Judicial Errors and Innovative Activity

Giovanni Immordino
Università di Salerno and CSEF

Michele Polo
Università Bocconi and IGIER

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Abstract: We analyze the effect of judicial errors on the innovative activity of firms. If successful, the innovative effort allows to take new actions that may be *ex-post* welfare enhancing (legal) or decreasing (illegal). Deterrence in this setting works by affecting the incentives to invest in innovation (*average deterrence).* Type-I errors, through over-enforcement, discourage innovative effort while type-II errors (under-enforcement) spur it. The *ex-ante* expected welfare effect of innovations shapes the optimal policy design. When innovations are *ex-ante* welfare improving, *laissez-faire* is chosen. When innovations are instead welfare decreasing, law enforcement should limit them through average deterrence. We consider several policy environments differing in the instruments available. Enforcement effort is always positive and fines are (weakly) increasing in the social loss of innovations. In some cases accuracy is not implemented, contrary to the traditional model where it always enhances (marginal) deterrence, while in others it is improved selectively only on type-II errors (asymmetric protocols of investigation).

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Acknowledgments: Giovanni Immordino Università di Salerno and CSEF, 84084 Fisciano (SA), Italy, giimmo@tin.it. Michele Polo, Università Bocconi, Via Sarfatti 25, 20136 Milan, Italy, michele.polo@unibocconi.it. We are indebted to Nuno Garoupa, Dilip Mookherjee, Massimo Motta, Marco Pagano, Matteo Rizzolli and Giancarlo Spagnolo for helpful discussions. Thanks also to seminar participants in Naples, Bonn, Milan Bicocca (Intertic Conference) the 2007 Italian Congress in Law and Economics (Milan) and C6 Conference (Capri). All usual disclaimers apply.

1 Introduction

The purpose of this paper is to study the effect of judicial errors on the innovative activity of firms. Quite often today the norms are required to rule delicate issues where innovative effort is involved, as in the design of liability rules for genetically modified organisms, or in the application of antitrust norms in high-tech industries. In these settings a widespread concern refers to the long term impact of the design and enforcement of norms on the rate of innovative activity. Indeed, a recurrent theme in competition policy is that antitrust should prevent abuses of dominant firms without chilling competition on the merits. In this context, the novelty of the issues brought to the attention of the enforcer by innovation makes the possibility of errors more likely than in standardized situations, and the analysis of judicial errors represents a major concern in enforcement.

The traditional approach of the Law and Economics literature is not fit to address these problems. It considers the choices of private agents among a set of feasible actions, some of which may cause a social damage and are therefore considered unlawful. In this setting the feasible actions are perfectly known and implementable by the individuals, the only restraint from taking harmful acts being the expected fines associated to illegal practices. The analysis focusses on the ability of law enforcement to discourage individuals from committing the most harmful actions, which represents the very notion of marginal deterrence[1].

This setting does not allow addressing the issues we want to analyze. Suppose, for instance, that the legislator wants to rule delicate areas as the liability issues in the production of genetically modified seeds, or the antitrust issues in the design of new products by dominant companies. In both cases the traditional problem, in which the private agents choose among a set of known actions, corresponds to the final stage of a process that requires initially to commit resources to research and to learning in order to identify the possible innovative solutions, among which the choice will be made in the end.[2], [3] Two features char-

[1] See Stigler (1970), Shavell (1992), Mookherjee and Png (1994), among others.

[2] To our knowledge, Kaplow (1995) is the only other paper where the design of the law affects agents' learning decisions. In his setting, more complex rules allow better control over individual behaviour but are harder for people to understand ex ante and for courts to apply ex post. In his setting, individuals can choose not to learn, and take actions ignoring the associated effects (and fines). Our model differs in that new actions can be taken only upon learning.

[3] This setting has some elements in common with the so called "activity level" model: see, for instance, Shavell (1980) and (2006) and Shavell and Polinsky (2000). According to this approach, private benefits and social harms depend on two different decisions of private agents: a level of activity (how long the individual drives) and a level of precaution (the speed). This literature has mainly focussed on a comparison of different liability rules (strict vs fault-based). In our paper the role of innovative effort resembles the activity while the choice of the new actions parallels precaution. The information structure, however, is different in our setting, since innovative effort is taken before uncertainty resolves, while in the "activity level" model uncertainty plays no role.

acterize these more complex situations: firstly, private agents have a richer set of decisions, as they initially choose whether and how much to invest in the innovative activity and then, if research has been successful, to pick out one of the innovative actions available; secondly, during the innovative process the private agents not only discover how to implement the new actions, but they also learn their social consequences and therefore whether *ex-post* they will be considered as lawful or unlawful according to the prescriptions of the legal rules.

In our analysis we associate the lawfulness or illegality of an action to its social consequences, i.e. whether it is welfare enhancing or welfare decreasing. Although we argue that in many instances the ultimate reason why legal rules consider an action as illegal lies with its social consequences, all the arguments we develop in the paper hold true even adopting a more formalistic definition of legality, based on whatever a norm prescribes to be lawful or not. All that matters in applying our analysis is that the elements that make an action lawful or unlawful according to the legal rule cannot be assessed with certainty at the time the investment in the innovative activity is chosen.

The inability to evaluate *ex-ante* with certainty the social consequences (or legality) of the innovative actions may depend on different reasons. Uncertainty may be rooted in the very nature of the research activity that the firm has to perform, so that the features of the innovation are unknown until discovery. For instance, in the example of the biotech firm, experiments with a new GM seed may promise higher yields but may also pose unknown risks to public health, that can be properly verified only once the research project has been concluded. Alternatively, uncertainty may derive from the interaction of the innovation, whose properties may be controlled and planned with sufficient confidence by the firm, with the economic or social environment at the time the innovation is introduced. The future features of this environment, in turn will depend on the decisions of a very high number of other agents and cannot be assessed *ex-ante* with certainty. In our second example, a dominant software company may invest in research to tie a new software application into a new operating system for PC's: beyond the initial intent of the company, the efficiency and foreclosure effects of this new software will depend, at the time of the commercial introduction, on the supply of alternative packages and applications by the competitors, that may be only imperfectly foreseen at the time of the research investment.

In this class of situations, deterrence works through an additional channel, by affecting the initial incentives to invest in research: if the private agents expect a very restrictive treatment of the results of their innovative effort, they will have lower incentives to commit resources to research. As a result, all the innovative actions will be discovered and possibly chosen with a lower probability. Deterrence in this case acts on the new actions "on average", reducing the likelihood that any of them will be taken, rather than selectively at the margin. For this reason we label this effect of law enforcement *average deterrence* to distinguish it from the traditional marginal deterrence effect. Notice that while more marginal deterrence is always welcome, and is therefore constrained only by the associated enforcement costs, average deterrence is only desirable when the innovative effort leads to new actions that *ex-ante* entail an expected welfare loss. Immordino, Pagano and Polo (2006), who first introduce this approach, show that when the positive welfare effect of innovations is *ex-ante* sufficiently large, average deterrence prevails on marginal deterrence and *laissez-faire* becomes the optimal policy.

In the present paper we adopt the same framework of Immordino, Pagano and Polo (2006) and consider the case of enforcers that may commit judicial errors.

Judicial errors and their reduction, i.e. accuracy, are a central concern in law enforcement: they have been analyzed in the standard model of law enforcement by Kaplow (1994), Kaplow and Shavell (1994, 1996), Polinsky and Shavell (2000) and Png (1986) among others, focussing on the (negative) impact or such errors on marginal deterrence. In this framework, accuracy is always desidable. Since reducing errors requires more enforcement resources, the optimal accuracy balances its marginal benefits and costs. In our approach, as discussed above, deterrence works (also) through the incentives for research and innovation. In this setting, more accuracy is not always desirable, contrary to the traditional result.

Following the literature, we can distinguish two types of errors: the enforcer may mistakenly convict an innocent or mistakenly acquit a guilty. The first case corresponds to a type-I error in statistical inference and it is labelled as a case of over-enforcement or false positive in the L&E jargon, while the second entails a type-II error and involves under-enforcement and false negative. Errors of different types affect average deterrence in opposite directions: type-I errors, inducing over-enforcement, reduce expected profits from learning effort and therefore limit the incentives to innovate. Conversely, type-II errors, through under-enforcement, boost innovative effort and reduce average deterrence. This asymmetry in the effects of errors does not emerge in the traditional model of law enforcement based on marginal deterrence, that is weakened by either type of error in the same way.

When we consider the investigation activity in our setting, the enforcer has two tasks: he first has to recognize properly the new actions chosen, and secondly he has to assess their lawfulness, which in short we connect to their welfare consequences. We argue that this latter task is the more compelling exercise, due to the novelty of the innovations: when examining the seeds, it is simpler to identify that they are a new GM variety rather than assessing their effects on public health. Moving to our second example, the enforcer can easily check that a new operating system has some applications tied in, but it is much harder to analyze the foreclosure potential of this tying strategy. To stress this point, we analyze the case where judicial errors may be committed only when analyzing the welfare consequences (lawfulness) of the innovative actions.

We consider different sets of instruments that the enforcer can use: the level of fines, the probability of recognizing the actions chosen (related to the enforcement effort) and the probability of correctly assessing the consequences (lawfulness) of the chosen actions (related to the accuracy effort). In this framework we distinguish different cases, that may better fit specific situations.

General (exogenous) versus specialized (endogenous) enforcement effort. If the enforcer monitors at the same time a wide and diversified range of private conducts (e.g. safety conditions of many types of GM vegetables and seeds, or industrial strategies of dominant firms), it cannot fine-tune its activity to the specific practice that a given norm is ruling (e.g. the GM seeds or the practice of tying). In this case the probability of identifying the practices chosen might be higher or lower, depending on the resources dedicated to general monitoring, but it will be the same for any illegal private behavior, i.e. it will be exogenous in our policy analysis. Alternatively, the enforcer might be able to devote specific resources to monitoring a certain kind of practices that are ruled by a given norm: by increasing or decreasing the resources committed to this specialized monitoring activity the enforcer can affect selectively the probability of identifying those given practices, (the GM seeds or the tying policy), making the enforcement effort endogenous in the analysis.[4]

General (common) accuracy on both types of error versus specific type-I and type-II accuracies. In this case we can analyze how accuracy can be improved, by committing more resources, on both types of errors at the same rate or selectively on each of them.

To illustrate this point, consider the following example, drawn from antitrust. Suppose that the welfare effects of a given practice of a dominant firm depend, in decreasing order, on four fundamental variables: market shares, entry conditions, demand elasticity and cost efficiencies. If we choose a very low level of accuracy, we might just consider the market shares, concluding that if the market share is high the practice is harmful while welfare is enhanced (or unaffected) by the practice if the firm has a small market share.

If however we opt for greater accuracy, we might proceed in different ways when assessing additional variables, improving accuracy selectively on one or the other type of error or opting for a common level of accuracy on both of them. A case of selective accuracy can be illustrated by this protocol of investigation:

1. The enforcer considers first the market share of the dominant firm: if this is above a

[4] The traditional example of general versus specialized monitoring refers to patrolling a highway, an activity that allows to identify with the same probability any breach of the driving rules, as opposed to the use of remote speed control facilities, that instead allow to elicit cases of excessive speed only. On this issue see Mookherjee, D. and Png, I. P. L. (1992).

certain threshold, the enforcer consider the firm guilty (the action socially harmful); if instead the market share falls short of the threshold

2. the enforcer moves on considering the entry conditions: it entry is hard, the firm is condemned while easy entry leads to the next step

3. where the enforcer analyzes demand elasticity: if demand is inelastic the firm is condemned while in case of elastic demand the enforcer proceed to the final step

4. where efficiencies are considered: low efficiencies lead to condemn the firm while substantial cost reductions justify to clear the case.

