Acquisitions, innovation and the entrenchment of monopoly

Vincenzo Denicolò*and Michele Polo†

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Abstract

We analyze a dynamic model of repeated innovation where inventors may either be acquired by an incumbent or else resist takeover and challenge for leadership. In the short run, acquisitions always spur innovation because of the invention-for-buyout effect. In the longer run, however, they may stifle it because of a countervailing effect, the entrenchment of monopoly. The latter occurs when the incumbent's dominance depends on past levels of activity and is therefore reinforced by recurrent acquisitions. We show that if the entrenchment effect is sufficiently strong, forward-looking policymakers should prohibit acquisitions in anticipation of the long-run negative impact on innovation. This argument sets out a new theory of harm that can be used to block acquisitions that could otherwise go unchallenged.

Keywords: Acquisitions; Innovation; Market power; Invention-for buyout; Entrenchment-of-monopoly

*Corresponding author, University of Bologna and CEPR. Address: Department of Economics, Piazza Scaravilli 2, I-40125 Bologna, Italy. E-mail address: vincenzo.denicolo@unibo.it; Tel.: +39 3281892059

†Bocconi University, GREEN and IGIER. E-mail address: michele.polo@unibocconi.it

1 Introduction

Technology giants often acquire innovative start-ups with high growth potential. This phenomenon is particularly apparent in the digital industry, with recent examples including Facebook's acquisitions of WhatsApp and Instagram, Google's acquisitions of YouTube and Waze, and Microsoft's acquisition of LinkedIn.[1] However, acquisitions of startups are also frequent in other innovative sectors, such as for instance pharma and biotech.

Most acquisitions of innovative startups today slip under the radar of antitrust authorities, or are cleared, because merger control focuses on the size of the firms at the time of the takeover, and these target firms are often still small. Wallmann (2019) refers to this phenomenon as "stealth consolidation." However, in the last decade this lenient policy has increasingly been called into question. Critics, such as Cremer et al. (2019), Furman et al. (2019), and Scott Morton et al. (2019), argue that this policy is ill-suited to innovative industries, where the acquisition of small entrants may hinder Schumpeterian competition (i.e., the replacement of market leaders by new firms) and stifle innovation. Advocates of the permissive policy counter that the prospect of such buyouts heightens the incentive to innovate for small enterprises that lack the assets required to effectively bring their innovations to the market – a mechanism known as the *invention-for-buyout* effect, named after Rasmusen's (1988) "entry-for-buyout."

Adding to this debate, we propose a Schumpeterian model of repeated innovation and acquisition in which acquisitions have both pro- and anti-competitive consequences. The former stem from the invention-for-buyout mechanism, while the latter derive from a mechanism that we define as *entrenchment of monopoly.*

The entrenchment-of-monopoly effect occurs when an acquisition increases the incumbent's competitive advantage over potential challengers, i.e., its level of "market dominance". This obstructs the entry of future inventors, reducing their incentives to innovate. The effect persists even if future inventors are later acquired themselves, because the incumbent's entrenchment worsens their outside options and therefore

[1] These prominent cases are just the tip of the iceberg. Focusing only on the "big five," Motta and Peitz (2021) report 42 acquisitions by Amazon, 33 by Apple, 21 by Facebook, 48 by Google, and 53 by Microsoft in the period 2015-2020. The FTC (2021) lists over six hundreds acquisitions by the same five firms that fall below the thresholds for notification in the period 2012-2019.

reduces the acquisition price they can negotiate with the incumbent.

The entrenchment-of-monopoly effect may result from various specific mechanisms, such as consumer inertia, dynamic economies of scale, exclusive access to more and better data, and similar factors. All of these mechanisms imply that acquisitions that increase the incumbent's size today may strengthen its market dominance in the future. To demonstrate the consequences of the entrenchment effect, in this paper we focus on the case in which the incumbent's competitive advantage is rooted in a form of consumer inertia, whereby consumers who have patronized the incumbent in the past become reluctant to switch to a different supplier. However, the entrenchment effect tends to arise more broadly, whenever the incumbent's strength depends on its past levels of activity.

Within this theoretical framework, we demonstrate that the effects of acquisitions depend on the time horizon. In the short run, they enhance the incentive to innovate due to the invention-for-buyout effect. However, in the longer term, if the entrenchment effect is sufficiently strong, they can diminish the rate of innovation and consumer welfare. In other words, the buyout effect prevails in the short run, but the entrenchment effect may dominate in the long run. The analysis also sheds light on the role of other factors, such as the speed at which innovations are imitated. Moreover, we demonstrate that the optimal policy may be state-dependent, permissive when market dominance is weak and restrictive once repeated acquisitions have made it too strong.

These results have significant implications for both empirical research and policy. From a policy perspective, they provide a theoretical foundation for certain policy changes outlined in the new horizontal merger guidelines issued by the Department of Justice and the Federal Trade Commission in 2023. Specifically, our analysis relates to Guideline 6 (*Mergers Can Violate the Law When They Entrench or Extend a Dominant Position*) and Guideline 8 (*When a Merger is Part of a Series of Multiple Acquisitions, the Agencies May Examine the Whole Series*). Concerning the latter, we elucidate why acquisitions should not be evaluated in isolation. Adopting such a myopic approach in our model would invariably lead to a lenient policy but is generally sub-optimal. Instead, forward-looking policymakers may lean towards a more restrictive policy. Regarding new Guideline 6, our results provide a theory of harm that justifies why the agencies might want to prevent the entrenchment of a monopoly even when the static allocative effects of the acquisition are negligible.

Importantly, our theory assumes that the sole objective of the antitrust authorities is consumer welfare. It is sometimes contended that this narrow focus is responsible for the leniency of merger policy. However, we show that if the authorities are forward looking and consider the cumulative dynamic effects of different policy rules, taking consumer surplus as the welfare criterion may justify a restrictive policy on acquisitions. After presenting our results, we discuss the policy implications more fully in the concluding section.

From an empirical perspective, our analysis can explain why acquisitions may hinder innovation by future entrants, a phenomenon documented in the enterprise software sector by Eisfeld (2023) using a structurally estimated dynamic model of the industry. More precisely, Eisfeld (2023) demonstrates that acquisitions by major industry incumbents are followed by a decline in entry, consistent with the entrenchment-of-monopoly effect. Instead, acquisitions by industry outsiders or financial companies stimulate future entry, consistent with the invention-for-buyout effect.

The remainder of the paper is structured as follows. In the next section, we provide a brief overview of the relevant literature. Section 3 presents a tractable model of repeated innovation and acquisitions, where the entrenchment of monopoly results from consumer inertia. Section 4 derives the equilibrium. Section 5 examines the impact of acquisitions on the rate of innovation, and Section 6 analyzes the optimal antitrust policy. Finally, in Section 7, we provide a summary and conclusion. The proofs are set out in the Appendix.

2 Relation to the literature

Although the risk of entrenchment of monopoly is often cited in the acquisition policy debate (see, for instance, Scott Morton et al., 2019 and Bryan and Hovenkamp, 2020b), to the best of our knowledge this paper presents the first formal analysis of this possibility. (In recent independent work, Fons-Rosen et al. (2022) analyze numerically an endogenous growth model that bears some resemblance to ours.)

Previous research on the impact of acquisitions on innovation has either focused on static models of isolated innovations or else posited that the degree of market dominance is time-invariant. In these settings, the entrenchment effect cannot arise.[2]

Models of isolated innovations have been used both to demonstrate the invention-for-buyout effect (see, for instance, Mason and Weeds, 2013, Phillips and Zhdanov, 2013, and Letina et al., 2021) and to uncover various adverse effects on innovation of acquisitions. In an important contribution, Cunningham et al. (2021) have shown, both theoretically and empirically, the profitability of "killer acquisitions," in which the new owner suppresses one or more research projects initiated by the takeover target in order to prevent the cannibalization of its own market. In a similar vein, Kamepalli et al. (2020) suggest the possibility of a "kill zone," where entrants, whose innovations would challenge the incumbent's dominance, are discouraged by the threat of an aggressive reaction; see also Fumagalli, Motta and Tarantino (2022). In contrast, Gilbert and Katz (2022) show that the incumbent can be biased against shutting down the entrant's project as having multiple products may facilitate price discrimination. Our analysis abstracts from these effects.

In addition, a recent literature uses static models to analyze the impact of acquisitions not only on the rate but also on the direction of technological progress. In particular, acquisitions can affect the diversity of research projects (Letina et al., 2021), the degree of horizontal product differentiation (Gilbert and Katz, 2022), whether innovators target substitutes or complements of the incumbent's product (Shelegia and Motta, 2021; Dijk, Moraga-Gonzàlez and Motchenkova, 2023), and whether they target the product of the market leader or of the follower (Bryan and Hovenkamp, 2020a).

The present paper, instead, forms part of the strand of the literature analyzing antitrust policy in dynamic models of repeated innovation. The pioneering contribution here is Segal and Whinston (2007). Below, we discuss the differences with their model at some length; for now, suffices it to say that they do not consider acquisitions and assume that the degree of market dominance is constant over time.

This latter assumption is also made by Cabral (2018, 2021). He distinguishes

[2] There is also an extensive literature on the impact of mergers on innovation, which analyzes the effect of mergers on post-merger innovation incentives. For an excellent synthesis, see Bourreau et al. (2021). In contrast, this paper focuses on the impact of acquisitions on the pre-merger incentives to innovate of potential targets.

between incremental and radical innovations. For incremental innovations, the buyout effect implies that acquisitions spur innovation. Radical innovations, however, are different: the invention-for-buyout effect is nil, insofar as these innovations would not be transferred to the incumbent anyway. Still, acquisitions are not neutral because innovators may choose which type of innovation to target. When acquisitions are permitted, incremental innovations may therefore crowd out radical ones. This crowding-out mechanism may reduce the overall rate of innovation. Clearly, it is quite different from the entrenchment of monopoly.

Another model of repeated innovation is presented in Katz (2021). Similar to our approach, Katz explores scenarios in which the inventor chooses the magnitude of quality improvement over the existing technology. He notes that in this case, the incentive to innovate is determined not by the level of the inventor's payoff but by the rate with which it increases with the size of the innovation. Katz further demonstrates that, even if acquisitions result in an increase in the inventor's profit level, they can lead to a decrease in the incentive to innovate. While this observation applies to our model, it is not the reason why acquisitions may hinder innovation. In our model, the buyout effect alone would lead to acquisitions increasing both the profit level and the profit slope for the inventor.

