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4

CONJECTURAL EQUILIBRIA AND RATIONALIZABILITY IN A GAME WITH INCOMPLETE INFORMATION

Pierpaolo BATTIGALLI and
Danilo GUAITOLI

Abstract

Conjectural equilibria and rationalizability are introduced in order to analyze extensive games with incomplete information when the common prior assumption is dropped. We provide a macroeconomic example which clearly illustrates the crucial role of players' structural and behavioral conjectures: different equilibrium and disequilibrium paths (some with keynesian type inefficiencies) are generated by different expectations patterns. Convergence of rationalizable paths to some conjectural equilibrium may occur, but not necessarily to a full information–rational expectations one.

> "We may therefore very well have a position of equilibrium only because some people have no chance of learning about facts which, if they knew them, would induce them to alter their plans. Or, in other words, it is only relative to the knowledge which a person is bound to acquire in the course of the attempt to carry out his original plan that an equilibrium is likely to be reached." (Hayek (1937))

Introduction[1]

An n-person game of incomplete information can be represented in the following way: the actual game to be played is determined by an unknown state $\theta = (\theta_0, \theta_1, \ldots, \theta_n) \in \Theta = \prod_i \Theta_i$ where each player i knows her "type" θ_i and θ_0 is an unknown environmental parameter. The standard solution concept for such games, the Bayes–Nash equilibrium introduced by Harsanyi (1967–68), assumes that players' interim beliefs about the unknown component of the state are mutually consistent in the sense that they can be derived from a *common prior* ρ on Θ. An equilibrium is given by a profile $[(s_1(\theta_1))_{\theta_1 \in \Theta_1}, \ldots, (s_n(\theta_n))_{\theta_n \in \Theta_n}]$ whereby for all (i, θ_i) the corresponding strategy $s_i(\theta_i)$ maximizes the expected payoff of i conditional on θ_i given $(s_{-i}(\theta_{-i}))_{\theta_{-i} \in \Theta_{-i}}$. In the finite case, assuming that the prior ρ is strictly positive, Bayes–Nash equilibria are equivalent to the Nash equilibria of a corresponding game with imperfect information whereby a chance move selects the state θ with probability $\rho(\theta)$ and each player i observes θ_i.[2]

Note that the common prior assumption should be interpreted as a technical device which allows to derive all the conditional beliefs which are necessary for game theoretic analysis. As a matter of semantics, if the state θ were actually chosen by some random mechanism, the strategic situation would simply be a game with imperfect and possibly asymmetric information about a chance move, such as Pocker.

A more general notion of Bayesian equilibrium replaces the common prior hypothesis with the weaker assumption that each player i of type θ_i knows which interim belief each player j would have conditional on each θ_j (of course, this does not mean that θ_i knows the *actual* interim beliefs of her opponents). In other words, the map from types to their interim beliefs is part of the parameters of the game theoretic model which are assumed to be (commonly) known by players. If types simply correspond to private information concerning technology and tastes, as is usually the case in economic applications, this generalization does not make the underlying epistemic assumptions of the model significantly more plausible. If the notion of type also encompasses all the

[1] This is a revised version of Battigalli and Guaitoli (1988) which in turn builds on the independent works of Battigalli (1987) and Guaitoli (1987). We have corrected and embellished the analysis without changing the essential ideas. Since 1988 the conjectural (or subjective, or self-confirming) equilibrium concept has been thoroughly analyzed, mostly by other authors and independently of our work. Also the notion of rationalizability for extensive games has been thoroughly explored by a few authors (including one of us). Had we tried to take all these theoretical developments into account, we would have written a new different paper. Therefore here we only mention the most related later works in some footnotes.

[2] According to another interpretation each θ_i is a separate player.

private beliefs of a player, including beliefs about beliefs about beliefs... as suggested by Harsanyi (1967–68), then this more general assumption becomes a near tautology. But in this case, the general types space is so wildly complex that in order to get a workable model it is necessary to consider suitably "small" belief closed subsets of types, thus losing the apparent gain in generality (see Mertens and Zamir (1985), Brandenburger and Dekel (1993) and references therein).

Consider a given normal form or extensive form game G and suppose that G is played repeatedly over time. A Nash equilibrium of G can be interpreted as a steady state of a learning process and it is possible to provide sufficient conditions such that, if the learning process converges at all, it eventually induces a Nash equilibrium outcome. For example, Battigalli (1987) shows that in a two-person extensive form game with (possibly) an initial chance move, if each player can perfectly observe ex post all the actions which have been actually taken in a given period, then any steady state of a plausible learning process must be observationally equivalent to a Nash equilibrium. This result can be generalized to n-person games with "observable deviators."[3] But the steady state interpretation is not valid for the Bayes–Nash equilibria of a game of incomplete information. If we take seriously the idea that there is incomplete (and possibly asymmetric) information about the true parameters of the model as specified by some state of the world, we should assume that "the world begins" at the interim stage at some *fixed* state θ. Let $G(\theta)$ be the "true game" corresponding to θ. Then we should assume that the particular game $G(\theta)$ is played repeatedly. Therefore the players at most can learn to play as in a Nash equilibrium of $G(\theta)$, i.e. an *ex post-Nash* equilibrium at θ.[4] They cannot learn to play a Bayes–Nash equilibrium of the incomplete information game, because they cannot observe the play that would obtain in states $\theta' \neq \theta$.

Thus we are left with the classic motivation for the Bayes–Nash equilibrium concept: equilibrium play is the result of strategic introspective reasoning by the players given common knowledge of the model describing the incomplete information game including the common prior or, more generally, the map from types to interim beliefs.

[3] The precise definition of games with observable deviators is contained in Section 2.2 and extensively discussed in Battigalli (1995). The generalization of the above mentioned result to games with observable deviators has been independently put forward by Fudenberg and Levine (1993a).

[4] For example, if the game of incomplete information has no environmental uncertainty (θ_0 known) and has private values (the payoff of player i does not depend on θ_{-i}) and if the "true game" $G(\theta)$ satifies the above mentioned assumptions, a steady state would be observationally equivalent to a Nash equilibrium of $G(\theta)$.

In many economic situations the common prior assumption is not plausible. Often the type of a player is best interpreted as a psychological parameter such that no reliable statistical evidence about it is available. This holds *a fortiori* when the type of a player also incorporates her beliefs. In such situations the Bayes-Nash equilibrium concept lacks a plausible justification. Here we consider a different approach pursuing two complementary themes. On one hand, following Hahn (1973) we insist on the steady state interpretation of equilibria, showing that it leads to consider the concept of a *conjectural equilibrium at* a given state θ, a generalization of the ex post-Nash equilibrium concept. On the other hand, we formalize a non-equilibrium solution concept for dynamic games of incomplete information, without common prior, played "one-shot." This solution concept is a modification of Pearce's (1984) extensive form *rationalizability* and characterizes the strategic choices which are consistent with strategic introspective reasoning.

We illustrate these equilibrium/solution concepts analyzing a stilized macroeconomic game with observed sequential actions and incomplete information about an aggregate productivity parameter. In particular we show that a rationalizable path of a multi-period version of the game can quickly converge to a conjectural equilibrium outcome of the single-period game, which need not be an ex post-Nash equilibrium outcome. These results are used to discuss potential sources of ineffectivness of monetary policy.

Conjectural Equilibria. As Hayek (1937) says in the above quoted passage, it is possible that some individual has incorrect conjectures about the environment and the opponents' behavior, but she is unable to find out that she is wrong and revise her behavior accordingly. Hahn (1973) proposes a general (and somewhat informal) notion of equilibrium which takes this possibilty into account. Here we provide a precise game-theoretic formalization of this notion of equilibrium. For a given extensive game with incomplete information without common prior and for each player in this game we specify what this player can learn ex post about the true state and the actual play when the game is over. The interpretation is that if the game were to be repeated this information would be available at the beginning of the next play. Each player has (probabilistic) conjectures about the state and the opponents' strategies. An array of strategies and conjectures is a *conjectural equilibrium at* a given *state* θ if (a) each player i maximizes her expected payoff given her type θ_i and her conjecture and (b) no player receives (ex post) information about the true state θ and the actual play induced by the strategies which is inconsitent with her conjectures.