This example shows a protocol of investigation in which further levels of accuracy are implemented asymmetrically, requiring a more compelling standard of proof for positive welfare effects and instead concluding for welfare losses (illegality) more easily. The enforcer is quite accurate in examining all the arguments before concluding in favour of the firm, and therefore reduces the probability of clearing the case when the firm is guilty (type-II errors); at the same time, the enforcer is quite biased towards condemning the firm, an outcome that can occur at any stage if any of the adverse circumstances is assessed. Hence, this protocol implements selectively type-II accuracy.[5]

Conversely, common accuracy would correspond to a protocol of investigation that prescribes to collect the evidence on, say, market share, entry conditions, demand elasticity and cost efficiency and then, considering all the evidence together, to verify the overall impact of the practice and therefore its lawfulness. We might conclude, for instance, contrary to the outcome of the asymmetric protocol considered above, that the practice is welfare enhancing even if the market shares are large because the absence of switching costs determine a very elastic demand.[6]

In both cases, more resources are needed to verify the wider set of evidence required. But the different protocols imply that investigations go more in depth symmetrically or asymmetrically. Arguably, an asymetric protocol of investigation may lead to a decision against the firm based on a biased selection of the factual elements. This feature may lead, given the precedents, to an overrule in the appeal phase, where completeness and balancedness of the arguments are evaluated. For this reason the enforcer might prefer to adopt a common accuracy framework rather than an asymmetric one.

[5] Different levels of type-II accuracies may be obtained then by interrupting the protocol of investigation at stage 3 or 2.

[6] Also in this case we can set the desired level of (common) accuracy deciding how many variables we want to consider in the overall assessment. For a detailed and interesting discussion of errors in antitrust enforcement against unilateral practices see Lear (2006).

In this paper we study the optimal policies starting from the environment in which the enforcer has the wider set of instruments, i.e. endogenous enforcement and specific accuracies (case 1); then we move to a second best environment in which the enforcer either cannot control enforcement effort, that is exogenous (case 2), or has to implement a common level of accuracy (case 3). A third best setting in which enforcement is exogenous and accuracy is common (case 4) concludes the analysis.

The main findings of our analysis can be summarized in the following way.

- Type-I and type-II errors have an opposite impact on average deterrence: the former, through over-deterrence, reduce the incentives to innovate while the latter work in the opposite direction. This is quite in contrast with the traditional model, where both types of error reduce in the same way marginal deterrence;
- When the *ex-ante* effect of innovation is welfare enhancing, due to a low probability that the innovation is socially damaging and/or to its limited social loss, the optimal policy requires to adopt a *laissez-faire* or *per-se legality* regime through zero enforcement effort and/or zero fines. This outcome occurs in the same region of parameters no matter which are the policy instruments available to the enforcer.
- When instead the innovative activity is *ex-ante* socially damaging, the optimal policy should limit it through average deterrence. The design of the policy depends on the policy instruments available.
  - When enforcement is endogenous and accuracies are separately set (case 1) the enforcer implements a mix of maximum fines, positive enforcement and type-II accuracy. It is therefore optimal to adopt an asymmetric protocol of investigation.
  - When enforcement is exogenous (case 2) and the innovative activity gives a negative but limited welfare loss the enforcer initially curbs the incentives to innovate by increasing the fine without spending on accuracies. Once hit the maximum fine, type-II accuracy is progressively improved.
  - The case of common accuracy and endogenous enforcement (case 3) adds a new twist to the optimal policy. The optimal policy entails a positive level of enforcement, increasing in the social loss. Common accuracy is pursued only when the bad state is relatively likely: in this case, in fact, type-II errors occur more often and drive the ex-ante effect. Common accuracy, therefore, reduces type-II errors, under-deterrence and innovation. When instead the bad state is relatively unlikely (although the ex-ante effect of innovation is welfare decreasing due to the huge losses in the bad state), errors lead more often to over-deterrence and

therefore work in the desired direction: in this case accuracy would not improve the enforcement policy.

– Finally, when enforcement is exogenous and accuracy is common (case 4), we obtain a combination of the previous results, with a region of parameters with no accuracy and increasing fines, one with maximum fine and no accuracy and the last where some accuracy is chosen together with maximum fines.

Compared to the existing literature, therefore, we show that when average deterrence is the driving effect in law enforcement type-I and type-II errors have a very different impact on the firm's choices and accuracy is not necessarily welcome: in some cases it is not implemented while in others it is improved selectively only on type-II errors (asymmetric protocols of investigation).

Our paper is organized as follows. Section 2 presents the model. Section 3 analyzes the firm's choices regarding the action and the investment in innovative activity. Section 4 and its subsections consider the different environments and the associated optimal polices (case 1-4). Section 5 concludes. All the proofs are in the Appendix.

2 The model

We consider a model with a profit-maximizing firm, and a benevolent enforcer that may commit mistakes. The firm can either choose one among several known and lawful actions or invest in learning to identify a new action, whose private and social effects are *ex-ante* unknown.

The key issue that we wish to explore is: what is the optimal design of fines, enforcement and accuracy when private innovative activity is important and enforcers are subject to judgement errors.

The firm can choose the *status-quo* action a_0 (planting traditional seeds, offering an untied application) with associated profits $\Pi(a_0)$ and welfare $W(a_0)$: we normalize these two measures to zero, i.e. $\Pi(a_0) = W(a_0) = 0$. Action a_0 is the most profitable among the known and legal actions that the firm is able to implement without investing in learning. It is correctly recognized by the enforcer in its own nature (a_0) and social consequences ($W(a_0)$).

Alternatively, the firm can consider a new action a (*innovation*), with associated profit $\Pi(a) = \Pi > 0$.[7] Depending on the state of nature s, the social consequences of the new

[7] In this paper we consider just one possible new action as a result of the learning effort, rather than a set

action differ. With probability β, a bad state $s = b$ occurs, where the new action has a negative social externality, $W(a) = W_b(a) = \underline{W} \leqslant 0$.[8] In this case, private incentives conflict with social welfare. With probability $1 - \beta$, instead, a good state $s = g$ materializes and the new action improves welfare, $W(a) = W_g(a) = \overline{W} > 0$. In this case, there is no conflict between private and social incentives, since the innovation improves both the profits of the firm and social welfare. Nature chooses which state of the world occurs; hence, the probability β of the bad state (social harm) is an *ex-ante* measure of the likelihood of misalignment between public interest and firms' objectives. In our example, β is the prior probability that the GM seeds will be hazardous to public health, or that the new tied application, when introduced in the market, will foreclose alternative software packages. The prior probability β, and its interpretative model of the issue, is summarized in our discussion of the parameters $(\beta, \underline{W}, \overline{W})$.

While the firm knows from the beginning how to implement the status-quo action a_0, carrying out the new action requires an investment in learning (experiment with GM seeds, create a new tied application), which accordingly will be referred to as "*innovative activity*". If the investment is successful, the firm will discover how to implement the new action a. In this case, the firm also learns the state of nature s, that is whether its innovation is socially harmful or beneficial. Proceeding with our example, the biotech company learns not only how to produce new GM seeds, but also the dangers that they pose to public health and the damages that it might face according to the current liability rules. And the software company, once the new application is created, is able to predict whether in the current market conditions it will enable to foreclose the alternative packages or not.

The amount of resources I that the firm invests in the innovative activity determines its chances of success: for simplicity, the firm's probability $p(I)$ of learning how to carry out the new action a is assumed to be linear in I, i.e. $p(I) = I$ with $I \in [0, 1]$. The cost of learning is increasing and convex in the firm's investment. For simplicity we assume $c(I) = c\frac{I^2}{2}$ with

$$c > \Pi \tag{1}$$

to ensure an internal solution.

The institutional framework in the design and enforcement of norms is as follows. The legislator writes the legal rule, which specifies under what circumstances the actions are legal or not, and the admitted range of fines. The enforcement officials commit to a certain

of new actions. This latter case, that is analyzed in Immordino, Pagano and Polo (2006), allows to consider also the traditional issue of marginal deterrence, i.e. the choice by the firm of one among many illegal actions. Since the distinguishing feature of our approach is in the effect of deterrence on innovative activity (what we later call as average deterrence), we drop the marginal deterrence issue considering a single new action.

[8] In the comparative statics of the equilibrium we will vary $\underline{W}$ to change the social loss of the new actions.

fine schedule and seek evidence on firms' actions (enforcement) and on the associated social consequences (accuracy). We assume that enforcers are benevolent but may make errors.[9]

The legal rule identifies some circumstances that make the new action legal or unlawful. In general we can adapt this setting to a wide range of formal frameworks: for instance, the norm may state that the new action is illegal whenever it occurs together with contingencies $x_1, ..., x_n$, a case that reminds more or less articulated *per-se* rules.[10] Alternatively, illegality may be related to the effects of the action, as required under a rule of reason approach.[11] It is important to stress that our analysis can be adapted to either of the two cases. All that matters is that, at the time of the innovative investment, the elements that the legal rule identifies in order to assess the legality of the new action are not known with certainty. With this important caveat in mind, we consider a legal rule written as follows, that allows us to simplify greatly the notation in the analysis:

> The action a_0 is lawful; the (new) action a is illegal if *ex-post* socially damaging, i.e. if $W(a) < 0$. The illegal action is sanctioned according to a fine f chosen in the interval $[0, F]$.

For instance, the legal rule prohibits to commercialize hazardous seeds or to adopt practices that foreclose the market to competitors.

In order to enforce the legal rule the enforcer has first to identify the action chosen (a_0 or a) and the social consequences of the action (0 or $W(a)$). Obtaining evidence on these elements requires to commit resources. We define respectively as *enforcement* and *accuracy* the activities devoted to obtain evidence on the action chosen and on its consequences (legality). By increasing the resources dedicated to enforcement (accuracy) the enforcer obtains with a higher probability hard evidence on the action chosen (on its consequences and legality).

Given the fine f, the expected fine depends on the probability of enforcement, i.e. on the ability of the enforcer to find hard evidence on the action chosen, and on the accuracy in assessing the social consequences of the action.

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[9] In the present setting there is no real difference between the authority and the official. Hence, we refer to them as "the enforcer".

[10] Drawing from antitrust, for instance, a very simple per-se rule would consider as illegal an action as the practice of resale price maintenance when adopted by a firm with a market share larger than $x\%$. A more articulated rule would consider resale price maintenance as illegal when adopted by a dominant firm, where this latter is identified by certain thresholds in market shares (x_1), entry conditions (x_2) and demand elasticity. (x_3), and when sales effort activities are irrelevant (x_4).

[11] For a discussion on an effect-based interpretation of antitrust norms, see Gual et al. (2005).

More specifically, the probability of enforcement is positively affected by the amount of resources E devoted to monitoring firms' actions: with probability $q(E)$ the enforcer obtains hard evidence that the firm took action a. For simplicity, we assume the probability $q(E)$ to be linear in E, i.e. $q(E) = E$. The cost of the enforcement effort is convex, implying decreasing returns to enforcement: $g'_E > 0$ and $g''_E > 0$ for $E \in [0, 1]$, with $g_E(0) = g'_E(0) = 0$ and $\lim_{E\to 1} g(E) = \lim_{E\to 1} g'(E) = \infty$. With probability $1 - q(E)$, instead, the enforcement effort does not produce enough evidence to prove that the firm took action a. In the benchmark model the level of enforcement effort is positive and exogenous while endogenous enforcement is considered later on.

Once the enforcer has successfully identified the action chosen by the firm, he still has to identify its social consequences (lawfulness). We assume that the enforcer is more accurate in assessing the effects of the status-quo rather than the new action. Judicial errors occur only when assessing the effects (legality) of the new action a, while the status-quo action a_0 is correctly recognized as legal. This different degree of accuracy reflects the more compelling task of assessing new rather then well known phenomena.

