Tick Size, Trading Strategies and Market Quality

Ingrid M. Werner* Barbara Rindi†

Sabrina Buti‡ Yuanji Wen§

31st January 2022

Abstract

We investigate the effects of a tick size reduction on market quality in a multi-period limit order book market. For illiquid stocks, reducing the tick size facilitates undercutting and discourages liquidity provision, resulting in deteriorating market quality but higher volume. For liquid stocks, reducing the tick size curtails queues, resulting in lower depth and volume but narrower spread. With a competing crossing network, a tick size reduction results in worse market quality for all stocks due to migration of order flows. We empirically test our model predictions and find support for recent tick size reductions in Japan and the U.S.

JEL classification: G10, G20, G24, D40

Keywords: Limit order markets, tick size, liquidity, market microstructure

*Fisher College of Business, Ohio State University, werner@fisher.osu.edu

†Bocconi University and IGIER and Baffi-Carefin, barbara.rindi@unibocconi.it

‡Université Paris Dauphine-PSL, CNRS, DRM, sabrina.buti@dauphine.psl.eu

§UWA Business School, yuanji.wen@uwa.edu.au

The tick size is the minimum price increment at which traders can place orders and it is one of the most important features in the design of modern trading platforms, worldwide. Hence, one of the topics attracting the highest level of attention of market regulators around the World is the effect of a change in the tick size.

Following the historical progressive reduction in tick size, in October 2016, the Securities and Exchange Commission (SEC) launched the U.S. Tick Size Pilot (USTSP), a two year pilot program aimed at investigating the effects of an increase in the tick size for the so called "emerging growth companies" (EGCs), proxied in the pilot by 1200 small capitalization stocks.[1] Those proposing an increase of the tick size argued that its reduction to one penny had been harmful for this group of stocks and therefore that "one size does not fit all."[2]

Our paper shows both theoretically and empirically that indeed "one (tick) size does not fit all." The impact of a variation in the tick size on market quality depends first on whether a stock is liquid or not, and second on whether a stock market is fragmented or not. On the theory side, we show that in a non-fragmented market, following a decrease in the tick size, depth always worsens while the effects on spread and volume depend on stock liquidity: for liquid stocks, spread improves and volume deteriorates, the opposite holding for illiquid stocks. Once we introduce market fragmentation, we show that when the tick size decreases all market quality indicators (i.e., spread, depth, and volume) worsen for both liquid and illiquid stocks, but effects are stronger for liquid stocks. We then test our model predictions by examining policy shocks on tick size in two of the World's most important financial markets, one of which is slightly fragmented (Japan) while the other is highly fragmented (U.S.). We find empirical evidence consistent with our cross-sectional predictions on both stock liquidity and market fragmentation by exploring the heterogeneous treatment effects

[1] For an analysis of the effects of early reductions in the tick size, see for example Angel (1997), Bessembinder (2003), Goldstein and Kavajecz (2000), and Harris (1994).

[2] In 2018, the European Securities and Markets Authority (ESMA) introduced a new tick size schedule for all European markets (ESMA, 2018), with the tick size varying as a function of both the liquidity and the price of the stocks. The Tokyo Stock Exchange also employs a tick-size schedule, and the schedule for TOPIX 100 index stocks was adjusted in a series of moves starting in 2014.

within the Japanese and the U.S. markets.

To build our theoretical model, we consider a Public Limit Order Book (PLB) that works like a continuous double auction market in the spirit of Parlour (1998) and compare a large tick size market (LTM) to a small tick size market (STM). Moreover, to analyze the effects of a change in the tick size on stocks characterized by different liquidity, we proxy the degree of liquidity of a stock by considering a combination of trading activity, i.e., the number of trading periods within a day, and traders' willingness to supply liquidity.

Our model shows that the effects of a tick size change can be decomposed in three different channels of transmission. Considering a smaller tick size - the effects for a larger tick size being symmetric - our model shows that:

1. undercutting is cheaper in that traders willing to undercut a standing limit order have to give up a smaller price improvement ("undercutting effect"). This leads to traders being less willing to supply liquidity in a STM, resulting in inside spread and depth deteriorating and volume improving;

2. the limit orders clustering at the old price levels tend to scatter at the new more numerous price levels thus deteriorating inside depth. As a result, the limit order queues at all price levels shorten ("layering effect") leading traders to be more willing to supply than to take liquidity, thus improving spread and deteriorating volume;

3. taking liquidity at the best bid-offer is cheaper due to the mechanical reduction in the spread ("mechanical effect") - which cannot be smaller than the tick size. This effect in general leads to an increase in traders' incentive to take liquidity thus deteriorating liquidity in the STM.

No other existing model derives the relevant channels of transmission of a change in the tick size from first principles. Even more importantly, our model shows that the relevance of these channels crucially depends on the degree of liquidity of the underlying asset. As mentioned above, we proxy such liquidity by both trading activity and traders' willingness to supply and demand liquidity: the combination of high (low) trading activity and high (low) willingness to provide liquidity characterizes liquid (illiquid) stocks.

Our model shows that when the tick size decreases the "undercutting effect" is stronger for illiquid stocks as when the book is less liquid there is generally more room to undercut existing limit orders.[3] All of the other two effects are stronger with liquid stocks. The "layering effect" is stronger as in a very deep book the clustering of liquidity is more severe, and the spreading of limit orders at the new price levels is more beneficial for liquidity provision. The "mechanical effect" is also stronger in liquid books in which liquidity supply tends to cluster at the first level of the PLB that becomes tick size constrained.

Combining these effects, our model shows that following a reduction in the tick size the "layering" and "mechanical" effects prevail on the "undercutting" effect for liquid stocks so that investors lean towards supplying as opposed to taking liquidity, with the result that spread improves, while depth and volume deteriorate. For illiquid stocks instead the strong "undercutting" effect reduces investors' willingness to offer liquidity via limit orders as opposed to take liquidity via market orders so that not only depth but also spread deteriorate while volume improves.

We then consider a dual market model in which the PLB competes with a CN ($PLB\&CN$) in order to check the robustness of our predictions to market fragmentation. Our CN is a transparent crossing network executing at the midquote of the competing PLB, and therefore it is a stylized protocol to capture the effects of migration of order flows towards a competing market. Such an extension creates a fourth channel of transmission: With a smaller tick size, the migration of order flows to a competing venue increases ("migration effect"), leading to a deterioration in the PLB of both order flows - liquidity supply (limit orders) and liquidity demand (market orders) - and market

[3] Symmetric results hold for an increase in the tick size.

quality.[4] Notice that deep books magnify the "migration effect", because the intense competition for liquidity supply strengthens the incentives to jump the queue by switching to the CN. Therefore, similarly to the "layering" and "mechanical" effects, the "migration effect" is stronger with liquid stocks.

We show that in a fragmented market, for liquid stocks the strong migration of both aggressive limit and market orders to the CN confirms the negative effects on depth and volume observed in the market with no fragmentation. Interestingly, the strong "migration effect" now counterbalances the "layering" effect, decreasing liquidity provision and slightly worsening the spread. For illiquid stocks, instead, the strong "undercutting" effect is now combined with the "migration effect", so that not only spread and depth worsen but also the volume deteriorates. Furthermore, our model shows that when taking into account the "migration effect", magnified in deep books, all the results obtained are stronger for liquid than for illiquid stocks.

Our theoretical results can be used to understand the channels at work whether the market is fragmented or not. We test our empirical predictions by investigating two recent events: the 2014 reduction in tick size on the Tokyo Stock Exchange (TSE), and the 2018 reduction in tick size at the end of the USTSP. These two markets allow us to analyze cross-market variations on the impact of tick size on market quality because one market is highly fragmented (U.S.) while the other isn't (TSE). Moreover, we proxy liquid/illiquid stocks respectively with a) "tick size constrained" (TSC) stocks, i.e., stocks which before the tick size change had liquidity clustered -constrained- at the old best bid-offer; and b) "tick size unconstrained" (TSU) stocks, i.e., stocks which before the tick size change did not have their liquidity clustered at the old inside spread. For the TSE our results are generally consistent with the theoretical predictions for a single market. In particular, we observe that for TSC stocks spread and trading volume reduce simultaneously following a tick size reduction. Results for the USTSP are instead broadly consistent with predictions of our $PLB\&CN$ framework

[4] Notice that we consider the most competitive scenario, a lit CN with midquote pricing. We discuss in Section 3.4 how the migration effect and, therefore, the effects on market quality may be impacted if the other market is less price-competitive being another PLB as opposed to a midpoint CN.

and the “migration effect”.

Our paper contributes to the theoretical literature on the relationship between the reduction of the tick size and market quality. Differently from previous models analyzing the effects of a tick size variation by Seppi (1997), Cordella and Foucault (1999), and Kadan (2006), we consider a pure order driven market in which liquidity provision is endogenously created by the competitive interaction between patient traders who supply liquidity via limit orders, and impatient traders who demand liquidity via market orders. The endogenous choice between supplying and demanding liquidity is present in Foucault, Kadan and Kandel (2005) but because traders must provide a price improvement when submitting limit orders, in their framework patient traders cannot join the queue at the existing best bid or offer. Therefore, the “layering effect” is not at work. Moreover, because the “undercutting effect” discourages liquidity provision and pushes traders to either demand liquidity or join the queue at the existing best bid or offer, we obtain the interesting result that in a non-fragmented market reducing the tick size harms the bid-ask spread for illiquid stocks. Our protocol is also close to Goettler, Parlour and Rajan (2005) who consider an infinite horizon limit order book and show numerically that, by reducing the tick size, both spread and depth decrease. We depart from such a framework because, first, our model has a closed form solution given a tick-to-support ratio, and second, we show that the effects of a tick size variation depend on both the liquidity of the PLB and market fragmentation. Finally, our model contributes to the literature on market fragmentation. Differently from Degryse, Van Acheter, and Wuyts (2009) who analyze competition between a dealer market and a CN, we consider a PLB with endogenous liquidity provision competing with a CN. Compared to Buti, Rindi, and Werner (2017), and Zhu (2011), we focus on how the “migration effect” to the competing CN is affected by tick size variations, and we also solve the model for a small tick size.[5]

We contribute also to the empirical literature analyzing the effects of a reduction of the tick

[5]We refer to our Online Appendix C for a more in detail discussion of how our model departs from previous literature.

size on market quality. Various studies analyzing markets around the World have found that a tick size reduction is associated with a reduction in both spread and depth, and that the spread is not equally affected across stocks.[6] We add to this literature by analyzing cross-market variations on the impact of a tick size on market quality and by showing that, in line with our theoretical predictions, the effects of a tick size reduction depend on both market fragmentation and stock liquidity. Moreover, we show that in fragmented markets effects are stronger for TSC stocks, as documented by the studies analyzing the SEC tick size pilot.[7] Finally, the focus on fragmented vs. non-fragmented markets adds to the existing literature on market fragmentation by showing the relationship between the effects of a reduction of the tick size and market fragmentation.[8]

1 The Model

We theoretically address two fundamental issues regarding the tick size. First, we investigate what are the effects of a change in the tick size on the quality of a limit order book for two different categories of stocks, one characterized by higher propensity to supply liquidity and higher trading activity, and the other one characterized by lower propensity to offer liquidity and lower trading activity. Second, we aim to understand whether the results obtained in a single market framework are robust to inter-market competition.

We develop a model of a PLB in which risk neutral traders with no private information have a personal evaluation of the asset and can endogenously decide whether to demand or take liquidity. To investigate the effects of a change in the tick size, we solve the model under two price grids: the first one is a large tick market with four price levels, two on the ask side and two on the bid side (PLB_{LTM}); the second one is a small tick market with ten price levels, five on the ask and five on the bid side (PLB_{STM}). To investigate whether a change in the tick size differs depending on the

[6] See for example Bessembinder (2003) and Goldstein and Kavajecz (2000).
[7] See for example Comerton-Forde, Gregoire and Zhong (2019) and Rindi and Werner (2019).
[8] See for example O'Hara and Ye (2011).

type of stocks considered, we solve both our LTM and STM frameworks first with 3 trading periods, and then with 4 trading periods. Finally, to investigate how the effects of a change in the tick size differ depending on investors' propensity to supply or demand liquidity, we solve our models under different dispersion of investors' beliefs. When the dispersion is large, traders' personal evaluations of the asset are extreme and therefore traders are more impatient and inclined to demand rather than supply liquidity; the opposite holding when the dispersion is small and traders are more willing to supply liquidity. Finally, we extend our model by introducing a CN executing orders at the midquote. In this new framework investors can choose between trading in the PLB or trading in the CN. We then compare the new large tick market ($PLB_{LTM}\&CN$) with the new small tick market ($PLB_{STM}\&CN$) to investigate whether the effects of a change in the tick size are robust to the presence of competition between trading venues.

