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SHOCKS AND UNEMPLOYMENT
VOLATILITY**

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FINANCIAL FRICTIONS, FINANCIAL SHOCKS AND UNEMPLOYMENT VOLATILITY[†]

Abstract

Financial market shocks and imperfections, alongside productivity shocks, represent both an impulse and a propagation mechanism of aggregate fluctuations. When labor and financial markets are imperfect, firms' funding and leverage respond to productivity changes. Models of business cycle with equilibrium unemployment largely ignore financial imperfections. The paper proposes and solves a tractable equilibrium unemployment model with imperfections in two markets. Labor market frictions are modeled via a traditional Diamond Mortensen Pissarides (DMP) model with wage posting. Financial market imperfections are modeled in terms of limited pledgeability, in line with the work of Holmstrom and Tirole. We show analytically that borrowing constraints increase unemployment volatility in the aftermath of productivity shocks. We calibrate the model to match key labor and financial moments of the US labor markets, and we perform two quantitative exercises. In the first exercise we ask whether the interaction between productivity shocks and borrowing constraints increase the volatility of unemployment with respect to models that focus only on the labor market imperfections. In the general specification of the model, both leverage and non pledgeable income move with the cycle. Our calibration exercise shows that the volatility of unemployment in response to productivity shock increases by as much as 50 percent with respect to a pure DMP model with wage posting. The second quantitative exercise explores the role of pure financial shocks on aggregate equilibrium. We calibrate pledgeability shocks to match the frequency of financial crisis and define financial distress as a situation in which internal liquidity completely dries up. The second exercise shows that full dry up of internal liquidity implies an increase in unemployment as large as 60 percent. These results throw new light on the aggregate impact of financial recessions.

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1 Introduction

Following the Great Recession, there is a new interest in the aggregate dynamics of labor market. On the one hand, the surge in US unemployment in 2008 despite moderate productivity fall challenged the view that labor market dynamics can be mainly accounted for by productivity shocks (Christiano et al, 2015). On the other hand, empirical research has established that unemployment volatility is larger during financial recessions (Boeri et al. 2013, IMF, 2010). These facts and regularities forced scholars into investigating the links between financial frictions and unemployment.

There are at least two ways to study the interlinks between labor and finance over the business cycle. The first is to ask whether financial frictions propagate and amplify standard business cycle fluctuations. The second is to study the effects of financial shocks on the labor market. This paper offers a contribution into both dimensions. The key prerequisite for studying the interactions between labor and finance is to develop a tractable general equilibrium model of unemployment with financial frictions. This paper proposes a stochastic dynamic model in which aggregate productivity shocks induce fluctuations in both investment opportunities as well as in financial constraints. We integrate the Diamond Mortensen-Pissarides (DMP) model with the work of Holmstrom and Tirole, in line with Boeri et al. (2014). A large part of a firm's funding come from pledgeable income and firms invest and build capacity within an imperfect labor market. Firms need funds to invest in both physical capital as well as in search capital. The latter is the accumulation of rents within an imperfect labor market. Funds come from external and internal liquidity. Internal liquidity comes from the internal cash flow of the firm, and depends on pledgeable income. External liquidity is an entrepreneur's own cash-flow and it is fully pledgeable. Both types of liquidity are subject to business cycle fluctuations.

The steady state of the model is calibrated to match key moments of the US labor market as well as the average leverage of non-financial non-listed firms in the US in 2006. The paper embarks into two quantitative exercises. We first investigate the amplification effects of productivity shocks induced by financial market imperfections. In this first exercise, we take productivity shocks as the driving force of business cycle fluctuations, in the spirit of the recent work by Shimer (2005) and (2010), Hall (2005) and Hagedorn and Manovski (2008). Unlike these quantitative versions of the (DMP) model, we study in close details fluctuations to borrowing constraints and firms' funding. In our model, aggregate productivity changes affect investment opportunities, the labor market and indirectly- firm funding. A natural amplification mechanism of aggregate shocks emerge from the simple theory. The analytical results show that financial frictions increase the responsiveness of the economy to productivity changes along two channels. The first amplification channel of productivity is a collateral effect. The second channel is a pledgeability effect. The economics of these two effects is clear, and operate in the same direction. A fall in productivity causes a fall in profit per worker, exactly as in the original Pissarides (1988) model. If borrowing constraints are not endogenously responding to the productivity fall, general equilibrium and free entry of firms is ensured through a fall in the welfare value of unemployment. If funding responds to the fall in productivity, as it does in our theory, a productivity shock reduces leverage and the size of entrant firms along the two channels. Such a response at the entry margin amplifies the response of unemployment. Quantitatively, the increase in the responsiveness of unemployment to productivity changes is as large as 50 percent, particularly when the pledgeability effect is fully operating.

The second quantitative exercise deals with pure liquidity shocks at given aggregate productivity. Stochastic expected changes to pledgeability do not have any direct effect on profit per worker, but they affect borrowing constraints. Changes in financing possibility affect the firm investment opportunities, its scale of operation and its search capital. The challenge is to calibrate the financial shocks around the steady state level of leverage. We define financial distress as a situation in which firms internal funding for new firms completely dries up, and we calibrate these events to match the (low) frequencies of financial crises, as documented by Reinhart and Rogoff (2009) and by the recent work of Boissay et al. (2013). The exercise shows that a very adverse and unlikely financial shocks has strong aggregate

effects. Technically, we show that a one-off full dry up of internal liquidity and the associated fall in leverage implies an increase in the level of unemployment as large as 60 percent. These results shed light on the aggregate impact of financial recessions, and are coherent with the main aggregate dynamics of the US in 2007-09, when vacancies plummeted, unemployment soared while output per hour barely moved. The time-line of the Great Recession is coherent with a realization of financial shocks early in 2007.

The paper proceeds as follows. Section 2 briefly summarizes the recent empirical literature on financial recessions and its links to the labor market. Section 3 introduces the stochastic model focusing on its labor and the finance dimensions. The idea of search capital, leverage and internal liquidity are spelled out in this section. The section derives the equilibrium and the analytical links between business cycle fluctuations and unemployment volatility. Section 4 calibrates the steady state of the economy and performs the first quantitative exercise. Section 5 focuses on pure financial shocks and performs the second exercise. Section 6 summarizes and concludes.

2 Financial Recessions and Labor Market Adjustment

2.0.1 Related Literature

Labor and finance is a growing field and various papers are now contributing to understanding the labor market impact of financial shocks and financial market imperfections on unemployment.

The links between, on the one hand, financial market imperfections and, on the other hand, labour demand and unemployment, have been traditionally investigated along four main channels by the literature.

The first channel is a risk adjustment effect highlighted by Greenwald and Stiglitz (1993), based on a intuition by Hart (1983), when studying optimal labour contracts under asymmetric information and moral hazard. Greenwald and Stiglitz argue that capital market imperfections tend to increase risk aversion of firms, and, consequently, reduce the risk-adjusted marginal product of labour. The reduction in the marginal product of labour, in turn, negatively affects employment and labour demand. Furthermore, imperfections in capital markets can give rise to potentially large fluctuations in the effective marginal product of labour in a way that would not happen if the capital markets were to be perfect. If firms pay efficiency wages, these risk adjustment effects turn out to be amplified, so that there is a complementarity between imperfect capital markets and imperfect labour markets in amplifying cyclical fluctuations. This mechanism operates mainly via the layoff side.

The second channel is a quasi-fixed investment effect of labour demand: insofar as labour involves some ex-ante costs, e.g., training and hiring costs (Oi, 1962), the financing of such investment costs within imperfect capital markets affects the demand for labour. The best indirect rigorous application of this simple mechanism can be found in Farmer (1985) model of optimal contracting under private information, where firms have to finance a machine in order to hire workers. The presence of limited liability induces an equilibrium relationship between interest rate shocks and labour demand. Limited liability implies that a creditor receives a low return if the firm goes bankrupt, and consequently requires a higher return if the firm does not go bankrupt. In Farmer model, an increase in interest rates reduces employment even in a frictionless labour market, since the optimal contract then guarantees a higher interest rate if the lender does not go bankrupt, and, as a result, more firms choose to file for bankruptcy. This drives up unemployment during financial crises, via an increase in job destruction.

A third channel linking finance to employment emphasizes the stickiness of the bank-borrower relationship resulting from asymmetric information. On the one hand, long term bank-borrower relationships can be the result of adverse selection in the market for borrowers that switch lenders (Sharpe, 1990). On the other hand, long term bank-borrower relationships act as signalling mechanism to overcome the moral hazard problem (Holmstrom and Tirole, 1997). The presence of sticky relationships implies that specific shocks to financiers transmit quickly to hiring firms, and thus to

labour adjustment. Wasmer and Weil (2004) use such a stickiness to integrate capital market imperfections and equilibrium unemployment theories. This strand of literature argues that labour market deregulation goes hand in hand with financial market liberalization (Bertola and Rogerson, 1997, Wasmer and Weil 2004; Koskela and Stebacka 2001). Greater access to financial markets by both firms and workers makes it possible to partly self-insure against labour market risk, reducing the demand for employment protection. This literature does not address the out-of-the-steady state properties of these interactions between labour and financial markets.

A fourth channel analyses financial frictions within a New-Keynesian modelling approach. Seminal contributions include Bernanke and Gertler (1989), Carlström and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999). Financial frictions are described as in Townsends (1979) model, in which costly verification in times of distress creates bankruptcy costs so that investors shy away from bankruptcy risk. Persistence and propagation is created through the investors income flows and/or the value of machines used as collateral. Unemployment is not modelled or it is caused by sticky wages and prices.

This literature does not offer many insights as to employment adjustment in the aftermath of financial recessions, the exception being research by Aghion, Hart, and Moore (1992) and Wruck (1990) on financial distress in the context of Chapter 11 bankruptcy procedures. In these models financial distress leads to excessive management control and excessive restructuring even when liquidation could be optimal. Empirical evidence provided by Gilson, John, and Lang (1990) is coherent with this view. Once more, employment adjustment is driven by the job destruction margin.

After the Great Recession, involving a very large financial shock at least judging from financial market spreads or asset prices, various papers contributed to the understanding of the interplay between labour and finance. The field is indeed very vibrant. Pagano and Pica (2010) studied the effects of financial market imperfections on labour adjustment in the context of a perfect labour market. Monacelli, Quadrini and Trigari (2011) investigated the interaction between the firms wage policy and its financial structure. They draw on an original intuition by Michelacci and Quadrini (2005 and 2009) who allow liquidity-constrained firms to borrow from their workers at relatively high costs (workers are risk-averse) depressing new hires. Finally, Kuehn, Simutin and Wang (2014) establish a link between search theory and the asset pricing literature.

More on the empirical side, Bentolila, Jansen and Ruano (2013) exploit a unique dataset on bank balance sheets to evaluate the consequences of credit restrictions on employment. Hall (2014), Kehoe, Midrigan and Pastorino (2014) look at the labour market impact of shocks to consumers' and firms' discount rate. Eckstein, Setty and Weiss (2014) and Christiano, Eichenbaum and Trabandt, (2015) examine the business cycle implications of financial shocks through corporate borrowing. Jermann and Quadrini (2012) find that financial shocks amplify cyclical fluctuations in hours worked. Braun and Larrain (2005), Boeri, Garibaldi and Moen (2013) as well as Giroud and Mueller (2015) find that it is highly leveraged firms to have experienced the largest employment losses in the aftermath of the financial recession of 2008-9. Pratap and Quintin (2011) consider the effects of financial crises on emerging markets, documenting that they involved large productivity and earning losses for workers moving across industries.