More precisely, the enforcer when investigating the effects of the new action receives a signal $\sigma = \{b, g\}$ on the state of nature, i.e. on the social consequences of the new action. With probability α_I the signal is incorrect when the true state of the world is the good one: in this case the enforcer considers action a as unlawful when the good state occurs, committing a "type-I error". Conversely, with probability α_{II} the signal is incorrect when the true state is the bad one, and a "type-II error" occurs, i.e. the enforcer will fail to identify a as unlawful when the true state is the bad one. Hence,

$$\alpha_I = \Pr(\sigma = b \,|s = g\,) \quad \text{and} \quad \alpha_{II} = \Pr(\sigma = g \,|s = b\,)$$

We assume that the signals received are informative, i.e. $\alpha_I \leqslant \frac{1}{2}$ and $\alpha_{II} \leqslant \frac{1}{2}$.

The level of accuracy of the enforcer can be improved by committing more resources to obtain a more precise assessment of the effects. As we argued in the introduction, accuracy can be refined regarding type-I, type-II or both types of errors. By adopting different protocols of investigation and standards of proof and by committing more resources, the enforcer can reduce selectively type-I or type-II errors or can symmetrically improve the assessment reducing both types of errors.

We assume that the cost of a given probability α_I of type-I error is $g_I(\frac{1}{2} - \alpha_I)$, where $\alpha_I = \frac{1}{2}$, i.e. a completely uninformative signal, corresponds to the lowest accuracy, with $g'_I > 0$ and $g''_I > 0$ for $\alpha_I \in \left[0, \frac{1}{2}\right]$, and with $g_I(0) = g'_I(0) = 0$ and $\lim_{\alpha_I\to 0} g(.) = \lim_{\alpha_I\to 0} g'(.) = \infty$. Similarly, for the cost of decreasing type-II errors we assume: $g_{II}(\frac{1}{2} - \alpha_{II})$ with $g'_{II} > 0$ and $g''_{II} > 0$ for $\alpha_{II} \in \left[0, \frac{1}{2}\right]$, with $g_{II}(0) = g'_{II}(0) = 0$ and $\lim_{\alpha_{II}\to 0} g(.) =$

$\lim_{\alpha_{II} \to 0} g'(.) = \infty$.[12]

The timing of the model is the following. At time 0 nature chooses the state of the world $s = \{g, b\}$ which is not observed by any agent. Agents know that the probability of the bad state is $\beta > 0$. At time 1, the legislator writes the norm and the enforcer commits to a certain fine f, to the effort devoted to enforcement E and to accuracy α_I and α_{II}. At time 2, the firm chooses the innovative activity I and learns with probability $p(I) = I$ how to implement the new action a and its payoffs $\Pi(a)$ and $W(a)$ (state of the world), knowing the norm, the fine schedule, the enforcement probability E and the probabilities of error α_I and α_{II}. At time 3, the firm chooses an action, conditional on what it learnt in the previous stage. Finally, at time 4 the action chosen determines the private profits and the social welfare; the enforcer collects evidence (with errors) and possibly levies fines.

Finally, we assume the following ranking among payoffs:

$$\overline{W} > \Pi \geqslant F > 0. \tag{2}$$

The first inequality implies that in the good state social gains exceed private ones, or, equivalently, that the new action in good state increases consumers' surplus as well as producer' surplus. According to the second inequality, the profits from the new action exceeds or is equal to the maximum fine, implying that the firm, if the innovative effort is successful, always prefers to choose the new action (incomplete deterrence)[13]. Even in this case, however, some room for deterrence remains through the effects of the enforcement policy on the innovative activity I and on the probability to take the new action.

In the next sections we analyze the firm's choices regarding the action and the innovative effort and then we move to considering different policy scenarios combining endogenous or exogenous enforcement effort and specific or common accuracy.

3 Firm's choices: actions and innovative activity

At stage 3, depending on whether its innovative activity was successful or not, the firm chooses an action. If the innovative activity was unsuccessful, under our assumptions the firm chooses the status quo action a_0 with associated profits $\Pi(a_0) = 0$ and welfare $W(a_0) = 0$. If instead the innovative activity was successful, the firm is able to take the new action a. If the action is not socially harmful ($s = g$) the action a is lawful. Nevertheless, with probability α_I the authority perceived state of the world is the bad one ($\sigma = b$). Then,

[12] When $\alpha_I = \alpha_{II} = \alpha$ the same assumptions apply to $g_\alpha(\frac{1}{2} - \alpha)$.

[13] The firm will choose the new action even when $\Pi = F$, because the fine will never be inflicted with certainty due to the Inada conditions on enforcement and accuracy.

when the firm chooses the profit maximizing action a (that gives also the maximum welfare $\overline{W}$) expected profits are equal to $\Pi - E\alpha_I f$. If, alternatively, the firm chooses the action a_0 profits are equal to 0 and there is no error in enforcement. Assumption (2) implies that $\Pi - E\alpha_I f > 0$ for any fine f, enforcement E and probability of type-I error α_I. The firm will then choose the new action a.

If instead the new action is socially harmful ($s = b$), and therefore unlawful, the fine is inflicted only with probability $E(1-\alpha_{II})$ since with probability α_{II} the enforcer receives the wrong signal $\sigma = g$. In this case when the firm chooses the new action a (that gives also the minimum welfare $\underline{W}$) expected profits are equal to $\Pi - E(1-\alpha_{II}) f$. Again due to assumption (2), $\Pi - E(1-\alpha_{II}) f > 0$ and the firm will choose the unlawful action a.

At stage 2, knowing the enforcement and accuracy efforts, the firm chooses its innovative activity I so as to maximize its expected profits, given the optimal actions that it will choose at stage 3. The firm learns how to carry out the new project with probability $p(I) = I$ and its expected profits at this stage are:

$$E\Pi = I\left\{\beta\left[\Pi - E(1-\alpha_{II}) f\right] + (1-\beta)\left[\Pi - E\alpha_I f\right]\right\} - c\frac{I^2}{2}, \tag{3}$$

where the first term is the expected gain from innovative activity (net of the expected fines), positive by assumption (2) and the second term is the cost of innovative activity. The optimal innovative activity is therefore[14]:

$$\widehat{I}(E, f, \alpha_I, \alpha_{II}) = \frac{\Pi - [\beta(1-\alpha_{II}) + (1-\beta)\alpha_I]Ef}{c} \tag{4}$$

Notice that $\widehat{I}(.)$ is greater than zero thanks to assumption (2) and smaller than one by assumption (1). The effect of the probability of type-I error α_I on innovative activity is given by:

$$\frac{\partial \widehat{I}}{\partial \alpha_I} = -\frac{(1-\beta)Ef}{c} \leqslant 0. \tag{5}$$

Since a type-I error corresponds to over-enforcement, when type-I errors become more likely the expected profits are reduced and the incentives to exert innovative activity fall accordingly. The effect of the probability of error α_{II} on innovative activity is given by:

$$\frac{\partial \widehat{I}}{\partial \alpha_{II}} = \frac{\beta Ef}{c} \geqslant 0. \tag{6}$$

In contrast to type-I, type-II errors correspond to an under-enforcement bias that favors the innovative activity.

When the enforcer cannot set the type-I and type-II accuracies separately and has to rely on a common level of accuracy $\alpha_I = \alpha_{II} = \alpha$, the optimal innovative effort becomes:

$$\widehat{I}(E, f, \alpha) = \frac{\Pi - [\beta(1-\alpha) + (1-\beta)\alpha]Ef}{c} \tag{7}$$

[14] The second order condition is clearly satisfied.

The effect of errors on the innovative activity is given in this case by:

$$\frac{\partial \widehat{I}}{\partial \alpha} = -\frac{(1-2\beta)Ef}{c} \gtreqless 0 \Leftrightarrow \beta \gtreqless \frac{1}{2}. \tag{8}$$

This result can be explained as follows: type-I errors occur in the good state and correspond to over-enforcement, lowering the expected profits; conversely, type-II errors occur in the bad state and entail under-deterrence and higher expected profits. When the probability α of committing an error is the same for the two errors, type-I errors are more frequent than type-II errors if the good state is more likely, i.e. $\beta < \frac{1}{2}$. Over-enforcement in this case is the predominant effect, reducing the expected profits and the investment $\widehat{I}$ in innovative activity. More (common) accuracy, that is lower α, reducing over-deterrence, spurs innovation. The opposite holds true if the bad state is relatively likely, i.e. $\beta > \frac{1}{2}$.

Finally, the enforcement effort E and the fine f depress the firm's innovative activity, both for common and specific accuracies[15]:

$$\frac{\partial \widehat{I}}{\partial E} = -\frac{[\beta(1-\alpha_{II}) + (1-\beta)\alpha_I]f}{c} \leqslant 0, \quad \frac{\partial \widehat{I}}{\partial f} = -\frac{[\beta(1-\alpha_{II}) + (1-\beta)\alpha_I]E}{c} \leqslant 0.$$

We summarize our main findings with the following Proposition.

Proposition 1: *The innovative activity is deterred by a higher enforcement effort E and a higher fine f. When type-I and type-II errors are set separately, the innovative activity is deterred by a higher level of type-II accuracy (lower α_{II}) and a lower level of type-I accuracy (higher α_I). When instead the enforcer can set only a common level of accuracy α, this latter deters the innovative activity if and only if the bad state is relatively likely, i.e. $\beta > \frac{1}{2}$.*

Before moving to the analysis of the optimal policies, it is useful to highlight the impact of type-I and type-II errors in the traditional model based on marginal deterrence compared to the opposite effects on average deterrence described above. The easiest way[16] to reshape our setting into the standard law enforcement model is to consider two actions a_g and a_b corresponding to the new action a in the good and in the bad state. The associated profits, net of the expected fines, are[17]

$$\Pi(a_g) = \pi(a_g) - E\alpha_I f$$

[15] The derivatives in the case of common accuracy have the same signs and can be easily obtained by imposing $\alpha_I = \alpha_{II} = \alpha$.

[16] We thank Matteo Rizzolli for suggesting this way of presenting the issue.

[17] In our model $\pi(a_g) = \pi(a_b) = \Pi$. Hence, the condition (9) always hold. In a more general setting, however, this condition might fail.

and

$$\Pi(a_b) = \pi(a_b) - E\,(1 - \alpha_{II})\,f$$

In the traditional setting both actions are feasible at the same time and the firm will take just one of them, while the enforcement policy aims at tilting the comparison in favour of a_g. This outcome can be reached as long as, after rearranging,

$$Ef(1 - \alpha_I - \alpha_{II}) \geqslant \pi(a_b) - \pi(a_g). \tag{9}$$

It is evident that both errors work in the same direction reducing $\Pi(a_g)$ and increasing $\Pi(a_b)$, making harder to meet such inequality. In other words, both types of error weaken in the same way marginal deterrence.

In our setting, instead, either a_g or a_b will be *ex-post* available, but at the time of the innovative investment the firm's expected profits from these new actions are $E\Pi = (1-\beta)\Pi(a_g) + \beta\Pi(a_b)$. Then, the two errors affect in opposite directions the expected profits from the new actions and the incentive to invest in innovative activity.

4 Optimal enforcement policies

Once identified the firm's choices regarding the action and the innovative effort, we can move to the analysis of the optimal policy in the different scenarios.

The monitoring activity on markets and practices, that allows to identify the actions chosen by the firm, is related to the enforcement effort E: we distinguish the case of general monitoring, that cannot be adapted practice by practice to specific conducts, and specialized monitoring that instead can be targeted to identifying a specific practice. For instance, the antitrust authority might exert general monitoring on the strategies of dominant firms, discovering with the same probability any of the practices realized, or alternatively it might focus on a specific strategy, e.g. tying, monitoring this practice with a dedicated team. In the former case, the enforcement effort E is exogenous with respect to the design of the optimal policy on a given practice (e.g. tying) while in the latter E is set endogenously when shaping the optimal policy towards that practice. Accuracy can be pursued in different ways as well. We may have a design of the protocols of investigation and of the standards of proof that determine a common or a specific level of accuracy for type-I and type-II errors. In the introduction we offered a practical example related to antitrust.