Rationalizability. Bernheim (1984) and Pearce (1984) pointed out that pure introspective reasoning relying on common knowledge of the game and of players' rationality in general does not yield equilibrium play. In two-person static games one can only iteratively eliminate strongly dominated strategies. All the remaining strategies are *rationalizable*. They also noticed that extensive form reasoning may yield sharper results in dynamic games. Pearce's notion of *extensive form rationalizability* tries to formalize this kind of extensive form introspective reasoning. As we argued above, the case for non-equilibrium analysis is even stronger when we consider incomplete information games, because the Bayes-Nash equilbrium concept relies on the unplausible assumption that the map from types to interim beliefs is common knowledge. Therefore we put forward a notion of extensive form rationalizability for dynamic games of incomplete information. In the special case of two-person incomplete information games with no environmental uncertainty (θ_0 known), our solution concept is equivalent to applying Pearce's (1984) extensive form rationalizability to a companion extensive game where a fictitious player 0 with a constant payoff function chooses the state of the world. This equivalence does not extend to n-person games because, unlike Pearce (1984), our solution concept assumes that each player updates her beliefs about different opponents independently of one another and that this is common knowledge. This implies that each player can make sharper inferences about opponents' subjective beliefs from the observation of their past actions and hence can better predict their future actions. Beside being interesting in its own right, this feature plays an important role in our analysis of the multi-period macroeconomic game.[5]

1 Conjectural Equilibria and Rationalizability in Extensive Form Games with Incomplete Information

1.1 Extensive Form Games with Incomplete Information

A finite *n-person extensive form game with incomplete information* and without chance moves is a tuple $\Gamma = (\Theta; T, \preceq; \iota; H; A, \alpha; u)$ whereby:

- $\Theta = \Theta_0 \times \Theta_1 \times \cdots \times \Theta_n$ is a finite Cartesian product; $\theta_i \in \Theta_i$, $i = 1, \ldots, n$, denotes the *type* of player i and θ_0 represents a residual environmental

[5] Here we are able to give a relatively simple definition of the solution concept because we only consider games with observable deviators. See Battigalli (1996) for a general analysis.

unknown parameter; the $(n+1)$-tuple $\theta = (\theta_0, \theta_1, \dots, \theta_1)$ is called *state of nature.*

- $(T, \preceq)$ is a finite *arborescence* with initial nodes $\theta \in \Theta$,[6] Z denotes the set of *terminal nodes* and $X = T \backslash Z$ denotes the set of *decison nodes.* Given $Y \subset X$, $Z(Y)$ denotes the set of terminal nodes following some node $x \in Y$, $f(x)$ denotes the set of *immediate successors* of a decision node $x \in X$.

- $\iota : X \to \{1, \dots, n\}$ is the player function: $\iota(x)$ is the player moving at x; $X_i(\theta_i) = \iota^{-1}(i) \cap \{x \in T \mid \exists \theta \in \{\theta_i\} \times \Theta_{-i}, \theta \preceq x\}$ is the set of decision nodes for type θ_i of player i.

- H is the information partition, i.e. a partition of the set of decision nodes X (H is such that for all.$h \in H$, $x, y \in h$, $\iota(x) = \iota(y)$ and $f(x) = f(y)$); it is assumed that each player has *perfect recall*[7] and knows her own type; the latter assumption means that H is a refinement of the partition $\{X_i(\theta_i), i = 1, \dots, n, \theta_i \in \Theta_i\}$; $H_i(\theta_i)$ denotes the subpartition of $X_i(\theta_i)$ induced by H, i.e. the collection of information sets for type θ_i.

- $\alpha : T \backslash \Theta \to A$ is the *action function* specifying the last action taken to reach every non initial node (for each decision node $x \in X$, $\alpha(\cdot)$ is one to one on the set of immediate successors $f(x)$, and for each pair of nodes x, y in the same information set h, $\alpha(f(x)) = \alpha(f(y))$).

- $u = (u_1, \dots, u_n)$ is a vector of *payoff functions* $u_i: Z \to R$ (the dependence of payoffs on the state of the world follows from the fact that each terminal node $z \in Z$ has a unique initial predecessor $\theta \in \Theta$).

Note the absence of a probability measure on the set of initial nodes, or states, Θ: the true state $\theta \in \Theta$ is given at the outset, no random device selects θ and the players' subjective beliefs about the state are not part of the description of the game.

We will sometimes refer to the *complete information game at state θ.* This is the extensive game $\Gamma(\theta)$ obtained by restricting the information partition H and the functions ι, α and u to the tree with root θ.

[6] This means that the precedence relation $\preceq \subset T \times T$ is a partial, reflexive, antisymmetric and transitive binary relation on the finite set T such that (a) $\Theta \subset T$ is the set of nodes without predecessors and (b) the set of predecessors of any node $x \in T \backslash \Theta$ is totally ordered by $\preceq$.

[7] A player with perfect recall remembers her previous actions and information (see Kreps and Wilson (1982, p. 867) for a precise definition).

1.2 Special Information Structures

Some special information structures are worth of notice. A game Γ has *symmetric information* about the state if Θ_i is a singleton for all players $i = 1, \dots, n$ (but Θ_0 need not be a singleton) and has *observed sequential actions* if each player observes the history of past moves at each information set.[8]

We now turn to a more general information structure, called *observable deviators.* But first we need to define some auxiliary concepts.

A *strategy for type θ_i* of player is a function $s_i : H_i(\theta_i) \to A$ which assigns a feasible action $s_i(h)$ to each information set $h \in H_i(\theta_i)$. Let $S_i(\theta_i)$ be the set of strategies for type θ_i. Then we can define the set $\mathcal{S}_i$ of *feasible type-strategy* pairs for player i, i.e.

$$\mathcal{S}_i = \{(\theta_i, s_i) \mid s_i \in S_i(\theta_i)\}.$$

It is useful to adopt the convention that θ_0 is the "type" of a fictitious player 0 and the only feasible strategy for θ_0 is θ_0 itself. Thus $\mathcal{S}_0$ is the main diagonal of $\Theta_0 \times \Theta_0$. Since player j's payoff may be affected by player i 's behavior as well as by player i's type, $\mathcal{S}_i$ represents the relevant uncertainty about player i. $\mathcal{S} = \mathcal{S}_0 \times \mathcal{S}_1 \times \dots \times \mathcal{S}_n$ is the set of type-strategy profiles and $\mathcal{S}_{-i} = \times_{j \neq i} \mathcal{S}_j$ is the set type-strategy profiles of i's opponents. With a slight abuse of notation we write $(\theta, s) \in \mathcal{S}$ and $(\theta_{-i}, s_{-i}) \in \mathcal{S}_{-i}$. Each type-strategy profile (θ, s) induces a terminal node $\zeta(\theta, s) \in Z$. For any information set h, the set of type-strategy profiles consistent with h is

$$\mathcal{S}(h) = \{(\theta, s) \in \mathcal{S} \mid \zeta(\theta, s) \in Z(h)\}.$$

The set of type-strategy pairs for player i consistent with information set h is

$$\mathcal{S}_i(h) = \{(\theta_i, s_i) \in \mathcal{S}_i \mid \exists (\theta_{-i}, s_{-i}) \in \mathcal{S}_{-i}, (\theta_i, \theta_{-i}, s_i, s_{-i}) \in \mathcal{S}(h)\}.$$

Intuitively, a game has observable deviators if the total information of player $\iota(h)$ at information set h is given by (her type and past moves and) n separate pieces of information respectively concerning the type and past behavior of each of her n opponents (including the fictitious player 0). This can be formalized as follows:

[8] Formally, for all $h \in H$, $x', x'' \in h$, $x' \neq x''$ (if and) only if there are $\theta', \theta'' \in \Theta$ such that $\theta' \preceq x'$, $\theta'' \preceq x''$ and $\theta' \neq \theta''$.