In a model without innovation, Nocke and Whinston (2010) analyze a sequence of mergers, demonstrating the optimality of a myopic merger policy that assesses each merger in isolation. A forward-looking policy is not necessary in their framework because the only change from one period to the next is the set of firms that remain independent. In contrast, our model is inherently more dynamic.

While we have chosen to keep the model simple enough to be tractable analytically, several papers have utilized numerical analysis to explore richer industry dynamics. In a partial equilibrium framework, Hollenbeck (2020) examines the trade-off between the static allocative effects of acquisitions and the positive effects on innovation due to the buyout effect. He finds that acquisitions lead to short-term welfare reduction but can be beneficial in the long run. In contrast, Mermelstein et al. (2020) argue that a forward-looking policy would be more restrictive than a myopic one. This is because in their model, entry may be inefficient due to economies of scale in production and investment. Acquisitions facilitate entry, once again because of the invention-for-buyout effect, but the inefficiency of entry implies that a restrictive policy may be optimal in the long run.

Other papers embed the sequence of acquisitions in a general equilibrium framework. Cavenaile et al. (2021) find that strengthening antitrust enforcement could yield substantial welfare gains in the long run, though the complexity of their computable general equilibrium model makes it challenging to identify the relevant channels. Similarly, Fons-Rosen et al. (2022) find that a stricter policy may boost the economy's growth rate in a model of endogenous growth that feature a form of the entrenchment-of-monopoly effect.

3 The model

We propose a highly stylized model of repeated innovation that allows for closed-form solutions but nevertheless can exhibit complex dynamics. In the absence of acquisitions in the model, incumbents would be systematically replaced by new innovators, in the spirit of Schumpeterian competition. However, the possibility of acquiring these challengers may lead to the persistence of monopoly.

The model is tailored to industries where the ability to innovate is diffuse, making it improbable for the same firm to innovate repeatedly and for successful innovators to be identified before the innovation is developed. Once the innovation has been developed, on the other hand, the incumbent can identify potential challengers.

An example of the industries we have in mind is the enterprise software industry analyzed by Eisfeld (2023). Eisfeld identifies 500 different product markets in this broad industry and studies approximately 3,000 acquisitions of startups that occurred between 2010 and 2019. The acquired firms are predominantly small, with around 70% of them having initiated just one or two research projects, which are subsequently developed by larger incumbents. Acquisitions are the prevailing pattern of growth for these startups, occurring in about 95% of the cases.[3]

[3] Acquisitions play a less prominent but still vital role in the development of startups in other innovative industries, such as pharma and biotech. For instance, Cunningham et al. (2021) report that nearly one-fourth of all new drugs were invented by small startups that were acquired by larger companies before product launch.

3.1 Timing and payoffs

We consider an infinite horizon game in discrete time. Each period t is divided into three stages. In the first stage (*ex ante*), an innovative entrant chooses its investment in R&D, which determines the size of its innovation. In the second (*interim*), the inventor enters the market, where an incumbent firm already operates. If acquisitions are permitted, the incumbent and the entrant bargain over the acquisition price and, if they reach an agreement, the acquisition occurs. In the third stage (*ex post*), firms compete in prices. This sequence is repeated in every period $t = 1, 2, ...$

Firms are forward looking and maximize intertemporal profits, future values being discounted by the common discount factor $\delta < 1$.

3.2 Demand and cost

A vertically differentiated product is bought by a mass of infinitely-lived homogeneous consumers (normalized to 1). The quality of the product, q_t, increases over time as a result of innovative activity. In each period t, a consumer may demand either 0 or 1 unit. One unit of a product of quality q_t purchased at price p_t yields a net utility of

$$U_t = q_t - p_t. \tag{1}$$

The utility of not purchasing is 0. The unit production cost c is independent of quality and is normalized to 0.

We have chosen this very simple set-up because it can accommodate a steady flow of innovations while ensuring a form of separability in the value functions, which, as we will discuss in more detail later, allows for closed-form solutions.

3.3 Innovation, entry and market structure

In each period t, the industry comprises an incumbent, a new entrant, and a competitive fringe. The entrant has an idea for improving the existing technology q_{t-1}. It then develops the idea into an innovation, i.e., a product of quality $q_t > q_{t-1}$, by investing in R&D.[4] (With a modest abuse of notation, we denote by q_t both the

[4] Our assumption that only outsiders can innovate aligns with standard Schumpeterian models, where the incumbent would not innovate even if it had the same capability for innovating as

quality and the identity of inventor.)

We assume that the magnitude of innovation, $\Delta_t = q_t - q_{t-1}$, depends on the entrant's R&D expenditure. This fits well industries where innovation is incremental, and uncertainty is limited, such as the software industry mentioned above. Specifically, we assume that the cost of raising quality by Δ_t, denoted as $C(\Delta_t)$, is given by:

$$C(\Delta_t) = \frac{1}{2}\Delta_t^2. \tag{2}$$

The fact that the cost is independent of the current quality level, q_{t-1}, ensures stationarity. We adopt the quadratic specification for simplicity, and setting the coefficient to $\frac{1}{2}$ is an innocuous normalization.

The entrant chooses its R&D investment, and hence the size of the innovation, to maximize profits. After developing its invention, it enters the market. In the absence of acquisitions, inventor q_t is the technological leader but faces competition from the incumbent, which is inventor q_{t-1}. In period $t+1$, inventor q_t becomes the new incumbent and competes with inventor q_{t+1}.

As time passes, inventions can be imitated by the competitive fringe. We assume that the innovation is used exclusively by the inventor for two periods and afterwards can be imitated freely. (For example, the invention might be protected by a patent that lasts for two periods.) As a consequence, in the absence of acquisitions, inventor q_{t-2} is absorbed by the competitive fringe in period t. Thus, in each period t there are three types of firm: an entrant (E), which supplies a product of quality $q_t^E = q_t$, an incumbent (I) with quality $q_t^I = q_{t-1}$, and a competitive fringe (F) with $q_t^F = q_{t-2}$.[5]

outsiders, due to Arrow's replacement effect. Recognizing that in certain industries, incumbents innovate repeatedly over time, a strand of the endogenous growth literature has attempted to modify the standard Schumpeterian framework to allow for innovation by leaders. For example, see Denicolò and Zanchettin (2012) and the references therein. This literature has demonstrated that the main qualitative properties of Schumpeterian models often extend to these more complex environments.

[5] One can allow imitation to be faster, say because intellectual property protection is imperfect. For example, continuing to assume that invention q_t is fully protected in period t and can be imitated freely in period $t+2$, the innovation could be imitated partially in period $t+1$. In this case, the competitive fringe's quality would be $q_t^F = q_{t-2} + \vartheta(q_{t-1} - q_{t-2})$, where the parameter ϑ is an index of the speed of imitation, or an inverse index of the strength of intellectual property protection. When $\vartheta = 1$, the competitive fringe imitates the innovation in just one period, whereas the baseline case in which it takes two periods is re-obtained for $\vartheta = 0$. With this more general formulation, the only change in our formulas is that the discount factor δ is replaced by $\delta(1-\vartheta)$. Thus, the parameter δ that we use throughout the model can be thought of as capturing both the private rate of time preference and the strength of intellectual property protection.

The presence of the competitive fringe ensures the stationarity of the model, even as the quality level continues to rise over time, thus guaranteeing the existence of a steady state. On the other hand, the assumption that innovative products are imitated within just two periods is not essential for our results but facilitates the analysis by reducing the order of the difference equations describing the industry's dynamics.

3.4 Market dominance

Following Segal and Whinston (2007), we assume that, despite being a technological laggard, the incumbent may have acquired some other competitive advantage, owing, for instance, to such factors as consumer inertia, intertemporal network externalities, dynamic economies of scale, or exclusive access to more and better data. Alternatively, entrants may face entry hurdles; for example, some consumers may be unwilling to try new products, or may not even be aware of their existence. For a discussion of various forms of incumbency advantage, see Biglaiser et al. (2019).

While Segal and Whinston (2007) assume that the incumbent's dominance is time-invariant, we allow it to change over time as a function of the industry's past history. This flexibility is essential in order for dominance to be fortifiable through acquisitions, thus potentially leading to the entrenchment of market power.

For tractability, we adopt a specific interpretation of market dominance, assuming that demand is not entirely contestable. This approach is commonly used in the literature on exclusionary conduct (e.g., Ide and Montero, 2020 and Oertel and Schmutzler, 2021). Specifically, we assume that a fraction μ_t of consumers are "captive" to the incumbent and cannot purchase from the new entrant, while the remaining consumers are "free" and can buy the new product. Both free and captive consumers can purchase from the fringe.

The entrant establishes its captive consumer base by capturing a fixed fraction, κ_F, of the customers it serves. It can then leverage this consumer base in the subsequent period when it becomes the new incumbent. The size of the captive consumer base, denoted as μ_t, represents, in our model, the degree of market dominance of the incumbent.

In the absence of acquisitions, μ_t evolves over time according to the following law of motion (the superscript NA stands for "no acquisition"):

$$\mu_{t+1}^{NA} = \kappa_F (1 - \mu_t) x_t^E, \tag{3}$$

where x_t^E is the fraction of free consumers who buy from the entrant. This is because in period t, there are $1 - \mu_t$ free consumers, so $(1 - \mu_t)x_t^E$ represents the number of consumers who purchase from the entrant. Equation (3) states that a fraction κ_F of them become captive in the next period.

This model of captive consumers can be justified by assuming that a fraction κ_F of consumers need two periods to become familiar with their current product before being ready to switch to a new and superior one. For the other $1 - \kappa_F$ consumers, however, learning is more expeditious, enabling them to freely adopt all products on offer, including the latest. In any case, all consumers can always patronize the competitive fringe because older products may be adopted easily if a customer is already familiar with more advanced ones.

The difficulty of learning how to use new and improved products seems to be a common occurrence. For example, Eisfeld (2023) notes that in the enterprise software industry she analyzes, users often exhibit some resistance to switching to new products due to learning costs or other biases in favor of the status quo products.[6]

Equation (3) can also be interpreted, more broadly, as a metaphor for various possible reasons for the emergence and evolution of market dominance over time. Its key feature is that the entrant's degree of market dominance in period $t + 1$, μ_{t+1}, depends on its past sales, $(1 - \mu_t)x_t^E$. A stronger incumbent in period t (a higher μ_t) poses a greater entry barrier, because it limits the entrant's current sales,and because it constrains the captive consumer base that the entrant can build for the next period.