We start from the less constrained environment, in which the enforcer can determine endogenously the fine f, the enforcement effort E and the two levels of accuracy α_I and α_{II}, moving then to second and third best cases where the set of instruments narrows.

4.1 Case 1: endogenous enforcement and specific accuracies

In this section we analyze the situation when the enforcer has the wider set of instruments available. This case corresponds, in our classification, to endogenous enforcement effort and accuracies that can be set separately for the two types of error. In this setting, therefore, the enforcer controls four instruments: E, α_I and α_{II}, which require to spend resources, and the fine f.

The expected welfare, once taken into account the firm's optimal choices, is:

$$\mathrm{E}W = \widehat{I}(E, f, \alpha_I, \alpha_{II})\Delta\mathrm{E}(W) - g_E(E) - g_I(\frac{1}{2} - \alpha_I) - g_{II}(\frac{1}{2} - \alpha_{II}) - c\frac{\widehat{I}(E, f, \alpha_I, \alpha_{II})^2}{2}, \quad (10)$$

where $\Delta\mathrm{E}(W) \equiv [\beta\underline{W} + (1-\beta)\overline{W}]$ is the expected welfare change due to the new action a, while the last four terms capture the public cost of enforcement and accuracy and the private costs of the innovative activity.

The policy instruments affect, through average deterrence, the investment in innovative activity described in (4). The relevant first order conditions are:

$$\frac{\partial \mathrm{E}W}{\partial f} = [\Delta\mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial f} \gtreqless 0, \quad (11)$$

$$\frac{\partial \mathrm{E}W}{\partial E} = [\Delta\mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial E} - g'_E \leqslant 0, \quad (12)$$

$$\frac{\partial \mathrm{E}W}{\partial \alpha_I} = [\Delta\mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha_I} + g'_I \geqslant 0, \quad (13)$$

and

$$\frac{\partial \mathrm{E}W}{\partial \alpha_{II}} = [\Delta\mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha_{II}} + g'_{II} \geqslant 0. \quad (14)$$

The four derivatives have the same structure. The first term captures the marginal effect of the policy variables on the innovative activity, *average deterrence*. The second term, which is zero in the case of fines, is the marginal cost of the policy. The optimal choice of the policy variables, therefore, depends on the sign of the marginal social value of the innovative activity, $\Delta\mathrm{E}(W) - c\widehat{I}$ that can be positive or negative according to the "economic model" that characterizes the enforcer's *ex-ante* assessment of the new action, summarized by the probability of the bad state β and the associated social loss $\underline{W}$.[18] Hence, in general we may expect internal as well as corner solutions.

To characterize the optimal policy, we start by substituting in the expression for the marginal social value of the innovative activity, $[\Delta\mathrm{E}(W) - c\widehat{I}]$, the optimal $\widehat{I}$ chosen by the

[18] We run our comparative statics excercises with respect to β and $\underline{W}$ keeping the welfare gain in the good state $\overline{W}$ constant. Considering variations in this latter parameter dos not add any insight to the analysis.

firm according to (4) and we solve for the social loss $\underline{W}$. The result is the following locus:

$$\underline{W}_0(\underset{-}{f}, \underset{-}{E}, \underset{-}{\alpha_I}, \underset{+}{\alpha_{II}}, \underset{+}{\beta}) = \frac{\Pi - (1-\beta)\overline{W} - [\beta(1-\alpha_{II}) + (1-\beta)\alpha_I]Ef}{\beta}$$

which describes, for given policy parameters, the combinations $(\underline{W}, \beta)$ corresponding to $[\Delta \mathrm{E}(W) - c\widehat{I}] = 0$. This locus is increasing and concave in the $(\underline{W}, \beta)$ space, above it the marginal social value of innovative activity is negative and below the locus it is positive. A central role in the analysis is played by the locus $\underline{W}_0(.)$ measured at the policy parameters $f = 0$, $E = 0$, $\alpha_I = \alpha_{II} = \frac{1}{2}$:

$$\underline{W}_0(0, 0, \frac{1}{2}, \frac{1}{2}, \beta) = \frac{\Pi - (1-\beta)\overline{W}}{\beta}. \tag{15}$$

Recalling that $\underline{W}$ has been assumed to be non-positive, we must keep account of a parametric restriction on β. More specifically, β must be smaller or equal than $\beta_1 = \frac{\overline{W}-\Pi}{\overline{W}}$.[19]

Finally, in order to ensure a maximum in the policy problem we assume that the cost of enforcement and type-II accuracy are sufficiently convex: more precisely, a sufficient condition is:

$$g''_E g''_{II} \geqslant [\Delta \mathrm{E}(W) - c\widehat{I}]^2 \left(\frac{\partial \widehat{I}}{\partial E \partial \alpha_{II}} \right)^2. \tag{16}$$

The following Proposition establishes the optimal policy in the different regions.

Proposition 2: *In the space ($\underline{W}$, β) we can distinguish the following regions:*

i) When the marginal social value of the innovative activity is non negative, i.e. for $0 \geqslant \underline{W} \geqslant \underline{W}_0(0, 0, \frac{1}{2}, \frac{1}{2}, \beta)$ and $\beta \in [0,\ \beta_1]$, the optimal policy entails "laissez faire", i.e. $E^* = 0$, $\alpha_I^* = \alpha_{II}^* = 1/2$.

ii) When the marginal social value of the innovative activity is negative, i.e. for $\underline{W} < \min\left\{\underline{W}_0(0, 0, \frac{1}{2}, \frac{1}{2}, \beta),\ 0\right\}$ and $\beta \in [0,\ 1]$, the optimal policy prescribes the maximum fine, a positive enforcement effort and type-II accuracy: $f^* = F$, $E^* > 0$, $\alpha_I^* = 1/2$ and $\alpha_{II}^* \in \left(0,\ \frac{1}{2}\right)$. Both the enforcement effort and type-II accuracy increase when the marginal social value of the innovative activity becomes more negative, with $E^* \to 0$ and $\alpha_{II}^* \to 1/2$ when $\underline{W} \underset{-}{\to} \underline{W}_0(0, 0, \frac{1}{2}, \frac{1}{2}, \beta)$.

Proof. See the Appendix. ■

[Figure 1 about here]

[19]Notice that $0 < \beta_1 < 1$.

In Figure 1 it is shown that whenever the innovative activity is *ex ante* welfare enhancing (below the locus $\underline{W}_0(0, 0, \frac{1}{2}, \frac{1}{2}, \beta)$), the optimal policy is aimed at sustaining the innovative effort. The enforcer does not monitor the firms concerning the practice a, i.e. $E^* = 0$. In this case there is no reason to spend in accuracy because no case is opened. Hence, even if the legal rule would consider in certain circumstances the new action a as illegal, the enforcer opts for a *laissez-faire* or *per-se legality* rule. Alternatively, the legislator might anticipate this policy and simply consider a as legal.

When instead the expected welfare is reduced by the innovative activity the enforcer aims at limiting the innovative effort through average deterrence. In this case the Becker argument on maximum fines applies: as long as some average deterrence is needed, the cheapest way to implement it is by setting $f = F$ and save on costly enforcement E. Moreover, both enforcement E and type-II accuracy α_{II} reduce the incentive to innovative activity affecting average deterrence in the desired direction. When the costs of enforcement and accuracy are sufficiently convex, as assumed, we prefer to use a mix of the two instruments rather than a single one. In this case when the social loss in the bad state gets worse, calling for less innovative activity, both enforcement and type-II accuracy are increased, i.e. they work as complements. For the same reason no type-I accuracy is pursued since the associated over-deterrence bias in the good state works in the desired direction on the innovative activity.

Notice that when average deterrence is needed, the enforcer does not implement accuracy on both types of error, but rather adopts an asymmetric protocol of investigation as illustrated above. Turning back to the antitrust example discussed in the introduction, if the innovative activity decreases the expected welfare, the enforcer should adopt a protocol of investigation that selectively proceeds with further investigations as long as the interim assessment suggests a social gain, while it stops the investigation and condemns the firm as long as a negative welfare effect can be argued. This procedure allows to reduce the type-II errors and at the same time is biased towards type-I errors. We argue that asymmetric protocols of investigation of this sort often characterize the way in which antitrust authorities handle cases in practice. Our result suggests that these asymmetric protocols are indeed consistent with the optimal policy.

We now turn to the optimal policy when the enforcer has some constraints on the tools available. We start from the case of exogenous enforcement that arises when the enforcer cannot fine tune the monitoring activity in the markets to each specific conduct (new action), increasing or decreasing selectively the enforcement effort practice by practice.

4.2 Case 2: exogenous enforcement and specific accuracies

When the enforcer can rely only on the level of fines f and on the two accuracies α_I and α_{II} the relevant first order conditions are:

$$\frac{\partial \mathrm{E}W}{\partial f} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial f} \gtreqless 0, \tag{17}$$

$$\frac{\partial \mathrm{E}W}{\partial \alpha_I} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha_I} + g_I' \geqslant 0, \tag{18}$$

and

$$\frac{\partial \mathrm{E}W}{\partial \alpha_{II}} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha_{II}} + g_{II}' \geqslant 0. \tag{19}$$

Following the same procedure of the previous case, we can define the locus $\underline{W}_0(\underset{-}{f}, \underset{-}{\alpha_I}, \underset{+}{\alpha_{II}}, \underset{+}{\beta})$ along which the marginal social value of the innovative activity is zero for given policy parameters. In the case of exogenous enforcement $E > 0$ the equilibrium analysis focusses on two relevant loci corresponding to different combinations of policy parameters:

$$\underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta) = \frac{\Pi - (1-\beta)\overline{W}}{\beta}$$

along which the innovative activity is welfare neutral when $f = 0$ and $\alpha_I = \alpha_{II} = \frac{1}{2}$, and

$$\underline{W}_0(F, \frac{1}{2}, \frac{1}{2}, \beta) = \frac{\Pi - (1-\beta)\overline{W} - EF/2}{\beta}$$

corresponding to the policy parameters $f = F$ and $\alpha_I = \alpha_{II} = \frac{1}{2}$. Once again we must keep account of the parametric restriction on β. The first locus implies that β must be smaller or equal than β_1, while the second locus implies that β must be smaller or equal than $\beta_2 = \frac{\overline{W} + EF/2 - \Pi}{\overline{W}}$.[20] It is useful to notice that $\underline{W}_0(F, \frac{1}{2}, \frac{1}{2}, \beta)$ is above $\underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$ in the $(\underline{W}, \beta)$ space. Moreover, this latter corresponds to the locus that delimits the *laissez-faire* region in case 1, when enforcement is endogenous. The following Proposition characterizes the optimal policy for different values of the social loss $\underline{W}$ and different likelihood of the bad state β.

Proposition 3: *In the space ($\underline{W}$, β) we can distinguish the following regions:*

i) When the marginal social value of the innovative activity is non negative, i.e. for $0 \geqslant \underline{W} \geqslant \underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$ *and* $\beta \in [0,\ \beta_1]$, *the optimal policy entails* $f = 0$ and $\alpha_I^* = \alpha_{II}^* = \frac{1}{2}$ *i.e. a laissez-faire regime.*

ii) When the marginal social value of the innovative activity is negative, i.e. for $\underline{W} < \min\left\{\underline{W}_0(F, \frac{1}{2}, \frac{1}{2}, \beta),\ 0\right\}$ *and* $\beta \in [0,\ 1]$, *the optimal policy entails* $f = F$, $\alpha_I^* = \frac{1}{2}$ *and*

[20] Notice that $\beta_2 \geqslant \beta_1$.

