INDUSTRIAL ORGANIZATION
Exercise book

Valeria Gattai

BOCCONI UNIVERSITY
A.Y. 2007-2008

1. MONOPOLY

Exercise 1.1
Consider a market with linear demand function $q=a-bp$, where q denotes total quantity and p is the price. There is only one firm in the market, with marginal (and average) costs constant and equal to c.
1) Compute the equilibrium price and quantity, and represent them in a graph.
2) Compute the demand elasticity to price (as a positive number). How does demand elasticity varies with the parameter b?
3) What is the so called Lerner index? Based on previous results, show that the Lerner index is a decreasing function of b.
4) Explain whether the monopoly equilibrium (calculated at 1) falls on the elastic or inelastic part of the demand curve. Is this a general result? Why?

Exercise 1.2
Consider a market with linear demand function $p=\frac{a}{b}-\frac{q}{b}$, where q denotes total quantity and p is the price; total costs are given by $C(q)=cq$.
1) Represent the demand function in a graph, indicating the values of its intercepts and slope.
2) Compute the perfect competition equilibrium and show it on the previous graph.
3) Compute consumer surplus, producer surplus and total surplus. What are the corrisponding areas on the graph?
4) Suppose now that the same market is populated by a single firm. Compute the monopoly equilibrium.
5) Compute consumer surplus, producer surplus and total surplus in this new situation, and show the corresponding areas on the graph.
6) Compute the welfare loss due to monopoly pricing. How does it vary with parameter b?

Exercise 1.3*
Consider a monopoly with marginal (and average) costs constant and equal to c. The demand function is not linear, and it is given by $q=p^{-\alpha}$, where q denotes tota quantity, p is the price and $\alpha>1$.
1) Compute the equilibrium price and quantity.
2) Compute the equilibrium price and quantity under perfect competition.
3) Compute the welfare loss due to monopoly pricing, and give a graphical representation of it.

Exercise 1.4
Consider the following demand function: $p=\alpha - \beta q^{\delta}$, where p is the price and q denotes total quantity.
1) Assuming that total costs of a monopolist equal $C(q)=cq$, compute the market equilibrium.
2) Compute Lerner index.

Exercise 1.5
A monopolist faces the following demand function: $p=a-b\ln q$, where p is the price and q denotes total quantity.
1) Compute the elasticity of demand to price.
2) Assuming marginal (and average) costs being constant and equal to c, compute the equilibrium price and quantity.

Exercise 1.6

Consider a monopoly producer of a single good. This good is sold in two consecutive periods: t=1, 2. Demand curve is $q_t=1-p_t$; marginal (and average) costs equal c in the first period, and $c-\lambda q_1$ in the second one.

1) Based on the information provided, do you think that demands are dependent or independent? Are costs dependent or independent?

2) Normalize to 1 the discount factor between the two periods, and derive the equilibrium price.

3) Compute the Lerner index, and the elasticity of demand (to price) in equilibrium for both periods. Discuss your findings.

Exercise 1.7

Consider a monopoly producer of a single good. This good is sold in two consecutive periods: t=1, 2. Marginal (and average) costs are constant and equal to c. Demand curve is $q_1=1-p_1$ in the first period, and $q_2=1-p_1-p_2$ in the second one.

1) Based on the information provided, do you think that demands are dependent or independent? Are costs dependent or independent?

2) Normalize to 1 the discount factor between the two periods, and derive the equilibrium price.

3) Compute the Lerner index, and the elasticity of demand (to price) in equilibrium for both periods. Discuss your findings.

Exercise 1.8

The camper market is polulated by 4 (groups of) consumers, with different valuations of the good:
Mr1 is willing to pay up to 6 for a camper
Mr2è is willing to pay up to 4 for a camper
Mr3 is willing to pay up to 2 for a camper
Mr4 is willing to pay up to 0.5 for a camper

1) Define durable goods? Do think that campers can be regarded as durable goods? Why?

2) Consider two periods, with discount factor $\delta \in (0,1)$. Suppose first that the monopolist is not expected to engage in any intertemporal discrimination of price (i.e. it is not expected to fix different prices over time). Derive the equilibrium price in the two periods.

3) Compute the present value of the monopolist profits.

4) Suppone, now, that the monopolist is expected to fix different prices over time. Do you think that concumers would change their behavious, compared to 2)?

5) Define the so-called Coase conjecture and illustrate a few solutions to it.

Exercise 1.9*

A durable-good monopolist has marginal (and average) costs equal to 0, and serves a market populated by two (groups of) consumers. The firts one has preferences: $U_1=6–p_1$, while the second $U_2=2-p_2$. Assume a two period-time horizon, with discount factor $\delta \in (0,1)$.

1) Suppose first that the monopolist is not expected to engage in any intertemporal discrimination of price (this means that consumers buy the good if and only if their net surplus U_i is non negative). Derive the optimal p_1 and p_2. Derive also the equilibrium quantities, consumer surplus and the present value of the monopolist profits.

2) Suppone, now, that the monopolist is expected to fix different prices over time.

a) Derive the optimal p_2, given that the monopolist has already sold the good to consumer 1, in the first period.
b) Compare consumer 1 utility if she buys the good in the first period, with her utility if she waits and buys the good in the second period, at the price computed before at a). What is the maximum value of p_1 that the monopolist can charge in the first period without discouraging consumer 1 from buying the good?

c) Compare p_1 from b) with that computer at 1) and comment. Derive the present value of the monopolist profits.

3) Suppose now that the monopolist offers the first consumer a refund of p_1-p_2 in case it decreases the price in the second period.

a) Derive again the condition on p_1 accorging to which the first consumer does not wait and buy the good in the second period. What is the optimal p_1 in this case?

b) Is it worth descreasing the price in the second period for the monopolist under the refund assumption? Why?

Exercise 1.10

Consider a market with linear demand function $q=1-p$, where q denotes total quantity and p is the price. This market is populated by a single firm, with marginal (and average) costs equal to 0.

1) Suppose that price discrimination is not allowed. Compute the market equilibrium, the monopolist profits, consumer surplus and total surplus. Offer a graphical representation of these results.

2) Define price discrimination. Which conditions are required in order to take place for the price discrimination?

3) Compute the market equilibrium, the monopolist profits, consumer surplus and total surplus in case of first type-price discrimination.

4) Compute the perfect competition equilibrium in the same market. Compare 3) and 4): do you think that the society as a whole would benefit from switching to perfect competition? Why?

Exercise 1.11

A monopolist serves two markets, l and h, corresponding to different regions of a country, with weights 1/3 and 2/3 respectively. The demand curve in l is given by $q_l = 1 - p_l$; the demand curve in h is $q_h = 2 - p_h$.