Definition 1. *A game has* observable deviators *if for all information sets* $h \in H$

$$\mathcal{S}(h) = \mathcal{S}_0(h) \times \mathcal{S}_1(h) \times \cdots \times \mathcal{S}_n(h).$$

1.3 Terminal Information Patterns and Revealing Outcomes

When the game is over the agents get some information about the state of the world and the opponents behavior, which may affect their behavior in future strategic interactions. In general this information is imperfect. For instance, in the macroeconomic game discussed in the sequel, the terminal node z corresponds to an economic outcome or signal $\sigma = \phi(z) \in \Sigma$. The outcome σ is observed by everybody, but the outcome function $\phi : Z \to \Sigma$ is not one to one. We say that σ is *revealing* if there is only one terminal node z corresponding to σ, i.e. if $\phi^{-1}(\sigma)$ is a singleton.

More generally, each player $i = 1, \dots, n$ has her own signal function $\phi_i : Z \to \Sigma_i$. A *terminal information pattern* is an n-tuple of signal functions $(\phi_1, \dots, \phi_n)$.

1.4 Conjectural Equilibria

A *conjecture* for type θ_i of player i is a probability measure $c^i \in \Delta(\mathcal{S}_{-i})$ on the opponents' type–strategy set. Given conjecture c^i type θ_i chooses a strategy $s_i \in S_i(\theta_i)$ in order to maximize her expected payoff

$$U_i(\theta_i, s_i, c^i) = \sum_{(\theta_{-i}, s_{-i}) \in \mathcal{S}_{-i}} c^i(\theta_{-i}, s_{-i}) u_i(\zeta(\theta_i, \theta_{-i}, s_i, s_{-i})).$$

Consider a *fixed* state θ and suppose that the players interact repeatedly and try to maximize their expected payoff given their subjective conjectures.[9] For the

[9] Situations in which the state or some component of the state is actually selected at random at the beginning of each round according to an *iid* probability distribution can be modeled in the present framework by introducing chance moves in the extensive game.

sake of simplicity assume that they neglect the consequences of their current strategic choices on the future interactions or, more simply, that they maximize the expected payoff in the current interaction. A player will change her strategy in the next interaction only if the signal she gets at the end of the current interaction makes her change her conjecture. Suppose that player i (of type θ_i) chooses the expected payoff maximizing strategy s_i given conjecture c^i and let $\phi_i : Z \to \Sigma_i$ be her signal function. Player i will change her conjecture only if she gets a signal σ_i and if her conjecture c^i assigns a positive probability to some profile (θ'_{-i}, s'_{-i}) such that $\phi_i[\zeta(\theta_i, \theta'_{-i}, s_i, s'_{-i})] \neq \sigma_i$, that is, only if player i did not expect to receive signal σ_i with probability one.

A conjectural equilibrium is a situation in which each player maximizes her expected payoff given some subjective conjecture about the other players and the state of nature, and the induced signals do not make any player change her conjecture and hence her strategy (cf. Hayek (1937) and Hahn (1973)). More formally:

Definition 2. *A* conjectural equilibrium at state $(\theta_0, \theta_1, \dots, \theta_n)$ *with respect to the terminal information pattern* $(\phi_1, \dots, \phi_n)$ *is a* $2n$*-tuple* $(s_1, c^1, \dots, s_n, c^n) \in S_1(\theta_1) \times \Delta(\mathcal{S}_{-1}) \times \cdots \times S_n(\theta_n) \times \Delta(\mathcal{S}_{-n})$ *such that for each player* $i = 1, \dots, n$ *(a)* s_i *maximizes* $U_i(\theta_i, \cdot, c^i)$ *in* $S_i(\theta_i)$ *and (b) for each* (θ'_{-i}, s'_{-i}) *in the support of* c^i, $\phi_i[\zeta(\theta_i, \theta'_{-i}, s_i, s'_{-i})] = \phi_i[\zeta(\theta, s)]$.[10]

If for each player i the conjecture c^i is degenerate at (θ_{-i}, s_{-i}) – the true strategies and types of the opponents –, then we have a Nash equilibrium of $G(\theta)$, the complete information game at state θ, or in other words an *ex post Nash*

[10] The extension to the case of extensive games with chance moves and players using randomized strategies is straightforward: (b) is replaced by the condition that the induced subjective distribution on signals coincides with the induced objective distribution (Battigalli (1987)). However randomized strategies make more sense in a framework where at each round the actual players are drawn at random from large populations. This also justifies the short run maximization assumption. The equilibrium mixed strategies are interpreted as stationary statistical distributions and it is natural to replace (a) with the condition that different pure strategies in the support of a player's mixed strategy are best responses to possibly different conjectures. This corresponds to the *self-confirming equilibrium* concept put forward by Fudenberg and Levine (1993a) (they assume that the state is selected at each round by a chance move and that the terminal node is observed). Also the extension to the case of long run maximization is quite straightforward. This corresponds to and Kalai and Lehrer's (1995) notion of *subjective equilibrium*. Fudenberg and Levine (1993b), Fudenberg and Kreps (1995) and Kalai and Lehrer (1995) explicitly characterize conjectural equilibria as steady states of learning dynamics and provide convergence results. For more on conjectural equilibria and learning see Battigalli *et al.* (1992). Rubinstein and Wolinsky (1994) define a notion of *rationalizable conjectural equilibrium* describing a situation in which it is common knowledge that no player has a compelling reason to change her conjecture.

equilibrium at θ. This, of course, is a special case of conjectural equilibrium. Conjectural equilibria coincide with ex post Nash equilibria in static games if each player's signal function is one to one. But, in general, players' conjectures in a conjectural equilibrium can be heterogeneous and incorrect. [11]

Note that we are not considering the possibility that players choose randomized strategies. This implies that a conjectural equilibrium at θ does not necessarily exist (consider the trivial case of a static game without pure ex post Nash equilibria and one to one signal functions).

1.5 Rationalizability in Extensive Games with Observable Deviators

In Subsection 1.4 we assumed that a player maximizes her expected payoff given some arbitrary subjective conjecture. But conjectures are not all equally reasonable. Suppose that uncertainty and private information about the game are already represented by the incomplete information structure with states of nature and types so that it can be assumed without loss of generality that the incomplete information game Γ as defined in Subsection 1.1 is common knowledge. Sometimes it is possible to exclude some conjectures (and hence some choices) showing that they are inconsistent with common knowledge of rationality. If Γ is a truly dynamic game, that is, if it is possible that some player actually observes something about her opponents' behavior before moving, then we may want to assume something more than mere ex ante common knowledge of rationality. The solution adopted here is a modification of Pearce's (1984) extensive form rationalizability.

Consider a two-person game. Suppose that player i has some conjecture c^i at the outset. If rationality is common knowledge this conjecture must assign zero probability to those type-strategy pairs (θ'_j, s'_j) such that s'_j is not "rational"

[11] However, it can be shown that if $(\hat{s}, \hat{c})$ is a conjectural equilibrium at $\hat{\theta}$, then $\zeta(\hat{\theta}, \hat{s})$ is also an ex post Nash equilibrium outcome of $G(\hat{\theta})$ provided that the following conditions are satisfied:

(a). *private values*: for all i, s, θ_i, θ'_{-i}, θ''_{-i}, $U_i(\theta_i, \theta'_{-i}, s) = U_i(\theta_i, \theta''_{-i}, s)$;

(b). *observable deviators*;

(c). *perfect monitoring of actions*: for all i, θ, s', s'', if $\zeta(\theta, s') \neq \zeta(\theta, s'')$, then $\phi_i[\zeta(\theta, s')] \neq \phi_i[\zeta(\theta, s'')]$.

(d). *uncorrelated conjectures*: for all i, c^i is a product measure on S_{-i}.