Within the DMP paradigm, at least three papers address the link between financial market frictions and labor market volatility. Petrosky-Nadeau and Wasmer (2013) provide an extension of the double search frictions of Wasmer and Weil (2004). They argue that financial frictions create an additional entry cost to productivity, this increasing the elasticity of labor market tightness to productivity shocks by a factor of five. Petrosky-Nadeau (2014) focuses on the financing of vacancy costs in a model in which firms accumulate net worth, and shows that these financing constraints generate persistence in market tightness. Another set of paper links the DMP literature to Kiyotaki and Moore type of financial frictions, where firms' capital acts as a collateral. Garin (2015) shows that in an environment in which borrowing limits are linked to the firms physical capital stock can quantitatively account for the sluggish response of labor market variables to productivity shocks

In spite of this flourishing literature, the underlying mechanisms, the way in which financial shocks

turned into employment losses are still far from being fully characterized. On the theory side, the interlink between the limited pledgeability of Holmstrom and Tirole, has not yet been studied within an imperfect labor market setting. Moreover, evidence on a number of other dimensions is still not clear. For example, Chari, Christiano and Kehoe (2008) show that even after the Great Recession there was a limited number of financially constrained firms. Moreover firms have, in any event, enough retained earnings to finance their operations without having to look for external financing (Bates, Kahle and Stultz, 2006; Chari, Christiano and Kehoe, 2008, Kararbounis and Neiman, 2012). Another problem with the literature reviewed above is that in most cases it considers only adjustment of employment/unemployment stocks and does not focus on the role of firm entry and firm size. When this literature yields predictions on gross job flows, finance affects employment almost uniquely via the job destruction margin. A partial exception is the model by Monacelli, Quadrini and Trigari (2011) for which there is, however, limited empirical support: Simintzi, Vikrant and Volpin (2010) in particular show that firms reduce debt when facing an increase in the bargaining power of workers. Empirical work, however, point to significant effects on the job creation margin as well, with financial markets deeply affecting firms' entry and post-entry growth (Arellano, Bai and Zhang, 2012). Our aim is to have a model that can generate large effects on employment, by operating along the job creation margin, allowing for entry and growth of firms, and where retained earnings are endogenous.

2.1 Key Definitions and Background Facts

The financial crisis of 2008 led to a surge in interest in the macroeconomics of financial recessions, over and beyond its labor market dimension. Reinhart and Rogoff (2009) documented a large body of stylized facts on banking crises and financial recessions. To put things in perspective, we begin with some definitions. A 'financial and banking crisis' is an event during which the financial sector experiences bank runs, increase in default rates, capital losses, bankruptcy, etc. A recession is defined as a 'financial' when it takes place in concomitant with a banking crisis.

While the interest of this paper is on the labor market consequences of these event, we simply recall the key findings of financial recessions, following the recent empirical work by Boissay et al. (2013) and Siemer (2014)

1. Financial recessions are deeper and last longer than ordinary recessions. Boissay et al. (2013) argue that the drop in real GDP per capita from peak to trough is 40% bigger during financial recessions than during normal recessions. Financial recessions are longer and last one extra year, on average.
2. Financial recessions, or banking crises during recessions, are rare events. Boissay et al. (2013) find that banking crises have a yearly frequency of less than 5%, which means that a country experiences a banking crisis every 22 years. Only half of those are financial recessions, so that they happen once every 40 years.
3. The 2007-2009, arguably a financial recession, features an un-precedent decline in firm entry. The number of entering firms during the great recessions dropped by more than 25 percent, an un-precedent drop over the last 30 years (Siemer, 2014). In addition, Siemer (2014) estimates a differential impact of financial constraints on employment growth and finds that in the 2007-2009 recession employment growth in (small) firms with larger financial constraints was 5 percent lower than employment growth in less constrained firms.

We will use these background facts to motivate our theory. Fact 1 is the basic starting point of our research into the amplification effects of financial imperfections. Fact 3 suggests that firm entry played a key role over the great recession, a feature that will be prominent in our theory. Fact 2 will help us calibrating the frequency of financial shocks. In terms of the labor market, our earlier

Table 1: Unemployment and GDP during financial recessions

Country	Type of recession	du	du/u	dy/y	ϵ^a
US	Financial rec	2.65	50%	-3.0%	16.66
	Other rec	1.93	33%	-2.6%	12.69
	<i>Difference</i>	<i>0.72</i>	<i>17%</i>	<i>-0.4%</i>	<i>3.97</i>
UK	Financial rec	2.10	36%	-3.2%	11.25
	Other rec	0.50	7%	-3.1%	2.25
	<i>Difference</i>	<i>1.60</i>	<i>28%</i>	<i>0.0%</i>	<i>9.00</i>

^a Apparent elasticity of unemployment with respect to GDP.

Notes: Episodes of recessions with financial crises: UK 1975, 1990, 2008; US 1990, 2008.

UK: Unemployment data starting from Q1-1983; Employment Q2-1992; GDP and data starting from 1970;

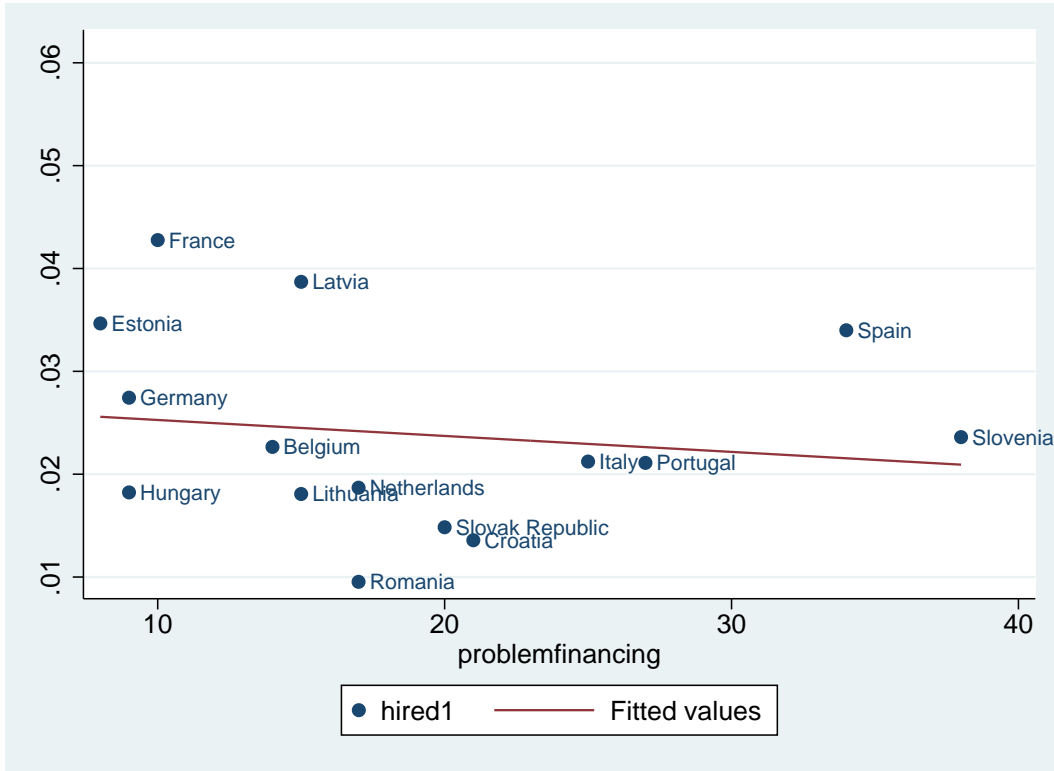
US: Unemployment rate and Employment data starting from Q1-1970; GDP data from 1970.

Sources: OECD, US Bureau of Labor Statistics.

work showed showed that recessions involving in their early stages a financial crisis feature a stronger employment and unemployment response than non-financial recessions (Boeri et al., 2013) .

Table 1 displays changes in unemployment during financial recessions. The Table shows that financial recessions are characterized by a larger unemployment response (in terms of both changes in the unemployment rate and percentage variations) than ‘ordinary’ recessions. The apparent elasticity of unemployment changes to GDP is as high as 16 during financial recessions in the US. While GDP contracts by some 3 percent, unemployment increase by as much as 50 percent. In normal recessions the same elasticity has a value of 12.69. Similar results hold for the United Kingdom, even though the absolute value of the elasticity is substantially lower. The same phenomenon is not observed in all other G7 economies. This is also not surprising, since labor market institutions clearly interact with the volatility of unemployment over the recessions. These findings are consistent with those of the IMF (2010).

There is also evidence of stronger falls in job creation in countries where firms faced a significant hardening of credit constraint, notably in terms of refinancing operations. In particular, evidence from a new European System of Central Banks survey (Wage Dynamics Network, WDN), covering 25 European countries, was addressed at measuring firms perceptions on the nature of shocks driving the Great Recession, suggest that the countries in which a highest proportion of firms stated to be facing serious problems of refinancing experienced the largest falls in monthly hiring as a proportion of employment (Fig [?]).



3 The Model

In presenting and solving the model we proceed in the following way. We first spell out the basic characteristics of the environment. Next, we present basic value functions, firms' profit and labor market search at given funding. Funding is then discuss in details, and we derive the endogenous borrowing constraints. Next we define general equilibrium and characterizes the steady state. Finally, we derive the analytical result on the amplification effect played by financial imperfections.

The basic environment

Preferences and Discounting. Time is discrete and all agents discount the future at rate β ; β represents both the time preferences as well as the discounted factor of budget constraints. The utility function is linear in the time t income y_t , and workers behave so as to maximize the net present value of their future income flow.

Entry Cost and Firm Size. Entrepreneurs set up a firm at effort cost K . This cost is the only part of investment that does not need financing and it is best thought of as pure effort or entrepreneur equity. Entrepreneurs decide the size or capacity A of the firm, and invest A in *physical* capital. The price of each unit of capital is exogenously fixed at ϕ . Labor and capital are perfect complements and the technology is Leontief. With a given probability rate λ , the project ends and the firm simply 'dies'. The workers survive in the unemployment pool and receive a fixed income z .

Output and Productivity Shocks. Output is given by a linear technology $f(y_t, A_t) = y_t A_t$. The linear technology y with the fixed cost K implies increasing returns to scale. Let y_t denote the aggregate state of the economy, as the outcome of stochastic process is described by

$$y_t = y e^{\epsilon_t},$$

where ϵ_t is a pure technological shock that follows an autoregressive path

$$\epsilon_t = \rho\epsilon_{t-1} + u_t$$

with $0 < \rho < 1$ and $u_t \approx N(0, \sigma^2)$. Throughout the paper we use a discrete approximation of y_t applying the Tauchen method (see Sargent and Stachurski, 2014) and work with a finite number of states y_1, \dots, y_n with $y_i < y_{i+1}$ and a stochastic matrix P such that

$$p_{ij} = \text{prob}[y_t = i | y_{t-1} = j]$$

is the probability that the economy transits from state i to state j in one period.

Matching Workers are hired in a search market that is open in the beginning of the period. A Cobb Douglas matching function $x(u, v) = Hu^\alpha v^{1-\alpha}$ relates the number of new matches to the stocks of searching workers u and firms with vacancies v .¹ The vacancy to unemployment rate is indicated with θ_t , and define q as $q = x(u, v)/v = H\theta_t^{-\alpha}$. Let c denote the per period cost of maintaining a vacancy. The firms fill a vacancy with probability 1 at cost c/q by positing infinitely many vacancies infinitely shortly.² The search cost plus the rent guaranteed to the worker will represent a firm *search capital*.

Funding and Borrowing Constraints Firms have access to two funding sources to finance their investment needs. First, the entrepreneur has a flow value of income $y_o y_t$. Such cash flow characterizes *external liquidity* and it is fully pledgeable. Second, firms may use the liquidity generated by their investment to finance their investments. This is called *internal liquidity* and has limited pledgeability per unit of investment. We assume that xA of the firm investment is non pledgeable and it is held as private benefit of the entrepreneur. We discuss the details of the financial contract and the nature of x below.

Asset values

Value functions are indicated in capital letters. Workers have an (endogenous) outside option U_t , where U is the expected net present income flow of a worker that is currently unemployed. The financial contract will determine the borrowing constraint and firm size A . We discuss its determination below and we now take its value as given.

The expected net present value of the joint income for the entrepreneur and the workers in a firm of size A can be expressed as

$$M(y) = yA + \beta \{(1 - \lambda)M(y'|y) + \lambda AU(y'|y)\} \quad (1)$$

where y' denote next period's aggregate productivity, which is stochastic the period before, and where $M(y'|y) = E^{y'|y}M(y)$. $U(y'|y)$ is defined analogously. The value function reflects that the firm continues with probability $(1 - \lambda)$, and with the complementary probability dies and the A workers get the outside option $U(y')$. The outside option of the firm is assumed to be zero.

It is convenient to define the joint surplus of the match, $S(y)$, as the npv joint income less the outside options of the workers. The firm's joint surplus is

$$S(y) = M(y) - AU \quad (2)$$

Using the joint income, the surplus can be written as

$$S(y) = yA - UA + \beta AU(y'|y) + \beta(1 - \lambda)S(y'|y)$$

¹For simplicity and to minimize notation in the exposition we work with matching parameter $H = 1$, but in the calibration we do use the constant to match unemployment rate and the average duration of unemployment.

