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Undisclosed orders and optimal submission strategies in a limit order market[☆]

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ABSTRACT

Reserve orders enable traders to hide a portion of their orders and now appear in most electronic limit order markets. This paper outlines a theory to determine an optimal submission strategy in a limit order book, in which traders choose among limit, market, and reserve orders and simultaneously set price, quantity, and exposure. We show that reserve orders help traders compete for the provision of liquidity and reduce the friction generated by exposure costs. Therefore, total gains from trade increase. Large traders always benefit from reserve orders, whereas small traders benefit only when the tick size is large.

1. Introduction

Electronic limit order markets are the primary venues for trading financial securities. In such markets, known as limit order books (LOBs), orders can be classified into two broad categories: market and limit orders. Market orders include instructions about the quantity to be bought or sold; limit orders also carry a limit price.

In addition to quantity and limit price, most trading platforms today allow traders to include instructions about the visibility of their orders. That is, traders can decide to hide a fraction of their trade by using a reserve, or iceberg, order.[2] In recent years, reserve orders have grown to account for a surprisingly large proportion of the trading volume in various markets, such as 44% of Euronext's volume, approximately 28% of the volume on the Australian Stock Exchange, and more than 16% of volume executed on Xetra.[3]

Reserve orders have costs and benefits compared with limit orders. The invisible portion loses time priority to the

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[2] Orders that enable traders to display only a fraction of the entire order are called either reserve (e.g., Nasdaq, BATS) or iceberg (e.g., LSE, BATS Chi-X Europe). Some trading platforms also allow traders to submit completely invisible orders.

[3] See Bessembinder, Panayides, and Venkataraman (2009), Aitken, Berkman, and Mak (2001), and Frey and Sandas (2009), respectively.

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visible part of the order.[4] As a consequence, it incurs a higher execution cost, that is, the cost traders face if their order is not executed. The advantage, however, is that reserve orders reduce exposure costs. Exposure costs arise because agents submitting large visible orders run the risk of being undercut by aggressive traders who quote more competitive prices on the same side of the market. Because reserve orders allow traders to reduce the visible part of their orders, they lower incentives for incoming traders to undercut.

This paper focuses on the economic rationales underlying traders' order submission decisions and derives a model in which agents choose among orders with different degrees of visibility. Therefore, it contributes to the current debate on pre-trade transparency that has gradually switched from the comparison of exogenous market structures with different degrees of transparency to the optimal endogenous degree of order visibility.[5] Our study also contributes to the discussion, originated by recent empirical research, on the type of traders who submit reserve orders.[6] We analyze whether traders rationally use these orders in a market in which no one holds private information about asset values. In this respect, our study complements the Moinas (2010) model, in which insiders use reserve orders.

We consider a LOB with standard price and time priority rules and, thereby, highlight the strategic choice that traders make among market, limit, and reserve orders. Traders simultaneously determine not only how aggressive their order should be in terms of price and size, but also which part of their order should remain undisclosed. By choosing a limit instead of a market order, traders forgo execution certainty to obtain a better price. In doing so, they increase their execution cost but reduce their price opportunity cost. This cost is associated with an execution at a price that is less favorable than other available investment opportunities. By choosing reserve orders, traders face this trade-off between execution costs and price opportunity costs, while also considering exposure costs.[7]

The novel contribution of our model is the focus on the market friction generated by exposure costs, which alters traders' optimal order submission strategy. In standard microstructure models in which traders are either liquidity suppliers or liquidity demanders, aggressive undercutting drives quotes toward reservation prices and makes market participants better off.[8] In our setting, however, traders can endogenously choose to demand or supply liquidity and, hence, aggressive undercutting can generate both a reduction in the supply of liquidity and less favorable execution prices. Because gains from posting large visible orders might not be fully realized due to the lower execution probability induced by undercutting, traders have an incentive to switch from limit to market orders. The consequence of this friction is that agents coming to the market in subsequent periods could be worse off due to the higher price opportunity costs.

The main goal of this paper is, therefore, to investigate the effects of reserve orders on traders' welfare, when traders could use these orders to prevent the friction generated by undercutting. Building on our model, we determine whether the use of reserve orders increases gains from trading in a market in which agents are classified as large or small, according to their order size. Our welfare analysis compares the LOB model with reserve orders against a benchmark model in which traders are not allowed to use this order type. The analysis also includes a first-best framework, in which there are no frictions and orders are executed with certainty at the fundamental value of the asset. With the first-best model, we assess the relative improvement in welfare when introducing reserve orders.

We find that in equilibrium traders choose reserve orders to compete for the provision of liquidity and they select a visible order size that just prevents undercutting. This result is consistent with findings by Aitken, Berkman, and Mak (2001), Bessembinder, Panayides, and Venkataraman (2009), De Winne and D'Hondt (2007), and Frey and Sandas (2009). Reserve orders reduce competition for the provision of liquidity and, even though they make small market orders more expensive, they increase the profitability of limit orders. Large traders always benefit from the introduction of reserve orders, and this positive effect increases with a larger tick size which enhances the incentive to submit limit instead of market orders. In contrast, small traders benefit from reserve orders only when the tick size is large. When the tick size shrinks and traders use market orders more extensively, small traders are harmed by reserve orders.

In addition to predictions about welfare changes, we deliver predictions about market quality and the components of the spread. Specifically, reserve orders should have mixed effects on market quality. Because they reduce undercutting and cluster liquidity at a single price level, their introduction should increase market depth at the top of the LOB but widen the spread. Our results suggest that

[4] On LOBs, orders get executed according to hierarchical order precedence rules. Price priority is the primary precedence rule; time and display status priorities are the secondary precedence rules. A visible limit order improving on the existing best price thus gains priority over the reserve and limit orders standing on the book. Conversely, a visible limit order posted at the existing best price joins the queue at that price and attains priority only over the invisible part of reserve orders.

[5] Previous research focuses on the social benefits of disclosing information on limit prices, associated quantities, and the identity of market participants (e.g., Baruch, 2005; Foucault, Moinas, and Theissen, 2007; Rindi, 2008). More recently, attention has been devoted to hidden liquidity on LOBs, as discussed in Section 2, and non-transparent trading venues (e.g., Buti, Rindi, and Werner, 2011; Ye, 2011; Zhu, 2011).

[6] See, for example, Aitken, Berkman, and Mak (2001), Bessembinder, Panayides, and Venkataraman (2009), De Winne and D'Hondt (2007), and Frey and Sandas (2009).

[7] Early research papers on this trade-off underline the effects of modeling financial markets with limit orders on inventory (Demsetz, 1968), order submission (Cohen, Maier, Schwartz, and Whitcomb, 1981), and adverse selection costs (Copeland and Galai, 1983). To limit the dimension of the possible investors' strategies, Glosten (1994) and Seppi (1997) assume that the choice between limit and market orders is exogenous. More recently, Parlour (1998), Foucault, Kadan, and Kandel (2005), Goettler, Parlour, and Rajan (2005, 2009), and Rosu (2009) propose multi-period models in which this choice becomes endogenous. For a more extensive survey, see Parlour and Seppi (2008).

[8] See, for example, Glosten (1994), Glosten and Milgrom (1985), and Biais, Martimort, and Rochet (2000).

empirical research should consider a new component of the bid–ask spread caused by exposure costs. Traders submitting large limit orders bear exposure costs as a result of undercutting. To reduce these costs, they select reserve orders, which decrease price competition and cause the inside spread to widen.

The remaining part of this paper is structured as follows. In Section 2, we outline previous literature on undisclosed orders. In Section 3, we focus on the benchmark model, and Section 4 contains the model with reserve orders. In Section 5, we discuss the effects of reserve orders on traders' welfare. We present the empirical implications in Section 6, and we draw conclusions in Section 7. All the proofs appear in the Appendix.

2. Literature on undisclosed orders

Most research on reserve orders is empirical and, thus, offers few theoretical insights. These empirical studies generally investigate the source of reserve orders and relate the results to market quality indicators. For example, Aitken, Berkman, and Mak (2001) show that, in the Australian stock market, no difference exists in the price reactions to disclosed and undisclosed limit orders. Traders use reserve orders more frequently when competition is intense (i.e., tick size is small and trade size is large) and volatility is high. Bessembinder, Panayides, and Venkataraman (2009) investigate the costs and benefits of iceberg orders in Euronext. They find that such orders have lower implementation shortfall costs and that patient traders value the option to hide, far more so than impatient traders do.[9] Furthermore, Bessembinder, Panayides, and Venkataraman (2009) and Harris (1996, 1997) show that traders are more likely to hide their orders when the tick size is small and the order size is large.

Studying market reactions to the presence of iceberg orders on the Madrid Stock Exchange, Pardo and Pascual (2012) find that hidden volume detection has no significant impact on returns or volatility. De Winne and D'Hondt (2007) show that traders become significantly more aggressive when the opposite side of the market signals hidden depth at the best quotes. Furthermore, traders tend to hide larger amounts when their order is large, relative to the displayed depth, which suggests that they use hidden quantity to manage exposure risk. Finally, Frey and Sandas (2009) find that iceberg orders facilitate the search for latent liquidity because they tend to attract market orders when discovered by market participants. The greater the fraction of an iceberg order executed, the smaller its price impact.

Even though the empirical evidence suggests that the choice of reserve orders is generally motivated by traders' concerns about market liquidity, there may also be some information content of reserve depth. In Nasdaq Small-Order Execution System market makers' quotes, Tuttle (2006) finds that hidden size adds liquidity to the market and is used more intensively in stocks that are more likely to experience an informational event. In addition, the presence of hidden depth at the time of a trade is a significant predictor of a midquote revision.

Furthermore, two recent experimental works (Bloomfield, O'Hara, and Saar, 2011; Gozluklu, 2009) use laboratory studies to investigate how the ability to hide orders affects traders' strategies and market outcomes. Both informed and uninformed traders use undisclosed orders, and their aggressiveness in demanding and supplying liquidity changes when undisclosed orders become available.