This is a quite straightforward adaptation of a result due to Fudenberg and Levine (1993a).

for type θ'_j. But conjecture c^i might assign zero probability to some "rational" type-strategy pairs as well. Suppose that c^i assigns zero probability to the fact that one her information sets, say h, will be reached. How will player i change her beliefs about player j if she actually observes h? We postulate that if h is consistent with some "rational" type-strategy pairs for player j – that is, if there exists some "rational" $(\theta_j, s_j) \in S_j(h)$ – then player i sticks to the belief that player j is rational. If player j will move again after h, this means that player i expects player j to choose "rationally" in the future (backward induction). If player j has already moved before h,[12] then player i believes that j has chosen rationally in the past (forward induction). This is illustrated by the simple signaling game depicted in Figure 1. Suppose that player's 2 payoff for (θ', R, d) and (θ'', R, u) is $x > 0$. Action R is strictly dominated for type θ' of player 1, but not for type θ''. At information set h player 2 is sure that R has been chosen by type θ'' and responds with u. Anticipating this type θ'' would indeed choose R. Therefore the solution outcome is (R, u) if $\theta_1 = \theta''$ and L if $\theta_1 = \theta'$.

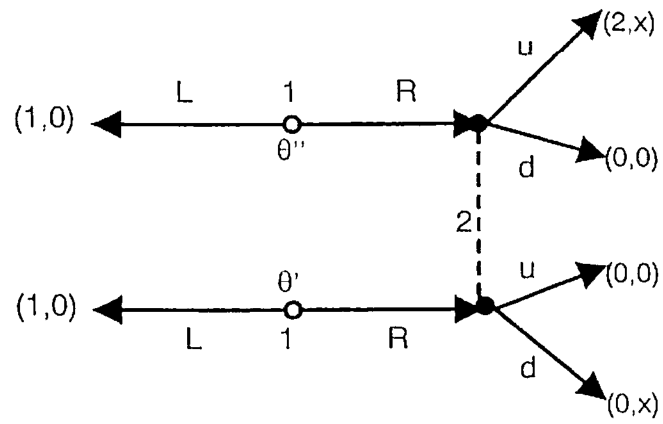

Figure 1 A signaling game.

We go even further and assume that even if h is not consitent with player j being rational and rationality being common knowledge, player i at information set h would assign probability zero to a type-strategy pair $(\theta'_j, s'_j) \in S_j(h)$ if there is some other pair $(\theta''_j, s''_j) \in S_j(h)$ such that there is a "better explanation" for θ''_j choosing s''_j than for θ'_j choosing s'_j. For example, consider again the

[12] Note that an information set h might merely represent information about j's type, i.e. we might have $S_j(h) = \{(\theta_j, s_j) \mid \theta_j \in \dot{\Theta}_j, s_j \in S_j(\theta_j)\}$ for some subset of types $\dot{\Theta}_j \subset \Theta_j$.

game depicted in Figure 1, but now suppose that $x < 0$. If player 2 is sure that R has been chosen by type θ'' then she responds with d. Anticipating this type θ'' would choose L. We assume that player 2 at h would indeed be sure that R has been chosen by θ'', because R is a best response to some conjecture for type θ'', but it is strictly dominated for type θ'. Thus R and h are inconsistent with player 1 being rational and rationality being common knowledge and player 2 belief at information set h reflects the fact that she looks for the "best rationalization" of R.

For two–person games with complete information or one sided incomplete information, the solution concept we advocate is essentially Pearce's (1984) extensive form rationalizability.[13] But we depart from Pearce's concept for general n–person games (with or without complete information). We assume that player i's beliefs about each opponent j are updated according to the "best rationalization" principle *independently* of what player i observes about other players k.[14] This feature of the solution concept will be illustrated by the analysis of the macroeconomic example. To simplify the analysis we focus here on incomplete information games with observable deviators. Since we are exploring the implications of common knowledge of rationality in a dynamic game, we need to consider players' beliefs at each information sets. A minimal rationality requirement is that such beliefs satisfy Bayes rule and each player always maximizes her conditional expected payoff.

Definition 3. *An* updating system of independent conjectures *for type θ_i of player i is an array of probability measures $[c_j^{\theta_i}(\cdot \mid \mathcal{S}_j(h))]_{j\neq i, h\in H_i(\theta_i)}$ such that (a) for all $h \in H(\theta_i)$, $c_j^{\theta_i}(\cdot \mid \mathcal{S}_j(h)) \in \Delta(\mathcal{S}_j(h))$ and (b) (Bayes rule) for all $h, g \in H_i(\theta_i)$, all $(\theta_j, s_j) \in \mathcal{S}_j(g)$,*

$$\mathcal{S}_j(g) \subseteq \mathcal{S}_j(h) \Rightarrow c_j^{\theta_i}((\theta_j, s_j) \mid \mathcal{S}_j(g))c_j^{\theta_i}(\mathcal{S}_j(g) \mid \mathcal{S}_j(h)) = c_j^{\theta_i}((\theta_j, s_j) \mid \mathcal{S}_j(h)).$$

A strategy $s_i \in S_i(\theta_i)$ is sequentially rational *for type θ_i given $[c_j^{\theta_i}(\cdot \mid \mathcal{S}_j(h))]_{j\neq i, h\in H_i(\theta_i)}$ if for each information set $h \in H(\theta_i)$ consistent with s_i (i.e. such that $(\theta_i, s_i) \in \mathcal{S}_i(h)$) s_i maximizes the conditional expected payoff $U_i(\theta_i, \cdot, c_{-i}^{\theta_i}(\cdot \mid \mathcal{S}_{-i}(h)))$ in $S_i(\theta_i, h)$, where $S_i(\theta_i, h) = \{s_i' \mid (\theta_i, s_i') \in \mathcal{S}_i(h)\}$ and*

[13] More precisely, in the two person, one–sided incomplete information case our solution concept is equivalent to Pearce's solution for a companion three–person game with a fictitious player 0 (Nature) choosing the state θ and having a constant payoff function u_0.

[14] This does not hold for Pearce's solution concept (see Battigalli (1996) for a discussion). In the first version of this paper we failed to notice this point. For this reason Propositions 3 and 4 of Battigalli and Guaitoli (1988) are formally incorrect.

$c_{-i}^{\theta_i}(\cdot \mid \mathcal{S}_{-i}(h)) = \prod_{j\neq i} c_j^{\theta_i}(\cdot \mid \mathcal{S}_j(h))$ is the joint product measure on $\mathcal{S}_{-i}(h) = \prod_{j\neq i} \mathcal{S}_j(h)$.

We are now ready to give our formal definition of rationalizability for extensive games of incomplete information with observable deviators. Define recursively the sets $R_i(k)$, $i = 1, \ldots, n$, $k = 0, 1, \ldots$ as follows:

(0) for all $i = 1, \ldots, n$, $R_i(0) = \mathcal{S}_i$,

$(k+1)$ for all $i = 1, \ldots, n$, $(\theta_i, s_i) \in R_i(k+1)$ if and only if there is an updating system of independent conjectures $[c_j^{\theta_i}(\cdot \mid \mathcal{S}_j(h))]_{j\neq i, h\in H_i(\theta_i)}$ such that

(a) for all $h \in H_i(\theta_i)$, $\ell = 0, \ldots, k$, $j \neq i$, if $R_j(\ell) \cap \mathcal{S}_j(h) \neq \emptyset$, then $c_j^{\theta_i}(R_j(\ell) \cap \mathcal{S}_j(h) \mid \mathcal{S}_j(h)) = 1$,

(b) s_i is sequentially rational for type θ_i given $[c_j^{\theta_i}(\cdot \mid \mathcal{S}_j(h))]_{j\neq i, h\in H_i(\theta_i)}$.

Condition (a) corresponds to the above mentioned "best rationalization" principle. It is easy to show by induction that $R_i(\ell) \subseteq R_i(\ell - 1)$. This implies that it is always possible to find an updating system satisfying condition (a).

