau-t}\big(u(A\, a_\tau x_\tau + B) + d\, a_\tau x_\tau\big)\right]$. Let e_g denote the expected value of a project of type P_g. Then, for any contract (A, B) that is acceptable to the agent and such that $A \geq 0$, it must be that

$$\underline{U} \leq V_0 = \mathbb{E}\left[\sum_{\tau=0}^{T-1} \beta^\tau \big(u(A\, a_\tau x_\tau + B) + d\, a_\tau x_\tau\big)\right] \leq \sum_{\tau=0}^{T-1} \beta^\tau \Big\{u\big(\mathbb{E}_{t-1}[A a_\tau x_\tau + B]\big) + d\mathbb{E}_{t-1}[a_\tau x_\tau]\Big\} \leq \sum_{\tau=0}^{T-1} \beta^\tau \big(u(Ae_g + B) + de_g\big).$$

Hence,

$$Ae_g + B \geq u^{-1}\left(\frac{1-\beta}{1-\beta^T}\underline{U} - de_g\right).$$

For the principal's expected profit, we have

$$\mathbb{E}\left[\sum_{t=0}^{T-1} \beta^t (p\, a_t x_t - w_t)\right] = \sum_{t=0}^{T-1} \beta^t \mathbb{E}[p\, a_t x_t - A\, a_t x_t - B] \leq \frac{1-\beta^T}{1-\beta}\left(p\, e_g - u^{-1}\left(\frac{1-\beta}{1-\beta^T}\underline{U} - de_g\right)\right).$$

This upper bound can be achieved only if all earlier inequalities hold as equalities. Given that u is strictly concave, it is possible only if $A = 0$ and the agent chooses to implement good projects in each period.

If $A = 0$ and $B = u^{-1}\left(\frac{1-\beta}{1-\beta^T}(\underline{U} - de_g)\right)$, then the agent accepts the contract and chooses to implement P_g projects in every period because this strategy maximizes

$$\mathbb{E}_{t-1}\left[\sum_{\tau=t}^{T-1} \beta^{\tau-t}\big(u(B) + d\,a_\tau x_\tau\big)\right]$$

for all $t = 0, \ldots, T-1$. Thus, the upper bound for the principal's profit is achievable. $\square$

Proof of Proposition 6. The parameters that yield the described optimal contract and the agent's strategy may be, for example, as follows:

$$u(c) = \frac{3c}{c+3.5} \quad d = 0.05 \quad k = 2 \quad \gamma = 3/4$$
$$\beta = 0.999 \quad p = 100 \quad \underline{U} = 10$$

and the outcome distributions

	-2	-1	1	2
P_g	0.01	0.53	0.12	0.34
P_b	0.34	0.01	0.64	0.01

Note that the expected value of x_t under P_g is 0.25, and the expected value of x_t under P_b is -0.03.

Given a contract (A, B), the agent's problem can be written recursively in a Bellman equation-type manner. In each period $t \in \{0, 1, 2, 3\}$, the variable μ_{t-1} and the variable $y_{t-1} = a_{t-1}x_{t-1}$ contain all information that is relevant for the agent's payoffs in the current and future periods, so they can serve as state variables. Therefore, the agent's optimal expected discounted value at time t is

$$\begin{aligned} V_t(\mu_{t-1}, y_{t-1}) = \max\Bigg\{ & u(B) + \beta V_{t+1}(\mu_{t-1}, 0), \\ & \max_{P\in\{P_g,P_b\}} \sum_{x_t\in\{-2,-1,1,2\}} \Big[u(Ax_t + B) + dx_t + \beta V_{t+1}\big(G(\mu_{t-1}, x_t), x_t\big)\Big]\Bigg\} \\ & + \kappa\big((1-\gamma)y_{t-1} + \gamma\mu_{t-1}\big) \end{aligned}$$

where $G(\mu_{t-1}, x_t)$ is the right-hand side of the recursive memory evolution formula (10), $G(\mu_{t-1}, x_t) = \max\{x_t, \mu_{t-1}\}$ if $x_t > 0$ and $G(\mu_{t-1}, x_t) = \min\{x_t, \mu_{t-1}\}$ if $x_t < 0$. The action that the agent chooses is determined by the term that delivers the maximum in the right-hand side of the Bellman equation — the maximizing distribution from $\{P_g, P_b\}$ or $a_t = 1$

if the first term in the first maximum gives a greater value. The recursion terminates with $V_T(\mu, y) \equiv 0$, and the ex ante value for the agent is $V_0(0,0)$.

