Strategic Differentiation by Business Models: Free-to-Air and Pay-TV*

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Abstract

Broadcasting markets are marked by the coexistence of outlets with radically different business models, some offering content free of charge and relying on advertising, others charging for access and airing few ads. We develop a model with competing broadcasters that leads to endogenous differentiation in business models. Differentiation is not driven, as in classic works, by the heterogeneity of agents. Rather it relates to the "two-sided" nature of these markets. A key driver is a strong form of strategic substitutability induced by natural properties of technology that allows advertisers to reach viewers.

Keywords: Platform Strategy, Asymmetric Equilibria, Attention Platforms, Endogenous heterogeneity

JEL: D85, L12, L13, L14, D42, D43.

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Media markets often display the coexistence of outlets following sharply different business models. For instance Free-to-Air (FTA) broadcasters distribute content to viewers free of charge and depend entirely on advertising revenue, whereas Pay-TV broadcasters rely mostly on subscription fees. Likewise, among the news media some outlets collect revenues solely from ads while others put their content behind paywalls. Why do these opposed regimes, with either advertisers or consumers footing the bill, coexist so frequently? Can these striking differences in business models be traced to competition among firms?

In this paper we study competition between television broadcasters. We argue that a principle of differentiation driven by strategic considerations helps account for these asymmetric outcomes. We design a model in which, under certain conditions, two originally identical stations, with the same set of potential viewers and advertisers, elect opposite pricing structures (business models) and raise most of their revenue from distinct sides of the market. We exclude agent heterogeneity as a cause, in order to distinguish our story from the classic results on differentiation. Instead we assume that all the agents on the same side of the market (viewers or advertisers), are homogeneous and relate differentiation to the two-sided nature of these markets.

We show that the key property that brings about this asymmetric equilibrium is a strong form of strategic substitutability. Loosely speaking, if one station supplies more advertising and decreases or eliminates subscription fees (i.e., shifts towards the FTA model), it heightens its competitor's incentive to raise fees and reduce advertising (moving towards the Pay-TV model), and vice-versa. To understand what drives this property, notice that in media markets consumers and advertisers typically satisfy their needs for content and advertising on multiple outlets (what the literature calls "multi-homing"). This means that if one broadcaster moves towards FTA and the competitor mimics this move, then the two stations, catering to the exact same viewers, turn out to be substitute means of delivering advertising messages to the same audience. Such an overlap induces competition for advertising money in the form of lower ad prices. This makes it more attractive for the competitor to shift the other way, towards Pay-TV. Strategic substitutability can also be illustrated the other way around. If a station moves towards the Pay-TV model, as by instituting a subscription fee and eliminating advertising, the other becomes advertisers' sole medium for reaching viewers, making the FTA option more attractive.

We maintain that this reasoning is sound only if the revenue potential of both the market for viewers and that for advertising is positive and balanced. If one side is too attractive, then the asymmetric equilibrium breaks down and both broadcasters cater to that side. We show that the degree of differentiation (i.e. the "distance" between the equilibrium business models) is hump-shaped in the revenue potential of one side relative to the other.

Further, we show that for an asymmetric outcome to always exist, strategic substitutability needs to be "strong enough". By this we mean that the change in strategy triggered by a change in that of one's rival must be big enough as measured by the slope of the best-response functions. We provide a mathematically simple, rather weak and intuitive sufficient condition for existence that can be traced to a property of the technological process that describes how advertising works. This property, namely strict log-concavity, captures a fact fully recognised by the industry, namely that concentrating the messages of an advertiser on a smaller number of outlets (one, in our model) increases the reach of the advertising campaign to the maximum by reducing wasteful duplication of exposure.

In section 4, we argue informally that this logic also applies in the richer setting with a continuum of heterogeneous viewers with idiosyncratic disutility from advertising.

Finally we draw policy lessons, in particular as regards the identification of relevant markets and the effect of advertising caps.

Relation to the literature. Our paper naturally relates to the literature on endogenous product differentiation in traditional markets as a means of moderating price competition.[1]

In these models ex-ante symmetric firms differentiate their products (for instance, offering high- and low-quality versions) to cater to different types of consumer. Likewise, in our model one firm supplies high-quality (for instance ad-free) paid content and another supplies low-quality (ad-supported) content free of charge. However, the classic results rely heavily on the heterogeneity of consumer tastes, which is necessary for screening purposes. In this paper we show that heterogeneity within either side of the market is not essential to the asymmetric outcome. Indeed, we obtain differentiation assuming homogeneous viewers and homogeneous advertisers throughout. What is essential is the presence of two distinct types of agents, one of the defining features of two-sided markets.

This paper contributes to a thriving literature on differentiation in media markets. Peitz and Valletti (2008) and Anderson *et al* (2016) have studied media outlets' choice of genre and content, extending the classic differentiation frameworks to two-sided outlets in the context, respectively, of single- and multi-homing consumers. We do not explicitly differentiate according to content, but in our framework one can consider quality to be better, the shorter and fewer the advertising breaks. A few recent works focus specifically on business models. Weeds (2013) provides an alternative case for the thesis that Pay-TVs and FTAs cohabit, in a framework akin to Shaked and Sutton (1982) with exclusive and heterogeneous consumers. In our paper the drivers of differentiation are different from hers (and, we speculate, complementary). Kind *et al* (2009) link symmetric business

[1] The classical references are Hotelling (1929) and d'Aspremont et al. (1979) for endogenous differentiation by variety and Shaked and Sutton (1982) for endogenous differentiation by quality.

models to the extent of content differentiation among firms. Content substitutability, they contend, makes it harder to extract rents through subscriptions and thus fosters FTA. Another related work is Dietl *et al* (2012), which takes the nature of the operators as given (one free, one pay) and draws implications on the quantity of ads. Unlike these papers, we posit generalized multi-homing agents and obtain endogenously asymmetric rather than symmetric outcomes.