The monopolist serves both regions from a single plant, with no marginal and transportation costs.

1) Suppose that a third type-price discrimination is allowed. Compute the equilibrium prices and quantities in each market. Compute also the monopolist profits, the consumer and the total surpluses.

2) Suppose now that price discrimination is not allowed. Compute the equilibrium price and quantities in the two markets. From the monopolist point of view, which of the two situations - 1) and 2) – is preferable? And from the point of view of the society as a whole?

3) Compute the equilibrium price and quantity in case price discrimination is not allowed, and market l is not served. Compare 2) and 3) in terms of social surplus.

Exercise 1.12

A monopolist serves two types of consumers, l and h, with demands given by $q_l = 1 - p_l$ and $q_h = 2 - p_h$ respectively; l-type consumers account for 1/3 of the entire population, while h-type consumers for 2/3. Assume producion costs equal to 0 for semplicity.

1) Derive the consumer surplus and the monopolist profits in case a two part-tariff $T + pq$ is offered (this means that a consumer that buys q units of the good pays a fixed amount T plus a variable amount p).

2) Compute the equilibrium price and quantities in case price discrimination is not allowed and both types of consumers buy the good. From the point of view of the monopolist, is better equilibrium 1) or 2)? Why?

Exercise 1.13*

A monopolist serves two markets, l and h, corresponding to different regions of a country, with weights λ and $(1-\lambda)$ respectively, with $0<\lambda<1$. The demand curve in market $i = l, h$ is given by $q_i = v_i - p_i$, with $v_h > v_l$.

The monopolist serves both regions from a single plant, with marginal (and average) costs $c<v_l$ and null transportation costs.
1) Suppose that the third type-price discrimination is allowed. Compute the equilibrium prices and quantities in each market. Compute also the monopolist profits, the consumer and the total surpluses.
2) Suppose now that price discrimination is not allowed. Compute the equilibrium price and quantities in the two markets. From the monopolist point of view, which of the two situations - 1) and 2) – is preferable? And from the point of view of the society as a whole?
3) Compute the equilibrium price and quantity in case price discrimination is not allowed, and market *l* is not served (i.e. $v_h + c > 2v_l$). Compare 1), 2) and 3) in terms of social surplus.

Exercise 1.14*
A monopolist serves two types of consumers, *l* and *h*, with demands given by $q_l = v_l - p_l$ and $q_h = v_h - p_h$ respectively; *l*-type consumers account for a share λ of the entire population, while *h*-type consumers for $(1- \lambda)$. Assume producion costs equal to 0 for semplicity.
The monopolist serves all consumers from a single plant, with marginal (and average) costs $c<v_l$ and null transportation costs.
1) Derive the consumer surplus and the monopolist profits in case a two part-tariff $T + pq$ is offered (this means that a consumer that buys q units of the good pays a fixed amount T plus a variable amount p).
2) Compute the equilibrium price and quantities in case price discrimination is not allowed and both types of consumers buy the good (i.e. $v_l>(c+v_h)/2$). From the point of view of the monopolist, is better equilibrium 1) or 2)? Why?

Exercise 1.15
A monopolist serves two markets, *l* and *h*, corresponding to different regions of a country, with weights 1/2 each. The demand curve in *l* is given by $q_l = 1 - p_l$; the demand curve in *h* is $q_h = 2 - p_h$.
The monopolist serves both regions from a single plant, with no marginal and transportation costs.
1) Suppose that the third type-price discrimination is allowed. Compute the equilibrium prices and quantities in each market. Compute also the monopolist profits, the consumer and the total surpluses.
2) Suppose now that price discrimination is not allowed. Compute the equilibrium price and quantities in the two markets. From the monopolist point of view, which of the two situations - 1) and 2) – is preferable? And from the point of view of the society as a whole?

Exercise 1.16
Consider a monopolist serving two markets – *a* e *b* – characterized by different demand curves and different number of consumers. Indeed, market *a* shows high demand and covers a share $(1-\lambda)$ of consumers, with demand $q_a=3-p_a$, while market *b* exhibits low demand and covers a λ of consumers, with demand curve $q_b=1-p_b$. For simplicity, assume 0 marginal (and average) costs.
1) Compute the optimal prices, quantities and profits, in case the monopolist is allowed to fix different prices in the two markets. What type of price discrimination is this?
2) Compute the optimal price, quantities and profits, in case the monopolist is not allowed to fix different prices, and it serves both markets.
3) Compute the optimal price, quantities and profits, in case the monopolist is not allowed to fix different prices, but it can decide to serve only one market.
4) Being able to choose, would the monopolist select option 2) or 3) when price discrimination is not allowed?

1. MONOPOLY – solutions

Exercise 1.1

1) The monopolist solves the following maximization problem:

$$\max_{p} \pi = (p - AC)q = (p - c)(a - bp)$$

from which we derive first order conditions *(FOC)* with respect to the price:

$FOC_p : a - bp - b(p\text{-}c) = 0$

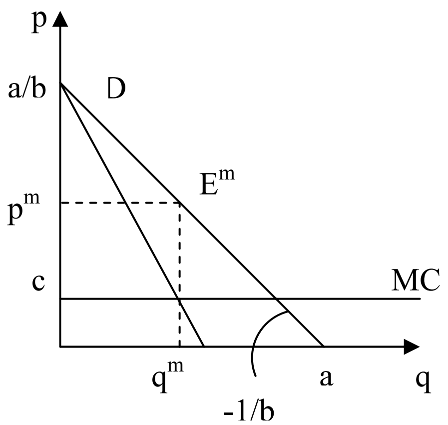
Solving the *FOC*, we obtain the monopoly equilibrium price:

$$p^m = \frac{a + bc}{2b}$$

Now, substitute the monopoly equilibrium price into the demand function, to obtain the monopoly equilibrium quantity:

$$q^m = \frac{a - bc}{2}$$

The graphical representation is as follows:



2) The elasticity of demand (to price), as a positive number, is given by the following formula:

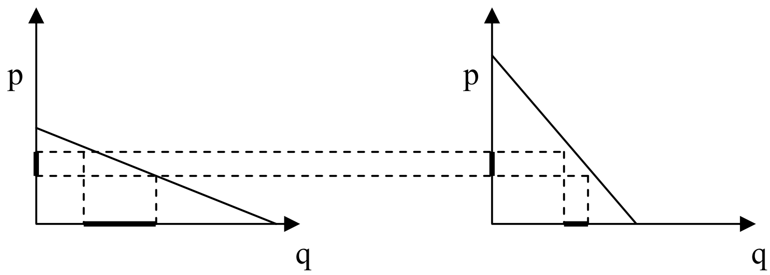
$$\varepsilon = -D(p)'\frac{p}{D(p)}$$

which yields, in this case:

$$\varepsilon = \frac{bp}{a - bp}$$

It is clear that, the lager *b*, the lager the numerator and the smaller the denominator; the demand elasticity is thus an increasing function of the parameter *b*.
This result can be derived also at a more intuitive level.