[6] Alternatively, one could imagine a scenario in which a fraction κ_F of consumers must bear learning costs substantial enough that they find it worthwhile to invest in learning only if they can leap ahead by two quality steps. As a consequence, a consumer who purchased the state-of-the-art product in period t, q_t, would not consider switching to product q_{t+1} in period $t + 1$. In other words, such a consumer would be captive in period $t + 1$ and revert to being free in period $t + 2$.

3.5 Pricing

In each period t, the competitive fringe prices product q_{t-2} at cost. Meanwhile, the incumbent and the entrant engage in price competition.

In our baseline model, we assume that firms cannot price discriminate and that they set prices sequentially, with the incumbent acting as price leader. As we will demonstrate, this implies that acquisitions do not affect consumer surplus for a given state of the technology. For our purposes, this is a conservative property that biases the analysis against prohibiting acquisitions.

Later on we will consider alternative assumptions. The case when the entrant acts as price leader is taken up in Section 6.3. The case of simultaneous moves is more challenging, as there is generally no pure-strategy pricing equilibrium. Intuitively, the existence of captive consumers is analogous to a capacity constraint, as the entrant cannot supply more than $(1 - \mu_t)$ units. However, a pure-strategy equilibrium reappears if, with either simultaneous or sequential moves, firms could engage in price discrimination. We will consider this case in footnote 11.

3.6 Acquisitions

If acquisitions are allowed, the incumbent may take over the inventor after it has fully developed its new product. The resulting merged entity is denoted as M. The entrant furnishes M with its new technology, q_t, which is ready for use without incurring in any further development cost. (This sets our model apart from models of killer acquisitions.) On the other hand, the incumbent brings its exclusive control over the old technology q_{t-1} and its captive consumer base. In a way, the incumbent is revitalized through the acquisition, which postpones its potential exit. The merged entity M faces no competitors apart from the fringe.

We assume that not only free but also captive consumers can purchase any product from M, including the newest one. This assumption is justified on the grounds that the merged entity may ensure backward compatibility, provide the same usage modes, or guarantee a seamless transfer of data to the new service, making it feasible for all consumers to transition to the newer product. A similar assumption is made in Kamepalli et al. (2020). The assumption implies that there are efficiencies from acquisitions. These effiencies make acquisitions profitable even though in our baseline model they do not lead to price increases.

Continuing to posit that a fraction κ_F of a firm's customers become captive, with acquisitions the law of motion of captive consumers becomes (the superscript A stands for "acquisition"):

$$\mu_{t+1}^A = \kappa_F x_t^M, \tag{4}$$

where x_t^M denotes the fraction of consumers served by firm M in period t. Since the merged entity can serve more consumers than the entrant alone, it can build a larger captive consumer base for the next period. This is how acquisitions increase market dominance in our model, creating the entrenchment-of-monopoly effect.

If acquisitions occur systematically, consumers who consistently purchase from the merged entity remain captive for multiple consecutive periods. The assumption that the same fraction of consumers becomes captive, regardless of their past purchase history, may then be questioned. It might be more reasonable to assume that consumers who are already captive are more susceptible to being captured again compared to free consumers. For instance, consumers who repeatedly purchased from the same firm in the past may have hindered their ability to learn. In this case, the law of motion for captive consumers becomes:

$$\mu_{t+1}^A = \kappa_F(1-\mu_t)x_t^{F,M} + \kappa_C \mu_t x_t^{C,M}, \tag{5}$$

where $x_t^{F,M}$ and $x_t^{C,M}$ represent the fraction of free and captive consumers, respectively, served by firm M in period t, and $\kappa_C \geq \kappa_F$.[7]

We assume that bargaining is efficient, so acquisitions will take place whenever they are jointly profitable. The acquisition price paid by the incumbent, P_t,[8] determines the division of the bargaining surplus between the two parties. We denote as α the share that accrues to the entrant, so α represents the entrant's "bargaining power." For example, if one of the two firms is randomly selected to make a take-it-or-leave-it offer to the other, then α is the probability that the entrant makes the offer and the incumbent receives it; with a probability of $1-\alpha$, these roles are

[7] In fact, the analysis that follows applies also to the case $\kappa_C < \kappa_F$. All that changes is that convergence to the steady state is oscillatory rather than monotonic.

[8] The assumption that the incumbent acquires the entrant, and not the other way around, is just an accounting convention. Nothing would change if the roles were reversed.

reversed.

3.7 Equilibrium

We analyze the Markov perfect equilibria of this game of complete information. Under our assumptions, at the beginning of each period t (the *ex ante* stage), the payoff-relevant variables are μ_t, q_{t-1} and q_{t-2}. At the *interim* stage, i.e. once the entrant has chosen the size of the innovation Δ_t, they also include q_t.

4 The acquisition game

In this section, we find the model's equilibrium under the assumption that acquisitions are always permitted. Since the merged entity can replicate any behavior of both entrant and incumbent, acquisitions are weakly profitable. In fact, we will show that they are always strictly profitable. This implies that acquisitions will always take place in equilibrium (see Corollary 1).

To ensure a perfect equilibrium, we start from the pricing subgames and proceed backwardly to the acquisition process and choice of innovation size. We denote total discounted profits as of period t by Π_t^i and current profits by π_t^i, with $i \in \{E, I, M\}$.

4.1 Pricing subgames

On path. We begin from the pricing subgame that is actually played on the equilibrium path, i.e., the one starting after the acquisition.

The merged entity's only competitor is the fringe, which supplies the best freely available quality, $q_t^F = q_{t-2}$, and prices it at cost, $p_t^F = 0$. The equilibrium strategy of the merged firm is given by the following lemma.[9]

Lemma 1 *The merged entity supplies only one product of quality $q_t^M = q_t$. It serves all consumers ($x_t^{F,M} = x_t^{C,M} = 1$) at price $p_t^M = q_t - q_{t-2} = \Delta_t + \Delta_{t-1}$, reaping a*

[9]To simplify the presentation, we adopt the following tie-breaking rule: when a consumer or a firm is indifferent among different actions, it chooses the one that maximizes aggregate profits. This assumption captures the idea that the stronger firm could shave the price marginally in order to break the indifference.

profit of

$$\pi_t^M = \Delta_t + \Delta_{t-1}. \tag{6}$$

The intuition is simple. The competitive fringe does not sell any output in equilibrium but exerts competitive pressure by providing an outside option to consumers. The merged entity undercuts the fringe in utility space, charging a price equal to the value of the quality differential.

Note that the presence of the competitive fringe prevents prices and profits from increasing without limit even though the quality level continues to rise over time. This implies that all benefits from technological progress eventually accrue to consumers.[10]

Off path. Next, we characterize the price equilibrium that arises, out of the equilibrium path, if the incumbent does not acquire the entrant. (The same equilibrium arises also on the equilibrium path if acquisitions are prohibited, as discussed below.) In this case, there are two active firms besides the fringe. Remember that in the baseline specification we assume that the firms price sequentially, with the incumbent acting as price leader.

Lemma 2 *If the incumbent acts as price leader, it serves all captive consumers and the entrant serves all free consumers ($x_t^E = 1$). The incumbent prices at $p_t^I = q_{t-1} - q_{t-2} = \Delta_{t-1}$ and obtains a profit of*

$$\pi_t^I(\mu_t) = \mu_t \Delta_{t-1}. \tag{7}$$

The entrant's equilibrium price is $p_t^E = q_t - q_{t-2} = \Delta_t + \Delta_{t-1}$, so the profit it earns in the first period of its life cycle is

$$\pi_t^E(\mu_t) = (1 - \mu_t)(\Delta_t + \Delta_{t-1}). \tag{8}$$

[10]Note also that equilibrium prices do not depend on the discount factor, despite the forward-looking nature of the firms. As the proof of the lemma demonstrates, even if the merged entity has an incentive to build a larger captive consumer base for the next period, when it sets the price myopically, it already serves all the consumers it can reach. Therefore, it is not necessary to further reduce the price below the myopic level. (This is also true for the entrant, in the equilibrium with no acquisition that we will analyze presently.) In a more general model, where the incentive to further reduce the price is consequential, antitrust policies aimed at countering the entrenchment of monopoly might target both acquisitions and firms' pricing strategies. However, this raises a number of delicate issues beyond the scope of this paper. Our policy analysis focuses solely on the entrenchment of monopoly through acquisitions.

When the incumbent acts as price leader, both the incumbent and the entrant slightly undercut the competitive fringe in utility space, and the entrant also slightly undercuts the incumbent. Consequently, consumers get the same net utility from any firm they may buy from.[11]

Implications. Lemmas 1 and 2 carry several significant implications. First, the per-period profit functions π_t^i are additively separable in the quality steps Δ_t and Δ_{t-1}. As discussed below, this property is crucial to allow for a closed-form solution, and our specific assumptions about demand and cost serve to guarantee it. In particular, separability might be lost with heterogeneous consumers.

Second, the lemmas imply the following:

Corollary 1 *Acquisitions are always strictly profitable.*

In our model, acquisitions do not increase current prices but are still profitable for two reasons. From a static perspective, they facilitate the diffusion of innovation: the state-of-the-art product is sold not only to free consumers but also to captive ones. This is a consequence of the efficiencies from acquisitions noted above. From a dynamic perspective, acquisitions increase the fraction of captive consumers that the merged entity can exploit in the next period, thus improving the outside options in negotiations over the acquisition price with the next inventor.

Third, the fact that acquisitions do not impact the price of the state-of-the-art product, which always remains at $\Delta_t + \Delta_{t-1}$, implies that the standard static allocative effects of mergers vanish in our model. As mentioned, this is a conservative property for our purposes. In a model where acquisitions also result in price increases, a more restrictive policy would likely be optimal.

In terms of consumer welfare, we have:

Corollary 2 *Either with or without acquisitions, consumers obtain exactly the surplus guaranteed to them by the fringe:*

$$CS_t = q_{t-2}. \tag{9}$$

[11] If firms could engage in price discrimination, irrespective of the timing, the incumbent would charge a price of Δ_{t-1} for the captive consumers and 0 for the free consumers. Therefore, the incumbent would earn a profit of $\pi_t^I = \mu_t \Delta_{t-1}$, as in baseline case, and the entrant would obtain a profit of $\pi_t^E = (1-\mu_t)\Delta_t$. It can be easily verified, following the same procedure as in the baseline case, that the level of innovation with price discrimination is exactly the same as in the baseline.