$\alpha^*_{II} \in \left(0,\ \frac{1}{2}\right)$; *moreover,* α^*_{II} *is increasing in* $\underline{W}$ *and converges to* $\alpha^*_{II} = \frac{1}{2}$ *when* $\underline{W} \to \underline{W}_0(F, \frac{1}{2}, \frac{1}{2}, \beta)$.

iii) When $\underline{W}_0(F, \frac{1}{2}, \frac{1}{2}, \beta) \leqslant \underline{W} < \underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$ *and* $\beta \in [0,\ \beta_2]$, *the optimal policy requires to set* $\alpha^*_I = \alpha^*_{II} = \frac{1}{2}$ *and*

$$f^* = 2\frac{\Pi - \beta\underline{W} - (1-\beta)\overline{W}}{E} \in (0, F)\,.$$

Proof. See the Appendix. ■

[**Figure 2 about here**]

The optimal policy is driven by the average deterrence effect. When the social loss and/or the likelihood of the bad state is small, i.e. below $\underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$, the innovative activity is socially desirable. With an exogenous level of enforcement E the optimal policy requires to depenalize the new action ($f = 0$) even when ex-post it is found to be welfare decreasing. The *ex-ante* positive effect on the incentives to innovate, indeed, more than counterbalances the *ex-post* limited losses. Accuracy in this case is irrelevant because the firm does not pay any fine no matter what is the final decision. Notice that the region where the *laissez-faire* regime is implemented is the same under case 1 and case 2 and it is implemented adopting $E = 0$ in the former and $f = 0$ in the latter situation.

Once we move to more serious social losses, in the region between $\underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$ and $\underline{W}_0(F, \frac{1}{2}, \frac{1}{2}, \beta)$, a *laissez-faire* regime is too lax and we want to marginally limit the incentives to innovative activity. This result is obtained using the less costly tool, i.e. increasing the fine without spending in accuracy. Once we hit the maximum admitted fine F, further social losses in the bad state require stronger average deterrence: we implement it by keeping the fine at the maximum level F and reducing under-deterrence, i.e. improving type-II accuracy. The asymmetric protocols of investigation are indeed optimal even in the case of exogenous enforcement. See Figure 2 for a representation of the optimal policy.

4.3 Case 3: endogenous enforcement and common accuracy

We now consider an alternative restriction in the set of policy instruments, when the enforcement effort is again endogenous but the enforcer does not implement separately type-I and type-II accuracies and, based on the prevailing jurisprudence, simply chooses a common level of accuracy α. The Becker argument applies to this case, as already did in the case of endogenous enforcement and separate accuracies: when average deterrence is desirable, the cheapest way to implement it is by setting the maximum fine $f = F$. When instead *laissez-faire* is the preferred regime, fines are irrelevant. The first order conditions to identify the optimal policy (f, E, α) are now

$$\frac{\partial \mathrm{E}W}{\partial f} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial f} \gtreqless 0,$$

$$\frac{\partial \mathrm{E}W}{\partial E} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial E} - g'_E \leqslant 0,$$

and

$$\frac{\partial \mathrm{E}W}{\partial \alpha} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha} + g'_\alpha \geqslant 0.$$

Recall that while $\frac{\partial \widehat{I}}{\partial E}$ and $\frac{\partial \widehat{I}}{\partial f}$ are always negative, the sign of $\frac{\partial \widehat{I}}{\partial \alpha}$ depends on the likelihood of the bad state β, as previously discussed: if the bad state is relatively likely ($\beta > \frac{1}{2}$) type-II errors are more frequent and accuracy reduces under-deterrence and the incentives to innovate ($\frac{\partial \widehat{I}}{\partial \alpha} > 0$).

Given the loci $\underline{W}_0(f, E, \alpha, \beta)$ corresponding to welfare neutral points in the $(\underline{W}, \beta)$ space, the relevant one for the analysis of the optimal policy is:

$$\underline{W}_0(0, 0, \frac{1}{2}, \beta) = \frac{\Pi - (1-\beta)\overline{W}}{\beta}.$$

Again we assume that the cost of enforcement and accuracy are sufficiently convex, in this case a sufficient condition is:

$$g''_E g''_\alpha \geqslant [\Delta \mathrm{E}(W) - c\widehat{I}]^2 \left(\frac{\partial \widehat{I}}{\partial E \partial \alpha}\right)^2. \tag{20}$$

Then, we can state the following result.

Proposition 4: *In the space $(\underline{W}, \beta)$ we can distinguish the following regions:*

i) When the marginal social value of the innovative activity is non negative, i.e. for $0 \geqslant \underline{W} \geqslant \underline{W}_0(0, 0, \frac{1}{2}, \beta)$ and $\beta \in [0, \ \beta_1]$, *the optimal policy entails "laissez faire", $E^* = 0$, $\alpha^* = 1/2$;*

ii) When the marginal social value of the innovative activity is negative and the bad state relatively likely, i.e. for $\underline{W} < \min\left\{\underline{W}_0(0, 0, \frac{1}{2}, \beta), \ 0\right\}$ and $\beta \in \left(\frac{1}{2}, \ 1\right]$, the optimal policy prescribes maximum fine and positive enforcement and accuracy, i.e. $f^* = F$, $E^* > 0$ and $\alpha^* \in \left(0, \ \frac{1}{2}\right)$. In this region both enforcement and accuracy increase when the marginal social value of the innovative activity becomes more negative, i.e. $\frac{dE^*}{d\underline{W}} < 0$ and $\frac{d\alpha^*}{d\underline{W}} > 0$ with $E^* \to 0$ and $\alpha^* \to \frac{1}{2}$ when $\underline{W} \underset{-}{\to} \underline{W}_0(0, \frac{1}{2}, \beta)$;

iii) When the marginal social value of the innovative activity is negative and the bad state relatively unlikely, i.e. for $\underline{W} < \min\left\{\underline{W}_0(0, 0, \frac{1}{2}, \beta), \ 0\right\}$ and $\beta \in \left[0, \ \frac{1}{2}\right]$ the optimal *policy requires maximum fines, positive enforcement and no accuracy, i.e.* $f^* = F$, $E^* > 0$ and $\alpha^* = 1/2$. *Enforcement increases as the welfare loss in the bad state increases, i.e.* $\frac{dE^*}{d\underline{W}} < 0$ *with* $E^* \to 0$ *when* $\underline{W} \to \underline{W}_0(0, \frac{1}{2}, \beta)$.

Proof. See the Appendix. ■

[Figure 3 about here]

As can be seen in Figure 3, when the bad state is very unlikely and/or the social loss $\underline{W}$ is small, i.e. when the marginal social value of the innovative activity is non negative (region *i*), we find the usual result of *laissez faire* or *per-se legality rule*: the locus that delimits this regime is the same as in case 1 and case 2.

When instead the social loss increases, the optimal enforcement E^* is positive and increasing in the social loss $\underline{W}$. In this case the main goal of the policy is to discourage the innovative activity. When the bad state is relatively likely (region *ii*) errors lead more often to under-enforcement and enforcement and accuracy both improve average deterrence, in a way similar to what we found in the case of endogenous enforcement and separate accuracies (case 1). Hence, common accuracy works in the right direction and it is implemented. However, when the bad state is relatively unlikely (region *iii*) the predominant effect of errors is over-enforcement and accuracy is undesirable since it would limit this bias. In this case the enforcer does not implement any level of accuracy, contrary to the usual result in the Law and Economics literature.

4.4 Case 4: exogenous enforcement and common accuracies

In this section we consider an even more constrained environment for the enforcement policy in which the enforcement effort is exogenous and only common accuracy can be implemented. The enforcer, therefore, can just choose the level of fine f and the level of common accuracy α.

The first order conditions for the optimal policy are given by:

$$\frac{\partial \mathrm{E}W}{\partial f} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial f} \gtreqless 0, \tag{21}$$

and

$$\frac{\partial \mathrm{E}W}{\partial \alpha} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha} + g'_{\alpha} \gtrless 0. \tag{22}$$

with the usual interpretation referred to average deterrence. The analysis parallels our findings in case 2 (exogenous enforcement and separate accuracies) and, among the loci $\underline{W}_0(\underset{-}{f}, \underset{+/-}{\alpha}, \underset{+}{\beta})$, we focus on:

$$\underline{W}_0(0, \frac{1}{2}, \beta) = \frac{\Pi - (1-\beta)\overline{W}}{\beta}$$

and

$$\underline{W}_0(F, \frac{1}{2}, \beta) = \frac{\Pi - (1-\beta)\overline{W} - EF/2}{\beta}$$

The following Proposition characterizes the optimal policy.

Proposition 5: *In the space ($\underline{W}$, β) we can distinguish the following regions:*

i) When the marginal social value of innovative activity is non negative, i.e for $0 \geqslant \underline{W} \geqslant \underline{W}_0(0, \frac{1}{2}, \beta)$ and $\beta \in [0,\ \beta_1]$, the optimal policy adopts laissez-faire by setting $f^* = 0$ and $\alpha^* = \frac{1}{2}$;

ii) When the marginal social value of innovative activity is negative and the bad state is relatively unlikely, i.e. for $\underline{W} < \min\left\{\underline{W}_0(F, \frac{1}{2}, \beta),\ 0\right\}$ and $\beta \in \left(\frac{1}{2},\ 1\right]$, the optimal policy requires to set $f^* = F$ and $\alpha^* \in \left(0,\ \frac{1}{2}\right)$; in this case $\alpha^* \to \frac{1}{2}$ when $\underline{W} \underset{-}{\to} \underline{W}_0(F, \frac{1}{2}\beta)$;

iii) When the marginal social value of innovative activity is negative and the bad state is relatively likely, i.e. for $\underline{W} < \min\left\{\underline{W}_0(F, \frac{1}{2}, \beta),\ 0\right\}$ and $\beta \in \left[0,\ \frac{1}{2}\right]$, the optimal policy requires to set $f^* = F$ and $\alpha^* = \frac{1}{2}$;

i*v) When $\underline{W}_0(F, \frac{1}{2}, \beta) \leqslant \underline{W} < \underline{W}_0(0, \frac{1}{2}, \beta)$ and $\beta \in [0,\ \beta_2]$, the optimal policy entails no accuracy, i.e. $\alpha^* = \frac{1}{2}$ and*

$$f^* = 2\frac{\Pi - \beta\underline{W} - (1-\beta)\overline{W}}{E} \in (0, F)\,.$$

Proof. See Appendix. ■

The third best environment of case 4 combines the results of the previous cases. *Laissez-faire* is chosen in the same region of parameters as in all the other cases. We have an intermediate region, as in case 2, where average deterrence is obtained by progressively increasing the fine without investing in accuracy. Once we hit the maximum fine we identify two regions, as in case 3: when the bad state is relatively likely common accuracy reduces more often under-enforcement and innovative effort and it is therefore desirable. When instead the bad state is unlikely, common accuracy would reduce more often type-I error sustaining the innovative activity and reducing the expected welfare: the optimal policy therefore does not implement any accuracy.

Figure 4 shows the optimal policies for the four environments considered in this Section.

[**Figure 4 about here**]

5 Conclusions

In this paper we have analyzed the effect of judicial errors on the innovative activity following the approach introduced in Immordino, Pagano and Polo (2006). The traditional model of law enforcement assumes that there is a set of privately convenient but socially damaging actions that are illegal, one of which is selected by the private agent by comparing the expected benefits and fine. Marginal deterrence, in this setting, is the key effect.