Recall that the slope of the demand curve is *–1/b*. Compare the two curves drawn in the graph below, assumine equal scale on the axis. The steeper one (on the right) exhibits lower elasticity because the variation of the quantity (measured on the horizontal axis), associated to a given variation of the price (measured on the vertical axis), is lower with respect to the flatter one (on the left).



3) The Lerner index measure the monopolist relative *mark up*. It is given by:

$$I(L) = \frac{p^m - MC}{p^m}$$

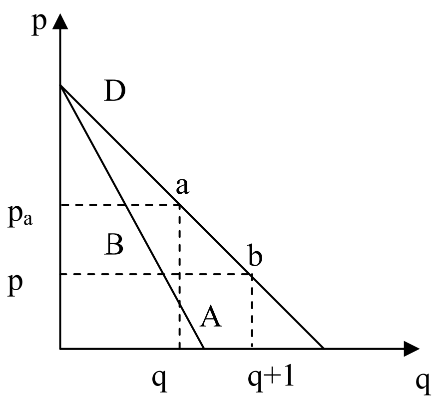
So, in this case:

$$I(L) = \frac{a - bc}{a + bc}$$

Notice that, the larger *b*, the smaller the Lerner index (indeed the numerator decreases, while the denominator increases). This result is in line with 2): we know that the Lerner index is inversely proportional to the demand elasticity, so an increase in *b*, while increasing *ε*, decreases *I(L)*.

4) Compute *ε**, i.e. the equilibrium value of the demand elasticity:

$$\varepsilon^* = \frac{a + bc}{a - bc}$$



Being the numerator clearly lager than the denominator, *ε*>1*; this means that the monopoly equilibrium falls on the elastic part of *D(p)*.
This is a general result. To have an intuitive explanation of it, look at the graph above.

In moving from a to b, namely increasing production by 1 unit, the monopolist marginal revenue conists of two terms: a positive area A (due to the increase in production) and a negative area B (due to the fact that q units, previously sold at price p_a are now sold at a lower price p).

Therefore: $MR= p-(p_a-p)q$

Which can be re-written as:

$MR=p(1-1/\varepsilon)$

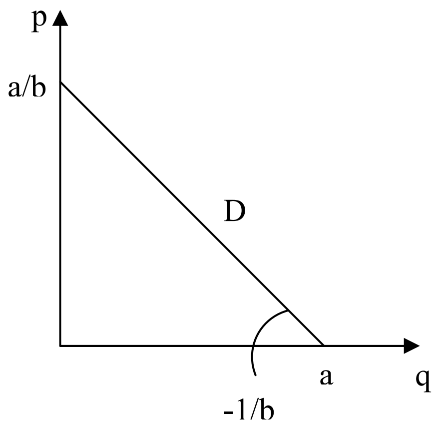
As a result of the maximizing behaviour of the monopolist we know that, in equilibrium, $MR=MC\geq 0$, so:

$p(1-1/\varepsilon)= MR=MC\geq 0$

As a result $(1-1/\varepsilon)\geq 0$, i.e. $\varepsilon\geq 1$: the monopolist equilibrium always falls on the elastic part of the demand curve.

Exercise 1.2

1)



2) Under perfect competition firms *price taker*, so they fix a price equal to their marginal cost:

$p=c$

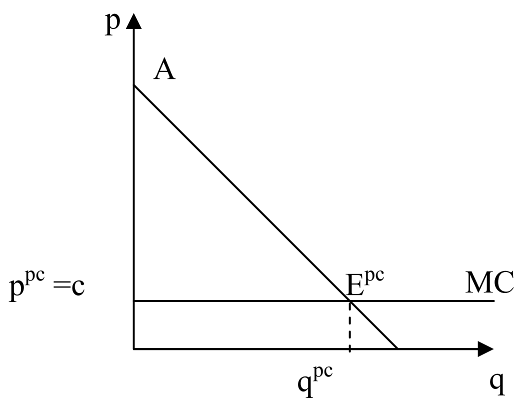
This can be substituted into the demand function, to obtain:

$$\frac{a}{b}-\frac{q}{b}=c$$

The equilibrium price and quantity, under perfect competition, are thus given by:

$q^{pc} = a-bc$
$p^{pc} = c$

The graphical representation is as follows:



3) The consumer surplus *(CS)* is the area between the demand curve and the equilibrium price. In this case:

$$CS^{pc} = area\ Ap^{pc}\ E^{pc} = \frac{(a-bc)^2}{2b}$$

The producer surplus coincides with the profit, so it is 0 under perfect competition:

$\pi^{pc}=0$

The total surplus *(W)* is the sum of consumer and producer surpluses, so it equals CS^{pc} under perfect competition:

$$W^{pc} = area\ Ap^{pc}\ E^{pc} = \frac{(a-bc)^2}{2b}$$

4) The monopolist solves the following maximization problem:

$$\max_p \pi = (p-AC)q = (p-c)(a-bp)$$

First order condition with respect to orice are given by:

$FOC_p : a - bp - b(p\text{-}c) = 0$

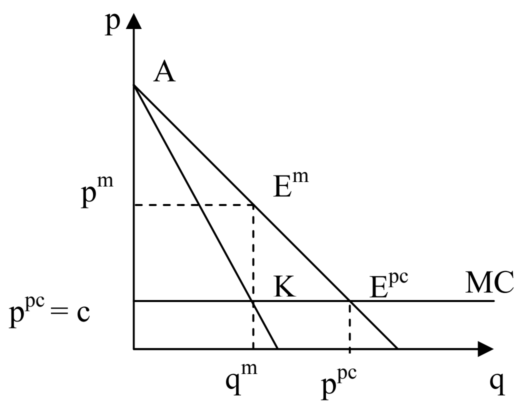
Solving the *FOC* we obtain the monopoly equilibrium price:

$$p^m = \frac{a+bc}{2b}$$

This can be substituted into the demand function to obtain the monopolist equilibrium quantity:

$$q^m = \frac{a-bc}{2}$$

The graphical representation is as follows:



5)

$$CS^m = area\ Ap^mE^m = \frac{(a-bc)^2}{8b}$$

$$\pi^m = area\ p^mE^mKp^{pc} = \frac{(a-bc)^2}{4b}$$

$$W^m = area\ Ap^{pc}KE^m = \frac{3(a-bc)^2}{8b}$$

6) The welfare loss (*WL*), due to monopoly pricing, is computed as the difference in equilibrium total surplus between perfect competition and monopoly:

$$WL = area\ E^mKE^{conc} = \frac{(a-bc)^2}{8b}$$

Notice that the larger *b*, the larger the denominator and the smaller the numerator, so *WL* decreases. This result is in line with those of Esercise 1.1. Indeed, with an increase in *b*, we showed that there is an increase in the demand elasticity, and a decrease in the Lerner index, which corresponds to a less concentrated market where total surplus is higher.