From the consumer's perspective, acquisitions affect only innovation, which, in turn, determines future consumer surplus.

Finally, Lemmas 1 and 2 show that in equilibrium the entrant serves all free consumers ($x_t^E = 1$), and the merged entity all consumers ($x_t^{F,M} = x_t^{C,M} = 1$). Therefore, with no acquisitions the share of captive consumers evolves over time according to:

$$\mu_{t+1}^{NA} = \kappa_F(1-\mu_t). \tag{10}$$

With acquisitions, the dynamics of μ_t is:

$$\mu_{t+1}^{A} = \kappa_F(1-\mu_t) + \kappa_C\mu_t. \tag{11}$$

4.2 The acquisition price

Proceeding with our backward induction, consider next the acquisition price.

Firms are forward looking and correctly anticipate all the future consequences of their choices. Since entrants are systematically acquired, the acquisition price must coincide with the entrant's value function (gross of the innovation cost). This is determined simultaneously with the value functions for the incumbent and the merged entity, as we shall see presently.

To proceed, it is important to keep in mind that in a Markov perfect equilibrium, the value functions depend only on the payoff-relevant variables. From the foregoing, it appears that profits depend on μ_t and the quality differentials Δ_t and Δ_{t-1}. Thus, the period-t payoff-relevant variables are $\{\mu_t, \Delta_{t-1}\}$ at the *ex ante* stage and $\{\mu_t, \Delta_{t-1}, \Delta_t\}$ at the *interim* stage. Accordingly, we denote by $V_t^i(\mu_t, \Delta_{t-1})$ the firms' *ex ante* value functions, and by $v_t^i(\mu_t, \Delta_{t-1}, \Delta_t)$ the *interim* functions, for $i \in \{E, I, M\}$.

These value functions must satisfy the following conditions (to simplify the notation, we suppress the dependence of the *interim* value functions on the relevant variables when this does not create confusion):

$$v_t^M = \pi_t^M + \delta V_{t+1}^I(\mu_{t+1}^A, \Delta_t) \tag{12}$$

$$v_t^E = (1-\alpha)\left[\pi_t^E + \delta V_{t+1}^I(\mu_{t+1}^{NA}, \Delta_t)\right] + \alpha\left(v_t^M - \pi_t^I\right) \tag{13}$$

$$\begin{aligned} v_t^I &= \alpha\pi_t^I + (1-\alpha)\left\{v_t^M - \left[\pi_t^E + \delta V_{t+1}^I(\mu_{t+1}^{NA}, \Delta_t)\right]\right\} \\ &= v_t^M - v_t^E. \end{aligned} \tag{14}$$

Equation (12) says that the merged entity obtains profits π_t^M in period t and then becomes the new incumbent with μ_{t+1}^A captive consumers, yielding a continuation value of $\delta V_{t+1}^I(\mu_{t+1}^A, \Delta_t)$. According to (13), the acquisition price (which, as noted, coincides with the entrant's value function) equals the entrant's disagreement payoff plus a fraction α of the bargaining surplus. The entrant's disagreement payoff is equal to the current profit if it resists the takeover, π_t^E, plus the continuation value, $\delta V_{t+1}^I(\mu_{t+1}^{NA}, \Delta_t)$. The "one-shot deviation principle" implies that the continuation value must be calculated on the expectation that even if there was no acquisition in period t, entrant q_t, once it has become the new incumbent in period $t+1$, will acquire entrant q_{t+1}. At that point, however, it will have only μ_{t+1}^{NA} captive consumers. As for the period-t incumbent, its disagreement payoff is simply π_t^I, given that with no agreement it would exit the market in the next period. The bargaining surplus is therefore $v_t^M - \left[\pi_t^E + \delta V_{t+1}^I(\mu_{t+1}^{NA}, \Delta_t) + \pi_t^I\right]$, from which there follows condition (13). The final condition says that the value of being the incumbent, which is given by its current profit plus a share $(1-\alpha)$ of the bargaining surplus, is equal to the value of the merged entity minus the acquisition price. In other words, the acquisition does not change the sum of the firms' values because the extra-profits created by the merger are already included in the forward-looking valuation of the firms.

4.3 The innovation size

The system of equilibrium conditions (12)-(14) cannot be solved for the *interim* value functions yet, because it also involves the *ex ante* value functions $V_{t+1}^I(\mu_{t+1}^A, \Delta_t)$ and $V_{t+1}^I(\mu_{t+1}^{NA}, \Delta_t)$, which depend on Δ_t. In turn, Δ_t might potentially depend on the future values Δ_{t+1}, Δ_{t+2} etc. To proceed, we must therefore consider the optimal choice of the size of the innovation.

Anticipating that it will be acquired at the *interim* stage for a price $P_t = v_t^E(\mu_t, \Delta_{t-1}, \Delta_t)$, the entrant's choice must satisfy the following condition:

$$\Delta_t^A(\mu_t, \Delta_{t-1}) = \arg\max_{\Delta_t} \left[v_t^E(\mu_t, \Delta_{t-1}, \Delta_t) - \frac{1}{2}\Delta_t^2 \right]. \tag{15}$$

In a Markov perfect equilibrium, the value of Δ_t is anticipated by all players, furnishing a link between the *ex ante* and *interim* value functions. That is, the *ex ante* value must be equal to the *interim* value calculated at the optimal innovation size:

$$V_t^i(\mu_t, \Delta_{t-1}) = v_t^i \left[\mu_t, \Delta_{t-1}, \Delta_t^A(\mu_t, \Delta_{t-1}) \right] \qquad \text{for } i \in \{E, I, M\}. \tag{16}$$

This completes the set of conditions that must all hold simultaneously in equilibrium.

4.4 Equilibrium

It is easy to see that the set of Markov perfect equilibria coincides with the set of solutions to the system of equilibrium conditions (12)-(16), given the profit functions (6), (7) and (8).

The solution can be calculated explicitly thanks to a key simplifying property of the model, which we noted above: the profit functions π_t^i are additively separable in Δ_t and Δ_{t-1}. This separability implies that while the value function $v_t^E(\mu_t, \Delta_{t-1}, \Delta_t)$ depends on Δ_{t-1}, the marginal value of increasing the innovation size, $\frac{\partial v_t^E(\mu_t, \Delta_{t-1}, \Delta_t)}{\partial \Delta_t}$, does not. Therefore, the optimal innovation size in period t, $\Delta_t^A(\mu_t, \Delta_{t-1})$, is independent of Δ_{t-1}, and in turn Δ_t does not affect the future values Δ_{t+1}, Δ_{t+2}, ..., in spite of the forward-looking nature of system (12)-(16).

These properties of the model imply that the marginal value $\frac{\partial v_t^E(\mu_t, \Delta_{t-1}, \Delta_t)}{\partial \Delta_t}$ can be calculated even without full knowledge of the value function $v_t^E(\mu_t, \Delta_{t-1}, \Delta_t)$, allowing a two-stage solution. In the first stage, we calculate the marginal value and find the equilibrium innovation size for any value of μ_t, $\Delta_t^A(\mu_t)$. With this function in hand, in the second stage we determine the value function $v_t^E(\mu_t, \Delta_{t-1}, \Delta_t)$ by a guess-and-verify method. Further details are provided in the proof of Lemma 3 in the Appendix. Applying this procedure, we get:

Lemma 3 *In the baseline model, the equilibrium innovation size depends on the share of captive consumers* μ_t *and is*

$$\Delta_t^A(\mu_t) = (1+\delta\kappa_F)(1-\mu_t) + \alpha(1+\delta\kappa_C)\mu_t. \tag{17}$$

The ex ante value functions are

$$\begin{aligned} V_t^E(\mu_t, \Delta_{t-1}) &= \phi_0 + \phi_1\mu_t + \phi_2\mu_t^2 + (1-\mu_t)\Delta_{t-1} & (18)\\ V_t^I(\mu_t, \Delta_{t-1}) &= \varphi_0 + \varphi_1\mu_t + \varphi_2\mu_t^2 + \Delta_{t-1} & (19)\end{aligned}$$

The coefficients ϕ_s and φ_s, for $s = 0, 1, 2$, depend on the exogenous parameters α, δ, κ_F and κ_C and are reported in the proof of Lemma 3 in the Appendix.[12] Given $V_t^E(\mu_t, \Delta_{t-1})$ and $V_t^I(\mu_t, \Delta_{t-1})$, one can easily recover $V_t^M(\mu_t, \Delta_{t-1})$ and the *interim* value functions $v_t^i(\mu_t, \Delta_t, \Delta_{t-1})$, $i = M, I, E$, from conditions (12)-(16).

5 Acquisitions and innovation

In this section, we analyze the impact of acquisitions on innovation. We show that prohibiting acquisitions always reduces the equilibrium size of innovation in the short run but may increase it in the long run if the entrenchment effect is large enough.

5.1 Innovation with no acquisitions

To proceed, we determine the innovation size when acquisitions are prohibited. In this case, the entrant's payoff is

$$\Pi_t^{E,NA} = \pi_t^E(\mu_t) + \delta\pi_{t+1}^I(\mu_{t+1}^{NA}), \tag{20}$$

where the profit functions are the same as in Lemma 2.[13] The equilibrium innovation size with no acquisitions then is $\Delta_t^{NA}(\mu_t) = \arg\max_{\Delta_t}\left[\Pi_t^{E,NA} - \frac{1}{2}\Delta_t^2\right]$. Simple

[12] The Appendix also verifies that $V_t^I(\mu_t, \Delta_{t-1})$ increases with μ_t, a property that we used in the derivation of the pricing equilibria.

[13] This is not self-evident, because firms are forward looking, and the entrant's continuation value is different with and without acquisitions. With them, the continuation value is $\delta V^I(\mu_{t+1}^A, \Delta_t)$; without, it is $\delta\pi_{t+1}^I(\mu_{t+1}^{NA})$. However, the proof of Lemma 2 shows that all that matters is that the continuation value is non-decreasing in μ_{t+1}, which is true in both cases.

calculations lead to the following:

Lemma 4 *If acquisitions are always prohibited, the equilibrium size of innovation is*

$$\Delta_t^{NA}(\mu_t) = (1 + \delta\kappa_F)(1 - \mu_t). \tag{21}$$

5.2 Market dominance and innovation

We start by showing that market dominance generally has an adverse effect on innovation.