1.1 PLB Market

A market for a security v operates over a trading day divided into either 3, $t = \{t_1, t_2, t_3\}$, or 4 trading periods, $t = \{t_1, t_2, t_3, t_4\}$. Each trader comes to the market with a personal evaluation of the asset, β, drawn from a truncated normal distribution, $\beta \sim TN(\mu, \sigma^2)$, with support $\beta \in (\underline{\beta}, \overline{\beta})$, $0 \leq \beta < 1 < \overline{\beta}$. Without loss of generality we set $\mu = 1$, $\underline{\beta} = 0$, $\overline{\beta} = 2$, and $v = 1$.[9] The parameter β can also be interpreted as traders' degree of impatience: a trader with a β close to either $\underline{\beta}$ or $\overline{\beta}$ has an extreme valuation of the asset and is more eager to trade immediately, whereas a trader with a β close to 1 is more patient and willing to supply liquidity. When σ is small, investors' personal valuations are more clustered around the asset value v, and hence the traders are more willing to supply liquidity. When instead σ is larger, beliefs are more dispersed and on average traders are more willing to take liquidity. There is no asymmetric information in that all investors have equal

[9] We opted for a truncated normal distribution rather than a uniform distribution as by changing the standard deviation of the truncated normal, we can compare markets for the same gains from trade but different clustering around the fundamental asset value.

access to public information.[10]

To investigate how changes in the tick size affect traders' aggressiveness in the provision of liquidity as well as depth at different price levels, we need at least two price levels on each side of the PLB. Therefore, we extend Parlour's (1998) model to include multiple price levels on each side of the market, symmetrically distributed around the asset value v. In the PLB_{LTM}, we have two prices on the ask ($A_1 < A_2$) and two prices on the bid side of the market ($B_1 > B_2$). The difference between two adjacent prices -the minimum price increment- is the tick size which also corresponds to the minimum inside spread and is set equal to τ. Thus the possible price levels are equal to $A_1 = v + \frac{\tau}{2}$, $A_2 = v + \frac{3\tau}{2}$, $B_1 = v - \frac{\tau}{2}$, and $B_2 = v - \frac{3\tau}{2}$. In the PLB_{STM}, we assume instead that there are five price levels on each side of the market (a_l, b_l with $l = 1, .., 5$, $a_l > a_{l-1}$ and $b_l < b_{l-1}$) and a tick size equal to $\frac{\tau}{3}$.[11] In particular, this value is consistent with an additional price level between A_1 and B_1 on both sides of the market, and, at the same time, it guarantees that there are four price levels on the PLB_{STM} grid (namely a_2, b_2, and a_5, b_5) that coincide with A_1, B_1, and A_2, B_2. Finally, as in Seppi (1997) and Parlour (1998), we assume that a trading crowd provides liquidity at the highest and lowest price levels of the limit order book (A_2 and B_2, and a_5 and b_5, for the PLB_{LTM} and the PLB_{STM}, respectively). Adding a trading crowd allows investors to use market orders also in the first period of the trading game.[12] Note that incoming traders are allowed to submit limit orders that queue up in front of the trading crowd.

For ease of exposition, we now present the dynamic of the order book over time, traders' strategies, and the market equilibrium by focusing on the PLB_{LTM}. The PLB_{STM} follows the same logic.

[10] See Riccò, Rindi and Seppi (2020) for an extension of this framework to a 5-period model including asymmetric information.

[11] This choice is purely technical in order for the small tick grid to fit realistically into the large one. In real markets when exchanges decrease the tick size they prefer to do so in a way that the prices that belonged to the large tick grid before the tick size change still belong to the new smaller tick grid.

[12] Colliard and Foucault (2012) assume the existence of a dealership market that serves the same purpose.

1.2 Order Types and Dynamic of the Limit Order Book

At each period t, a trader is drawn from the β-distribution and, before submitting his order of unitary size, he observes the state of the book, $S_{t-1} = [Q_{t-1}^{A_2}, Q_{t-1}^{A_1}, Q_{t-1}^{B_1}, Q_{t-1}^{B_2}]$, which is characterized by the Q_t shares available at each level of the price grid. In this market, which enforces price and time priority, traders can choose between two types of orders that cannot be modified or cancelled after submission: limit orders that we indicate by $+1$, and market orders that we indicate by -1. We define H_t a trader's strategy at time t. Traders can submit limit orders to sell or to buy one share respectively at the two levels of the ask and of the bid side of the book, $H_t = \{+1^{A_k}, +1^{B_k}\}$, where $k = 1, 2$. These orders stay on the book until they are executed against a market order of opposite sign. Alternatively, traders can submit market orders which hit the best bid or ask prices available on the book and are hence executed immediately, $H_t = \{-1^{B_{k'}}, -1^{A_{k'}}\}$, where $k' = 1, 2$ refers to the best price. Finally, traders can decide not to trade, $H_t = \{0\}$. The trader's strategy space is therefore $H_t = \{-1^{A_{k'}}, +1^{A_k}, 0, +1^{B_k}, -1^{B_{k'}}\}$. The change in the limit order book induced by the trader's strategy H_t is indicated by h_t and defined as:

$$h_t = [h_t^{A_2}, h_t^{A_1}, h_t^{B_1}, h_t^{B_2}] = \begin{cases} [\pm 1, 0, 0, 0] & \text{if } H_t = \pm 1^{A_2} \\ [0, \pm 1, 0, 0] & \text{if } H_t = \pm 1^{A_1} \\ [0, 0, 0, 0] & \text{if } H_t = 0 \\ [0, 0, \pm 1, 0] & \text{if } H_t = \pm 1^{B_1} \\ [0, 0, 0, \pm 1] & \text{if } H_t = \pm 1^{B_2} \end{cases} \tag{1}$$

The state of the book is hence characterized by the following dynamics:

$$S_t = S_{t-1} + h_t \tag{2}$$

where S_{t-1} is the state of the book that the trader arriving at t observes before he submits his order, and S_t is the state of the book after his order submission. We assume that when a trader arrives at

t_1 he observes the initial state of the book denoted by S_0, which is by assumption empty.[13]

1.3 Order Submission Decision

To select his order submission strategy, a trader needs to choose an order type and, if he chooses to submit a limit order, a limit price. His goal is to maximize utility, which in this risk neutral setting is equivalent to maximizing his payoff, considering all the available strategies. Market orders guarantee immediate execution, while limit orders enable traders to obtain a better price. When traders choose a limit rather than a market order, they increase their non-execution costs (NEC) as, to potentially obtain a better price, they forgo execution certainty. At the same time, however, they reduce their price opportunity cost (POC), which is the cost associated with an execution at a less favorable price. Hence in this market traders face a trade-off between NEC and POC.

Table 1: Large tick public limit order book (PLB_{LTM}): Order Submission Strategy Space
This table reports the payoffs, $U(\cdot)$, of the order strategies H_t available on a PLB_{LTM}.

Strategy: PLB_{LTM} only	H_t	$U(\cdot)$
Market Sell Order	$-1^{B_{k'}}$	$B_{k'} - \beta v$
Limit Sell Order	$+1^{A_k}$	$p_t(A_k\|S_t) \cdot (A_k - \beta v)$
No Trade	0	0
Limit Buy Order	$+1^{B_k}$	$p_t(B_k\|S_t) \cdot (\beta v - B_k)$
Market Buy Order	$-1^{A_{k'}}$	$\beta v - A_{k'}$

The payoffs of the different strategies available to traders, $U(\cdot)$, are listed in Table 1. Market orders are always executed at the best ask or bid price and their payoff depends on the traders' personal evaluation of the asset β. Limit orders' payoff also depends on the execution probability that we denote by $p_t(A_k|S_t)$ and $p_t(B_k|S_t)$ for a limit sell and for a limit buy order respectively submitted at the ask price A_k, or at the bid price B_k. As mentioned before, in our model the trader's own β can also be interpreted as his degree of impatience. If a trader comes to the market

[13] A book opens empty when there are no orders posted by traders at any level of the book, and there is only the trading crowd offering liquidity at the second price level.

with an extreme β, all else equal, his NEC will be high compared to those of a trader with a β around 1, who is instead more concerned about POC. For this reason, he will be more inclined to opt for a market order and, therefore, to buy or sell the asset immediately.[14]

In our model the order execution probability of a limit order depends both on the state of the own side and on the state of the other side of the book - e.g., the bid and the ask size for a limit buy order, respectively. For a potential buyer, long queues on the bid side decrease the execution probability of a newly posted limit buy order thus making a market buy order more attractive, whereas long queues on the ask side increase the execution probability of a limit buy as it increases the incentive for an incoming seller to post market rather than limit orders. The execution probability of a limit order also depends on the future states of the book because order choices of incoming traders will impact the NEC of existing limit orders.[15]

1.4 Market Equilibrium

Traders use information from the state of the PLB to rationally compute the execution probabilities of different orders. Having done so, they are then able to choose the optimal strategy consistent with their own β by simply comparing the expected payoff from each order.

Given the tick size, and the moments and the support of the truncated normal distribution, we solve our model in closed form by backward induction. At time t_4, the execution probability of limit orders is zero, and traders will therefore either submit market orders or decide not to trade.

[14] Note that we refine the strategy space compared to Parlour's (1998) by relaxing the assumption that nature exogenously selects buyers and sellers at the beginning of each trading period. In our model traders determine whether to buy or sell, or not to trade, based on their asset valuation. Hence, for example, a trader with a β next to $\overline{\beta}$ would opt for a market buy order, rather than refrain from trading as would happen in Parlour's model had nature selected him as a seller.

[15] For instance, Figure D.1 in the Online Appendix D shows that if a trader arriving at the market at t_1 decides to submit a limit sell order at A_2, the execution probability of this order will decrease if in the next periods another trader submits a limit sell order at A_1.

It is straightforward to show that traders' equilibrium strategies are:

$$H^*_{t_4}(\beta|S_{t_3}) = \begin{cases} -1^{B_{k'}} & \text{if } \beta \in [\underline{\beta}, \frac{B_{k'}}{v}) \\ 0 & \text{if } \beta \in [\frac{B_{k'}}{v}, \frac{A_{k'}}{v}) \\ -1^{A_{k'}} & \text{if } \beta \in [\frac{A_{k'}}{v}, \overline{\beta}] \end{cases} \quad (3)$$

By using these equilibrium strategies together with the distribution of β, we calculate the equilibrium execution probabilities of limit orders submitted at t_3 that are the dynamic link between periods t_3 and t_4:

$$p^*_{t_3}(A_k|S_{t_3}) = \begin{cases} \int\limits_{\beta \in \{\beta : H^*_{t_4} = -1^{A_{k'}}\}} \frac{\phi(1,\sigma^2;\beta)}{\Phi(1,\sigma^2;\overline{\beta}) - \Phi(1,\sigma^2;\underline{\beta})} d\beta & \text{if } A_k = A_{k'} \text{ and } Q^{A_k}_{t_1} = 0 \\ 0 & otherwise \end{cases} \quad (4)$$

$$p^*_{t_3}(B_k|S_{t_3}) = \begin{cases} \int\limits_{\beta \in \{\beta : H^*_{t_4} = -1^{B_{k'}}\}} \frac{\phi(1,\sigma^2;\beta)}{\Phi(1,\sigma^2;\overline{\beta}) - \Phi(1,\sigma^2;\underline{\beta})} d\beta & \text{if } B_k = B_{k'} \text{ and } Q^{B_k}_{t_1} = 0 \\ 0 & otherwise \end{cases} \quad (5)$$

where $\frac{\phi(1,\sigma^2;\beta)}{\Phi(1,\sigma^2;\overline{\beta}) - \Phi(1,\sigma^2;\underline{\beta})}$ is the probability density function of the β truncated normal distribution with support $\beta \in (\underline{\beta}, \overline{\beta})$. Because there is only one period left in the trading game, the execution probability of a limit order submitted at t_3 is positive only if the order is posted at the best ask ($A_{k'}$) or bid price ($B_{k'}$), and if there are no other orders already standing in the book at that price. As an example, here we discuss the equilibrium strategies for the book opening at t_3 with room for limit orders on both sides of the market, i.e., $p^*_{t_3}(A_k|S_{t_3}) \neq 0$ and $p^*_{t_3}(B_k|S_{t_3}) \neq 0$.[16]

[16] We refer to the Online Appendix A and B for a more detailed discussion.