The paper also contributes to the broader literature on the exercise of market power and the effect of competition in two-sided markets. So far, theories of "price skewness" have focused on the reasons why all the platforms in a given market may tilt their pricing structure one way or the other. By now we have a good understanding of symmetric business model equilibria characterized by asymmetric price structures, with all platforms cross-subsidizing the same side at the expense of the other. Which side is favored, then, depends on the relative elasticity and the strength of indirect network externalities (Rochet and Tirole (2006), Armstrong (2006), Bolt and Tienman (2008) and Schmalensee (2011), Spiegel (2013)). This result offers a good explanation for one fundamental feature of two-sided markets, namely the unbalanced price structure, but it neglects another, namely the coexistence of opposing price structures. To our knowledge, only Ambrus and Argenziano (2010) study asymmetric network equilibria with single-homing consumers in a general setting. They show that asymmetric networks arise endogenously in equilibrium: each one relatively cheaper and larger on one side. Their argument depends on heterogeneity in consumers' valuation of the network good and is thus different from but complementary to ours, which relies on multi-homing. Finally, our paper relates to a recent strand of theoretical and empirical work revisiting some classic results in media economics (e.g. Anderson and Coate (2005), Crampes *et al* (2009)), allowing consumers to satisfy their content needs on multiple platforms: Anderson *et al* (2016), Anderson and Peitz (2016), Ambrus *et al* (2016) and Athey *et al* (2018). We share these works' thesis that multi-homing viewers are less valuable, in that they can be served by advertisers via different operators, so the associated rents are competed away. We also share with Anderson and Peitz (2016) a time-use model of consumer choice among media.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 illustrates the equilibrium and comparative statics in duopoly. Section 4 informally discusses the case of viewers with heterogeneous preferences for content quality. Section 5 sets out concluding remarks and policy lessons.

1 The model

Two broadcasting stations, denoted $i = 1, 2$, offer their content and advertising services to a continuum (mass one) of identical viewers and advertisers. Agents, if they wish, may patronize more than one station. Following the literature, we call this "multi-homing" (as opposed to "single-homing"). To simplify the notation we do not index viewers or advertisers.

Viewer preferences and choices. Viewers choose which station, if any, to subscribe to, allocating a finite amount of time to the subscribed stations, which for simplicity we assume equal to 2 units. Let $v_i \in \{0, 1, 2\}$ denote the units of time spent watching station i (referred to as "viewing time"). Let $a_i \in [0, \overline{a}]$ denote the (endogenous) quantity of advertising on station i where $\overline{a} > 0$ is a positive real number. Finally let $U(v_1, v_2, a_1, a_2)$ be the gross utility from watching. U can be decomposed without loss of generality as the sum of the gross utilities from single-homing on station 1 and 2, denoted $u(\cdot)$ and a function $g(\cdot)$ capturing the fact that content substitutability reduces the overall utility for those who multi-home:

$$U(v_1, v_2, a_1, a_2) := \sum_{i=1,2} u(v_i, a_i) - g(v_1, v_2). \tag{1}$$

Assume that u, g are non-negative, twice continuously differentiable in all arguments with $u(0, a_i) = 0$. $g(v_1, v_2)$ also symmetric in v_1, v_2. Further:

(Love for Content with Diminishing Returns) $\frac{\partial u}{\partial v_i} > 0$ and $\frac{\partial^2 u}{\partial v_i^2} < 0$.

(Love for variety) $u(2, a_i) \leq U(1, 1, a_i, a_j)$.

(Advertising Aversion) $\frac{\partial u}{\partial a_i} = -v_i$.

(Content Substitutability) $g(v_1, v_2) = 0$ if and only if $v_1 v_2 = 0$.

These assumptions capture two key features of this market. First, content is preferred to advertising: willingness to pay decreases in a_i and it does so in proportion to viewing time, so a_i can be thought of as a vertical (quality) dimension of differentiation. Second, viewers also prize variety: spreading one's attention over different outlets always increases utility. This naturally implies a tendency to multi-home. Differentiation along a horizontal dimension (variety) is captured by our content substitutability assumption. Of course, this plays a role only if both stations are actually viewed. All viewers get a payoff equal to their utility less all fees. Online appendix A illustrates these assumptions with a quadratic utility function in the spirit of Levitan and Shubik (1980).

Viewers choose which station to subscribe to, and conditional on this choice they optimally allocate their viewing time slots to the accessible stations. It is convenient to anticipate their choice at this stage. Love for content and love for variety immediately imply that the optimal time allocation is equal to $(v_i = 2, v_j = 0)$, $(v_i = 0, v_j = 2)$ and $(v_i = 1, v_j = 1)$ given single-homing on i, single-homing on j and multi-homing respectively.

Advertising technology. Advertisers choose which station to patronize, if any. The stations allow advertisers to inform viewers for a price t_i according to a technology described below. Informing one viewer is assumed to generate an expected profit of $k > 0$, so the advertisers' payoff is equal to k times the number of viewers informed. How do viewers get informed? The probability of a given advertiser's informing a given viewer depends on which station(s) the advertiser patronizes, the quantity of advertising on each one (a_i, a_j) and the viewing time on each station (v_i, v_j). A reasonable assumption is that a larger v_i, given a_i, enhances the probability of a viewer's being exposed to the advertiser's message on station i and thus informed. Similarly, a larger a_i given v_i increases the probability of exposure.[2] To capture this in the simplest possible way we assume that the probability of informing a given viewer *through station* i is a function of $a_i \times v_i$ denoted $\phi(a_i v_i)$. To streamline the notation let us set $\phi'(a_i v_i) = \frac{d\phi(a_i v_i)}{da_i v_i}$ and $\phi''(a_i v_i) = \frac{d^2\phi(a_i v_i)}{d(a_i v_i)^2}$. Then the following properties hold: $\phi(a_i v_i)$ is a twice continuously differentiable, strictly increasing, strictly concave function ($\phi'(\cdot) > 0$ and $\phi''(\cdot) < 0$ for $a_i v_i \geq 0$) with $\phi(0) = 0$ and $\phi(\cdot) < 1$. As a multi-homing viewer can be informed through either of the two stations, the probability that a viewer characterized by (v_i, v_j) is informed *at least once on some station* is denoted Φ and assumed equal to 1 minus the probability that the viewer is not informed on either station. That is:

