Journal of Financial Economics 124 (2017) 244–265

Contents lists available at ScienceDirect

Journal of Financial Economics

journal homepage: www.elsevier.com/locate/finec

# Dark pool trading strategies, market quality and welfare☆

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ARTICLE INFO

*Article history:*
Received 21 October 2013
Revised 1 July 2015
Accepted 27 July 2015
Available online 20 February 2016

*JEL Classification:*
G10
G20
G24
D40

*Keywords:*
Dark pool
Limit order book
Trading strategies

ABSTRACT

We show that when a continuous dark pool is added to a limit order book that opens illiquid, book and consolidated fill rates and volume increase, but spread widens, depth declines, and welfare deteriorates. The adverse effects on market quality and welfare are mitigated when book-liquidity builds but so are the positive effects on trading activity. All effects are stronger when traders' valuations are less dispersed, access to the dark pool is greater, horizon is longer, and relative tick size larger.



## 1. Introduction

Dark pools are Alternative Trading Systems (ATSs) that do not provide their best-priced orders for inclusion in the consolidated quotation data. They offer subscribers venues where anonymous, undisplayed orders interact away from the lit market yet execute at prices no worse than the National Best Bid Offer (NBBO). Dark pools today represent a considerable fraction of volume (Fig. 1). In the U.S. there are over 50 dark pools, and the 19 of them for which data are available (from Rosenblatt Securities Inc.) account for more than 14% of consolidated volume. In Europe the 16 dark markets which report to Rosenblatt account for approximately 4.5% of volume, and in Canada they represent 2% of volume.

The most active types of dark pools in the U.S., Europe, and Canada are Bank/Broker pools followed by Independent/Agency pools (Fig. 1). The Bank/Broker pools are operated by banks and are used both for agency and proprietary trading. These pools generally offer continuous execution and execute at prices derived from the NBBO. The Independent/Agency pools, like ITG POSIT, are instead operated by agency brokers and offer periodic executions at the midpoint of the NBBO. In Market Maker pools, liquidity can only be provided by the manager of the pool. Consortium-Sponsored pools are owned by several banks which already own their dark pool and use the

☆ We thank Bruno Biais, Hans Degryse, Dan Li, Albert Menkveld, Maureen O'Hara, Sugata Ray, Gideon Saar, and in particular Duane Seppi and an anonymous referee for extremely useful comments as well as participants at 6th Early Career Women in Finance Conference (2010), 4th IIROC DeGroote Conference on Market Structure and Market Integrity (2010), Northern Finance Association Meeting (2010), Western Finance Association Meeting (2011), European Finance Association Meeting (2011), Vanderbilt Conference on Institutional Investors and Price Efficiency (2013), Econometric Society European Meeting (2014), Financial Intermediation Research Society Conference (2015), and seminar participants at Bocconi University, Toulouse University and Toronto University. The usual disclaimer applies.

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http://dx.doi.org/10.1016/j.jfineco.2016.02.002
0304-405X/

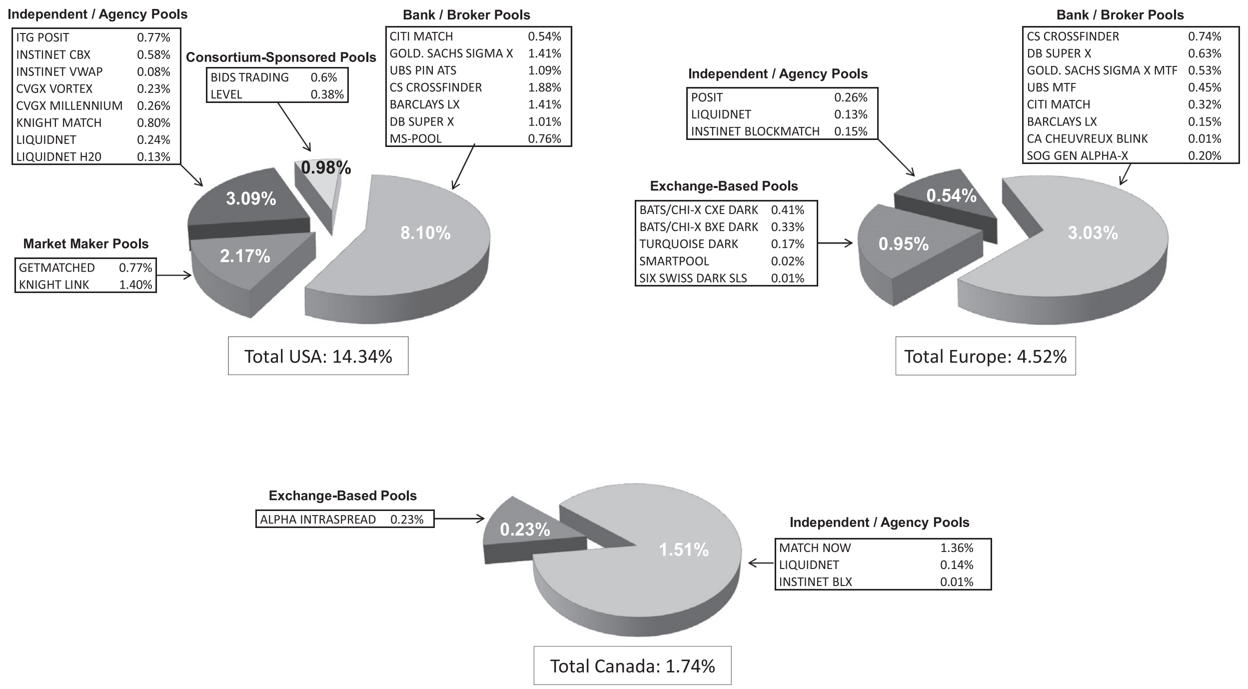


**Fig. 1.** Dark pools volume. Percentage of consolidated U.S., European, and Canadian equity volume, December 2012. Data source: Rosenblatt Securities Inc.

Consortium-Sponsored pools as trading venues of last resort. Finally, Exchange-Based dark pools are owned by exchanges and offer continuous execution.

The rising market share of dark trading recently prompted three major U.S. exchanges to publicly urge the Securities and Exchange Commission (SEC) to put rules in place to curb dark pool trading. Exchange officials are concerned that dark pools divert volume away from lit venues, rather than attracting new order flow to the market. With declining trading volumes worldwide, such a diversion of order flow is a real threat to exchanges' bottom lines. Consequently, it is important for exchanges to understand which factors cause order flow to go dark, and under what circumstances dark pools are likely to primarily divert volume away from lit venues as opposed to create more opportunities for trades to take place. Regulators are concerned about the effects of dark trading on market quality and welfare. Order migration away from lit markets to dark pools may adversely influence the incentive for traders to provide liquidity in the lit market, potentially resulting in higher trading costs. Dark pools may also affect the distribution of welfare between retail and institutional investors, as dark venues are primarily used by institutional traders.

In this paper we build a theoretical model that enables us to address the concerns raised by exchanges and regulators in a realistic market setting. Specifically, we populate our model with fully rational traders who form their optimal trading strategies based on their private valuations. All traders in our model can choose to submit a one-share market or limit order to a transparent limit order book (LOB) with a discrete price grid. In addition, some traders may submit orders to a dark pool. If sufficient two-sided trading interest is routed to the dark pool, orders are executed at the midquote of the prevailing NBBO.[1] The dark pool executes orders continuously, meaning that traders with access to the dark pool can simultaneously access the lit and the dark markets. To model this simultaneous access, we introduce an additional order type, Immediate-or-Cancel (IOC) orders. These orders are first routed to the dark pool, and if they do not execute are routed back to the lit market as a market order. Our model closely resembles real world order book markets competing with Bank/Broker dark pools, and this group of dark pools executes 57%, 67%, and 87% of dark volumes in the U.S., Europe, and Canada, respectively (Rosenblatt Securities Inc., 2012). We use this rich setup to address the concerns raised by exchange officials and regulators, market participants, and media about order migration, market quality, and welfare.

Our theoretical model builds on Parlour (1998), but in the spirit of Buti and Rindi (2013) we extend her model to include a price grid, a dark pool, and additional order types. We also differentiate between traders with and without access to the dark pool. We start by modeling a benchmark LOB where traders decide whether to submit a market order, a limit order, or to refrain from trading based on the information they infer about future execution probabilities from the current state of the LOB. The model runs for four periods, and the LOB starts empty. We then introduce a dark pool which also starts empty, accepts orders from traders with access, and attempts to

[1] What is important is that the price in the dark pool is derived from the NBBO, not whether or not trades execute at the midquote. However, for tractability, our dark pools always execute at the midquote of the NBBO.

execute submitted orders continuously at the prevailing LOB midpoint. Note that the opacity of the dark pool effectively works as a friction in that it adds an inference problem to the traders' optimization problem. Traders with access cannot see orders resting in the dark pool, and also do not know what the execution price will be for an order sent to the dark pool as it depends on the state of the future LOB. Hence, traders use the lit LOB to make inferences about the potential price improvement (midquote price) and the execution probability in the dark pool compared to the trading opportunities on the LOB.

By comparing results from the benchmark LOB model without a dark pool to the results from the model with a LOB competing with a dark pool, we are able to address the concerns raised by exchange officials and regulators discussed above. We show that the introduction of a dark pool to a LOB market results in higher consolidated fill rates, but also higher LOB fill rates. We also show that the higher LOB fill rates are associated with wider LOB spread and lower LOB depth. The intuition for this result is that the consequences for LOB fill rates and market quality of the introduction of a dark pool depend on whether it is predominately traders that would have used limit orders or market orders that go dark. When the LOB starts empty traders are more likely to use limit orders, and it is therefore predominantly limit orders that migrate to the dark venue. Moreover, everyone knows that the dark pool traps market orders submitted to the lit market, and since this means that the execution probability of limit orders in the LOB declines, traders remaining in the lit market on the margin switch from limit to market orders. As a result of both these effects, LOB liquidity supply declines and LOB fill rates increase causing spreads to widen.

It follows that our model suggests that exchanges are actually better off in the presence of dark pools because the higher fill rate allows them to harvest additional trading fees. However, since the increase in fill rates is associated with a wider LOB spread and lower LOB depth, the concerns raised by regulators that dark trading may undermine the liquidity of the lit market book are warranted. Note that the reason for lit market depth to decline and spreads to widen is that traders use more marketable orders, resulting in higher fill rates. Our model therefore illustrates that there is a trade-off between displayed liquidity and trading volume. Ultimately, the question then becomes whether traders are better or worse off. We find that while the fill rate increases, this occurs at worse trading conditions and as a result welfare both for traders with and without access deteriorates.

We derive cross-sectional predictions for our model by varying the model parameters. Dark pool activity is decreasing in the dispersion of valuations around the common value of the asset, and the effects of dark pool activity on LOB and consolidated fill rates, market quality, and welfare are weaker when the dispersion in traders' valuations is larger. Not surprisingly, dark pool activity is increasing in the proportion of traders with access, and dark pool activity is associated with a stronger positive effect on LOB and consolidated fill rates, but also a stronger adverse effect on spread and depth when more traders have access to the dark pool. Finally, dark pool activity decreases when the stock price is higher and therefore the relative tick size is smaller, and the effects of dark pool activity on LOB and consolidated fill rates, market quality, and welfare are weaker when the relative tick size is small.

The discrete time nature of the model allows us to analyze the equilibrium order submission strategies from period two onwards. By comparing our original four-period model to one where the book and dark pool also open empty but there are only three periods in which to trade, we find that if there are more future periods, the value of having access to a dark pool is higher, and the effects on traders' optimal LOB strategies are stronger. Therefore, all the effects are magnified.

We also study the time-series dynamics of our model and find that dark pool fill rates increase when liquidity builds up in the order book. The reason is that when there is an order queue, a new limit order submitted to the LOB has lower execution probability and hence the possibility of obtaining a midquote execution in the dark pool becomes relatively more attractive. As more orders migrate to the dark venue, the execution probability of dark orders increases thus making these orders more profitable. Consequently, our model predicts that order migration and dark pool market share increase in liquidity. This prediction is confirmed in recent empirical work on dark pool data by Buti, Rindi, and Werner (2011) and Ready (2013).

While dark pool fill rates increase in liquidity, LOB and consolidated fill rates actually decline when the order book liquidity builds. This happens because traders tend to make a greater use of market orders at the expense of limit orders in deeper books, and therefore it is predominantly market orders that migrate to the dark pool. As the execution probability of market orders is higher than that of dark orders, LOB fill rates are lower when the book becomes liquid. Hence, while the introduction of a dark pool boosts lit market fill rates when LOB liquidity is low, it actually hurts lit market fill rates when LOB liquidity builds.

In terms of market quality, we find that the spread widens less and depth declines more when order book liquidity builds than when the book remains illiquid. When book liquidity builds, both market and limit orders switch to the dark venue. The migration of market orders reduces the drain of lit market liquidity and spreads therefore widen less. However, the limit order queue is long, and limit orders migrate to the dark venue intensively. This migration does not impact the spread because the book is already deep but results in a decline in LOB depth.

Finally, we study two additional variations in market structure and trading protocols. First, we allow the traders with access to a dark pool to submit larger orders, and to engage in order splitting both between order types and across venues. Our conclusions about the effects of dark pools on volume creation, market quality, and welfare are robust to this extension. Second, we alter the trading protocol of the dark pool from a continuous dark pool to a periodic dark pool. We find that the effects of introducing a dark pool are reduced when the dark pool crosses orders periodically.

Our theoretical model contributes to the literature in several ways. Previous models of dark trading focus on studying the effects of introducing a periodic dark crossing network on a lit dealer market (DM) where public traders are restricted to using market orders (e.g., Degryse, Van Achter, and Wuyts, 2009; and Zhu, 2014). By contrast, we study the effects of introducing a continuous dark pool on a lit market that operates as a LOB where all traders rationally decide whether to supply or demand liquidity, and traders with access to the dark pool decide whether they will route their order to the lit or the dark market (or both). Previous models of LOB either make simplifying assumptions that severely restrict traders' ability to choose order type freely (e.g., Foucault, 1999; and Foucault, Kadan, and Kandel, 2005) or assume that prices are continuous (Rosu, 2009; 2014) in order to characterize a stationary equilibrium.[2] In our model instead all traders can choose to supply or demand liquidity and the price grid is discrete with a positive minimum price increment–the tick size. A positive tick size forces liquidity suppliers to price improve by a significant economic amount, which guarantees that price and time priority are enforced. It is precisely these market structure features that create a LOB. Models of LOB in stationary equilibrium also cannot study how traders' optimal strategies are adjusted dynamically over the course of the trading day. By contrast, our model can be thought of as the evolution of a market over the course of a trading day and starts with an empty LOB and (when available) an empty dark pool, allows liquidity to build, and also captures the effects of the market close on equilibrium trading strategies. Previous models focus on dark pools that execute periodically (crossing networks) that resemble real world Independent/Agency dark pools. However, the most popular real world Bank/Broker dark pools in the U.S. and Europe execute continuously (Fig. 1) and we therefore model a dark pool that runs in parallel with the transparent market and where orders may execute continuously. Finally, to model the continuous interaction between a lit and a dark venue, we allow traders to simultaneously access lit and dark venues using IOC orders. This feature has not previously been modeled, yet several dark pools, for example, Sigma X in the U.S. and Match Now in Canada, offer this type of functionality.