The trader's optimal strategies are:

$$H^*_{t_3}(\beta|S_{t_2}) = \begin{cases} -1^{B_{k'}} & \text{if } \beta \in [\underline{\beta}, \beta_{-1^{B_{k'}},+1^{A_k},t_3|S_{t_2}}) \\ +1^{A_k} & \text{if } \beta \in [\beta_{-1^{B_{k'}},+1^{A_k},t_3|S_{t_2}}, \beta_{+1^{A_k},+1^{B_k},t_3|S_{t_2}}) \\ +1^{B_k} & \text{if } \beta \in [\beta_{+1^{A_k},+1^{B_k},t_3|S_{t_2}}, \beta_{+1^{B_k},-1^{A_{k'}},t_3|S_{t_2}}) \\ -1^{A_{k'}} & \text{if } \beta \in [\beta_{+1^{B_k},-1^{A_{k'}},t_3|S_{t_2}}, \overline{\beta}] \end{cases} \tag{6}$$

where

$\beta_{-1^{B_{k'}},+1^{A_k},t_3|S_{t_2}} = \frac{B_{k'}}{v} - \frac{p^*_{t_3}(A_k|S_{t_3})}{1-p^*_{t_3}(A_k|S_{t_3})} \cdot \frac{A_k - B_{k'}}{v}$, $\beta_{+1^{A_k},+1^{B_k},t_3|S_{t_2}} = \frac{p^*_{t_3}(A_k|S_{t_3})A_k + p^*_{t_3}(B_k|S_{t_3})B_k}{p^*_{t_3}(A_k|S_{t_3}) + p^*_{t_3}(B_k|S_{t_3})} \cdot \frac{1}{v}$, and $\beta_{+1^{B_k},-1^{A_{k'}},t_3|S_{t_2}} = \frac{A_{k'}}{v} + \frac{p^*_{t_3}(B_k|S_{t_3})}{1-p^*_{t_3}(B_k|S_{t_3})} \cdot \frac{A_{k'} - B_k}{v}$.

These thresholds are derived by taking into account that the trader arriving at the beginning of period t_3 observes the state of the book S_{t_2}. For instance $\beta_{-1^{B_{k'}},+1^{A_k},t_3|S_{t_2}}$ denotes the threshold between a market sell order hitting the best bid price, $B_{k'}$, and a limit sell order posted at the ask price A_k, and it is derived by equating the payoffs of the two orders:

$$(B_{k'} - \beta) = (A_k - \beta) \times p^*_{t_3}(A_k|S_{t_3}) \tag{7}$$

Rearranging terms, we can express the equilibrium condition (7) in terms of POC and NEC:

$$(A_k - B_{k'}) \times p^*_{t_3}(A_k|S_{t_3}) = (B_{k'} - \beta) \times (1 - p^*_{t_3}(A_k|S_{t_3})) \tag{8}$$

The left hand side of this equation is the POC that the trader faces if he chooses a market sell order with immediate execution rather than a limit sell order at A_k. The right hand side is the NEC that the trader faces if instead he chooses a limit sell order rather than a market sell order, and is equal to the loss he realizes if his order is not executed, i.e., the profit he would get from a market order sale, times the probability that the limit sell order is not executed. Therefore, if the execution probability is high enough for NEC to be lower than POC, the trader will submit a limit order. If instead the execution probability is low, he will choose a market order. This trade-off crucially depends on the value of the tick size, τ.

When a trader chooses a limit order, he also has to decide how aggressively to price this order relative to v. The optimal price at which a trader submits a limit order is once more the result of the trade-off between NEC and POC: a more aggressive price implies a higher execution probability due to both the lower risk of being undercut by incoming traders and the fact that the order becomes more attractive for traders on the opposite side of the market. However, this is obtained at the cost of lower profits once the order is executed.

From the equilibrium strategies at t_3, we can derive the execution probabilities for limit orders submitted at t_2 and the corresponding equilibrium strategies. We can reiterate the procedure to derive both the execution probabilities for limit orders submitted at t_1 and the equilibrium strategies at t_1. Due to the recursive structure of the game and because traders are indifferent between orders with a zero execution probability, a unique equilibrium always exists and is defined as follows:

Definition. *An equilibrium is a set of order submission decisions* $\{H_t^*\}$ *and states of the limit order book* $\{S_t\}$ *such that at each period the trader maximizes his payoff* $U(\cdot)$ *according to his belief over the execution probabilities* $p^*(\cdot)$*, i.e.,*

$$\{H_t^* := \arg\max\ U(\cdot \,|\, S_{t-1})\}$$

$$\{S_t := S_{t-1} + h_t^*\}$$

where h_t^* *is defined by (1)*

Without loss of generality we solve the model for $\tau = 0.1$ and for different values of $\sigma = \{0.5, 1, 1.5\}$.[17]

[17] We do not present results for smaller values of σ because no trading becomes a dominant strategy when in the market nearly all traders are willing to supply liquidity.

1.5 Market Quality Indicators

We build standard indicators of market quality, i.e., depth at the inside quotes, spread and volume, to compare the PLB_{LTM} and the PLB_{STM} and assess the effects of a variation in the tick size.[18]

For each period t, the conditional expected depth at the inside quotes for the PLB_{LTM} is measured as the number of shares available at the first level of the book net of the number of shares executed by market orders conditional on the state of the book S_{t-1}:[19]

$$
\begin{aligned}
DIQ_t^{LTM} = Q_{t-1}^{A_1} + Q_{t-1}^{B_1} + E\left[\left(\int_{\beta \in \{\beta : H_t = +1^{A_1}, +1^{B_1}\}} \frac{\phi(1,\sigma^2;\beta)}{\Phi(1,\sigma^2;\overline{\beta}) - \Phi(1,\sigma^2;\underline{\beta})} d\beta \right) | S_{t-1} \right] \\
- E\left[\left(\int_{\beta \in \{\beta : H_t = -1^{A_1}, -1^{B_1}\}} \frac{\phi(1,\sigma^2;\beta)}{\Phi(1,\sigma^2;\overline{\beta}) - \Phi(1,\sigma^2;\underline{\beta})} d\beta \right) | S_{t-1} \right]
\end{aligned}
\tag{9}
$$

The average inside spread, SP_t, is computed as the expected difference between the best ask and the best bid prices conditional on the state of the book S_{t-1}:

$$
SP_t^{LTM} = E[(A_{k'} - B_{k'})|S_{t-1}] \tag{10}
$$

Finally, volume in period t, VL_t, is measured by the expected number of orders executed, conditional on the state of the book S_{t-1}. Because at each period at most one unitary order is submitted, VL_t is computed as the probability that a trader submits a market order at all price levels:

$$
VL_t^{LTM} = E\left[\left(\int_{\beta \in \{\beta : H_t = -1^{A_{k'}}, -1^{B_{k'}}\}} \frac{\phi(1,\sigma^2;\beta)}{\Phi(1,\sigma^2;\overline{\beta}) - \Phi(1,\sigma^2;\underline{\beta})} d\beta \right) | S_{t-1} \right] \tag{11}
$$

Indicators of market quality for the PLB_{STM} are computed in a similar way, but using $j = \{a_{1:5}, b_{1:5}\}$. Note that in our analysis we focus either on the first three periods (of the 4-period model) or the first two periods (of the 3-period model), since in the last period of the trading game

[18] We focus on depth at the inside quotes because it is endogenous to the model, while at the outside quotes the trading crowd provides exogenous depth.

[19] For the PLB_{STM} we consider instead depth at the first two levels, a_l and b_l with $l = 1, 2$, so that we can compare the PLB_{LTM} and the PLB_{STM} since $A_1 = a_2$ and $B_1 = b_2$.

- t_4 and t_3 respectively - traders only consume liquidity. Therefore, from now on all market quality indicators and order submission probabilities are computed as expected conditional averages over t_1, t_2 and t_3 for the 4-period model, and over t_1 and t_2 for the 3-period model. Only for volume, to take into consideration the different trading activity of the asset, we compute the sum over the trading periods considered.

2 Tick Size Variation

In this Section we discuss the effects of a variation in the tick size for the market model with only a PLB. Our aim is to investigate how these effects change with both the trading activity, and the degree of liquidity provision in the PLB. We start by discussing traders' equilibrium order submission strategies and then we move to study the effects of a change in the tick size on the quality of the market by comparing the results obtained from the large tick market with those from the small tick market framework (PLB_{LTM} vs. PLB_{STM}). Following Riccó, Rindi and Seppi (2021), we proxy the intensity of trading activity by the number of trading periods investors can arrive at the market, and we solve each of our model (LTM or STM) as both a 4-period and a 3-period protocol. We proxy the degree of liquidity provision in the PLB by the parameter σ, thus solving each model under three different values of σ. To economize on space, we present results for the sell side only, results for the buy side being symmetric.

2.1 Effects of an Increase in Trading Activity

Figures 1a and 1b show that in a PLB_{LTM} when the trading activity increases, i.e., when we move from a 3-period to a 4-period model, traders provide and take liquidity more aggressively so that both limit and market orders at A_1 or B_1 increase, while limit and market orders at A_2 or B_2 decrease. An increase in trading activity generates two opposite effects on the NEC of limit orders: on the one side, NEC for all limit orders decreases because there are more periods to trade and execute orders; on the other side, NEC for orders at the outside quotes increases because there are now more incoming traders who can potentially gain price priority on the PLB by undercutting with more aggressive limit orders. As a result, liquidity provision in general increases but clusters at the most aggressive quotes, A_1 and B_1 (Table 2).[20] As liquidity provision increases with trading activity, spread decreases and depth at the inside quotes increases. Trading volume increases with trading activity as the number of periods to trade and execute increases.

If we now consider the equilibrium order submission strategies in a small tick market, PLB_{STM}, Figures 1c and 1d show that when the trading activity increases, limit and market orders move away from the outside quotes (a_5 and b_5) and cluster at the first level of the book (a_1 and b_1). So the effects on market quality of an increase in trading activity in the STM are similar to those we observe in the LTM, as summarized in the following Corollary:

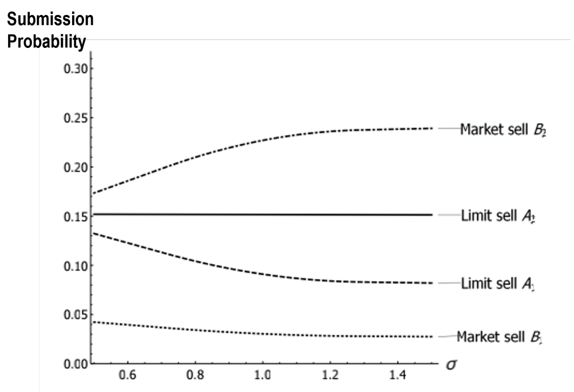
Corollary 1. *When trading activity increases, volume increases and traders become more aggressive in their provision of liquidity, so that spread decreases and depth at the inside quotes increases.*

A general effect of the increase in trading activity is that it moves the order submission strategies towards the inside quotes.

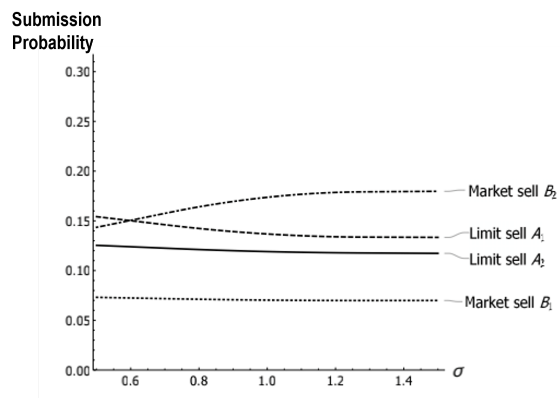
[20]Notice that for tiny values of σ (e.g., $\sigma = 0.5$), liquidity provision decreases in the PLB_{LTM}: when the price grid is large and there are few traders demanding liquidity (σ is small), the improvement in NEC due to the extra period is limited and the POC of a more aggressive limit order is high, so that traders decrease their limit orders overall. Nevertheless, as liquidity provision becomes more aggressive and clusters at A_1 and B_1, we observe the same effects on market quality indicators (spread, depth, volume) as for the other parameterizations.