$$\Phi(a_i v_i, a_j v_j) := 1 - (1 - \phi(a_i v_i))(1 - \phi(a_j v_j)) = \phi(a_i v_i) + \phi(a_j v_j) - \phi(a_i v_i)\phi(a_j v_j). \quad (2)$$

This formulation captures the fact that advertisements on i and j are substitute means

[2]For concreteness, think of a_i as the number of commercial breaks during the season, i.e. the number of times a television or radio program is interrupted to broadcast a sequence of advertisements. By paying the advertising fee, an advertiser gets to run its commercial in every break (or, equivalently, in a given fraction of them, say one in five or one in ten). So the greater the number of breaks, the more times the ad runs, the larger the exposure. Similarly the higher the viewing time, the larger the probability that a viewer will be watching during a break, and the larger the exposure.

for informing a multi-homing viewer. For a single-homing viewer, say on station i, then $a_j v_j = 0$ and $\Phi = \phi(a_i v_i)$. In this case i becomes a competitive bottleneck in reaching this viewer. If (v_i, v_j) is identical across all viewers (as is the case in equilibrium) then Φ reads also as the fraction of the population of viewers informed.

Stations. The stations' profit is equal to the sum of subscription and advertising revenues. Stations choose the quantity of ads $a_i \in [0, \overline{a}]$, the advertising fee $t_i \geq 0$ and the subscription fee $f_i \geq 0$.[3]

Timing and Equilibrium. At stage 1 stations simultaneously choose quantities a_i and a_j. At stage 2, having observed quantities, viewer subscription and advertising fees f_i, t_i and f_j, t_j are set simultaneously. At stage 3, viewers and advertisers observe the stations' choices and choose which station(s) to patronize (if any), and viewers allocate their attention. The equilibrium concept is Subgame Perfect Nash Equilibria.[4]

The assumption that quantity is predetermined when agents make their choices captures the idea that broadcasting content production and program schedules (hence the quantity of commercial breaks) are set in advance. In the United States, for instance, broadcasters and advertisers meet on a seasonal basis at an "upfront" event to sell commercials on the networks' upcoming programs, whose length is usually already set. Unsold airtime, if any, is filled with tune-in spots.

[3]Observe that subscription fees cannot be contingent on viewing choices $\{v_i, v_j\}$, that is on the viewing time actually spent on each station, as we deem this to be unrealistic. For instance, this rules out equilibrium outcomes in which viewers are basically *paid* to watch commercials ($f_i < 0$ and a_i large). Indeed, in this case one would expect viewers to just grab the subsidy and choose $v_i = 0$.

[4]We resolve indifferences as follows. Agents who are indifferent between multi-homing or not choose to multi-home. If a station is indifferent between a fee that induces no participation and one that induces some participation, it always chooses the latter.

2 Competition leads to differentiation

Stage 2: Viewers' choices and equilibrium subscription fees. Let $\mathbb{I}_i(v_i > 0)$ be an indicator function equal to 1 if its argument holds true and 0 otherwise. The viewer's problem is:

$$\max_{v_1, v_2} U(v_1, v_2, a_1, a_2) - \sum_{i=1,2} f_i \mathbb{I}_i(v_i > 0), \tag{3}$$

$$\text{subject to } v_i \in \{0, 1, 2\} \text{ for } i = 1, 2 \text{ and } v_1 + v_2 \leq 2.$$

The above formulation encompasses two problems. First, given (f_i, a_i) and (f_j, a_j), viewers need to choose which (if any) stations to subscribe to. Second, given subscription choices, they must allocate their time. We have already established that the optimal choice at the intensive margin is $(v_i = 2, v_j = 0)$ or $(v_i = 0, v_j = 2)$ for single-homers and $(v_i = 1, v_j = 1)$ for multi-homing viewers. Consider subscription choices on this basis. A key object in what follows is a viewer's incremental utility from subscribing to station i after having already subscribed to $j \neq i$:

$$\Delta U_i(a_i, a_j) := U(1, 1, a_i, a_j) - u(2, a_j) \geq 0. \tag{4}$$

Observe that if i chooses a fee no greater than this incremental utility then all viewers necessarily subscribe to i and this holds true for all f_j. We now argue that in equilibrium each station charges a subscription fee equal to its incremental utility, inducing all viewers to multi-home. This is what Anderson *et al* (2016) in a closely related setting refer to as the "incremental pricing principle". Intuitively, station i cannot raise f_i unilaterally above U_i without losing all viewers in the subsequent subgame. Similarly, no station can unilaterally lower its fee below U_i without leaving money on the table. The following claim formalizes this argument (proof in online appendix B.1).

Claim 1 *The unique subgame perfect subscription fees are equal to the respective incremental utilities:*

$$f_i(a_i, a_j) := \Delta U_i(a_i, a_j) \quad \textit{for } i = 1, 2. \tag{5}$$

Note that when station i increases the quantity of advertising, it raises the incremental utility of the other station ΔU_j, inducing j to increase its subscription fee.