Definition 4. *A strategy s_i is* rationalizable *for type θ_i if $(\theta_i, s_i) \in R_i(\infty) := \bigcap_{k\geq 0} R_i(k)$. A node $x \in T$ is rationalizable if there is some $[(\theta_i, s_i)] \in \prod_{i=1}^{i=n} R_i(\infty)$ such that $\zeta(\theta, s) \in Z(x)$.*

Since the game is finite, it can be shown by standard arguments that each $R_i(k) \neq \emptyset$ and there is some K_i such that $R_i(k) = R_i(K_i)$ for all $k \geq K_i$. Thus $R_i(\infty) = R_i(K_i) \neq \emptyset$ (cf. Battigalli (1996)).

The example depicted in Figure 2 shows how the proposed notion of rationalizability can yield forward induction results. We now establish a simple relation between rationalizability and backward induction.

Theorem 1. *Let Γ_x be a perfect information subgame of Γ with root x. Suppose that Γ_x has a unique subgame perfect equilibrium and that no player moves more than once in any path in Γ_x. Then, if x is a rationalizable node in Γ, every rationalizable profile consistent with x (that is, every $(\theta, s) \in \mathcal{S}(\{x\}) \cap R(\infty)$) coincides with the subgame perfect equilibrium on Γ_x.*[15]

[15] This proposition can be generalized as follows: Consider any proper subform $\hat{\Gamma}$ of Γ corresponding to an incomplete information game such that information sets are ordered (i.e.

Proof. (sketch). Since x is rationalizable, at x every player ascribes the "highest degree of strategic rationality" to her opponents. Since no player moves more than once and conjectures about different players are updated independently of one another, at each node y in the subgame the player $\iota(y)$ moving at y continues to ascribe the "highest degree of strategic rationality" to all those who might move after her. Therefore backward induction can be used to compute the rationalizable continuation strategies in the subgame. □

2 Conjectural Equilibria and Rationalizable Paths in a Macroeconomic Game with Incomplete Information[17]

To illustrate the solution concepts proposed in the previous section, we present and analyze a stylized macroeconomic game that exemplifies the role of players' conjectures in determining economic outcomes and policy effectiveness. Figure 2 shows the extensive form of the game, Figures 3 and 4 describe the economic meaning of initial and terminal nodes as well as players' actions. A formal model of the economy that can be represented in the stylized form of the game is discussed in the Appendix. The first player is a Central Bank, which controls the money supply; the second player is a Union, which controls nominal wages; the third player represents a large number of Firms that set prices (and determine employment). We first consider a one-period version of the game and then a two-period version. We will show how the rationalizable paths of the repeated game are related to the conjectural equilibria of the one-period game.

the precedence relation over nodes induces a unique partial order over information sets) and no player moves more than once in any path of $\hat{\Gamma}$. If the corresponding game can be solved by backward induction (using dominance when information sets are not singletons) and every initial node of $\hat{\Gamma}$ is rationalizable in Γ, then every rationalizable profile consistent with $\hat{\Gamma}$ coincides with the backward induction solution on $\hat{\Gamma}$.

[17] The reader should note that although we relate conjectural equilibria to rationalizability, we do not explicitely discuss a notion of "rationalizable conjectural equilibrium." Rubinstein and Wolinsky (1994) show that, given common knowledge of rationality, even if players' strategies and conjectures are rationalizable and consitute a conjectural equilibrium, the players may change their conjectures ex post. In this case, the given profile is not a rationalizable conjectural equilibrium. But Battigalli (1991) shows that if there is a common signal function such that actions are perfectly observed ex post (as in the games analyzed here), then a conjecural equilibrium in rationalizable strategies is also a rationalizable conjectural equilibrium.

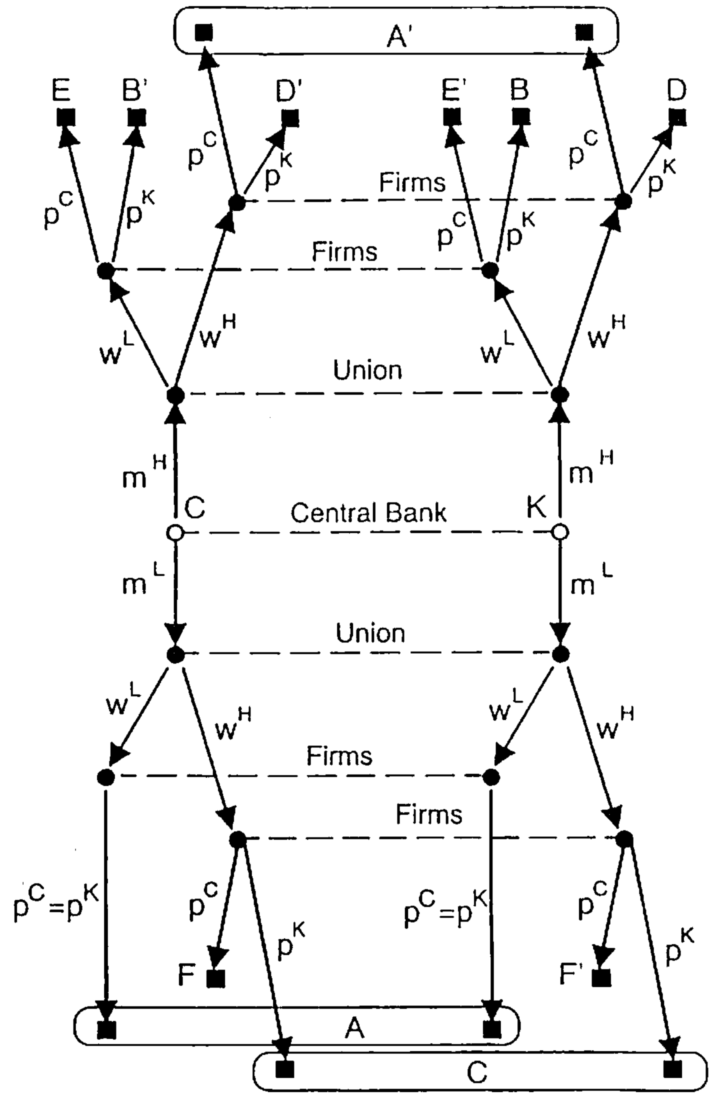

Figure 2 A macroeconomic game with symmetric incomplete information and observed sequential actions.

The problem we want to emphasize is not one of credibility, time consistency or players' conflicting objectives (as in Barro and Gordon (1983), Tabellini (1985),

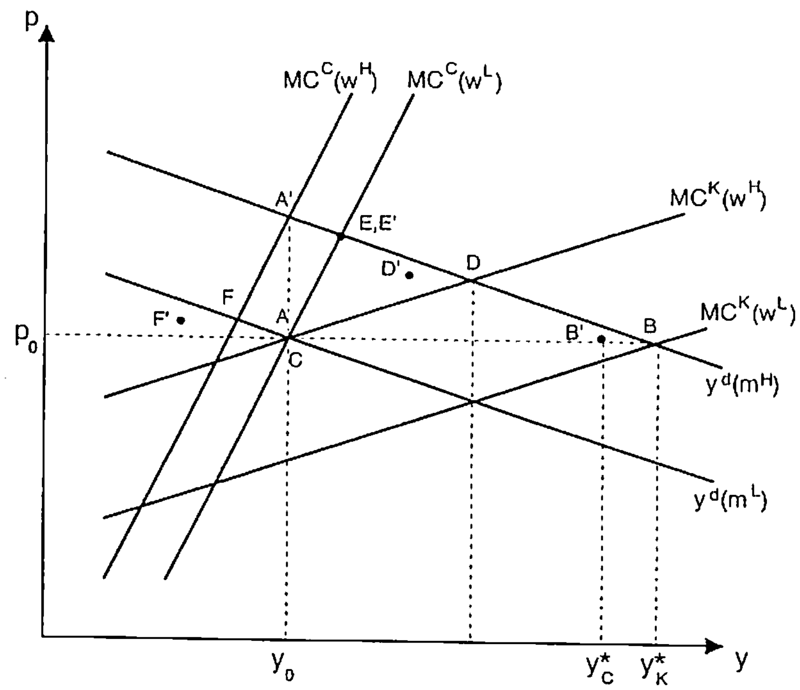

Figure 3 Economic characterization of the game: p price level, y output, MC^{θ} marginal cost schedule in state θ, y^d aggregate demand.

and related literature). We make this clear by assuming that the Central Bank moves first, thereby committing to its choice of the money supply. Furthermore, the Central Bank is sufficiently averse to inflation while having essentially the same objectives in terms of employment as the Union.