Figure 1: Large and Small Tick Single Market Model.
The figures show the equilibrium trading strategies as a function of traders' dispersion of belief, σ, in the PLB_{LTM} with two prices on the ask ($A_1 < A_2$) and two prices on the bid side of the market ($B_1 > B_2$), and in the PLB_{STM} with five prices on the ask ($a_1 < a_2 < a_3 < a_4 < a_5$) and five prices on the bid side of the market ($b_1 > b_2 > b_3 > b_4 > b_5$). Strategies for the 3-period model are expected conditional averages over periods t_1 and t_2, while strategies for the 4-period model are expected conditional averages over periods t_1, t_2 and t_3. We present strategies for the sell side, the buy side being symmetric.

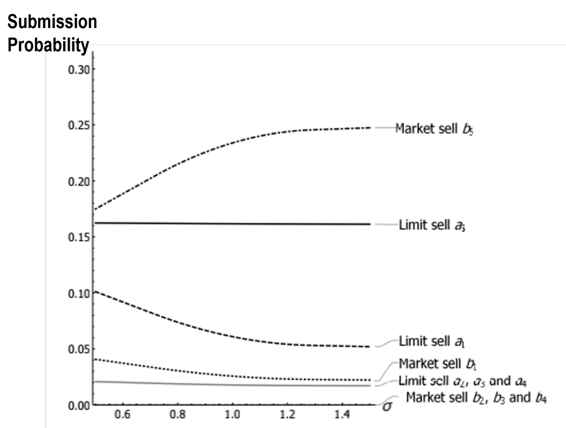
(a) $\mathbf{PLB_{LTM}}$: 3-period model



(b) $\mathbf{PLB_{LTM}}$: 4-period model



(c) $\mathbf{PLB_{STM}}$: 3-period model



(d) $\mathbf{PLB_{STM}}$: 4-period model

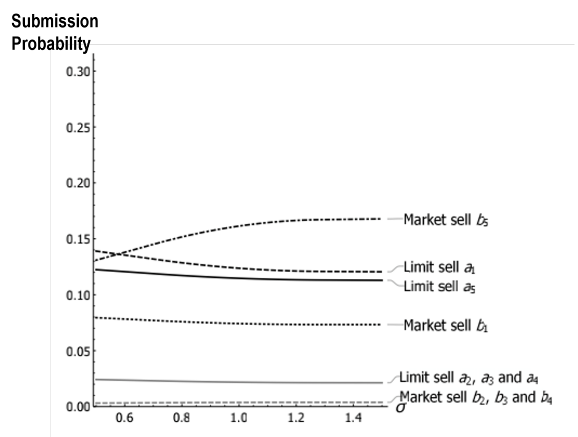


Table 2: Market Quality Indicators for a PLB only and a PLB&CN

This table reports values for spread, inside quotes depth, volume and liquidity provision for both a market with only a PLB (Panel A) and a market in which the PLB competes with a CN (Panel B). In this last case, we include also values for consolidated volume, i.e., the sum of volume of PLB and CN. Values are presented for both a large tick PLB market (LTM) and a small tick PLB market (STM). Moreover, we consider both a 4-period model ($t = 4$) and a 3-period model ($t = 3$) as a function of traders' dispersion of belief, σ. For the 4-period model values are expected conditional averages computed over periods $\{t_1, t_2, t_3\}$, while for the 3-period model expected conditional averages are computed over periods $\{t_1, t_2\}$.

		Spread			Inside Quotes Depth			Volume			Liquidity Provision			Consolidated Volume		
		$\sigma = 0.5$	$\sigma = 1$	$\sigma = 1.5$	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 1.5$	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 1.5$	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 1.5$	$\sigma = 0.5$	$\sigma = 1$	$\sigma = 1.5$
A. PLB																
$t = 4$	LTM	0.238	0.246	0.247	0.307	0.271	0.264	1.300	1.463	1.497	0.559	0.512	0.501			
	STM	0.224	0.233	0.235	0.279	0.244	0.237	1.281	1.436	1.469	0.571	0.520	0.509			
$t = 3$	LTM	0.258	0.272	0.275	0.193	0.126	0.111	0.863	1.029	1.067	0.568	0.485	0.467			
	STM	0.256	0.273	0.277	0.162	0.096	0.082	0.863	1.038	1.078	0.569	0.481	0.461			
B. PLB&CN																
$t = 4$	LTM	0.295	0.295	0.295	0.028	0.027	0.027	0.887	1.159	1.222	0.330	0.327	0.328	1.297	1.480	1.522
	STM	0.296	0.298	0.298	0.000	0.000	0.000	0.801	1.011	1.058	0.312	0.293	0.290	1.272	1.435	1.471
$t = 3$	LTM	0.296	0.296	0.297	0.022	0.018	0.017	0.715	0.972	1.031	0.405	0.402	0.401	0.857	1.040	1.082
	STM	0.298	0.299	0.299	0.000	0.000	0.000	0.710	0.960	1.017	0.390	0.383	0.381	0.862	1.044	1.085

2.2 Effects of an increase in σ

Figures 1a and 1b show that in a PLB_{LTM} when σ increases the main effect on traders' order submission strategies is that in equilibrium investors gradually switch from limit to market orders. When σ is small, traders are eager to supply liquidity and the market is liquid to the point that there is a clustering of liquidity at the best bid and offer, A_1 and B_1. This is because the β distribution of traders is squeezed towards the mean and traders know that fewer people are willing to take liquidity, so they need to post aggressive limit orders. As σ increases, traders will have more heterogeneous gains from trade and switch from limit orders at A_1 and B_1 to market orders at the outside quotes, A_2 or B_2. Because market orders increase, volume increases with σ (Table 2).[21] At the same time spread widens as the demand for liquidity increases compared to the supply of liquidity. Similarly, as limit orders at the inside quotes decrease, depth at the inside quotes decreases with σ.

When we investigate how the dynamics of the equilibrium strategies in a PLB_{STM} change following an increase in σ, we observe that, as in the PLB_{LTM}, traders gradually switch from limit to market orders (Figures 1c and 1d), and orders start clustering at the outside quotes (a_5 and b_5). We therefore conclude that the effects of an increase in σ are very similar in the two markets (Table 2) leading to the following Corollary 2:

Corollary 2. *When σ increases, liquidity demand increases and liquidity supply decreases, so that spread and volume increase, and depth at the inside quotes decreases.*

A general effect of the increase in σ is that it moves the order submission strategies towards the outside quotes.

2.3 $\mathbf{PLB_{LTM}}$ vs. $\mathbf{PLB_{STM}}$: Effects of a Tick Size Variation

We have now set the stage to answer our main question of how a change in the tick size affects traders' strategies and market quality. To present our results, we focus on a switch from a PLB_{LTM} to a

[21] Recall that in our model orders are of unitary size so that volume and market orders coincide.

PLB_{STM}, i.e., a decrease in the tick size. Moreover, based on our Corollaries 1 and 2, we consider that high trading activity and small σ are consistent with higher market liquidity at the inside quotes, whereas low trading activity and large σ are consistent with lower market liquidity at the inside quotes.

2.3.1 Effects on Liquidity Provision

In equilibrium a decrease in the tick size generates three main effects. First, undercutting becomes less expensive because to gain price priority a limit order has to give away a smaller price improvement. We refer to this effect as the "undercutting effect". From the point of view of a trader posting a limit order, the "undercutting effect" increases NEC as a higher probability of undercutting results in a lower probability of limit order execution, and the increase in NEC decreases the trader's willingness to supply liquidity.[22] The effect is strongest (weakest) when trading activity is low (high) and σ is large (small), as there is more room in the book to undercut the existing quotes.

Second, limit orders clustering at the few price levels of the PLB_{LTM} tend to spread over the larger number of price levels of the PLB_{STM}. This means that traders willing to post limit orders have now more chances to be at the top of the queue of the new price levels, and therefore this effect, differently from the "undercutting effect", decreases NEC, incentivizing liquidity provision. We refer to this effect as the "layering effect". It is strongest (weakest) when trading activity is high (low) and σ is small (large), so that markets are heavily populated by agents willing to supply liquidity who could potentially create long queues on the PLB_{LTM}.

Finally, a smaller tick size generates a mechanical decrease of the inside spread and for traders deciding whether to post a market or a limit order at the top of the book it lowers the POC of switching from limit to market order thus increasing traders' incentives to take liquidity. This "mechanical effect" is strongest when trading activity is high and σ is small, because trading con-

[22] Dyhrberg, Foley and Svec (2019) show that a change in tick size significantly affects undercutting in the Kraken cryptocurrency exchange.

centrates at the first level of the book and the inside spread is binding, whereas it is weakest when trading activity is low and σ is large, because limit orders do not generally cluster at the smallest possible price increment.

Table 3 summarizes the relevance of the three main effects of a change in the tick size depending on the trading activity and on the dispersion of beliefs, σ. In line with our Corollaries 1 and 2, we consider liquid a book characterized by both high trading activity and small σ and we consider illiquid a book characterized by both low trading activity and large σ. Therefore, we will concentrate our analysis on Regions A and D of Table 3. The two cases with mixed liquidity scenarios, i.e., low trading activity and small σ (Region B), or high trading activity and large σ (Region C), cannot be considered representative of liquid or illiquid markets because our two liquidity parameters are moving in opposite directions, so that the overall effect is mixed.

Going back to our analysis, the effects of a decrease in the tick size on traders' order submission strategies depend on the liquidity of the stock. In markets characterized by a concentration of liquidity at the inside quotes, the "undercutting" effect is the weakest (Region A in Table 3) as there is little room for undercutting, whereas the "layering effect" is the strongest as the reduction in the tick size allows traders to spread liquidity on the thinner price grid. In addition, for liquid stocks the "mechanical effect" is the strongest as a reduction in the tick size increases traders' incentive to hit the liquidity that becomes available at the new inside quotes thus increasing the execution probability of limit orders posted at the inside quotes. In illiquid markets instead, in which liquidity does not tend to concentrate at the inside quotes, the "undercutting effect" is the strongest (Region D in Table 3) and the "layering effect" is the weakest as there is no clustering of liquidity at the inside quotes to spread across the new price levels. As a result also the "mechanical effect" is the weakest as there are fewer orders posted at the new inside quotes to hit via market orders.

This leads to the following Corollary (Table 4):

Corollary 3. *When all else equal we evaluate the effects of a reduction in the tick size by comparing*

Table 3: Tick Size Change: Main Effects

This table summarizes how the magnitude of the main effects generated by a tick size change depends on trading activity, 4-period model ($t = 4$) vs. 3-period model ($t = 3$), and dispersion of beliefs, σ small vs. σ large. The first magnitude refers to σ, the second one to trading activity. For example, in Region B "undercutting effect: weak-strong" implies that the effect weakens when σ is small but strengthens for $t = 3$. Notice that in the framework with only a PLB we observe effects 1-3, while in the framework with both a PLB and a CN we observe effects 1-4.

	t=4	t=3
Small σ	Region A: Liquid market 1. Undercutting effect: weak-weak 2. Layering effect: strong-strong 3. Mechanical effect: strong-strong 4. Migration effect: strong-strong	Region B: Mixed 1. Undercutting effect: weak-strong 2. Layering effect: strong-weak 3. Mechanical effect: strong-weak 4. Migration effect: strong-weak
Large σ	Region C: Mixed 1. Undercutting effect: strong-weak 2. Layering effect: weak-strong 3. Mechanical effect: weak-strong 4. Migration effect: weak-strong	Region D: Illiquid market 1. Undercutting effect: strong-strong 2. Layering effect: weak-weak 3. Mechanical effect: weak-weak 4. Migration effect: weak-weak

a PLB_{LTM} with a PLB_{STM}, changes in traders' order submission strategies depend on the liquidity of the stock, which in turn depend on both the trading activity and the dispersion of beliefs, σ:

- *When the market is liquid with high trading activity and small σ, liquidity provision increases and therefore liquidity demand decreases.*

- *When the market is illiquid with low trading activity and large σ, liquidity provision decreases and therefore liquidity demand increases.*

In our PLB traders can either supply or demand liquidity or decide not to trade, so when liquidity supply increases, generally liquidity demand decreases. A reduction in the tick size incentivizes liquidity provision in liquid stocks characterized by intense trading activity and stronger incentive to provide liquidity; whereas a reduction in the tick size disincentivizes liquidity provisions in illiquid stocks.

Table 4: Summary on the Effects of a Tick Size Reduction: PLB only and PLB&CN
This table summarizes the effects of a reduction in the tick size, i.e., a switch from a large tick PLB market (LTM) to a small tick PLB market (STM), on market quality indicators for both a market with only a PLB and a market in which the PLB competes with a CN ($PLB\&CN$). We focus on a liquid stock ($t = 4$ and $\sigma = 0.5$) and an illiquid stock ($t = 3$ and $\sigma = 1.5$), so on Region A and Region D of Table 3, respectively.