Stage 2: Equilibrium advertising fee. Let $\Delta\Phi_i$, referred to as the *incremental* probability of station i, denote the increase in the probability of informing a multi-homing viewer if the advertiser were to patronize station i *in addition* to station j. Hence, $\Delta\Phi_i$ is equal to the expected probability that a multi-homing viewer will be informed through i but not through j:

$$\Delta\Phi_i(a_i, a_j) := \Phi(a_i, a_j) - \Phi(0, a_j) \tag{6}$$

$$= \phi(a_i)(1 - \phi(a_j)) \geq 0. \tag{7}$$

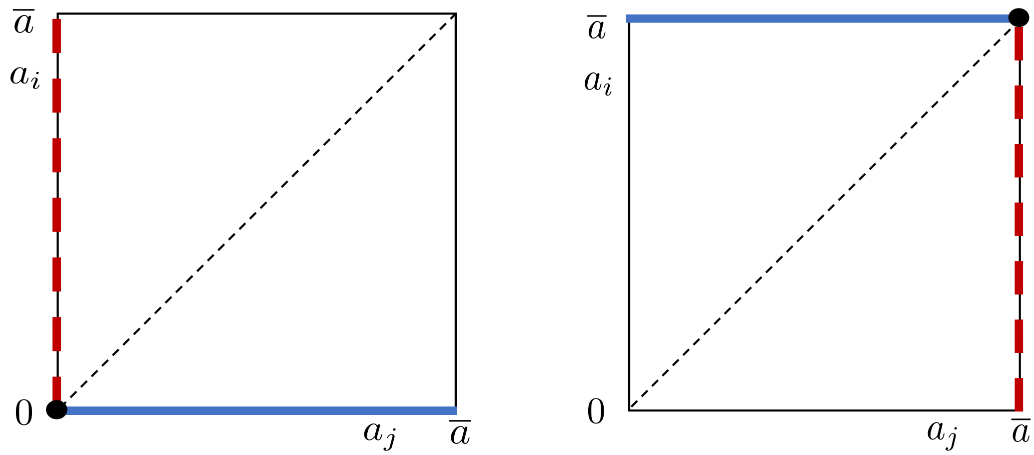
By an argument analogous to the foregoing, in equilibrium t_i and t_j must be equal to k times the respective incremental probabilities:

$$t_i(a_i, a_j) = k \cdot \Delta\Phi_i(a_i, a_j) \quad \text{for } i = 1, 2. \tag{8}$$

Stage 1: Equilibrium advertising quantities. Given a_j, the problem for station i is:

$$\max_{a_i \in [0, \overline{a}]} \pi_i := \pi(a_i, a_j, k) = \Delta U_i(a_i, a_j) + k \cdot \Delta\Phi_i(a_i, a_j). \tag{9}$$

k parametrizes the relative profitability of the two sides of the market. When choosing the quantity of advertising, stations trade off revenues from subscription against revenues from advertising. Indeed if the profit maximizing quantity lies in the interior of the choice



(a) $k \leq \overline{k}$ (b) $k \geq \overline{k}$

Figure 1: i's best responses (solid line - blue) and j's (dotted line - red)

set, it is characterized by the familiar first order condition equating marginal revenues on opposite sides:

$$\frac{\partial \pi_i}{\partial a_i} = \frac{\partial \Delta U_i}{\partial a_i} + k\frac{\partial \Delta \Phi_i}{\partial a_i} = 0. \tag{10}$$

The solution to problem (9) is i's best response to the choice of station j, a_j. In online appendix B.2 we show that such a solution exists and is unique. A Subgame Perfect Nash Equilibrium is basically a vector of fees and quantities satisfying (5) and (8) and solving problem (9) for $i = 1, 2$. As asymmetric equilibria always come in pairs, without loss of generality we restrict attention to equilibria with $a_1 \geq a_2$. Thus an asymmetric equilibrium is one in which 1 supplies more ads, $a_1^\star > a_2^\star \geq 0$, and hence has lower subscription prices $0 \leq f_1^\star < f_2^\star$ and greater advertising revenue $t_1^\star > t_2^\star \geq 0$.

What does this equilibrium look like? Clearly, if the value of informing a viewer is $k = 0$, then the game has a trivial unique symmetric equilibrium in which both stations forego advertising altogether: $a_1^\star = a_2^\star = 0$ and set the same subscription fee. Figure 1 (a) depicts this situation, showing the shape of the best responses and their intersection in the origin. On the contrary, if informing viewers is arbitrarily profitable (that is, k is sufficiently large) then the game has another straightforward symmetric equilibrium, which is unique and in which both stations advertise as much as feasible $a_1^\star = a_2^\star = \overline{a}$ and set the same subscription fee (Figure 1 (b)). It follows that an asymmetric equilibrium can obtain in loose terms only for "intermediate" values of k, that is when the revenue potential of the two sides is relatively balanced. The next claim formalizes this requirement, providing a *necessary* condition on k in order for an asymmetric outcome to exist (proof in online appendix B.3).

Claim 2 *An asymmetric equilibrium exists only if the profit k that advertisers expect when informing a viewer is neither too low nor too high:*

$$\underline{k} := [\phi'(0)]^{-1} < k < [\phi'(\overline{a})(1 - \phi(\overline{a}))]^{-1} := \overline{k}. \tag{11}$$

From now on in the analysis we assume that $k \in (\underline{k}, \overline{k})$.