In our model, the state of the book observed by traders when they come to the market, as well as the price and priority rules that govern trading, influence traders' strategic choice between trading venues and between order types, and hence affect market quality and welfare. This feature is consistent with empirical evidence that shows how order submission strategies depend on the state of the order book and the price/time priority rules (Griffiths, Smith, Turnbull, and White, 2000; Handa, Schwartz, and Tiwari, 2003; and Ranaldo, 2004), and that dark trading is affected by the spread, the depth, and the tick-to-price ratio (Buti, Rindi, and Werner, 2011; and Ready, 2013).

Note that our predictions are very different from those made by, for instance, Degryse, Van Achter, and Wuyts (2009) and Zhu (2014) who model the lit market as a DM. In their models, traders who are unwilling to pay the spread cannot submit limit orders and hence either stay out of the market or move to the dark pool to execute at the midquote. By contrast, traders in our model do not need to move to the dark as they can post their limit orders on the LOB. As a result, we find less order migration to the dark venue than what is predicted by DVW and Zhu. Our model also generates very different predictions about the factors that drive orders to go dark. DVW and Zhu find that the smaller the spread, the fewer orders go dark because the price improvement offered by the dark pool is small. When instead the spread is large, traders are more likely to route their orders to the dark venue since it offers a larger price improvement compared to dealer quotes. Our model predicts the opposite, i.e., that dark pools are more actively used when order books are liquid and therefore the limit order queue at the best ask or bid price is longer.

The paper is organized as follows. In Section 2 we review the related literature. In Section 3 we present both the benchmark framework and the framework with a continuous dark pool. In Section 4 we report the results on factors that affect order flows and dark pool market share and in Section 5 on the effects on market quality and welfare. Section 6 briefly discusses the two extensions of the model. Section 7 is dedicated to the model's empirical implications and Section 8 to the conclusions and policy implications. All proofs are in the Appendix.

## 2. Literature review

The literature on multimarket competition is extensive.[3] Our paper is related in particular to the branch of this literature which deals with competition between trading venues with different levels of pre-trade transparency. The paper which is closest to ours is Degryse, Van Achter, and Wuyts (2009), who investigate the interaction of a crossing network (CN) and a DM and show that the composition and dynamics of the order flow on both systems depend on the level of transparency.[4] However, as we discuss in depth later in the paper, our contribution differs substantially from Degryse, Van Achter, and Wuyts (2009). First of all, we consider the interaction between a LOB and a dark venue where traders can both demand liquidity (via market orders) and compete for the provision of liquidity (via limit orders). By contrast, in a DM traders are only allowed to demand liquidity. Second, we consider a dark pool with

[2] Foucault (1999) assumes that the book is always either empty or full, and traders cannot compete to provide liquidity. Foucault, Kadan, and Kandel (2005) adopt a set of simplifying assumptions: limit orders must price improve by narrowing the spread by at least one tick, buyers and sellers alternate with certainty, and traders cannot access both markets simultaneously. Rosu's 2009; 2014 frameworks are not suitable to discuss competition between dark and lit markets as he assumes that prices are continuous.

[3] Works on competition among trading venues include: Barclay, Hendershott, and McCormick (2003), Baruch, Karolyi, and Lemmon (2007), Bessembinder and Kaufman (1997), Easley, Kiefer, and O'Hara (1996), Karolyi (2006), Lee (1993), Pagano (1989), Parlour and Seppi (2003), Reiss and Werner (2004), and Subrahmanyam (1997).

[4] Hendershott and Mendelson (2000) model the interaction between a CN and a DM and show costs and benefits of order flow fragmentation. Donges and Heinemann (2004) model intermarket competition as a coordination game among traders and investigate when a DM and a CN can coexist; Foster, Gervais, and Ramaswamy (2007) show that a volume-conditional order-crossing mechanism next to a DM Pareto improves the welfare of additional traders.

a continuous execution system where traders have simultaneous access both to the LOB and to the dark pool.

Another related paper is Zhu (2014) who uses the Glosten and Milgrom (1985) model to show that when the dark market is introduced to a DM, price discovery on the lit venue improves. The reason is that informed traders choose to send their market orders to the DM and not to the CN because they would all submit orders on the same side in the dark venue and no executions would take place.[5] By contrast, in our LOB model traders can act as liquidity suppliers and earn the spread. Moreover, in our setting traders have different valuations, but they have symmetric information.

We conjecture that if we were to extend our model to include asymmetric information, informed traders would use both the LOB and the dark pool. The reason is that a LOB unlike a DM does not offer infinite supply of liquidity, and informed traders therefore have no incentive to concentrate in the LOB. Moreover, since in our setting informed traders would be able to use limit orders, we conjecture that dark trading would not necessarily cause a wider spread even if asymmetric information were introduced. This is especially likely to be the case in books with a limited supply of liquidity at the inside LOB spread.

In addition to the traditional trade-off between price opportunity costs and non-execution costs facing uninformed traders, informed traders also need to take into account that their limit orders may partly reveal their information. Boulatov and George (2013), using a Kyle (1989) setting to compare a dark to a transparent venue, formally show that this consideration makes informed traders compete more aggressively for the provision of liquidity in the dark than they do in the lit market. As a result, they predict that the market quality facing uninformed traders may be better, and therefore profits for informed traders lower, in a dark venue.[6] However, while informed traders in Boulatov and George (2013) can choose between market and limit orders, they cannot choose between the lit and the dark venue. If informed traders could choose between the lit and the dark venue, they would have an incentive to move away from the dark to the lit market as their trading profits are lower in the dark. Therefore, it is still an open question for future research how dark trading would affect market quality and price discovery in a market where asymmetrically informed traders choose both between market and limit orders and between a lit and a dark venue.

Our model is also closely related to Foucault and Menkveld (2008) who focus on the competition between two transparent LOBs. They show that when brokers can apply Smart Order Routing Technology (SORT), the execution probability of limit orders (i.e., the liquidity provision) in the incumbent LOB increases. In our model traders can use IOC instructions when submitting an order to the dark pool and we suggest that this routing technology enhances the competition from the new trading venue.

Finally, dark pools are currently competing with other dark options offered by exchanges to market participants and this provides a link to the recent literature on hidden orders. In Buti and Rindi (2013) and Moinas (2010) traders active in a LOB can choose between disclosed and undisclosed orders, whereas in our model they can choose between lit and dark trading venues.[7]

## 3. The model

In this section we present a model of a LOB and we use it as a benchmark protocol. We then add a dark pool that crosses orders continuously (*CDP*) and analyze the competition between the LOB and the *CDP*. In this framework, traders with access to the dark pool not only may submit orders to the *CDP* but may also use more sophisticated trading strategies; for example, liquidity demanders may submit IOC orders to the *CDP* that can immediately bounce back to the lit LOB market in case of non-fill or partial execution.[8] We believe that the *CDP* captures the most relevant microstructure features of the Bank/Broker and Exchange-Based dark pools.

### 3.1. Benchmark model (B)

We consider a trading protocol over a trading day divided into four periods ($t = t_1, t_2, t_3,$ and $t_4$). The protocol features a LOB for a security which pays $v$ at the end of the trading day. The LOB is characterized by a set of four prices and associated quantities, denoted by $\left\{p_i^z \& q_i^z\right\}$, where $z = \{A, B\}$ indicates the ask or bid side of the market, and $i = \{1, 2\}$ the level on the price grid. The prices are defined relative to the common value of the asset, $v$:

$$p_2^A = v + \frac{3}{2}\tau \tag{1}$$

$$p_1^A = v + \frac{1}{2}\tau \tag{2}$$

$$p_1^B = v - \frac{1}{2}\tau \tag{3}$$

$$p_2^B = v - \frac{3}{2}\tau \ , \tag{4}$$

where we assume that the minimum price increment that traders are allowed to quote over the existing prices is equal to a constant $\tau$. Hence $\tau$ is the minimum spread that can prevail on the LOB. The associated quantities denote the number of shares that are available at each price level. Following Parlour (1998) and Seppi (1997), we assume that a trading crowd absorbs any amount at the highest ask and

[5] Ye (2011) instead models competition between a Kyle (1985) auction market and a dark pool and finds opposite results on price discovery. Ye assumes that only informed traders – but not noise traders – can strategically opt to trade in the dark pool, and finds that dark pools harm price discovery.

[6] Because dark pools are characterized by limited or no pre-trade transparency, our model is also related to the vast literature on anonymity and transparency. See, for example, the theoretical works by Admati and Pfleiderer (1991), Baruch (2005), Fishman and Longstaff (1992), Forster and George (1992), Madhavan (1995), Pagano and Röell (1996), Rindi (2008), and Röell (1991).

[7] See also the experimental papers by Bloomfield, O'Hara, and Saar (2015) and Gozluklu (2009).

[8] The IOC option is crucial to allow traders to realistically fully exploit the possibility to continuously execute orders in the *CDP*.

**Table 1**
Order submission strategies.

This table reports the trading strategies, $\varphi$, available to traders for the two different frameworks considered: a benchmark model (*B*) with a limit order book (LOB), and a continuous dark pool (*L&C*) competing with a LOB. Notice that only a percentage $\alpha$ of traders has access to the dark pool in the (*L&C*). The LOB is characterized by a set of four prices, denoted by $p_i^z$, where $z = \{A, B\}$ indicates the ask or bid side of the market, and $i = \{1, 2\}$ the level on the price grid. In the *L&C*, $\widetilde{p}_{Mid,t}$ indicates the spread midquote on the LOB prevailing in period *t*. IOC indicates Immediate-or-Cancel orders. The agents trade up to one share.

| Strategies | Notation |
|---|---|
| (B) and (L&C) | |
| Market order | $\varphi_M(1, p_i^z)$ |
| Limit order | $\varphi_L(1, p_1^z)$ |
| No trading | $\varphi(0)$ |
| (*L&C*) | |
| Dark pool order | $\varphi_D(\pm 1, \widetilde{p}_{Mid,t})$ |
| IOC on dark pool or market order | $\varphi_D(\pm 1, \widetilde{p}_{Mid,t}, p_i^z)$ |

lowest bid on the price grid, which in our model are $p_2^A$ and $p_2^B$. Therefore, the book depth is unlimited at the second level, whereas the number of shares available at $p_1^A$ ($p_1^B$) forms the state of the book at each time *t* and is defined as $b_t = [q_1^A q_1^B]$.

In our model all agents are fully rational, risk-neutral, and trade because they wish to trade. In each period *t*, upon arrival, a trader selects an optimal order type. The trader's personal valuation of the asset, $\beta v$, is captured by a multiplicative parameter, $\beta$, drawn from a uniform distribution with support [0, 2]: traders with extreme valuations of the asset ($\beta$ next to zero or next to two) perceive large gains from trade and will tend to use market orders; traders with a $\beta$ next to one perceive smaller gains from trade as their valuation of the asset is close to the common value and will primarily use limit orders.[9]

Traders observe the state of the LOB but not the identity of market participants. To select the optimal order type the incoming trader compares the expected profits from each of the different order strategies, $\varphi(.)$, presented in Table 1. He can submit a market order to the first two levels of the price grid, $\varphi_M(1, p_i^z)$; he can post a limit order to the first level, $\varphi_L(1, p_1^z)$, and he can choose not to trade, $\varphi(0)$. The profitability of the orders depends on the state of the book, $b_t$, and on the personal valuation of the trader arriving in period *t*, $\beta_t$.

Fig. 2 shows the extensive form of the trading game and to keep it as simple as possible, we present only the equilibrium strategies. We refer to this extensive form to discuss the strategies available to traders conditional on the state of the LOB, as well as their payoffs and the effects of different orders on the state of the LOB. There are four possible trading strategies that a trader may choose when he arrives at the market at $t_1$ and observes an empty LOB, $b_{t_1} = [00]$. A trader with a low valuation will opt for a market sell order that hits the trading crowd standing on the second level of the bid side, $\varphi_M(1, p_2^B)$. This order pays the spread and executes with certainty with the following payoff:

$$\pi_{t_1}[\varphi_M(1, p_2^B)] = (p_2^B - \beta_{t_1} v) \ . \quad (5)$$

After this order is executed, the book at $t_2$ will still open with no shares on the first level of the book, $b_{t_2} = [00]$. The same book will open at $t_2$ if a trader instead submits a market buy order, $\varphi_M(1, p_2^A)$, at $t_1$. A trader with a valuation closer to *v* will instead choose a limit order to buy, $\varphi_L(1, p_1^B)$, or to sell, $\varphi_L(1, p_1^A)$, at $p_1^B$ and $p_1^A$, respectively, and the book will open at $t_2$ with one share at the best bid or ask price, $b_{t_2} = [01]$ or $b_{t_2} = [10]$. Consider, for example, the strategy of a limit order to sell at $t_1$, $\varphi_L(1, p_1^A)$, and move to the next periods. The expected profit of the trader that opts for a limit sell order depends on the probability of the order being executed in the following trading rounds, $t_2$, $t_3$, and $t_4$:

$$\begin{aligned}\pi_{t_1}^e[\varphi_L(1, p_1^A)] &= (p_1^A - \beta_{t_1} v)\{\Pr_{w_{t_2}=1}(p_1^A|b_{t_2}) + \Pr_{w_{t_2}=0}(p_1^A|b_{t_2}) \\ &\quad [\Pr_{w_{t_3}=1}(p_1^A|b_{t_3}) + \Pr_{w_{t_3}=0}(p_1^A|b_{t_3}) \Pr_{w_{t_4}=1}(p_1^A|b_{t_4})]\}, \end{aligned} \quad (6)$$

where $\Pr_{w_t}(p_1^A|b_t)$ is the probability that $w_t$ shares posted at $p_1^A$ get executed at *t*.

When the book opens with one share at the best ask price, $b_{t_2} = [10]$, the trader arriving at $t_2$ can choose among the strategies shown in Fig. 2. For example, consider a trader who arrives at $t_2$, observes the book $b_{t_2} = [10]$, and submits another limit sell order of one share so that the book will open at $t_3$ as $b_{t_3} = [20]$. The incoming trader at $t_3$ knows that any additional limit order submitted on the ask side will have zero execution probability because only one trader is left before the market closes. Therefore, if a trader willing to sell arrives, he will either submit a market order to sell, or refrain from trading and get no profits, $\pi_{t_3}[\varphi(0)] = 0$. In both cases, the book will open at $t_4$ as $b_{t_4} = [20]$. In the last period, traders never submit limit orders because the market closes and their execution probability is zero. Therefore, the incoming trader after observing $b_{t_4} = [20]$, submits either a market buy order to $p_1^A$ , or a market sell order to $p_2^B$, or refrains from trade.

To summarize, at each trading round, the arriving risk-neutral trader selects the optimal order submission strategy which maximizes his expected profits, conditional on the state of the LOB, $b_t$, and on his type captured by his personal valuation of the asset, $\beta_t$:

$$\max_{\varphi} \pi_t^e[\varphi_M(1, p_i^z), \varphi_L(1, p_1^z), \varphi(0)|\beta_t, b_t] \ . \quad (7)$$

Note that in this model the standard trade-off between non-execution costs and price opportunity costs applies. Traders with very high or very low valuations relative to *v* generally choose market orders to minimize non-execution costs, whereas traders with a valuation close to *v* generally choose limit orders to minimize the risk of trading at an unfavorable price.