		PLB		**PLB&CN**	
		Liquid	Illiquid	Liquid	Illiquid
Spread	LTM	0.238	0.275	0.295	0.297
	STM	0.224	0.277	0.296	0.299
	Dif (STM-LTM)	-0.014	0.002	0.001	0.002
Depth at Inside Quotes	LTM	0.307	0.111	0.028	0.017
	STM	0.279	0.082	0.000	0.000
	Dif (STM-LTM)	-0.028	-0.029	-0.028	-0.017
Volume	LTM	1.300	1.067	0.887	1.031
	STM	1.281	1.078	0.801	1.017
	Dif (STM-LTM)	-0.019	0.011	-0.086	-0.014
Liquidity Provision	LTM	0.559	0.467	0.330	0.401
	STM	0.571	0.461	0.312	0.381
	Dif (STM-LTM)	0.012	-0.006	-0.018	-0.020
Consolidated Volume	LTM			1.297	1.082
	STM			1.272	1.085
	Dif (STM-LTM)			-0.025	0.003

2.3.2 Effects on Market Quality

After analyzing the effects of a reduction in the tick size on liquidity provision, we now discuss the effects of a tick size change on market quality.

We consider first the effects on spread. When the tick size is reduced, the inside spread may be potentially smaller as on the new finer price grid the difference between the best potential ask and the best potential bid will decrease "mechanically". However, the inside spread will be smaller in the PLB_{STM} only if, following the reduction in the tick size, order submission strategies - of both limit and market orders - will actually move to the new more aggressive price levels on the finer grid. When the market is liquid, with high trading activity and low σ, Corollary 1 and Corollary 2 show that it is also characterized by a clustering of limit and market orders at the inside quotes. This means that when the tick size decreases (moving from the PLB_{LTM} to the PLB_{STM}), limit orders will move from A_1 and B_1 to the new more aggressive price levels a_1 and b_1 ("layering effect") and therefore the inside spread will be effectively smaller (Table 4). When instead the market is illiquid, Corollary 1 and 2 show that both limit and market orders will cluster away from the inside quotes. Hence, a reduction of the tick size will have the main effect of increasing the risk of undercutting for limit orders posted at the outside quotes. As a result, the limit and market orders will not move down to populate the new more aggressive price levels but rather will consolidate away from the inside quotes so that the inside spread in the PLB_{STM} will be larger than in the PLB_{LTM}.

The effects of a reduction in the tick size on the depth at the inside quotes is a direct consequence of Corollary 3. When the stock is liquid, the "layering effect" induces traders to spread the limit orders from the inside quotes of the PLB_{LTM} to the new more aggressive price levels of the PLB_{STM}, thus decreasing the queue at the new inside spread.[23] In addition, the "mechanical reduction" in the inside spread induces investors - who evaluate the trade-off between submitting a limit order or posting a market order crossing the inside spread - to privilege market orders which are now cheaper thus consuming liquidity at the top of the book. As a result, depth at the inside quotes decreases. When the market is illiquid instead, both the "layering effect" and the "mechanical effect" weaken, and the "undercutting effect" strengthens. Therefore, investors consolidate the clustering of limit orders away from the inside quotes with the result that depth at the inside quotes decreases even more than for a liquid market.

The difference in traders' equilibrium strategies in the PLB_{LTM} and the PLB_{STM} also explains the different pattern in volume. As stated in Corollary 3, a reduction in the tick size increases (decreases) liquidity supply and decreases (increases) liquidity demand when the market is liquid (illiquid), so that volume decreases when the market is liquid whereas it increases when it is illiquid.

[23] In our 3 and 4-period models the queues at each price level can never be long as they may involve two orders at the most. Therefore, we proxy these queues with the probability of order submission at each price level.

The following Proposition summarizes the results obtained from the comparative static analysis of the PLB_{LTM} and PLB_{STM}:

Proposition 1. *When all else equal we consider a reduction in the tick size by comparing a PLB_{LTM} with a PLB_{STM}:*

- *When the market is liquid with high trading activity and small σ, spread at the inside quotes decreases, depth at the inside quotes worsens, and volume decreases.*
- *When the market is illiquid with low trading activity and large σ, spread at the inside quotes increases, depth at the inside quotes worsens, and volume increases.*

The results obtained so far allow us to generate an empirical prediction for a market that does not face competition from other trading venues.

Empirical Prediction 1. Following a reduction in the tick size, we expect for all stocks depth at the inside quotes deteriorates. For liquid stocks we expect spread at the inside quotes to improve and volume to deteriorate, whereas for illiquid stocks we expect spread at the inside quotes to deteriorate and volume to improve.

3 Extension: PLB&CN Market

Having analysed the effects of an increase of the tick size in a stand alone PLB, we now investigate whether our results change in a framework in which a PLB competes with a CN ($PLB\&CN$). We study this extension as we need to understand whether inflows and outflows of orders across the PLB and the CN may influence the overall effects of a change in the tick size. Analyzing the possible impact of this leakage of order flows is important as most of markets nowadays are fragmented. Adding competition from a CN is a simple way of capturing a more general setting with competition from another venue as modelled in Panayides, Rindi and Werner (2019).

3.1 CN Market

The CN is modelled as a transparent market which enforces time priority and executes orders continously at the midquote of the PLB, $(A_{k'} + B_{k'})/2$.[24] Traders can supply liquidity to the CN by submitting a limit order to buy or to sell one share, $H_t = \{+1^{B_{CN}}, +1^{A_{CN}}\}$, that will stand in the CN until executed. Otherwise, traders can demand liquidity to the CN by submitting a market order to buy or to sell, $H_t = \{-1^{A_{CN}}, -1^{B_{CN}}\}$. Note that, differently from the PLB, in a CN there is no trading crowd offering liquidity.[25] Therefore a market order can be submitted only if a trader previously submitted to the CN a limit order of opposite sign that has not been executed yet. The trader's strategy space becomes:

$$H_t = \{-1^{A_{k'}}, +1^{A_k}, 0, +1^{B_k}, -1^{B_{k'}}, -1^{A_{CN}}, +1^{A_{CN}}, +1^{B_{CN}}, -1^{B_{CN}}\} \tag{12}$$

The state of the market observed by the trader at each period t now includes the state of both the PLB and the CN: $S_t = [Q_t^{A_2}, Q_t^{A_1}, Q_t^{B_1}, Q_t^{B_2}, Q_t^{A_{CN}}, Q_t^{B_{CN}}]$, where $Q_t^{A_{CN}}$ and $Q_t^{B_{CN}}$ represent, respectively, the number of shares to sell or to buy available on the CN.[26] It follows that the change h_t in the state of the market induced by the trader's strategy H_t becomes:

$$h_t = [h_t^{A_2}, h_t^{A_1}, h_t^{B_1}, h_t^{B_2}, h_t^{A_{CN}}, h_t^{B_{CN}}] = \begin{cases} [\pm 1, 0, 0, 0, 0, 0] & \text{if } H_t = \pm 1^{A_2} \\ [0, \pm 1, 0, 0, 0, 0] & \text{if } H_t = \pm 1^{A_1} \\ [0, 0, 0, 0, 0, 0] & \text{if } H_t = 0 \\ [0, 0, \pm 1, 0, 0, 0] & \text{if } H_t = \pm 1^{B_1} \\ [0, 0, 0, \pm 1, 0, 0] & \text{if } H_t = \pm 1^{B_2} \\ [0, 0, 0, 0, \pm 1, 0] & \text{if } H_t = \pm 1^{A_{CN}} \\ [0, 0, 0, 0, 0, \pm 1] & \text{if } H_t = \pm 1^{B_{CN}} \end{cases} \tag{13}$$

[24] For ease of exposition and for consistency purposes with the previous Section, we assume that when not otherwise specified the PLB is a PLB_{LTM}.

[25] Because the CN executes at the midquote of the PLB and $A_{CN} = B_{CN}$, it is impossible to offer liquidity on both sides of the market as limit orders would be instantaneously executed against each other.

[26] Given that the CN executes orders continuously, and since $A_{CN} = B_{CN}$, either $Q_t^{A_{CN}} > 0$ and $Q_t^{B_{CN}} = 0$, or $Q_t^{A_{CN}} = 0$ and $Q_t^{B_{CN}} > 0$, or $Q_t^{A_{CN}} = Q_t^{B_{CN}} = 0$.

The state of the market is still characterized by Equation (2) but now h_t is given by Equation (13). Consistently with the single PLB framework, we assume that when a trader arrives at t_1 both the PLB and the CN are empty.

3.2 Order Submission Decision and Market Equilibrium

In the $PLB\&CN$ framework, a trader needs to choose not only an order type but also a trading venue. We refer to Section 1.3 for a discussion of the payoffs of the strategies available on the PLB, and we focus on the payoffs of the CN strategies that are presented in Table 5. Market CN orders are executed immediately provided there is liquidity available in the CN, while limit CN orders payoffs depend on their execution probability that we denote by $p_t(A_{CN}|S_t)$ and $p_t(B_{CN}|S_t)$ for a limit sell and a limit buy order, respectively. Compared to PLB market orders, CN market orders are executed at the midquote and, therefore, have lower POC. If liquidity is available on the CN, traders will always prefer $H_t = -1^{A_{CN}}$ to $H_t = -1^{A_{k'}}$, and $H_t = -1^{B_{CN}}$ to $H_t = -1^{B_{k'}}$.[27] When comparing PLB to CN limit orders, traders face the usual trade-off between NEC and POC: CN limit orders have lower NEC than PLB limit orders because they are more attractive for liquidity takers, however, they have a higher POC because the more aggressive price implies that orders are executed at the midquote. So, if the execution probability of orders posted at the midquote is high enough to compensate the higher POC, the trader will opt for a limit order in the CN. If instead the execution probability at the midquote is low, the trader will opt for a limit order in the PLB which has lower POC.

The market equilibrium is derived by backward induction following the same logic presented in Section 1.4: traders use now information from both the state of the PLB and the CN to compute the expected payoffs of the different orders, and then optimally select the trading venue and the trading strategy that maximizes their payoff given their personal evaluation of the asset, β. We

[27] Notice that the CN will not intercept all market orders because not always there will be liquidity available to consume. Therefore, limit PLB orders are still competitive, since they have lower POC than CN limit orders.

Table 5: Crossing Network - Order Submission Strategy Space.
This table reports the payoffs, $U(\cdot)$, of the order strategies H_t available on a crossing network (CN) competing with a large tick public limit order book (PLB_{LTM}).

Strategy	H_t	$U(\cdot)$
Market Sell Order	$-1^{B_{CN}}$	$(A_{k'} + B_{k'})/2 - \beta v$
Limit Sell Order	$+1^{A_{CN}}$	$p_t(A_{CN}\|S_t) \cdot [(A_{k'} + B_{k'})/2 - \beta v]$
No Trade	0	0
Limit Buy Order	$+1^{B_{CN}}$	$p_t(B_{CN}\|S_t) \cdot [\beta v - (A_{k'} + B_{k'})/2]$
Market Buy Order	$-1^{A_{CN}}$	$\beta v - (A_{k'} + B_{k'})/2$

refer again to Section 1.4 for a definition of the equilibrium of the trading game.