To the best of our knowledge, only two theoretical models explicitly include reserve orders. Moinas (2010) proposes a two-period signaling game, in which liquidity suppliers appear first, followed by liquidity demanders, who hit the limit orders posted in advance. In her model, insiders can only be liquidity suppliers and, therefore, choose reserve orders to trade large volumes without divulging private information to liquidity demanders. In our framework, such an argument might not hold, because insiders would have the additional option to choose market orders.

Esser and Mönch (2007) extend the literature pertaining to optimal liquidation strategies (e.g., Bertsimas and Lo, 1998) to include iceberg orders. They determine the optimal limit price and peak size for an iceberg order in a static framework with no strategic interaction among traders.

3. General framework and benchmark model

We build a discrete time model without asymmetric information in which, following Bessembinder, Panayides, and Venkataraman (2009), Pardo and Pascual (2012), De Winne and D'Hondt (2007), and Frey and Sandas (2009), large traders choose reserve orders to compete for liquidity provision. In this model, small and large traders select their order placement strategies accounting for the strategic behavior of their possible counterparts, as well the interaction between the two sides of the LOB.

We begin by focusing on the general features that guide the choice of traders' optimal order submission strategies. This assessment provides a benchmark model (B) against which we evaluate traders' welfare and market quality. In turn, we extend this framework to include reserve orders (R).

3.1. The market

A market for a security is conducted over four periods: $t = t_1, \ldots, t_4$. The common value of the asset v is publicly known. Two categories of risk-neutral agents are active: large traders (L), who can choose to trade up to j units, with $\alpha \le j \le 10$, where α, $j \in \mathbb{N}$; and small traders (S), who trade α units, equal to the equilibrium undisclosed portion of the reserve order. In each trading round, nature selects a large or small trader with equal probability, $\Pr(L) = \Pr(S) = \frac{1}{2}$. The incoming agent maximizes expected profits by choosing an optimal trading strategy that cannot be modified thereafter, though traders can cancel their orders. Similar to Parlour (1998), we define each agent's private evaluation of the asset as β_t, symmetric around

[9] Shortfall costs are the sum of the price impact and the opportunity cost, weighted by the filled and unfilled portions of the order, respectively.

$\beta = 1$ and drawn from the uniform distribution

$$\beta_t \sim U[\underline{\beta}, \overline{\beta}] \quad \text{where } 0 \leq \underline{\beta} < 1 < \overline{\beta}. \tag{1}$$

This parameter indicates the willingness to trade of an agent who arrives in the market at time t. Traders with extreme values of β_t value the asset either very high or very low, and they are accordingly the most eager buyers (high β_t) or the most eager sellers (low β_t). For example, if a trader arrives in the market with a very low β_t, he likely sells the asset, because his profits are given by ($price - \beta_t v$). In contrast, traders with a β_t close to one exhibit the lowest willingness to trade.

Each trader arriving in the market observes the LOB, which consists of a grid of six prices, three on the ask and three on the bid side. The prices at which each trader can buy or sell the asset are, thus, A_i (ask prices) and B_i (bid prices), where $i \in \{1,2,3\}$, $A_1 < A_2 < A_3$ and $B_1 > B_2 > B_3$. For simplicity, we assume that these prices are symmetric around the common value of the asset, v.

At A_3 and B_3, a trading crowd absorbs the amount of the asset that is demanded or offered. Incoming traders can demand liquidity over the whole price grid, but they can offer liquidity only at the first two levels of the book. In line with Seppi (1997) and Parlour (1998), the trading crowd prevents traders from bidding prices that are too distant from the inside spread, which constitutes a theoretical shortcut to limit the price grid. We also assume that the minimum difference between the ask and bid prices ($A_1 - B_1$) equals the tick size τ, that is, the minimum price variation.

The state of the book at each period t, $b_t = [q_2^A, q_1^A, q_1^B, q_2^B]$, reflects the number of shares available at each price. Traders compete on prices when room exists in the book to allow for undercutting. To enforce competition, we assume that at t_1 the LOB opens empty and then gradually fills up as traders post their orders. We also assume that traders, when observing b_t, have been aware of the state of the book since the beginning of the trading game.

3.2. Order types

The market we model features a standard LOB, regulated by price and time priority rules. When a trader arrives in the market, he chooses an order that maximizes expected profits, given his type (β_t) and the state of the LOB (b_t). In Table 1, we present the possible orders a large trader (Panel A) and a small trader (Panel B) might choose.

An aggressive large trader who wants to sell can demand liquidity by submitting a market sell order of size j, which matches the limit buy orders with the highest precedence on the bid side. If the size j of this order is smaller than (or equal to) the number of shares available at the best price (B_i) on the opposite side of the market, we label the order MO_jB_i. If the size j is greater than the depth available at B_i, such that the order must walk down the book in search of execution, we label the strategy MO_jB.[10] A less aggressive trader could choose a limit sell order of size j at either A_1 or A_2 ($LO_jA_{1,2}$). This order is executed only when one or more market buy orders hit the limit price, after all other orders on the book with either a lower price or a higher time priority have been executed. Finally, the trader can decide not to trade (NTL). Analogous strategies are available to a large trader who wants to buy. In real-world financial markets, traders can also split their limit orders by submitting them at different price levels or different times of the day. We do not consider these strategies here because they are dominated.

An aggressive small trader who wants to sell can demand liquidity with a market sell order ($MO_\alpha B_i$). A less aggressive small trader could act as a liquidity supplier by submitting a limit sell order to either the first ($LO_\alpha A_1$) or the second ($LO_\alpha A_2$) level of the LOB. Finally, if the trader finds no profitable strategies, he can decide to refrain from trading (NTS). Small buyers select among similar strategies.

Table 1
Order submission strategies: benchmark.
This table presents the possible orders that a large trader (Panel A) and a small trader (Panel B) can choose upon their arrival in the market. Large traders can submit orders of up to ten shares in size, and small traders can trade only α shares. A large trader can submit a market sell order (MO_jB_i) of size j at price B_i or a market sell order that walks down the buy side in search of execution (MO_jB). A large trader can also choose to submit a limit sell order of size j to either A_1 or A_2 (LO_jA_i) or can decide not to trade (NTL). Small traders can submit a market sell order ($MO_\alpha B_i$) that is executed at the first price level at which liquidity is available (B_i). In addition, small traders can opt to submit a limit sell order to the first ($LO_\alpha A_1$) or the second ($LO_\alpha A_2$) level of the ask side of the LOB, and they can also decide not to trade (NTS). Strategies for the buy side are symmetrical.

Panel A: Large trader ($j \in [\alpha, 10]$, $i \in \{1,2,3\}$ for MO, $i \in \{1,2\}$ for LO)	
Strategy	
Market sell order	MO_jB_i or MO_jB
Limit sell order	LO_jA_i
No trade	NTL
Limit buy order	LO_jB_i
Market buy order	MO_jA_i or MO_jA
Panel B: Small trader ($i \in \{1,2,3\}$ for MO, $i \in \{1,2\}$ for LO)	
Strategy	
Market sell order	$MO_\alpha B_i$
Limit sell order	$LO_\alpha A_i$
No trade	NTS
Limit buy order	$LO_\alpha B_i$
Market buy order	$MO_\alpha A_i$

3.3. Equilibrium submission strategies

A trader determines an optimal order submission strategy by simultaneously choosing the sign, the size, and the aggressiveness of an order. Formally, a risk-neutral large trader chooses the optimal strategy o_L that maximizes expected profits, conditional on the state of the LOB b_t and the trader's type β_t:

$$\max_{o_L \in [\Omega^L_{sell}, \Omega^L_{buy}, NTL]} \mathrm{E}[\pi_t(o_L)]$$

$$\Omega^L_{sell} = \{MO_jB_i, MO_jB, LO_jA_i\}$$
$$\Omega^L_{buy} = \{MO_jA_i, MO_jA, LO_jB_i\}, \tag{2}$$

[10] In general, a market order that walks up or down the book until it is entirely executed crosses various prices, so we do not use an index for the level of the book, as we do for the other order types.

where Ω^L_{sell} are selling strategies and Ω^L_{buy} are buying strategies. Profits from not trading equal zero, $\pi_t(NTL)=0$. In contrast, profits from a market sell order of size $j\in[\alpha,10]$ that hits the quantity available at B_i equal $\pi_t(MO_jB_i)=j(B_i-\beta_t v)$. The profits from a j-market order that walks down the book are $\pi_t(MO_jB)=\sum_i f_i(B_i-\beta_t v)$, where f_i is the number of shares executed at B_i when $\sum_i f_i=j$. Finally, the expected profits from a limit sell order of size j are given by

$$\begin{aligned}E[\pi_{t_q}(LO_jA_i)] = (A_i-\beta_t v)\Bigg\{ &\sum_{w_{t_{q+1}}=\alpha,j} w_{t_{q+1}} \Pr_{w_{t_{q+1}}}(A_i|b_{t_{q+1}}) \\ &+ \sum_{w_{t_{q+1}}=0,\alpha} \Pr_{w_{t_{q+1}}}(A_i|b_{t_{q+1}}) \Bigg[\sum_{w_{t_{q+2}}=\alpha}^{j-w_{t_{q+1}}} w_{t_{q+2}} \Pr_{w_{t_{q+2}}}(A_i|b_{t_{q+2}}) \\ &+ \sum_{w_{t_{q+2}}=0,\alpha} \Pr_{w_{t_{q+2}}}(A_i|b_{t_{q+2}}) \sum_{w_{t_{q+3}}=\alpha}^{j-w_{t_{q+1}}-w_{t_{q+2}}} w_{t_{q+3}} \Pr_{w_{t_{q+3}}}(A_i|b_{t_{q+3}}) \Bigg] \Bigg\},\end{aligned} \tag{3}$$

where $\Pr_{w_t}(A_i|b_t)$ is the probability that w_t shares get executed at t and $q\in\{1,2,3\}$. In this formula, the first term indicates profits from shares executed in the period immediately following the order submission. The other terms denote expected profits from the execution in the subsequent periods. Profits for buying strategies are computed similarly and omitted here.