We find the solution of this game by backward induction, and from now on for simplicity we assume without

[9] In Parlour (1998), traders are assumed to arrive at the market with an exogenous probability of being a buyer or a seller. As a result, traders may refrain from trading because they have a high (low) valuation of the asset but nature selects them as a seller (buyer). By contrast, in our model the individual $\beta$ determines whether a trader buys, sells, or does not trade.

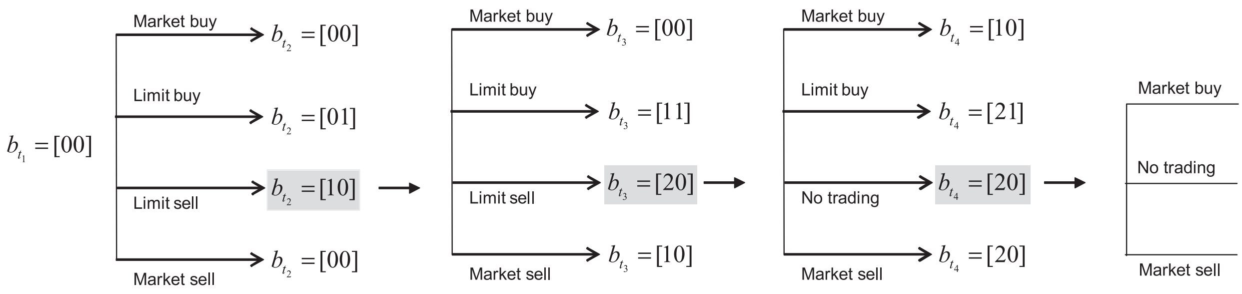


**Fig. 2.** Benchmark model of limit order book ($B$). Example of the extensive form of the game for the benchmark model when the opening book at $t_1$ is $b_{t_1} = [00]$. Only equilibrium strategies are presented.

loss of generality that $\tau = 0.08$ and $\nu = 1$.[10] We start from the end-nodes at time $t_4$ and for all the possible states of the book we compare trading profits from the traders' optimal strategies. This allows us to determine the probability of the equilibrium trading strategies at $t_4$, which are to submit market orders on the buy or sell side, or not to trade. Using this information, we can compute the execution probabilities of limit orders placed at $t_3$, which in turn allows us to derive the equilibrium order submission strategies for period $t_3$. Given the probability of market orders submitted at $t_3$, we can finally compute the equilibrium order submission strategies at $t_2$ . The same procedure is then reiterated to obtain the equilibrium order submission strategies at $t_1$.

In this model, traders are indifferent between orders with zero execution probability and therefore a unique equilibrium always exists due to the recursive structure of the game.

*Definition 1.* An equilibrium of the trading game is a set of $n \in N_t$ order submission decisions, $\{\varphi^n\}$, such that at each period the trader maximizes the expected payoff $\pi_t^e$ according to his Bayesian updated beliefs over the execution probabilities, $\Pr_{w_t}(p_1^z | b_t)$.

### 3.2. Limit order book and continuous dark pool (L&C)

We now extend the model to include a continuous dark pool that operates alongside our benchmark LOB. A *CDP* is organized like an opaque crossing network that offers continuous execution using a time priority rule. In our discrete model, this means that the *CDP* crosses orders at each trading round at the spread midquote prevailing on the LOB in that period, $\widetilde{p}_{Mid,t}$. Hence, in a *CDP* not only the execution probability is uncertain but also the execution price, because the midquote changes dynamically with the bid and ask quotes. As the dark pool is opaque, traders are unable to observe the orders previously submitted by the other market participants to the dark pool. It follows that they can only infer the state of the dark pool by monitoring the LOB and by Bayesian updating their expectations. We assume that at $t_1$ the dark pool opens empty, $CDP_{t_1} = 0$.

To match the situation in real markets, we assume that only a percentage $\alpha$ of traders has access to the *CDP*. Their action space includes, in addition to the orders presented for the benchmark model, the ability to submit orders to buy or to sell the asset on the *CDP* and a more sophisticated strategy that allows them to simultaneously send orders to the LOB and to the dark venue, as shown in Table 1. Therefore, each trader decides not only his optimal order type, as in the *B* framework, but also his preferred trading venue. In particular, traders can submit their orders exclusively to the *CDP*, $\varphi_D(\pm 1, \widetilde{p}_{Mid,t})$ , or send an IOC order to the *CDP*. If the IOC order does not execute immediately, it is automatically routed to the LOB as a market order, $\varphi_D(\pm 1, p_{Mid,t}, p_i^z)$. As in the benchmark model, all traders compare the expected profits from the different order types but now the profitability of these orders depends also on the expected state of the *CDP* at the time of the order submission, $\widetilde{CDP}_t$.

In Fig. 3 we present the extensive form of the game, and we include again only the equilibrium strategies to keep it as simple as possible. At $t_1$ the *L&C* is similar to the benchmark case. The reason is that with our limited horizon (four periods), the execution probability of dark orders is not high enough to induce traders to submit orders to the *CDP* instead of submitting a limit order to the empty LOB. However, as liquidity builds up in the LOB, traders with access to the *CDP* switch to the dark venue both because the execution probability of limit orders declines and because liquidity also builds up in the dark thus attracting market orders. Assume for example that at $t_1$ a trader submits a limit sell order at $p_1^A$, $\varphi_L(1, p_1^A)$, so that the book opens at $t_2$ as $b_{t_2} = [10]$. An incoming trader with access to the dark pool who is willing to sell will now, in addition to the benchmark strategies, consider submitting an order to the *CDP*, $\varphi_D(-1, \widetilde{p}_{Mid,t})$, that has the following expected payoff:

$$\pi_{t_2}^e[\varphi_D(-1, \widetilde{p}_{Mid,t})] = E[(\widetilde{p}_{Mid,t} - \beta_{t_2}\nu) \Pr_{-1}(\widetilde{p}_{Mid,t} | \Omega_{t_2})] \ , \tag{8}$$

where $\Omega_{t_2} = \{b_{t_2}, CDP_{t_2}\}$ is the information set of the trader and $\Pr_{-1}(\widetilde{p}_{Mid,t} | \Omega_{t_2})$ is the probability that one share to sell will be executed in the *CDP*. Clearly, this order

[10] Results are robust for different tick size values, for instance, $\tau \in [0.01, 0.1]$.

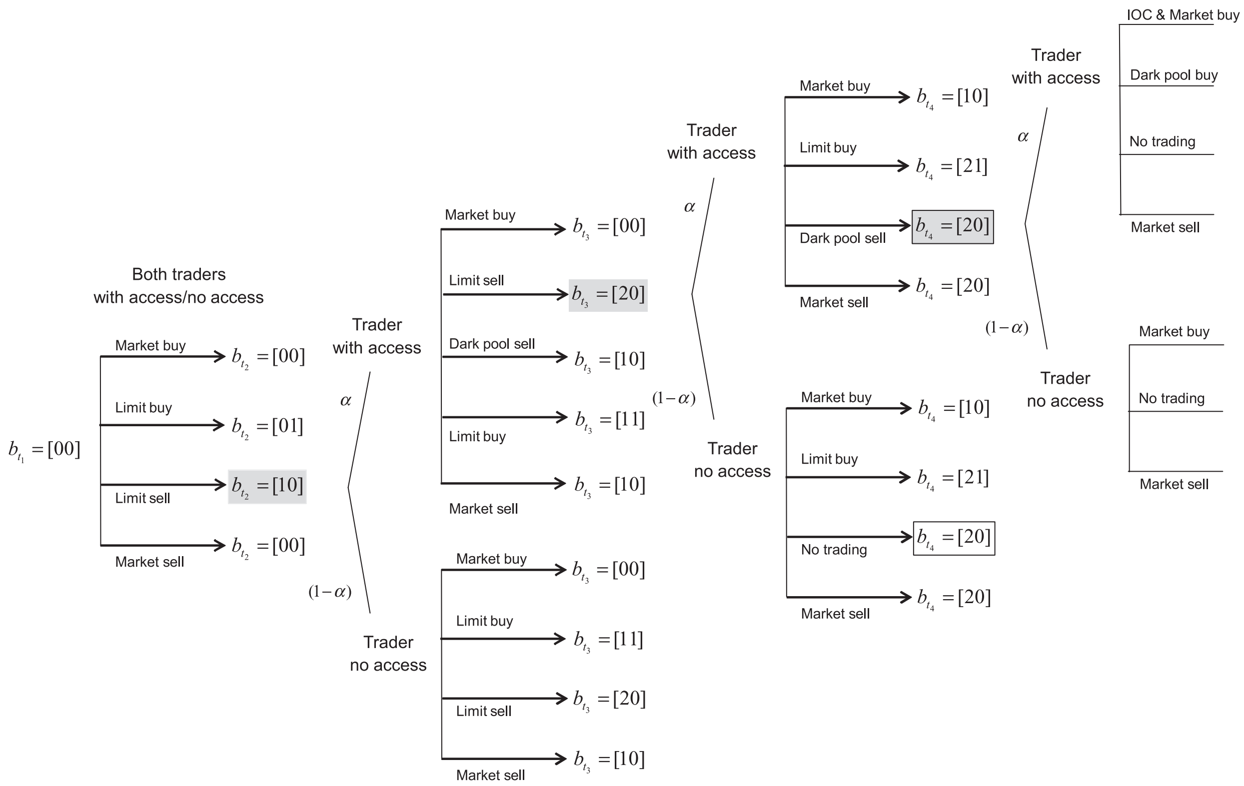


**Fig. 3.** Limit order book and continuous dark pool (*L&C*). Example of the extensive form of the game for the model with a continuous dark pool when the opening book at $t_1$ is $b_{t_1} = [00]$. Books that belong to the same information set, and hence are undistinguishable, are inside a squared box. For example, $b_{t_4} = [20]$ can be observed when either a trader with access to the dark pool arrives at $t_3$ and submits a dark pool order to sell, or a trader with no access arrives and refrains from trading. Only equilibrium strategies are presented.

submitted to the dark venue changes the state of the dark pool from $CDP_{t_2} = 0$ to $CDP_{t_3} = -1$.[11]

Alternatively, assume that a trader arrives at $t_2$ and submits a limit order to sell, so that the book opens in the following period as $b_{t_3} = [20]$. In this case, an incoming seller at $t_3$ with dark pool access not only has the options to submit a market order or refrain from trading as in the *B* framework but he can also submit a dark pool order to sell, $\varphi_D(-1, \widetilde{p}_{Mid,t})$. If he opts for a dark pool order, the trader arriving at $t_4$ observes the resulting opening book, $b_{t_4} = [20]$, and Bayesian updates his expectations on the state of the *CDP*, taking into account that this state of the LOB could be the result of two equilibrium strategies at $t_3$, either a dark pool sell order, or a decision not to trade by a trader without access to the *CDP*:

$$\widetilde{CDP}_{t_4} = \begin{cases} 0 & with\ prob = \dfrac{(1-\alpha)\Pr_{t_3}\varphi(0)}{\alpha \Pr_{t_3}\varphi_D(-1,\widetilde{p}_{Mid,t}) + (1-\alpha)\Pr_{t_3}\varphi(0)} \\ -1 & with\ prob = \dfrac{\alpha \Pr_{t_3}\varphi_D(-1,\widetilde{p}_{Mid,t})}{\alpha \Pr_{t_3}\varphi_D(-1,\widetilde{p}_{Mid,t}) + (1-\alpha)\Pr_{t_3}\varphi(0)} \end{cases}. \quad (9)$$

If the trader arriving at $t_4$ wants to buy the asset and has access to the *CDP*, he can choose depending on his personal valuation $\beta_{t_4}$ among submitting a regular market order, $\varphi_M(1, p_1^A)$, a dark pool order, $\varphi_D(+1, \widetilde{p}_{Mid,t})$, or an IOC order to the *CDP*, $\varphi_D(+1, p_{Mid,t}, p_1^A)$. This last strategy allows him to look for execution first on the *CDP* and then on the LOB, and provides the following payoff:

$$\begin{aligned}\pi^e_{t_4}[\varphi_D(+1, p_{Mid,t_4}, p_1^A)] &= \Pr_{+1,t_4}(p_{Mid,t_4}|\Omega_{t_4})(\beta_{t_4}v - p_{Mid,t_4}) \\ &+ [1 - \Pr_{+1,t_4}(p_{Mid,t_4}|\Omega_{t_4})](\beta_{t_4 v - p_1^A}), \quad (10)\end{aligned}$$

where $\Pr_{+1,t}(p_{Mid,t}|\Omega_t)$ is the probability that one share to buy will be executed in the *CDP* at *t*.

To summarize, the existence of the dark pool influences market participants' order submission decisions whether or not they themselves have access to the dark pool. Specifically, everyone uses the LOB to infer the state of the dark pool, and this affects the estimates of the execution

[11] Even though the *CDP* is dark, in this case a trader at $t_3$ can infer that there is a dark order standing in the pool. Specifically, if the book opened with $b_{t_2} = [10]$ at $t_2$ and there was no trade executed at $t_2$, and the book opens with [10] again at $t_3$, the trader will infer that the previous trader must have submitted a dark pool sell order, we label this case $b_{t_3} = [10h]$. The reason is that all traders know that "no trading" is not an equilibrium strategy at $t_2$, and all traders also know that a buyer would not use the dark pool when the book is $b_{t_2} = [10]$ as he would rather buy at a better price using a limit order.

probability of limit orders for everyone and the execution probability of dark orders for those with access to the dark pool.

More generally, in each trading round the fully rational risk-neutral trader with access (*WA*) to the *CDP* takes all these effects into account and chooses the optimal order submission strategy which maximizes his expected profits, conditional on his valuation of the asset, $\beta_t$, and his information set, $\Omega_t$, respectively:

$$\max_{\varphi} \pi_t^e[\varphi_M(1, p_i^z), \varphi_D(\pm 1, p_{Mid,t}, p_i^z), \varphi_D(\pm 1, \widetilde{p}_{Mid,t}), \varphi_L(1, p_1^z), \varphi(0) \mid \beta_t, \Omega_t]. \tag{11}$$

Traders with no access (*NA*) to the *CDP* still solve problem (7), however, they now condition their strategies not only on their own valuation relative to $v$ and on the state of the LOB but also on the inferred state of the *CDP*. The game is solved as before by backward induction starting from $t_4$ and assuming that $\alpha = 0.5$.[12]

## 4. What drives volume into the dark?

We solve both the benchmark model (*B*) and the model (*L&C*) with a continuous dark pool alongside a LOB numerically. Our model allows us to analyze if orders migrate from the lit market to the dark venue, and if they do, how lit market and consolidated execution rates are affected by the introduction of a dark pool. Our model also allows us to investigate which factors attract order flow away from the lit market and into the dark pool. To emphasize the dynamics of our model, we solve the model both at $t_1$ with four periods remaining and at $t_2$ when we condition on the different equilibrium states of the book at $t_2$. When we report results at $t_1$, these are averages across future periods ($t_1$, $t_2$, $t_3$, and $t_4$) and include all nodes of the tree. The results at $t_2$ instead report the average across future periods ($t_2$, $t_3$, and $t_4$) and nodes of the remaining tree, conditional on the two possible ask-side order book outcomes after $t_1$, $b_{t_2} = [00]$, and [10]. The model is symmetric, and we do not spell out the bid-side strategies to conserve space.