In our model the agents first have to invest resources in learning and research effort - which we call the innovative activity - and then, if successful, are able to choose a new action that, at the time of the investment, may be welfare enhancing (legal) or reducing (illegal). The enforcement and accuracy policy, determining the probability of being fined, affects the expected profits from the new action and the incentives to exert the innovative activity. The focus of the analysis is therefore shifted to the impact of enforcement policy on the innovative activity, what we call average deterrence, since it influences the probability of taking the new action whether legal or not. The basic instruments of the enforcer are the level of fines, the enforcement effort, which affects the probability of finding hard evidence on the actions chosen, and the accuracy effort, which reduces the probability of wrongly assessing the social consequences (legality) of the actions. We consider four different environments, combining exogenous versus endogenous enforcement effort and common accuracy on any type of error versus different levels of type-I and type-II accuracy.

In this framework we analyze the impact of judicial errors and accuracy and their optimal setting. The traditional Law and Economics benchmark states that both types of error add up to weaken marginal deterrence and therefore accuracy is always desirable. In our setting, instead, average deterrence is affected differently by type-I and type-II errors. Type-I errors, which imply over-enforcement, reduce the expected profits from the new actions and discourage the innovative activity, while type-II errors, through under-enforcement, sustain the incentive to invest in learning. The expected welfare effect of the innovative activity drives the design of the optimal policy: when the innovative activity is *ex-ante* welfare enhancing the policy should sustain it, while it should reduce the incentives to innovate when this activity is *ex-ante* socially damaging.

When innovation is socially desirable, the optimal policy prescribes *laissez-faire* either by not exerting any enforcement effort and/or by not imposing any fine. As shown in

Immordino, Pagano and Polo (2006), when innovative activity plays an important role and average deterrence is a tool of enforcement policy, *laissez-faire* is chosen to sustain welfare-improving innovations whereas in the traditional setting some positive level of enforcement would be always maintained to preserve marginal deterrence.

When the innovative activity is *ex-ante* welfare-reducing the enforcer designs the policy to discourage it. This result can be reached in different ways according to the available instruments. When enforcement effort is endogenous, the optimal monitoring activity is increasing in the social loss from innovations and it is paired with type-II accuracy, adopting an asymmetric protocol of investigation. With exogenous enforcement, average deterrence is initially ensured through increasing fines and it is then paired, when the maximum fine is hit, with type-II accuracy. Finally, with common accuracy its prevailing effect depends on the likelihood of the bad state: since type-II errors occur in this case, if the new action more often leads to a social loss, accuracy predominantly reduces under-deterrence and curbs the incentives to innovate. In this case, therefore, accuracy is welcome. In the complementary case when the good state is relatively likely, accuracy would reduce mostly type-I errors and over-deterrence: with welfare reducing innovations, therefore, accuracy is not desirable, contrary to the traditional result.

Appendix

Proof of Proposition 2. In case 1 the enforcer has to design the optimal policy with respect to f, E, α_I and α_{II}. The four first order conditions for the optimal policy are:

$$\frac{\partial \mathrm{E}W}{\partial f} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial f} \gtreqless 0, \tag{23}$$

$$\frac{\partial \mathrm{E}W}{\partial E} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial E} - g'_E \leqslant 0, \tag{24}$$

$$\frac{\partial \mathrm{E}W}{\partial \alpha_I} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha_I} + g'_I \geqslant 0, \tag{25}$$

and

$$\frac{\partial \mathrm{E}W}{\partial \alpha_{II}} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha_{II}} + g'_{II} \geqslant 0, \tag{26}$$

where $\frac{\partial \widehat{I}}{\partial f} \leqslant 0$, $\frac{\partial \widehat{I}}{\partial \alpha_I} = -\frac{(1-\beta)Ef}{c} \leqslant 0$ and $\frac{\partial \widehat{I}}{\partial \alpha_{II}} = \frac{\beta E f}{c} \geqslant 0$.

Let us consider the four first order conditions at $f = F$, $E = 0$, $\alpha_I = \alpha_{II} = \frac{1}{2}$ below the locus, i.e. for $\underline{W} > \underline{W}_0(0, 0, \frac{1}{2}, \frac{1}{2}, \beta)$: since $\Delta \mathrm{E}(W) - c\widehat{I} < 0$, (24) is negative at $E = 0$, corresponding to a corner solution. When $E = 0$ we have $\frac{\partial \widehat{I}}{\partial \alpha_I} = \frac{\partial \widehat{I}}{\partial \alpha_{II}} = \frac{\partial \widehat{I}}{\partial f} = 0$. (23) is solved as an equality for any fine, including $f^* = 0$. (25) and (26) are solved as equalities at $\alpha_I^* = \alpha_{II}^* = \frac{1}{2}$ since $g'_I(0) = g'_{II}(0) = 0$. Hence, when the innovative activity is welfare enhancing, the public policy sustains it by not enforcing any prohibition. Accuracy is not needed since no case is opened. Moreover, for any $\underline{W} > \underline{W}_0(0, 0, \frac{1}{2}, \frac{1}{2}, \beta)$ the optimal policies $f^* = 0$, $E^* = 0$, $\alpha_I^* = \alpha_{II}^* = \frac{1}{2}$ are consistent with the definition of the threshold.

Along the locus, i.e. for $\underline{W} = \underline{W}_0(0, 0, \frac{1}{2}, \frac{1}{2}, \beta)$, for given policy parameters $\Delta \mathrm{E}(W) - c\widehat{I} = 0$. Then (24) is solved as an equality at $E^* = 0$ since $g'_E(0) = 0$. Then $\frac{\partial \widehat{I}}{\partial \alpha_I} = \frac{\partial \widehat{I}}{\partial \alpha_{II}} = \frac{\partial \widehat{I}}{\partial f} = 0$ as before and the same equilibrium values apply to this case.

Finally, above the locus, $\underline{W} < \underline{W}_0(0, 0, \frac{1}{2}, \frac{1}{2}, \beta)$, for given policy parameters $\Delta \mathrm{E}(W) - c\widehat{I} < 0$. Now (24) is solved as an equality at $E^* > 0$ and $\frac{\partial \widehat{I}}{\partial \alpha_I} < 0$, $\frac{\partial \widehat{I}}{\partial \alpha_{II}} > 0$ and $\frac{\partial \widehat{I}}{\partial f} < 0$. (23) and (25) are always positive and imply the corner solutions $f^* = F$ and $\alpha_I^* = \frac{1}{2}$. (26) is instead solved internally for $\alpha_{II}^* \in (0, \frac{1}{2})$. The second order conditions for E and α_{II} are

$$\begin{aligned} \frac{\partial^2 EW}{\partial E^2} &= -c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 - g''_E < 0, \\ \frac{\partial^2 EW}{\partial \alpha_{II}^2} &= -c\left(\frac{\partial \widehat{I}}{\partial \alpha_{II}}\right)^2 - g''_{II} < 0, \end{aligned}$$

and the Hessian matrix

$$|H_2| = \left[c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 + g''_E\right]\left[c\left(\frac{\partial \widehat{I}}{\partial \alpha_{II}}\right)^2 + g''_{II}\right] - \left[[\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial^2 \widehat{I}}{\partial E \partial \alpha_{II}} - c\frac{\partial \widehat{I}}{\partial E}\frac{\partial \widehat{I}}{\partial \alpha_{II}}\right]^2,$$

that is positive, as required, in a left neighborhood of $\underline{W}_0(0,0,\frac{1}{2},\frac{1}{2},\beta)$ and, for $\underline{W} < \underline{W}_0(0,0,\frac{1}{2},\frac{1}{2},\beta)$, when, g''_E and g''_{II} are sufficiently large. More formally, a sufficient condition for the second order conditions to hold is given by (16). Finally, the comparative statics with respect to the social loss in the bad state gives

$$\frac{dE^*}{d\underline{W}} = \beta\frac{\frac{\partial\widehat{I}}{\partial E}g''_{II} + [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial^2\widehat{I}}{\partial E\partial\alpha_{II}}\frac{\partial\widehat{I}}{\partial\alpha_{II}}}{|H_2|} < 0,$$

and

$$\frac{d\alpha^*_{II}}{d\underline{W}} = \beta\frac{\frac{\partial\widehat{I}}{\partial\alpha_{II}}g''_E + [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial^2\widehat{I}}{\partial E\partial\alpha_{II}}\frac{\partial\widehat{I}}{\partial\alpha_E}}{|H_2|} > 0.$$

Hence, when the social loss in the bad state gets worse ($\underline{W}\downarrow$) the enforcement effort and the type-II accuracy are increased, showing a complementarity relationship.

To complete the proof we need to show that once we move above the locus $\underline{W}_0(0,0,\frac{1}{2},\frac{1}{2},\beta)$ the adjustments in the policy parameters E and α_{II} still induce a negative marginal social value of innovative activity as supposed when deriving the optimal policies. To show this, let us totally differentiate the marginal social value of the innovative activity with respect to the social loss in the bad state $\underline{W}$. Substituting the equilibrium expressions we get:

$$\begin{aligned}\frac{d\left[\Delta E(W) - c\widehat{I}(.)\right]}{d\underline{W}} &= \frac{\partial\Delta \mathrm{E}(W)}{\partial\underline{W}} - c\left[\frac{\partial\widehat{I}}{\partial E}\frac{\partial E^*}{\partial\underline{W}} + \frac{\partial\widehat{I}}{\partial\alpha_{II}}\frac{\partial\alpha^*_{II}}{\partial\underline{W}}\right] \\ &= \beta\left[g''_E g''_I - [\Delta \mathrm{E}(W) - c\widehat{I}]^2\left(\frac{\partial\widehat{I}}{\partial E\partial\alpha_{II}}\right)^2\right]/\,|H_2| > 0,\end{aligned}$$

given condition (16). Hence, when the social loss in the bad state gets worse ($\underline{W}\downarrow$) the marginal social value of the innovative activity $\Delta E(W) - c\widehat{I}$ becomes more negative, consistently with what assumed when deriving the optimal policy. ■

Proof of Proposition 3. In case 2 the enforcer has to design the optimal policy with respect to f, α_I and α_{II}. In order to analyze the optimal policies let us consider the first order conditions (23), (25) and (26).

Below $\underline{W}_0(0,\frac{1}{2},\frac{1}{2},\beta)$ when $f = 0$ and $\alpha_I = \alpha_{II} = \frac{1}{2}$ the marginal social value of the innovative activity $[\Delta \mathrm{E}(W) - c\widehat{I}]$ is positive: in this region increasing the fine f reduces the incentive to innovate and the expected welfare. Consequently, the optimal policy is to set the fine $f^* = 0$, i.e. to depenalize the new action. In this case $\frac{\partial\widehat{I}}{\partial\alpha_I} = \frac{\partial\widehat{I}}{\partial\alpha_{II}} = 0$ and it is optimal to set $\alpha^*_I = \alpha^*_{II} = 1/2$ since $g'_I(0) = g'_{II}(0) = 0$. This outcome is therefore equivalent to the case when the new action is lawful, i.e. *laissez-faire* of *per-se legality* rule. Notice that the optimal policy parameters are indeed those corresponding to the locus $\underline{W}_0(0,\frac{1}{2},\frac{1}{2},\beta)$: this ensures that below this locus, i.e. for $\underline{W} \geqslant \underline{W}_0(0,\frac{1}{2},\frac{1}{2},\beta)$, the marginal social value of innovative activity is indeed positive when the optimal policy is chosen.

Consider now the region between the two loci, i.e. $\underline{W}_0(F, \frac{1}{2}, \frac{1}{2}, \beta) < \underline{W} < \underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$. Once we move above $\underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$, i.e. when $\underline{W} < \underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$ for given policy parameters the marginal social value of innovative activity becomes negative and $\frac{\partial \mathrm{E}W}{\partial f} > 0$, suggesting to increase the fine. Notice that a higher fine affects $[\Delta \mathrm{E}(W) - c\widehat{I}]$ reducing the investment $\widehat{I}$ and its marginal cost and increasing the marginal social value of innovation. The optimal fine f^* is determined by the condition $\Delta \mathrm{E}(W) - c\widehat{I}(f^*) = 0$. When $f = f^*$ the first order conditions for α_I^* and α_{II}^* are solved for $\alpha_I^* = \alpha_{II}^* = 1/2$ since $g_I'(0) = g_{II}'(0) = 0$. Substituting the expressions of $\Delta \mathrm{E}(W)$ and $\widehat{I}$ and solving we obtain

$$f^* = 2\frac{\Pi - \beta \underline{W} - (1-\beta)\overline{W}}{E}.$$

Notice that f^* is decreasing in $\underline{W}$, it tends to 0 when $\underline{W} \underset{-}{\to} \underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$ and tends to F when $\underline{W} \underset{+}{\to} \underline{W}_0(F, \frac{1}{2}, \frac{1}{2}, \beta)$.