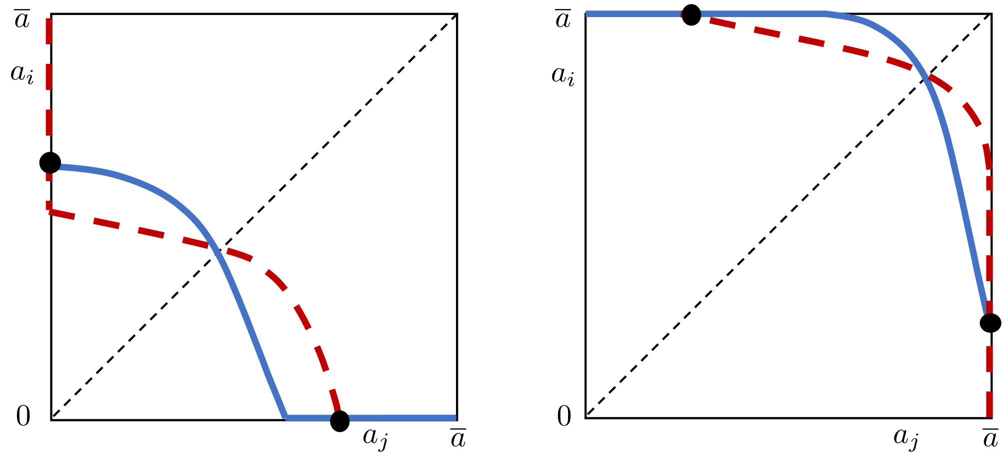
How does change in a_j affect the incentives of firm i? The following Claim (see online appendix B.2 for a formal proof) highlights a fundamental property of the game.

Claim 3 *Advertising quantities are strategic substitutes. That is to say, the quantity set by station i to maximize its profits is non-increasing in the quantity set by station j.*

To see this, notice that at any interior solution of i's problem, denoted $a_i^\star(a_j)$, by the implicit function theorem, differentiating (10) with respect to a_j gives:

$$\frac{da_i^\star}{da_j} = \frac{\phi'(a_i^\star(a_j))\phi'(a_j)}{\phi''(a_i^\star(a_j))(1 - \phi(a_j))} < 0. \tag{12}$$

The negative dependence results from the basic assumptions of positive and diminishing returns from advertising: $\phi' > 0$, $\phi'' < 0$. Loosely speaking strategic substitutability means that if one station moves towards the 'pay-TV' model (i.e. decreases quantity a_i and increases subscription fee f_i) then the other has a stronger incentive to move the other



(a) Case $k \leq \hat{k}$: one station supplies. (b) Case $k > \hat{k}$: Both stations supply.

Figure 2: Asymmetric equilibria

way towards the 'free-to-air' model, and vice-versa. Intuitively, if a competitor supplies a larger quantity of ads, the marginal returns of providing advertising diminish as the probability of the viewer's not being informed by the competitor, and thus potentially being informed by the station, shrinks. To see this via an extreme example note that if a_j is such that $\phi(a_j)$ is close to 1 then $\Delta\Phi_i$ is close to 0 so that i has basically no incentive to trade subscription against advertising revenues and the optimal a_i is close to 0. When $k \in (\underline{k}, \overline{k})$ an interior solution $a_i^\star(a_j)$ exists for some a_j. By (12) in the neighborhood of such a solution the best response function is strictly decreasing. It follows that when $k \in (\underline{k}, \overline{k})$ best responses are not flat (like those depicted in Figure 1), with at least a portion that is negatively sloped.

The fact that the *symmetric* best responses are negatively rather than positively sloped makes it possible that they intersect at least once *away* from the diagonal. Strategic substitutability by itself, however, does not imply that an asymmetric equilibrium exists.[5]

[5] See Amir *et al* (2010) for an excellent discussion of asymmetric outcomes in static games with global strategic substitutes. They also provide sufficient conditions for asymmetric outcomes (diagonal non-concavities) that are different from those offered in this paper.

Nonetheless we now argue that if strategic substitutability, as measured by the slope of the best response functions, is "strong enough" then an asymmetric equilibrium *must* exist. To build geometric intuition, consider Figure 2 which shows the qualitative shape of the best response of firm i (solid blue) and j (dotted red) in two notable cases. These cases are presented in detail below. For the purpose at hand, simply note that in both panels the symmetric best responses drawn satisfy the (local) property that they are very steep (slope lower than -1) when they cross the diagonal. This property immediately guarantees the existence of an asymmetric outcome. To see this, observe that in a neighborhood to the left of the intersection along the diagonal, i's best response (solid line) must lie above the inverse of j (dotted line) because the slope is greater than 1 in absolute value. But then the two lines, being continuous, must intersect again, giving rise to an asymmetric equilibrium. The same property implies that such an asymmetric equilibrium is stable while the symmetric equilibrium (which always exists) is not stable for a wide range of best response dynamics. In this sense the symmetric outcome is less compelling.[6,7]

In our problem, the slope being less than -1 means that the response of the optimal a_i to a 1-unit decrease in the quantity supplied by station j is greater than one unit. As (12) points out, the response along the diagonal $a_i = a_j = a$ depends on the ratio $\phi'^2(a)/(\phi''(a)(1-\phi(a)))$. So the response depends on the relative strength of two opposing effects. The numerator captures "avoided duplication." A 1-unit reduction in a_j decreases by $\phi'(a_j)$ the probability that a viewer is informed through station j making station i's

[6]In fact with continuous best responses, a symmetric pure strategy equilibrium always exist. See Vives (1990) footnote 7 and theorem 4.2 (iii.) for a formal argument in a related context.