The principal maximizes his profit over all contracts (A, B) such that $A \geq 0$ and $V_0(0,0) \geq \underline{U}$. This is an optimization problem that, for any given values of the parameters, can be solved numerically. For the values given above, the optimal contract (up to four digits after the decimal point) is $A = 0.1881$ and $B = 3.5132$, and the agent's strategy has the features described in the statement of the proposition. □

While the existential claim of the proposition is established purely numerically, we provide some intuition and illustrate the tradeoffs that shape the agent's strategy and the optimal contract.

Let us start with investigating the optimal agent's action at $t = 2$ after a history that leads to $\mu_1 = -2$ and $x_1 = -2$, that is, the worst possible memory. We observe that, at $t = 3$, the agent always chooses $a_3 = 1$ and the efficient P_g-type project. Indeed, the outcome of that period's project, x_3, enters her utility via the terms $u(Ax_3 + B) + dx_3$ and nothing else because $V_4 \equiv 0$, and, hence, the agent simply chooses the project that maximizes the expected value of $u(Ax_3+B)+dx_3$. For the specified parameter values and under the optimal contract, we compute

$$\begin{aligned} &V_3(-2,-2) = -2.4768 \quad &&V_3(-2,-1) = -1.9768 \\ &V_3(1,1) = 3.5232 &&V_3(2,2) = 5.5232. \end{aligned}$$

If we break down the agent's total expected utility at $t = 2$ into three components — the current utility term $u(Ax_t + B) + dx_t$, the memorable utility from the past $\kappa\big((1-\gamma)x_{t-1} + \gamma\mu_{t-1}\big)$, and the continuation value $\beta V_{t+1}(\mu_t, x_t)$, we find the following numbers for her choice of P_g and P_b projects:

	P_g	P_b
current	1.5232	1.4979
memorable	−4	−4
continuation	1.2270	1.4468
total	−1.2498	−1.0554

In this case, a P_b-type project gives the agent a greater continuation value because of a greater probability of positive outcomes under P_b that is not fully offset by the high probability of the +2 outcome under P_g. In turn, the continuation value here outweighs the advantage of P_g in the current utility.

Next, let us discuss the agent's action at $t = 2$ after a history that leads to $\mu_1 = 2$ and $x_1 = 2$, that is, the best possible memory. If she chooses $a_2 = 0$, then she is sure that her memory will be preserved and she will get $V_3(2,0) = V_3(2,2) - 1 = 4.5232$. If she chooses $a_2 = 1$ and a good project, then her memory cannot improve; instead, with the 54% chances, it will turn negative. The breakdown of the agent's total utility is

	P_g	$a_2 = 0$
current	1.5232	1.5028
memorable	4	4
continuation	2.2010	4.5187
total	7.7242	10.0215

and the agent's choice of $a_2 = 0$ is clearly superior. Similarly, the agent chooses $a_1 = 0$ if $\mu_0 = 2$ and $x_0 = 2$.

Finally, let us study the optimal contract. It has $A > 0$ for the following reason: if $A = 0$, then the agent an inefficient P_b project at $t = 1$ if $\mu_0 = -2$ and $x_0 = -2$ for reasons similar to the reasons for choosing P_b at $t = 2$ after $\mu_1 = -2$ and $x_1 = -2$ that were discussed above. In fact, for the values of the parameters that we specified, the optimal value of $A = 0.1881$ is the minimal value that induces the agent to choose P_g project instead of P_b project at $t = 1$ if $\mu_0 = -2$ and $x_0 = -2$. If the principal chooses $A > 0$, he will need to compensate the risk-averse agent for the variation in her payoffs when she chooses $a_t = 1$. In our case, the mechanical increase of A from 0 to 0.1881 reduces the principal's expected profit from 55.98 to 55.95 due to this compensation, but the agent's switch from P_b to P_g at $t = 2$ after $\mu_1 = -2$ and $x_1 = -2$ more than counterbalances and bumps the expected profit to 56.22.

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