In the output market firms are spatially disperse and consumers have imperfect price information (e.g. they know the price distribution but not each individual firm's price). With convex search costs, the individual firm will perceive a kinked demand curve at the price set by the other firms (as in Stiglitz (1987)): the elasticity is greater for price increases than price reductions, because a larger number of customers will leave in the first case than will arrive in the second. Increases in demand or costs will leave prices unchanged as long as mark–ups remain positive; beyond that prices will rise following marginal costs. Decreases in demand or costs will leave prices unchanged as long as mark–ups

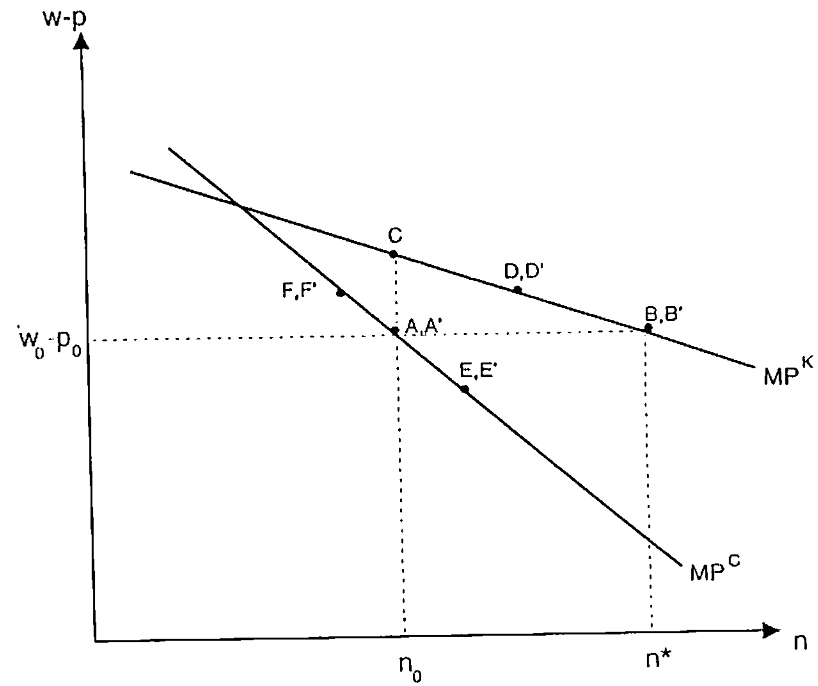

Figure 4 Economic characterization of the game: $w - p$ real wage, n employment, MP^{θ} marginal productivity schedule in state θ.

do not exceed the monopoly level; beyond that prices will fall following marginal revenues.

With this type of microstructure (or possibly others with anticyclical mark–ups) the economy can find itself in states of "Keynesian" as well as "classical" unemployment. By "Keynesian" we mean a state in which marginal productivity is greater than real wages (prices greater than marginal cost) and the level of real wages is potentially compatible with full employment. In "classical" states the marginal productivity is equal to real wages (prices equal marginal cost) and a higher employment requires a lower real wage. We model incomplete information by assuming that the economy is in one of two possible states (C for classical, K for Keynesian) corresponding to different marginal productivity schedules, but none of the players knows the parameters of the aggregate production function that define the state (the initial observable levels of output and employment reveal only the average, not the marginal productivity).

Each of the first two players can choose between two actions (which capture qualitatively the relevant policy issues).[18] The Central Bank can either keep the money supply constant at the initial level (m^L) or expand to the Keynesian full employment level of aggregate demand (m^H). The Union can either keep nominal wages constant at the initial level (w^L) or increase them proportionally to the monetary expansion (w^H). For different choices of m and w, Firms set different prices according to a classical (p^C) or a Keynesian (p^K) price function, which correspond to profit maximization under the assumptions made on the microstructure (see the Appendix and Figure 3).

Preference rankings are defined in the following way. The Central Bank has preferences over price stability and employment given by the following function V (here f_k denotes the partial derivative of f with respect to the kth argument):

$$V(|p_t - p_{t-1}|, n_t), \text{ with } V_1 = 0 \text{ if } p_t - p_{t-1} = 0,\ V_1 > 0 \text{ otherwise},\ V_2 > 0.$$

We assume it is sufficiently averse to inflation to generate the following ordering:

$$\begin{aligned} u_1(B) &= u_1(B') > u_1(D) = u_1(D') > u_1(A) = u_1(C) > \\ &> u_1(E) = u_1(E') > u_1(F) = u_1(F') > u_1(A'). \end{aligned}$$

The Union has preferences over real wages and employment given by the following function W:

$$W(w_t - p_t, n_t), \text{ with } W_1 > 0, W_2 > 0.$$

We assume it cares sufficiently about employment to generate the following ordering:

$$\begin{aligned} u_2(B) &= u_2(B') > u_2(D) = u_2(D') > u_2(C) > u_2(A) = \\ &= u_2(A') > u_2(F) = u_2(F') > u_2(E) = u_2(E'). \end{aligned}$$

[18] These actions actually correspond to the subgame perfect equilibrium strategies of the complete information game when players can choose actions in a continuum.

Firms do not behave strategically as an aggregate. We capture that by assuming that their payoff is always greater if they choose the price function corresponding to the true state:

$$u_3(C, p^C) > u_3(C, p^K), \qquad u_3(K, p^K) > u_3(K, p^C).$$

With *complete* information, as can be easily verified by backward induction, the game would have a unique subgame perfect Nash equilibrium for each initial state: (C, m^L, w^L, p^C) and (K, m^H, w^L, p^K). No variable would change if the initial state is classical, while in a Keynesian state a monetary expansion with wage and price stability would reach full employment. With incomplete information players choose their strategies on the basis of their conjectures (about the state and the strategies of the other players). We refer to a *conjectural equilibrium path* as a sequence of actions which are part of a conjectural equilibrium at a particular state. This is convenient because there may be many conjectures that supports the same equilibrium sequence at a given state.

Proposition 1. *In the one-period game, the only conjectural equilibrium path at state 0 = C is (m^L, w^L, p^C); both (m^H, w^L, p^K) and (m^L, w^L, p^C) are conjectural equilibrium paths at state 0 = K.*

Proof.

(i) At state C the sequence (m^L, w^L, p^C) can be supported for example by the following deterministic conjectures: each player believes the state to be C, the first two players expect Firms to play p^C, the Central Bank expects the Union to play w^L after m^L. Each player's choice is a best reply to these conjectures and (being observed ex post) is consistent with the other players' conjectures, while no new information is revealed about the state. But note that the same conjectural equilibrium path is obtained even if all players believe the state to be K, as long as they have the above conjectures about the players that follow. This is because $p^C = p^K$ after (m^L, w^L), and no new information is revealed about the state, so none of the conjectures would have to be changed. It can be easily checked that no other sequence of actions can be a conjectural equilibrium path at the state C, because the information revealed ex post would always be inconsistent with at least one of the players' conjectures supporting the sequence.

(ii) The actions in the sequence (m^H, w^L, p^K) at state K are best replies to these conjectures: each player believes the state to be K, the first two players expect Firms to play p^K, the Central Bank expects the Union to play w^L after m^H. In this case the outcome is revealing, so these are the only possible conjectures along the equilibrium path.[19]

(iii) At state K the actions (m^L, w^L, p^C) constitute another conjectural equilibrium path for the same conjectures that support the equilibrium at $\theta = C$: players' conjectures about the state do not matter, as long as the first two players expect Firms to play p^C and the Central Bank expects the Union to play w^L after m^L. No information is in fact revealed ex post about the state nor about Firms' strategy ($p^C = p^K$ along this sequence). Again it can be checked that no other conjectural equilibrium path exists at the state K.