[7]In oligopoly models, stability is often used as a selection criterion for a number of reasons. For instance, stable equilibria are "more compelling" in that they allow one to think of the static equilibrium as the rest point of some dynamic adjustment process that captures some learning or bounded rationality of the players. Stability-guaranteeing assumptions are also needed to ensure "natural" comparative statics results such as output going down with marginal costs (Dixit (1986)). See Vives (2001), chapter 2, for a formal definition and a thorough discussion of this property).

advertising more effective (recall that i's marginal probability of informing a viewer goes up proportionally to $\phi'(a_i)\phi'(a_j) = \phi'^2(a)$ along the diagonal). The denominator captures the intensity of decreasing returns. If ϕ' decreases very fast (i.e. ϕ'' is large), then it takes only a small adjustment of a_i to restore the equality of marginal revenues on opposite sides (10) . On the contrary, the weaker the decreasing returns, the larger the necessary adjustment in a_i. At the limit if ϕ'' were close to 0 then, other things being equal, the slope of the best response would be arbitrarily large.[8]

The next proposition goes one step further introducing a property of the technology that allows stations to inform viewers (strict log-concavity) which in our setting can be considered intuitive and natural. Crucially, it implies the slope property discussed above, guaranteeing the existence of an asymmetric equilibrium.

Proposition 1 *Suppose that the probability of not informing a viewer $1 - \phi$ is strictly log-concave and the profit k that advertisers expect when informing a viewer is neither too low nor too high ($k \in (\underline{k}, \overline{k})$). Then an asymmetric equilibrium exists and is unique.*

What feature of the industry does log-concavity capture and how does it relate to the slope of the best response? The following claim shows a key, telling property of log-concave technologies: multi-homers are easier to inform by 'concentrating' advertising on one station than by spreading it around.

Claim 4 *If $1 - \phi(a)$ is strictly log-concave then for all $a_i \geq a_j$ and $\Delta > 0$,*

$$\Phi(a_i, a_j) < \Phi(a_i + \Delta, a_j - \Delta). \tag{13}$$

[8] Decreasing returns affect the marginal probability of informing those viewers who are not informed by station j, so $\phi''(a_i)$ is weighted by $(1 - \phi(a_j))$. Along the diagonal then, this term becomes $\phi''(a)(1-\phi(a))$.

That is, for a given total advertising quantity $a_i + a_j$, concentrating it more on one station increases the total reach.

Under (13) total reach is trivially maximized when, given a total (and feasible) quantity of advertising $a \leq \overline{a}$ we have $a_i = a$ and $a_j = 0$. Log-concavity captures a very widely assumed thesis in the broadcasting industry, namely that multi-station advertising campaigns are wasteful, as individual stations cannot tell which ads multi-homing viewers have seen on rival stations. Multi-outlet campaigns lead to some individuals not being exposed on either station and others being reached multiple times.[9,10] If concentration pays off in terms of reach, then intuitively the avoided duplication effect must be greater than that of the intensity of diminishing returns. Indeed on the locus $a_i = a_j$, strict log-concavity is equivalent to the slope of the best response being lower than -1.[11]

What does the asymmetric equilibrium look like? The following proposition completes the description of the equilibrium, providing a full characterization of the quantities (proof in online appendix B.6).

Proposition 2 *Suppose that the probability of not informing a viewer $1 - \phi$ is strictly log-concave. Then there are thresholds $\tilde{k}$ and $\hat{k}$ in the profit k that advertisers expect when informing a viewer with $\underline{k} < \tilde{k} \leq \hat{k} < \overline{k}$, such that:*

[9]Tying ads to content and synchronizing airings are simple strategies that TV stations use to enhance reach. For a simple illustration, suppose each station has 2 units of content each requiring 1 unit of attention and supplies only 1 advertising message tied to each piece of content. Suppose there are 2 advertisers, each purchasing 2 messages. Viewers consume 1 random piece of content on each station. If advertisers concentrate all messages on the same station (i.e. purchase 2 messages on it) then all consumers are exposed. If advertisers purchase 1 message on each outlet then on average a quarter of consumers are not informed.

[10]The reason why we do observe multi-outlet campaigns in reality, despite the argument that these are wasteful, is that some viewers single-home. So in concentrating their advertising effort, advertisers trade efficiency for reach on these single-homers.

[11]Formally, if $\log(1 - \phi(a))$ is strictly concave in a then $-\phi''\phi - (\phi')^2 < 0$, and therefore the slope of the best response is lower than -1 along the diagonal (see the proof of Proposition 1 in online appendix B.4).

(i.a) If $(\underline{k}, \tilde{k}]$ *then* $a_2^\star = 0$ *and* $a_1^\star \in (0, \overline{a})$

(i.b) If $(\tilde{k}, \hat{k}]$ *then* $a_2^\star = 0$ *and* $a_1^\star = \overline{a}$*,*

(ii.) If $(\hat{k}, \overline{k}]$ *then* $a_2^\star > 0$ *and* $a_1^\star = \overline{a}$*.*

Depending on the relative profitability of k we can have basically either of two regimes. If k is "small" then only one station is active on the advertising side of the market (Figure 2(a)). Furthermore if $a_1^\star < \overline{a}$ then station 1 chooses the quantity that equates marginal returns on opposite sides of the market:

$$1 = k \cdot \phi'(a_1^\star). \tag{14}$$

If k is "large" then both stations are active, with station 1 at capacity and $a_2^\star$ equating revenues on opposite sides of the market (Figure 2(b)):

$$1 = k \cdot \phi'(a_2^\star)(1 - \phi(\overline{a})). \tag{15}$$

Intuitively the threshold between the two regimes, denoted $\hat{k}$, is such that station 2 is exactly indifferent between supplying and not supplying one unit of advertising when the rival station supplies at capacity $\overline{a}$. Comparing the two cases, notice that in the former total advertising is allocated in such a way as to maximize total reach and in the latter the constraint on a_1^* implies an allocation of total advertising that, in accordance with Claim 4, does not maximize total reach. Since station 1 cannot expand advertising beyond the threshold $\overline{a}$ it is profitable for station 2 to provide a limited amount of advertising $a_2^* > 0$, with some inefficient duplication.