We define order migration (*OM*) as the average probability that in equilibrium an order migrates to the dark pool. The average is computed over a number of periods, *T*, of the game and over all of the equilibrium states of the LOB and of the dark pool:

$$OM = \frac{1}{T}\sum_t \alpha\ E_{\Omega_t}\left[\int_0^2 \varphi_{WA}^n \cdot f(\beta_t) d\beta_t\right], \tag{12}$$

where $\varphi_{WA}^n = \{\varphi_D(\pm 1, \widetilde{p}_{Mid,t}), \varphi_D(\pm 1, p_{Mid,t}, p_i^z)\}$.[13]

We define trade creation (*TC*) as the average difference between the sum of the fill rates on the LOB and the dark pool, and the fill rate in the benchmark model:

$$TC = \frac{1}{T}\sum_t (FR_t^{L\&C} - FR_t^B), \tag{13}$$

where

$$FR_t^{L\&C,B} = \sum_{a=WA,NA} \Pr(a)\ E_{\Omega_t}\left[\int_0^2 \varphi_a^n \cdot f(\beta_t) d\beta_t\right]. \tag{14}$$

The equilibrium strategies ($\varphi_a^n$) considered in Eq. (14) include all market orders for the *B* framework, and both market orders and executed dark pool orders for the *L&C* framework.

*Proposition 1. In equilibrium, the introduction of a dark pool that competes with a limit order book which both open empty on average produces order migration, and trade creation. Within this framework, compared to a limit order book that remains empty after the first period, a limit order book that has a resting order after the first period on average produces stronger order migration, and trade destruction. The effects of the introduction of a dark pool on order migration and trade creation are on average stronger when,*

- *there are more trading periods T;*
- *the support of the $\beta$ parameter is smaller so that the dispersion of traders' personal valuations around the common asset value is smaller;*
- *the parameter $\alpha$ is larger so that the proportion of traders with access to the CDP is larger;*
- *the relative tick size $\tau/v$ is larger.*

Fig. 4, Panel A, reports results on *OM* for the model evaluated at $t_1$ with an empty book and four periods remaining, and at $t_2$, respectively, with an empty book, and one share on the first level of the ask side, and three periods remaining. The results evaluated both at $t_1$ and at $t_2$ show that when traders who are active on a LOB are offered the additional option to trade in the dark at a better price but with execution uncertainty, orders migrate to the dark pool.

Results evaluated at $t_2$ further show that migration is more intense when the book becomes more liquid with one share posted at the top of the ask side of the LOB and competition for the provision of liquidity increases: as liquidity increases, some traders find dark pool orders more attractive than limit and market orders (Table 2). The reason is that when competition for the provision of liquidity increases, the queue becomes longer due to time priority and there is less room on the LOB; so, dark orders become attractive for aggressive liquidity suppliers. At the same time, as the liquidity of the book increases and traders start using the dark pool, the execution uncertainty of the dark pool decreases and dark orders become more attractive for liquidity demanders. Because both the LOB and the dark pool open empty, traders start moving to the dark pool only at $t_2$ when some depth builds up in the book; it is important to notice, though, that traders go dark even if liquidity builds up only on one side of the LOB.[14] Because the horizon is short, the incentive to go dark is limited in our model. Nevertheless, Table 2 shows that by

[12] Results are robust for different values of the percentage $\alpha$ of *WA* traders, for instance, $\alpha \in [0.4, 0.8]$.

[13] When we solve the four-period model starting at $t_1$, we average over four periods and therefore, $T = 4$. When instead we solve the three-period models by conditioning on the different equilibrium states of the opening book at $t_2$, $T = 3$.

[14] More precisely, investors choose to trade in the dark pool at $t_2$ only when there is one share at the top of the ask side to the book. It should also be noticed that in this framework in which traders can only use one-share orders, we cannot distinguish the effects of a smaller spread from that of a deeper opening book.

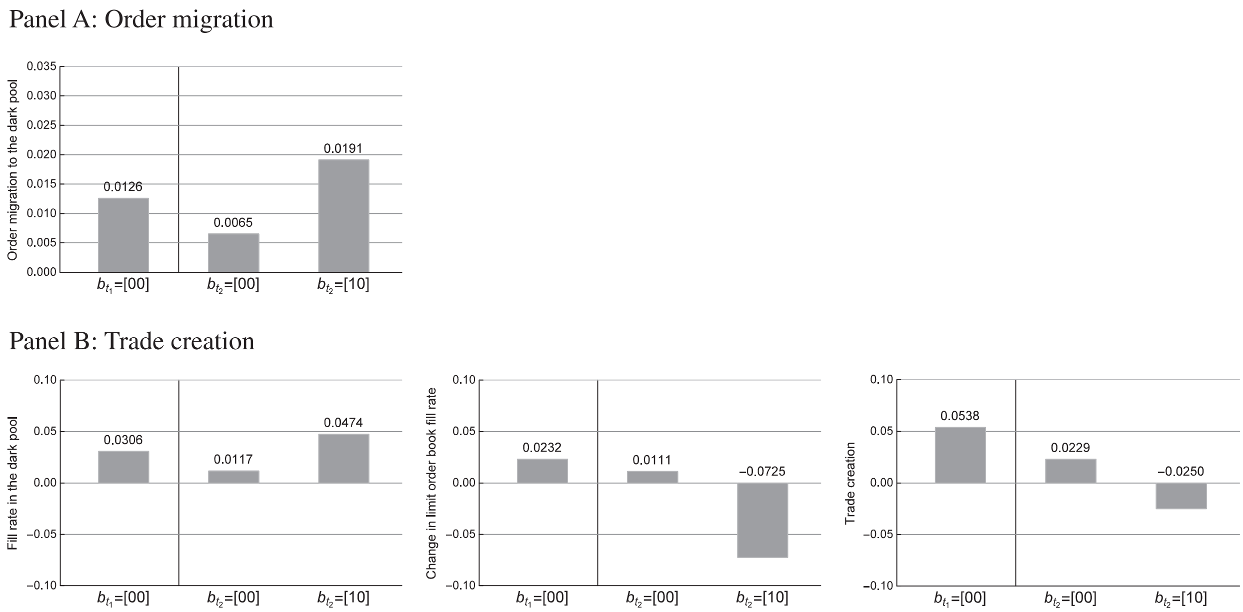


**Fig. 4.** Order migration and trade creation. This figure presents results for the *L&C* framework that combines a limit order book and a continuous dark pool. We report in Panel A order migration which is the average probability that an order migrates to the dark pool, and in Panel B trade creation that is the sum of two components. The first one is the average fill rate in the dark pool. The second one is the difference between the average limit order book fill rate in the *L&C* and the average limit order book fill rate in the benchmark. We report results for both the four-period model that opens with an empty book at $t_1$, $b_{t_1} = [00]$ , and for the three-period models that open at $t_2$ according to the two equilibrium opening books (at $t_2$) from the four-period model, and that differ for the number of shares at the first level of the ask side of the book: $b_{t_2} = [00]$, and $b_{t_2} = [10]$. Results are computed assuming a tick size $\tau = 0.08$ and a probability that a trader with dark pool access arrives $\alpha = 0.5$.

**Table 2**
Order submission probabilities at $t_1$, $t_2$, and $t_3$.

This table reports the submission probabilities of traders for the orders listed in column 1 for the benchmark framework (*B*), and for the model with a limit order book and a continuous dark pool (*L&C*). Notice that in the *L&C* model, we differentiate between traders with (*WA* ) or with no access (*NA*) to the dark pool. We consider the $t_1$ equilibrium strategies from the four-period model that opens with an empty book at $t_1$, $b_{t_1} = [00]$, and the $t_2$ equilibrium strategies from the three-period models that open at $t_2$ according to the two equilibrium opening books (at $t_2$) from the four-period model, and that differ for the number of shares at the first level of the ask side of the book: $b_{t_2} = [00]$, and $b_{t_2} = [10]$. We also present equilibrium books at $t_3$ that derive from those two opening books at $t_2$. Note that $b_{t_3} = [10h]$ is the state of the book that opens with both a limit sell order at the best ask price and a dark sell order on the dark pool. Results are computed assuming that the tick size is equal to $\tau = 0.08$ and the probability that a trader with dark pool access arrives is $\alpha = 0.5$.

| *Panel A: $t = \{t_1, t_2\}$* | $b_{t_1} = [00]$ | | | $b_{t_2} = [00]$ | | | $b_{t_2} = [10]$ | | |
|---|---|---|---|---|---|---|---|---|---|
| Trading strategy | *B* | *L&C* | | *B* | *L&C* | | *B* | *L&C* | |
| | | *NA* | *WA* | | *NA* | *WA* | | *NA* | *WA* |
| $\varphi_M(1, p_2^B)$ | 0.0735 | 0.0934 | 0.0934 | 0.2438 | 0.2498 | 0.2498 | 0.4184 | 0.4184 | 0.4049 |
| $\varphi_D(-1, \widetilde{p}_{Mid})$ | | | | | | | | | 0.0447 |
| $\varphi_L(1, p_1^A)$ | 0.4265 | 0.4066 | 0.4066 | 0.2562 | 0.2502 | 0.2502 | 0.0706 | 0.0707 | 0.0395 |
| $\varphi_L(1, p_1^B)$ | 0.4265 | 0.4066 | 0.4066 | 0.2562 | 0.2502 | 0.2502 | 0.1389 | 0.1362 | 0.1362 |
| $\varphi_M(1, p_1^A)$ | | | | | | | 0.3721 | 0.3747 | 0.3747 |
| $\varphi_M(1, p_2^A)$ | 0.0735 | 0.0934 | 0.0934 | 0.2438 | 0.2498 | 0.2498 | | | |

| *Panel B: $t = t_3$* | $b_{t_3} = [00]$ | | | $b_{t_3} = [10]\&[20]$ | | | $b_{t_3} = [11]$ | | | $b_{t_3} = [10h]$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Trading strategy | *B* | *L&C* | | *B* | *L&C* | | *B* | *L&C* | | *B* | *L&C* | |
| | | *NA* | *WA* | | *NA* | *WA* | | *NA* | *WA* | | *NA* | *WA* |
| $\varphi_M(1, p_2^B)$ | 0.3662 | 0.3662 | 0.3662 | 0.4400 | 0.4400 | 0.4259 | | | | 0.4400 | 0.4400 | 0.4400 |
| $\varphi_M(1, p_1^B)$ | | | | | | | 0.4800 | 0.4800 | 0.4733 | | | |
| $\varphi_D(-1, \widetilde{p}_{Mid})$ | | | | | | 0.0541 | | | 0.0267 | | | |
| $\varphi_L(1, p_1^A)$ | 0.1338 | 0.1338 | 0.1338 | | | | | | | | | |
| $\varphi(0)$ | | | | 0.0400 | 0.0400 | | 0.0400 | 0.0400 | | 0.0400 | 0.0400 | 0.0400 |
| $\varphi_L(1, p_1^B)$ | 0.1338 | 0.1338 | 0.1338 | 0.0769 | 0.0769 | 0.0769 | | | | 0.0769 | 0.0769 | |
| $\varphi_D(+1, \widetilde{p}_{Mid})$ | | | | | | | | | 0.0267 | | | 0.5200 |
| $\varphi_M(1, p_1^A)$ | | | | 0.4431 | 0.4431 | 0.4431 | 0.4800 | 0.4800 | 0.4733 | 0.4431 | 0.4431 | |
| $\varphi_M(1, p_2^A)$ | 0.3662 | 0.3662 | 0.3662 | | | | | | | | | |

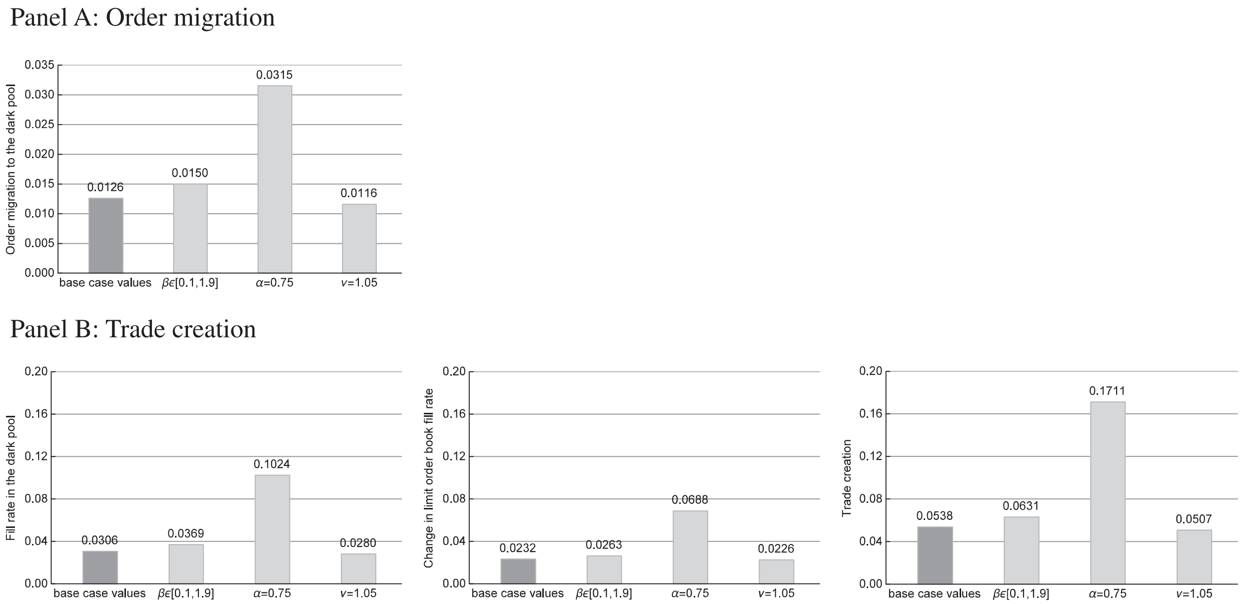


**Fig. 5.** Order migration and trade creation – comparative statics. This figure presents results for the *L&C* framework that combines a limit order book and a continuous dark pool. We report in Panel A order migration which is the average probability that an order migrates to the dark pool, and in Panel B trade creation that is the sum of two components: the first one is the average fill rate in the dark pool; and the second one is the difference between the average limit order book fill rate in the *L&C* and the average limit order book fill rate in the benchmark. Results are reported first (bar labeled "base case values" ) assuming that $\beta \in [0, 2]$, $\alpha = 0.5$, and $\nu = 1$. Then, they are computed assuming that just one parameter value changes, so for $\beta \in [0.1, 1.9]$, $\alpha = 0.75$ and $\nu = 1.05$, respectively.

moving from the benchmark to the market with a dark pool in the $b_{t_2} = [10]$ book, both limit orders and market orders migrate to the dark pool.[15]

Having discussed the migration of orders away from the lit market into the dark pool, we now consider the model's results on trade creation. *TC* measures the overall increase in the execution rate following the introduction of the dark pool and hence, it is the sum of the orders executed in the dark and on the LOB in excess of the *B* framework. First of all we study executions in the dark pool and notice that consistently with the pattern of *OM*, the dark orders' fill rate increases with the liquidity of the book at $t_2$. When instead we consider the total fill rate (*TC*) that also includes the net fill rate of the LOB, we find that *TC* decreases with the liquidity of the book (Fig. 4, Panel B). This result is driven by the different effect that the migration of limit and market orders has on executions. When limit orders migrate from the LOB into the dark, executions overall increase, whereas when market orders migrate to the dark pool executions decrease as the execution probability of dark orders is larger than that of limit orders and smaller than that of market orders. When the book becomes deeper, traders use more market than limit orders and the second effect is stronger, so that total executions relative to the benchmark model decrease. This effect is evident by looking at the fill rate of the LOB relative to the benchmark that becomes substantially negative in the $b_{t_2} = [10]$ book and counterbalances the positive increase of the fill rate (*FR*) in the dark pool.[16]

Notice in Fig. 4 that *OM* and *TC* for the four-period model are stronger than that of the three-period one, conditional on the same initial state of the book, $b_t = [00]$. When there are more trading periods and dark orders have more chances to execute, the incentives to use the dark pool are higher, and therefore the four-period model has more *OM* than the model with only three periods. These are general properties of the model which hold for all our results, and suggest that when we increase the number of periods, all the effects are magnified.