The small trader solves an analogous problem:

$$\begin{aligned}&\max_{o_S\in[\Omega^S_{seller},\Omega^S_{buyer},NTS]} E[\pi_t(o_S)] \\ &\Omega^S_{sell}=\{MO_\alpha B_i, LO_\alpha A_i\} \\ &\Omega^S_{buy}=\{MO_\alpha A_i, LO_\alpha B_i\},\end{aligned} \tag{4}$$

where, for example, profits for the selling strategies are given by

$$\begin{aligned}E[\pi_{t_q}(LO_\alpha A_i)] = \alpha(A_i-\beta_t v)\Big\{ &\Pr_\alpha(A_i|b_{t_{q+1}}) \\ &+ \Pr_0(A_i|b_{t_{q+1}}) \Big[\Pr_\alpha(A_i|b_{t_{q+2}}) + \Pr_0(A_i|b_{t_{q+2}}) \Pr_\alpha(A_i|b_{t_{q+3}}) \Big] \Big\}\end{aligned} \tag{5}$$

$$\pi_t(MO_\alpha B_i)=\alpha(B_i-\beta_t v). \tag{6}$$

Equilibrium definition: An equilibrium of the trading game is a set of orders o^*_L and o^*_S that solve Eqs. (2) and (4) when the expected execution probabilities, $\Pr_{w_t}(A_i|b_t)$, are computed, assuming that traders submit orders o^*_L and o^*_S.

We solve the model by backward induction using tick sizes equal to $\tau=\{0.025, 0.05, 0.075, 0.1\}$. We present results for $\tau=0.05$, because the optimal trading strategies are qualitatively robust to various values of τ.[11] We assume that β is uniformly distributed with support $[0,2]$ and that $v=1$. The equilibrium strategies resulting from the benchmark model have crucial relevance, because we compare them with the strategies that emerge from the protocol with reserve orders.

[11] The value of the tick size changes only the width of the β_t ranges, that is, the probability associated with different order types. With a lower tick size, traders tend to use more market orders. For larger values of the tick size, they opt for limit and reserve orders more frequently.

4. Model with reserve orders

In our extended framework, large traders have the option to hide part of their order by choosing a j-reserve order ($RO_jA_{1,2}$ or $RO_jB_{1,2}$). Therefore, they simultaneously choose the sign, size, and aggressiveness of the order, as well as the degree of exposure. To determine an optimal trading strategy, large traders solve the following problem, which includes reserve orders:

$$\begin{aligned}&\max_{o_L\in[\Omega^L_{sell},\Omega^L_{buy},NTL]} E[\pi_t(o_L)] \\ &\Omega^L_{sell}=\{MO_jB_i, MO_jB, LO_jA_i, RO_jA_i\} \\ &\Omega^L_{buy}=\{MO_jA_i, MO_jA, LO_jB_i, RO_jB_i\}.\end{aligned} \tag{7}$$

Profits from a j-market order that walks down the book thus become uncertain, because hidden liquidity could be available in the book. Therefore, traders rationally compute the expected profits $E[\pi_t(MO_jB)]=\sum_i f_i(B_i-\beta_t v)\Pr_{f_i}(B_i|b_t)$, with $\sum_i f_i=j$, where $\Pr_{f_i}(B_i|b_t)$ is the probability that f_i shares are available at B_i. Analogously, they compute expected profits for the limit orders, as shown in Eq. (3). However, to estimate the probability that w_t shares are executed at A_i, $\Pr_{w_t}(A_i|b_t)$, they also take into account the presence of hidden depth.

Reserve orders are a type of limit orders, so traders still compute expected profits using Eq. (3). However, the hidden part of the undisclosed order has a lower execution probability than the corresponding visible part of a limit order posted at the same price. Thus, the two strategies do not return the same profits.

Small traders solve Eq. (4), but they must rationally compute the probability of hidden liquidity, similar to large traders. We solve this new model with incomplete information by backward induction. Traders are fully rational, so, in equilibrium, their conjectures about hidden liquidity are consistent with optimal order submission strategies.

Finally, to determine the optimal peak size, we solve Eqs. (4) and (7) for different values of α ($0<\alpha<j$) and choose α^* to maximize the expected profits from reserve orders. When large traders choose the peak size of their reserve orders, they attempt to camouflage their choices behind small traders, such that α^* equals the small trader's order size. This prevents other market participants from easily detecting the undisclosed depth.[12]

4.1. Traders' strategies: an example

Fig. 1 offers an example of the extensive form of a game in which $\alpha=1$ and $j=10$. Assume, for example, that nature selects a large trader at t_1 who decides to submit $LO_{10}A_2$. In this case, the profits equal the difference between the price at which the trader sells A_2 and his evaluation of the asset $\beta_{t_1}v$, multiplied by the probability that the order is

[12] Ideally, a search for an optimal peak size would allow small agents to trade any quantity $\alpha\in[1,9]$, such that regardless of the equilibrium value of α, reserve orders would still hide behind orders submitted by small traders. Analytically, this allowance is cumbersome. To simplify the algebra, every time we consider a new value of α, we also set the order size of small traders equal to that value.

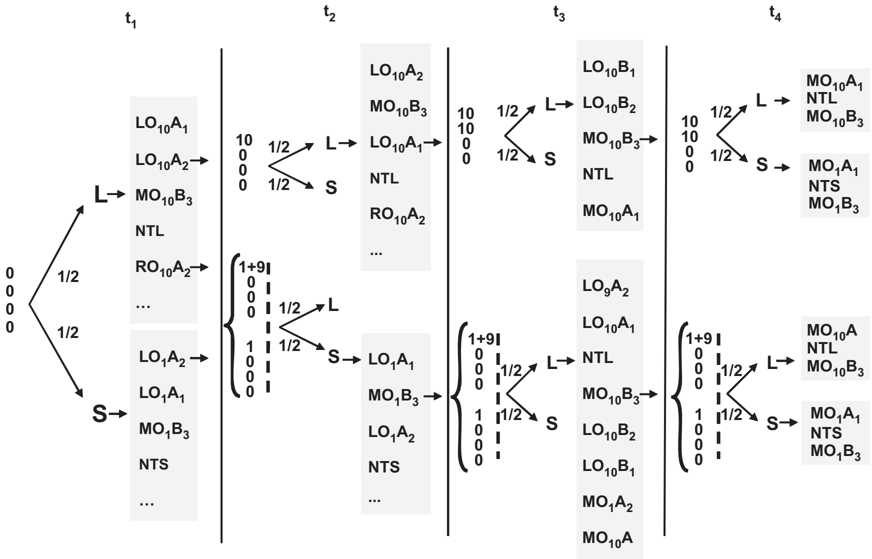

Fig. 1. Extensive form of the game. This example of the extensive form of the game refers to the case in which $j=10$ and $\alpha=1$. At t_1, the book opens empty and $b_{t_1}=[0,0,0,0]$. Nature chooses with equal probability a large trader (L) or a small trader (S) who decides an optimal submission strategy among all feasible orders. If at t_1 a large trader chooses $LO_{10}A_2$, at t_2 the book is $b_{t_2}=[10,0,0,0]$. If at t_2 another large trader chooses $LO_{10}A_1$, and then the next trader at t_3 submits $MO_{10}B_3$, then at t_4 the book opens as [10,10,0,0]. In this last period, the large trader submits either $MO_{10}B_3$ or $MO_{10}A_1$ or else does not trade (*NTL*). A small trader chooses between MO_1B_3 and MO_1A_1 or else decides not to trade (*NTS*). However, if at t_1 a large trader chooses $RO_{10}A_2$, traders arriving in the following periods are uncertain of the actual depth of the book.

executed in the subsequent periods and the associated trade sizes:

$$\begin{aligned}
\mathrm{E}[\pi_{t_1}(LO_{10}A_2)] &= (A_2-\beta_{t_1}v)\Big\{\Pr_{10}(A_2|b_{t_2})10+\Pr_1(A_2|b_{t_2})\\
&\Big\{1+\Pr_9(A_2|b_{t_3})9+\Pr_1(A_2|b_{t_3})\Big[1+\Pr_1(A_2|b_{t_4})+\Pr_8(A_2|b_{t_4})8\Big]\\
&+\Pr_0(A_2|b_{t_3})\Big[\Pr_1(A_2|b_{t_4})+\Pr_9(A_2|b_{t_4})9\Big]\Big\}+\Pr_0(A_2|b_{t_2})\\
&\Big\{\Pr_{10}(A_2|b_{t_3})10+\Pr_1(A_2|b_{t_3})\Big[1+\Pr_1(A_2|b_{t_4})+\Pr_9(A_2|b_{t_4})9\Big]\\
&+\Pr_0(A_2|b_{t_3})\Big[\Pr_1(A_2|b_{t_4})+\Pr_{10}(A_2|b_{t_4})10\Big]\Big\}\Big\}. \qquad (8)
\end{aligned}$$

In this formula, the three terms on the right-hand side refer to the three following possible execution paths at t_2: (1) an incoming trader buys the whole order of size ten at A_2, with a probability $\Pr_{10}(A_2|b_{t_2})$; (2) a trader buys one unit, $\Pr_1(A_2|b_{t_2})$; and (3) no trader hits the order at t_2, or $\Pr_0(A_2|b_{t_2})$. The unfilled part of the order gets executed at t_3 or at t_4, or both, provided a market order arrives from the opposite side of the market that hits A_2. Therefore, uncertainty remains about the execution of a limit order.

If instead the large trader chooses a market sell order ($MO_{10}B_3$), the order is executed with certainty, and the payoff equals:

$$\pi_{t_1}(MO_{10}B_3)=10\times(B_3-\beta_{t_1}v). \qquad (9)$$

Our model, therefore, includes the trade-off between market and limit orders. Market orders are executed with certainty, but at the most aggressive price on the opposite side of the book. Limit orders obtain better prices, but at the expense of an uncertain execution.