Proposition 1 *The equilibrium size of innovation when acquisitions are prohibited, Δ_t^{NA}, is a decreasing function of the degree of market dominance μ_t; that when acquisitions are permitted, Δ_t^A, is also decreasing provided that $\frac{1+\delta\kappa_F}{1+\delta\kappa_C} > \alpha$.*

To understand why Proposition 1 holds, consider first the case of no acquisitions. Innovator q_t's marginal benefit from increasing its innovation size is the increase in the discounted sum of its profits in the two stages of its life cycle. Inspection of the profit functions (8) and (7) shows that the marginal profit is equal to the number of free consumers $1 - \mu_t$ in the first period, and to the number of captive consumers $\mu_{t+1}^{NA} = \kappa_F(1 - \mu_t)$ in the second. Both decrease with μ_t.

With acquisitions, the mechanism is analogous. The q_t innovator's outside option when bargaining on the acquisition price is the profit $\pi_t^E(\mu_t)$ that it would obtain if it resisted the takeover plus the continuation value $\delta V_{t+1}^I(\mu_{t+1}^{NA}, \Delta_t)$. An increase in μ_t leads to a decrease in $\pi_t^E(\mu_t)$. If $\kappa_C < \kappa_F$, it also reduces μ_{t+1}^{NA}, and consequently $V_{t+1}^I(\mu_{t+1}^{NA}, \Delta_t)$. In this case, Δ_t^A is always a decreasing function of the degree of market dominance μ_t.

However, when $\kappa_C > \kappa_F$, an increase in μ_t raises μ_{t+1}, and thus $V_{t+1}^I(\mu_{t+1}^{NA}, \Delta_t)$. If the entrant secures a sufficiently large share of the efficiency gains from the merger, an increase in μ_t may actually enhance the incentive to innovate, as those gains become more substantial when μ_t is higher. In this rather special case, the entrenchment effect may have a positive impact on innovation.

5.3 The short run

Comparing (17) and (21) one immediately obtains:

Proposition 2 *For any given $\mu_t > 0$, prohibiting acquisitions reduces the equilibrium size of innovation in period t:*

$$\Delta_t^A(\mu_t) > \Delta_t^{NA}(\mu_t). \tag{22}$$

This result reflects the invention-for-buyout effect. Intuitively, the innovation is more valuable in the hands of the incumbent, which can supply the state-of-the-art product not only to the free but also to the captive consumers. By transferring the new technology to the incumbent, acquisitions create a surplus, a share of which goes to the inventor. The prospect of being bought out thus increases the value of the innovation to forward-looking inventors. The greater the entrant's bargaining power α, the stronger this invention-for-buyout effect.[14]

5.4 The long run

However, acquisitions also affect the dynamics of μ_t. Starting from an arbitrary μ_t, if acquisitions are permitted, μ will converge to its steady state level

$$\bar{\mu}^A = \frac{\kappa_F}{1 + \kappa_F - \kappa_C}, \tag{23}$$

whereas if acquisitions are prohibited, the steady state is:

$$\bar{\mu}^{NA} = \frac{\kappa_F}{1 + \kappa_F}. \tag{24}$$

Clearly, $\bar{\mu}^A > \bar{\mu}^{NA}$. This inequality reflects the entrenchment of monopoly due to acquisitions.

The strength of the entrenchment effect can be measured by the percentage increase in the long-run degree of market dominance:

$$\frac{\bar{\mu}^A - \bar{\mu}^{NA}}{\bar{\mu}^{NA}} = \frac{\kappa_C}{1 + \kappa_F - \kappa_C}. \tag{25}$$

[14] This conclusion is not foregone, however. The incentive to innovate is not determined by the impact of acquisitions on the inventor's profit but by the marginal profitability of the innovation size. Proposition 2 guarantees that, in our model, the marginal and total effects go hand in hand. For a model where this property does not necessarily hold, refer to Katz (2021).

When $\kappa_C = \kappa_F$, the entrenchment effect is simply κ_C. In general, the effect increases with κ_C, which can therefore be regarded as the entrenchment parameter.

In the steady state, if acquisitions are always prohibited the level of innovation is

$$\Delta^{NA}(\bar{\mu}^{NA}) = \frac{1 + \delta\kappa_F}{1 + \kappa_F}. \tag{26}$$

If acquisitions are always permitted, on the other hand, it is

$$\Delta^{A}(\bar{\mu}^{A}) = \frac{(1 + \delta\kappa_F)(1 - \kappa_C) + \alpha\kappa_F(1 + \delta\kappa_C)}{1 + \kappa_F - \kappa_C}. \tag{27}$$

Comparing (26) and (27), it appears that the positive short-run effect of acquisitions on innovation may be reversed in the long run.

Proposition 3 *In the steady state, prohibiting acquisitions increases the equilibrium size of innovation if*

$$\kappa_C > \frac{\alpha(1 + \kappa_F)}{1 + \delta\kappa_F - \alpha\delta(1 + \kappa_F)}. \tag{28}$$

Intuitively, the long-run effect of acquisitions is the sum of two components: the difference between Δ_t^A and Δ_t^{NA} for any given μ_t, and the difference between $\bar{\mu}^A$ and $\bar{\mu}^{NA}$. The first component reflects the buyout effect and is always positive. The second component reflects the entrenchment effect and is negative if Δ_t^A is a decreasing function of μ_t. Condition (28) determines when the second component prevails over the first. Naturally, this can only occur if the entrenchment effect is negative: indeed, it is easy to verify that condition (28) implies that $\frac{1+\delta\kappa_F}{1+\delta\kappa_C} > \alpha$.

Condition (28) simplifies considerably in the special case $\kappa_F = \kappa_C$, when it reduces to:

$$\kappa_C > \frac{\alpha}{1 - \alpha}. \tag{29}$$

Intuitively, the entrenchment parameter κ_C must be large and the entrant's bargaining power α, which determines the magnitude of the invention-for-buyout effect, must be small.

When $\kappa_C > \kappa_F$, other factors come into play. Prohibiting acquisitions is more likely to raise the long run level of innovation the lower the private discount factor (and hence the higher the speed of imitation, or the weaker the protection of intellectual property, as explained in footnote 5), and the lower the fraction of free consumers that are turned into captive.

[insert Figure 1 around here]

The effects that we have identified are depicted in Figure 1, where the case of acquisitions being permitted is represented by the continuous lines, the case where they are prohibited by the dashed lines. First, both with and without acquisitions, the rate of innovation decreases as market dominance increases, reducing innovators' ability to appropriate the returns from their innovations (Proposition 1). Second, for any given level of dominance, the rate of innovation is higher when acquisitions are permitted (Proposition 2), reflecting the invention-for-buyout effect. Third, the long-run degree of dominance is higher if acquisitions are permitted than if they are prohibited, reflecting the entrenchment-of-monopoly effect. The figure represents the case where condition (28) holds, and hence, by Proposition 3, the long-run level of innovation is higher when acquisitions are prohibited.

5.5 Transitory dynamics

Our model is tractable enough to allow explicit calculation of the equilibrium dynamics of the innovation size Δ_{t+n} for $n = 1, 2, ...$, starting from an arbitrary μ_t. When acquisitions are permitted, the degree of market power evolves over time as follows:

$$\mu_{t+n}^{A} = \frac{\kappa_F}{1+\kappa_F-\kappa_C} + \left(\mu_t - \frac{\kappa_F}{1+\kappa_F-\kappa_C}\right)(\kappa_C-\kappa_F)^n, \tag{30}$$

and thus the level of innovation is:

$$\begin{aligned}\Delta_{t+n}^{A} \quad = \quad & \frac{1+\kappa_F\left[\alpha(1+\delta\kappa_C)+\delta(1-\kappa_C)\right]-\kappa_C}{1+\kappa_F-\kappa_C} + \\ & -\left[1+\delta\kappa_F-\alpha(1+\delta\kappa_C)\right]\left(\mu_t - \frac{\kappa_F}{1+\kappa_F-\kappa_C}\right)(\kappa_C-\kappa_F)^n. \end{aligned} \tag{31}$$

When, on the contrary, acquisitions are prohibited, we have:

$$\mu_{t+n}^{NA} = \frac{\kappa_F}{1+\kappa_F} + \left(\mu_t - \frac{\kappa_F}{1+\kappa_F}\right)(-\kappa_F)^n \tag{32}$$

and

$$\Delta_{t+n}^{NA} = \frac{1+\delta\kappa_F}{1+\kappa_F} - (1+\delta\kappa_F)\left(\mu_t - \frac{\kappa_F}{1+\kappa_F}\right)(-\kappa_F)^n . \tag{33}$$

While the level of market dominance with acquisitions is monotonic (provided that $\kappa_C > \kappa_F$), it oscillates in the absence of acquisitions. This is because a stronger incumbent today not only diminishes the profits of today's entrant but also constrains the market power that it can build and leverage in the next period, when it becomes the new incumbent.

The above equations allow us to determine the dynamic effects of a policy change. To fix ideas, consider a shift from a lenient policy to a restrictive one, starting from a degree of market dominance μ_t that lies somewhere in between $\bar{\mu}^{NA}$ and $\bar{\mu}^{A}$. Immediately with the policy change, the level of innovation drops, as the invention-for-buyout effect vanishes. In subsequent periods, however, the share of captive consumers μ_{t+n} tends to shrink, reducing the degree of market dominance, with a positive effect on the entrant's innovative effort. In the counterfactual where acquisitions are permitted, on the other hand, μ_{t+n} increases towards its steady state level $\bar{\mu}^{A}$, with a negative effect on innovation. This dynamics is represented by the arrows pointing to the steady states in Figure 1. If condition (28) holds, at some point in time the innovation size with acquisitions banned becomes larger than if acquisitions continued to be permitted.

6 Antitrust policy

We now analyze the optimal antitrust policy in our model, assuming that the agencies take consumer surplus as their objective and discount future values by the social discount factor δ_S.