²Formally, this can be obtained by assuming that the matching market is set in continuous time in the beginning of the period and ends arbitrarily quickly.

In what follows we define

$$\rho(y) = \frac{U(y) - \beta U(y|y')}{U(y)}$$

as a time varying returns to the unemployed. $S(y)$ can then be expressed as

$$S(y) = (y - \rho U)A + \beta(1 - \lambda)S(y'|y)$$

Let's indicate with $C(U)$ all the labor related costs that the firm faces for a single worker, over and above compensating him for his outside option. Hence $C(U)$ includes the search costs as well as the rent to be paid in excess of the worker's outside option. Firm profits at given investment size A read

$$V(U(y), y) = [S(y) - \phi - C(U(y))] A \quad (3)$$

The profits are linear in the aggregate productivity y and the size of the investment. The firm maximize profits subject to the search constraint and the borrowing constraint. We move to those in turn.

Search and Worker's Rent

Let $W(y)$ denote the expected net present income of a worker hired in a firm, and let $R(y)$ denote the rent offered per employee, $R \equiv W - U$. In order to attract a worker, the firm has to spend $c/q(\theta)$ on search and $R + U$ on wages. In competitive search equilibrium the firm faces a trade-off between offering a high wage, or equivalently, a high R , and paying a high search cost.

We define the search capital associated with acquiring a worker as $C = c/q + R$. Without financial frictions, the firm will choose R so as to minimize C given the trade-off between search costs and worker rents. As we show below, this also holds in the presence of financial frictions. Hence C is given by (4) as

$$C = \min [c\theta^\alpha + R] \quad \text{S.T.} \quad \rho(y)U = z + p(\theta)R$$

It follows that

$$R = \frac{c\theta^\alpha \alpha}{1 - \alpha}$$

Substituting the rent into the objective function, total labor costs are

$$C = \frac{c\theta^\alpha}{1 - \alpha}$$

Over and beyond the rent, the firm guarantees to the worker the present discounted value of unemployment U . This implies that θ is given by

$$\theta(y) = \left[\frac{\rho(y)U - z}{c} \right] \frac{1 - \alpha}{\alpha} \quad (4)$$

Substituted into (4) this gives

$$C(U) = \kappa [\rho(y)U - z]^\alpha \quad (5)$$

where

$$\kappa = \frac{c}{1 - \alpha} \left[\frac{1 - \alpha}{\alpha c} \right]^\alpha$$

We have so far been silent on the time profile of wages. As all agents have utility functions that are linear in NPV income, the time profile of wages does not matter. One possibility is that R can be paid out immediately, and that the worker so receives $(1 - \beta)U = \rho(y)U$ thereafter. If a constant wage is preferred, the important condition is that no payment to the worker happens after the firm dies at rate λ .

Finance

As stated initially, an entrepreneur invests in capacity and buys machines. We assume that the entrepreneurs have no funds of their own, and use effort to finance the entry cost K . The rest of the funds has to be financed through borrowing. The financial friction we impose is that the entrepreneur can commit to repay his exogenous income $y_t y_o$ plus a part but not all the income the project is expected to generate. The liquidity generated by the income flow will be labeled as *external*, while the one generated by the investment is *internal*. Note that the latter can not be fully used to finance investment needs. This limited *pledgeability* emerges for several reasons. Firstly, one may assume that part of the gain from running a business is a private, non-pecuniary benefit. This can not easily be transferred to the creditor. Second, in order to have the proper incentives to make the right decisions, taking properly care of the machines and so on, the entrepreneur must have a sufficiently large stake in the project. We refer to the part of the income that the entrepreneur can commit to repay as the entrepreneur's pledgeable income. We do not propose an explicit model of pledgeability and assume the the income flow that is not pledgeable is simply an income flow $x A_t$. This implies that per each machine the entrepreneur has a flow $x()$ that is akin to private benefits. The basic model of Holmstrom and Tirole (2011) uses this reduced form approach, while they present various microeconomic mechanisms coherent with such simplified approaches. While we are agnostic on the exact microeconomic mechanism behind the function $x()$, our model is flexible enough to consider two drivers of pledgeability. The first driver is a pure financial shock z while the second driver is aggregate productivity y . In the general form we can this write

$$x = x(z, y) \quad (6)$$

with $x_z > 0$ and $x_y < 0$. In the paper we will say that the effect x_z is a *pure financial shock* and will be the focus of Section 6. In this section we are interested in the second effect, mainly the link between productivity and pledgeability x_y , a channel that we will call it the *pledgeability effect of productivity*. The idea is that x can be negatively linked to productivity, so that financial frictions increase during periods of low aggregate productivity. Such negative link may be due to a simple time varying lender's availability of funds with the cycle. The idea of negative link between fund and the overall has been studied in details by Geanakoplos (2011), through an effect that he calls *The leverage cycle*. In this respect, the link between x and y is a reduced form of such effect.

While in this section we keep z completely fixed, we explore specifications of the model with and without the direct dependence of x to y . yet, in deriving the analytical results we will work with a $x = x(y)$ with $x' < 0$. Finally, we assume that the entrepreneur cannot save his non-pledgeable income. The assumption is easily rationalized if the non-pledgeable income is private benefits. The time t pledgeable income from an ongoing project of size A is the following flow value

$$\tilde{p} = y y_0 + (y - w)A - x(y)A$$

The first term is the flow from the exogenous income y_o , the second term is the amount of resources left once wages are paid, while the third term is the private benefit of the entrepreneur. The NPV of the pledgeable for a firm that is entering the market is thus

$$\begin{aligned} P(\tilde{y}) &= y_0 y + (y - w)A_t - x(y)A + (1 - \lambda)\beta P(y'|y) \\ &= Y_0(y) + A(S(y) - R - X(y)) \end{aligned} \quad (7)$$

where

$$\begin{aligned} X(y) &= x(y) + (1 - \lambda)\beta X(y'|y) \\ Y_0(y) &= y y_0 + \beta Y_0(y'|y) \end{aligned} \quad (8)$$

The value of \tilde{P} is the total amount of liquidity available to firm. One interpretation of equation (7) is that the first term is the total amount the entrepreneur can borrow from his own private wealth; the second term is the amount he can borrow on the additional income he receives from the investments. In any event, equations (7) and (8) highlight the two forces driving the dynamics of financial resources over and beyond the surplus $S(y)$, whose expression was already discussed. The value $Y_o(y)$ is called the *collateral effect* and refers to the fact that the outside funding of the entrepreneur moves alongside productivity. The value $X(y)$ is called the *pledgeability effect* and refers to the fact that the amount of resources that the firm can pledge depends on the dynamics of the non pledgeable income $x(y)$. These two effects will play a key role in the discussions on volatility of the next section.

Total resource \tilde{P} can be used to finance machines and search costs, hence $\tilde{P} = A(\phi + c/q)$. Inserting for \tilde{P} thus gives

$$Y_0(y) + S(y)A - X(y)A = A(\phi + C)$$

Now define $P(y) = Y_0(y) + S(y)A - X(y)A$. The borrowing constraint thus writes

$$P = A(\phi + c/q + R)$$

It follows that firm size on entry is obtained as

$$A(y) = \frac{Y_0(y)}{\phi + C + X(y) - S(y)} \quad (9)$$

Define

$$k(y) = \frac{1}{\phi + C + X(y) - S(y)}$$

as the financial multiplier, which reflects how many units of worker-machine pair the firm can invest in per unit of collateral $Y_o(y)$.

Before moving to the general equilibrium we define two summary statistics that will be very handy in our quantitative exercise. The firm relative pledgeable income is the amount of resources that the firm can pledge relative to total productivity and it is simply indicated with $\nu(y) = \frac{y-x(y)}{y}$. The firm internal liquidity is the instantaneous value of financial resources that the firm will generate per unit of investment if the worker receives its outside flow $\rho(y)U$, as we argued in the previous section. Internal liquidity thus reads

$$l(y) = y\nu(y) - \rho(y)U \quad (10)$$

General equilibrium

Definition 3.1 *General equilibrium is a set of value functions $U(y), C(U), V(U, y)$, a labor market tightness $\theta(U)$ and a firm size $A(y, U)$, stochastic process y and transition matrix P such that such that*

1. $C(U)$ minimizes the firms' search and hiring costs and $\theta(y)$ is the resulting labor market tightness
2. $A(y, U)$ satisfies the borrowing constraint without slack
3. $V(U(y), y) = K$ for all y .

Note that equilibrium unemployment u_t is given by the recursive flow equation

$$u_{t+1} = \lambda(1 - u_t) - \theta(U)^{1-\alpha}u_t \quad (11)$$

with u_0 given.

4 Deterministic equilibrium

Suppose the distribution of $y'|y$ is degenerate, so that $y'|y = y$ with probability 1. Then $\rho = (1 - \beta)$ and equilibrium simplifies to

$$\begin{aligned} S &= \frac{y - (1 - \beta)U}{1 - \beta(1 - \lambda)} A \\ C &= \kappa [(1 - \beta)U - z]^\alpha \\ A &= \frac{\frac{y_0 y}{1 - \beta(1 - \lambda)}}{\phi + C(U) - \frac{y - x(y) - (1 - \beta)U}{1 - \beta}} \end{aligned} \quad (12)$$

$$K = \left[\frac{y - (1 - \beta)U}{1 - \beta(1 - \lambda)} - \phi - C(U) \right] A \quad (13)$$

Note that in the deterministic equilibrium the surplus is given simply by equation 12. In addition, $X(y) = \frac{x(y)}{1 - \beta(\lambda)}$ and $Y_o(y) = \frac{y_0 y}{1 - \beta}$. The steady state unemployment reads

$$u = \frac{\lambda}{\lambda + \theta(U)^{1 - \alpha}} \quad (14)$$

For the equilibrium to exist, it must be sufficiently "productive" so that firms can recoup K . If we define net output as productivity y , net of non pledgeable income x and the worker outside option z , or $y - z - x$ the following simple existence theorem on few parameters ensures existence and uniqueness

Theorem 4.1 *If the present discounted value of netoutput is larger than the price of capital, so that*

$$\frac{y - z - x}{1 - \beta(1 - \lambda)} > \phi \quad (15)$$

equilibrium exists and it is unique.

Proof: Using equations (12) and (13) and isolating the search cost in the right and side, one easily obtains

$$\tilde{y} - \phi - \tilde{x} \frac{\tilde{K}}{y + \tilde{K}} - \tilde{z} - \tilde{\delta}U = C(U) \quad (16)$$

where we expressed $\tilde{y} = \frac{y}{1 - \beta(1 - \lambda)}$, $\tilde{x} = \frac{x}{1 - \beta(1 - \lambda)}$, $\tilde{z} = \frac{z}{1 - \beta(1 - \lambda)}$, $\tilde{K} = \frac{K(1 - \beta)}{Y_o}$ and $\tilde{\delta} = \frac{1 - \beta}{1 - \beta(1 - \lambda)}$. The search cost is an upward sloping function defined for $U \geq \frac{z}{1 - \beta}$. Since a viable market clearly requires $y > z$, the maximum value in the right hand side of (16) is $C(\frac{y}{1 - \beta})$ and $\frac{z}{1 - \beta} < U < \frac{y}{1 - \beta}$. The left hand side of equation 16 is a downward sloping linear function $H(U) = \Delta - \tilde{\delta}U$, where $\Delta = \tilde{y} - \phi - \tilde{x} \frac{\tilde{K}}{y + \tilde{K}} - \tilde{z}$. The equilibrium exist and it is unique if $H(\frac{z}{1 - \beta}) > 0$ and $H(\frac{y}{1 - \beta}) < 0$. Since $H(\frac{y}{1 - \beta}) < 0$ is true for any set of positive parameter values, existence and uniqueness reduces to $H(\frac{z}{1 - \beta}) > 0$ which simple algebra implies shows that requires

$$\frac{y - z - x}{1 - \beta(1 - \lambda)} > \frac{y - z - x \frac{\tilde{K}}{y + \tilde{K}}}{1 - \beta(1 - \lambda)} > \phi$$

where the first inequality immediately follow since $\frac{\tilde{K}}{y + \tilde{K}} < 1$. *QED* Having established existence, we can state a basic result of financial friction x .

Result 4.1 *Financial frictions reduce unemployment value and increase the unemployment rate at costant productivity.*

From 12 it follows that an increase in x , holding constant y , shifts A down. Hence, from (13), U shifts down, and from 14 the unemployment rate increases. The economics of Result 4.1 works entirely through the borrowing constraints. An increase in x (holding y fixed) reduces pledgeable income and the value of the firm. To ensure free entry it is necessary to have lower welfare value for the unemployed. Market tightness moves linearly with the value U .