□

The conjectural equilibrium in state C and the first of the two equilibria in state K correspond to ex post Nash equilibria: no change occurs in a classical state, while a monetary expansion with wage and price stability reaches full employment in a Keynesian state. But the other conjectural equilibrium in state K does not correspond to an ex post Nash equilibrium, since players' choices are not best replies to the true state. Even if each player believes the economy to be in a Keynesian state, this is not common knowledge (there is no common prior imposed by the game structure). So if the Central Bank and the Union are convinced that Firms have classical beliefs (and the Central Bank expects the Union not to increase wages unless there is an expansion), they will be induced to play "classical" as well: the Central Bank will not expand, fearing inflation, and the Union will stick to w^L. But since in this case Firms keep prices unchanged (no matter what they believe), there is no way to test and prove those conjectures wrong.

Conjectural equilibria are not the only possible outcomes of the one-period game. There are many more conjectures that players can have which will not generate a conjectural equilibrium path. By applying our definition of rationalizability we can easily derive the following result.

Proposition 2. *Any sequence of actions in the one-period game is a rationalizable path at each state.*

[19] Note that with the assumed payoff structure neither this equilibrium nor the previous one impose any restriction on the central bank's conjecture about the union's response to an out-of-equilibrium monetary choice.

Proof. First note that rationalizable strategies include: both p^C and p^K for Firms; both (w^L if m^L, w^H if m^H), as well as (w^H if m^L, w^L if m^H), for the Union; both m^L and m^H for the Central Bank. It is easy to find combinations of deterministic conjectures to induce any sequence of actions at each state. Except for the conjectural equilibrium paths, all other sequences will generate ex post information inconsistent with some conjectures, but ex ante all of them are rationable paths. □

We now turn to the analysis of the rationalizable outcomes of a two-period version of the macroeconomic game. A complete analysis of the two-period game depends on fine details such as the numerical values of the payoffs. But, fortunately, we can draw some intersting conclusions without a very detailed specification of the game. The essential features of the game are the following:

- After the first period game has been played, player 1, 2 and 3 observe the first period actions and move again in the same order as before, but this time their information about the state of nature is given by the terminal information pattern of the one shot game. One-period preferences over economic outcomes are the same as before.
- The two-period payoffs of player 1 and 2 (Central Bank and Union) are the discounted sum over the two periods. Player 3 (representative Firm) has *lexicographic* preferences over the expected payoff of period one and the expected payoff of period two respectively, i.e. she maximizes the current period expected payoff in each period. The rationale for this assumption is that otherwise player 3 might try to deceive the other players behaving as if she or he had a "classic" conjecture about the state, while the actual conjecture is "Keynesian." Clearly such behavior does not make sense when player 3 represents a very large number of independent firms. This lexicographic maximization assumption allow us to avoid a more complex mathematical formulation with a continuum of firms.

To simplify the analysis we need an additional assumption:

- Player 3 always has deterministic (degenerate) conjectures about the state and this is common knowledge at every point of the game.[20]

[20] More formally, the iterative definition of rationalizability is modified by considering for player 3 a restricted set of admissible updating systems of conjectures such that she or he always assigns probability one to one state $\theta \in \{C, K\}$.

Note that also the two-period game has observed sequential actions. Each revealing outcome of the one-period game corresponds to a game with complete and perfect information in the second period. Each non-revealing outcome corresponds to a game of incomplete information, either similar or equal to the one-period game analyzed above. This information structure, the fact that each player updates her or his conjectures about the state and other players' strategies independently of one another and the above mentioned assumptions about preferences have the following implications for the second period rationalizable behavior:

Proposition 3. *Every revealing rationalizable path corresponds to the backward induction outcome in the second period.*

Proof. Apply Theorem 1. □

Remark 1. In every *non-revealing* rationalizable path of the two-period game player 3 chooses the same action (either p^C or p^K) as in the first period.

Proof. Let p^θ be the price rule adopted by player 3 in the first period. In a rationalizable path, p^θ is a conditional best response given player 3's conjecture about the state, which must be degenerate at θ. In the second period the true state is not observed (non revealing path) and the conjecture does not change (independent updating). Thus p^θ is chosen in the rationalizable path also in the second period. □

In the analysis of the one-period game we have seen that the "classical" sequence of choices (m^L, w^L, p^C) is part of some conjectural equilibrium at both states $\theta = C$ and $\theta = K$. Player 1, the Central Bank, may decide not to expand the stock of money even if she has a "Keynesian" structural conjecture, as long as she expects that players 2 and 3 play in a "classical" way. Now we show how this conjectural equilibrium outcome may result from "rational learning", that is in the second period along a rationalizable path.

Proposition 4. *The two nodes in the two-period game corresponding to (C, m^H, w^H, p^C) and (K, m^H, w^H, p^C) and yielding the first period-outcome A' are rationalizable. Independently of the true state there is only one rationalizable continuation in the second period, that is (m^L, w^L, p^C) which is part of a conjectural equilibrium of the second period incomplete information game at both states $\theta = C$ and $\theta = K$.*

Proof. First note that the situation at the beginning of the second period game is the same as in the one-period game, except that all nominal variables are proportionally higher. Thus the players in the second period have the same kind of incentives as in the one-period game: there is a trade-off between inflation and unemployment for the Central Bank and a trade-off between wage and employment for the Union with the same preferences over economic outcomes in real terms. The representative firm chooses according to her conjecture about the state.

Suppose that (m^H, w^H, p^C) is indeed a rationalizable first-period sequence of actions. Then, by the remark above, Central Bank and Union rationalizable expectation about the representative Firm after (m^H, w^H, p^C) (which corresponds to the non-revealing outcome A') is P^C for every continuation. The Union maintains this conjecture also after the new move by the Central Bank (independent updating). Consider the rationalizable choice of the Union after (m^H, w^H, p^C, m^L). Independently of the conjecture on the state, the Union expects to get the same real wage and employment by choosing w^L and a slightly higher real wage, but lower employment by choosing w^H. We have assumed that the Union prefers the former outcome to the latter $(u_2(A) > \max\{u_2(F), u_2(F')\})$. Thus w^L is the rationalizable action after (m^H, w^H, p^C, m^L). On the other hand, whatever the conjecture about the state the Union prefers to raise the nominal wage (w^H) after a second monetary expansion, i.e. after (m^H, w^H, p^C, m^H), because an unchanged nominal wage would induce an unacceptably low real wage $(u_2(A') > \max\{u_2(E), u_2(E')\})$.

Thus the rationalizable expectations of the Central Bank after (m^H, w^H, p^C) are that a new monetary expansion (m^H) would only cause new inflation (w^H, p^C) while a restrictive monetary policy would at least maintain the staus quo. This implies that *if* (m^H, w^H, p^C) is rationalizable, then the only rationalizable continuation is (m^L, w^L, p^C).

To see that (m^H, w^H, p^C) is a rationalizable first-period outcome, note that $(K, m^H, w^L, p^K, m^L, w^L, p^K)$ corresponds to a rationalizable terminal node (it is correctly and commonly believed that everybody is "Keynesian", an initial monetary expansion induces full employment and then nothing changes in the second period). Thus m^H is a rationalizable action in the first period. But for the Union it is rationalizable to believe that the state is C and the representative Firm, believing this too, will always choose the classical price rule p^C. Such expectations induce the Union to raise immediately the nominal wage after a first period monetary expasion. It is (trivially) rationalizable for the representative Firm to believe that the state is C. Thus (m^H, w^H, p^C) is rationalizable in the first period. □

Appendix

Here is the example of a macroeconomic model that can generate the economic environment of the game analyzed in Section 2 (and which we have used to characterize the terminal nodes and the outcome function $\sigma = \phi(z)$). It is a "short-term" model, since we do not explicitely consider capital accumulation, investment and intertemporal allocations. It is also a "reduced-form" model, in the sense that aggregate relationships are introduced and motivated, but not all explicitely derived here from individual optimization. All variables are logarithms.