6.1 Consumer welfare

As noted, under our assumptions, acquisitions do not have a direct impact on consumer surplus. They influence consumers solely to the extent that they affect innovation, which, in turn, determines future consumer surplus. This property implies the following:

Lemma 5 *At any period t, and for any arbitrarily given μ_t, the policy that maximizes discounted consumer surplus is the one that maximizes:*

$$W_t = \sum_{n=0}^{\infty} \delta_S^n \Delta_{t+n}. \tag{34}$$

In other words, in our model, social welfare comparisons boil down to the comparison of the discounted sum of current and future sizes of innovation. This property holds true because our assumptions eliminate the price effects of acquisitions. But even if these effects were present, they would be temporary, whereas the effects on innovation size are permanent. Therefore, if the social discount factor δ_S were sufficiently close to 1, welfare comparisons would rest almost only on how acquisition policy affects the long-run level of innovation.[15]

6.2 Optimal policy

We now examine the choice between a fixed policy that consistently prohibits acquisitions and one that consistently permits them. In the next subsection, we briefly discuss contingent rules that base antitrust policy on the level of market dominance.

Applying the consumer welfare criterion, let us compare the two fixed policy rules, lenient and restrictive, for an arbitrary initial μ_t.

Proposition 4 *Prohibiting acquisitions increases consumer welfare if and only if*

$$\kappa_C > \frac{\alpha\,(1+\delta_S\kappa_F)}{(1-\alpha)\delta_S\delta\kappa_F + (\delta_S - \alpha\delta)}. \tag{35}$$

The condition simplifies considerably in the case $\kappa_C = \kappa_F$, when it reduces to:

$$\kappa_C > \frac{\alpha}{(1-\alpha)\,\delta_S}. \tag{36}$$

The effects of κ_C and α are the same as in Proposition 3, and for the same reasons. That is, prohibiting acquisitions is the more likely to be optimal, the higher the entrenchment-of-monopoly parameter κ_C and the lower the invention-for-buyout

[15] The same observation applies to the learning costs that may explain the existence of captive consumers, as discussed in footnote 6 above. If δ_S is sufficiently large, the impact of these learning costs on the social welfare calculation becomes negiglibe.

parameter α. Furthermore, prohibiting acquisitions is the more likely to be optimal, the higher the social discount factor δ_S. This makes intuitive sense: in our model, prohibiting acquisitions is socially costly in the short run because it temporarily hinders innovation but may lead to long-term benefits. It is therefore logical that a restrictive policy may be optimal only if the policymaker is sufficiently farsighted. When $\delta_S \to 1$ condition (35) collapses to (28): the weight of the transitory dynamics in the social welfare calculation becomes negligible, so the welfare comparison depends only on the steady-state levels of innovation.

When $\kappa_C > \kappa_F$, two more parameters come into play. Prohibiting acquisition is the less likely to be optimal, the lower the private discount factor δ, and the lower the fraction of free consumers that are turned into captive κ_F. Since a higher discount factor δ also captures the possibility of slower imitation, as discussed in footnote 5, Proposition 4 suggests that in our model acquisition policy and patent policy may be interconnected: when entrants are better protected against imitation, acquisition policy should be more lenient, while weaker patent protection calls for stricter antitrust rules.

6.3 State-dependent policy

If antitrust authorities can observe the state of the industry μ_t, they may base acquisition policy on it. This raises the question of whether the optimal policy may be state-dependent, being permissive when market dominance is weak and restrictive once repeated acquisitions have made it too strong. We now briefly discuss this possibility. For a proof of the results presented here and for additional details, we refer the reader to the working paper version of this article (Denicolò and Polo, 2021).

In the baseline model, it turns out that this added flexibility is unnecessary: restricting attention to fixed policy rules that either always permit acquisitions or always prohibit them does not result in any loss of generality. However, this property of the baseline model is rather special and rests on the profit functions being linear in the degree of market dominance, μ_t. When profits are nonlinearly dependent on μ_t, state-dependent policies may become optimal.

A simple way to generate non-linear profit functions within our theoretical framework is to assume that if the acquisition does not occur, the entrant, instead of the incumbent, acts as price leader. Under this assumption, it continues to be true that acquisitions consistently stimulate short-term innovation but may impede long-term innovation if the monopoly entrenchment effect is substantial. However, the optimal policy may become more nuanced.

We have specifically examined a class of simple state-dependent policy rules, where the policy-maker allows acquisitions as long as $\mu_t < \hat{\mu}$ and bans them when $\mu_t \geq \hat{\mu}$ for some critical threshold $\hat{\mu}$, which can be interpreted as the degree of leniency in acquisition policy. The case where acquisitions are always prohibited is re-obtained when $\hat{\mu} < \bar{\mu}^{NA}$, and the case where they are always permitted when $\hat{\mu} > \bar{\mu}^{A}$.

These cutoff policies result in cycles where the industry alternates between periods of low market dominance, during which acquisitions are permitted, and periods of high market dominance, during which they are prohibited. These cycles occur because, as long as acquisitions are allowed, the degree of market dominance μ_t continues to increase until it surpasses the threshold $\hat{\mu}$. At that point, the new acquisition is prohibited, the old incumbent is replaced, and μ_t jumps down, marking the beginning of a new cycle. The length of the cycles is an increasing function of $\hat{\mu}$.

In this setting, we demonstrate that as the strength of the entrenchment effect κ_C increases, the optimal policy becomes increasingly restrictive. When κ_C is sufficiently low, acquisitions are always permitted. As κ_C increases, it becomes optimal to adopt a state-contingent policy with a large threshold $\hat{\mu}$, which generates long incumbency cycles. As κ_C further increases, the threshold gets lower, and the incumbency cycles become shorter and shorter, until acquisitions are no longer allowed.

7 Conclusion

We have analyzed a tractable model of repeated innovation, where incumbents may either compete with innovative entrants or else acquire them. Acquisitions have both positive and negative effects on innovation. The former stems from the invention-for-buyout mechanism: inventors earn more by transferring their innovations to the incumbent than by exploiting them themselves, so their incentive to innovate is greater when such technology transfers are permitted. The negative effect, on the other hand, derives from the entrenchment of monopoly due to acquisitions. When these are permitted, that is to say, incumbents come to enjoy a higher degree of market dominance, which in turn reduces the entrants' incentive to innovate.

We have shown that the invention-for-buyout effect always prevails in the short run but can be outweighed in the long run by the entrenchment effect. As a result, if policymakers are sufficiently farsighted and the entrenchment effect is sufficiently strong, prohibiting acquisitions may be the optimal policy. In some cases, the optimal policy may be state-dependent. In other words, it may be best to permit acquisitions as long as market dominance is weak and prohibit them once repeated acquisitions have made it too strong.

Implementing such a restrictive policy likely requires a reorientation of merger regulation. So far, merger regulation has primarily focused on the prospective change in the degree of market concentration and thus has looked at the size of the target firm as a key determinant of this change. However, in our framework, the size of the acquisition target is largely irrelevant. Strictly speaking, in the model, the acquisition takes place before the inventor starts commercializing its new product, and thus the inventor's market share is always zero. In practice, incumbents may not be able to identify potential challengers until after commercialization has started, and the longer it takes for the acquisition to occur, the larger the size of the target is likely to be. However, this factor seems accidental and, as such, should not carry much weight in the policy assessment.

The agencies should instead focus on the incumbent's degree of market dominance. To the extent that, in innovative industries, this correlates with the size of the incumbent, then the incumbent's size should be the primary consideration in the antitrust assessment. Consequently, our results imply that when the risk of entrenchment is sufficiently high, and the incumbent's size is sufficiently large, the acquisition of potential competitors should be prohibited, regardless of their size.

The new Horizontal Merger Guidelines recently adopted by the Department of Justice and the Federal Trade Commission seem to pave the way for such a more restrictive approach. While we specifically mentioned Guidelines 6 and 8 in the introduction, the entire document seems to open the door to assessing mergers in a more dynamic manner, with a particular focus on potential "*pattern or strategy of* *growth through acquisition*" (p. 23).

The European merger regulation also appears to be evolving in the same direction. For example, the Digital Market Act has recently spurred more proactive enforcement of merger control in digital markets, requiring that all acquisitions of start-ups by gatekeeper platforms be notified to the European Commission, irrespective of the target size.

In light of these policy changes, it becomes crucial to analyze in greater detail the potential sources of the entrenchment effect. In this paper, this effect arises in a specific model of consumer inertia. Nevertheless, we contend that a similar effect tends to arise whenever the incumbent's strength depends on its past levels of activity. In previous versions of the paper (Denicolò and Polo 2021), we explored cases where the entrenchment effect arises because the incumbent has a cost advantage over entrants that depends on the length of its incumbency, as in Stein (1999), or because it can imitate the entrants the better, the longer it has been active, and we demonstrated that our qualitative results extend to these settings. However, incorporating other potential sources of entrenchment while maintaining tractability is challenging due to the difficulty of obtaining closed-form solutions. In particular, to get closed-form solutions the profit functions must be separable in the current and past innovation sizes, Δ_t and Δ_{t-1}, a property that appears rather fragile.

If this property fails, one must resort to other analytical approaches. One possibility is to continue using a fully dynamic model but resort to numerical solutions. Another option is to adopt a two-period model, where the first period serves as a stylized representation of the short run, and the second period represents the long run. While these alternative approaches are less satisfactory than the one used in this paper, they can facilitate the analysis of richer model variations. We leave these possible extensions for future work.

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Appendix

The Appendix collects the proofs omitted in the main text.

Proof of Lemma 1. The merged entity's objective function is

$$\Pi_t^M = \pi_t^M + \delta V_{t+1}^I(\mu_{t+1}^A, \Delta_t),$$

where $\pi_t^M = \left[(1-\mu_t)x^{F,M} + \mu_t x^{C,M}\right] p_t^M$ is the current period profit, and $\delta V_{t+1}^I(\mu_{t+1}^A, \Delta_t)$ is the continuation value, i.e., the discounted value of being the incumbent in the next period with $\mu_{t+1}^A = \kappa_F x_t^M$ captive consumers. Note that $x^{F,M} = x^{C,M} = x^M$ as the firm cannot price discriminate.

To begin with, assume that the merged entity prices myopically, i.e., $\delta = 0$. All consumers have the same willingness to pay for quality, so the merged firm has no incentive to price discriminate by supplying different quality levels and hence will supply only the highest quality, q_t. Since the competitive fringe guarantees to all consumers an outside option of $U_t^F = q_{t-2}$, the merged entity must match this utility level: $U_t^M = q_t^M - p_t^M = U_t^F$ (with a tiny price discount to break the indifference, if necessary). Therefore, $p_t^M = \Delta_t + \Delta_{t-1}$. In this myopic equilibrium, $x^{F,M} = x^{C,M} = 1$.