Suppose a planner faces the same financial constraint as the entrepreneurs. Then the following holds

Remark 4.1 *The equilibrium of the model is constrained efficient.*

The proof is also omitted, since it is just an application of Moen (1997) competitive search.

Comparative statics on Productivity

Before moving to the calibration of the model, we investigate analytically how financial frictions interact in the model to shape the responsiveness of unemployment to productivity changes.

To derive the comparative static of the model and understand the economics of our theory, it is very useful to write the model compactly as

$$K = \pi(y, U)A(y, x(y)) \quad (17)$$

$$A = \frac{\frac{yoy}{1-\beta}}{\frac{x(y)}{1-\beta} - \pi(y, U)} \quad (18)$$

where π is profit per worker, $A(y, U)$ is firm size and $\tilde{\beta} = \beta(1 - \lambda)$. Consistent with (13) it follows that

$$\pi(y, U) = \frac{y - rU}{1 - \beta(1 - \lambda)} - C(U) - \phi$$

From the system of equation 17 and 18 the equilibrium firm size is analytically determined as

$$A = \frac{yoy + K(1 - \beta) \frac{1 - \tilde{\beta}}{1 - \beta}}{x(y)} \quad (19)$$

Remark 4.2 *Productivity is positively related to firm size of entrants firms. Both the collateral effect and the pledgeability effect increase size of new firm in response to a change in y*

Proof. To proof the previous remark just totally differentiating firm size to obtain

$$\frac{dA}{dy} = \frac{\tilde{\beta}}{1 - \beta} \frac{yoy}{x(y)} + \eta(y) \frac{A}{y} \quad (20)$$

where $\eta(y) > 0$ is the absolute value of the elasticity of pledgeability $x(y)$ to productivity y . The first component in equation 20 is the *collateral effect* of productivity, the second effect is the *pledgeability effect*. In other words, productivity interacts with financial frictions in two forms. The collateral effect refers to the link between the entrepreneur outside income and productivity while the pledgeability effect refers to the effect of productivity on the size of the firm pledgeable income

We are now in the position to derive a key comparative static result

Result 4.2 *An increase in productivity leads to higher profits per worker, higher firm size A as well as higher value of unemployment.*

Proof The effect of firm size is obtained by equation (20). Totally differentiating equation (17) with respect to y one has

$$-\frac{K}{A^2(y)} \frac{dA}{dy} = \pi_y + \pi_u \frac{dU}{dy}$$

where π_y and π_u are the partial derivatives of the profits per worker and respectively read

$$\pi_y = \frac{1}{1-\beta} > 0; \quad \pi_u = -\frac{1}{1-\beta} - C'(U) < 0;$$

The total derivative $\frac{dU}{dy}$ reads

$$\frac{dU}{dy} = \frac{\pi_y + \frac{K}{A^2} \frac{dA}{dy}}{-\pi_u} > 0 \quad (21)$$

QED

The previous result deserves a comment. In Pissarides (1988) an increase in output leads to an increase in U up to the point where the productivity increase is balanced by higher search and wage costs. In our model, there is an additional effect, as firm size A also grows. The increase in A , in turns, depends on the pledgeability and on the collateral effect. As will be clear below, this allows for a bigger response in U after a change in output than in the standard Pissarides model.

The Excess Volatility of Financial Frictions

The elasticity of unemployment to productivity is defined as

$$\Psi_{u,y} = \frac{du}{dy} \frac{y}{u}$$

The elasticity of unemployment to productivity has the same dynamics of the elasticity of the unemployment value to productivity, and in this section we study $\Psi_{U,y} = \frac{dU}{dy} \frac{y}{U}$.³ Conversely, the numerical simulations rely on the quantitative value of $\Psi_{u,y}$. It is very useful to introduce a benchmark model in which there are no borrowing constraints and optimal capacity is fixed \bar{A} . When capacity \bar{A} is fixed, the firm can not expand beyond \bar{A} . When productivity changes, the fixed capacity can not be adjusted. This baseline alternative model is a pure imperfect labor market model, since the search part of the model is identical to the model of the previous section. In terms of the economic literature, the model with fixed size \bar{A} is a version of the DMP model coherent with Pissarides (1988 and 2000), chapter 2). We can say that in such model the value of unemployment is determined by only one equation

$$K = \pi(y, U) \bar{A} \quad (22)$$

In the model described by equation (22), the elasticity $\Psi_{U,y}(\bar{A})$ read

$$\psi_{U,y}(\bar{A}) = -\frac{\pi_y}{\pi_U} \frac{y}{U} \quad (23)$$

where footscript denotes the partial derivatives introduced above. With endogenous firm size, making use of the result in equation 21 we have that

$$\begin{aligned} \psi_{U,y} &= \frac{\pi_y + \frac{K}{A^2} \frac{dA(y)}{dy}}{-\pi_U} \frac{y}{U} \\ &> \psi_{U,y}(\bar{A}) \end{aligned} \quad (24)$$

³To see this, write $u \equiv u(p(\theta(U(y))))$. Now $el_p u = \frac{p}{p+\lambda} \approx 1$, $el_\theta p(\theta) = 1 - \alpha$, and $el_U(1-\beta)U - z\theta = 1$. Hence $el_y u \approx (1-\alpha)el_y[(1-\beta)U(y) - z]$. Since $el_U[(1-\beta)U - z] = \frac{(1-\beta)U}{1-\beta U + z}$ it follows that

$$el_{uy} \approx (1-\alpha) \frac{(1-\beta)U + z}{(1-\beta)U} el_{Uy}$$

The analytical experiment that we undertake is to consider the value of the elasticity when the two models operate at the same level of capacity and at the same welfare value of unemployment. In other words we want to discuss the two models when $A(\cdot) = \bar{A}$ and $U(x, A) = U(\bar{A})$. The analytical results immediately follow.

Result 4.3 *Financial frictions increase the elasticity of unemployment to productivity changes*

Proof. Since $\frac{K}{A^2} \frac{dA(y)}{dy} >$ the result is obvious and is also illustrated in equation (24).

The intuition for the result is clear. With a fixed size, an increase in y is matched by an increase in U so that the value of a worker net of search costs stays constant. With endogenous A , this is not sufficient, as the size of the firm also increases. Hence, in order to satisfy the zero profit condition, the value of a worker net of search costs has to fall. Finally, we can also study the analytics of the excess volatility and show that

Result 4.4 *The excess volatility of financial frictions is the sum of two effects, the pledgeability effect and the collateral effect*

Proof The excess volatility is just obtained by substituting from $\frac{dA}{dy}$ in the elasticity result

$$\frac{K}{A^2} \frac{dA(y)}{dy} = \frac{\eta(y)A + \frac{1-\bar{\beta}}{1-\beta} \frac{y \sigma y}{x(y)}}{U \pi_u}$$

The question is at this point quantitative. Note that also the economics discussion of the previous results is taken up in section 4.1 when we discuss the results of the simulations.

5 Calibration

We calibrate the model around the steady. The labor market parameters are taken either from standard reference values in the literature or from baseline empirical moments for the US labor market. The main references for calibrating the labor market side of the model are Shimer (2005) and Hagedorn and Manovski (2008). The financial parameters are novel in the model and we proposed an original calibration. Note that our basic calibration is for normal times and does not particularly aims at modeling crises time. Nevertheless, in section 6 we deal with financial shocks and we deal with crisis situations.

There are 10 parameters to be set for the steady-state: $\beta, y, z, \alpha, \lambda, c, M, \phi, K/y_0, x,$. Note that we point out to the ratio K/y_0 as the fraction of the two parameters are what matter in the key labor steady state conditions of equations (13) and (12).

Basic Values

Let us begin with 4 parameters that are taken from standard values in the literature: β, y, z, α

We calibrate the model to quarterly values and we accordingly set the discount rate β to 0.99. The baseline productivity is set as a reference value to 1. The elasticity of the matching function α is a parameter that shows wide variation in the literature, from Shimer (2005) that sets 0.72 to Hall (2008) that sets 0.28. We position ourself in the the central value of the range and we set $\alpha = 0.5$, a value that is also coherent with the central estimate of Pissarides and Petrongolo (2001). The unemployed income z is 0.5. It is now well known that z is a key parameter for simulating business cycle. Shimer sets 0.4 while Hagedorn and Manovski (2008) set 0.955. We do not want to enter in such a debate and fix a value of 0.5, knowing that our baseline calibration will display insufficient volatility with respect to the US labor market.

Key Labor Market Moments

The first key moment that we match is the job finding rate. Shimer (2005) shows that the average monthly probability of not finding a job in the US is 0.55. This means that the average probability of not finding a job in a quarter is 0.1663 and the quarterly job finding probability is 0.8336. The second key labor market moment that we match is market tightness. We refer to Hagedorn and Manovski (2008) and target $\theta = 0.663$. Finally, we also match an average unemployment rate of 0.06, higher than the 0.57 in Shimer for the period 1951 to 2003. The main reasons is that our calibration include the last surge in unemployment that followed the great recession.

These targets allow us to pin down three key labor market parameters: m , the matching function elasticity, c , the search cost and the separation rate λ . Specifically, the separation λ ensures the average unemployment rate at given job finding rate since $u = \frac{s}{s+0.8336}$. The matching function constant is set so as to ensure that market tightness matches the Hagedorn and Manovski reference value of 0.663. Given θ , there is a one to one correspondence between the search cost and the equilibrium value of unemployment. From equation

$$\theta = \left[\frac{U(1-\beta) - z}{c\beta} \right] \frac{1-\alpha}{\alpha}$$

there is a one to one correspondence between c and U once z is set at 0.5. Shimer (2005) proposes a value of $c = 0.228$, while Hagedorn and Manovski (2008) work with endogenous capital stock which imply a larger value of c . Since in our model there are increasing returns to scale linked to the entry cost K , the value of c is larger than in Shimer. In Table 2 and 3 the value of c corresponds to a value of unemployment $U^* = 80$.

Accounting for Leverage

Before entering the calibration strategy of financial variables we need to be more specific on the institutional details we are modeling. The distinction between internal and external liquidity is very clear in our theoretical perspective, but it is not readily immediately available from financial data.

In real life financial institutions, the credit relationship with a given client is specified by three different variables. The *credit plafond* refers to the maximum amount of credit that an institutions is willing to invest in the client. Such credit plafond is typically not revealed to the client/firm. The *contractual line* refers to contractual arrangement between the financial institution and the firm. Such contractual lines take typically different financial forms, including mortgages, long and short term loans, pure cash lines as well as covenants of derivative products. Finally, the *drawn line* is the amount of liquid cash that has left the financial institution and is directly available within the firm's asset. In our theoretical perspective, the credit plafond is not modeled. Conversely, our pledgeability concept gets closer to the total agreed line, independently of whether it is actually drawn. In Boeri et al. (2014) we explore the role of liquidity and we present a model in which firms distinguish between cash reserves and other forms of debt. For the purpose of this paper we abstract from such difference. Let's introduce some definitions

Definition 5.1 *Total Asset (TA) are the amount of assets, either in the form of cash or real investment physically within the firms. Total Borrowing (TB) is the amount of resources borrowed by the firm. Firm equity (E) is the firm value.*

If we indicate with TA total assets within the firm we have that

$$TA = (C(U) + \phi)A + K$$

where A is the size of the firm as well as the investment size whose price is indicated with ϕ . K is the effort cost necessary to enter the market. It is sunk but it has been invested in the firm. If the

we indicate with TB total borrowing we have that

$$TB = (C(U) + \phi)A,$$

so that the firm borrows all the resources beyond the entry cost K .

Leverage in the financial literature (Adrian and Shin, 2008) is typically indicated as the fraction of total asset to the firm equity. The equity ratio is the inverse of leverage. We follow such definitions. In the context of our accounting total equity ratio ER is the fraction between the value of the firm and its total assets so that

$$ER = \frac{V}{(C(U) + 1)A + K} = \frac{K}{(C(U) + 1)A + K}$$

where V is the value of the firm. In equilibrium, as we have seen, such value is equal to the entry cost. Our key financial variable is thus leverage and we define it as

$$lev = \frac{(C(U) + \phi)A + K}{K} \quad (25)$$

Targeting Leverage

We are left with the key financial market parameters: x the non pledgeable income, the price of capital ϕ and the entry cost K/y_o . Without loss of generality we set $y_o = 1$ so that we are left with 3 parameters: x , ϕ and K .