If the incoming trader at t_1 decides to submit $LO_{10}A_2$, then at t_2, the book opens with ten shares on A_2, $b_{t_2}=[10,0,0,0]$. If a trader arriving at t_2 chooses to undercut this order with $LO_{10}A_1$, the expected profits are as follows:

$$\begin{aligned}
\mathrm{E}[\pi_{t_2}(LO_{10}A_1)] &= (A_1-\beta_{t_2}v)\Big\{\Pr_{10}(A_1|b_{t_3})10+\Pr_1(A_1|b_{t_3})\\
&\Big[1+\Pr_1(A_2|b_{t_4})+\Pr_9(A_2|b_{t_4})9\Big]\\
&+\Pr_0(A_2|b_{t_3})\Big[\Pr_1(A_2|b_{t_4})+\Pr_{10}(A_2|b_{t_4})10\Big]\Big\}. \qquad (10)
\end{aligned}$$

In this sequence, the strategies available to the trader who arrives in the market at t_3 are $MO_{10}B_3$, $LO_{10}B_2$, $LO_{10}B_1$, *NTL*, and $MO_{10}A_1$ for a large trader and MO_1B_3, LO_1B_2, LO_1B_1, *NTS*, and MO_1A_1 for a small trader. At time t_4, the market closes, and traders either submit market orders or refrain from trading, because the execution probability of limit orders reaches zero.

If, instead, at t_1 the large trader chooses a reserve order, from t_2 onward the depth remains uncertain. For example, if the trader elects a reserve order to sell ($RO_{10}A_2$), then at t_2 the book opens as $b_{t_2}=[1+9,0,0,0]$. Alternatively, if at t_1 nature selects a small trader who submits LO_1A_2, the opening book is $b_{t_2}=[1,0,0,0]$. In both cases, the LOB at t_2 shows one unit on A_2, and the incoming trader is uncertain about whether the book has any undisclosed depth. This trader then rationally computes the probability of each possible state of the LOB and trades accordingly.

4.2. Optimal undisclosed orders

We find the solution of this game by backward induction. We begin from the end nodes to compute the probabilities of market orders at time t_4, which are the execution probabilities of limit orders placed at t_3 and enable us to compute the equilibrium order submission strategies in that period. Similarly, we compute the equilibrium order submission strategies at t_2 and t_1. We then solve the game for all possible values of α to determine the optimal visible size of reserve orders.

When choosing an optimal submission strategy, a large trader weighs the pros and cons of selling the asset by using reserve orders instead of limit orders. Because the difference between A_1 and B_1 is equal to the tick size, orders on the top of the book are not exposed to price competition. Therefore, reserve orders posted, for example, to A_1 have no advantage over limit orders, because they cannot be undercut. Moreover, they lose time priority for the hidden part, so they have a lower execution probability and are dominated strategies.

An undisclosed order on A_2 instead offers advantages and disadvantages compared with a j-share limit order on A_2 or A_1, which are the other two alternatives available to non-aggressive traders. Compared with LO_jA_2, an undisclosed order might induce the next trader to refrain from undercutting by submitting an order at A_1. Compared with LO_jA_1, the undisclosed order gains tick size but pays the cost of lower execution probability.

Proposition 1 summarizes results regarding traders' optimal choice of reserve orders.

Proposition 1. Patient large traders optimally select reserve orders as equilibrium strategies at t_1. Traders choose the maximum disclosed size of reserve orders that prevents undercutting.

In Fig. 2, we report the β thresholds for the equilibrium strategies at t_1 and t_2, conditional on different states of the book.[13] Reserve orders are optimal submission strategies, selected by patient traders who come to the market at time t_1 with a β close to one. When opting for a reserve order, a trader must choose the optimal disclosed and undisclosed portions. On the one hand, the trader prefers that the largest possible part of the order is visible, to increase execution probability. On the other hand, by increasing the visible size at A_2, the trader increases the incentive for the next trader to undercut at A_1.

According to our model parameterization, the optimal ratio of visible-to-undisclosed size is one to nine shares. Specifically, $\alpha = 1$ is the disclosed size that induces a subsequent trader to join the queue. When a ten-unit reserve order is posted at A_2 with one visible share, the next large trader arriving at t_2 or t_3 does not undercut with $LO_{10}A_1$ but rather submits $LO_{10}A_2$. The protection offered by reserve orders, however, comes at a cost of lower execution probability. The beta range associated with a market order to buy at t_2 ($MO_{10}A/MO_{10}A_2$) grows smaller when b_{t_2} is $[1+9,0,0,0]$ instead of $[10,0,0,0]$.

Traders arriving at t_2 do not choose reserve orders. With only two periods left, the lower execution probability becomes too costly compared with the smaller undercutting

[13] The β thresholds for the equilibrium strategies at t_3 and t_4 are available upon request.

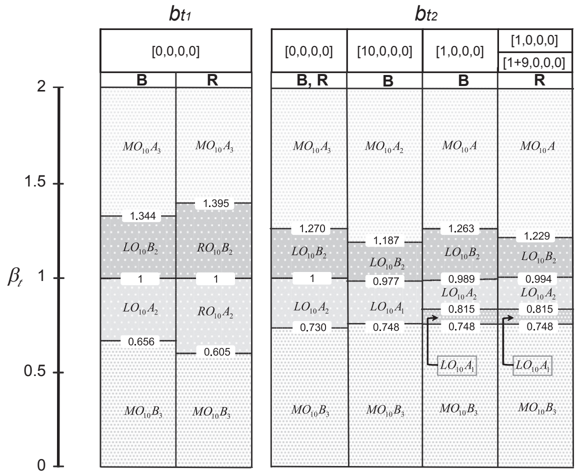

Fig. 2. β thresholds and equilibrium trading strategies, large trader at $\tau = 0.05$. This figure reports the equilibrium strategies and β thresholds of large traders for different states of the book (b_{t_1}, b_{t_2}). Two model specifications are presented. The first is a benchmark model (B) with no undisclosed orders, and the second introduces reserve orders (R). When the book opens at t_2 with one share at A_1, in the R framework traders cannot distinguish between $b_{t_2} = [1+9,0,0,0]$ and $b_{t_2} = [1,0,0,0]$, so they use the same trading strategy.

that reserve orders entail. Likewise at t_3, traders do not use reserve orders as they anticipate that at time t_4 no undercutting occurs.

A final observation refers to the widespread practice of splitting orders, which is not included in this model as an available strategy. With a time priority rule, splitting orders over time at the same price level of the book always is dominated by reserve orders. The hidden portion of the reserve order is immediately disclosed by the execution of the visible part. Therefore, it gains priority over the second part of the split, submitted only after the first one has been executed. Splitting different portions of the order into different price levels is never optimal, because it induces competitors to join the queue at the more aggressive price.

4.3. Discussion

The main purpose of this study is to investigate the role of exposure costs in securities trading. We show that these costs can be reduced through reserve orders. To this end, we must build a framework in which traders can submit orders of different sizes, because without trades of at least two sizes the detection of hidden quantities is straightforward, and reserve orders are always dominated by limit orders.

Existing models that include a stationary equilibrium cannot incorporate this essential feature. As Rosu (2009) suggests, a stationary Markov equilibrium might allow for multiple submissions of one-unit orders but not for block trading. Similarly, neither the Foucault (1999) nor the Foucault, Kadan, and Kandel (2005) framework is adequate to model undisclosed orders. For example, the Foucault (1999) model does not allow for different order sizes, and traders cannot compete to provide liquidity, because the book is always either empty or full. Foucault, Kadan, and

Kandel (2005) make the crucial assumption, necessary to find a stationary solution, that traders always improve the price when submitting a one-unit order. This assumption precludes the possibility that an incoming trader can join the queue, and it eliminates by construction all potential benefits of using undisclosed orders to reduce competition.

We instead model the market as a four-period trading game that can be solved by backward induction. Our finite-horizon model has a closed-form solution for a market in which traders' strategies include orders of different sizes, reserve orders, and the freedom to choose between price improvements and joining the queue. In this framework, traders not only condition their order submission decisions on the current state of the LOB but also strategically account for the effects of their own orders on the dynamics of the book. By contrast, a fully recursive model could not embed the evolution of the LOB and, therefore, would not allow for the crucial interaction of traders with different states of the book.

Our model relies on the exogenous probability (equal to one) of the initial state of the book, which opens empty on the first two levels. That is, with a four-period model, the assumption of no depth at the top of the book enables us to account for a richer set of trading strategies. With the short trading horizon, a deeper top of the book would reduce the set of available strategies.[14] At the beginning of the following periods, however, the state of the book becomes endogenous, depending on the order submission strategies in previous periods. In this sense, our model improves on existing stationary equilibrium protocols in which state probabilities are exogenous.

To determine the outcome of an ideal model with fully endogenous probabilities, we could increase the number of trading periods, such that the dynamic interaction of traders would deplete and replenish depth, eventually creating liquidity cycles. Although a numerical solution could be found to this model with several periods, the size of the state space and the complexity of the analysis would significantly increase. In this scenario, it would be extremely difficult to keep track of all the effects that the interaction of different traders with different states of the book generates.

5. Welfare analysis

Our welfare analysis investigates whether and how the additional instruction that gives traders the option to hide part of their orders increases the welfare of market participants. Measuring welfare is crucial to assess the overall impact of the introduction of reserve orders, because the market structure we consider is that of a LOB. In traditional market-maker models (e.g., Glosten, 1994, 1998), agents are naturally categorized into customer-investors, who consume price quotes, and market makers, who produce quotes. In that setting, market makers typically compete on prices down to the zero-profit condition, so spread and depth measure the distribution of transaction costs among different customers. In a limit order market, this theoretical categorization does not exist, because customer-investors produce price quotes themselves. Therefore, traditional market quality measures are not exhaustive indicators of the impact of a structural change, because agents submitting limit instead of market orders can assess the quality of the book only in terms of the expected gains from trade.

To this end, we begin by introducing the concept of welfare for a pure LOB, in which all traders can submit both market and limit orders. Next, we compute the aggregate welfare of all market participants, as well as the expected profits of small and large traders separately. We provide measures of welfare for different values of agents' willingness to trade (β) and the minimum price change (τ). Furthermore, to investigate the effect of the introduction of reserve orders, we compare all measures of welfare in the benchmark model with the measures computed for the model with reserve orders. Finally, we measure relative welfare by considering a market without friction that proxies for a first-best allocation of resources.