At $\underline{W} = \underline{W}_0(F, \frac{1}{2}, \frac{1}{2}, \beta)$ the three first order conditions are solved for $f^* = F$ and $\alpha_I^* = \alpha_{II}^* = 1/2$. For lower values of the welfare loss in the bad state, $\underline{W} < \underline{W}_0(F, \frac{1}{2}, \frac{1}{2}, \beta)$, the marginal social value of innovative activity for given policy parameters becomes negative and $\frac{\partial \mathrm{E}W}{\partial f} > 0$. Since the fine is already at F we get $f^* = F$ as a corner solution. In this case $\frac{\partial \widehat{I}}{\partial \alpha_I} < 0 < \frac{\partial \widehat{I}}{\partial \alpha_{II}}$ and we have the corner solution $\alpha_I^* = \frac{1}{2}$ and an internal solution $\alpha_{II}^* \in \left(0, \frac{1}{2}\right)$. The SOC therefore boil down to $\frac{\partial^2 EW}{\partial \alpha_{II}^2} = -c\left(\frac{\partial \widehat{I}}{\partial \alpha_{II}}\right)^2 - g_{II}'' < 0$ since the other variables are set at the corner solutions. Moreover, in the region $\underline{W} < \underline{W}_0(F, \frac{1}{2}, \frac{1}{2}, \beta)$ we have

$$\frac{d\alpha_{II}^*}{d\underline{W}} = -\frac{\partial^2 \mathrm{E}W/\partial \alpha_{II}\partial \underline{W}}{\partial^2 \mathrm{E}W/\partial \alpha_{II}^2} = \frac{\beta \frac{\partial \widehat{I}}{\partial \alpha_{II}}}{c\left(\frac{\partial \widehat{I}}{\partial \alpha_{II}}\right)^2 + g_{II}''} > 0$$

since $\frac{\partial^2 EW}{\partial \alpha_{II}\partial \underline{W}} = \beta \frac{\partial \widehat{I}}{\partial \alpha_{II}} > 0$. Hence, when we decrease $\underline{W}$ below $\underline{W}_0(F, \frac{1}{2}, \frac{1}{2}, \beta)$ the constrained optimum entails $\frac{df^*}{d\underline{W}} = \frac{d\alpha_I^*}{d\underline{W}} = 0$ (corner solutions) and $\frac{d\alpha_{II}^*}{d\underline{W}} > 0$ (internal solution). In order to check the consistency of this exercise, let use consider how the marginal social value of the innovative activity varies at the boundary of this region:

$$\frac{d\left[\Delta E(W) - c\widehat{I}(.)\right]}{d\underline{W}} = \left\{\frac{\partial \mathrm{E}W}{\partial \underline{W}} - c\frac{\partial \widehat{I}}{\partial \alpha_{II}}\frac{d\alpha_{II}^*}{d\underline{W}}\right\} = \beta\left[\frac{g_I''}{c\left(\frac{\partial \widehat{I}}{\partial \alpha_{II}}\right)^2 + g_{II}''}\right] > 0.$$

Hence, when the social loss gets worse the marginal social value of innovative activity corresponding to the optimal policies becomes more negative. ■

Proof of Proposition 4. In case 3 the enforcer has to design the optimal policy with respect to f, E and α. The first order conditions for an internal maximum are (23), (24) and

$$\frac{\partial \mathrm{E}W}{\partial \alpha} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha} + g_\alpha' \geqslant 0, \tag{27}$$

where $\frac{\partial \widehat{I}}{\partial \alpha} = -\frac{(1-2\beta)Ef}{c} \gtreqless 0 \Leftrightarrow \beta \gtreqless \frac{1}{2}$. Below the locus $\underline{W}_0(0,0,\frac{1}{2},\beta)$ the marginal social value of innovative activity is positive when $f = 0$, $E = 0$ and $\alpha = \frac{1}{2}$. Then we have a corner solution at $E^* = 0$ and consequently $\frac{\partial \widehat{I}}{\partial f} = \frac{\partial \widehat{I}}{\partial \alpha} = 0$. Then $\frac{\partial \mathrm{E}W}{\partial \alpha} = 0$ is obtained for $\alpha^* = \frac{1}{2}$ since $g'_\alpha(0) = 0$. $\frac{\partial \mathrm{E}W}{\partial f} = 0$ holds for any fine including $f^* = 0$. Given the definition of $\underline{W}_0(0,0,\frac{1}{2},\beta)$ we know that in the region $\underline{W} > \underline{W}_0(0,0,\frac{1}{2},\beta)$ we have $[\Delta \mathrm{E}(W) - c\widehat{I}] > 0$ at the optimal policies, as assumed when deriving them.

At the locus $\underline{W}_0(0,0,\frac{1}{2},\beta)$ the marginal social value of innovative activity is zero and $E^* = 0$ is the internal solution. Then, the same argument above applies.

Above the locus, i.e. for $\underline{W} < \underline{W}_0(0,0,\frac{1}{2},\beta)$, the marginal social value of innovative activity becomes negative. Then $\frac{\partial \mathrm{E}W}{\partial f} > 0$ and we have the corner solution $f^* = F$. The enforcement effort admits an internal solution $E^* > 0$.

When $\beta \in (0, \min\left\{\frac{1}{2}, \beta_1\right\}]$ the predominant effect of accuracy is to limit over-deterrence, i.e. $\frac{\partial \widehat{I}}{\partial \alpha} \leqslant 0$ and we have a corner solution $\alpha^* = \frac{1}{2}$. The second order condition is required only for E:

$$\frac{\partial^2 EW}{\partial E^2} = -c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 - g''_E < 0$$

Moreover, totally differentiating $\frac{\partial \mathrm{E}W}{\partial E} = 0$ we obtain

$$\frac{dE^*}{d\underline{W}} = \frac{\beta \frac{\partial \widehat{I}}{\partial E}}{c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 + g''_E} < 0.$$

In order to complete the proof, we have to check that, once we move above the locus, i.e. when $\underline{W} < \underline{W}_0(0,0,\frac{1}{2},\beta)$, the optimal policies induce a negative marginal social value of innovation as supposed when deriving them. We have:

$$\frac{d\left[\Delta E(W) - c\widehat{I}(.)\right]}{d\underline{W}} = \frac{\partial \Delta \mathrm{E}(W)}{\partial \underline{W}} - c\left[\frac{\partial \widehat{I}}{\partial E}\frac{dE^*}{d\underline{W}}\right] = \beta\frac{g''_E}{c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 + g''_E} > 0.$$

Hence, when the social loss get worse, the marginal social value of innovative activity at the optimal policies becomes (more) negative.

When instead $\beta \in (\frac{1}{2}, \beta_1]$ type-II errors are relatively more frequent and accuracy reduces under-deterrence and innovation. In this case $\frac{\partial \widehat{I}}{\partial \alpha} > 0$ and we have an internal solution for both $E^* > 0$ and $\alpha^* < \frac{1}{2}$. The second order conditions are now

$$\begin{aligned}
\frac{\partial^2 EW}{\partial E^2} &= -c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 - g''_E < 0, \\
\frac{\partial^2 EW}{\partial \alpha^2} &= -c\left(\frac{\partial \widehat{I}}{\partial \alpha}\right)^2 - g''_\alpha < 0,
\end{aligned}$$

and the determinant of the Hessian matrix

$$|H| = \left[c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 + g''_E\right]\left[c\left(\frac{\partial \widehat{I}}{\partial \alpha}\right)^2 + g''_\alpha\right] - \left[[\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial^2 \widehat{I}}{\partial E \partial \alpha} - c\frac{\partial \widehat{I}}{\partial E}\frac{\partial \widehat{I}}{\partial \alpha}\right]^2 > 0,$$

under condition (20). Totally differentiating the first order conditions we obtain the comparative statics results:

$$\frac{dE^*}{d\underline{W}} = \frac{\beta}{|H|}\left\{\frac{\partial \widehat{I}}{\partial E}g''_\alpha + [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial^2 \widehat{I}}{\partial E \partial \alpha}\frac{\partial \widehat{I}}{\partial \alpha}\right\} < 0, \tag{28}$$

and

$$\frac{d\alpha^*}{d\underline{W}} = \frac{\beta}{|H|}\left\{\frac{\partial \widehat{I}}{\partial \alpha}g''_E + [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial^2 \widehat{I}}{\partial E \partial \alpha}\frac{\partial \widehat{I}}{\partial E}\right\} > 0, \tag{29}$$

since $\frac{\partial^2 \widehat{I}}{\partial E \partial \alpha} = -\frac{(1-2\beta)f}{c} > 0$ when $\beta > \frac{1}{2}$.

We conclude the proof running the consistency check on the sign of the marginal social value of innovative activity at the optimal policies when $\underline{W} < \underline{W}_0(0, 0, \frac{1}{2}, \beta)$:

$$\frac{d\left[\Delta E(W) - c\widehat{I}(.)\right]}{d\underline{W}} = \frac{\partial \Delta \mathrm{E}(W)}{\partial \underline{W}} - c\left[\frac{\partial \widehat{I}}{\partial E}\frac{dE^*}{d\underline{W}} + \frac{\partial \widehat{I}}{\partial \alpha}\frac{d\alpha^*}{d\underline{W}}\right],$$

that given (20) simplifies to

$$\frac{d\left[\Delta E(W) - c\widehat{I}(.)\right]}{d\underline{W}} = \beta\left\{g''_E g''_\alpha - [\Delta \mathrm{E}(W) - c\widehat{I}]^2\left(\frac{\partial \widehat{I}}{\partial E \partial \alpha}\right)^2\right\} / |H| > 0.$$

■

Proof of Proposition 5. In case 4 the enforcer has to design the optimal policy with respect to f and α. The first order conditions are (23) and (27).

For $\underline{W} > \underline{W}_0(0, \frac{1}{2}, \beta)$, when $f = 0$ and $\alpha = \frac{1}{2}$ the marginal social value of the innovative activity $[\Delta \mathrm{E}(W) - c\widehat{I}]$ is positive. Then $\frac{\partial \mathrm{E}W}{\partial f} < 0$ and $f^* = 0$. Consequently, $\frac{\partial \widehat{I}}{\partial \alpha} = 0$ and $\alpha^* = \frac{1}{2}$ since $g'_\alpha(0) = 0$. Hence, the optimal policy parameters are those corresponding to the locus $\underline{W}_0(0, \frac{1}{2}, \beta)$: this ensures that below this locus, i.e. for $\underline{W} > \underline{W}_0(0, \frac{1}{2}, \beta)$, the marginal social value of innovative activity is indeed positive when the optimal policy is chosen. Along the locus, i.e. for $\underline{W} = \underline{W}_0(0, \frac{1}{2}, \beta)$, the same optimal policies apply.

Consider now the region between the two loci, i.e. $\underline{W}_0(F, \frac{1}{2}, \beta) < \underline{W} < \underline{W}_0(0, \frac{1}{2}, \beta)$. Once we move above $\underline{W}_0(0, \frac{1}{2}, \beta)$, i.e. when $\underline{W} < \underline{W}_0(0, \frac{1}{2}, \beta)$ for given policy parameters the marginal social value of innovative activity becomes negative and $\frac{\partial \mathrm{E}W}{\partial f} > 0$, suggesting to increase the fine. The optimal fine is determined by the condition $\Delta \mathrm{E}(W) - c\widehat{I}(f^*) = 0$.