The key moment that we match is leverage of US non-listed non-financial firms in 2006. Kalemili-Ozcab et al. (2011) in their vast empirical work on leverage around the great recession show that leverage in non listed non financial firms was 2.4 in 2006. We thus target lev from equation (25) to be 2.4. We proceed in the following way. Starting from the two equations (17) and (18), we obtain the following relationship

$$\phi = (lev^* - 1)\pi(U^*) - C(U^*)$$

where $U^* = 80$ is the reference value for the welfare value of unemployment set coherently with the labor market moments.⁴ We thus get a value of ϕ . In addition, the K/A ratio is uniquely determined by the following condition

$$\left(\frac{\hat{K}}{A}\right) = \frac{C(U^*) + \phi}{lev^* - 1}$$

For given value of \hat{K}/A ratio, to obtain an actual level of K and A we need a value of x . One can further show that

$$K = \frac{\left(\frac{\hat{K}}{A}\right) \frac{yoy}{1-\beta}}{\frac{x}{1-\beta(1-\lambda)} - \left(\frac{\hat{K}}{A}\right)}$$

We then set as a reference value $x = 0.1$ and obtain the value of K and A indicated in Table 3

Productivity Shock

The stochastic process is calibrated alongside the standard in the business cycle literature

$$\rho = 0.97; \quad \sigma^2 = 0.007 \quad (26)$$

For discretizing the state space, Sargent and Stuchurski (2014) show that Tauchen method requires the specification of two key parameters: n the number of states of the discrete approximation and b , an integer that parameterizes the width of the state space. We choose b and n in a parsimonious

⁴Productivity implies a gross value for 100 with a .99 discount rate

way to ensure that number of states is limited and the model with endogenous borrowing constraints converges smoothly. This amounts in setting $n = 3$ and $b = 1.2$. In the simulation, the productivity shocks are detrended around a HP filter trend with smooth parameter 1600.

The productivity shocks affects the borrowing constraints in two channels. The collateral effects is specified by the recursive equation

$$Y_o(y_t) = yoye^{\epsilon_t} + \beta Y_o(y_{t+1}|y_t)$$

while the pledgeability effect is driven by

$$X(y_t) = xy e^{-\gamma \epsilon_t} + (1 - \lambda)\beta Y_o(y_{t+1}|y_t)$$

with γ is a summary parameters for the elasticity $\eta(y)$. We do not have strong a priory bias for the values of γ . We will show the results of two models with $\gamma = 0$ and $\gamma = 6$. Pledgeable $\nu(y)$ moves from a steady state value of .9 when $\gamma = 0$ to a minimum value of .88 when $\gamma = 6$.

Table 2: Matching the Calibration Target

Target	Source	Value	
		Data	Model
1. Average Job Finding Rate, $M\theta^{(1-\alpha)}$	Shimer (2005)	0.8336	0.8366
2. Average Market tightness, θ	Hagedorn Manovski (2008)	0.634	0.6634
3. Firm Leverage, lev	Kalemli-Ozcan et al. (2011)	2.4	2.3990
Based on Shimer (2005) monthly probability of not finding a job set at 0.55			
Source: Authors' calculation			

Table 3: Calibrated Values and Steady State Statistics

Parameter	Notation	Value
Pure Discount Rate	β	0.990
Baseline productivity	y	1.000
unemployed income	z	0.500
Death Rate	λ	0.053
matching function elasticity	α	0.500
matching function parameter	m	1.027
search cost parameter	c	0.457
own income flow	y_o	0.010
Financial Friction	x	0.100
entry cost	k	4.878
Price of Capital	ϕ	1.137
<i>Productivity Values</i>		
Persistence of productivity process	ρ	0.970
Variance of innovation in productivity process	σ	0.007
Number of states	n	3.000
Withd of the state space	b	1.200
Minimum pledgeability effect	γ	0.000
Maximum pledgeability effect	$\gamma(max)$	6.000
<i>Equilibrium Values</i>		
value of unemployment	U	80.000
firm size	A	3.695
labor market frictions	$C(U)$	0.724
job finding probability	$p(\theta(U))$	0.837
vacancy unemployment ratio	$\theta(U)$	0.663
unemployment rate	u	0.060
Leverage	lev	2.410
Source: Authors' calculation		

5.1 Productivity Shocks and Financial Frictions: Quantitative Results

The first quantitative exercise deals with the amplification power obtained by the financial frictions. Analytically we did show that the model implies an increase in the volatility of unemployment and market tightness to productivity change along two channels that we labeled collateral and pledgeability effect. The question is quantitative.

In the model productivity shocks induce fluctuations to firm's profitability as well as to the financial multiplier. The effect on firm profitability is the standard fluctuation modeled in the business literature of the DMP model. It has attracted the key attention of Shimer (2005), Hagedorn and Manovski (2008) and Hall (2005). We basically ask what financial shocks imply with respect to such baseline models. We thus run two different version of our models as a way to understand the quantitative impact of both the collateral effect and the pledgeability effect.

Steady State Comparison

Specifically, our method is as follows. First, we obtain a pure DMP model by considering an economy where firm size is fixed at \bar{A} and there are no financial frictions. All firms reach the optimal size upon entry and productivity affects profits per worker. The model with fixed is obtained as

$$\left[\frac{y - U(1 - \beta)}{1 - \beta(1 - \lambda)} - C(U) - \phi \right] \bar{A} = K \quad \text{Model I: Basic DMP}$$

In the steady state model with financial frictions and only the collateral effect we have

$$\left[\frac{y - U(1 - \beta)}{1 - \beta(1 - \lambda)} - C(U) - \phi \right] \frac{\frac{yY_o}{1 - \beta}}{\phi + C(U) - \frac{y - U(1 - \beta)}{1 - \beta(1 - \lambda)} + \frac{x}{1 - \beta(1 - \lambda)}} = K \quad \text{Model II: collateral effect}$$

When we consider also the pledgeability effect the model reads

$$\left[\frac{y - U(1 - \beta)}{1 - \beta(1 - \lambda)} - C(U) - \phi \right] \frac{\frac{yY_o}{1 - \beta}}{\phi + C(U) - \frac{y - U(1 - \beta)}{1 - \beta(1 - \lambda)} + \frac{x(y)}{1 - \beta(1 - \lambda)}} = K$$

Model III: collateral and pledgeability

Note that formally the difference between Model II and Model III is the fact that the latter includes also the collateral effect of productivity $x(y)$. Note also that the calibration exercises imposes that the initial size be identical across the three models.

$$\bar{A} = \frac{\frac{yY_o}{1 - \beta}}{\phi + C(U) - \frac{y - U(1 - \beta)}{1 - \beta(1 - \lambda)} + \frac{x}{1 - \beta(1 - \lambda)}} = \frac{\frac{yY_o}{1 - \beta}}{\phi + C(U) - \frac{y - U(1 - \beta)}{1 - \beta(1 - \lambda)} + \frac{x(y)}{1 - \beta(1 - \lambda)}} \quad (27)$$

Equation (*Model I: Basic DMP*) is the baseline DMP representation of our model. Firm size is fixed and there are no financial frictions. Equation (*Model III: collateral and pledgeability*) is the model with endogenous financial frictions. It is natural to consider the responsiveness of the two models starting from the same initial size, as we do in equation (27).

The size of the collateral and the pledgeability effect in steady state

Table 4 uses the steady state calibration and consider a fall in productivity by four percent in the three Models. We start with the equation of *Model I: Basic DMP*. First, recall that in equilibrium profit per worker multiplied by the financial multiplier must ensure that the zero profit conditions hold at all times. The first line of Table 4 shows that the entry cost $K = 4.87$ is obtained by the product between a profit per worker equal to 1.32 and a size/ financial multiplier equal to 3.69. The associated market clearing value of unemployment is 80. The second line of the Table considers a fall

Table 4: Amplification with Endogenous Leverage

Model	Prod	Plead. income	profits worker	Size New Firms	Entry Cost	Welfare Unempl.	Lev	Int Liq	Mkt Tightness
	y	$\nu(y)$	π	A	K	U	$lev.$	$l(y)$	θ
(1) Fixed Size ^a	1	-	1.320	3.69	4.87	80	-	-	0.66
Fixed Size ^a	0.96	-	1.320	3.69	4.87	76.85	-	-	0.594
(2) Endog. Lev ^b	1	0.90	1.320	3.69	4.87	80	2.41	0.10	0.66
(2) Endog. Lev ^b	0.96	0.90	1.327	3.673	4.87	76.80	2.37	0.097	0.593
(3) Endog. Lev ^c	0.96	0.87	1.63	2.98	4.87	75.02	2.11	0.092	0.55

^a Model with fixed and maximum capacity of new firms $A = \bar{A}$

^b Model with endogenous leverage and and capacity of new firms $A = A(U)$ and fixed non pledgeable income x . Pure collateral effect

^c Model with endogenous leverage and and capacity of new firms $A = A(U)$ and non pldgeable income $x(y)$. Collateral effect and pledgeability effect

See main text for steady state equations

Sources: Author's calculation.

in average productivity by 4% to a low value of $y = 0.96$.⁵ Since financial markets and borrowing constraints are not operating and firm size is fixed, a new equilibrium requires a fall in the value of unemployment to ensure that profits per worker remain constant at 1.32. This is obtained by a fall in U to 76.85. This is a fall approximately equal to 4% which implies that the model has not enough volatility to match the data. This is the well known Shimer paradox.

The key finding of our quantitative exercise are the model (2) and (3) in the Table 4. Consider first model 2. The starting point is identical to that of model 1, as indicated by the condition of equation (27). Financial constraints are now binding and the same fall in productivity to 0.96 implies a fall in the size of new entrants firms to 3.675. In this model the collateral effect is operating. Consider for a moment profit per workers equal to 1.32 as they are in the second line. Such level of profits can not be an equilibrium since the value of a new firm is lower than the entry cost. To ensure an equilibrium it is necessary to obtain an *additional fall in the welfare value of unemployment*. Such additional fall is exactly the amplification power of financial market imperfections that we are investigating. Table 4 makes clear that the additional fall in the value of unemployment is tiny and negligible, and lower than 1 percent.

Remark 5.1 *The additional fall in the welfare value of unemployment implied by the model with only the collateral effect is is quantitatively negligible.*

Consider now the model 3 in Table 4. With respect to the model 2 the pledgeability effect is now operating. This can be seen from the fact that alongside a fall in productivity to 0.96 there is also a fall in non pledgeable income to 0.87. The fall in the unemployment value is now amplified to 6 percent against a fall in productivity around 4 percent, thus there is an additional increase in volatility of 50 percent with respect to the baseline model. Note also that this increase in volatility is associated to a drop in the size of new entrant firms by 20 percent. The drop in the new entrant size is thus driven by both internal liquidity and leverage.

⁵Incidentally, this is not too distant from the lowest value of the grid in the calibrated state space of productivity

Quantifying Elasticities in Steady State

Before moving to the dynamic simulations, we can use the results from the existence theorem to get some estimate of the size of the steady state elasticity of the three effects. The existence proof showed that the equilibrium can be described by equation 16, where the welfare value of unemployment is the crossing between a downward sloping line in U and an upward sloping function described by $C(U)$.

$$\tilde{\delta}U + \kappa [(1 - \beta)U - z]^\alpha = \frac{ye^\epsilon}{1 - \beta} - \phi + \frac{\tilde{K}}{ye^\epsilon + \tilde{K}}xye^{-\gamma\epsilon} \quad (28)$$

Taking a linear approximation around the steady states $U = U^*$ we have

$$\left[\tilde{\delta} + \alpha(1 - \beta)C(U^*) \right] dU = \tilde{y}e^\epsilon d\epsilon + \frac{\tilde{K}}{(y + \tilde{K})} \gamma xy e^{-\gamma\epsilon} d\epsilon + \frac{\tilde{K}}{(y + \tilde{K})^2} yxy e^\epsilon d\epsilon \quad (29)$$

We can thus quantify the elasticity of U with respect to a pure productivity shock ϵ and evaluate its value at steady state ($\epsilon = 0$). If we indicate the elasticity with Ψ to distinguish it from the productivity shock with ϵ we have that

$$\Psi_{U,y(\epsilon=0)} = \frac{1}{U^*} \left[\frac{y}{1 - \beta} + \frac{\tilde{K}}{(y + \tilde{K})^2} yxy + \frac{\tilde{K}}{(y + \tilde{K})} \frac{\gamma x}{1 - \beta} \right] \quad (30)$$

In the previous equation the three elements inside the square brackets correspond to measures of the elasticities of the three model (*Model I*: Basic DMP), (*Model III*: collateral and pledgeability) and (*Model III*: collateral and pledgeability). In other words, the total elasticity is a sum of three effects. Since, we have already shown that

$$el_{uy} \approx (1 - \alpha) \frac{(1 - \beta)U + z}{(1 - \beta)U} el_{Uy}$$

, we can obtain steady state estimates of the elasticity of unemployment to productivity.