Our model also shows that all the effects of the introduction of a dark pool on *OM, FR*, and *TC* are stronger when we solve the model for smaller support for the $\beta$ parameter, i.e., [0.1, 1.9] (Fig. 5). Such a smaller dispersion of trader valuations around the common value of the asset implies that traders perceive smaller gains from trade and therefore their comparative valuation of the price improvement offered by limit orders increases, with the result that they become more inclined to post limit orders than market orders. In equilibrium, this also means that traders use the dark pool more intensively so that the effects on *OM,*

[15] Table 2 also shows that in general, by moving towards the end of the trading day the proportion of market vs. limit orders increases. This prediction about the time-patterns in flow of different types of order agrees with the standard empirical results on intraday patterns of order flows (Bae, Jang, and Park, 2003).

[16] Note that *TC* reflects the impact that the introduction of the dark pool has on the LOB at $t_1$. In fact, even though orders do not migrate to the dark in that very first period as both the book and the dark pool open empty, traders switch from limit to market orders anticipating that in the future periods liquidity will build up in the dark pool, and with the dark pool attracting market orders, the execution probability of limit orders posted on the LOB will decrease.

*FR* , and *TC* are magnified. A similar effect arises when, all else equal, we solve the model for a larger proportion of traders having access to the *CDP* ($\alpha = 0.75$). In this case, the overall effect on *OM* and *TC* is stronger as we assume that more traders have access to the dark pool. Finally, our numerical simulations show that when we solve the model for a higher asset value ($v = 1.05$) resulting in a smaller relative tick size, traders' incentive to post limit orders diminishes (as limit orders are less profitable) so that when the dark market is introduced fewer limit orders switch to the dark pool. Furthermore, the higher the asset value, the smaller the relative inside spread, and the less expensive market orders are compared to dark pool orders. Hence, all the effects on *OM* and *TC* previously described are reduced with a smaller relative tick size.

In Degryse, Van Achter, and Wuyts (2009), the introduction of a CN alongside a DM leads to the creation of new orders, and it generates *OM* only as a secondary effect. This is due to the fact that in a DM traders cannot post limit orders, and the introduction of a CN has the main effect of attracting investors willing to supply liquidity who otherwise would refrain from trading. By contrast, in our benchmark LOB model traders can post limit orders and therefore when competition for the provision of liquidity becomes strong the introduction of a dark pool attracts limit orders away from the LOB into the dark pool. The effect of the creation of new orders that DVW obtain starts taking place only when depth at the top of the book induces traders in the *B* framework to refrain from trading rather than posting a limit order behind the queue with zero execution probability (e.g., $b_{t_3} = [10]$, and [20]). When this happens the market resembles a DM because limit orders have zero execution probability so that when a dark pool becomes available, as in the *L&C* framework, traders opt for dark pool orders rather than refraining from trading.

However, our model shares with Degryse, Van Achter, and Wuyts (2009), as well as with Hendershott and Mendelson (2000), a feedback effect generated by traders' perception of dark pool liquidity which influences traders' estimate of the execution probability of dark pool orders and hence their use. When traders perceive that liquidity is building in the dark pool, they update their estimate of the dark pool depth and assign a higher probability of execution to dark orders. The result is that they are more likely to opt for dark trading. For instance, Table 2 shows that when the book opens $b_{t_3} = [10h]$ so that traders infer that there is a dark sell order standing in the *CDP*, if they have access to the dark pool they post dark buy orders, $\varphi_D(+1, \widetilde{p}_{Mid,t})$, rather than limit or market buy orders, $\varphi_L(1, p_1^B)$ or $\varphi_M(1, p_2^A)$. This positive liquidity-externality effect intensifies at $t_4$ when traders perceive that dark volume is growing. This prediction is consistent with the empirical results by Buti, Rindi, and Werner (2011) that show the existence of a positive autocorrelation between contemporaneous and lagged dark activity.

## 5. Who benefits from a dark pool?

Even though dark trading has existed for several decades, it is only recently that dark pool volume relative to consolidated equity volume has increased to more than 14% in the U.S. and almost 5% in Europe (Fig. 1). It is therefore understandable that regulators are concerned about the effects on market quality and traders' welfare of the widespread use of dark pools. We first investigate how the introduction of a dark pool affects the quality of the LOB market. Because changes in market quality influence agents' gains from trade, we also study how welfare of market participants changes after the introduction of a dark pool.

### 5.1. Market quality

To evaluate the effect of dark trading on the quality of the LOB, we consider two standard measures of market quality, i.e., inside spread (*S*) and market depth (*D*). We compute expected spread and depth in period $t_{i+1}$ by weighing the realized values in the equilibrium states of the book by the corresponding order submission probabilities in the previous periods:

$$y_{t_{i+1}} = \sum_{a=WA,NA} \Pr(a)\ E_{\Omega_{t_i}}\left[\int_0^2 y_{t_{i+1}} \cdot \varphi_a^n \cdot f(\beta_{t_i}) d\beta_{t_i}\right], \quad (15)$$

where $y_{t_{i+1}} = \left\{S_{t_{i+1}}, D_{t_{i+1}}\right\}$. We then compute the percentage difference between these indicators of market quality for the *L&C* and the *B* frameworks, and average them across periods:[17]

$$\Delta y = \frac{1}{T}\sum_t (y_t^{L\&C} - y_t^B)/y_t^B \quad , \quad (16)$$

where $y = \{S, D\}$. The following proposition summarizes our results.

*Proposition 2. In equilibrium, the introduction of a dark pool that competes with a limit order book which both open empty on average produces wider inside book spread and lower inside order book depth. Within this framework, compared to a limit order book that remains empty after the first period, a limit order book that has a resting order after the first period on average produces:*

- *a weaker negative effect on inside book spread, and*
- *a stronger negative effect on inside order book depth.*

*The effects of the introduction of a dark pool on spread and order book depth are on average stronger with a larger T, a smaller support for $\beta$, a greater $\alpha$, and a larger relative tick size, $\tau/v$.*

Our results in Fig. 6, Panel A, show that the introduction of a *CDP* that competes with a LOB has a negative effect on the liquidity of an empty LOB as both the inside spread and the depth at the best bid and offer worsen. However, while the negative effect on spread is dampened when the liquidity of the book builds, the negative effect on depth is amplified. We explain the dynamic of spread and depth by considering once again how traders react to the introduction of the dark pool, and therefore how

[17] As in the initial period, spread and depth are exogenous, $T = 3$ for the four-period model, and $T = 2$ for the three-period models.

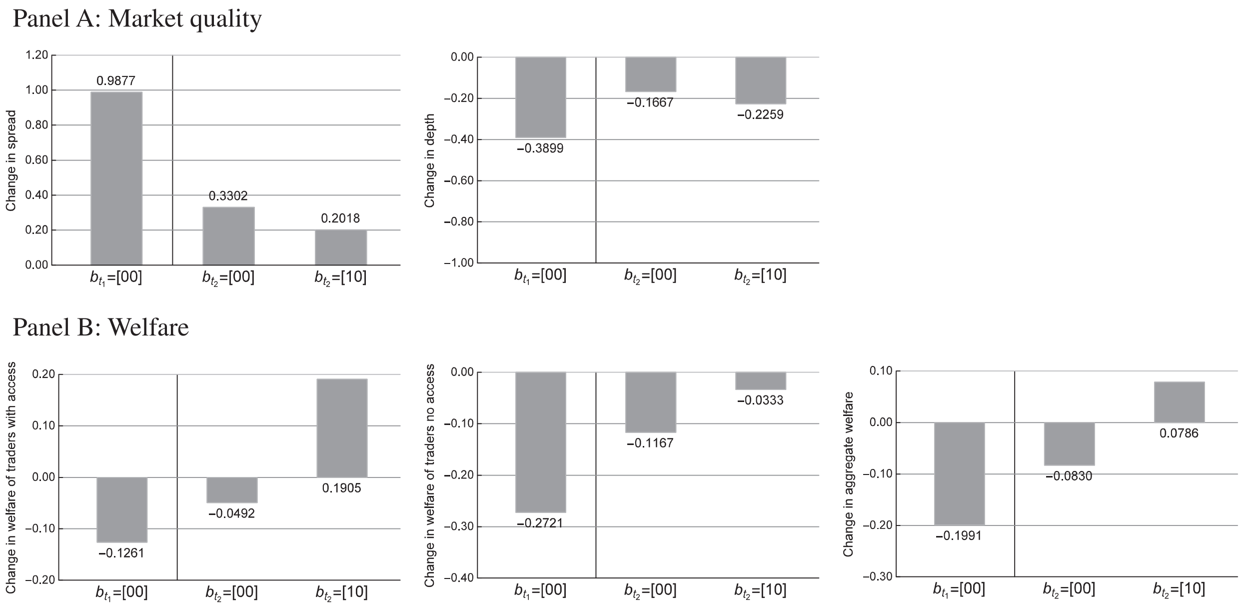


**Fig. 6.** Market quality and welfare. This figure presents results for spread, depth, and welfare for the *L&C* framework that combines a limit order book and a continuous dark pool (*CDP*). All measures are computed as the average percentage difference between their value for the *L&C* framework and the benchmark framework. As spread and depth are exogenous in the initial period we do not include it in the average, while for welfare we consider three measures: the welfare of a trader with no access to the *CDP*, the welfare of a trader with access to the *CDP*, and aggregate welfare. We report results for both the four-period model that opens with an empty book at $t_1$, $b_{t_1} = [00]$, and for the three-period models that open at $t_2$ according to the two equilibrium opening books (at $t_2$) from the four-period model, and that differ for the number of shares at the first level of the ask side of the book: $b_{t_2} = [00]$, and $b_{t_2} = [10]$. Results are computed assuming a tick size $\tau = 0.08$ and a probability that a trader with dark pool access arrives $\alpha = 0.5$.

the resulting change in traders' order submission strategies, which are reported in Table 2, affects the quality of the LOB.

At the beginning of the trading game, $t_1$, both the book and the dark pool open empty and the introduction of a *CDP* induces traders to switch from limit to market orders. Traders anticipate that on average in future periods (not yet at $t_2$ but rather at $t_3$ and at $t_4$ when orders move heavily to the dark pool) dark pool orders will attract liquidity demand from the LOB, and therefore the execution probability of their limit orders decreases.

To explain this result we compare the execution probability of a limit sell order posted at $t_1$ under the *B* and the *L&C* frameworks, respectively. If a trader posts a limit sell order at $t_1$, $\varphi_L(1, p_1^A)$, the book will open at $t_2$ with one share on the best ask, $b_{t_2} = [10]$, and the execution probability of this limit order will depend on the probability that traders (in $t_2$ or in subsequent periods) will submit a market buy order at $p_1^A$, $\varphi_M(1, p_1^A)$. Table 2 shows that at $t_2$ this probability is actually slightly higher under the *L&C* framework (0.3747) than under the *B* framework (0.3721). The reason is that at the opening of $t_2$ the LOB is still empty on the bid side, so the only effect of the dark option on the buyers' strategies is – as at $t_1$ – to induce them to switch from limit to market buy orders. This actually raises the execution probability of the resting limit sell order submitted at $t_1$.

In the subsequent periods, however, the execution probability of the initial limit sell order (as well as of the limit orders posted at $t_2$) decreases substantially. As mentioned above, the execution probability of the limit sell order submitted at $t_1$ is the probability that in the subsequent periods an incoming buyer submits a market buy order at $p_1^A$. If the order has not been executed yet, there are four possible opening books at $t_3$ with at least one share on $p_1^A$: $b_{t_3} = [10]$, [20], [11], and [10*h*]. When the book opens at $t_3$ with one or two shares on the ask side, $b_{t_3} = [10]$ or [20], the probability of $\varphi_M(1, p_1^A)$ is the same for the *B* and the *L&C* frameworks, so in this case the execution probability of the initial limit order does not differ in the two frameworks.[18] When instead the book opens with one share on both sides of the LOB, $b_{t_3} = [11]$, or a one-share order on the ask side of the LOB and a one-share order to sell in the dark pool, $b_{t_3} = [10h]$, the probability of $\varphi_M(1, p_1^A)$ decreases for the *WA* traders from 0.48 to 0.4733 in the [11] book and from 0.4431 to zero in the [10*h*] book, respectively. The reason is that when there are resting orders both at the bid and the offer of the LOB, $b_{t_3} = [11]$, an incoming *WA* buyer moves from market to dark pool orders (and also never chooses not to participate).[19] Similarly, when there is a resting sell order that was submitted at $t_1$ and there was no trade in $t_2$ so traders can infer that

[18] Compared to the same book at period $t_2$, at $t_3$ buyers know that with only one period left no trader will post a dark buy order in front of their limit buy order, thus intercepting incoming market sell orders. Therefore, the execution probability of a limit buy order in the *L&C* framework is the same as in the *B* framework and traders do not switch, as in period $t_2$, from limit to market orders.

[19] Recall that the execution probability of limit orders posted behind the queue at $t_3$ is zero and therefore a buyer never opts for a limit buy order at $t_3$ when the book opens with already one share on the bid side, $b_{t_3} = [11]$.

there is a sell order in the *CDP*, $b_{t_3} = [10h]$, the incoming *WA* buyer switches entirely to the dark pool. A similar effect is observed at $t_4$. Therefore, the reduction of the execution probability of the initial limit sell order, $\varphi_M(1, p_1^A)$, in periods $t_3$ and $t_4$ outweighs the slight increase in the execution probability we observe at $t_2$.