5.1. Measuring welfare in a pure LOB

We concentrate on a measure of net variation in welfare that helps us show whether and how different traders benefit from the introduction of reserve orders. Specifically, following Goettler, Parlour, and Rajan (2005), we consider the consumer surplus accruing to submitters of different order types when trading an asset with common value v. The surplus from an order submitted at time t is equal to the gains from trade obtained from the execution of that order.

Consider first a large market order of size j, in which f_i ($\sum_i f_i = j$) indicates the number of shares executed at the i-th price, p_i, where $p_i = \{A_i, B_i\}$ indicates the ask or bid prices. The average execution price for this order is $\overline{p}_t(j) = [(1/j)\sum_{i=1}^{3} p_i \ f_i]$. It follows then that the surplus from this order for a large trader coming into the market at time t with a private evaluation β_t is given by

$$w_{t,L}^{MO} = j[\beta_t v - \overline{p}_t(j)] \ \mathrm{sign}(order), \tag{11}$$

where sign($order$) takes a value of +1 for a buy order and −1 for a sell order.

In a LOB, each market order is executed against a limit order, so the surplus accruing to the limit order submitters who take the other side of the trade is equal to

$$w_{t,L}^{LO} = j[\overline{\beta}_t v - \overline{p}_t(j)] \ \mathrm{sign}(order), \tag{12}$$

where $\overline{\beta}_t$ is the share-weighted average of the private values of all limit order submitters who took the other side of the previous large market order. The gains from trade obtained by a small trader submitting a market order, $w_{t,S}^{MO}$, or a limit order, $w_{t,S}^{LO}$, can easily be computed by setting $j = \alpha$.

To build a measure of total surplus that gathers the gains from trades by all market participants, we must consider all possible realizations of β that characterize incoming traders. These realizations can represent both

[14] For example, if there were 40 shares at B_1, traders arriving in the market in any of the four periods could not submit limit or reserve orders to buy, because their execution probability would be zero.

impatient traders willing to buy or sell the asset through market orders or patient traders wishing to submit limit buy or sell orders. We, therefore, compute the expected change in welfare for traders submitting both market and limit orders, then compute the aggregate welfare measures for both small and large traders.

Formally, the expected gains from trade for investor $a=\{L,S\}$ arriving in the market at time t are equal to the sum of the expected change in welfare that this investor obtains by submitting either market or limit orders:

$$E[W_{t,a}] = \int_0^{\underline{\beta}_t} w_{t,a}^{MO} f(\beta_t)\, d\beta_t + \int_{\underline{\beta}_t}^{\overline{\beta}_t} w_{t,a}^{LO} f(\beta_t)\, d\beta_t + \int_{\overline{\beta}_t}^{2} w_{t,a}^{MO} f(\beta_t)\, d\beta_t, \tag{13}$$

such that $\underline{\beta}_t$ and $\overline{\beta}_t$ are the endogenously determined thresholds between market and limit orders to sell and to buy, respectively.

This measure can calculate only the absolute change in welfare. To obtain a relative measure, we compute the expected gains from trade accruing to agents in a market without friction, which achieves a first-best allocation of resources. In a frictionless market, all orders are executed at the fundamental value of the asset. Therefore, the expected change in welfare for a large trader is equal to

$$\overline{W}_{t,L} = j \int_0^2 |v - \beta v| \quad f(\beta_t)\, d\beta_t. \tag{14}$$

To obtain an analogous measure for a small trader, $\overline{W}_{t,S}$, it is sufficient to set $j=\alpha$.

Finally, to measure the expected change in total welfare, we add the large ($W_{t,L}$) and the small ($W_{t,S}$) traders' surplus over the four periods:

$$E[W] = \sum_{t=t_1}^{t_4} E[W_t] = \sum_{t=t_1}^{t_4} \{\Pr(L)E[W_{t,L}] + \Pr(S)E[W_{t,S}]\}, \tag{15}$$

and use $W_{t,a} = \overline{W}_{t,a}$ to compute welfare in the first-best allocation.

5.2. Welfare results

In Propositions 2 and 3, we summarize the results, as depicted in Figs. 3 and 4, pertaining to the change in welfare generated by the introduction of reserve orders.

Proposition 2. The introduction of reserve orders affects traders' welfare, such that total welfare improves, both in absolute value and relative to the first-best allocation, and the increase relates positively to the value of the tick size.

When traders have the opportunity to use reserve orders as a new trading option (R regime), total welfare increases in Fig. 3. This positive effect is confirmed when we consider the change in gains from trade measured relative to the first-best allocation. In a standard LOB, the comparative gain from submitting limit, instead of market, orders increases with the tick size. Because reserve orders are a type of limit orders, when tick size is large, traders use them more extensively, so the positive effect on traders' surplus increases.

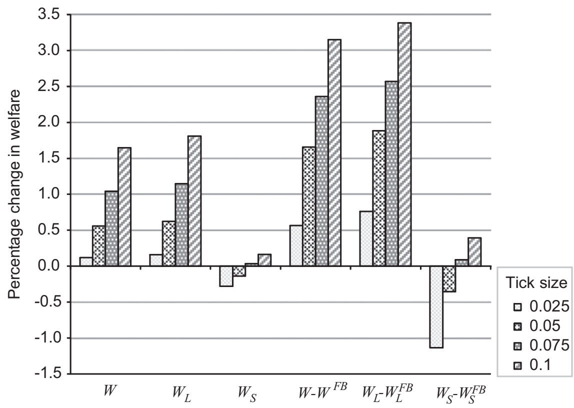

Fig. 3. Percentage change in welfare. This figure reports the percentage variation in expected gains from trade between the model with reserve orders and the benchmark model for all traders (W), as well as for large (W_L) and small traders (W_S) separately. We also present the relative percentage variations of W, W_L, and W_S, with respect to the corresponding first-best (FB) values. We consider the following tick size values: $\tau=\{0.025, 0.05, 0.075, 0.1\}$.

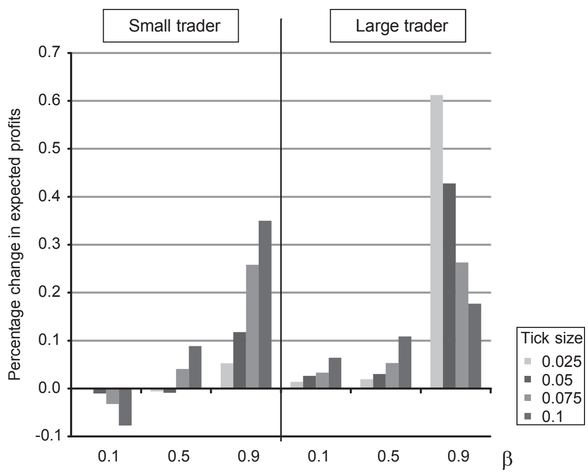

Fig. 4. Percentage change in expected profits for different types of traders. This figure reports, for three values of $\beta=\{0.1, 0.5, 0.9\}$, the percentage variation in expected profits between the model with reserve orders and the benchmark model, for both small and large traders. For each value of β, we consider the following tick size values: $\tau=\{0.025, 0.05, 0.075, 0.1\}$.

To evaluate the effect of the introduction of new instruments, such as reserve orders, on the welfare of market participants, it is important to investigate the change in the distribution of expected profits among agents of different types. The first relevant distinction refers to large versus small traders. Because reserve orders are used by large market participants, regulators should assess whether their introduction generates a Pareto improvement or an enhancement of total welfare. A second distinction within each category of traders describes patient and impatient agents, which could in principle result in different allocations of the gains from trade. Furthermore, in Proposition 2 we show that the change in welfare depends on the deep parameters that govern the interaction between traders and the LOB, so we need to confirm whether the effects on different agent types still depend on the value of the tick size.

Proposition 3. The expected profits of large traders always increase when they can use the option of reserve orders, and the expected profits of small traders increase only for large values of the tick size. In terms of willingness to trade, large traders are always better off, and small traders are better off only for values of β close to one and worse off for extreme values of β.

To explain the results for the aggregate expected profits of small and large traders, we consider patient and impatient traders separately. As Fig. 4 shows, large traders always benefit from the introduction of reserve orders, and those who are patient benefit the most. For small traders, expected profits increase when they are patient and worsen when they come to the market with an extreme value of β.

The interpretation of these results is straightforward for patient traders who compete for the provision of liquidity by using large orders. When the new option becomes available, they take advantage of reserve orders to reduce undercutting by incoming traders. Consistent with Bessembinder, Panayides, and Venkataraman (2009), the advantage of reserve orders for large patient traders decreases for larger values of the tick size because protection from undercutting becomes less relevant. The clustering of orders at one price reduces the price impact of large market orders and, therefore, enables large traders to benefit from reserve orders even when they are impatient. For these traders, however, the advantage of reserve orders increases with tick size, which amplifies the costs of walking up the book. Fig. 3 shows that overall the expected change in the profits of large traders increases with the tick size, the reason being that they use reserve orders more intensively when the tick size is large.

The effect of reserve orders on the expected profits of small traders is less intuitive. When they are patient, they benefit from the ability to submit limit orders at more profitable prices, behind the disclosed part of the reserve orders, and this price effect increases with tick size. When they are impatient, they suffer from the reduced competition for the provision of liquidity and, thus, execute their orders at less favorable prices. If we consider patient and impatient traders together, in Fig. 3, we can show that when the tick size is large, the positive effect prevails, because limit orders are used more extensively. However, when the tick size is small, the use of market orders increases, and small traders are worse off overall.

Our results have significant implications for market designers. The recent increase in competition among trading platforms has prompted regulatory authorities to decrease the tick size (e.g., BATS, 2009; U.S. Securities and Exchange Commission, 2010). This reduction is likely to amplify the possible negative effects of reserve orders on the expected profits of small traders and could even mean that markets offering reserve orders become contestable for retail volume. At very low tick sizes, a rival platform competing for volume could lure away small investors.