Table 5 reports the decomposition of the elasticity in the various effect. Three results are immediately evident. First, the baseline model features an unemployment productivity elasticity of 1, a value that can not match the unemployment volatility of the US, as we show in the dynamic simulation that follows. Second, the collateral effect is negligible. Third, the pledgeability effect is sizeable, and total elasticity increase up to 1.5.

Dynamic Simulations

To begin with, we generate quantitative simulations for a pure DMP model. The first set of simulations refers to the general equilibrium of the model when firm size is fixed at its steady state A . We perform 500 simulations of a labor market draw of 20000 periods, out of which we get rid of the first 2000 observations to reduce the effect of initial conditions. All simulated data are logged and detrended with an Hodrick Filter parameter of 1600. In the Table we report the correlation coefficients and standard deviations of these simulated cyclical variations.

Let us focus on Table 6. The baseline model with fixed size can not match the US labor market statistics. Remember that in the US data compiled by Shimer (2005) and Hagedorn and Manovski (2008) the standard deviations of they key variables are approximately the following $\sigma_y = 0.01$, $\sigma_u = 0.1$, $\sigma_v = 0.1$ and $\sigma_\theta = 0.2$, so that unemployment fluctuates 10 times as much as productivity and market tightness as much as twenty times. The volatility of the basic model is an order of magnitude lower than that presented in the data. Such lack of volatility of the standard DMP model is labeled in the literature as the *Shimer Paradox*.

Figure 1: Decomposition of Productivity Effects: Steady State

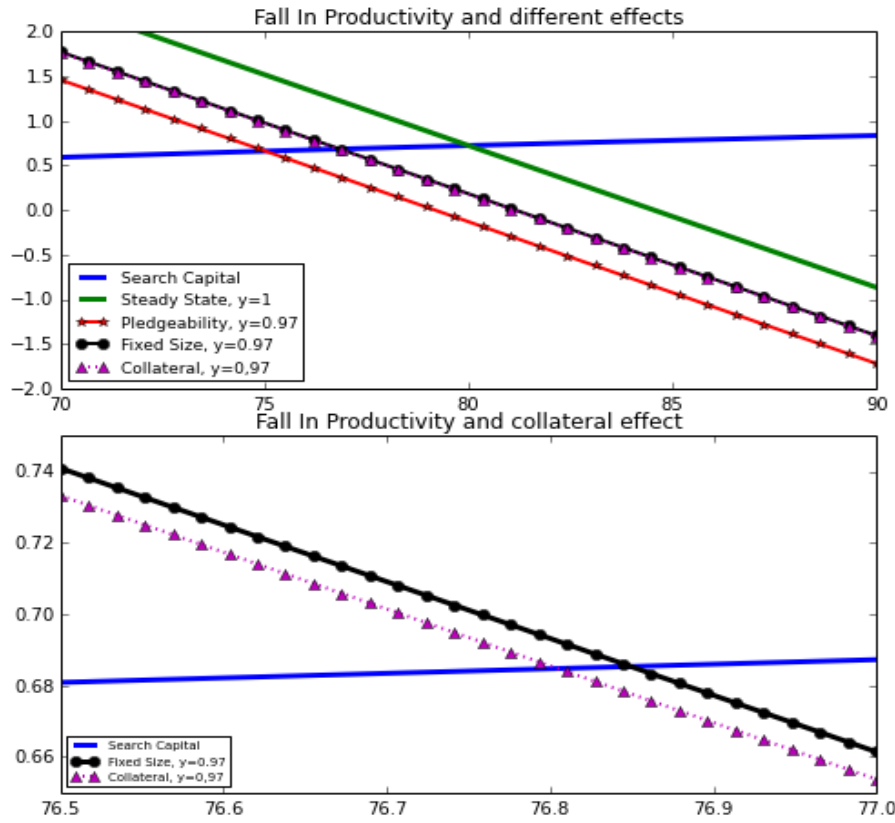


Table 5: Amplification with Endogenous Leverage

Model	Effect	$\Psi_{U,y}$ Welfare vs Productivity	$\Psi_{u,y}$ Unemployment Rate vs productivity
(1) Fixed Size ^a	$\frac{1}{1-\beta}$	1.25	1.01
(2) Endog. Lev ^b	$\frac{\tilde{K}}{(y+\tilde{K})^2} y^2 x$	0.001	0.002
(3) Endog. Lev ^c	$\frac{\tilde{K}}{(y+\tilde{K})} \frac{\gamma xy}{1-\beta}$	0.62	0.50
Total Effect		1.872	1.521

^a Model with fixed and maximum capacity of new firms $A = \bar{A}$

^b Model with endogenous leverage and and capacity of new firms $A = A(U)$ and fixed non pledgeable income x . Pure collateral effect

^c Model with endogenous leverage and and capacity of new firms $A = A(U)$ and non pledgeable income $x(y)$. Collateral effect and pledgeability effect

See main text for steady state equations

Sources: Author's calculation.

In light of literature that followed the Shimer findings, the results on the relative standard deviations in Table 6 are hardly surprising. Our basic model with a fixed firm size and time invariant financial multiplier is just a variant of Pissarides (1988). The two main differences are the fixed entry cost K and the fact that wages are modeled in the spirit of competitive search (Moen, 1997). When the Hosios conditions are satisfied, competitive search is identical to the Nash bargaining solution of Pissarides (1988).

In principle, increasing the value of non market activity $z = .5$ to $y = 1$ would generate enough volatility in the model. That is what Hagedorn and Manovski (2008). Shimer (2010) has also shown that modeling rigid wages can lead the model to feature as much volatility as in the data. That is not the point of our exercise. We want to ask how much additional volatility is introduced in the model when borrowing constraints are endogenous and are subject to the business cycle fluctuations. In other words, we are interested in understanding how much *amplification* power is obtained by the financial frictions.

The results are reported in Table 7 and Table 8. The procedure of the simulation is identical to that of Table 6 but financial constraints are now operating. Table 7 deals only with the pledgeability effect. Comparing the simulations of Tables 6 7 it is clear that the difference is negligible. The increase in the volatility between the two models is around 1 percent. Fluctuations in firms' leverage due to the collateral effect basically do not amplify the unemployment volatility.

Table 8 reports the simulation when both the collateral effect and the pledgeability effect are operating. Results are quantitatively significant. Since the quantitative impact of the collateral effect is negligible, as illustrated in Table 7 all the additional volatility featured by Table 8 is due to the pledgeability effect. The relative standard deviation of unemployment increase from 1.4 of Table 6 and 7 to 2.08 in Table 8. This amounts to an increase in volatility by 50 percent. Similar results are obtained for vacancies and market tightness. Note that the increase in volatility is driven by the volatility of the financial multiplier and firm size at entry, which appears in Table 8 to be 20 times more volatile than in the model of Table 7. Recall that in Table 8 pledgeable income fluctuates around 3 percent with respect to the baseline calibration. The model thus appears very sensitive to small cyclical changes in pledgeability $x(y)$, as we show in the following discussion.

Remark 5.2 *The additional fall in the welfare value of unemployment implied by the model when the pledgeability effect is operating is around 50 percent.*

To summarize, we learned that

- the value of unemployment U moves marginally more than y in the baseline model with fixed size;
- the value of unemployment U moves significantly more than that in the baseline model with fixed size when the pledgeability effect is operating.
- labor market variables are highly responsive to changes in pledgeable income.

Table 6: Simulation statistics:
Baseline DMP Model with fixed size

	y	U	$C(\cdot)$	A	k	v	θ	u	$p(\theta)$	lev
y	1.000	1.000	0.999	0.005	-	0.904	0.999	-0.717	0.999	1.000
U		1.000	0.999	0.005	-	0.904	0.999	-0.717	0.999	1.000
C			1.000	0.005	-	0.905	1.000	-0.717	1.000	1.000
A				1.000	-	0.004	0.005	-0.005	0.005	0.005
k					-	-	-	-	-	-
v						1.000	0.905	-0.352	0.905	0.904
θ							1.000	-0.717	1.000	1.000
u								1.000	-0.717	-0.717
$p(\theta)$									1.000	1.000
<i>Relative Standard Deviations (%)</i>										
	1.000	0.573	1.537	0.000	-	2.296	3.075	1.400	1.537	0.349
<i>Source: Authors' calculation</i>										

Table 7: Simulation Statistics:
Model with Collateral effect

	y	U	$C(\cdot)$	A	k	v	θ	u	$p(\theta)$	lev
y	1.000	1.000	0.999	1.000	-1.000	0.904	0.999	-0.713	0.999	1.000
U		1.000	0.999	1.000	-1.000	0.904	0.999	-0.713	0.999	1.000
C			1.000	0.999	-1.000	0.904	1.000	-0.713	1.000	1.000
A				1.000	-1.000	0.903	0.999	-0.713	0.999	1.000
k					1.000	-0.904	-1.000	0.713	-1.000	-1.000
v						1.000	0.904	-0.346	0.904	0.904
θ							1.000	-0.713	1.000	1.000
u								1.000	-0.713	-0.713
$p(\theta)$									1.000	1.000
<i>Relative Standard Deviations (%)</i>										
	1.000	0.577	1.548	0.115	0.383	2.312	3.096	1.410	1.548	0.419
$\gamma = 0$ See main text for equations on the pledgeability effect										
<i>Source: Authors' calculation</i>										

6 Unemployment Response to financial shocks

In this section we embark in the second quantitative exercise of the paper and focuses on the labor market effects of financial and liquidity shocks. In the previous section we showed that labor market variables are highly responsive to pledgeability shocks. The idea of this section is to consider a pure shock to pledgeable income, independently of productivity changes.

Holmstrom and Tirole (2011) show in great details the effects of sudden changes in liquidity generated by the investment of size A . In terms of our pledgeability function $x = x(y, z)$, this section is concerned with the effect x_z , where z is a pure financial shock. Most importantly, a financial shock z does not have any direct impact on firms' profitability. It only affects its borrowing constraint. In the aftermath of a liquidity shocks z , new entrants face lower resources for investments at given productivity level. The action in the model comes from the entry margin. The evidence reported by Siemer (2014) is coherent with such an approach. ⁶

⁶From the labor market perspective, the action in the model is all on the the job creation margin emphasized by Pissarides (1988). We do not model the effect of liquidity on the job destruction margin. In Boeri et al. (2014) we study the effects of refinancing shocks on the firm choice of liquid assets and we also show that such shocks can lead firms to liquidate assets and destroy jobs.