In terms of market quality, the expected reduced execution probability of limit orders that induces traders in early periods to switch from limit to market orders reduces liquidity supply and increases liquidity demand thus explaining why spread and depth initially deteriorate. In the subsequent periods the effects of the introduction of a dark pool on spread and depth differ depending on the state of the book. To explain this result, we consider the effect of introducing a *CDP* to a LOB market on spread and depth based on books with different liquidity at $t_2$ and three periods remaining. Fig. 6, Panel A, shows that the negative effect on spread dampens as the liquidity of the book increases, but at the same time the negative effect on depth is exacerbated. The switch from market and from limit orders to dark pool orders, which takes place when the book is deeper, explains this result. When the book opens empty, the introduction of a dark pool induces limit orders to switch to market orders so that liquidity is consumed and both spread and depth deteriorate. When instead the book opens deeper, both market and limit orders move to the dark pool and the migration of each order type has an opposite effect on market quality. Specifically, the migration of market orders to the dark pool helps preserve LOB liquidity and the migration of limit orders reduces LOB liquidity. Therefore, the negative effect on spread is alleviated by the migration of market orders, whereas the effect on depth becomes more intense as overall the migration of limit orders is stronger.[20]

Fig. 6 , Panel A, also shows that the effect on spread and depth for the four-period model is stronger than that of the three-period one, conditional on the same initial state of the book, $b_t = [00]$. At $t_2$ the switch from limit to market orders induced by future expectation of smaller limit order execution probability is weaker than it was at $t_1$ as there are only two rather than three periods before the end of the trading game. Therefore, average spread and depth deteriorate less conditional on $t_2$ than on $t_1$ when the book opens empty in each period, $b_t = [00]$. We can conclude that when we increase the number of periods, the effects on market quality are magnified.

We compare the effect on spread and depth following the introduction of a dark pool for different values of $\beta$, $\alpha$, and $v$ in Fig. 7, Panel A. When we solve the model either for a smaller $\beta$ support or for a larger $\alpha$, we find that all the effects of the introduction of a *CDP* on spread and depth are magnified. When we assume that the LOB is populated by traders who are more willing to submit limit orders, or by a larger proportion of traders with dark pool access, traders move to the dark pool more intensively and all the effects at work become stronger. All the effects of the introduction of a *CDP* on spread and depth diminish in intensity when the asset value is higher and therefore the relative tick size is smaller. This is due to the fact that the narrower relative inside spread reduces the benefit from the dark pool execution at the midquote and, therefore, in general it reduces order migration to the dark.

To conclude, our model shows that the effects of the dark pool on spread and depth depend on which type of orders—market or limit—are diverted from the LOB and therefore depend in a very subtle way on market conditions.

### *5.2. Welfare analysis*

In our model traders with and without access to the *CDP* have a private motive to trade and consequently, we can fully characterize their welfare. In light of our results on *OM* and *TC*, and on market quality, we can assess to what extent dark pools enable these traders to realize welfare gains, and therefore address the policy question of whether in a competitive setting the dark trading option enhances their welfare.

Following Goettler, Parlour, and Rajan (2005) and Degryse, Van Achter, and Wuyts (2009), we measure welfare for a trader with or without access to the *CDP* as:

$$W_{a,t} = \int_0^2 \pi_t^e(\varphi_a^n)\,d\beta_t \ . \tag{17}$$

Aggregate welfare of traders with and without access at period $t$ is equal to the sum of the gains from trade for both trader's types:

$$W_t = \sum_{a=WA,NA} \Pr(a) W_{a,t} \ . \tag{18}$$

We then compute the percentage difference between the *L&C* and the *B* frameworks for each trader type and in aggregate for each period, and average them out across periods. The following proposition summarizes our results.

*Proposition 3. In equilibrium, the introduction of a dark pool that competes with a limit order book on average produces lower welfare for both traders with and without access to the dark venue. Within this framework, compared to a limit order book that remains empty after the first period, a limit order book that has a resting order after the first period on average produces:*

- *a smaller reduction in welfare for traders with no access to the dark pool,*
- *an increase in welfare for traders accessing the dark pool, and*
- *an increase in the aggregate welfare of traders with and without access to the dark pool.*

*The effects of the introduction of a dark pool on spread and order book depth are on average stronger with a larger T, a smaller support for $\beta$, a greater $\alpha$, and a larger relative tick size $\tau/v$.*

*Welfare of traders with no access (NA traders).* In our framework changes in traders' welfare are mainly driven by the variation in spread: because agents only trade one unit, depth only marginally affects the execution quality of their market orders. This is particularly true for traders

[20] Note that the migration of limit orders has a limited impact on spread when the book is already deep.

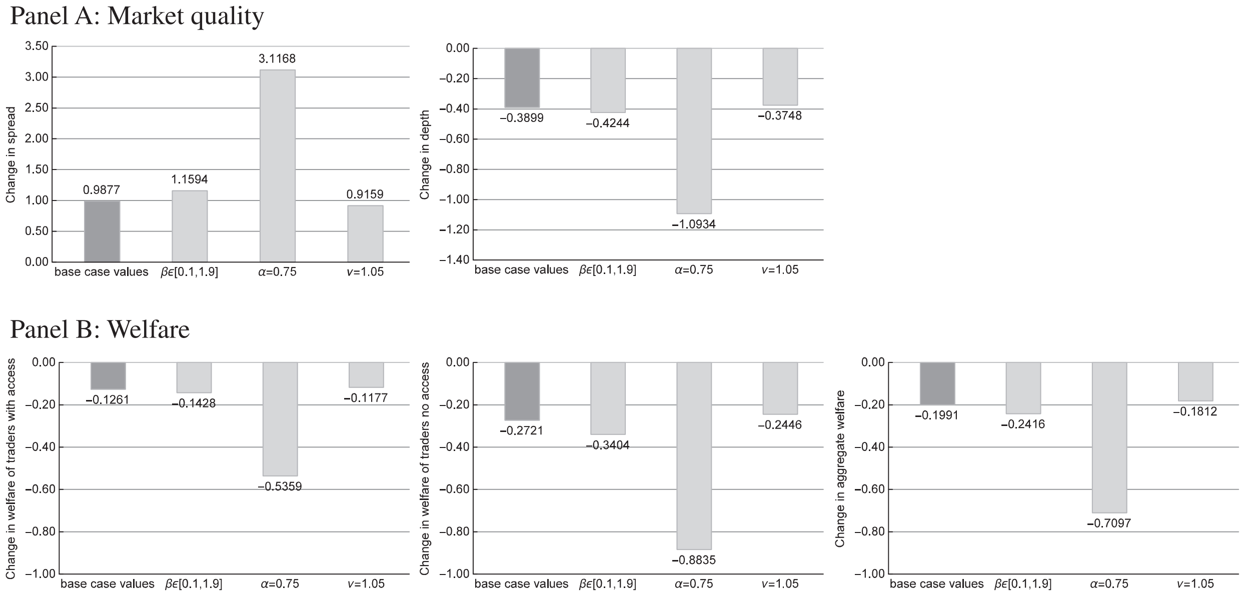


**Fig. 7.** Market quality and welfare – comparative statics. This figure presents results for spread, depth, and welfare for the *L&C* framework that combines a limit order book and a continuous dark pool (*CDP* ). All measures are computed as the average percentage difference between their value for the *L&C* framework and the benchmark framework. As spread and depth are exogenous in the initial period we do not include it in the average, while for welfare we consider three measures: the welfare of a trader with no access to the *CDP*, the welfare of a trader with access to the *CDP*, and aggregate welfare. Results are reported first (bar labeled "base case values" ) assuming that $\beta \in [0, 2]$, $\alpha = 0.5$, and $\nu = 1$. Then, they are computed assuming that just one parameter value changes, so for $\beta \in [0.1, 1.9]$, $\alpha = 0.75$ and $\nu = 1.05$, respectively.

who do not access the dark pool because they cannot enjoy the benefit of midquote executions. Moreover, as discussed in the previous section on market quality, when the dark pool is introduced, if a change takes place in the trading strategies of the *NA* traders, it is a switch from limit to market orders which magnifies once more the effect of the introduction of the dark pool on spread (Table 2). Because spread deteriorates, *NA* traders' welfare declines (Fig. 6, Panel B). Even market conditions affect *NA* traders' welfare exactly in the same way as they affect the spread: because the negative effect on spread is decreasing in the liquidity of the LOB, due to the fact that in deep books traders' strategies switch from market to dark pool orders rather than from limit to market orders as per empty books, so do traders' losses, and the result is that the welfare of *NA* traders deteriorates less when the book opens with some depth at the inside spread.

*Welfare of traders with access (WA traders). WA* traders do not benefit from the introduction of a dark pool when both the book and the dark pool open empty (Fig. 6, Panel B). As discussed for *NA* traders, with empty LOB and dark pool also *WA* traders switch from limit to market orders, anticipating the future lower limit order execution probability. This switch deteriorates both spread and depth and hence has a negative effect on welfare. However, as the book becomes deeper and at the same time some liquidity builds up in the dark pool, *WA* traders start using the dark pool intensively and benefit from dark trading. Hence, once order book depth increases from zero to one and *WA* traders can take advantage of the cheap midquote executions, the welfare gains become substantial so that the introduction of the dark pool increases their welfare.

*Aggregate welfare of NA and WA traders.* As we noted above, when both the book and the dark pool open empty, welfare decreases overall for both *NA* traders and *WA* traders, and the introduction of a dark pool deteriorates their aggregate welfare. However, because the negative effect on the spread of the introduction of a dark pool is decreasing in the liquidity of the LOB, so do traders' losses and in general we observe that following the pattern of spread, aggregate welfare deteriorates when the book opens empty but improves conditional on deeper books. The reason being that when liquidity increases, traders use the dark pool intensively and therefore aggregate welfare increases. This means that as spread deteriorates less following the introduction of a dark pool, the losses of the *NA* traders become smaller, and are more than compensated by the increasing welfare gains of the *WA* traders who enjoy trading in the *CDP*.

Interestingly, our results when the model is evaluated at $t_2$ for the three remaining periods show that the welfare effects improve when the liquidity of the book builds up. The more liquid the LOB is, the more the market resembles a DM in which dealers provide infinite liquidity on both sides of the book. Therefore, our result is reminiscent of Degryse, Van Achter, and Wuyts (2009) who find that when a dark pool is introduced alongside a DM welfare increases because the dark pool attracts some traders who would otherwise not trade. In this regard, Table 2 shows that when at $t_3$ the book opens with some liquidity at the inside spread so that traders refrain from trading rather than posting limit orders, the introduction of the *CDP* attracts this latent demand of liquidity to the dark pool thus increasing aggregate welfare.

Consistent with previous findings that all the effects are magnified when either $T$ or $\alpha$ is larger, or the $\beta$ support or $v$ is smaller, we also find that all the welfare effects are stronger when there are more trading periods, when we assume that the proportion of traders with dark pool access is larger, when the dispersion of traders' valuations around the current asset value is smaller, or when the relative tick size is larger and traders' incentive to use the dark pool increases (Fig. 7, Panel B).

## 6. Extensions

As a robustness check, we extend the original model to allow for traders that come to the market with different order sizes. Specifically, the traders with access to the dark pool may trade up to two shares, and those without access may still trade up to one share. As before, traders with access may use IOC orders, but in addition they may split their two shares across venues and also combine a market and a limit order. We find that even in this richer setting, the results from the original model on the effects of the introduction of a dark pool on order flows, market quality, and welfare hold.[21]

To discuss how the design of the dark market affects the dynamics of order flow migration, we further extend this richer setting by considering a Periodic Dark Pool (*PDP*) that crosses orders periodically and resembles the Independent/Agency dark pools. The core difference with the *CDP* is that with a *PDP* traders have to wait until the next cross to see if their orders are executed in the dark venue. The main finding from this extension of the model is summarized in the following proposition.

*Proposition 4. The effects induced by the introduction of a dark pool to a limit order book market are weaker when the dark pool is periodic rather than continuous.*

Dark pool activity is higher for a *CDP* than for a *PDP*, and the effects of dark pool activity on LOB and consolidated fill rates are stronger for the *CDP*. With a *CDP* more orders migrate to the dark both because executions take place at each trading round and because traders use IOC orders that allow them to access simultaneously the LOB and the dark pool. An order submitted to the dark pool has more chances of getting executed under the continuous protocol, which encourages dark order submissions. This may explain the popularity of the mostly continuous Bank/Broker pools relative to the mostly periodic Independent/Agency pools in Fig. 1.

## 7. Empirical implications

In this section, we translate our theoretical propositions into empirically testable hypotheses. We also discuss the empirical findings in the literature to date that speak to our hypotheses.

Our model studies the introduction of a dark pool to a LOB market. In markets today, there are many dark pools and also often several competing order books trading the same stocks. To bring the theoretical hypotheses to the data, we therefore reinterpret the model's introduction of a dark pool as an increase in dark pool activity. This is of course consistent with the equilibrium outcome of the model.

A key distinguishing feature of our model is that we derive predictions that relate dark trading to both the LOB depth and spread for a given stock. However, stocks have characteristics that generate varying LOB depth and spread even absent any influence of dark trading. To capture this variation we conduct comparative statics in the original model with respect to three exogenous variables: the dispersion of $\beta$, the magnitude of $\alpha$, and the asset value $v$ to capture the effect of changes in the relative tick size $\tau/v$. We also compare the model solved over four periods to the model solved over three periods also starting with the same state of the book to study the effect of the time horizon. We discuss empirically observable proxies for these exogenous variables below.

In the model, the dispersion of $\beta$ reflects the dispersion of trader valuations around the fundamental value of the asset and dictates whether a trader wants to buy or sell as well as her willingness to supply liquidity. The more (less) disperse the valuations are, the more (less) eager are buyers and sellers to trade and hence the more likely they are to submit market (limit) orders. As a result, stocks where traders have dispersed valuations generate order books characterized by low depth and wide spreads and stocks with less dispersed valuations generate order books with high depth and narrow spreads in our framework.

To map this variation into stock characteristics, a researcher can either rely on LOB depth and spread directly, or use variables such as the dispersion of analyst forecasts, degree of analyst following, and degree of fundamental (cash flow) uncertainty. Similarly, natural proxies for variation in $\alpha$ across stocks are, for example, measures of institutional ownership as well as the number of active dark pools. Cross-sectional variation in the asset value, or changes in tick size regimes in the time-series, can be used as proxies for the model's relative tick size. Finally, the horizon maps directly into the number of traders that come to the market, and can therefore be reinterpreted as a measure of the trader interest in a stock and proxies such as share volume, market capitalization, and measures of investor attention for a particular stock could be used to capture this variation.

### 7.1. Factors driving dark pool activity in the cross-section

*Prediction 1: Dark pool activity is increasing in LOB depth and decreasing in LOB spread (decreasing in the dispersion of trader valuations), and is increasing in the proportion of traders with access to dark pools, the relative tick size, and trader interest in a stock (the trading frequency/volume).*

Consistent with this prediction, Buti, Rindi, and Werner (2011) find that dark pool activity is higher for stocks with

[21] We refer to the Online Appendix for a more detailed analysis of the frameworks discussed in this section and for the proof of Proposition 4.

higher LOB depth and narrower LOB spread based on a self-reported sample of daily dark pool activity data for 11 dark pools that covers a large cross-section of U.S. securities in 2009. The authors also find that measures that characterize stocks with high depth and narrow spreads, such as higher market capitalization, higher volume, and lower volatility, are positively related to dark pool activity. Similarly, Ready (2013) studies monthly dark pool volume stock-by-stock for two dark pools (Liquidnet and ITG POSIT) from June 2005 to September 2007, and he finds that dark pools execute most of their volume in liquid stocks which he defines as stocks with low spreads and high share volume.