Exercise 1.3*

1) The monopolist solves the following maximization problem:

$$\max_p \pi = (p - AC)q = (p-c)p^{-\alpha}$$

from which we derive first order condition with respect to the price:

$$FOC_p : 1 - \alpha + \frac{\alpha c}{p} = 0$$

Solving the *FOC*, the equilibrium price is given by:

$$p^m = \frac{\alpha c}{\alpha - 1}$$

This can be substituted into the demand function to compute the equilibrium quantity:

$$q^m = \left(\frac{\alpha c}{\alpha - 1}\right)^{-\alpha}$$

Under perfect competition, firms are *price taker* and they charge a price equal to their marginal cost:

$p=c$

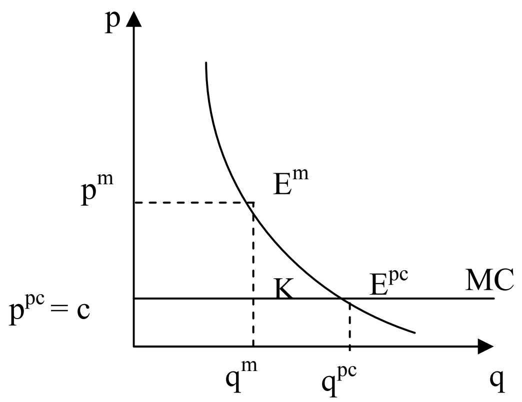
By substituting into the demand function, we obtain the following:

$$q^{\frac{-1}{\alpha}} = c$$

Therefore:

$q^{pc}=c^{-\alpha}$
$p^{pc}=c$

3) The graphical representation of 1) and 2) is as follows.



The welfare loss, due to monopoly pricing, is again the difference in equilibrium total surplus between perfect competition and monopoly. Differently from Exercise 1.2, the demand curve is not linear here, so the welfare loss cannot be computed as the area of a triangle. Indeed, it equals KE^mE^{conc}, which is calculated as the integral of the difference between demand and marginal costs, from q^m and q^{conc}:

$$\int_{q^m}^{q^{conc}} \left(q^{\frac{-1}{\alpha}} - c\right) dq = \left[\frac{q^{\frac{-1}{\alpha}+1}}{\frac{-1}{\alpha}+1} - cq\right]_{\left(\frac{\alpha c}{\alpha-1}\right)^{-\alpha}}^{c^{-\alpha}} = \frac{c^{1-\alpha}}{\alpha - 1}\left[1 - \left(\frac{\alpha}{\alpha - 1}\right)^{-\alpha}\left(\frac{2\alpha - 1}{\alpha - 1}\right)\right]$$

Exercise 1.4
1) The monopolist solves the following maximization problem (Notice that an analogous maximization can be solved with respect to the price):

$$\max_{q} \pi = (p - AC)q = (\alpha - \beta q^{\delta} - c)q$$

First order conditions with respect to the quantità are given by:

$$FOC_q : \alpha - \beta q^{\delta} - c - \beta\delta q^{\delta - 1} q = 0$$

which yields the equilibrium monopoly quantity:

$$q^m = \left(\frac{\alpha - c}{\beta(\delta + 1)} \right)^{\frac{1}{\delta}}$$

This can be substituted into the demand function, to compute the equilibrium price:

$$p^m = \alpha - \left(\frac{\alpha - c}{(\delta + 1)} \right)$$

2) The Lerner index is:

$$I(L) = \frac{p^m - MC}{p^m}$$

So, it equals:

$$I(L) = \frac{\alpha - \frac{\alpha - c}{(\delta + 1)} - c}{\alpha - \frac{\alpha - c}{(\delta + 1)}}$$

Exercise 1.5
1) First of all, derive the direct demand function:

$$q = e^{\frac{a - p}{b}}$$

The elasticità of demand (to price), as a positive number, is computed as:

$$\varepsilon = -D(p)' \frac{p}{D(p)}$$

which yields the following, in this case:

$$\varepsilon = \frac{p}{b}$$

2) The monopolist solves the following maximization problem (Notice that an analogous maximization can be solved with respect to price):

$$\max_q \pi = (p - AC)q = (\alpha - \beta \ln q - c)q$$

First order conditions, wit respect to quantity, are:

$$FOC_q : \alpha - \beta \ln q - c - \beta = 0$$

Solving the *FOC* we obtain the equilibrium monopoly quantity:

$$q^m = e^{\frac{a-b-c}{b}}$$

This can be substituted into the demand function to compute the equilibrium price:

$$p^m = b + c$$

Exercise 1.6

1) Demands are independent because the quantity in each period depends only on that period price. Costs are, instead, dependent because the first period quantity influences both TC_1 and TC_2. Thsi is an example of the so called *learning by doing*: the more a firm produces at time 1, the lower its costs at time 2.

2) The monopolist maximizes the present value of its profits, i.e. the sum of the two periods profits (recall that the discount factor is normalized to 1).

$$\max_{p_1, p_2} \pi = (p_1 - AC_1)q_1 + (p_2 - AC_2)q_2 = (p_1 - c)(1 - p_1) + [p_2 - c - \lambda(1 - p_1)](1 - p_2)$$

First order conditions are as follows:

$$FOC_{p_2} : 1 - p_2 - p_2 + c - \lambda(1 - p_1) = 0$$

$$FOC_{p_1} : 1 - p_1 - p_1 + c - \lambda(1 - p_2) = 0$$

From which we compute the equilibrium prices in both periods:

$$p_2^m(p_1) = \frac{1 + c - \lambda(1 - p_1)}{2}$$

$$p_1^m(p_2) = \frac{1 + c - \lambda(1 - p_2)}{2}$$

3) Now we calcolate the equilibrium Lerner index and demand elasticity at time 1 and time 2:

$$I(L)_2 = \frac{p_2^m - MC_2}{p_2^m} = \frac{1-c+\lambda(1-p_1)}{1+c-\lambda(1-p_1)}$$

$$\varepsilon_2 = -D_2'(p_2)\frac{p_2}{D_2} = \frac{1+c-\lambda(1-p_1)}{1-c+\lambda(1-p_1)}$$