Next suppose that $\delta > 0$. It is intuitive (and we shall confirm below) that the continuation value $V_{t+1}^I(\mu_{t+1}^A, \Delta_t)$ is a non-decreasing function of μ_{t+1}^A, which is in turn a non-decreasing function of $x^{F,M}$ and $x^{C,M}$. Therefore, a forward-looking firm would have an incentive to further reduce the price so as to increase $x^{F,M}$ and $x^{C,M}$, if possible. But since they are already equal to 1, the myopic price remains optimal also for a forward-looking firm. ■

Proof of Lemma 2. Plainly, all firms supply the highest quality level that they control: $q_t^E = q_t$, $q_t^I = q_{t-1}$, and $q_t^F = q_{t-2}$, and the fringe prices at marginal cost (i.e., 0). The incumbent and the entrant, on the other hand, price so as to maximize their respective profits, $\pi_t^I = \mu_t x_t^I p_t^I$ and $\Pi_t^E = \pi_t^E(\mu_t) + \delta V_{t+1}^I(\mu_{t+1}^{NA}, \Delta_t)$, where $\pi_t^E(\mu_t) = (1-\mu_t)x_t^E p_t^E$ denotes the entrant's profit in period t, and $V_{t+1}^I(\mu_{t+1}^{NA}, \Delta_t)$ is the value of being the incumbent in the next period with $\mu_{t+1}^{NA} = \kappa_F(1-\mu_t)x_t^E$ captive consumers. The incumbent, which is due to exit in the next period, prices myopically.

A forward-looking entrant, in contrast, must keep into account the impact of its current price on the number of captive consumers that it will inherit in the second period of its life-cycle, as this affects the profits that it will earn in its capacity as the new incumbent.

To begin with, however, suppose that the entrant prices myopically ($\delta = 0$). Given the behavior of the fringe, consider the entrant's best response to p_t^I. Free consumers choose to purchase from the entrant if $U_t^E > \max\left\{U_t^I, U_t^F\right\}$, that is, if $p_t^E < \min\left\{p_t^I + \Delta_t, \Delta_t + \Delta_{t-1}\right\}$. Therefore, the entrant's best response is

$$p_t^E(p_t^I) = \begin{cases} p_t^I + \Delta_t & \text{if } p_t^I \leq \Delta_{t-1}, \\ \Delta_t + \Delta_{t-1} & \text{if } p_t^I > \Delta_{t-1}. \end{cases}$$

Next, consider the incumbent's strategy as a price leader. The incumbent makes no sales if $p_t^I > \Delta_{t-1}$. On the other hand, it anticipates that if it reduces the price below Δ_{t-1}, it would always be undercut by the entrant and would therefore serve only the captive consumers anyway. Therefore, the incumbent must price exactly at Δ_{t-1} (with a tiny discount to break the captive consumers' indifference, if necessary). By doing so, it gets a profit of $\pi_t^I = \mu_t \Delta_{t-1}$. In response, the entrant prices at $p_t^E = \Delta_t + \Delta_{t-1}$ (again with with a tiny discount if necessary) and will serve all free consumers.

If $\delta > 0$, so that the entrant is forward looking, it would have a further incentive to reduce the price to increase x_t^E if that were possible, as the continuation value V_{t+1}^I is increasing in μ_{t+1}. However, x_t^E is already equal to 1, so the myopic price remains optimal also for a forward-looking firm. ■

Proof of Corollary 1. With no acquisition, the firms' aggregate payoff is

$$(1 - \mu_t)(\Delta_t + \Delta_{t-1}) + \mu_t \Delta_{t-1} + \delta V_{t+1}^I(\mu_{t+1}^{NA}, \Delta_t).$$

If the incumbent acquires the entrant, in contrast, the aggregate payoff becomes

$$(\Delta_t + \Delta_{t-1}) + \delta V_{t+1}^I(\mu_{t+1}^A, \Delta_t).$$

The lemma then immediately follows by comparing the above expressions, keeping in mind that V^I_{t+1} increases with the fraction of captive consumers μ_{t+1}, and that $\mu^A_{t+1} = \kappa_F \geq \mu^{NA}_{t+1} = \kappa_F(1-\mu_t)$. ■

Proof of Lemma 3. From the optimization problem (15) it appears that the equilibrium innovation size depends only on the derivative of $v^E_t(\mu_t, \Delta_{t-1}, \Delta_t)$ with respect to Δ_t, which is the marginal profitability of increasing the size of the innovation. To calculate the derivative, let us substitute (12) into (13), obtaining

$$\begin{aligned} v^E_t(\mu_t, \Delta_{t-1}, \Delta_t) &= (1-\alpha)\left[\pi^E_t + \delta V^I_{t+1}(\mu^{NA}_{t+1}, \Delta_t)\right] + \alpha[\pi^M_t + \delta V^I_t(\mu^A_{t+1}, \Delta_t) - \pi^I_t] \\ &= (1-\alpha)\pi^E_t + \alpha(\pi^M_t - \pi^I_t) + \delta\left[(1-\alpha)V^I_{t+1}(\mu^{NA}_{t+1}, \Delta_t) + \alpha V^I_t(\mu^A_{t+1}, \Delta_t)\right]. \end{aligned}$$

The on-path continuation value is

$$\begin{aligned} V^I_{t+1}(\mu^A_{t+1}, \Delta_t) &= v^I_{t+1}(\mu^A_{t+1}, \Delta_t, \Delta_{t+1}) = v^M_{t+1}(\mu^A_{t+1}, \Delta_t, \Delta_{t+1}) - v^E_{t+1}(\mu^A_{t+1}, \Delta_t, \Delta_{t+1}) \\ &= \pi^M_{t+1} + \delta V^I_{t+2}(\mu^{A,A}_{t+2}, \Delta_{t+1}) + \\ &\quad -(1-\alpha)\left[\pi^E_{t+1} + \delta V^I_{t+2}(\mu^{A,A}_{t+2}, \Delta_{t+1})] - \alpha[v^M_{t+1}(\mu^A_{t+1}, \Delta_t, \Delta_{t+1}) - \pi^I_{t+1}\right], \end{aligned}$$

where Δ_{t+1} is the innovation size in period $t+1$, which is correctly anticipated in period t, and $\mu^{A,A}_{t+2}$ is the fraction of captive consumers in period $t+2$ if the entrant is acquired both in period t and in period $t+1$. Likewise, the off-path continuation value is

$$\begin{aligned} V^I_{t+1}(\mu^{NA}_{t+1}, \Delta_t) &= v^I_{t+1}(\mu^{NA}_{t+1}, \Delta_t, \Delta_{t+1}) = v^M_{t+1}(\mu^{NA}_{t+1}, \Delta_t, \Delta_{t+1}) - v^E_{t+1}(\mu^{NA}_{t+1}, \Delta_t, \Delta_{t+1}) \\ &= \pi^M_{t+1} + \delta V^I_{t+2}(\mu^{NA,A}_{t+2}, \Delta_{t+1}) + \\ &\quad -(1-\alpha)\left[\pi^E_{t+1} + \delta V^I_{t+2}(\mu^{NA,A}_{t+2}, \Delta_{t+1})] + \alpha[v^M_{t+1}(\mu^A_{t+1}, \Delta_t, \Delta_{t+1}) - \pi^I_{t+1}\right]. \end{aligned}$$

where $\mu^{NA,A}_{t+2}$ is the fraction of captive consumers in period $t+2$ if the entrant is not acquired in period t but is acquired in period $t+1$. (It follows from the one-shot deviation principle that this is indeed the relevant value of μ.)

Next, note that all current-period profit functions (i.e., π^M_t, π^I_t and π^E_t) are additively separable in Δ_{t-1} and Δ_t, and that all other terms in the expression for $v^E_t(\mu_t, \Delta_{t-1}, \Delta_t)$ do not depend on Δ_{t-1}. This implies that $v^E_t(\mu_t, \Delta_{t-1}, \Delta_t)$ depends on Δ_{t-1} in an additively separable way and that, as a result, the optimal choice of

Δ_t does not depend on Δ_{t-1}.

Since a similar argument applies to all subsequent periods, it follows that Δ_{t+1} does not depend on Δ_t, and the same is true of Δ_{t+2}, Δ_{t+3} etc. These future values depend only on μ_t. In particular

$$\begin{aligned} V^I_{t+1}(\mu^A_{t+1}, \Delta_t) &= (1-\alpha)\left[\pi^M_{t+1} - \pi^E_{t+1}(\mu^A_{t+1})\right] + \alpha\pi^I_{t+1}(\mu^A_{t+1}) + \\ &\quad + \text{terms that depend only on } \mu_t \end{aligned}$$

and

$$\begin{aligned} V^I_{t+1}(\mu^{NA}_{t+1}, \Delta_t) &= (1-\alpha)\left[\pi^M_{t+1} - \pi^E_{t+1}(\mu^{NA}_{t+1})\right] + \alpha\pi^I_{t+1}(\mu^{NA}_{t+1}) + \\ &\quad + \text{terms that depend only on } \mu_t \end{aligned}$$

Thus, we have

$$\begin{aligned} v^E_t(\mu_t, \Delta_{t-1}, \Delta_t) &= (1-\alpha)\pi^E_t(\mu_t) + \alpha\left[\pi^M_t - \pi^I_t(\mu_t)\right]) + \\ &\quad +\delta\,\{\ (1-\alpha)\left[\pi^M_{t+1} - \pi^E_{t+1}(\mu^{NA}_{t+1})\right] + \alpha\pi^I_{t+1}(\mu^{NA}_{t+1}) + \\ &\quad +\alpha\left[\pi^M_{t+1} - \pi^E_{t+1}(\mu^A_{t+1})\right] + \alpha\pi^I_{t+1}(\mu^A_{t+1})\ \} \\ &\quad + \text{terms that depend only on } \mu_t \end{aligned}$$

Collecting all terms that depend on Δ_{t-1} and Δ_t, we finally have

$$\begin{aligned} v^E_t(\mu_t, \Delta_{t-1}, \Delta_t) &= (1-\mu_t)\left[\alpha + (1-\alpha)\mu_t\right]\Delta_{t-1} + \\ &\quad + \left[(1+\delta\kappa_F)(1-\mu_t) + \alpha\mu_t(1+\delta\kappa_C)\right]\Delta_t + \\ &\quad + \text{terms that depend only on } \mu_t \end{aligned} \tag{37}$$

From this expression, (17) follows immediately, proving the first part of the lemma.