To test whether the proportion of traders with dark pool access in the cross-section of stocks affects dark pool activity, an instrument such as the degree of institutional ownership is needed. This prediction can also be interpreted over time, and it is certainly the case that as dark pool access has increased over time to include not just institutions but also retail orders and High Frequency Traders, we have seen more dark pool activity.

Our model also suggests that the relative tick size affects dark pool trading. Because in the U.S. the tick size is one penny for all stocks priced above $1, this empirical prediction can be tested by considering changes in the price of the stock which affect the relative tick size. Indeed, O'Hara, Saar, and Zhong (2014) use order-level data from the NYSE to study how the relative tick size affects liquidity. While they do not have access to information on external dark pool trading, they do study hidden orders within the NYSE's trading systems. Consistent with our prediction, they find that stocks with larger relative tick size are more likely to have hidden orders. Note that the empirical predictions regarding the effect of the relative tick size should be tested with caution, as our model does not include subpenny trading which may take place in some dark markets and is particularly sensitive to relative tick size variations (Buti, Consonni, Rindi, Wen, and Werner, 2015).

### *7.2. The effect of dark pool activity on LOB fill rates and volume, and market quality in the cross-section*

*Prediction 2: More dark pool activity produces higher LOB fill rates and volume, and higher consolidated fill rates and volume. The positive effects of dark pool activity on LOB fill rates and volume, and consolidated fill rates and volume are increasing in LOB depth and decreasing in LOB spread (decreasing in the dispersion of trader valuations), and increasing in the proportion of traders with dark pool access, the relative tick size, and trader interest in a stock (the trading frequency/volume).*

To our knowledge no attempt has been made in the literature so far to test predictions regarding how fill rates are affected by dark pool trading.

*Prediction 3: More dark pool activity produces wider LOB spread and lower depth. The negative effect of dark pool activity on LOB spread and depth is increasing in LOB depth and decreasing in LOB spread (decreasing in the dispersion of trader valuations), and increasing in the proportion of traders with dark pool access, the relative tick size, and trader interest in a stock (the trading frequency/volume).*

To empirically isolate the effect of dark trading on market quality is challenging because, as our model shows, dark pool activity is in itself determined by LOB depth and spread. Another challenge is that data on dark pool activity have not until recently been generally available.[22]

Various data sets and empirical approaches have been used in the literature to address this question, and it is perhaps not surprising that the evidence is mixed. O'Hara and Ye (2011) find that the overall effect of fragmentation on Nasdaq and NYSE market quality is positive. As a proxy of volume on off-exchange venues they use trades reported to the Trade Reporting Facilities (TRFs). Unfortunately, the TRF data do not distinguish between dark markets, internalization by broker-dealers, and fully transparent LOBs like BATS or Direct Edge. Buti, Rindi, and Werner (2011) find that more dark pool activity is on average associated with improved market quality.

More recently, Comerton-Forde, Malinova, and Park (2015) and Foley and Putnins (2015) study the introduction of a minimum price improvement rule in Canada in October 2012 to gauge the influence of dark trading on market quality.[23] Both papers document that the rule dramatically reduces volume in dark venues. However, while Comerton-Forde, Malinova, and Park (2015) find no evidence that the reduction in dark volume is detrimental for lit market quality, Foley and Putnins (2015) find that the lower dark volume is associated with significantly worse lit market quality.

Degryse, de Jong, and van Kervel (2015) consider a sample of 52 Dutch stocks and analyze both lit fragmentation, (dark) internalized trades, and trades sent to dark pools. They find that when the two sources of dark liquidity are combined, the overall effect on market quality is detrimental. By contrast, lit fragmentation is associated with improved aggregate liquidity (although it is associated with poorer liquidity at the listing venue). Gresse (2014) in her recent study on the effect of the Markets in Financial Instruments Directive (MiFID) on U.K.- and Euronext-listed stocks finds that lit fragmentation increases depth and narrow spreads, whereas (dark) internalized trades are associated with greater depth but wider spreads.[24] Unfortunately, Gresse does not have access to dark pool trading for her sample stocks.

[22] The SEC now mandates disclosure of aggregate volume by stock per day for U.S. dark pools.

[23] The rule implies that dark venues have to improve on the lit market quotes by at least one penny (0.5 pennies if the spread is at 1 cent) when executing orders up to 5000 shares.

[24] There are several additional papers on MiFID: Soltani, Mai, and Jerbi (2011) on Euronext stocks, Kohler and von Wyss (2012) on Swiss stocks, Aitken, deB. Harris, and Sensenbrenner III (2012) on U.K. stocks. See also Riordan, Storkenmaier, and Wagener (2011) and Gentile and Fioravanti (2011).

### 7.3. The effect of dark pool activity on LOB fill rates and volume, and market quality in the time-series

*Prediction 4: Dark pool activity for a given stock is higher when the LOB depth is high and LOB spread is narrow. More dark pool activity produces lower (higher) LOB fill rates and volume, and lower (higher) consolidated fill rates and volume when LOB depth is high and LOB spread is narrow (LOB depth is low and LOB spread is wide).*

*Prediction 5: More dark pool activity is associated with a smaller (larger) widening of LOB spread and a larger (smaller) deterioration of LOB depth when LOB depth is high and LOB spread narrow (LOB depth is low and LOB spread wide).*

In other words, our model generates predictions that acknowledge that variation in LOB depth and spread over time for a particular stock affects both dark pool activity and the effect of dark pool activity on fill rates, volume, and LOB depth and spread. To test these predictions, it is important to control for endogeneity. Buti, Rindi, and Werner (2011) attempt to control for endogeneity using a two-stage least square (2SLS) regression framework and, as predicted by our model, find that dark pool activity for a particular stock is increasing in LOB depth and decreasing in LOB spread for all subsamples by listing venue and size. We are not aware of any other empirical work that has tested how variation in LOB depth and spread affects dark pool activity.

The second part of Predictions 4 and 5 emphasizes that the effect of dark pool activity on fill rates, volume, and market quality varies over time as a function of LOB depth and spread. To formally test these predictions is again complicated because of endogeneity. However, suggestive evidence for how the relationship between dark pool activity and the variables of interest (volume, LOB depth, and LOB spread) changes when LOB depth and spread vary can be obtained using market capitalization to proxy for LOB depth and spread. For example, Buti, Rindi, and Werner (2011) find that more dark pool activity is associated with lower consolidated volume for large cap stocks and with higher consolidated volume for small cap stocks, which supports the second part of Prediction 4. They also find that the economic magnitude of the beneficial effect of dark pool activity on market quality is larger for small cap stocks than for large cap stocks, which is consistent with Prediction 5.

## 8. Conclusions and policy implications

Regulators, exchange officials, media, and even some market participants are voicing concerns about the growing level of dark trading in U.S. and European equity markets. They worry that the presence of dark venues reduces the incentives for liquidity provision in the lit market, potentially reducing the depth at the best bid offer, widening the displayed spread, and discouraging traders from participating in the market. In addition, exchange officials see their franchise threatened as more trading moves off-exchange.

In this paper, we develop a theoretical model to address these concerns. The model attempts to capture the salient features of the real market and permits traders to use a rich set of order submission strategies and venues. While our model is dynamic and trader strategies are complex, the basic intuition is that when a dark pool is introduced, both market and limit orders migrate to the dark venue. It is the balance of limit and market orders that migrate to the dark pool that is crucial for determining the effects of a dark pool on trade creation, market quality, and welfare. We find that the introduction of a dark pool that competes with an illiquid order book is on average associated with trade creation, but also a deterioration of market quality and welfare. These effects are generally reduced if the order book starts out more liquid. Overall, our model illustrates that there is a trade-off between trade and volume creation on the one hand, and displayed liquidity such as depth and spread on the other hand.

After deriving the optimal strategies of all traders, and the resulting equilibrium outcomes, we study the determinants of dark pool activity in the cross-section. By evaluating equilibrium outcomes for different values of the model's parameters, we then study how dark pool activity depends on different asset and market characteristics. The model shows that dark pool activity increases in the dispersion of trader valuations around the common value of an asset, the fraction of traders that may access the dark pool, the relative tick size, and the trader interest in the asset. Since stocks with less disperse trader valuations have more liquid books in our framework, our model predicts that stocks with narrow spreads and high depth should have more dark pool activity. We extend the model and show that effects are weaker when the dark pool executes periodically, and that results are robust to introducing large traders splitting orders across venues.

Our results suggest that the regulatory objective to preserve retail traders' welfare could clash with the objective of dark pool operators to maximize trade and volume-related revenues. The reason is that when traders have access to a dark pool the lit market spread tends to widen, and investors restricted to trading in the lit market face higher trading costs as a result. Further, since fill rates and share volume in the public LOB increase when a dark pool is available, the operator of the dark pool has an incentive to boost dark trading even when the operator is the exchange which also runs the lit market. Our model also shows that managers of dark pools seeking to maximize revenues would prefer continuous executions to periodic crossings as this further enhances executions and share volume, but comes at an even higher cost to traders who lack access to the dark pool in terms of a wider lit market spread. Finally, our model shows that regulators should be wary of widening the relative tick size. While a wider relative tick size results both in more dark pool and more order book trading activity, this comes at the price of wider spreads, less depth, and a reduction in welfare.

We develop a model in this paper that allows us to discuss a wide range of policy issues which are currently on the agenda of financial regulators. However, there are several caveats that should be kept in mind when deriving policy conclusions from our results. First, the model does not include asymmetric information, so we cannot say anything about whether dark markets affect price discovery. However, this topic is addressed in complementary theoretical work by Ye (2011) and Zhu (2014). Unfortunately, their models reach opposite conclusions: Ye (2011) finds that informed traders are attracted to the dark pool while Zhu (2014) finds that informed traders avoid the dark pool.

Second, we do not discuss price manipulation. While smart traders could in principle trade on the lit market in advance to manipulate the execution price in the dark, we conjecture that this would primarily be an issue for illiquid stocks. Therefore, the possibility of manipulation provides a further incentive for the regulator to limit dark pool volumes for illiquid stocks.

Third, our model does not embed sub-penny trading as our dark pool trades execute at the midpoint of the lit market spread. Buti, Consonni, Rindi, Wen, and Werner (2015) show, however, that sub-penny trading also harms illiquid rather than liquid stocks. Therefore, our main policy implications are supported even for market structures where dark pools offer sub-penny trading.

Finally, our model focuses on the competition between a transparent LOB and a dark market. However, some exchanges also allow traders to use hidden orders, thus offering an alternative to dark pool trading. Among the wide range of existing undisclosed orders, the closest competitors to dark pool orders are Hidden Mid-Point Peg orders which are totally invisible and are submitted at the spread midpoint. Compared to dark pool orders, Hidden Mid-Point Peg execute against the LOB order flow and therefore have a higher execution probability than dark pool orders. Tackling the issue of competition for the provision of dark venues between exchanges and ATSs is therefore an interesting issue that we leave for future research.

## Appendix A

### A.1. Proof of Proposition 1

#### A.1.1. B framework

Consider first the benchmark case. The model is solved by backward induction, starting from $t = t_4$. The $t_4$-trader solves a simplified version of Eq. (7):

$$\max_{\varphi} \pi^e_{t_4}\left\{\varphi_M(1, p^B_i), \varphi(0), \varphi_M(1, p^A_i) \mid \beta_{t_4}, b_{t_4}\right\}. \quad (7')$$

Without loss of generality, assume that depending on $\beta_{t_4}$ and the state of the book $b_{t_4}$ the trader selects one of the equilibrium strategy $\varphi^n$, with $n \in N_{t_4}$, being $N_{t_4}$ the number of the equilibrium strategies at $t_4$. The $\beta$-thresholds between two different strategies are determined as follows:

$$\beta^{\varphi^{n-1},\varphi^n}_{t_4} \therefore \pi^e_{t_4}(\varphi^{n-1} \mid b_{t_4}) - \pi^e_{t_4}(\varphi^n \mid b_{t_4}) = 0\,. \quad (19)$$

These strategies are ordered in such a way that the $\beta$-thresholds are increasing, $\beta^{\varphi^{n-1},\varphi^n}_{t_4} < \beta^{\varphi^n,\varphi^{n+1}}_{t_4}$. Hence, the ex ante probability that a trader submits a certain order type at $t_4$ is determined as follows:

$$\Pr_{t_4}(\varphi^n \mid b_{t_4}) = F(\beta^{\varphi^n,\varphi^{n+1}}_{t_4} \mid b_{t_4}) - F(\beta^{\varphi^{n-1},\varphi^n}_{t_4} \mid b_{t_4})\,. \quad (20)$$

Consider now period $t_3$. The incoming trader solves Eq. (7), and uses $\Pr_{t_4}(\varphi^n \mid b_{t_4})$ to compute the execution probabilities of his limit orders. Given the optimal strategies at $t_4$, the $\beta$-thresholds and the order type probabilities at $t_3$ are derived using the same procedure as for period $t_4$, which is then reiterated for periods $t_2$ and $t_1$. When a trader is indifferent between strategies $\varphi^{n-1}$ and $\varphi^n$ , i.e., $\beta_t = \beta_t^{\varphi^{n-1},\varphi^n}$, we assume without loss of generality that he chooses $\varphi^{n-1}$.

We provide an example of how the model is solved. We start by considering all the possible opening LOBs at $t_4$, and following Fig. 2 we present as an example $b_{t_4} = [20]$. From now onwards to ease the notation we omit that all profits are conditional to the state of the book. Trader's profits are:

$$\pi_{t_4}[\varphi_M(1, p^B_2)] = (p^B_2 - \beta_{t_4}\upsilon) = \left(1 - \frac{3\tau}{2} - \beta_{t_4}\right) \quad (21)$$

$$\pi_{t_4}[\varphi_M(1, p^A_1)] = (\beta_{t_4}\upsilon - p^A_1) = \left(\beta_{t_4} - 1 - \frac{\tau}{2}\right) \quad (22)$$

$$\pi_{t_4}[\varphi(0)] = 0\,. \quad (23)$$

By solving Eq. (7') for this case, it is straightforward to show that all strategies are optimal in equilibrium ($N_{t_4} = 3$): $\varphi^1_{b_{t_4}} = \varphi_M(1, p^B_2)$, $\varphi^2_{b_{t_4}} = \varphi(0)$, and $\varphi^3_{b_{t_4}} = \varphi_M(1, p^A_1)$. As an example we compute the probability of $\varphi^1_{b_{t_4}}$ and to ease the notation in the following formula we omit the subscript "$b_{t_4}$" :

$$\beta^{\varphi^1,\varphi^2}_{t_4} \therefore \pi_{t_4}[\varphi^1] - \pi_{t_4}[\varphi^2] = 0,$$

$$\text{and therefore } \beta^{\varphi^1,\varphi^2}_{t_4} = 1 - \frac{3\tau}{2} \quad (24)$$

$$\Pr_{t_4}\varphi^1 = F(\beta^{\varphi^1,\varphi^2}_{t_4}) = \frac{1}{2}\left(1 - \frac{3\tau}{2}\right). \quad (25)$$

For the other periods we only specify the profit formulas, as the derivation of both the $\beta$-thresholds and order probabilities follows the same steps presented for period $t_4$.