Notice that $I(L)_2 = 1/\varepsilon_2$, as expected from the theory.

$$I(L)_1 = \frac{p_1^m - MC_1}{p_1^m} = \frac{1-c-\lambda(1-p_2)}{1+c-\lambda(1-p_2)}$$

$$\varepsilon_1 = -D_1'(p_1)\frac{p_1}{D_1} = \frac{1+c-\lambda(1-p_2)}{1-c+\lambda(1-p_2)}$$

Differently from before, $I(L)_1<1/\varepsilon_1$ in the first period, because the denominator is the same in the two ratios, but the numerator of $I(L)_1$ is smaller. This means that the relative *mark up* is lower compared to the one expected on good 1 only-monopoly. Put another way, the monopolist produces more in the first period because it internalizes the positive effect of q_1 on the second period costs.

Exercise 1.7

1) Demands are dependent. Notice that the first period quantity depends only on the first period price, while the second period quantity depends on both prices. Costs are independent and constant. This is an exemple of goodwill.

2) The monopolist maximizes the present value of its profits, i.e. the sum of the two period profits (recall that the discount factor is normalized to 1).

$$\max_{p_1, p_2} \pi = (p_1 - AC_1)q_1 + (p_2 - AC_2)q_2 = (p_1 - c)(1-p_1) + (p_2 - c)(1-p_1-p_2)$$

First order conditions are as follows:

$$FOC_{p_2} : 1 - p_2 - p_2 + c - p_1 = 0$$

$$FOC_{p_1} : 1 - p_1 - p_1 + c - p_2 + c = 0$$

Therefore, the optimal prices in the two periods are:

$$p_2^m(p_1) = \frac{1+c-p_1}{2}$$

$$p_1^m(p_2) = \frac{1-p_2+2c}{2}$$

3) Now we compute the Lerner index and the demand elasticity in equilibrium, for both periods:

$$I(L)_2 = \frac{p_2^m - MC_2}{p_2^m} = \frac{1-c-p_1}{1+c-p_1}$$

$$\varepsilon_2 = -D_2'(p_2)\frac{p_2}{D_2} = \frac{1+c-p_1}{1-c-p_1}$$

Notice that $I(L)_2 = 1/\varepsilon_2$, as expected from the theory.

$$I(L)_1 = \frac{p_1^m - MC_1}{p_1^m} = \frac{1-p_2}{1+2c-p_2}$$

$$\varepsilon_1 = -D_1'(p_1)\frac{p_1}{D_1} = \frac{1+2c-p_2}{1-2c+p_2}$$

Differently from before, $I(L)_1<1/\varepsilon_1$ in the first period, because the denominator is the same in the two ratios, but the numerator of $I(L)_1$ is smaller for $c<p_2$, which is always satisfied (no firm charges a price lower than its marginal cost). This means that the relative *mark up* is lower compared to the one expected on good 1 only-monopoly. Put another way, the monopolist produces more in the first period because it internalizes the positive effect of q_1 (an increase in q_1 means a decrease in p_1) on the second period demand.

Exercise 1.8

1) Durable goods are those that have a lifetime exceeding the basic period (for instance a car, a camera, a house etc.) and can be utilized many times. Notice that a customer who buys a durable good today is unlikely to buy the same good tomorrow. Campers can be regarded as durable goods.

2) If consumers do not expect any intertemporal price discrimination, they buy the good simply if the price is not higher than their willingness to pay. So, at time 1:
if the monopolist charges a price p_1=6, only Mr1 buys the good, so the overal profit is 6
if the monopolist charges a price p_1=4, Mr1 and Mr2 buy the good, so the overal profit is 8
if the monopolist charges a price p_1=2, Mr1, Mr2, and Mr3 buy the good, so the overal profit is 6
if the monopolist charges a price p_1=0.5, all consumers buy the good, so the overal profit is 2
In order to maximize its profit in first period, the firm thus chooses p_1*=4.
At time 2, given that Mr1 and Mr2 have already bought the good, the monopolist looks at Mr3 and Mr4, as a residual demand.
if the monopolist charges a price p_2=2, only Mr3 buys the good, so the overall profit is 2
if the monopolist charges a price p_2=0,5, Mr3 and Mr4 by the good, so the overall profit is 1.
In order to maximize its profit in second period, the firm thus chooses p_2*=2.

3) The present value of the monopolist profits is è π*=8+ δ2.

4) Consumers behaviour changes because those with higher willingness to pay might decide to wait and buy the good later, if they expect a future decrease in the price.

5) The so called Coase conjecture states that, if consumers and goods last forever and prices can be adjusted instantaneously, when price adjustments become more and more frequent, the monopolist profit converges to 0. There are several ways to escare from the Coase conjecture, such as leasing and commitment.

Exercise 1.9*

1) Durable goods are usually bought once for all by consumers. If consumers do not expect intertemporal price dicrimination, they do not have incentives to postpone the purchase. Therefore they simply buy the good if their utility U_i is non negative.

At time 1:
If the monopolist charges a price:
p_1 =1, both consumers buy the good, so the overall profit is 2
p_1 =2, both consumers buy the good, so the overall profit is 4
p_1 =3, only consumer 1 buys the good, so the overall profit is 3
p_1 =4, only consumer 1 buys the good, so the overall profit is 4
p_1 =5, only consumer 1 buys the good, so the overall profit is 5
p_1 =6, only consumer 1 buys the good, so the overall profit is 6
It is clear that the monopolist chooses p_1*=6 in the first period, to maximize its profits.

Now, given that consumer 1 has already bought the good, the monopolist chooses p_2*=2 in the second period. Notice that this is the best the monopolist can do, given the residual demand it faces.

At p_1* and p_2*, consumers utilities are U_1*=0 and U_2*=0.
The present value of the monopolist profits is $\pi^*=6+\delta 2$.

2) If consumers expect intertemporal price discrimination, they might decide to wait and buy the good after the price decreases.

a) If the monopolist has already sold the good to consumer 1, she will not buy the good again. Therefore the situation is completely analogous to 1): $p_2^{**}=2$.
b) Buying the good in the first period, consumer 1 utility is $6-p_1$. Waiting and buying the good in the second period, her utility is $\delta(6-p_2^{**})= \delta 4$.
Consumer 1 thus decides to buy the good in the first period if the following condition holds: $6-p_1 \geq \delta 4$. In the end, it is clear that the maximum price that the monopolist can charge, without discouraging purchase in the first period, is $p_1^{**}=6- \delta 4$.
c) The difference is that the monopolist cannot charge a price p_1 equal to 6 in the first period, otherwise consumer 1 postpones her purchase. Put another way, $p_1^*>p_1^{**}$.
The present value of the monopolist profits, in this new situation, is $\pi^{**}=6- \delta 2$.