With the equilibrium innovation size at hand, we can now determine the equilibrium value function, and hence the acquisition price. To this end, we make a guess on the functional form of the value functions and find them by the method of undetermined coefficients. Since Δ^A_t is linear in μ_t and (37) shows that the expression for the value function $v^E_t(\mu_t, \Delta_{t-1}, \Delta_t)$ involves the product $\mu_t \times \Delta_t$, we conjecture that the *ex ante* value functions are polynomials of degree 2:

$$V_t^E(\mu_t, \Delta_{t-1}) = (1-\mu_t)\Delta_{t-1} + \phi_0 + \phi_1\mu_t + \phi_2\mu_t^2 \tag{38}$$

$$V_t^M(\mu_t, \Delta_{t-1}) = \Delta_{t-1} + \varphi_0 + \varphi_1\mu_t + \varphi_2\mu_t^2. \tag{39}$$

Given $V_t^E(\mu_t, \Delta_{t-1})$ and $V_t^M(\mu_t, \Delta_{t-1})$, we have $V_t^I(\mu_t, \Delta_{t-1}) = V_t^M(\mu_t, \Delta_{t-1}) - V_t^E(\mu_t, \Delta_{t-1})$. We then identify the coefficients $\phi_0, \phi_1, \phi_2, \varphi_0, \varphi_1$ and φ_2 by imposing the condition that (38)-(39) must be identically satisfied. Here we report the solution for the special case $\kappa_C = \kappa_F$ (the general solution is available from the authors upon request):

$$\begin{aligned}
\phi_0 &= \frac{(2-\alpha)(1+\delta\kappa_F)^2\{1+(1-\alpha)\delta[1-\alpha(1-\kappa_F)]\kappa_F\}}{[1+(1-\alpha)\delta][1-(1-\alpha)\delta\kappa_F][1+(1-\alpha)\delta\kappa_F^2]} \\
\phi_1 &= -\frac{(1-\alpha)(1+\delta\kappa_F)^2[3-\alpha-(1-\alpha)\delta\kappa_F+(1-\alpha^2)\delta\kappa_F^2+(1-\alpha)^2\delta^2\kappa_F^3]}{[1-(1-\alpha)\delta\kappa_F][1+(1-\alpha)\delta\kappa_F^2]} \\
\phi_2 &= \frac{(1-\alpha)^2(1+\delta\kappa_F)^2}{[1+(1-\alpha)\delta\kappa_F^2]}
\end{aligned}$$

and

$$\begin{aligned}
\varphi_0 &= \frac{(1+\delta\kappa_F)^2\{1+(1-\alpha)\delta[1-\alpha(1-\kappa_F)]\kappa_F\}}{[1+(1-\alpha)\delta][1-(1-\alpha)\delta\kappa_F][1+(1-\alpha)\delta\kappa_F^2]} \\
\varphi_1 &= -(1-\alpha)(1+\delta\kappa_F)^2 \\
\varphi_2 &= 0.
\end{aligned}$$

This proves the second part of the lemma.

It is simple to verify that $V_t^I(\mu_t, \Delta_{t-1})$ is increasing in μ_t – a property the was used repeatedly in the proof of Lemma 1 and 2. ■

Proof of Proposition 1. From (17) and (21) we have

$$\frac{d\Delta_t^A}{d\mu_t} = -(1+\delta\kappa_F) + \alpha(1+\delta\kappa_C) \lesseqgtr 0 \Longleftrightarrow \alpha \lesseqgtr \frac{1+\delta\kappa_F}{1+\delta\kappa_C}$$

and

$$\frac{d\Delta_t^{NA}}{d\mu_t} = -(1+\delta\kappa_F) < 0. \quad ■$$

Proof of Proposition 2. From (17) and (21) we have:

$$\Delta_t^A(\mu_t) - \Delta_t^{NA}(\mu_t) = \alpha(1+\delta\kappa_C)\mu_t,$$

whence the result follows immediately.■

Proof of Proposition 3. Using the steady state values (23) and (24) we get

$$\Delta_t^{NA}(\bar{\mu}^{NA}) - \Delta_t^A(\bar{\mu}^A) = \kappa_F \frac{\left[1+\delta\kappa_F - \alpha\delta\left(1+\kappa_F\right)\right]\kappa_C - \alpha\left(1+\kappa_F\right)}{\left(1+\kappa_F\right)\left(1+\kappa_F-\kappa_C\right)},$$

whence the result follows immediately. ■

Proof of Lemma 5. In view of (9), discounted consumer surplus as of period t is

$$\begin{aligned}
\sum_{n=0}^{\infty} CS_{t+n}\delta_S^n &= \sum_{n=0}^{\infty} \delta_S^n q_{t+n-2} \\
&= \frac{1}{1-\delta_S}\left(q_{t-2} + \delta_S q_{t-1} + \delta_S^2 \sum_{s=0}^{\infty} \delta_S^n \Delta_{t+n}\right). \qquad (40)
\end{aligned}$$

The first two terms inside brackets are pre-determined, so the welfare function effectively reduces to W_t.■

Proof of Proposition 4. If acquisitions are always permitted, using (31)and (30) one can calculate the equilibrium value of the social welfare index W_t:

$$\begin{aligned}
W_t^A(\mu_t) &= \Delta_t^A(\mu_t) + \sum_{n=1}^{\infty} \delta_S^n \Delta_{t+n}^A(\kappa_F) \\
&= \frac{\left(1+\delta\kappa_F\right)\left(1-\kappa_C\delta_S\right) - \alpha\kappa_F\delta_S(1+\delta\kappa_C)}{\left(1-\delta_S\right)\left[1-\delta_S(\kappa_C-\kappa_F)\right]} - \frac{1-\alpha+\delta\kappa_F - \alpha\delta\kappa_C}{1-\delta_S(\kappa_C-\kappa_F)}\mu_t.
\end{aligned}$$

If instead acquisitions are always prohibited, using (33) and (32) the social welfare index becomes:

$$\begin{aligned}
W_t^{NA}(\mu_t) &= \Delta_t^{NA}(\mu_t) + \sum_{n=1}^{\infty} \delta_S^n \Delta_{t+n}^{NA}(\mu_{t+n}^{NA}) \\
&= \frac{1+\delta\kappa_F}{\left(1-\delta_S\right)\left(1+\delta_S\kappa_F\right)} - \frac{1+\delta\kappa_F}{1+\delta_S\kappa_F}\mu_t.
\end{aligned}$$

Comparing $W_t^{NA}(\mu_t)$ and $W_t^{A}(\mu_t)$, we get:

$$W_t^{NA} - W_t^{A} = \frac{(1+\delta\kappa_F)\,\kappa_C\delta_S - \alpha(1+\kappa_F\delta_S)(1+\delta\kappa_C)}{(1-\delta_S)(1+\kappa_F\delta_S)\,[1-\delta_S(\kappa_C-\kappa_F)]}\,[\delta_S(\kappa_F-\mu_t)+\mu_t]\,,$$

whence the result follows immediately. ■

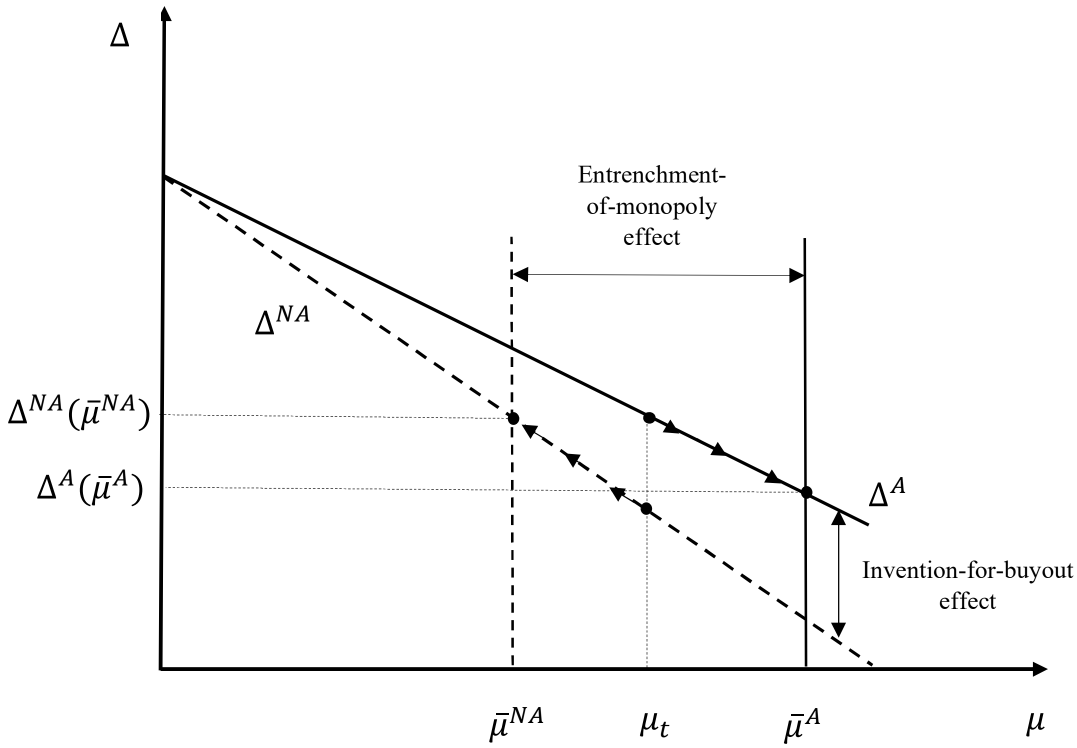

Figure 1. The decreasing lines represent the equilibrium rate of innovation when acquisitions are permitted (continuous) or prohibited (dashed). The distance between the two lines measures the invention-for-buyout effect. The vertical lines represent the long-run degree of market dominance, which is higher when acquisitions are pemitted because of the entrenchment effect. The arrows represent the process of convergence to the steady state, starting from the current level of market dominance.