Table 8: Simulation Statistics:
Model with Collateral and Pledgeability effect

	y	U	$C(\cdot)$	A	k	v	θ	u	$p(\theta)$	lev
y	1.000	1.000	0.995	0.996	0.996	0.899	0.995	-0.697	0.995	0.992
U		1.000	0.998	0.994	0.993	0.902	0.998	-0.699	0.998	0.988
C			1.000	0.983	0.982	0.904	1.000	-0.700	1.000	0.976
A				1.000	1.000	0.888	0.983	-0.690	0.983	0.999
k					1.000	0.887	0.982	-0.689	0.982	0.999
v						1.000	0.904	-0.327	0.904	0.880
θ							1.000	-0.700	1.000	0.976
u								1.000	-0.700	-0.685
$p(\theta)$									1.000	0.976
<i>Relative Standard Deviations (%)</i>										
	1.000	0.845	2.297	7.217	6.721	3.435	4.595	2.087	2.297	4.764
$\gamma = 6$ See main text for equations on the pledgeability effect <i>Source: Authors' calculation</i>										

The Model with Pledgeability Shocks

In this section we assume that productivity is constant at its steady state value y . Conversely, the economy is subject to stochastic liquidity shock in the form of pledgeability shock. Formally, the pledgeability function of equation(6), at time t is defined as

$$x_t = yx e^{\gamma_z z_t} \quad (31)$$

where y and x are the steady state values of productivity and pledgeability, $\gamma_z > 0$ is a parameter and z_t is a pure financial shock that follows an autoregressive path

$$z_t = \rho_z z_t + \omega_t$$

with $\rho_z < 1$ and ω_t is $N(0, \sigma_z^2)$. We use a discrete approximation of x_t and work with a finite number of states x_1, \dots, x_n and a stochastic matrix P^x such that

$$p_{if}^x = \text{prob}[x_t = i | x_{t-1} = j]$$

The value functions need to be specified with respect to x and x' . Note that financial shocks do not enter directly in the value of the surplus, nor in the value of unemployment. Conversely, they affect the financial multiplier and the firm size. The borrowing constraint now reads

$$A(\phi + C) = Y + A(S + \tilde{\rho}(x)) \quad (32)$$

where $Y = \frac{yoy}{1-\beta}$ while $\tilde{\rho}(x)$ solves the recursive equation

$$\tilde{\rho}(x) = x + \beta(1 - \lambda)\tilde{\rho}(x'|x)$$

Firm size along the borrowing constraint becomes

$$\begin{aligned} A &= \frac{\frac{yoy}{1-\beta}}{\phi + C + \tilde{\rho}(x) - S} \\ &= \frac{yoy}{1-\beta} k(x) \end{aligned} \quad (33)$$

The firms' maximization problem and the equilibrium value of unemployment can now be written as

$$\begin{aligned} V(U(x), x) &= K \\ \text{s.t.} \quad A(\phi + C) &= Y + A(S + \tilde{\rho}(x)) \end{aligned} \quad (34)$$

where

$$V(U(x), x) = [S(x) - \phi - C(U(x))] A$$

Frequency and Intensity of Liquidity Shocks

The quantitative challenge is to calibrate the financial shocks. With respect to the steady state calibration offered in Section 4 we need to calibrate two additional parameters: ρ_z , the persistence of the liquidity shock as well as σ_ω^2 , the variance of the innovation of the financial shock. The choice of the persistence parameter ρ_z is obtained in using the evidence on the financial crises outlined in Section 2. The idea is that the most severe adverse financial conditions take place at very low frequencies, since financial crises are rare events. In section 2 we argued that financial crises take place every 22 years while systemic financial crises take place every 45 years.

Formally, the economy can settle in s liquidity and financial states $x_i, i = 1, \dots, s$, so that x_i is the value of liquidity in financial state i . We thus indicate with $p(x)^s$ the steady state distribution of such financial shock. Let's begin with the calibration of variance σ_ω^2 . To this purpose we introduce the definition of financial distress.

Definition 6.1 *A firm (and the economy) is in financial distress when internal funding completely dries up.*

In light of the previous definition, we are interested in the lowest tails of the distribution of x^s . Working with the steady state definition of liquidity we can say that a distress level of pledgeability x_d is such that

$$\frac{y - (1 - \beta)U + x_d}{1 - \beta(1 - \lambda)} \approx 0 \quad (35)$$

In other words, we set $\sigma_\omega^2(x)$ to be large enough to satisfy the financial distress condition. This exercise- with a grid size of width $n = 9$ implies that x'_i varies from a $x_1 = 0.02$ to $x_9 = 0.42$. The values of $\sigma_z^2 = 0.007$ and $\gamma_z = 2$

While the definition of financial distress may seem ad hoc, few remarks are in order. First, it is a natural definition from the model, especially once the steady state definition of x was calibrated to match the leverage of the non financial US firms. The exact empirical counterpart of x_d is not obvious. We do know that during the great recession there was a sizable fall in financial flows to the non financial business. Using Flow of Funds Accounts Amaral (2011) show that no other post-WWII recession comes remotely close to the great recession in terms of the magnitude of the reduction in borrowing flows. While in most recessions recessions borrowing flows fell by at most 2 percent of GDP, in the great recessions it fell by 5 percent. This means that during the financial recession the reduction was twice as large than in normal recession. Nevertheless, the empirical counterpart of our x_d applies to the financial flows of *new* business rather than to the average business firm. The work of Siemer (2014) reviewed at the beginning gives support to this idea.

To set the persistence parameter ρ we work with the idea that the firm and the economy end up in financial distress at very low frequency, and such frequency correspond to the low probability events of a crisis. We thus choose ρ so that $1 - p(x_d)$ matches the low frequency of financial crisis. In terms of quarterly specification, ρ_z must implies a crisis probability coherent with the rare events outlined in Section 2, so that

$$1 - p(x_d) = \frac{1}{4 * 45}$$

The steady state quarterly frequency of systemic crisis is around 0.005. This exercise implies $\rho_z = 0.978$, so that liquidity shock have higher persistence than productivity shock.

The unemployment Elasticity of a pure financial shock

Before moving to the dynamic simulations, we can replicate the estimates provided in Table 5 for a pure financial shock. Using the same logic of productivity shock, and the existence theorem, the value

Table 9: Steady States with average liquidity and with Financial Distress

Model	Prod	Plead. income	profits worker	Size New Firms	Entry Cost	Welfare Unempl.	Lev	Int Liq	Mkt Tightness
	y	$\nu(x)$	π	A	K	U	$lev.$	$l(y)$	θ
(4) Average liquidity ^a	1	0.90	1.320	3.69	4.87	80	2.41	0.10	0.66
(5) Financial Distress ^b	1	0.57	5.62	0.86	4.87	55.51	1.21	0.01	0.12

^a Model with endogenous leverage and pledgeable income calibrated as in the baseline model of Table 3

^b Model with endogenous leverage and a pledgeable income to distress level.

See main text for steady state equations

Sources: Author's calculation.

of unemployment can be approximated as

$$U \approx \frac{\Lambda}{\tilde{\delta}} + \frac{y}{1-\beta} - \frac{\phi}{\tilde{\delta}} + \frac{\tilde{K}}{y + \tilde{K}} \frac{x}{1-\beta} y e^{-\gamma_z z} \quad (36)$$

We can quantify the elasticity of U with respect to a pure shock z and evaluate its value at steady state ($z = 0$). If we indicate the elasticity with Ψ to distinguish it from the productivity shock with ϵ we have that

$$\begin{aligned} \Psi_{U,x(z=0)} &= \left[\frac{\tilde{K}}{(y + \tilde{K})} \frac{x\gamma_z}{1-\beta} \right] U \\ &= 0.42 \end{aligned} \quad (37)$$

This elasticity value is particularly large when the economy is in financial distress. Note that in our calibration non pledgeable income increase for 0.1 to 0.4, which implies that x triple. With an unemployment elasticity of 0.42, the increase in unemployment can be larger than 100 percent.

Result 6.1 *If non pledgeable income jumps to financial distress level, the unemployment rate can potentially double.*

Impulse Response of Financial Shocks

The calibrated level of x allows us to embark enter in the the second exercise of the paper. We do not aim at accounting for the average fluctuations. Distress is a rare event and can not account for the average business cycle shock. Yet, we are still interested in the macro effects of extreme financial shocks. These rare events are best analyzed in terms of impulse response functions, to which we turn next. The exercise we perform is the following.⁷ We assume that the stochastic economy is currently settled around its steady state, and defines such condition as quiet. Given a particular time path time of T periods, we assume that at time t^* , x_{it} jumps from a quiet situation to a crisis situation. Specifically, we construct a time path in which at quarter $t = 10$ the financial shocks jumps from a comfort zone with $x_t = x$ to a situation in which $x_t = x e^{z_d}$. In light of the persistence parameter ρ_z and the stochastic matrix P_x such jump is almost a zero probability event. At time $t = 20'$ the value

⁷The time path in Figure 2 uses $\gamma_z = 5.5$.

of x_t goes back one step at a time toward its quiet level. The economy is back in quiet zone at time $t = 45$

Figure 2 reports the time profile of key macro variables. The first panel is just the shock profile of non pledgeable income. The second chart is the unemployment rate. The level of unemployment increases up to 10 percent in 4 quarters, with an increase as large as 60 percent percent. Vacancies fall by 50 percent and market tightness by 70 percent. Note that the second row of Figure 2 highlights the one to one link between market tightness and unemployed welfare. Leverage, the financial multiplier and the long run size of new firms follow similar patterns.

US Labor market around 2007 and the Time Profile of the Financial Recession

In light of the dynamics of Figure 2, it is tempting to recall the behavior of the US economy around 2007 and 2009. This is done in Figure 4. Job openings fell from a peak of 3.2 percent in the second quarter of 2007 to a through of 1.8 percent in 2009. In the same second quarter of 2007 unemployment started to rise and reached a maximum of 10 percent three years later, but it was already above 9 percent in 2009. Most importantly, productivity did not fell and actually increased after a mild fall in 2008. In terms of financial time line of the financial crisis compiled at the Federal Reserve Bank of St. Louis starts in February 2007, when Freddie Mac announced that would stop buy the most risky sub-prime mortgages. In April 2007 New Century Financial Corporation, a leading sub-prime lender, filed for Chapter 11. In June 2007 Bear Stearns- a leading investment bank, informed investors that it was suspending redemptions from one of its Structured Leveraged Funds. Given these facts, it is not surprising that Christiano et al. (2015) use financial shocks to rationalize the behavior of the US economy in the great recession.

For pure comparison, Figure 3 reports the same time path experiment that we performed on pledgeability also for productivity shock. In Figure 3 the driving force is a drop in productivity of 10 percent. We basically have a fall in labor productivity from its steady state value to a value such that internal liquidity dries up. In Figure 3 productivity shock induces a much milder increase in unemployment and fall in vacancies. Overall, the shape of the dynamics in the two shocks is similar, but the the intensity of the reaction of the economy is much smaller with respect to the pledgeability shock of Figures 2. In addition, we should also recall that such fall in productivity did not take place during the last great recession.

7 Final Remarks

There is an ongoing vibrant research on the labor market consequences of financial market imperfections. This paper has offered a contribution to such research along three lines.

First, it has presented and solved a simple aggregate model of financial constraints, internal liquidity and labor that relies on two key contributions in the existing literature. The labor market is a traditional DMP model with wage posting, in the spirit of Pissarides (1998), Mortensen and Pissarides (1994) and Moen (2007). The financial frictions rely on limited pledgeability in the spirit of Holmstrom and Tirole (1997) and (2011). The model is very simply and can be applied to different issues in labor and finance, as we are currently doing in Boeri et al. (2014) where we study the effect of liquid assets on firms hiring and firing decisions in the presence of refinancing shocks.

Second, we asked whether borrowing constraints enhance and amplify the labor market response of aggregate productivity shocks. The theoretical is clear, and we showed that fluctuations in productivity affect pledgeable income and amplify shocks over and beyond what a standard Pissarides (1988) model would imply. The analytical results identify two channels through which productivity changes affect financial frictions and enhance aggregate fluctuations. The first effect is a collateral effect and the second effect is a pledgeability effect. In the calibration exercise, we match not only the key labor market statistics, but also the average leverage of US non financial non listed companies. The