To compare a large and a small tick market in the $PLB\&CN$ framework, apart from the standard indicators of market quality presented in Section 1.5, i.e., depth at the inside quotes, spread and volume in the PLB, we define a measure of volume in the CN, VL_t^{CN}, given by:

$$VL_t^{CN} = E\left[\left(\int_{\beta \in \{\beta : H_t = -1^{A_{CN}}, -1^{B_{CN}}\}} \frac{\phi(1, \sigma^2; \beta)}{\Phi(1, \sigma^2; \overline{\beta}) - \Phi(1, \sigma^2; \underline{\beta})} d\beta \right) | S_{t-1} \right] \tag{14}$$

We also define consolidated volume as the sum of volume in the PLB_{LTM} and in the CN:

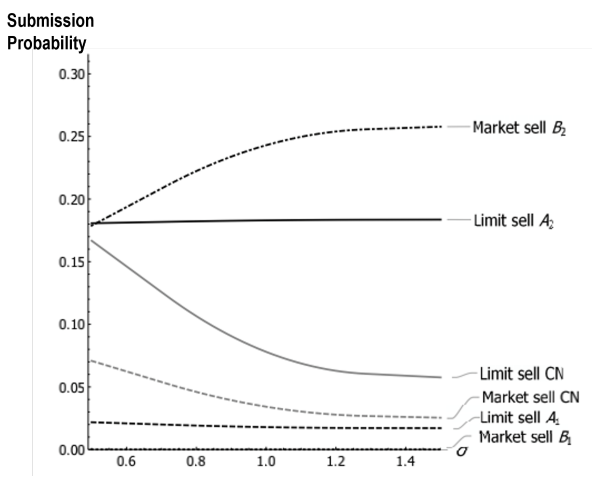
$$VL_t^{CON} = VL_t^{LTM} + VL_t^{CN} \tag{15}$$

3.3 The migration effect

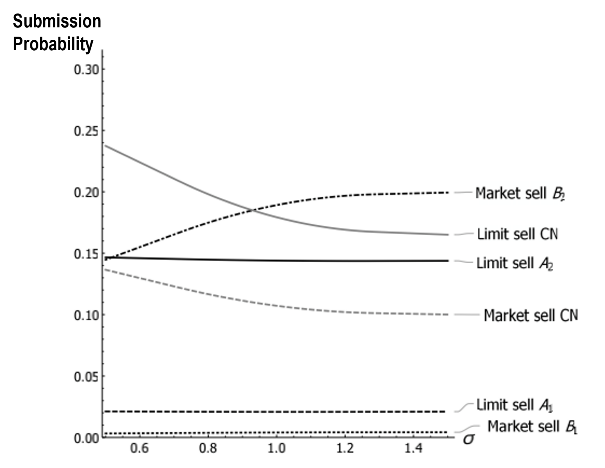
We now analyze whether the equilibrium trading strategies change once agents are given the option to submit orders to the CN. For the PLB_{LTM}, by comparing Figures 1a-1b and Figures 2a-2b we note that there is just one main difference between the single PLB and the $PLB\&CN$ frameworks: traders switch from $H_t = \{+1^{A_1}, +1^{B_1}\}$ and $H_t = \{-1^{A_1}, -1^{B_1}\}$ to $H_t = \{+1^{A_{CN}}, +1^{B_{CN}}\}$ and $H_t = \{-1^{A_{CN}}, -1^{B_{CN}}\}$, meaning that when a CN is available, traders move their most aggressive orders -both limit and market- to the CN. Posting limit orders in the CN allows traders to jump the queue at the PLB's price levels and improves NEC; moreover, executing market orders at the midquote in the CN is cheaper than crossing the spread in the PLB, decreasing POC. Therefore, in equilibrium the CN attracts both market and limit orders creating a "migration effect".[28]

Figure 2: Large and Small Tick PLB&CN Market Model.
The figures show the equilibrium trading strategies in the $PLB\&CN$ as a function of traders' dispersion of belief, σ. The PLB_{LTM} has two prices on the ask ($A_1 < A_2$) and two prices on the bid side of the market ($B_1 > B_2$), while the PLB_{STM} has five prices on the ask ($a_1 < a_2 < a_3 < a_4 < a_5$) and five prices on the bid side of the market ($b_1 > b_2 > b_3 > b_4 > b_5$). Strategies for the 3-period model are over periods t_1 and t_2, while strategies for the 4-period model are over periods t_1, t_2 and t_3. We present strategies for the sell side, the buy side being symmetric.

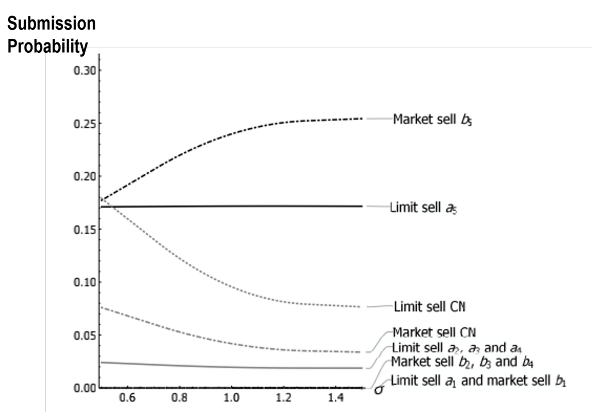
(a) PLB&CN$_{\text{LTM}}$: 3-period model



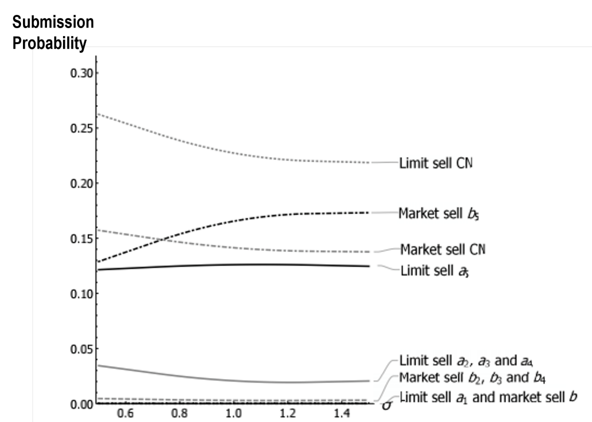
(b) PLB&CN$_{\text{LTM}}$: 4-period model



(c) PLB&CN$_{\text{STM}}$: 3-period model



(d) PLB&CN$_{\text{STM}}$: 4-period model



[28] Notice that differently from the framework with a PLB only, in which when liquidity supply increases, generally liquidity demand decreases, once we introduce a CN both liquidity demand and supply on the PLB can decrease or increase at the same time.

We now focus on how trading activity and the dispersion of beliefs, σ, affect this new "migration effect". When trading activity increases, traders become more aggressive in liquidity provision and spread tightens (Corollary 1). The stronger competition among liquidity suppliers increases NEC for aggressive limit orders and magnifies incentives to jump the queue by switching to the CN. As liquidity increases in the CN, impatient traders will take advantage of the lower POC of a CN market order and switch to the CN too. Therefore, a high trading activity magnifies the "migration effect". Similarly, when σ is reduced, liquidity provision increases and spread tightens because the market becomes heavily populated by agents willing to supply liquidity aggressively (Corollary 2). As for an increase in trading activity, the stronger competition in liquidity supply pushes both limit and market orders to the CN. Hence, the "migration effect" is stronger when σ is small (Table 3).

For the PLB_{STM}, by comparing Figures 1c-1d and Figures 2c-2d we note a similar effect: traders move limit and market orders from the first levels of the single PLB, a_1 and b_1, to the CN, and the "migration effect" increases with trading activity and decreases with σ.

Corollary 4. *When all else equal we compare a PLB to a $PLB\&CN$, both aggressive liquidity provision and aggressive liquidity demand on the PLB migrate to the CN. This effect is increasing in trading activity and decreasing in σ.*

3.4 PLB&CN$_{\mathbf{LTM}}$ vs. PLB&CN$_{\mathbf{STM}}$: Effects of a Tick Size Variation

We now discuss whether the effects of a reduction in the tick size may change once we introduce a CN by analyzing a switch from a $PLB\&CN_{LTM}$ to a $PLB\&CN_{STM}$. When the PLB competes with a CN the "undercutting effect", the "layering effect" and the "mechanical effect" are still at work. However, now we need to consider their interaction with the "migration effect". Table 3 summarizes how the magnitude of these four effects depends on trading activity and on σ. Also for a $PLB\&CN$, we will focus on Regions A and D, i.e., on a liquid book characterized by both high trading activity and small σ and on an illiquid book characterized by both low trading activity and large σ.

When the market is liquid, the “layering effect” (positive effect on limit orders) is strong but it is now overwhelmed by the “migration effect” (negative effect on limit orders) so that liquidity provision on the PLB decreases (Table 4). As a result, despite the mechanical decrease of the spread, in the $PLB\&CN$ framework the spread slightly increases after a tick size reduction. The incentive to spread limit orders over more price levels due to the “layering effect” combined with the migration of aggressive limit orders to the CN, implies that depth at the inside quotes falls. Finally, volume on the PLB decreases because aggressive market orders migrate to the CN too since the CN executes at the midquote.

When the market is illiquid, a reduction in the tick size generates the same effects we discussed for the single PLB, and the reduced cost of “undercutting” decreases investors’ incentive to supply liquidity as opposed to demand liquidity as in the PLB framework. Consistently with the “migration effect”, inside liquidity demand increases, spread widens, depth at the inside quotes decreases and volume decreases (Table 4).

Note that a strong “layering effect” - as opposed to a strong “undercutting effect” - is the reason why in the single PLB protocol spread improves for liquid stocks and not for illiquid stocks. In the $PLB\&CN$ framework, spread does not improve either for liquid or for illiquid stocks due to the “migration effect”. However, the deterioration of spread for liquid stocks is still attenuated by the “layering effect” with the result that such a deterioration is smaller than for illiquid stocks. The same line of reasoning explains the relative effect of the reduction in the tick size on depth at the inside that tends to decrease more for liquid than for illiquid stocks. The intuition is subtle here as when the tick size decreases in the $PLB\&CN$ limit orders posted at the new potential inside quotes almost disappear for both liquid and illiquid stocks, since investors are attracted by the cheaper CN midquote. The somewhat stronger effect we notice for liquid stocks - however - is due to the fact that before the reduction in the tick size - in the LTM - the incentive to post orders at the inside quotes was stronger for liquid than for illiquid stocks, as explained in Corollary 1 and 2.

The pattern of the market order migration from the PLB to the CN also explains the pattern of the consolidated volume (sum of market orders executed both in the PLB and in the CN). When the market is liquid, consolidated volume decreases with the reduction of the tick size as the "layering effect", which encourages liquidity provision across the new price levels, is strong enough to counterbalance the increase of market orders towards the cheaper CN. When instead the market is illiquid, the "undercutting effect" prevails, which act as a disincentive to supply liquidity as opposed to the "layering effect", with the result that the "migration effect" is magnified rather than being attenuated and consolidated volume slightly increases.

The following Proposition summarizes how competition from a CN impacts the effects of a reduction in the tick size in the $PLB\&CN$ framework:

Proposition 2. *When all else equal we consider a reduction in the tick size in the $PLB\&CN$ framework in which a PLB competes with a CN, we observe that:*

- *Spread at the inside quotes increases, depth at the inside quotes worsens, and volume decreases.*
- *Consolidated volume of PLB and CN decreases when the market is liquid, while it slightly increases when the market is illiquid.*

All in all, the effects of a reduction in the tick size in the $PLB\&CN$ compared to the single PLB differ due to the migration of both limit and market orders to the cheaper CN that following the reduction in the tick size increases in particular for liquid stocks, and induces spread to deteriorate not only for illiquid but also for liquid stocks. The effect on volume for liquid stocks is negative for both the PLB and the $PLB\&CN$; however, in the $PLB\&CN$ volume decreases due to the migration to the CN rather than as a consequence of the increase in liquidity provision in the PLB.

Our competing market is stylized and aims to show that due to the migration channel the effects of a tick size change in a framework with competition may be different compared to the effects discussed for a single market. The stronger the migration of order flows, the stronger the deterioration of spread, liquidity provision and volume is. Notice that in our set-up the competing market is a lit crossing network executing at the midquote, hence its price competition is very aggressive and represents an extreme case of potential migration to a competing venue following a reduction in the tick size. Clearly, if the competing venue were another PLB or a dark pool, the migration effect would be weaker and possibly - in the case of a competing PLB - also revert with market orders moving to the cheaper PLB that reduced its tick size.[29] Consequently the absolute value of the variation in spread and volume could change depending on the intensity of the "migration effect".

The extreme competition from the CN with the strong "migration effect" results in spread increasing also for liquid stocks as opposed to spread increasing only for illiquid stocks following a tick size reduction as in the single PLB framework. However, the relatively larger improvement (or smaller deterioration) of spread and liquidity provision obtained in the PLB framework for liquid stocks compared to illiquid stocks is confirmed in the $PLB\&CN$; likewise, the relatively stronger negative effect on volume for liquid stocks is also confirmed. In addition, the migration effect in the $PLB\&CN$ protocol makes the negative effect on depth stronger for liquid stocks. Taken together these results lead to our second empirical prediction:

Empirical Prediction 2. *Following a reduction in the tick size in a PLB that competes with a CN, we expect the relative effect on market quality to be stronger for liquid stocks. More precisely, with migration we expect - for liquid stocks - a stronger improvement (or smaller deterioration) of spread and liquidity provision, and a stronger deterioration of depth and volume.*

[29] Buti, Rindi and Werner (2017) suggest when analyzing a PLB competing with a dark CN that opacity is just a friction: the unobservability of the dark venue imbalance adds an inference problem to the trader's optimization problem. As a result, investors become more reluctant to send orders to the dark CN in fear of no execution.