When $f = f^*$ the first order condition $\frac{\partial \mathrm{E}W}{\partial \alpha} = 0$ is solved for $\alpha^* = 1/2$ since $g'_\alpha(0) = 0$. Substituting the expressions of $\Delta\mathrm{E}(W)$ and $\widehat{I}$ and solving we obtain

$$f^* = 2\frac{\Pi - \beta\underline{W} - (1-\beta)\overline{W}}{E}.$$

Notice that f^* is decreasing in $\underline{W}$, it tends to 0 when $\underline{W} \underset{-}{\to} \underline{W}_0(0, \frac{1}{2}, \beta)$ and tends to F when $\underline{W} \underset{+}{\to} \underline{W}_0(F, \frac{1}{2}, \beta)$.

At $\underline{W} = \underline{W}_0(F, \frac{1}{2}, \beta)$ the two first order conditions are solved for $f^* = F$ and $\alpha^* = 1/2$. For lower values of the welfare loss in the bad state, $\underline{W} < \underline{W}_0(F, \frac{1}{2}, \beta)$, the marginal social value of innovative activity for given policy parameters becomes negative and $\frac{\partial \mathrm{E}W}{\partial f} > 0$. Since the fine is already at F we get $f^* = F$ as a corner solution.

The sign of $\frac{\partial \widehat{I}}{\partial \alpha}$ depends on β. When $\beta \in (0, \min\{\frac{1}{2}, \beta_2\}]$ we have $\frac{\partial \widehat{I}}{\partial \alpha} \leqslant 0$ and the corner solution $\alpha^* = \frac{1}{2}$. When instead $\beta \in (\frac{1}{2}, \beta_2]$ we have $\frac{\partial \widehat{I}}{\partial \alpha} > 0$ and an internal solution $\alpha^* \in \big(0, \frac{1}{2}\big)$. In this case the comparative statics gives:

$$\frac{d\alpha^*}{d\underline{W}} = \frac{\beta \frac{\partial \widehat{I}}{\partial \alpha}}{c(\frac{\partial \widehat{I}}{\partial \alpha})^2 + g''_\alpha} > 0$$

implying that when the social loss get worse the enforcer improves general accuracy. Finally, the usual consistency check can be easily verified along the same lines of the previous proofs.

■

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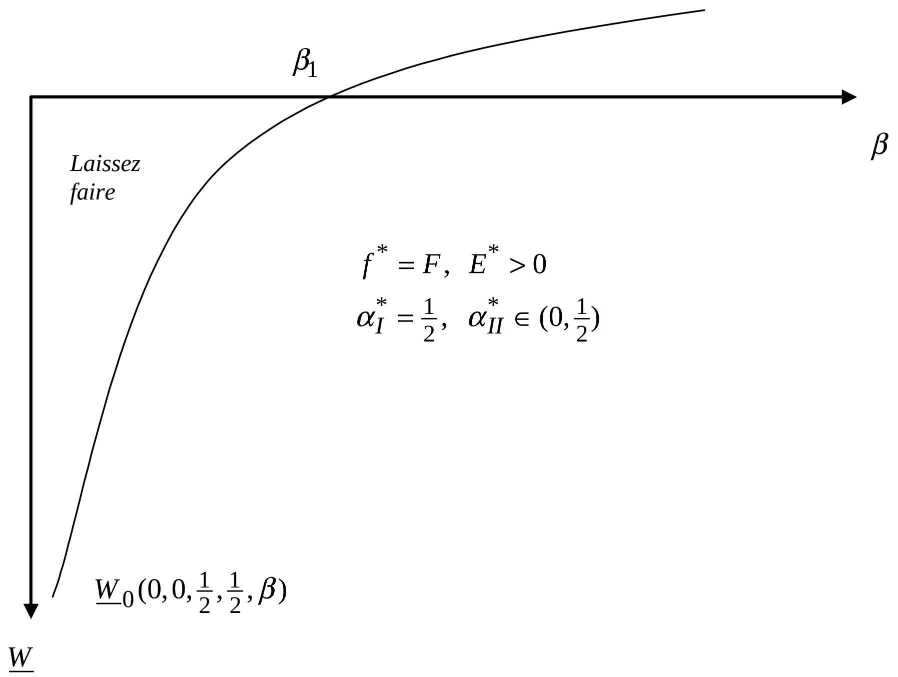

Figure 1: Endogenous enforcement and specific accuracy

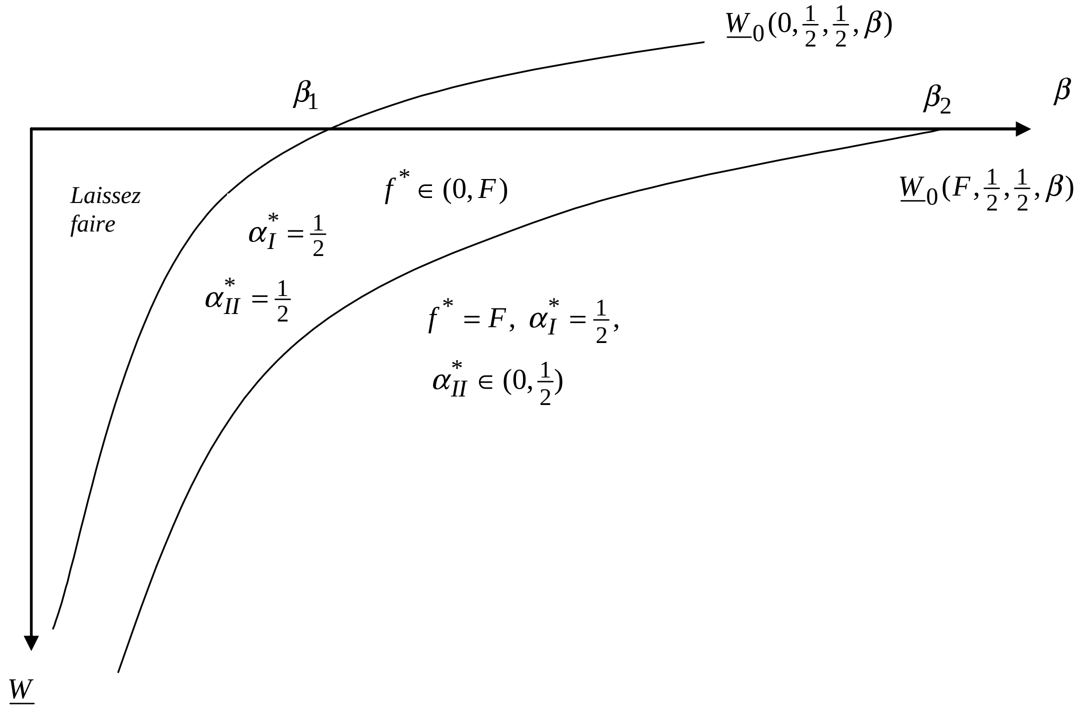

Figure 2: Exogenous enforcement and specific accuracy

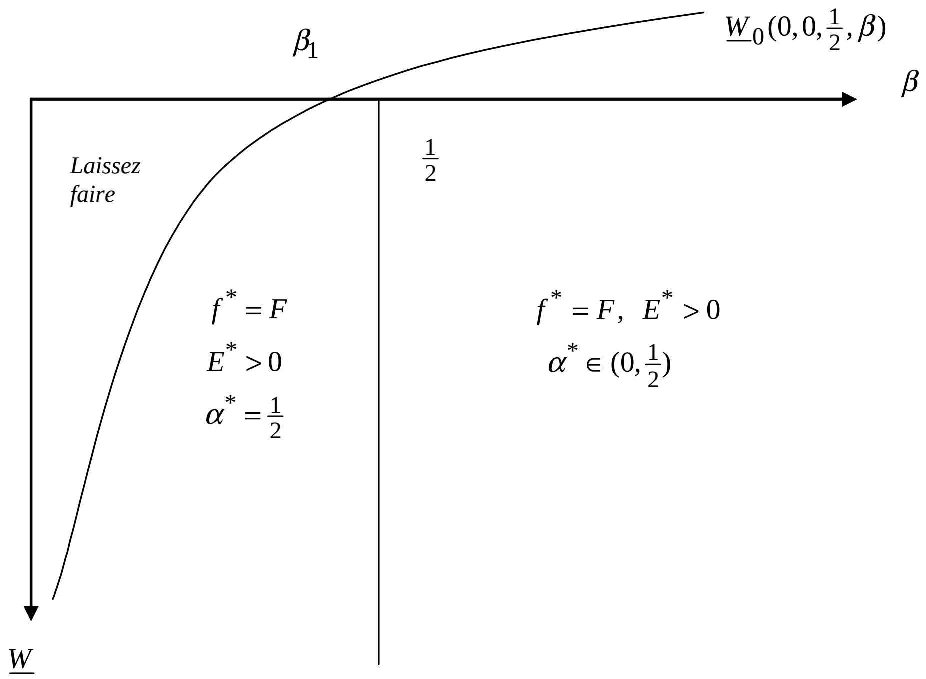

Figure 3: Endogenous enforcement and common accuracy

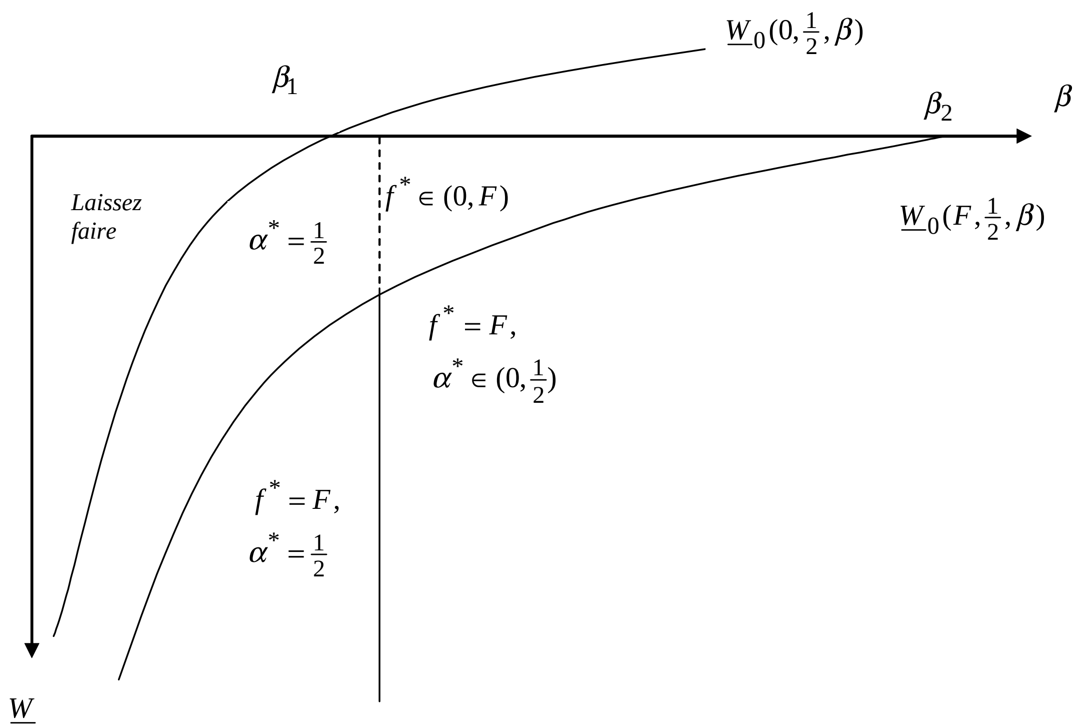

Figure 4: Exogenous enforcement and common accuracy