Aggregate demand for goods is a function of real money balances (e.g. through wealth effects on consumption, with money injected in the economy through lump-sum transfers to households):

$$y_t^d = b_0 + b_1(m_t - p_t), \qquad b_1 > 1.$$

Given a fixed stock of capital, and assuming that each household (in a continuum) supplies one indivisible unit of labor, output is related to employment through the aggregate production function

$$y_t = \theta_0 + \theta_1 n_t, \qquad \theta_1 \in (0,1),$$

with diminishing marginal productivity of labor

$$MP_t = g_0(\theta) - g_1(\theta) n_t,$$

where $\theta = (\theta_0, \theta_1)$, $g_0(\theta) = \theta_0 + \ln\theta_1$, $g_1(\theta) = 1 - \theta_1$.

Marginal costs, for a given (nominal) wage rate w_t, are

$$MC_t = w_t - \gamma_0(\theta) + \gamma_1(\theta) y_t,$$

where $\gamma_0(\theta) = (\theta_0/\theta_1) + \ln\theta_1$, $\gamma_1(\theta) = (1-\theta_1)/\theta_1$.

In the output market firms are spatially disperse and consumers have imperfect price information (e.g. they know the price distribution but not each individual firm's price). With convex search costs, the individual firm will perceive a

kinked demand curve at the price set by the other firms (as in Stiglitz (1987)): the elasticity is greater for price increases than price reductions, because a larger number of customers will leave in the first case than will arrive in the second. The gap in the marginal revenue makes that price a best reply even after small changes in demand or marginal costs; it also generates indeterminacy, with a continuum of possible "equilibrium" or rationalizable prices. Since we cannot study here the whole micro-game among firms, and in order to keep the analysis of the aggregate macro-game tractable, we make the following simplifying assumptions: 1) the range of possible profit-maximizing prices coincides with the whole interval between the competitive and the monopoly price; 2) each firm's conjecture on the other competitors is that they will keep the previous period price unchanged unless it now falls below the competitive level or above the monopoly level, in which case they will respectively increase it up to the competitive level or decrease it down to the monopoly level.

Aggregating over identical firms, the competitive price level is defined by the intersection between aggregate demand curve and marginal cost, provided the corresponding demand $\hat{y}_t^d$ does not exceed the full employment capacity $y_\theta^* = \theta_0 + \theta_1 n^*$, otherwise it is given by the inverse demand function at y_θ^* :

$$\hat{p}_t = \begin{cases} a_0 + a_1 m_t + a_2 w_t & \text{if } \hat{y}_t^d \le y_\theta^* \\ b_1^{-1}(b_0 + b_1 m_t - y_\theta^*) & \text{if } \hat{y}_t^d > y_\theta^* \end{cases}$$

where $\hat{y}_t^d = b_0 - b_1 a_0 + b_1 a_2(m_t - w_t)$, $a_0 = \frac{\gamma_1(\theta)b_0 - \gamma_0(\theta)}{1+\gamma_1(\theta)b_1}$, $a_1 = \frac{\gamma_1(\theta)b_1}{1+\gamma_1(\theta)b_1}$, $a_2 = \frac{1}{1+\gamma_1(\theta)b_1}$.

The monopolistic price level is given by the inverse demand function at the output level $\tilde{y}_t^d$ corresponding to the intersection between marginal revenue and marginal cost or at y_θ^*, whichever is smaller:

$$\tilde{p}_t = \begin{cases} \tilde{a}_0 + a_1 m_t + a_2 w_t & \tilde{y}_t^d \le y_\theta^* \\ b_1^{-1}(b_0 + b_1 m_t - y_\theta^*) & \tilde{y}_t^d > y_\theta^* \end{cases}$$

where $\tilde{y}_t^d = b_0 - b_1\tilde{a}_0 + b_1 a_2(m_t - w_t)$ and $\tilde{a}_0 = \frac{\gamma_1(\theta)b_0 - \gamma_0(\theta) - \ln(1-1/b_1)}{1+\gamma_1(\theta)b_1} > a_0$.

Under these assumptions, the actual price level will be given by

$$\begin{aligned} p_t &= p(m_t, w_t, p_{t-1}, \theta) \\ &= \min[\bar{p}_t, \max(\hat{p}_t, p_{t-1})] \end{aligned}$$

i.e.

$$p_t = \begin{cases} \hat{p}_t & \text{if } p_{t-1} \leq \hat{p}_t \\ p_{t-1} & \text{if } p_{t-1} \in [\hat{p}_t, \bar{p}_t] \\ \bar{p}_t & \text{if } p_{t-1} > \bar{p}_t \end{cases}$$

Increases in demand or costs will leave prices unchanged as long as mark–ups remain positive; beyond that prices will rise following marginal costs. Decreases in demand or costs will leave prices unchanged as long as mark–ups don't exceed the monopoly level; beyond that prices will fall following marginal revenues.

In the labor market, each period the union sets a nominal wage rate w_t at which n^* units of labor are supplied by households. Labor demand is derived (through the inverse production function) as the amount needed to produce the output demanded (for given m_t) at the price level set by firms. Since the price rule already takes into account n^* as an upper bound to labor input, employment is determined by labor demand as

$$n_t = \frac{b_0 + b_1(m_t - p_t) - \theta_0}{\theta_1}.$$

When $n_t < n^*$, we call the unemployment "classical" if the marginal productivity of labor is equal to the real wage (prices equal marginal costs), "Keynesian" if the marginal productivity is higher than the real wage (prices higher than marginal costs). For our game we assume $n_0 < n^*$ and $\theta \in \{C, K\}$, where C is such that $y_0 = C_0 + C_1 n_0$ and $w_0 - p_0 = MP(n_0, C)$ (classical unemployment), while K is such that $y_0 = K_0 + K_1 n_0$ and $w_0 - p_0 = MP(n^*, K) < MP(n_0, K)$ (Keynesian unemployment).

Assuming for simplicity $b_1 = 1/\gamma_1(K)$, players' actions are defined as:

$$\begin{aligned} m^L &= m_0, & m^H &= m_0 + (y_K^* - y_0)/b_1 \\ w^L &= w_0, & w^H &= w_0 + (y_K^* - y_0)/b_1 \\ p^C &= p(m, w, p_0, C), & p^K &= p(m, w, p_0, K). \end{aligned}$$

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5

LEXICOGRAPHIC RATIONALITY ORDERINGS AND ITERATIVE WEAK DOMINANCE

Pietro VERONESI

Abstract

We apply the concept of *Rationality Orderings*, introduced by Battigalli (1991), to a game–theoretic setting where players' rationality is defined according to the axiomatic system put forward by Blume *et al.* (1991a), i.e. each player is endowed with a vector of probability measures on her opponents' strategy space and act to lexicographically maximize her expected utility. The theory of rationality orderings entails that players' strategy sets are partitioned into cells indexed by "degree of rationality." It is observed that when players' utility functions are common knowledge, then players' maximally rational strategies are those which survive the iterated deletion of weakly dominated strategies. Under the weaker assumption that only players' preference relations on a given set of outcomes are common knowledge, we show for a restricted class of games that players' maximally rational strategies coincide with the strategies that survive the iterated deletion of strategies which are dominated by *pure* strategies.

Introduction

The iterated deletion of weakly dominated strategies is a procedure that has attracted some attention in the game–theoretic literature. One of the main reasons of interest in this procedure is that it seems to be the natural consequence of the basic principle that a rational player should never choose a weakly dominated strategy, once it is also assumed that players' rationality is common knowledge. On the other hand, it is also well known that the customary definition of a rational player as an expected-utility maximizer does not entail such a basic principle and therefore that the assumption of common knowledge of rationality does not imply the iterative weak dominance procedure (see