Applying the implicit function theorem to (14) and (15) establishes that whenever $a_i^\star$ is interior then it must be continuous and monotonically increasing in k. Coupled with

Proposition 2, this implies that the greatest enhancement of differentiation comes when the revenue potential of the two sides, as captured by the value of k, is intermediate.

Proposition 3 *The extent of strategic differentiation, as measured by $a_1^{\star} - a_2^{\star}$ (or equivalently by $f_2^{\star} - f_1^{\star}$) is continuous and hump-shaped in the expected profit from informing a viewer k.*

Specifically $a_1^{\star} - a_2^{\star}$ is equal to 0 for $k \notin [\underline{k}, \overline{k}]$, increasing for $k < \tilde{k}$, flat in $\left[\tilde{k}, \hat{k}\right]$ and decreasing for $k > \hat{k}$.

Relationship with classic results on product differentiation. The result on asymmetric business models is related to the theoretical literature on endogenous product differentiation. Following Hotelling (1929), these studies typically analyze equilibrium models of oligopoly with a product-choice stage preceding price competition, as in D'Aspremont et al. (1979) and Shaked and Sutton (1982). Product differentiation emerges in equilibrium, with ex-ante identical firms supplying different products. Differentiation is strategic in that it is driven by the need to attenuate price competition, not to cater to demand. These differentiation results depend crucially on the assumption that consumers have different tastes for either quality (vertical differentiation) or variety (horizontal differentiation). So consumers with different characteristics (or of different "types") patronize different firms in equilibrium. Heterogeneity is crucial, in that it allows firms to set positive mark-ups. While the heterogeneity of consumers and advertisers is certainly an important feature of media markets, a key insight of our analysis is that it is not indispensable to the existence of the asymmetric business model equilibrium. Indeed, in our setting viewers and advertisers are all homogeneous and in equilibrium consume the same bundle of products. However, having two groups of agents implies that each firm has two different sources of profit, corresponding to the two sides. In keeping with the classic result, here too differentiation is strategic, in that opting to raise revenues from the opposite side of the market from one's rival permits the relaxation of price competition on that side; and vice-versa. But the mechanism is different: it is heterogeneity across rather than within sides of the market that leads to different business models.[12] In this sense our result is more than the simple extension of a familiar result to two-sided platforms. Instead, in a multi-sided environment it highlights an additional, specific source of differentiation that cannot arise in one-sided settings.

This analysis has maintained the assumption that the stations have substitutable but otherwise symmetric content. Assuming instead that distaste for advertising is correlated with taste for content quality in other dimensions, the stations clearly have an additional incentive to differentiate along these other dimensions, with the Pay-TV opting for premium content. The analysis of the interactions between choice of business model and choice of content is left to future work.

3 Heterogeneous preferences and viewer sorting

To highlight the role of two-sidedness in shaping the asymmetric outcome, our model forecloses the classic demand-driven incentives to differentiate by assuming that all agents share the same preferences. In Calvano and Polo (2016) we analyze a richer model allowing for a continuum of heterogeneous viewers with idiosyncratic marginal disutility

[12] We are indebted to Helen Weeds and Patrick Rey for raising this issue and helping us develop this argument.

from advertising (taste for quality).[13,14] Under conditions analogous to those posited in Proposition 1, an asymmetric equilibrium exists with the pay TV station raising revenue only from subscription fees and the free-to-air station solely from advertising.

How does the heterogeneity of preferences affect incentives? With heterogeneity, the choice to subscribe can differ, some viewers single-homing and others accessing both stations. An important (and realistic) equilibrium feature in this richer setting is that the two stations cater to different subsets of viewers: the FTA station serves all viewers while the pay station serves only some, those with strong distaste for advertising or taste for quality. In this setting, in contrast with the analysis in section 3, the FTA serves a mix of single-homing and multi-homing viewers. A key insight from previous work is that the composition of demand is a relevant factor for profit and hence for incentives, the reason being that single-homing viewers are more valuable for advertising purposes, as the outlet serving them becomes a bottleneck, monopolizing and accordingly monetizing their attention. So in the richer model stations have preferences both over the level of demand (how many viewers?) and over its composition (single- or multi-homers?). In the rest of this section we offer a broad intuition as to why the new effects in play here actually reinforce and enrich our baseline logic. For a more detailed analysis, see Calvano

[13] We posit a utility function in the spirit of Levitan and Shubik (1980):

$$U(a_1, a_2, v_1, v_2; \theta) = \sum_{i=1}^{2} \left[\theta(1 - a_i)v_i - \frac{2-\sigma}{2} v_i^2 \right] - \sigma v_1 v_2, \tag{16}$$

with $v_i \in [0, 1]$ denoting viewing time on station i, θ denoting idiosyncratic marginal utility from exposure to content and $\sigma \in [0, 1)$ measuring the degree of substitutability between stations (contents).

[14] Another element we did not consider is heterogeneity of advertisers in, say, the expected profit from informing. Athey et al. (2018) show that an equilibrium in the sorting of advertisers across outlets arises in a setting with exogenous viewer demand and log-concave technology. The advertisers whose opportunity cost of not informing viewers is greatest multi-home, while low-value advertisers single-home on the outlet with the larger audience. We speculate that the externalities discussed above – leading to a negatively sloped best response function with the amount of advertising on one station reducing the incremental probability on the other – would carry over to this richer setting. A full analysis of this case is left for future work.

and Polo (2016).