Still as an example, and following Fig. 2, we consider the opening LOB $b_{t_3} = [20]$. Traders' profits are as follows:

$$\pi_{t_3}[\varphi_M(1, p^B_2)] = (p^B_2 - \beta_{t_3}\upsilon) \quad (26)$$

$$\pi^e_{t_3}[\varphi_L(1, p^A_1)] = \pi_{t_3}[\varphi(0)] = 0 \quad (27)$$

$$\pi^e_{t_3}[\varphi_L(1, p^B_1)] = (\beta_{t_3}\upsilon - p^B_1)\Pr_{t_4}(\varphi_M(1, p^B_1) \mid b_{t_4} = [21]) \quad (28)$$

$$\pi_{t_3}[\varphi_M(1, p^A_1)] = (\beta_{t_3}\upsilon - p^A_1)\,. \quad (29)$$

At $t_2$, again following Fig. 2, we present as an example the book $b_{t_2} = [10]$. We refer to Eqs. (21)–(23) for the profits of a market order to sell, to buy, or no trading, respectively. Profits of the other possible strategies are:

$$\pi^e_{t_2}\left[\varphi_L\left(1, p^A_1\right)\right] = \left(p^A_1 - \beta_{t_2}\upsilon\right)\Pr_{t_3}\left(\varphi_M\left(1, p^A_1\right) \mid b_{t_3} = [20]\right) \times \Pr_{t_4}\left(\varphi_M\left(1, p^A_1\right) \mid b_{t_4} = [10]\right) \quad (30)$$

$$\pi^e_{t_2}\left[\varphi_L\left(1,p^B_1\right)\right] = \left(\beta_{t_2}v - p^B_1\right)\left[\Pr_{t_3}\left(\varphi_M\left(1,p^A_1\right)\mid b_{t_3}=[11]\right)\right.$$
$$\times \Pr_{t_4}\left(\varphi_M\left(1,p^B_1\right)\mid b_{t_4}=[01]\right)$$
$$+ \Pr_{t_3}(\varphi(0)\mid b_{t_3}=[11])\Pr_{t_4}\left(\varphi_M\left(1,p^B_1\right)\mid b_{t_4}=[11]\right)$$
$$\left.+ \Pr_{t_3}\left(\varphi_M\left(1,p^B_1\right)\mid b_{t_3}=[11]\right)\right]. \quad (31)$$

We do not present profit formulas for period $t_1$, in which the opening LOB is $b_{t_1}=[00]$, because they are similar to the ones presented for period $t_2$, but now limit orders have an additional period to get executed.

*A1.2. L&C framework*

The solution of the *L&C* framework follows the same methodology, but now the trader with dark pool access solves Eq. (11). For brevity, we provide examples only for the last two periods of the trading game. The remaining two periods are solved in a similar way. To ensure the uniqueness of the equilibrium, we assume that when traders are indifferent between trading on the LOB or on the dark pool, they choose the LOB. We define the information set of the trader at $t$ as $\Omega_{t_i}=[b_{t_i}, y_{t_{i-3}}, y_{t_{i-2}}, y_{t_{i-1}}]$, where $y_{t_i}$ is the observed strategy on the LOB in period $t_i$.

Following Fig. 3, at $t_4$ we consider again the book $b_{t_4}=[20]$ with the following information set, $\Omega_{t_4}=[20, \varphi_L(1,p^A_1), \varphi_L(1,p^A_1), \varphi(0)]$. In this case when at $t_3$ a trader observes $\varphi(0)$, he doesn't know whether a trader with no access refrained from trading, or whether a trader with access to the *CDP* submitted a dark pool order, either to buy or to sell, or decided to refrain from trading. Therefore, traders coming at $t_4$ Bayesian update their expectations on the state of the dark pool as follows (we omit that all probabilities are conditional to $\Omega_{t_4}$):

$$\widetilde{CDP}_{t_4} = \begin{cases} -1 \;\; with\ prob = \dfrac{\alpha \Pr_{t_3}\varphi_D(-1,\tilde{p}_{Mid,t})}{\alpha[\Pr_{t_3}\varphi_D(-1,\tilde{p}_{Mid,t})+\Pr_{t_3}\varphi_D(+1,\tilde{p}_{Mid,t})+\Pr_{t_3}\varphi(0)]+(1-\alpha)\Pr_{t_3}\varphi^{NA}(0)} \\ 0 \;\; with\ prob = \dfrac{\alpha \Pr_{t_3}\varphi(0)+(1-\alpha)\Pr_{t_3}\varphi^{NA}(0)}{\alpha[\Pr_{t_3}\varphi_D(-1,\tilde{p}_{Mid,t})+\Pr_{t_3}\varphi_D(+1,\tilde{p}_{Mid,t})+\Pr_{t_3}\varphi(0)]+(1-\alpha)\Pr_{t_3}\varphi^{NA}(0)} \\ +1 \;\; with\ prob = \dfrac{\alpha \Pr_{t_3}\varphi_D(+1,\tilde{p}_{Mid,t})}{\alpha[\Pr_{t_3}\varphi_D(-1,\tilde{p}_{Mid,t})+\Pr_{t_3}\varphi_D(+1,\tilde{p}_{Mid,t})+\Pr_{t_3}\varphi(0)]+(1-\alpha)\Pr_{t_3}\varphi^{NA}(0)} \end{cases}. \quad (32)$$

Notice that when traders with and without access could choose the same equilibrium strategy, we add the superscript "*NA*" to indicate the order submitted by traders with no access. We refer to the *B* framework for the profits and the equilibrium strategies of the traders with no access, and discuss the profits of traders with access. We omit regular market orders that we have already presented in the *B* framework, Eqs. (21) and (22), and focus only on orders that involve the use of the *CDP*:

$$\pi^e_{t_4}[\varphi_D(-1,p_{Mid,t_4},p^B_1)] = (p^B_1-\beta_{t_4}v)\Pr(\widetilde{CDP}_{t_4}=0,-1)$$
$$+\left(\frac{p^A_1+p^B_2}{2}-\beta_{t_4}v\right)\Pr(\widetilde{CDP}_{t_4}=+1) \quad (33)$$

$$\pi^e_{t_4}[\varphi_D(-1,p_{Mid,t_4})] = \left(\frac{p^A_1+p^B_2}{2}-\beta_{t_4}v\right)\Pr(\widetilde{CDP}_{t_4}=+1) \quad (34)$$

$$\pi^e_{t_4}[\varphi_D(+1,p_{Mid,t_4})] = \left(\beta_{t_4}v-\frac{p^A_1+p^B_2}{2}\right)\Pr(\widetilde{CDP}_{t_4}=-1) \quad (35)$$

$$\pi^e_{t_4}[\varphi_D(+1,p_{Mid,t_4},p^A_1)] = (\beta_{t_4}v-p^A_1)\Pr(\widetilde{CDP}_{t_4}=0,+1)$$
$$+\left(\beta_{t_4}v-\frac{p^A_1+p^B_2}{2}\right)\Pr(\widetilde{CDP}_{t_4}=-1). \quad (36)$$

By comparing, for example, Eqs. (22) and (36), we observe that market orders are always dominated by IOC dark pool orders, unless the probability that the order executes on the *CDP* is zero:

$$\pi^e_t[\varphi_D(\pm1,\tilde{p}_{Mid,t},p^z_i)] \ge \pi_t[\varphi_M(1,p^z_i)]\ . \quad (37)$$

To determine the equilibrium strategies $\varphi^n_{WA,\Omega_{t_4}}$ at $t_4$ for $n\in N_{t_4}$, the model has to be solved up to period $t_3$. We anticipate that because in equilibrium $\Pr_{t_3}\varphi(0)=0$, $\Pr_{t_3}\varphi^{NA}(0)>0$, $\Pr_{t_3}\varphi_D(+1,\tilde{p}_{Mid,t})=0$, and $\Pr_{t_3}\varphi_D(-1,\tilde{p}_{Mid,t})>0$, $N_{t_4}=4$ and the strategies of the trader with access are as follows: $\varphi^1_{WA,\Omega_{t_4}}=\varphi_M(1,p^B_1)$, $\varphi^2_{WA,\Omega_{t_4}}=\varphi(0)$, $\varphi^3_{WA,\Omega_{t_4}}=\varphi_D(+1,p_{Mid,t_4})$, and $\varphi^4_{WA,\Omega_{t_4}}=\varphi_D(+1,p_{Mid,t_4},p^A_1)$.

As for the *B* framework, we now consider the case $b_{t_3}=[20]$ with the following information set, $\Omega_{t_3}=[20,\varphi_L(1,p^A_1),\varphi_L(1,p^A_1)]$. Traders know that the *CDP* is still empty, $CDP_{t_3}=0$, because no order has been submitted to the dark pool. Compared to the *B* framework, equilibrium strategies for the *NA* trader are the same, but the *WA* trader has now the additional possibility to submit a pure dark pool order to sell or buy (in this case, IOC dark orders are equivalent to market orders because the dark pool is empty):

$$\pi^e_{t_3}[\varphi_D(-1,p_{Mid,t_4})]$$
$$=\left(\frac{p^A_1+p^B_2}{2}-\beta_{t_3}v\right)\alpha\left[\Pr_{t_4}(\varphi_D(+1,p_{Mid,t_4},p^A_1)\mid\Omega_{t_4})\right.$$
$$\left.+\Pr_{t_4}(\varphi_D(+1,p_{Mid,t_4})\mid\Omega_{t_4})\right] \quad (38)$$

$$\pi^e_{t_3}[\varphi_D(+1,p_{Mid,t_4})]$$
$$=\left(\beta_{t_3}v-\frac{p^A_1+p^B_2}{2}\right)\alpha\left[\Pr_{t_4}(\varphi_D(-1,p_{Mid,t_4},p^B_2)\mid\Omega_{t_4})\right.$$
$$\left.+\Pr_{t_4}(\varphi_D(-1,p_{Mid,t_4})\mid\Omega_{t_4})\right] \quad (39)$$

where in both cases $\Omega_{t_4}=[20,\varphi_L(1,p^A_1),\varphi_L(1,p^A_1),\varphi(0)]$.

*A.1.3. OM*

Results for *OM* presented in Fig. 4, Panel A, are derived by straightforward comparison of the equilibrium

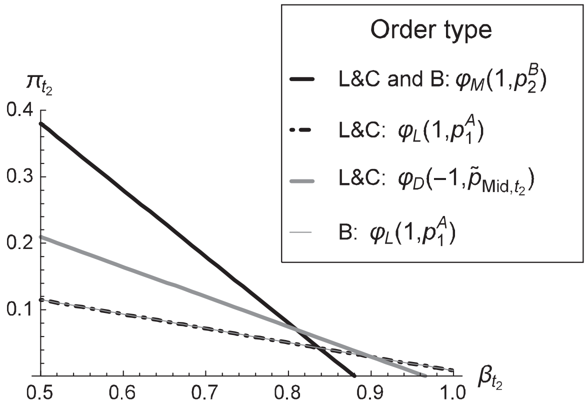


**Fig. A1.** Order migration on the L&C - $b_{t_2} = [10]$.

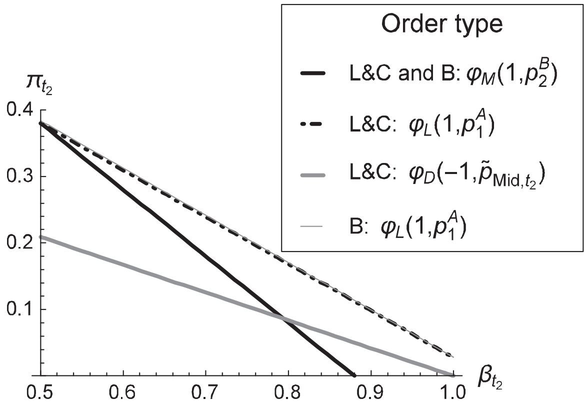


**Fig. A2.** Order migration on the L&C - $b_{t_2} = [00]$.

strategies for the two frameworks: *B*, and *L&C*. In Figs. A1 and A2 we provide plots at $t_2$ for *WA* trader's profits as a function of $\beta$ for the *L&C* and *B* frameworks. Following the exposition in the main text, we focus on selling strategies. Each figure provides a graphical representation of the traders' optimization problem. Fig. A1 shows how the introduction of a *CDP* changes the optimal order submission strategies of *WA* traders by crowding out both market and limit orders, and generating *OM*. To observe the effect of market depth and spread on *OM*, compare Figs. A1 and A2.

*A.1.4. TC*

Results for *TC* presented in Fig. 4, Panel B, are obtained by comparing fill rates for the *B* and *L&C* frameworks, as shown in Eq. (13). As an example, we consider period $t_1$ of the *B* model and specify the formula for the estimated fill rate in this period. Equilibrium strategies for the trader arriving at $t_1$ are: $\varphi^1 = \varphi_M(1, p_2^B)$, $\varphi^2 = \varphi_L(1, p_1^A)$, $\varphi^3 = \varphi_L(1, p_1^B)$, $\varphi^4 = \varphi_M(1, p_2^A)$.

$$FR^B_{t_1,[00]} = \Pr_{t_1} \varphi^1 + \Pr_{t_1} \varphi^4 \ . \tag{40}$$

*A.2. Proof of Proposition 2*

Results for spread and depth presented in Fig. 5, Panel A, are obtained by comparing the two market quality measures for the *B* and *L&C* protocols. As an example, we consider again the *B* model and specify formulas for the estimated spread and depth at $t_1$. We refer to the proof of Proposition 1 for a list of the equilibrium strategies in this case.

$$S^B_{t_1,[00]} = (p_2^A - p_2^B)(\Pr_{t_1} \varphi^1 + \Pr_{t_1} \varphi^4) + (p_1^A - p_2^B) \Pr_{t_1} \varphi^2 + (p_2^A - p_1^B) \Pr_{t_1} \varphi^3 \tag{41}$$

$$D^B_{t_1,[00]} = \Pr_{t_1} \varphi^2 + \Pr_{t_1} \varphi^3 \ . \tag{42}$$

Similar computations make it possible to derive the market quality measures for all the other cases.

*A.3. Proof of Proposition 3*

Results for welfare presented in Fig. 5, Panel B, are obtained by comparing welfare values for the *NA* and *WA* traders, and on aggregate in the *B* and *L&C* protocols. To provide an example, we consider again the *B* model and specify the welfare formula at $t_1$. We refer again to the proof of Proposition 1 for a list of the equilibrium strategies in this case.

$$W^B_{t_1,[00]} = \int_0^{\beta_{t_1}^{\varphi^1,\varphi^2}} \pi_{t_1}(\varphi^1) d\beta_{t_1} + \int_{\beta_{t_1}^{\varphi^1,\varphi^2}}^{\beta_{t_1}^{\varphi^2,\varphi^3}} \pi_{t_1}(\varphi^2) d\beta_{t_1} + \int_{\beta_{t_1}^{\varphi^2,\varphi^3}}^{\beta_{t_1}^{\varphi^3,\varphi^4}} \pi_{t_1}(\varphi^3) d\beta_{t_1} + \int_{\beta_{t_1}^{\varphi^3,\varphi^4}}^{2} \pi_{t_1}(\varphi^4) d\beta_{t_1} . \tag{43}$$

Similarly, we can derive welfare values for all the other cases.

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