3) Now we compare consumer 1 utility when she buys the good immediately, with her utility in case she waits:

a) $6-p_1+ \delta(p_1-p_2) \geq \delta(6-p_2)$, this is always satisfied for $p_1 \leq 6$. Therefore the monopolist chooses $p_1^{***}=6$ and, given the refund, consumer 1 buys the good immediately.

b) Now we compare the monopolist profits in the two cases:
is $p_1^{***}=p_2^{***}=6$, the good purchase takes place only in the first period, so the overall profit is 6.
If $2=p_2^{***}<p_1^{***}=6$, both consumers buy the good, so the overall profit is $6+ \delta[p_2 - (6-p_2)]=6-2\delta<6$. It is clear that the monopolist will not engage in intertemporal price discrimination.

Exercise 1.10

1) The monopolist solves the following maximization problem:

$$\max_{p} \pi = (p - AC)q = p(1-p)$$

from which we derive first order conditions:

$$FOC_p : 1 - p - p \ = 0$$

Solving *FOC*, we get the equilibrium price:

$$p^m = \frac{1}{2}$$

which can be substituted into the demand function to obtain the equilibrium quantity:

$$q^m = \frac{1}{2}$$

The monopolist profit is π^m = ¼, corresponding to the area $p^m E^m p^{pc} q^m$ on the graph.
Consumer surplus is CS^m=1/8, corresponding to the area $Ap^m E^m$.
Total surplus is W^m=3/8, corresponding to the area $AE^m q^m p^{pc}$.

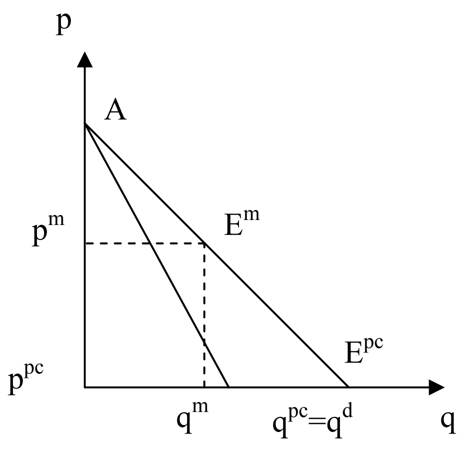
2) Price discrmination takes place when the monopolist charges different consumers different prices for the same good. In order for price discrimination to take place, the monopolist needs to distinguish among consumers, and arbitrage cannot is not allowed.
3) First type-price discrimination takes place when consumers are charged a price equal to their willingness to pay. By doing so, the monopolist is able to extract all consumers' surplus.
The equilibrium quantity equals that of perfect competition:

$q^d = q^{pc} = 1$

Consumer surplus is 0:

$CS^d = 0$
The profit is π^d=1/2 corresponding to the area $Ap^{pc}E^{pc}$ and total surplus W^d equals π^d.

The graphical representation is as follows:



4) Under perfect competition, total surplus coincides with that of 2). Indeed, by imposing *p=MC*, p^{pc}=0 and q^{pc}=1. The difference between first type-price discrimination and perfect competition lays in the balance between consumers and producers surpluses: in the first case the area $Ap^{pc}E^{pc}$

accrues entirely to the monopolist, while in the second case it accrues to consumers. It follows that consumers strictly prefer perfect competition, firms strictly prefer monopoly with price discrimination, while the society as a whole is indifferent between the two cases.

Exercise 1.11

1) Under the thrid type-price discrmination, the monopolist charges different prices to different – although clearly identifiable - groups of consumers. In particolar, in market *l*, it solves the following maximization problem:

$$\max_{p_l} \pi_l = (p_l - AC_l)q_l = p_l(1-p_l)$$

First order conditions are as follows:

$$FOC_{p_l} : 1 - p_l - p_l = 0$$

Solving the *FOC*, and substituing into the demand function, we compute the equilibrium price and quantity in market *l*, under third type-price discrimination:

$$p_l^d = \frac{1}{2}$$

$$q_l^d = \frac{1}{2}$$

Therefore, the monopolist profit, consumer surplus and total surplus are given by:

$$\pi_l^d = \frac{1}{4}$$

$$CS_l^d = \frac{1}{8}$$

$$W_l^d = \frac{3}{8}$$

Through analogous reasoning, in market *h*, we find:

$$p_h^d = 1$$

$$q_h^d = 1$$

$$\pi_h^d = 1$$

$$CS_h^d = \frac{1}{2}$$

$$W_h^d = \frac{3}{2}$$

Considering the two markets, and their respective weights, the overal monopolist profit equals:

$$\pi^d = \frac{1}{3}\pi_l^d + \frac{2}{3}\pi_h^d = \frac{3}{4}$$

The overall consumer surplus is:

$$CS^d = \frac{1}{3}CS_l^d + \frac{2}{3}CS_h^d = \frac{3}{8}$$

The overal total surplus is given by:

$$W^d = \frac{1}{3}W_l^d + \frac{2}{3}W_h^d = \frac{9}{8}$$

2) If price discrimination is not allowed, and both markets are served, the monopolist charges a uniform price everywhere. So, it solves the following maximization problem:

$$\max_p \pi = \frac{1}{3}(p - AC_l)q_l + \frac{2}{3}(p - AC_h)q_h = \frac{1}{3}p(1-p) + \frac{2}{3}p(2-p)$$

which yields the following *FOC:*

$$FOC_p : 6p - 5 = 0$$

In equilibrium:

$$p^u = \frac{5}{6}$$

$$q_l^u = \frac{1}{6}$$

$$q_h^u = \frac{7}{6}$$

The monopolist profit, consumer surplus and total surplus equal:

$$\pi^u = \frac{25}{36}$$

$$CS^u = \frac{33}{72}$$

$$W^u = \frac{249}{216}$$

By comparing these results with those of 1), we see that the monopolist prefers price discrimination ($\pi^d > \pi^u$) while having a uniform price is better for the society as a whole ($W^d < W^u$). Notice that this is a general result, not depending on the parameter values (see Exercise 1.13).