Figure 2: One Time Financial Shock

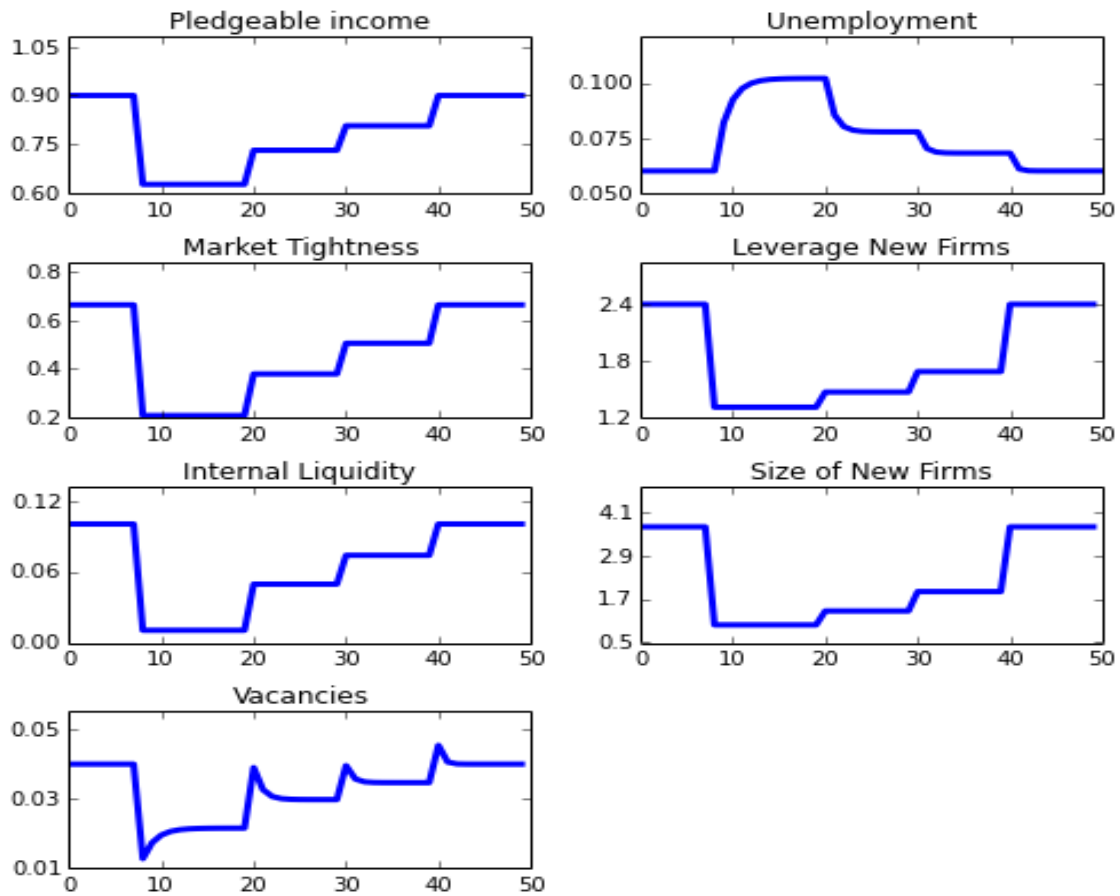


Figure 3: One Time Productivity Shock

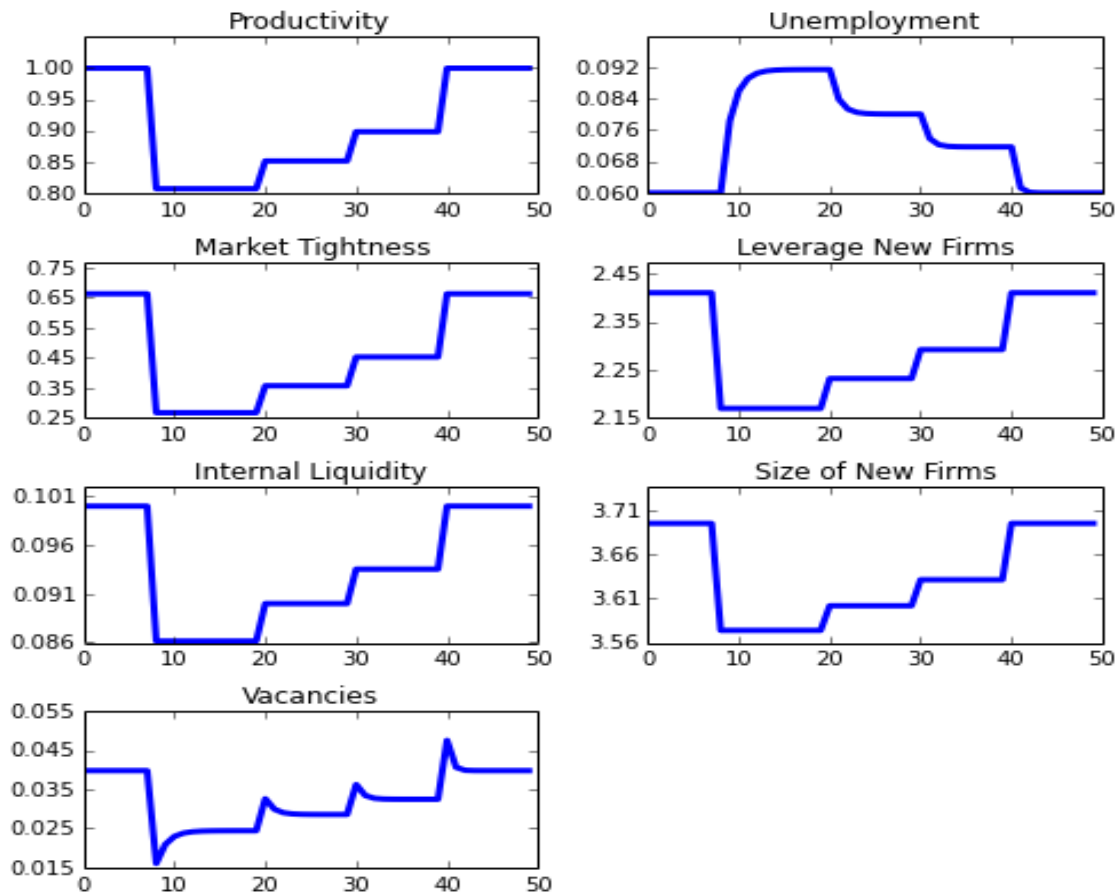
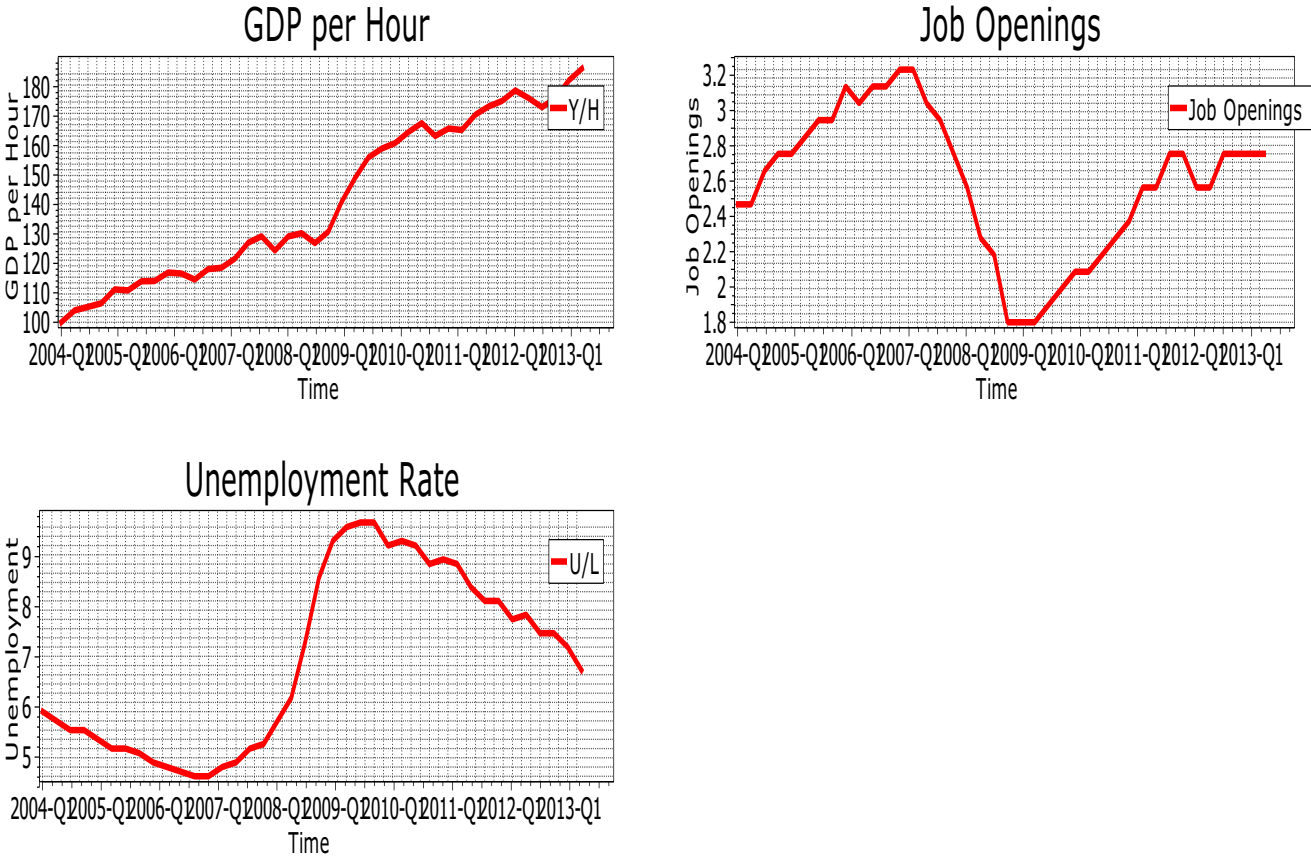


Figure 4: US Key Labor Market Statistics; 2004-2013



Source: Bureau of Labor Statistics at www.bls.gov

calibration of the model clearly suggests that the collateral effect is negligible while the pledgeability effect increases the response of unemployment to productivity changes as much as 50 percent.

Third, we ask whether financial shocks can help improve our understanding of labor market dynamics. Extreme financial shocks are a very rare event, as documented by the literature on systemic financial crises. Using the low frequency of such events, we asked what happens into the aggregate labor market when shocks to pledgeable income completely dry up firms' internal liquidity. The definition of such shock is very natural in the context of the model. The quantitative results show that these adverse financial shocks can not account for the average fluctuation in the US labor market, but can easily rationalize dramatic increases in unemployment and falls in vacancies, throwing new light on what happened in the US between 2007 and 2009.

Appendix

Solution Algorithm

The model needs to determine three vectors: $U(y)$, $A(y)$ and $S(y)$. The most stable vector is the following

1. Assume a vector of A . Assume a vector of $U(y) = \begin{bmatrix} U(y_1) \\ \vdots \\ U(y_i) \\ \vdots \\ U(y_n) \end{bmatrix}$ and obtain immediately $U(y'|y)$

$$\text{as } \begin{bmatrix} \sum_{j=1}^{j=n} p_{1j} U(y_j) \\ \vdots \\ \sum_{j=1}^{j=n} p_{ij} U(y_j) \\ \vdots \\ \sum_{j=1}^{j=n} p_{nj} U(y_j) \end{bmatrix}$$

2. Obtain the vectors $\theta(U, U'(y'|y_i))$, $C(U, U'(y'|y_i))$
3. Assume $S(U, U')$ and iterate over surplus value function to obtain $\hat{S}(U, U'(y'|y))$
4. Given S calculate profit to obtain $\pi(\hat{S}(U, U'(y'|y)), \hat{U}, \hat{U}')$
5. Calculate difference $\Delta = \pi(\hat{S}(U, U'(y'|y)), \hat{U}, \hat{U}') - K$ and obtain $\hat{U} = U + \delta \Delta$ where δ is an adjustment factor
6. Update $\hat{U}'(\hat{U}(y'|y))$ and go to 2 until convergence
7. Update A with respect to the values of U, U' and go to 1 until convergence

Restrictions on parameters

We require that $y - z > 0$ and either that $\phi < \frac{y-x-z}{1-\beta(1-\lambda)}$ or that

$$\frac{Y_0}{\phi - \frac{y-x-z}{1-\beta(1-\lambda)}} \left[\frac{y-z}{1-\beta(1-\lambda)} - \phi \right] > K \quad (38)$$

Numerical solutions to asset values

$$R = w - U + \beta U(y'|y) + \beta(1-\lambda)R(y'|y).$$

This can be solved forward to yield

$$R = \sum_{j=0}^{\infty} \beta^j (1-\lambda)^j \{E[w_{t+j}|y_t] - E[U(t+j|y_t)] - \beta E[U(t+j+1|y_t)]\}$$

In words the rent is the expected value of the wages in excess of the value of unemployment. Using the resolvent operator (See Sargent Ljungqvist, 2012) we have that the expected value of productivity is

$$\begin{aligned} \sum_{j=0}^{\infty} R^j (1-\lambda)^j E[y_{t+j}] &= (I - \beta(1-\lambda)P - \beta^2(1-\lambda)^2P^2 - \dots)y_i \\ &= [(I - \beta(1-\lambda)P)^{-1}\bar{y}]_i = \Delta(y_i) \end{aligned}$$

where P is the stochastic matrix and \bar{y} is a column vector of the state space. In other words $\Delta(y_i)$ is the i -th row of the square matrix $[(I - \beta(1-\lambda)P)^{-1}\bar{y}]$. Note that $\beta(1-\lambda) < 1$ ensures convergence of $\Delta(y_i)$. Applying the resolvent operator to the outside flow y_o , we compactly define $\Delta_o(y_i)$ as

$$\sum_{j=0}^{\infty} \beta^j (1-\lambda)^j E[y(t+j|y_t)] = [(I - \beta P)^{-1}\bar{y}]_i = \Delta_o(y_i).$$

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