We maintain that even in this richer setting, when one station moves towards a given business model, say by increasing advertising and reducing subscription fees, the other station has an incentive to move in the opposite direction. As above, we label the advertising-abundant station as station 1 and the one with the higher subscription fee station 2: that is $a_1 > a_2$ and $f_1 < f_2$. At some initial allocation, then, station 1 is closer to the free-to-air model and station 2 to the pay-TV model. Now consider what happens when firm 2 further increases f_2 and decreases a_2 intensifying its for-pay character and further reducing its audience. Some of the previously shared viewers are now served exclusively by firm 1, as individuals formerly at the margin between single-homing and multi-homing now strictly prefer to single-home.[15] This selection is favorable to firm 1: other things constant, its incentive to provide advertising and move in the opposite direction strengthens. We call this the "composition effect," since the effect of the pay station's strategy on the free station's incentives works through its effect on the composition of the rival's audience. Second, consider the impact on firm 2's incentives of firm 1 strengthening its FTA character by increasing a_1. In addition to the effects already identified in section 3, which induce station 2 to reduce a_2,[16] there is also an increase in the demand for station 2 subscriptions, as the former marginal viewer now strictly prefers to subscribe to 2. This "level effect" pushes station 2 further towards setting a higher f_2, thus sharpening its pay-TV character.[17]

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[15]15 This new effect comes on top of the one already found in the benchmark model. When a_2 is reduced, the incremental probability of informing viewers on station i increases, leading this station to increase the quantity of advertising a_1.

[16]An increase in a_1 raises the incremental utility of station 2 and viewers' willingness to subscribe, leading to a lower response of a_2.

[17]In Calvano and Polo (2016), viewers are also allowed to choose their viewing time on a continuous set rather than a discrete grid. Then we identify additional effects at the intensive margin that work in the same direction as those in the benchmark model. Single-homers spend more time on the station patronized than multi-homers. Hence, when f_2 is raised, inducing some multi-homers to watch only

In summary, elastic demand due to heterogeneous preferences *preserves* and *reinforces* the property of strategic substitutability, which is crucial for differentiation.

4 Conclusions and policy lessons

IIn this paper we have analyzed the business models chosen in equilibrium by two ex-ante identical platforms serving two groups of homogenous agents, viewers and advertisers. We have shown that there exists a (pair of) asymmetric equilibrium in which one station raises most of the revenues from viewers and the other from advertisers. Differentiation is not driven by the traditional heterogeneity of agents, that in our model are homogeneous on each side, but by the two-sidedness of the competitive environment. Indeed, when the revenue potential of the two sides is sufficiently balanced, it is optinal to cater from different sides to relax competition on the same group of agents.

The result arises due to a strong form of strategic substitutability, such that when one station goes towards a "free" business model by raising advertising quantity and reducing subscription fees the rival station has the opposite incentive, enhancing its "pay" nature. We connect this property to a feature of the advertising technology, say log-concavity, that is quite natural in our market environment. Log-concavity of the advertising technology implies that concentrating advertising on one station increases total reach by reducing wasteful duplication.

We argue that our result has some interesting policy implications on competition policy and regulation of the media markets. It is an established practice in antitrust

station 1, the latter gains exclusive viewers that spend more time watching its programs and advertisers' willingness to pay increases further. Conversely, when station 1 increases the quantity of advertising a_1, the multi-homing viewers of station 2 allocate more time to its programs, and their greater willingness to pay allows it to raise f_2.

and media regulation to treat operators with opposite business models as belonging to different relevant markets (Filistrucchi et al. (2014)). For instance, in the merger cases BSkyB/Kirch Pay-TV[18] and News Corporation/Premiere[19] the European Commission has ruled that FTA and Pay-TV operators belong to separate product markets. The German Bundeskartellamt reached similar conclusions in the Springer/ProSieben/Sat1 case. The common view is that a Pay-TV broadcaster deals only with viewers, whereas an FTA deals only with advertisers, with no overlapping or competitive constraints. This argument is then extended to the case where a Pay-TV broadcaster raises most of its revenues from subscription but also offers some advertising. To the best of our knowledge BSkyB/ITV is the only case in which an authority has taken a different position; here the UK Competition Commission has recognized that:

> In two-sided markets suppliers can compete with one another at different price points, given the ability to generate revenues in two separate markets. For instance, FTA services may compete directly for viewers with pay services, with higher viewing figures indirectly generating higher advertising revenues. (UK Competition Commission (2007), par. 4.6)

Supporting this latter position, we have shown that the asymmetric business models adopted by initially symmetric broadcasters are the result of strategic interaction. The broader market definition that follows carries strong implications for every area of

[18]See case COMP/JV.37, BSkyB/Kirch Pay TV (Mar. 21 2000). The merger involved BSkyB, whose main activity was pay-TV broadcasting in the UK, and KirchPayTV Gmbh offering pay-TV services in Germany and Austria. The Commission distinguished two product markets, one for pay-TV and one for interactive digital TV, according to the nature of the business model, without considering the advertising and viewer sides of the market.

[19]See case COMP/M.5121, 2008 O.J. (C 219) 2. The concentration involved the acquisition of a 25% stake in Premiere, a pay-TV operator active in Germany and Austria, by News Corporation, a large international media company active in the pay-TV segment. The Commission considered the pay-TV services only, expressing some concern for vertical relationships but ignoring the impact of FTA operators.

competition policy,

If we interpret $\overline{a}$ as an advertising cap set by a regulator, then an additional and novel effect in our model is the potential allocative distortion when such limits are too strict. Recall that given any such cap $\overline{a}$, when advertising is sufficiently profitable ($k > \hat{k}$), Pay-TV operators become active on the advertising side of the market. However, under reasonable assumptions concerning the advertising technology total reach would be greater if all advertising messages were concentrated on one station only. So by inducing the entry of additional stations on the advertising side of the market, tighter caps (smaller $\overline{a}$) reduce the overall surplus in the economy due to the inefficient use of consumer attention.

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