3) If price discrimination is not allowed, and only one market is served, the monopolist focuses on the high demand region (h in this example) and selects a price :

$p^n = 1$

(This is the monopoly equilibrium price in region h, as calculated in 1)).

It is easy to show that, with $p^n = 1$, market l is not served, indeed $q_l^n = 1 - 1 = 0$.
Given $p^n = 1$, the equilibrium profit, consumer surplus and total surplus are given by:

$$\pi^n = \frac{2}{3}$$

$$CS^u = \frac{1}{3}$$

$$W^u = 1$$

By comparing these results with those of 2), notice that total welfare is higher under a uniform price ($W^u > W^n$). This is a general result, not depending on the parameter values (see Exercise 1.13).

Exercise 1.12
1) Under the second type-price discrimination, the monopolist offers a two parts tariff *(T+pq)*, thus having two choice variables (T and p) to be determined.
From the theory, we know that T equals consumer surplus in the low demand market (l in this example), while p is derived from the maximization problem.
Therefore:

$$T = \frac{(1-p)^2}{2}$$

$$\max_p \pi = \frac{1}{3}(p - AC_l)q_l + \frac{2}{3}(p - AC_h)q_h + T = \frac{1}{3}p(1-p) + \frac{2}{3}p(2-p) + \frac{(1-p)^2}{2}$$

From the *FOC* with respect to the price, we compute:

$$p^d = \frac{2}{3}$$

$$q_l^d = \frac{1}{3}$$

$$q_h^d = \frac{4}{3}$$

$T = \frac{1}{18}$

Consumer and producer surpluses are as follows:

$CS^d = \frac{5}{9}$

$\pi^d = \frac{13}{18}$

2) This case is analogous to 2) in Exercise 1.11.
Comparing the monopolist profit with and without price discrimination, we see that $\frac{13}{18} = \pi^d > \pi^u = \frac{25}{36}$, so price discrimination is preferable for the firm. This is a general result, not depending on the parameter values (see Exercise 1.14).

Exercise 1.13*
1) Under the thrid type-price discrmination, the monopolist charges different prices to different – although clearly identifiable - groups of consumers. In particolar, in market *l*, it solves the following maximization problem:

$$\max_{p_l} \pi_l = (p_l - AC_l)q_l = (p_l - c)(v_l - p_l)$$

Solving the FOC_p and substituting into the demand function, we compute the equilibrium price and quantity in market *l*, under the third type-price discrimination

$$p_l^d = \frac{v_l + c}{2}$$

$$q_l^d = \frac{v_l - c}{2}$$

Based on these results, the monopolist profit and consumer surplus are given by:

$$\pi_l^d = \frac{(v_l - c)^2}{4}$$

$$CS_l^d = \frac{(v_l - c)^2}{8}$$

Through analogous reasoning, in market *h,* we find:

$$p_h^d = \frac{v_h + c}{2}$$

$$q_h^d = \frac{v_h - c}{2}$$

$$\pi_h^d = \frac{(v_h - c)^2}{4}$$

$$CS_h^d = \frac{(v_h - c)^2}{8}$$

Considering the two markets, and their respective weights, the overal monopolist profit is:

$$\pi^d = \lambda \frac{(v_l - c)^2}{4} + (1-\lambda)\frac{(v_h - c)^2}{4}$$

Consumer surplus equals:

$$CS^d = \lambda \frac{(v_l - c)^2}{8} + (1-\lambda)\frac{(v_h - c)^2}{8}$$

Total surplus is given by:

$$W^d = \frac{3}{8}\lambda(v_l - c)^2 + \frac{3}{8}(1-\lambda)(v_h - c)^2$$

2) If price discrimination is not allowed, and both markets are served, the monopolist charges a uniform price to all consumers.
So, it solves the following maximization problem:

$$\max_p \pi = \lambda(p - AC_l)q_l + (1-\lambda)(p - AC_h)q_h = \lambda(p-c)(v_l - p) + (1-\lambda)(p-c)(v_h - p)$$

Solving the FOC_p we calcolate the equilibrium price:

$$p^u = \frac{\lambda v_l + (1-\lambda)v_h + c}{2}$$

and substituting into the demand function, we are able to determine the equilibrium quantities in the two markets:

$$q_l^u = \frac{2v_l - \lambda v_l - (1-\lambda)v_h - c}{2}$$

$$q_h^u = \frac{2v_h - \lambda v_h - (1-\lambda)v_h - c}{2}$$

By means of straightforward substitutions, the monopolist profit is given by:

$$\pi^u = \frac{(\lambda v_l + (1-\lambda)v_h - c)^2}{4}$$

This is equivalent to:

$$\pi^u = \frac{\left[\lambda(v_l - c) + (1-\lambda)(v_h - c)\right]^2}{4}$$

(The second expression orginates from the first one throught the following substitution: $\pi^u = \frac{(\lambda v_l + (1-\lambda)v_h - c)^2}{4} = \frac{(\lambda v_l + (1-\lambda)v_h - (\lambda + (1-\lambda)c)^2}{4} = \frac{\left[\lambda(v_l - c) + (1-\lambda)(v_h - c)\right]^2}{4}$)

By means of straightforward substitution, consumer surplus is:

$$CS^u = \frac{(\lambda v_l + (1-\lambda)v_h - c)^2}{8} + \frac{\lambda(1-\lambda)(v_h - v_l)^2}{2}$$

Total surplus, as the sum of consumer and producer surpluses, is:

$$W^u = \frac{3(\lambda v_l + (1-\lambda)v_h - c)^2}{8} + \frac{\lambda(1-\lambda)(v_h - v_l)^2}{2}$$

To see whether the monopolist prefers price discrimination or not, we compare its profits in the two cases. In particular, π^d- π^u>0 ends up in the following condition: $\left[(v_l - c) - (v_h - c)\right]^2 > 0$ which is always verified, because the square of a number is surely positive. This means that the monopolist prefers price discrimination.

To see whether the society as a whole prefers price discrimination or not, we compare total welfare in the two cases. In particular, W^u-W^d>0 ends up in the following condition: $\frac{1}{8}\lambda(1-\lambda)(v_h - v_l)^2 > 0$ which is always verified, because the product of positive terms is surely positive. This means that having a uniform price is welfare enhancing from the society point of view.

3) If price discrimination is not allowed, and only one market is served, the monopolist focuses on the high demand region (*h* in this example) and selects a price:

$p^n = \frac{v_n + c}{2}$, namely the monopoly equilibrium price in *h*, as computed in 1)

It is easy to show that, with $p^n = \frac{v_n + c}{2}$, market *l* is not served. Indeed $q_l^n = \frac{2v_l - v_h - c}{4}$ is negative for $v_l > (c+v_h)/2$, as indicated among the data.