To prevent this effect, managers of incumbent platforms could offer investors who are classified as retail traders lower participation fees. Alternatively, they could offer a subsidy for small market orders when setting the optimal make or take fee structure. The first alternative is preferable, because it does not extend the subsidy to algorithmic trading programs that typically split large orders into smaller sizes. This approach is advisable, because the overall effect of algorithmic trading on market quality and traders' welfare remains an open question.[15] Finally, to reach target groups more closely, exchanges could tax reserve orders and reduce overall fees to redistribute income.

6. Empirical implications

The comparative analysis of a LOB with reserve orders and a benchmark LOB shows how large traders use reserve orders to compete for the provision of liquidity, which helps them avoid being undercut by incoming traders on the same side of the market. With this result, we can make some important empirical predictions pertaining to both the source of reserve orders and their effects on the welfare of traders.

Prediction 1. Traders use reserve orders to compete for the provision of liquidity.

Evidence to support this prediction is provided by a substantial number of empirical works. Aitken, Berkman, and Mak (2001) for the Australian stock market, Pardo and Pascual (2012) for the Madrid Stock Exchange, Frey and Sandas (2009) for the German stock market Xetra, and Bessembinder, Panayides, and Venkataraman (2009) and De Winne and D'Hondt (2007) for Euronext all indicate that traders use reserve orders for liquidity-related issues.

Prediction 2. Total gains from trade increase with the introduction of reserve orders, and this improvement correlates positively with tick size.

This prediction could be investigated empirically by considering different markets and using event studies to review periods before and after the introduction of reserve orders. To this end, researchers could follow the method proposed by Hollifield, Miller, Sandas, and Slive (2006), who estimate the gains from trade in limit order markets. Another finding relates to the value of the tick size, which affects the gains made by traders. According to our results, we expect that as the tick size increases, the overall gains of both large and small traders increase. This hypothesis can be tested empirically by comparing stocks with different price-to-tick ratios.

Prediction 3. In a regime with reserve orders, gains from trade in non-marketable orders increase. Gains from marketable orders instead increase when their size is large but decrease when their size is small.

[15] Algorithmic trading can have a positive effect on market quality (e.g., Hasbrouck and Saar, 2010; Hendershott, Jones, and Menkveld, 2011), but it can also enhance price instability (e.g., Kirilenko, Kyle, Samadi, and Tuzun, 2010).

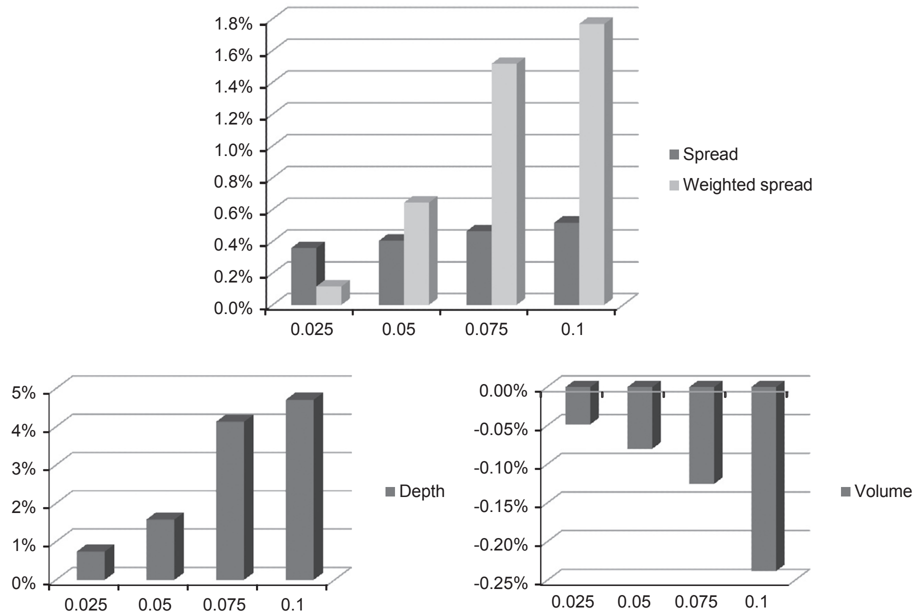

Fig. 5. Percentage change in market quality measures. This figure reports the percentage difference in market quality indicators between the model with reserve orders and the benchmark model. We consider best semi-spread, quoted and weighted by associated depth, the estimated depth at the best ask price, and volume generated by orders hitting the ask side. All measures are reported for four different values of the tick size, $\tau = \{0.025, 0.05, 0.075, 0.1\}$.

Empirical research could further assess the effects of reserve orders on the gains that agents obtain by providing or taking liquidity. This issue is particularly relevant because it provides guidance on the optimal design of make or take fees. We expect that the change in market conditions associated with the introduction of reserve orders increases gains from trade for all investors who submit non-marketable orders but reduces the gains enjoyed by small investors submitting marketable orders.

This model also enables us to make empirical predictions about the effects of reserve orders on market quality. To this end, we look at the regime with reserve orders and compare the average values of depth (disclosed plus undisclosed) for the best quotes, the semi-inside spread (effective and weighted), and volumes against the values obtained from the benchmark model (for the formulas, see the Appendix). The results in Fig. 5 imply Proposition 4.

Proposition 4. When traders use reserve orders to compete for liquidity provision, depth at the best quotes increases, inside spread widens, and volume decreases.

That is, when traders post reserve orders on the book, for example, on the ask side, they prevent other market participants from undercutting their orders and starting a price war that would drive the best ask price down. Therefore, a clustering of depth occurs at this price level, which, in turn, widens the average inside spread and reduces the surplus from market orders. This negative effect is reinforced by the lower visibility of existing liquidity, which makes the execution price of market orders uncertain. Overall, volumes decrease even though the price impact of large market orders declines. In this model, a reduction in volumes is consistent with an increase in total welfare, because the introduction of reserve orders increases the gains derived from supplying liquidity, thus reducing the incentive to take liquidity off the market. In turn, we can predict how reserve orders affect market conditions.

Prediction 4. When traders use reserve orders, the effects on market liquidity are mixed. The book becomes less tight in terms of spread but deeper in the number of shares associated with the best quotes. Overall volume, measured by the number of shares executed, decreases.

When traders use reserve orders to prevent undercutting by other market participants, the inside spread widens. A readily testable implication of this result is that the presence of undisclosed orders increases the size of the quoted and effective spread. Empirically, we also expect to observe that the increased spread worsens liquidity for small trades and that the depth clusters at the best bid-offer and compensates for the wider spread in large trades. To capture these effects, researchers should use a proxy of the semi-spread that can measure the price impact associated with different trade sizes.

These empirical predictions are consistent with the results Anand and Weaver (2004) obtain regarding the introduction of reserve orders to the Toronto Stock Exchange. Depth at the inside spread increased significantly when traders were allowed to use reserve orders. The results are also consistent with the Bessembinder and Venkataraman (2004) findings that reserve orders augmented depth and lowered the implicit transaction costs of block trades on the Paris Bourse.

Our last empirical prediction relates to the impact of reserve orders on the various components of the spread.

Prediction 5. In markets in which traders use reserve orders, empirical estimations of the spread should account for an additional component related to exposure costs.

Traders use reserve orders to minimize exposure costs and prevent undercutting by incoming, more aggressive orders. This behavior leads to an increase in the inside spread. Therefore, empirical assessments of the spread should include a new component that is related to competition for the provision of liquidity. This component differs from Copeland and Galai (1983), which shows that dealers set a wider spread when they fear being picked off by insiders. Our proposed component instead is independent of the presence of asymmetric information and, thus, also applies to bond and currency markets.

7. Concluding remarks

Dark liquidity lies at the heart of current regulatory debates, in both the United States and Europe (U.S. Securities and Exchange Commission, 2010; Committee of European Securities Regulators, 2010). A growing body of empirical literature shows how uninformed traders use reserve orders in electronic limit order platforms. However, no theory has explained how reserve orders can be used to control exposure costs or how they affect market conditions and the trading gains of different types of investors. To fill this gap and answer these questions, we offer a theory of reserve orders, in which traders choose between reserve and a range of other order types.

By reducing the visibility of their orders, investors generate two effects. First, they decrease the probability that they can be observed by other market participants, which increases their execution costs. Second, they reduce the incentive for incoming traders to undercut their orders and, thus, decrease their exposure costs. In equilibrium, traders solve this trade-off by choosing a disclosure size of their reserve orders that just prevents undercutting.

The use of undisclosed orders is relevant not only for traders, to determine their optimal order strategies, but also for exchange managers, who can use this instrument to fine-tune the optimal degree of pre-trade transparency. Allowing undisclosed orders decreases market transparency, because investors looking at the screen are not necessarily informed about the true depth at the posted quotes.

To validate the use of undisclosed orders, it is necessary to identify the benefits and costs in terms of traders' welfare. We address this important issue in market design by comparing a model with reserve orders against a benchmark model without them. Our results show that the introduction of reserve orders improves total welfare. Because reserve orders benefit from larger tick sizes, the increase in total gains from trade relates positively to the value of the tick size.

Large traders, who make direct use of reserve orders to reduce exposure costs, always profit from the opportunity to submit this order type. Conversely, the expected gains of small traders increase for large values of the tick size and decrease when the tick size is small. When the tick size is large, small traders tend to use limit orders more extensively. Thus, they benefit from the possibility of joining the queue at more profitable limit prices, behind the visible part of the reserve orders. When the tick size is small, and small traders switch to more aggressive market orders, they are harmed by the reduced competition for the provision of liquidity that the use of reserve orders entails. The consequence in terms of market competition is that, given the current tendency among regulators to reduce tick sizes, an exchange manager might decide to subsidize small traders, such as by designing an appropriate make or take fee structure.

However, undisclosed orders are not the only alternative available to traders to reduce exposure costs. When competition on the LOB becomes tough, large traders can take advantage of other existing market mechanisms that allow them to hide large orders. These are non-transparent crossing networks, such as dark pools (DP), that work like batch or continuous auctions and in which orders are executed either at a derivative price (usually the current mid-quote on the regular LOB) or at the best price on the DP price grid. The growing number of dark markets around the world provides evidence that they are the most realistic alternative to undisclosed orders. A topic for further research might be the development of a model that investigates how competition for the provision of liquidity works, between a LOB offering undisclosed orders and a DP.