4 Empirical Analysis

Our model generates testable empirical predictions for the effects on market quality of changes in the tick size that differ for liquid compared to less liquid stocks. In this section, we proceed to test these predictions based on two samples: the 2014 TSE reductions of tick sizes, and the 2018 end of the USTSP. To map our theoretical predictions into the data, we use TSC stocks, i.e., stocks with at or below median pre-event average quoted spread to tick, to proxy for liquid stocks and TSU stocks, i.e., stocks with above median pre-event average quoted spread to tick, to proxy for less liquid stocks. Figure 3 supports our assumption and shows in both experiments that TSC stocks are characterized by both larger liquidity provision (not only smaller spread but also larger depth) and higher trading activity (volume) compared to TSU stocks.[30]

4.1 The TSE Tick Size Reduction

The TSE tick size reduction targeted highly liquid large capitalization Japanese stocks that were members of the TOPIX 100 index because the minimum ¥1 price increment was binding for several low price stocks.[31] The program included two phases: the first phase launched on January 14, 2014, lowered tick sizes for TOPIX stocks with price above ¥3,000, while the second phase launched on June 22, 2014, lowered tick sizes for TOPIX stocks with price above ¥5,000.[32]

For our analysis, we use Refinitiv (former Thomson Reuters Tick History - TRTH) microsecond time-stamped data on trades and quotes to calculate measures of market quality daily for each venue.[33] Our tests are based on daily panel data for a 4-month window around the tick size experiment and we analyze separately TSC and TSU stocks. Notice that TOPIX 100 stocks are

[30]The details of both the Japanese and U.S. experiments, sample construction, as well as the summary statistics are in the Online Appendix E and F, respectively.

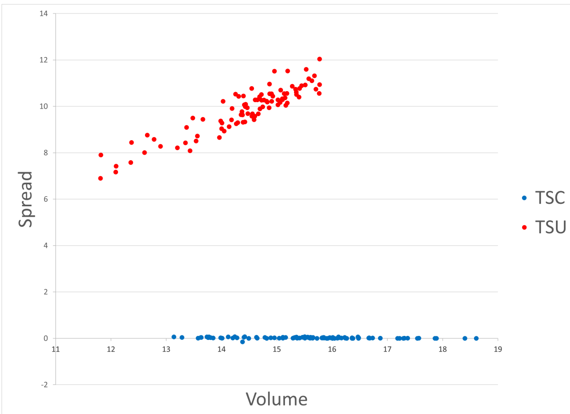
[31]TSE uses a tick-size schedule, where the tick size is dynamically adjusted depending on the stock's price.

[32]During the same period, a few stocks experienced a switch to smaller lot sizes. As a robustness check, we re-run our analysis excluding all stocks that either reduced their lot size, or did a stock split, in the relevant time window. All results are robust and qualitatively very similar. We do not present such results in the paper but they are available upon request from the authors.

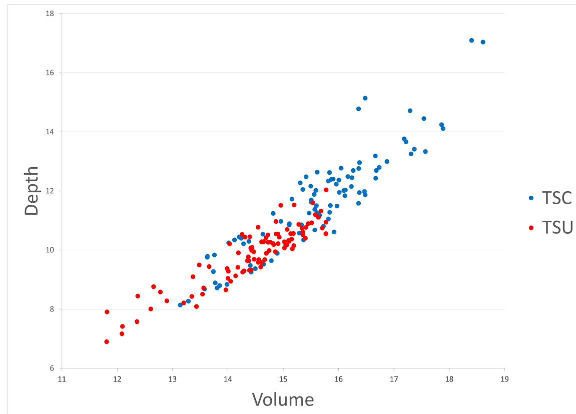
[33]We use relative quoted spread defined as the quoted spread divided by the midquote, and logarithm of depth and volume to mitigate the significant cross-sectional heterogeneity in the magnitude of these variables.

Figure 3: Tick size constrained/unconstrained stocks in the TSE and in the USTSP. This Figure plots for both tick size constrained stocks (TSC, in blue) and tick size unconstrained stocks (TSU, in red) the relationship between a liquidity indicator (spread or depth) and trading activity (volume). Spread is measured as the logarithm of the time-weighted quoted spread divided by the midquote in basis points. Depth is the sum of the time-weighted BBO bid and ask sizes and is in log shares. Volume for TSE and for lit exchanges in the U.S. Tick Size Pilot includes open and close and is measured in log shares.

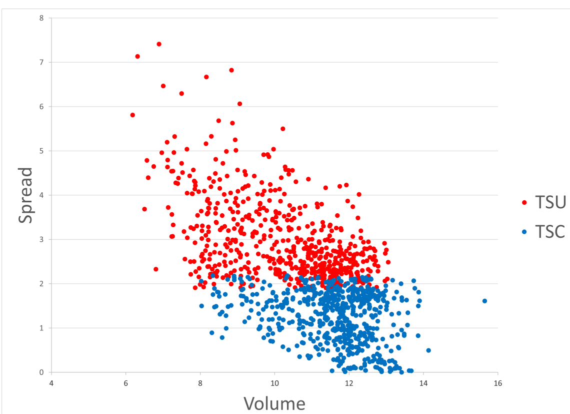
(a) TSE: Spread



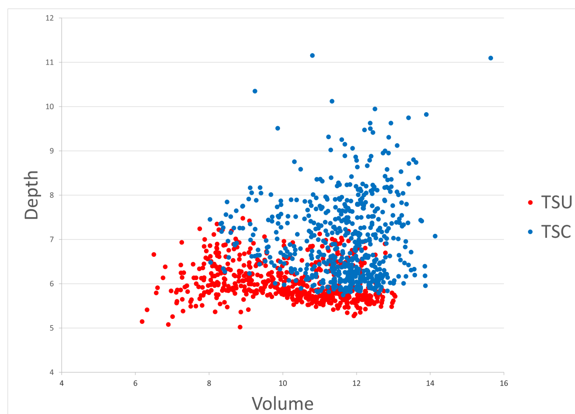
(b) TSE: Depth



(c) USTSP: Spread



(d) USTSP: Depth



all very liquid, and TSU stocks still present tight spreads relative to their tick size, however tick-constraints are less binding than for TSC stocks. The sample consists of 34 treated stocks and 57 controls for Phase 1, and 77 treated stocks and 19 controls in Phase 2. Hence, the stacked sample has 187 firms, 111 firms that are treated and 76 firms that are controls.

Following Gormley and Matsa (2011) we stack the two phases of the TSE experiment, and conduct difference-in-difference analysis for TSC and TSU stocks separately based on the following specification:

$$MQ_{i,j,t} = f_{i,j} + d_{j,t} + b \cdot Post_t \cdot Treated_{i,j} + \varepsilon_{i,j,t} \tag{16}$$

where $MQ_{i,j,t}$ is a market quality measure for stock i, in phase j on day t, $f_{i,j}$ is a firm fixed effect for firm i in phase j, and $d_{j,t}$ is a fixed effect that takes on a value of one for day t if the stock is in phase j and zero otherwise, $Post_t$ is a dummy variable that takes on a value of one for days following the respective phases, and is zero otherwise, $Treated_{i,j}$ is a dummy variable that takes on a value of one for stocks i that are subject to the treatment in phase j. b measures the effect of the experiment on treated stocks relative to control stocks.[34] Standard errors are clustered by firm-phase and day-phase. Results are presented in the first set of columns of Table 6. Panel A reports the results for TSC stocks, results for TSU stocks are in Panel B. Table 6 also reports results for the triple difference-in-difference panel regression:

$$MQ_{i,j,t} = f_{i,j} + d_{j,t} + b_1 \cdot Post_t \cdot TSC + b_2 \cdot Post_t \cdot Treated_{i,j} + b_3 \cdot Post_t \cdot Treated_{i,j} \cdot TSC + \varepsilon_{i,j,t}, \tag{17}$$

where the dummy variable, TSC, takes on a value of one for stocks that are tick-size constrained in the pre-event period and zero for those that are unconstrained. We control for firm-phase fixed effects, $f_{i,j}$, and day-phase fixed effects, $d_{j,t}$, and standard errors are clustered by firm-phase and day-phase. b_1 captures any change for constrained control stocks relative to unconstrained control stocks, b_2 measures the change in market quality for unconstrained treated stocks relative to unconstrained controls, and b_3, the differential treatment effect for constrained stocks, captures how different the difference-in-difference estimate is for tick-constrained treated stocks compared to unconstrained treated stocks. In the interest of space we only report b_3 in Panel C of Table 6.

The Japanese market is not highly fragmented, with the TSE attracting more than 80% of consolidated volume, and our results show that when the tick size is reduced, for TSC stocks (Panel A) depth deteriorates, while relative quoted spread and trading volume decrease. This result is

[34] We use firm-phase and day-phase fixed effects as in Gormley and Matsa (2011), but the results are virtually identical if we instead use group dummies for treated firms in each phase and phase-specific post dummies.

in line with our Empirical Prediction 1 for the single PLB, and it is interesting as usually spread and volume move in opposite directions. For TSU stocks spread improves, depth deteriorates and volume does not change significantly (Panel B). Our model for a single PLB predicts that spread should deteriorate and volume should improve for TSU stocks. However, the effect on volume is not significant, and as reported in Panel C the spread improvement is smaller for TSU than for TSC stocks. The TSE faces limited competition, and following the tick size reduction the TSE market share increases (Panel B) indicating a small reverse migration from the competing parallel trading systems to the primary market. An inflow - although moderate - of orders helps explain the spread improvement for TSU stocks.[35] A possible additional reason why spread does not deteriorate following a reduction in the tick size for TSU stocks is that the price grid that characterizes TSE stocks is too coarse for the "undercutting effect" to be binding. Dyhrberg, Foley and Svec (2020) consider TSU cryptocurrencies instruments characterized by much smaller tick sizes than the TSE stocks and find support for our model's predictions.

4.2 The End of the USTSP

In this second experiment, we focus on the tick size reduction at the end of the USTSP: from May 6, 2016, to September 28, 2018, a subset of U.S. stocks experienced an increase in the tick size from one to five cents. When the pilot expired, the tick size for these same stocks was reverted from five cents back to one cent. The USTSP designated three different treatments, but we are interested only in the effect of the tick size on quotes and therefore focus on test group G1.[36] For the analysis we use Daily Trade and Quote (DTAQ) data to calculate daily market quality measures. As for the TSE experiment, we base our tests on daily panel data for a 4-month window around the tick size experiment, and we analyze separately TSC and TSU stocks. Our sample includes 292 treated

[35] Credit Suisse (2014) shows that the TSE tick size reduction has reduced the Parallel Trading Systems' market share from 8.2% to 5.4% thus levelling the preceding more granular tick sizes that constituted - before the 2014 tick size reduction - their most important competitive advantage.

[36] For a more in depth analysis of the pilot, see for example Comerton-Forde, Gregoire, and Zhong (2019).

stocks, and 876 control stocks.[37]

We first run a difference-in-difference analysis based on the regression model:

$$MQ_{i,t} = f_i + d_t + b \cdot Post_t \cdot Treated_i + \varepsilon_{i,t} \tag{18}$$

where $MQ_{i,t}$ is the market quality metric for stock i on day t, f_i is a firm fixed effect, d_t is a day fixed effect, $Post_t$ is a dummy variable that takes on a value of one for days in the post period, October 1 - November 30, 2018, and is zero otherwise, and $Treated_i$ is a dummy variable that takes on a value of one for G1 stocks for which the tick size was reduced.[38] Standard errors are clustered by firm and day. b measures the effect of the tick size reduction of G1 stocks relative to control stocks, and we henceforth call it the treatment effect.

The final set of columns in Table 6 Panel A shows that TSC stocks experience a significant improvement in relative quoted spreads, a significant deterioration in depth and a significant - although negligible - increase in volume and market share. For TSU stocks Panel B shows that relative quoted spread and depth deteriorate significantly, while volume and market share do not change significantly. Results for TSU stocks are in line with our model's predictions for illiquid stocks, whereas results for TSC stocks differ from our model's prediction for liquid stocks that spread should increase and volume should decrease because we find that spread decreases and volume increases.

[37]There are some stock-days with very large quoted spreads and relative spreads, especially for TSU Control stocks. As a robustness check, we re-run all the analyses screening out any stock-day with a relative spread exceeding 500 basis points. Results are unaffected and are available upon request from the authors.