Given $p^n = \frac{v_n + c}{2}$, we compute the monopolist profit, consumer surplus and total welfare:

$$\pi^n = (1-\lambda)\frac{(v_h - c)^2}{4}$$

$$CS^u = (1-\lambda)\frac{(v_h - c)^2}{8}$$

Comparing these results with those of 1) and 2), notice that total welfare is higher under a uniform price: $W^u > W^d > W^n$. The monopolist profit is, instead, maximum under price discrimination.

Exercise 1.14*

1) Under the second type-price discrimination, the monopolist offers a two parts tariff *(T+pq)*, thus having two choice variables (*T* and *p*) to be determined.
From the theory, we know that *T* equals consumer surplus in the low demand market (*l* in this example), while *p* is derived from the maximization problem.
Therefore:

$$T = \frac{(v_l - p)q_l}{2}$$

$$\max_p \pi = \lambda(p - AC_l)q_l + (1-\lambda)(p - AC_h)q_h + T = \lambda(p-c)(v_l - p) + (1-\lambda)(p-c)(v_h - p) + \frac{(v_l - p)q_l}{2}$$

From which we derive:

$$p^d = c + (1-\lambda)(v_h - v_l)$$

$$q_l^d = v_l - c - (1-\lambda)(v_h - v_l)$$

$$q_h^d = v_h - c - (1-\lambda)(v_h - v_l)$$

$$T = \frac{[v_l - c - (1-\lambda)(v_h - v_l)]^2}{2}$$

Consumer and producer surpluses are given by:

$$CS^d = (1-\lambda)\left\{\frac{[v_h - c - (1-\lambda)(v_h - v_l)]^2}{2} - \frac{[v_l - c - (1-\lambda)(v_h - v_l)]^2}{2}\right\}$$

$$\pi^d = \frac{(1-\lambda)^2(v_h - v_l)^2}{2} + \frac{(v_l - c)^2}{2}$$

2) The case in which price discrimination is not allowed is completely analogous to 2) in Exercise 1.13.

In particular, π^d-π^u>0, end up in the following condition $[(v_l - c)(2-\lambda) - (1-\lambda)(v_h - c)]^2 > 0$ which is always verified since the square of a number is surely positive. Therefore, price discrimination is preferable from the poit of view of the firm.

Exercise 1.15

1) Under the thrid type-price discrmination, the monopolist charges different prices to different – although clearly identifiable - groups of consumers. In particolar, in market *l*, it solves the following maximization problem:

$$\max_{p_l} \pi_l = (p_l - AC_l)q_l = p_l(1-p_l)$$

FOC are as follows:

$$FOC_{p_l} : 1 - p_l - p_l = 0$$

They yield the equilibrium price and quantity in market *l*, under the third type-price discrimination:

$$p_l^d = \frac{1}{2}$$

$$q_l^d = \frac{1}{2}$$

Through analogous reasoning, in market *h*, we find:

$$p_h^d = 1$$

$$q_h^d = 1$$

Considering the two markets, and their respective weights, the monopolist makes overal profit:

$$\pi^d = \frac{1}{2}\pi_l^d + \frac{1}{2}\pi_h^d = \frac{5}{8}$$

Overall consumer surplus is:

$$CS^d = \frac{1}{2}CS_l^d + \frac{1}{2}CS_h^d = \frac{5}{16}$$

Overall total welfare equals:

$$W^d = \frac{1}{2}W_l^d + \frac{1}{2}W_h^d = \frac{15}{16}$$

2) If price discrimination is not allowed, and both markets are served, a uniform price is set in both regions. The monopolist solves the following maximization problem:

$$\max_{p} \pi = \frac{1}{2}(p - AC_l)q_l + \frac{1}{2}(p - AC_h)q_h = \frac{1}{2}p(1-p) + \frac{1}{2}p(2-p)$$

From which, we find:

$$p^u = \frac{3}{4}$$

$$q_l^u = \frac{1}{4}$$

$$q_h^u = \frac{5}{4}$$

The monopolist's profit, consumer surplus and total welfare are:

$$\pi^u = \frac{9}{16}$$

$$CS^u = \frac{13}{32}$$

$$W^u = \frac{31}{32}$$

Comparing these results with those of 1), it is clear that price discrimination is better for the monopolist ($5/8=\pi^d > \pi^u=9/16$), not for the entire society ($15/16=W^d<W^u=31/32$). This is a general result, not depending on the parameter values (see Exercise 1.13).

Exercise 1.16

1) This is an example of thrid type-price discrmination, because the monopolist charges different prices to different – although clearly identifiable - groups of consumers. In particolar, it solves the following maximization problem in market *a*:

$$\max_{p_a} \pi_a = (p_a - AC_a)q_a = p_a(3 - p_a)$$

From the *FOC* we get:

$$p_a^d = \frac{3}{2}$$

This ca be substituted into the demand function to derive:

$$q_a^d = \frac{3}{2}$$

An analogous reasoning, in market *b*, yields the followings:

$$p_b^d = \frac{1}{2}$$

$$q_b^d = \frac{1}{2}$$

Considering the two markets, and their respective weights, the monopolist makes overall profit:

$$\pi^d = \frac{9-8\lambda}{4}$$

2) If price discrimination is not allowed, and both markets are served, the monopolist charges all consumers a uniform price. Therefore, it solves the following maximization problem:

$$\max_p \pi = (1-\lambda)(p - AC_a)q_a + \lambda(p - AC_b)q_b = (1-\lambda)p(3-p) + \lambda p(1-p)$$

From the *FOC* and the demand function, we obtain:

$$p^u = \frac{3-2\lambda}{2}$$

$$q_a^u = \frac{3+2\lambda}{2}$$
$$q_b^u = \frac{2\lambda - 1}{2}$$

$$\pi^u = \frac{(3-2\lambda)^2}{4}$$

3) If price discrimination is not allowed, and only one market is served, the monopolist focuses on the high demand region (*a* in this example) and charges a price:

$p^n = \frac{3}{2}$, i.e. the monopoly price in *a*, as in 1)

(It is easy to show that, with $p^n = \frac{3}{2}$ market *b* is not served, because q_b would be negative).

Thus:

$$q_a^n = \frac{3}{2}$$

$$\pi^n = \frac{9}{4}(1-\lambda)$$

4) To understand which option the monopolist would select, we compare its profits in 2) and 3).
Studying $\pi^n > \pi^u$ yields the following condition $\lambda(4\lambda\text{-}3)<0$, that is always verified for $\lambda \in \left(0, \frac{3}{4}\right)$.