An alternative approach to reduce exposure costs could be to use algorithmic trading programs that aim to build synthetic instruments that split large orders into smaller, visible portions. Our model shows though that splitting large orders is a dominated strategy. Although co-location enables ultra-high frequency trading, when the visible part of a reserve order is executed, the hidden shares that are immediately disclosed hold time priority on any order subsequently submitted to the LOB.[16]

A second alternative could be to explore sophisticated mixed strategies that enable traders to hide their intention to trade. These strategies aim to provide early investors with access to a random generator that mixes different order sizes or types and, thus, makes inference by later investors more difficult. This option also suggests an interesting pathway for further research.

Appendix A

A.1. Proof of Proposition 1

We first solve the benchmark framework and then the R framework which includes reserve orders.

A.1.1. Benchmark framework

If a risk-neutral agent optimally decides to trade, he submits an order for the largest possible number of shares. For example, for the strategy MO_jB_i, traders' profits are $\pi_t(MO_jB_i) = j(B_i - \beta_t v)$. Provided that $\pi_t(MO_jB_i) > 0$, a larger j

[16] Co-location is a service provided by exchanges that allows customers to locate their servers in the same data center where the central matching engine operates and, thus, guarantees extremely fast connectivity.

Table A1
Opening limit order book at t_2.
This table shows the possible opening books at t_2 conditional on the sell orders available to small and large traders at t_1. The small and the large trader can submit a market order, respectively $MO_\alpha B_3$ and $MO_{10}B_3$, a limit order at A_1, $LO_\alpha A_1$ and $LO_{10}A_1$, or at A_2, $LO_\alpha A_2$ and $LO_{10}A_2$, or refrain from trading, NTS and NTL.

Strategies at t_1	$MO_{10}B_3, MO_\alpha B_3$	$LO_{10}A_2$	$LO_{10}A_1$	$LO_\alpha A_2$	$LO_\alpha A_1$	NTS, NTL
A_2	0	10	0	α	0	0
A_1	0	0	10	0	α	0
B_1	0	0	0	0	0	0
B_2	0	0	0	0	0	0

leads to larger profits and, therefore, j is equal to its maximum possible value, given the available depth.

Period $t=t_4$: To compute the equilibrium strategies, we must compare traders' profits and find the β thresholds, such that traders with an asset valuation within this range submit a certain order type. The β_{t_4} threshold that makes a small trader indifferent between submitting a market order to sell or not trading at all, for example, can be determined by solving $\pi_{t_4}(MO_\alpha B_i)-\pi_{t_4}(NTS)=0$. More generally, at time t_4, a small trader submits a market sell order if the price is higher than his own evaluation of the asset ($B_i \geq \beta_{t_4} v$, such that $\beta_{t_4} \leq B_i/v$), a market buy order in the opposite case ($\beta_{t_4} v \geq A_i$, such that $\beta_{t_4} \geq A_i/v$), and no trade for intermediate values of β_{t_4}. If $b_{t_4}=[\alpha,0,0,0]$, the probabilities are

$$\Pr_{t_4}(MO_\alpha B_3|b_{t_4}) = \Pr(S)\frac{\beta_{(MO_\alpha B_3,NTS)}}{2} = \frac{2-5\tau}{8}$$
$$\Pr_{t_4}(NTS|b_{t_4}) = \Pr(S)\frac{\beta_{(MO^s_\alpha A_2,NTS)}-\beta_{(MO_\alpha B_3,NTS)}}{2} = \tau$$
$$\Pr_{t_4}(MO^s_\alpha A_2|b_{t_4}) = \Pr(S)\frac{2-\beta_{(MO^s_\alpha A_2,NTS)}}{2} = \frac{2-3\tau}{8}, \tag{16}$$

where, for example, $\beta_{(MO_\alpha B_3,NTS)}$ is the threshold between a market sell order of size α executed at B_3 and no trading. When small and large traders optimally choose the same equilibrium strategy, we add superscript s to indicate an order submitted by small traders.

As for large traders, if j shares are available at the best bid and ask spreads, the β_{t_4} thresholds are the same as those of small traders. However, if only $f_i<j$ shares are available at the best ask A_i and $n \geq j-f_i$ shares are available at $A_l>A_i$, large traders have the option to submit either a market sell order of size f_i at A_i or a larger market order of size j that walks up the book in search of execution. Thus, the large trader submits MO_jA if $\beta_{t_4} \geq A_l/v$, submits $MO_{f_i}A_i$ if $A_l/v \leq \beta_{t_4} < A_i/v$, and prefers not to trade if $1<\beta_{t_4} \leq A_i/v$.

Period $t=t_3$: We focus on the large trader's problem and present the case with $b_{t_3}=[\alpha,0,0,0]$ as an example. The feasible large trader's strategies and the associated profits are

$$\pi_{t_3}(MO_{10}B_3) = 10(B_3-\beta_{t_3} v)$$
$$\mathrm{E}[\pi_{t_3}(LO_jA_i)] = (A_i-\beta_{t_3} v)\sum_{w_{t_4}=\alpha,j} w_{t_4} \times \Pr_{w_{t_4}}(A_i|b_{t_4})$$
$$\pi_{t_3}(MO_\alpha A_2) = \alpha(\beta_{t_3} v-A_2)$$
$$\pi_{t_3}(MO_{10}A) = 10\beta_{t_3} v-\alpha A_2-(10-\alpha)A_3$$
$$\mathrm{E}[\pi_{t_3}(LO_{10}B_i)] = (\beta_{t_3} v-B_i)\sum_{w_{t_4}=\alpha,10} w_{t_4} \times \Pr_{w_{t_4}}(B_i|b_{t_4}), \tag{17}$$

where for LO_jA_i, $j=10$ for A_1 and $j=10-\alpha$ for A_2. Because limit orders eventually can be executed at t_4, traders must compute the associated order execution probabilities. For example, the profit formula for $\pi_{t_3}(LO_{10-\alpha}A_2)$ is

$$\mathrm{E}[\pi_{t_3}(LO_{10-\alpha}A_2)] = (A_2-\beta_{t_3} v)(10-\alpha)\Pr_{t_4}(MO_{10}A_2|b_{t_4}), \tag{18}$$

where $b_{t_4}=[10,0,0,0]$. The equilibrium intervals of β_{t_3} are obtained by comparing the preceding profits and finding the ranges of β_{t_3} associated with the large trader's optimal strategies.

Period $t=t_2$: We start by considering the possible opening states of the LOB at t_2, as summarized in Table A1. At t_1, the book opens empty, and we use the branch of the trading game in which an agent willing to sell arrives for this example. Equilibrium strategies for the other branch of the trading game, starting with a buyer arriving at t_1, are perfectly symmetric and, therefore, not presented in this proof.

We focus again on a large trader's strategies. Consider the expected profits from limit orders for $b_{t_2}=[\alpha,0,0,0]$:

$$\mathrm{E}[\pi_{t_2}(LO_{10}p_i)] = |p_i-\beta_{t_2} v| \times \left[\sum_{w_{t_3}=\alpha,10} w_{t_3}\Pr_{w_{t_3}}(p_i|b_{t_3}) + \sum_{w_{t_3}=0,\alpha}\Pr_{w_{t_3}}(p_i|b_{t_3})\sum_{w_{t_4}=\alpha}^{10-w_{t_3}} w_{t_4}\Pr_{w_{t_4}}(p_i|b_{t_4})\right], \tag{19}$$

where $p_i=\{A_i,B_i\}$. The profits from market orders are the same as in $t=t_3$.

Period $t=t_1$: We compute and compare the profits associated with traders' strategies on the sell side, assuming that the initial book is empty. Strategies on the bid side are symmetric. We focus again on a large trader's strategies:

$$\pi_{t_1}(MO_{10}B_3) = 10(B_3-\beta_{t_1} v)$$

$$\mathrm{E}[\pi_{t_1}(LO_{10}A_i)] = (A_i-\beta_{t_1} v)\left\{\sum_{w_{t_2}=\alpha,10} w_{t_2}\Pr_{w_{t_2}}(A_i|b_{t_2}) + \sum_{w_{t_2}=0,\alpha}\Pr_{w_{t_2}}(A_i|b_{t_2})\left[\sum_{w_{t_3}=\alpha}^{10-w_{t_2}} w_{t_3}\Pr_{w_{t_3}}(A_i|b_{t_3}) + \sum_{w_{t_3}=0,\alpha}\Pr_{w_{t_3}}(A_i|b_{t_3})\sum_{w_{t_4}=\alpha}^{10-w_{t_2}-w_{t_3}} w_{t_4}\Pr_{w_{t_4}}(A_i|b_{t_4})\right]\right\} \tag{20}$$

A.1.2. R framework

Period $t=t_4$: There are two possible cases. If there is no uncertainty about available depth, we return to the benchmark framework. If uncertainty arises due to reserve orders, traders must rationally estimate the probability of hidden depth and compute the expected execution prices.

Because small traders' order size is equal to the peak size of reserve orders, they face no uncertainty about the execution price of their market orders. Large traders face uncertainty when they assign a positive probability to the presence of a reserve order. We present two possible cases for the ask side (the bid side is derived similarly and, hence, omitted from this proof).