[38]We use firm and day fixed effects for ease of comparison with the TSE experiment, but the results are identical if we instead use a dummy for treated firms and a post dummy.

Table 6: Change in Market Quality and in Market Shares following a Tick Size Reduction
This table reports the estimated treatment effect on market quality of the tick size reductions for each experiment and for treated stocks in sub-samples by tick-constraint. Panel A reports results for stocks with average quoted spread divided by the tick size in the pre-event period at the median or below (TSC), while Panel B reports results for stocks with average quoted spread divided by the tick size in the pre-event period above the median (TSU). Panel C reports the results from the triple difference-in-difference analysis of changes in TSC stocks relative to TSU stocks. Relative spread is the time-weighted quoted spread divided by the midquote in basis points. Depth is the sum of the time-weighted BBO bid and ask sizes and is in log shares. Volume for TSE and for lit exchanges in the U.S. Tick Size Pilot includes open and close and is measured in log shares. Market share is volume of the market(s) interested by the tick size change over total trading volume. Standard errors are reported in parentheses under each coefficient, and * next to a coefficient indicates significance at 10%, ** at 5%, and *** at 1%.

	TSE Tick Size Reduction				US Tick Size Pilot			
	Rel. Spread	logDepth	logTSEVolume	MktShare	Rel. Spread	logDepth	logLitVolume	MktShare
A. TSC								
Post*Treated	-9.964***	-1.003***	-0.084**	9.308***	-33.455***	-1.347***	0.150***	6.065***
	(0.734)	(0.087)	(0.039)	(0.771)	(4.032)	(0.037)	(0.045)	(0.492)
Observations	8,742	8,742	8,742	8,556	48,566	48,566	48,566	48,566
R-squared (within)	0.316	0.202	0.004	0.102	0.048	0.369	0.003	0.016
B. TSU								
Post*Treated	-3.989***	-0.561***	-0.009	5.196***	16.055***	-0.556***	-0.016	-0.473
	(0.730)	(0.076)	(0.036)	(0.573)	(5.518)	(0.031)	(0.035)	(0.519)
Observations	8,649	8,649	8,649	8,644	47,777	47,777	47,777	47,777
R-squared (within)	0.239	0.095	0.000	0.041	0.002	0.096	0.096	0.000
C. Triple Diff-in-Diff								
Post*Treated*TSC	-6.630***	-0.549***	-0.060	4.913***	-49.522***	-0.791***	0.166***	6.541***
	(1.200)	(0.123)	(0.054)	(1.054)	(6.838)	(0.048)	(0.057)	(0.698)
Observations	17,391	17,391	17,391	17,200	96,343	96,343	96,343	96,343
R-squared (within)	0.380	0.207	0.003	0.088	0.019	0.271	0.001	0.008

What explains this difference is the migration of order flows towards the exchanges with a smaller tick size: the competing venues are less price aggressive than our stylized CN executing at the midquote, and therefore may experience an outflow of orders once the tick size on exchanges is reduced. Market share indeed improves for TSC stocks suggesting a small reverse "migration effect" (Panel A in Table 6). Such effect compounds with the increased incentive to supply liquidity induced by the "layering effect" for TSC stocks thus explaining why spread falls and volume increases for these stocks following the tick size reduction.

The U.S. markets are highly fragmented and for this reason we pay particular attention to the "migration effect", which according to our Empirical Prediction 2 particularly affects TSC stocks. When a stock's liquidity clusters at the inside quotes, it is more sensitive to inflows and outflows of limit and market orders. For this reason, our model offers predictions for the Diff-in-Diff effect on market quality metrics suggesting that the effects on market quality of a tick size reduction should be stronger for liquid than for illiquid stocks:

$$[M_s(PLB\&CN_{STM}) - M_s(PLB\&CN_{LTM})]_{Liq-Illiq} < 0 \tag{19}$$

where M_s is our metric of market quality with $s \in \{spread, depth, volume\}$, and $[M_s(PLB\&CN_{STM}) - M_s(PLB\&CN_{LTM})]_{Liq}$ is the difference between M_s computed for the $PLB\&CN$ framework when the tick size is small (STM) and M_s computed for the same framework but when the tick size is large (LTM), both metrics computed under the regime with high trading activity and low σ.[39]

In line with our Diff-in-Diff predictions, Table 6 shows that the treatment effects are larger in magnitude for TSC than for TSU stocks. To formally test whether these differences between TSC and TSU stocks are statistically significant, we conduct a triple difference-in-difference panel regression:

$$MQ_{i,t} = f_i + d_t + b_1 \cdot Post_t \cdot TSC + b_2 \cdot Post_t \cdot Treated_i + b_3 \cdot Post_t \cdot Treated_i \cdot TSC + \varepsilon_{i,t}, \tag{20}$$

[39] Indeed, from Table 4 we can compute the Diff-in-Diff effects on spread, depth at the inside quotes, and volume that are equal respectively to -0.001, -0.011, and -0.072.

where the dummy variable, TSC, takes on a value of one for stocks that are tick-size constrained in the pre-event period and zero for those that are unconstrained. We control for firm fixed effects, f_i, and day fixed effects, d_t, and standard errors are clustered by firm and day. b_1 captures any change for constrained control stocks relative to unconstrained control stocks and b_2 measures the change in market quality for unconstrained treated stocks relative to unconstrained controls, and in the interest of space we will not discuss these results. Finally, b_3 is the main coefficient of interest for these tests, and it captures how different the difference-in-difference estimate is for tick-constrained treated stocks compared to unconstrained treated stocks.

The differential treatment effect for TSC stocks - reported in Panel C of Table 6 - is a significant reduction of both the relative quoted spread and depth, in line with our Empirical Prediction 2. By contrast, the differential treatment effect on volume for TSC stocks is positive (although minor). As discussed above, this effect, which is still stronger for liquid stock, is due to competition from less price aggressive trading venues than a simple CN and therefore to the migration of volume to the cheaper U.S. platforms with a reduced tick size.

5 Conclusions

Motivated by several recent attempts by exchanges around the world to right-size their tick sizes, we study the relationship between the tick size and market quality. We develop a model of a multi-period PLB where liquidity builds endogenously as rational traders with private evaluations determine whether to supply or demand liquidity, or refrain from trading, based on the state of the PLB and on the trade-off between POC and NEC.

Our main contribution to the existing literature is to single out four transmission channels - layering, undercutting, mechanical change in spread and migration - that explain the effects of a tick size variation on the quality of a PLB, and to show which of these channels are predominantly at work both when the book is liquid as opposed to illiquid, and when the market is fragmented as opposed to non-fragmented.

We show that in a non-fragmented market, when the book is liquid, following a reduction in the tick size limit orders clustering at the inside quotes tend to spread over the now larger number of price levels thus increasing traders' incentive to supply as opposed to take liquidity ("layering effect"). Coupled with the mechanical reduction in spread that is strong with liquid books that are tick size constrained, spread improves and depth at the inside deteriorates, while volume decreases - as in a single PLB investors have only the options to supply, take liquidity or no trade. When instead the book is illiquid, the main effect at work is that undercutting becomes cheaper and traders become less inclined to supply liquidity as opposed to take liquidity with the result that both spread and depth deteriorate, while volume improves.

When the PLB competes with a CN that executes at the midquote, the migration of order flows away from the PLB deteriorates both spread, depth, and volume. The "migration effect" is stronger for liquid stocks thus magnifying the effects of the tick size reduction for liquid books compared to illiquid books. We therefore show that the overall effects on market quality is stronger when the stock is liquid.

We investigate empirically the effects of a tick size reduction by choosing two major markets, one with low fragmentation (TSE) and the other one (U.S.) with high fragmentation. We proxy liquid and illiquid assets with TSC and TSU stocks, respectively. When the tick size is reduced in a market with low fragmentation, the TSE, our results are generally consistent with our prediction for the single PLB with spread improving and depth and volume deteriorating for TSC stocks.

When instead the market is highly fragmented (USTSP), we observe an inflow of orders instead of an outflow of orders as predicted by our model because the competing venues of the U.S. PLB markets differ from our stylized CN. However, as discussed above, the reverse migration of order flows should still induce stronger effects for TSC stocks. This is precisely what we observe in the USTSP where the - although small - inflow of orders seems to have a weaker effect for TSU stocks.

Notice that a decrease in the tick size has a detrimental effect on the liquidity of TSU stocks as both spread and depth worsen, while the effect on TSC stocks is more ambiguous as spread improves but depth worsens. This result suggests that in the U.S. market, illiquid stocks should have a larger tick than liquid stocks because for illiquid stocks an increase in the tick size would be liquidity enhancing.

Overall, our empirical results confirm that "one size does not fit all": the determination of the liquidity-maximizing tick size level should take into consideration both stock liquidity and market fragmentation. Given the importance to tailor the tick size to stock characteristics, the TSE proportional tick-size system seems better suited than the U.S. uniform tick size system to tackle differences across stocks.

References

Angel, J. J., 1997. Tick Size, Share Prices, and Stock Splits. *Journal of Finance,* 52:655-681.

Bessembinder, H., 2003. Trade, execution costs and Market quality after decimalization. *Journal of Financial and Quantitative Analysis*, 38:747-777.

Buti, S., B. Rindi and I. M. Werner, 2017. Dark Pool Trading Strategies, Market Quality and Welfare. *Journal of Financial Economics,*, 124:244-265.

Colliard, J.-E., and T. Foucault, 2012. Trading Fees and Efficiency in Limit Order Markets. *Review of Financial Studies,* 25:3389-3421.

Comerton-Forde, C., V. Gregoire, and Z. Zhong, 2019. Inverted fee structures, tick size, and market quality. *Journal of Financial Economics,* 134:141-164.

Cordella, T., and T. Foucault, 1999. Minimum Price Variations, Time Priority and Quote Dynamics. *Journal of Financial Intermediation,* 8:141-173.

Credit Suisse, 2014. The TSE Tick Size Reduction Phase I - A Month in Review. *Trading Strategy,* 8:141-173.

Degryse, H., M. Van Achter, and G. Wuyts, 2009. Dynamic order submission strategies with competition between a dealer market and a crossing network. *Journal of Financial Economics,* 91: 319-338.

Dyhrberg, A.H.. S. Foley and J. Svec, 2019. When bigger is better: The impact of a tiny tick size on undercutting behavior. SSRN Working Paper.

Foucault, T., O. Kadan, and E. Kandel, 2005. Limit Order Book as A Market for Liquidity. *Review of Financial Studies,* 18:1171-1217.

ESMA, 2018. Amendment to Commission Delegated Regulation (EU) 2017/588 (RTS 11).

Goettler, R. L., C. Parlour, and U. Rajan, 2005. Equilibrium in a Dynamic Limit Order Market. *Journal of Finance,* 60:2149-2190.

Goldstein, M. A., and K.A. Kavajecz, 2000. Eighths, sixteenths, and the market depth: changes in tick size and liquidity provision on the NYSE. *Journal of Financial Economics*, 56:125-149.

Gormley, T. A., and D. A. Matsa, 2011. Growing out of trouble? Corporate responses to liquidity risk. *Review of Financial Studies*, 24:2781-2821.

Harris, L., 1994. Minimum Price Variations, Discrete Bid-ask Spreads and Quotation Sizes. *Review of Financial Studies,* 7:149-178.

Kadan, O., 2006. So, Who Gains from a Small Tick Size? *Journal of Financial Intermediation,* 15:32-66.

O'Hara, M., and M. Ye, 2011. Is market fragmentation harming market quality?. *Journal of Financial Economics,* 100:459-474.

Panayides, M., B. Rindi and I. Werner (2019).Trading Fees and Intermarket Competition. SSRN Working Paper.

Parlour, C., 1998. Price Dynamics in Limit Order Markets. *Review of Financial Studies,* 11:789-816.

Riccó, R., B. Rindi and D. Seppi, 2020. Information, Liquidity and Dynamic Limit Order Markets. IGIER Working Paper n. 660.

Riccó, R., B. Rindi and D. Seppi, 2021. Optimal Market Access Pricing. IGIER Working Paper n. 675.

Rindi, B., and I. M. Werner, 2019. U.S. tick size pilot. Fisher College of Business Working Paper.

Seppi, D. J., 1997. Liquidity Provision with Limit Orders and a Strategic Specialist. *Review of Financial Studies,* 10:103-150.

Zhu, H., 2014. Do Dark Pools Harm Price Discovery?. *Review of Financial Studies,* 27:747-789.