1. If $f_i < j$ shares are visible at A_i and $n \geq j - f_i$ shares are available at $A_l > A_i$, large traders have the option to submit a market order of size f_i at price A_i or a larger market order of size j that walks up the book, whose execution price is uncertain for $j - f_i$ shares. The large trader's β_{t_4} thresholds for the ask side are as follows. Submit MO_jA if $\beta_{t_4} \geq (\Lambda_m/v)$, submit $MO_{f_i}A_i$ if $(A_i/v) \leq \beta_{t_4} < (\Lambda_m/v)$, and do not trade if $1 < \beta_{t_4} < (A_i/v)$. Here, $\Lambda_m = \sum \Pr_{j-f_i}(A_m|b_{t_4})A_m$, with $m = \{i, l\}$, is a weighted average of the possible prices, and $\Pr_{j-f_i}(A_m|b_{t_4})$ are the probabilities that the remaining $j - f_i$ shares are executed at price A_m. The weights depend on the traders' strategies in previous periods. For example, suppose a large trader who observed the following sequence of orders arrives at t_4: at t_1 a small limit order of α shares at A_2, $b_{t_2} = [\alpha, 0, 0, 0]$; at t_2 a small limit order of α shares at B_2, $b_{t_3} = [\alpha, 0, 0, \alpha]$; and at t_3 a small market order of α shares at B_2, $b_{t_4} = [\alpha, 0, 0, 0]$. In this case, Λ_m is
$$\Lambda_m = \frac{A_2 \Pr_{t_1}(RO_{10}A_2|b_{t_4}) + A_3 \Pr_{t_1}(LO_\alpha A_2|b_{t_4})}{\Pr_{t_1}(RO_{10}A_2|b_{t_4}) + \Pr_{t_1}(LO_\alpha A_2|b_{t_4})}. \tag{21}$$

2. If there are $f_i < j$ visible shares on A_i for both $i=1$ and $i=2$, with $f_1 + f_2 < j$, the large trader's β_{t_4} thresholds for the ask side is as follows. Submit MO_jA if $\beta_{t_4} \geq (\Lambda_k/v)$, submit $MO_{f_1+f_2}A$ if $(\Lambda_m/v) \leq \beta_{t_4} < (\Lambda_k/v)$, submit $MO_{f_1}A_1$ if $(A_1/v) \leq \beta_{t_4} < (\Lambda_m/v)$, and do not trade if $1 \leq \beta_{t_4} < (A_1/v)$. Here, $\Lambda_m = \sum \Pr_{f_2}(A_m|b_{t_4})A_m$, with $m = \{1, 2\}$, and $\Lambda_k = \sum \Pr_{j-f_1}(A_k|b_{t_4})A_m$, with $k \in \{1, 2, 3\}$.

Period $t = t_3$: We again consider the visible book $b_{t_3} = [\alpha, 0, 0, 0]$, where traders assign a positive probability to the existence of a reserve order submitted at t_1 and focus on the large traders' problem. Profits from those feasible strategies, which differ from the benchmark, are

$$\mathrm{E}[\pi_{t_3}(LO_jA_i)] = E\left[(A_i - \beta_{t_3} v) \sum_{w_{t_4} = \alpha, j} w_{t_4} \times \Pr_{w_{t_4}}(A_i|b_{t_4})\right]$$

$$\mathrm{E}[\pi_{t_3}(MO_{10}A)] = \left[\alpha + (10-\alpha) \Pr_{10-\alpha}(A_2|b_{t_3})\right](\beta_{t_3} v - A_2)$$
$$+(10-\alpha)\left[1 - \Pr_{10-\alpha}(A_2|b_{t_3})\right](\beta_{t_3} v - A_3). \tag{22}$$

In the case of LO_jA_i, $j = 10$ for A_1 and $j = 10 - \alpha$ for A_2. We specify the profit formula for $\pi_{t_3}(MO_{10}A)$:

$$\pi_{t_3}(MO_{10}A) = (\beta_{t_3} v - A_2)\left[\alpha + (10-\alpha)\frac{\Pr_{t_1}(RO_{10}A_2|b_{t_3})}{\Pr_{t_1}(RO_{10}A_2|b_{t_3}) + \Pr_{t_1}(LO_\alpha A_2|b_{t_3})}\right]$$
$$+(\beta_{t_3} v - A_3)(10-\alpha)\frac{\Pr_{t_1}(LO_\alpha A_2|b_{t_3})}{\Pr_{t_1}(RO_{10}A_2|b_{t_3}) + \Pr_{t_1}(LO_\alpha A_2|b_{t_3})}. \tag{23}$$

Period $t = t_2$ *and* $t = t_1$: The large trader solves Eq. (7) in both periods. We do not report the general profit formulas, which only differ from the benchmark model for the uncertainty that characterizes the state of the book.

Optimal exposure size for reserve orders (α^*): We solve the model for different values of α. When α shares are visible at A_2, we find that for $\alpha > 1$ incoming traders prefer to undercut at A_1. Therefore, reserve orders do not protect against price competition. For $\alpha = 1$, incoming traders join the queue at A_2. The optimal disclosed size is thus the one compatible with traders joining the queue: $\alpha^* = 1$.

A.2. Proof of Proposition 2

The results in Fig. 3 come from a comparison of the equilibrium welfare values for both the benchmark and the reserve order protocol. As an example, we consider the benchmark case and provide the expected profits formulas for both a large and a small trader arriving at t_4, when the LOB opens at $b_{t_4} = [1, 0, 0, 0]$. Thresholds for the equilibrium order strategies are derived as shown in the proof of Proposition 1, denoted as follows: $\beta^1_{t_4,S} = \beta_{(MO_1B_3,NTS)}$, $\beta^2_{t_4,S} = \beta_{(NTS,MO_1A_2)}$, $\beta^1_{t_4,L} = \beta_{(MO_{10}B_3,NTL)}$, $\beta^2_{t_4,L} = \beta_{(NTL,MO_1A_2)}$, and $\beta^3_{t_4,L} = \beta_{(MO_1A_2,MO_{10}A)}$. Consider the expected gains from trade by a small trader:

$$\begin{aligned}\mathrm{E}[W_{t_4,S}|b_{t_4}] &= \mathrm{E}[W^{MO}_{t_4,S}|b_{t_4}] + \mathrm{E}[W^{LO}_{t_4,S}|b_{t_4}] \\ &= \int_0^{\beta^1_{t_4,S}} (B_3 - \beta_{t_4} v) f(\beta_{t_4})\, d\beta_{t_4} \\ &\quad + \int_{\beta^2_{t_4,S}}^2 (\beta_{t_4} v - A_2) f(\beta_{t_4})\, d\beta_{t_4}\end{aligned} \tag{24}$$

and a large trader:

$$\begin{aligned}\mathrm{E}[W_{t_4,L}|b_{t_4}] &= \mathrm{E}[W^{MO}_{t_4,L}|b_{t_4}] + \mathrm{E}[W^{LO}_{t_4,L}|b_{t_4}] \\ &= \int_0^{\beta^1_{t_4,L}} 10(B_3 - \beta_{t_4} v) f(\beta_{t_4})\, d\beta_{t_4} \\ &\quad + \int_{\beta^2_{t_4,L}}^{\beta^3_{t_4,L}} (\beta_{t_4} v - A_2) f(\beta_{t_4})\, d\beta_{t_4} \\ &\quad + \int_{\beta^3_{t_4,L}}^2 [(\beta_{t_4} v - A_2) + 9(\beta_{t_4} v - A_3)] f(\beta_{t_4}) d\beta_{t_4}.\end{aligned} \tag{25}$$

Total gains from trade for agent $a = \{S, L\}$ are computed as the sum of the expected gains from trade for different equilibrium states of the LOB, b_{t_4}:

$$\mathrm{E}[W_{t_4,a}] = \sum_{b_{t_4}} \mathrm{E}[W_{t_4,a}|b_{t_4}]\Pr(b_{t_4}). \tag{26}$$

A.3. Proof of Proposition 3

The results in Fig. 4 come from a comparison of the equilibrium expected profits for both the benchmark and the reserve order protocol for three representative values of β, $\beta=\{0.1,0.5,0.9\}$, and considering four different values of the tick size, $\tau=\{0.025,0.05,0.075,0.1\}$. Expected profits are computed as shown in the proof of Proposition 2.

A.4. Proof of Proposition 4

The expected value of the inside spread at the opening of period t_{i+1} is computed by weighting the inside semi-spread $S_{t_{i+1}}$ by the period t_i equilibrium order submission probabilities associated with each possible state of the book:

$$\mathrm{E}[S_{t_{i+1}}] = \sum_{a=S,L} \Pr(a) E_{b_{t_i}}\left[\int_0^2 (A^*_{t_{i+1}}(o^*_a) - v) \times f(\beta_{t_i})\, d\beta_{t_i}\right], \tag{27}$$

where o^*_a is the optimal trading strategy of agent a, conditional on b_{t_i} and β_{t_i}, and $A^*_{t_{i+1}}(o^*_a)$ is the best ask price available at time t_{i+1} as a function of the equilibrium strategies of the traders. The expected value of the weighted inside semi-spread $WS_{t_{i+1}}$ is computed in a similar way, except that now spreads are multiplied by the quantity available at the best ask A^*, $q^{A^*}_{t_{i+1}}$:

$$\mathrm{E}[WS_{t_{i+1}}] = \sum_{a=S,L} \Pr(a) E_{b_{t_i}}\left[\int_0^2 q^{A^*}_{t_{i+1}}(o^*_a) \times S_{t_{i+1}}(o^*_a) \times f(\beta_{t_i})\, d\beta_{t_i}\right]. \tag{28}$$

Similarly, at the opening of period t_{i+1}, the expected value of total market depth on the first level of the book, both disclosed and undisclosed, is computed as

$$\mathrm{E}[D_{t_{i+1}}] = \sum_{a=S,L} \Pr(a) E_{b_{t_i}}\left[\int_0^2 q^{A^*}_{t_{i+1}}(o^*_a) \times f(\beta_{t_i})\, d\beta_{t_i}\right]. \tag{29}$$

Expected LOB semi-volume is estimated in each period t_i by averaging the sizes of market buy orders hitting the ask side of the LOB by the associated equilibrium probabilities:

$$\mathrm{E}[V_{t_{i+1}}] = \sum_{a=S,L} \Pr(a) E_{b_{t_i}}\left[\int_0^2 q^{A}_{t_i}(o^*_a) \times f(\beta_{t_i})\, d\beta_{t_i}\right], \tag{30}$$

where $q^A_{t_i}(o^*_a)$ is the traded quantity on the ask side of the market, which is a function of both the agent type a and the state of LOB. The results, presented in Fig. 5, are derived by comparing the values of these market quality measures for the B and R frameworks, across four tick size values: $\tau=\{0.025,0.05,0.075,0.1\}$.

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